Assignment-4

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***Abstract*—This assignment finds the application of affine transformation.**

Download all python codes from

https://github.com/satyam463/matrix−theory−Assignment3/blob/master/assignment3.py

1 Problem Statement

What does the equation $\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$ becomes when the axes are turned through 45°.

2 solution

$$\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 1 & 1 \end{pmatrix}, f = 0 \tag{2.0.2}$$

The axes are turned around 45°then

$$\mathbf{P} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \mathbf{x} \tag{2.0.3}$$

$$\mathbf{P} = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \mathbf{x} \tag{2.0.4}$$

$$\mathbf{P} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} \tag{2.0.5}$$

when $\mathbf{P}$ passes through the equation (2.0.1) we get

$$\mathbf{x}^T \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.6}$$

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} - 4a \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.7}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \implies |\mathbf{V}| = 0 \tag{2.0.8}$$

Therefore it is parabola with vertex at (0,0).

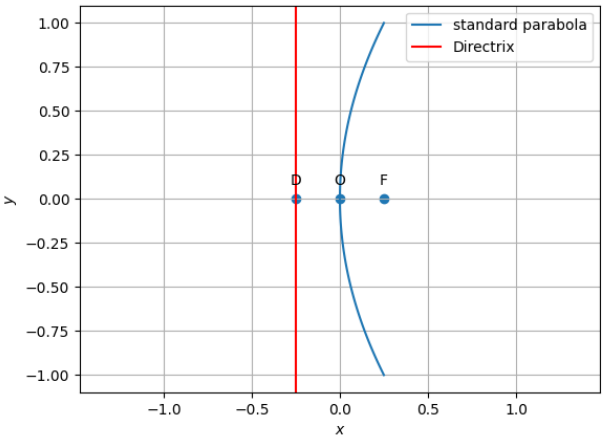

Fig. 0: standard parabola$y^2 = 4ax$