1   Problem Statement

Find the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines

$$(1 - 7)\vec{x} = -5 \, and \, (31)\vec{x} = 0$$

2   Theory

consider the equation of the system of lines

$$x - 7y = -5$$
$$3x + y = 0$$

Now matrix equation is

$$AX = B \implies X = A^{-1}B$$

$$\begin{bmatrix} 1 & -7 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$$

$A \, must \, be \, nonsingular \implies \det A \neq 0 \, and \det A = 1 \times 1 - -3 \times 7 = 22$
to find solution (point of intersection)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -7 \\ 3 & 1 \end{bmatrix}^{-1} = \frac{1}{|22|} \begin{bmatrix} 1 & 7 \\ -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|22|} \begin{bmatrix} 1 & 7 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|22|} \begin{bmatrix} -5 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-5}{22} \\ \frac{15}{22} \end{bmatrix}$$

The above value of x and y is the point of intersection of lines
Now the equation of line parallel to y-axis through the point of intersection

$$x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{-5}{22} \\ \frac{15}{22} \end{bmatrix} = \frac{-5}{22}$$

the required equation of line parallel to y - axis ; $\vec{x} = \frac{-5}{22}$