

Assignment-4

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Abstract—This assignment finds the application of affine transformation.

Download all python codes from

<https://github.com/satyam463/Matrix-Theory-Assignment4/blob/master/Assignment4.py>

1 PROBLEM STATEMENT

What does the equation $\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$ becomes when the axes are turned through 45° .

2 SOLUTION

$$\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.1)$$

$$\mathbf{V} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \Rightarrow |\mathbf{V}| = 0, \mathbf{u} = \begin{pmatrix} -2\sqrt{2}a \\ -2\sqrt{2}a \end{pmatrix} \quad (2.0.2)$$

The characteristics equation of \mathbf{V}

$$|\lambda \mathbf{I} - \mathbf{V}| = \begin{vmatrix} \lambda - 1 & 1 \\ 1 & \lambda - 1 \end{vmatrix} = 0 \quad (2.0.3)$$

$$\Rightarrow \lambda^2 - 2\lambda = 0 \quad (2.0.4)$$

The eigen values are

$$\lambda_1 = 0, \lambda_2 = 2 \quad (2.0.5)$$

(2.0.1) is the equation of parabola as $\lambda_1 = 0$ and $|\mathbf{V}| = 0$. The eigen vector \mathbf{p} is defined as

$$\mathbf{V}\mathbf{p} = \lambda\mathbf{p} \quad (2.0.6)$$

$$\Rightarrow (\lambda \mathbf{I} - \mathbf{V})\mathbf{p} = 0 \quad (2.0.7)$$

for $\lambda_1 = 0$

$$(\lambda_1 \mathbf{I} - \mathbf{V}) = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \quad (2.0.8)$$

$$\mathbf{p}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.9)$$

such that $\|\mathbf{p}_1\| = 1$ similarly the eigen vector for $\lambda_2 = 2$ can be find

$$\mathbf{p}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{P} = (\mathbf{p}_1 \ \mathbf{p}_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad (2.0.11)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \quad (2.0.12)$$

The parabola parameters are given by

$$f = \frac{|\eta|}{|\lambda_2|}; \eta = 2\mathbf{p}_1^T \mathbf{u} \quad (2.0.13)$$

$$f = \frac{8a}{2} = 4a \quad (2.0.14)$$

$$\begin{pmatrix} 0 & -6\sqrt{2}a \\ 0 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -4a \\ -2\sqrt{2}a \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ where } \mathbf{c} \text{ is vertex} \quad (2.0.16)$$

The axes are turned around 45° then

$$\mathbf{P} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \mathbf{x} \quad (2.0.17)$$

$$\mathbf{P} = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \mathbf{x} \quad (2.0.18)$$

$$\mathbf{P} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} \quad (2.0.19)$$

when \mathbf{P} passes through the equation (2.0.1) we get

$$\mathbf{x}^T \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.20)$$

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} - 4a \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.21)$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \Rightarrow |\mathbf{V}| = 0 \quad (2.0.22)$$

Therefore it is parabola..

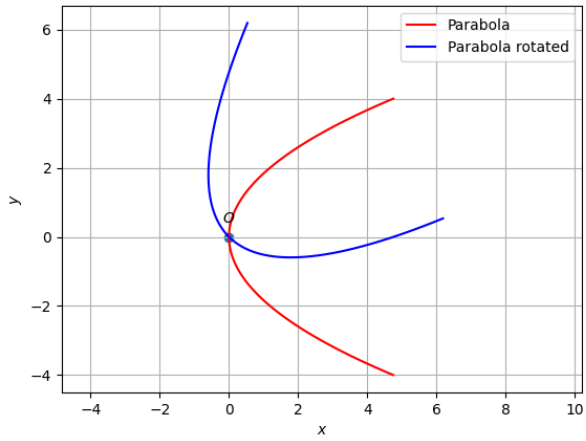


Fig. 0: parabola with $a = 1$