1 Problem Statement

Find the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines

$$\begin{pmatrix}1 & -7\end{pmatrix}\mathbf{x} = -5 \tag{1}$$

$$\begin{pmatrix}3 & 1\end{pmatrix}\mathbf{x} = 0 \tag{2}$$

2 Theory

consider the equation of the system of lines

$$x - 7y = -5 \tag{3}$$

$$3x + y = 0 \tag{4}$$

consider the augmented matrix

$$\begin{pmatrix}1 & -7 & -5\\ 3 & 1 & 0\end{pmatrix} \tag{5}$$

By applying row reduction reduction technique

$$\begin{pmatrix}4 & -7 & -5\\ 3 & 1 & 0\end{pmatrix} \xleftrightarrow[R_2 \to R_2/22]{R_2 \to R_2 - 3R_1} \begin{pmatrix}1 & -7 & -5\\ 0 & 1 & \frac{15}{22}\end{pmatrix} \xrightarrow{R_1 \to R_1 + 7R_2} \begin{pmatrix}1 & 0 & \frac{-5}{22}\\ 0 & 1 & \frac{15}{22}\end{pmatrix} \tag{6}$$

The value of $\mathbf{A}$ is the point of intersection.

$$\mathbf{A} = \begin{pmatrix}\frac{-5}{22}\\ \frac{15}{22}\end{pmatrix}; \tag{7}$$

Now the equation of line parallel to y-axis through the point of intersection

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \tag{8}$$

where$\mathbf{n}$ is the vector normal to the Y - axis and $\mathbf{A}$ is the point of intersection.

$$\mathbf{n}^T\mathbf{x} = \mathbf{n}^T\mathbf{A}; where \mathbf{n}^T = \begin{pmatrix}1 & 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}\frac{-5}{22}\\ \frac{15}{22}\end{pmatrix} \tag{9}$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = -\frac{5}{22} \tag{10}$$

$$\mathbf{x} = \begin{pmatrix}\frac{-5}{22}\\ 0\end{pmatrix} \tag{11}$$

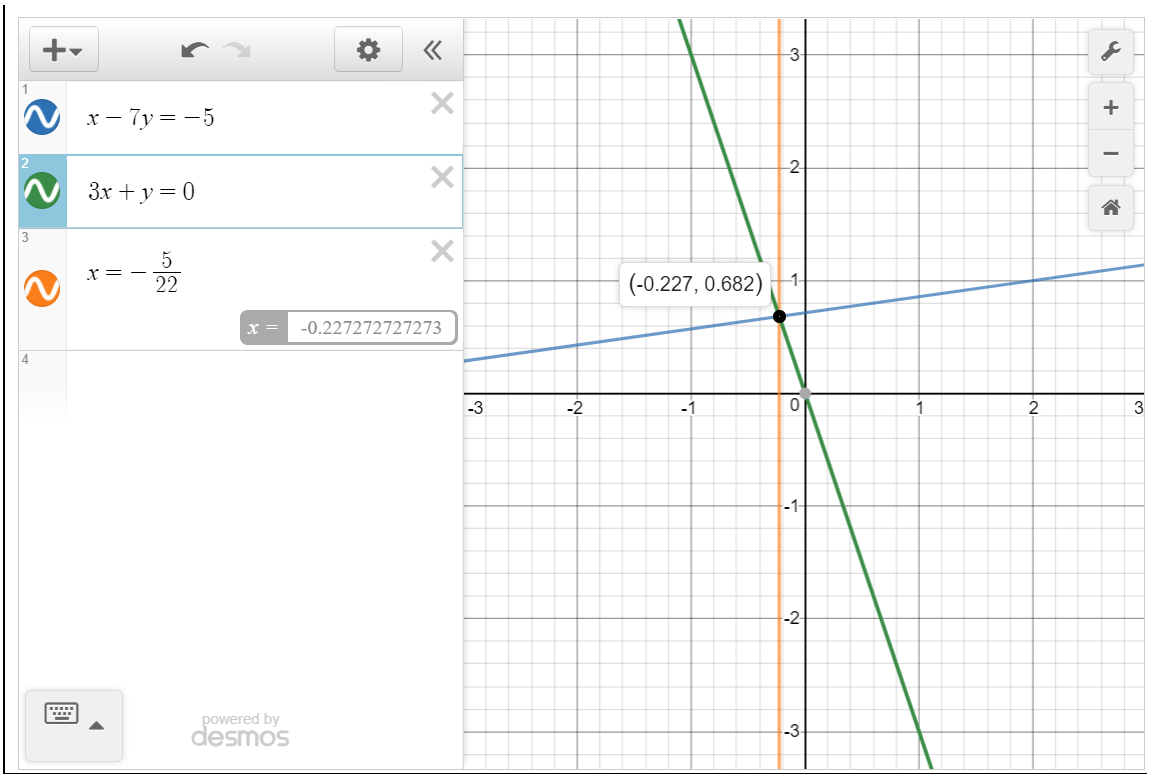

Figure 1: graphical representation of systems of lines
Shown in figure 1 is the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines