1 Problem Statement

Find the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines

$$\begin{pmatrix}1 & -7\end{pmatrix}\mathbf{x} = \mathbf{-5}\,\textbf{and}\,\begin{pmatrix}3 & 1\end{pmatrix}\mathbf{x} = \mathbf{0} \tag{1}$$

2 Theory

consider the equation of the system of lines

$$x - 7y = -5 \tag{2}$$
$$3x + y = 0 \tag{3}$$

consider the augmented matrix

$$\begin{pmatrix}1 & -7 & -5\\ 3 & 1 & 0\end{pmatrix} \tag{4}$$

By applying row reduction reduction technique

$$\begin{pmatrix}4 & -7 & -5\\ 3 & 1 & 0\end{pmatrix} \xleftrightarrow[R_2 \to R_2/22]{R_2 \to R_2 - 3R_1} \begin{pmatrix}1 & -7 & -5\\ 0 & 1 & \frac{15}{22}\end{pmatrix} \xrightarrow{R_1 \to R_1 + 7R_2} \begin{pmatrix}1 & 0 & \frac{-5}{22}\\ 0 & 1 & \frac{15}{22}\end{pmatrix} \tag{5}$$

$$\textbf{The value of}\mathbf{A} = \begin{pmatrix}\frac{-5}{22}\\ \frac{15}{22}\end{pmatrix}; \textbf{A is the point of intersection}. \tag{6}$$

Now the equation of line parallel to y-axis through the point of intersection

$$\mathbf{n^T}(\mathbf{x} - \mathbf{A}) = \mathbf{0} \tag{7}$$

where $\vec{n}$ *is the vector normal to the* $Y - axis$ *and* A *is the point of intersection.*

$$\boldsymbol{n^T x} = \boldsymbol{n^T A}; \textit{where } \boldsymbol{n^T} = \begin{pmatrix}1 & 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\end{pmatrix}\boldsymbol{x} = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}\frac{-5}{22}\\ \frac{15}{22}\end{pmatrix} \tag{8}$$
$$\boldsymbol{x} = \begin{pmatrix}\frac{-5}{22}\end{pmatrix} \tag{9}$$

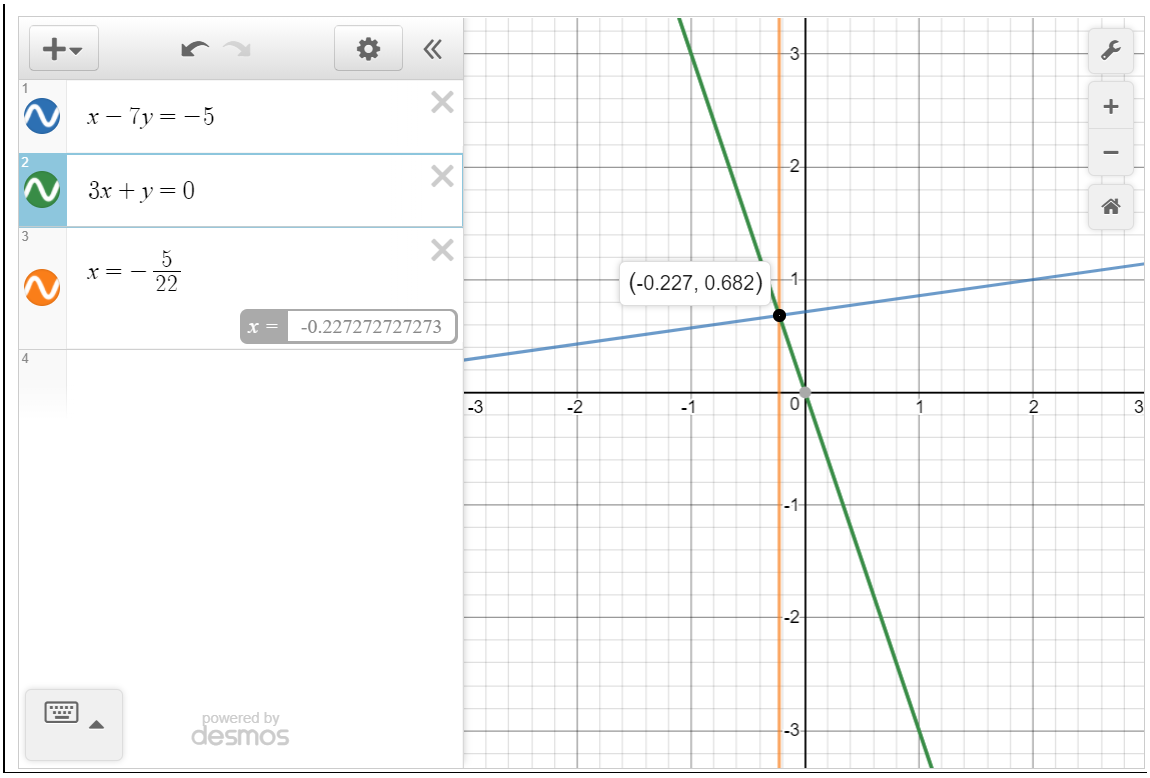

Figure 1: graphical representation of systems of lines