

Assignment-4

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Abstract—This assignment finds the application of affine transformation.

Download all python codes from

<https://github.com/satyam463/Matrix-Theory-Assignment4/blob/master/Assignment4.py>

1 PROBLEM STATEMENT

What does the equation $\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0$ becomes when the axes are turned through 45° .

2 SOLUTION

$$\mathbf{x}^T \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.1)$$

$$\mathbf{V} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 1 & 1 \end{pmatrix}, f = 0 \quad (2.0.2)$$

The axes are turned around 45° then

$$\mathbf{P} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \mathbf{x} \quad (2.0.3)$$

$$\mathbf{P} = \begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \mathbf{x} \quad (2.0.4)$$

$$\mathbf{P} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} \quad (2.0.5)$$

when \mathbf{P} passes through the equation (2.0.1) we get

$$\mathbf{x}^T \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} - 4\sqrt{2}a \begin{pmatrix} 1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.6)$$

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} - 4a \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.7)$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \Rightarrow |\mathbf{V}| = 0 \quad (2.0.8)$$

Therefore it is parabola with vertex at (0,0).

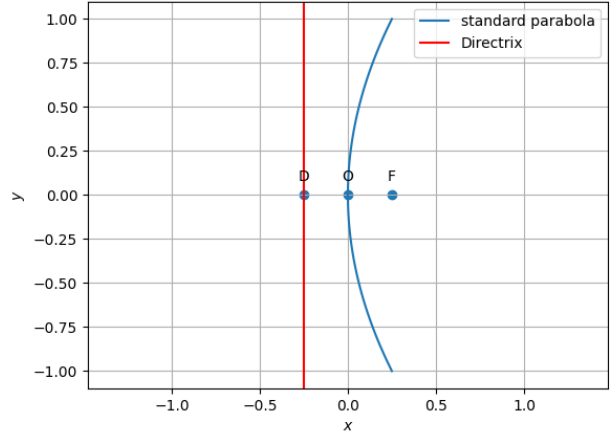


Fig. 0: standard parabola $y^2 = 4ax$