Assignment-4

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EE20MTECH14015

***Abstract*—This assignment finds the application of affine transformation.**

Download all python codes from

https://github.com/satyam463/Matrix−Theory−Assignment4/blob/master/Assignment4.py

1 Problem Statement

What does the equation $\mathbf{x}^T\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}\mathbf{x} - 4\sqrt{2}a\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 0$ becomes when the axes are turned through 45°.

2 solution

$$\mathbf{x}^T\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}\mathbf{x} - 4\sqrt{2}a\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 0 \tag{2.0.1}$$

$$\mathbf{V} = \begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix} \implies \left|\mathbf{V}\right| = 0, \mathbf{u} = \begin{pmatrix}-2\sqrt{2}a\\-2\sqrt{2}a\end{pmatrix} \tag{2.0.2}$$

The characteristics equation of $\mathbf{V}$

$$\left|\lambda\mathbf{I} - \mathbf{V}\right| = \begin{vmatrix}\lambda - 1 & 1\\1 & \lambda - 1\end{vmatrix} = 0 \tag{2.0.3}$$

$$\implies \lambda^2 - 2\lambda = 0 \tag{2.0.4}$$

The eigen values are

$$\lambda_1 = 0, \lambda_2 = 2 \tag{2.0.5}$$

(2.0.1) is the equation of parabola as$\lambda_1 = 0 and \left|\mathbf{V}\right| = 0$. The eigen vector $\mathbf{p}$ is defined as

$$\mathbf{V}\mathbf{p} = \lambda\mathbf{p} \tag{2.0.6}$$

$$\implies (\lambda\mathbf{I} - \mathbf{V})\mathbf{p} = 0 \tag{2.0.7}$$

for $\lambda_1 = 0$

$$(\lambda_1\mathbf{I} - \mathbf{V}) = \begin{pmatrix}-1 & 1\\1 & -1\end{pmatrix} \xleftrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix}-1 & 1\\0 & 0\end{pmatrix} \tag{2.0.8}$$

$$\mathbf{p_1} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \tag{2.0.9}$$

such that $\|\mathbf{p_1}\| = 1$ similarly the eigen vector for $\lambda_2 = 2$ can be find

$$\mathbf{p_2} = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix} \tag{2.0.10}$$

$$\mathbf{P} = \begin{pmatrix}\mathbf{p_1} & \mathbf{p_2}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix} \tag{2.0.11}$$

$$\mathbf{D} = \begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 2\end{pmatrix} \tag{2.0.12}$$

The parabola parameters are given by

$$f = \frac{|\eta|}{|\lambda_2|}; \eta = 2\mathbf{p_1}^T\mathbf{u} \tag{2.0.13}$$

$$f = \frac{8a}{2} = 4a \tag{2.0.14}$$

$$\begin{pmatrix}-6\sqrt{2}a & -6\sqrt{2}a\\0 & 0\\0 & 2\end{pmatrix}\mathbf{c} = \begin{pmatrix}-4a\\-2\sqrt{2}a\\-2\sqrt{2}a\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{c} = \begin{pmatrix}\frac{4\sqrt{2}}{3}\\-\sqrt{2}a\end{pmatrix} where\mathbf{c}isvertex \tag{2.0.16}$$

The axes are turned around 45°then

$$\mathbf{P} = \begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}\mathbf{x} \tag{2.0.17}$$

$$\mathbf{P} = \begin{pmatrix}\cos 45° & -\sin 45°\\\sin 45° & \cos 45°\end{pmatrix}\mathbf{x} \tag{2.0.18}$$

$$\mathbf{P} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\mathbf{x} \tag{2.0.19}$$

when $\mathbf{P}$ passes through the equation (2.0.1) we get

$$\begin{aligned}\mathbf{x}^T\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\mathbf{x}\\ -4\sqrt{2}a\begin{pmatrix}1 & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\mathbf{x} = 0\end{aligned} \tag{2.0.20}$$

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x} - 4a \begin{pmatrix} 2 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.21}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \implies \left|\mathbf{V}\right| = 0 \tag{2.0.22}$$

Therefore it is parabola..

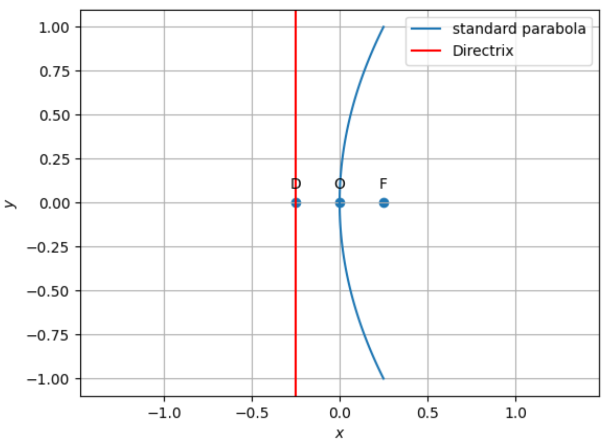

Fig. 0: standard parabola