1 Problem Statement

Find the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines

$$\begin{pmatrix} 1 & -7 \end{pmatrix} \mathbf{x} = -5 \tag{1}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

2 Theory

consider the equation of the system of lines

$$x - 7y = -5 \tag{3}$$

$$3x + y = 0 (4)$$

consider the augmented matrix

$$\begin{pmatrix} 1 & -7 & -5 \\ 3 & 1 & 0 \end{pmatrix} \tag{5}$$

By applying row reduction reduction technique

$$\begin{pmatrix} 4 & -7 & -5 \\ 3 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{pmatrix} 1 & -7 & -5 \\ 0 & 1 & \frac{15}{22} \end{pmatrix} \xrightarrow{R_1 \to R_1 + 7R_2} \begin{pmatrix} 1 & 0 & \frac{-5}{22} \\ 0 & 1 & \frac{15}{22} \end{pmatrix}$$
(6)

The value of A is the point of intersection.

$$\mathbf{A} = \begin{pmatrix} \frac{-5}{22} \\ \frac{15}{22} \end{pmatrix}; \tag{7}$$

Now the equation of line parallel to y-axis through the point of intersection

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \tag{8}$$

where \mathbf{n} is the vector normal to the Y - axis and \mathbf{A} is the point of intersection.

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A}; where \mathbf{n}^T = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-5}{22} \\ \frac{15}{22} \end{pmatrix} \tag{9}$$

$$\mathbf{x} = \begin{pmatrix} \frac{-5}{22} \\ 0 \end{pmatrix} \tag{10}$$

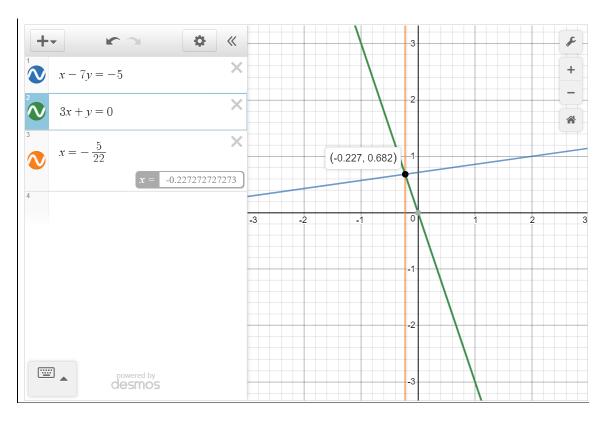


Figure 1: graphical representation of systems of lines Shown in figure 1 is the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines