1 Problem Statement

Find the equation of the line parallel to the Y-axis drawn through the point of intersection of the lines

$$\begin{pmatrix} 1 & -7 \end{pmatrix} \mathbf{x} = -\mathbf{5} \, \text{and} \, \begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{1}$$

2 Theory

consider the equation of the system of lines

$$x - 7y = -5 \tag{2}$$
$$3x + y = 0 \tag{3}$$

consider the augmented matrix

$$\begin{pmatrix} 1 & -7 & -5 \\ 3 & 1 & 0 \end{pmatrix} \tag{4}$$

By applying row reduction reduction technique

$$\begin{pmatrix} 4 & -7 & -5 \\ 3 & 1 & 0 \end{pmatrix} \xleftarrow[R_2 \to R_2/22]{R_2 \to R_2 - 3R_1} \begin{pmatrix} 1 & -7 & -5 \\ 0 & 1 & \frac{15}{22} \end{pmatrix} \xrightarrow{R_1 \to R_1 + 7R_2} \begin{pmatrix} 1 & 0 & \frac{-5}{22} \\ 0 & 1 & \frac{15}{22} \end{pmatrix} \tag{5}$$

$$\textbf{The value of} \, \tilde{\mathbf{A}} = \begin{pmatrix} \frac{-5}{22} \\ \frac{15}{22} \end{pmatrix} ; \tilde{\mathbf{A}} \, \textbf{is the point of intersection.} \tag{6}$$

Now the equation of line parallel to y-axis through the point of intersection

$$\tilde{\mathbf{n}}^\mathbf{T} \tilde{\mathbf{x}} - \tilde{\mathbf{A}} = \mathbf{0} \tag{7}$$

$where \vec{n} is the vector normal to the Y-axis and \vec{A} is the point of intersection.$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \vec{x} = \begin{pmatrix} \frac{-5}{22} \\ \frac{15}{22} \end{pmatrix} \tag{8}$$

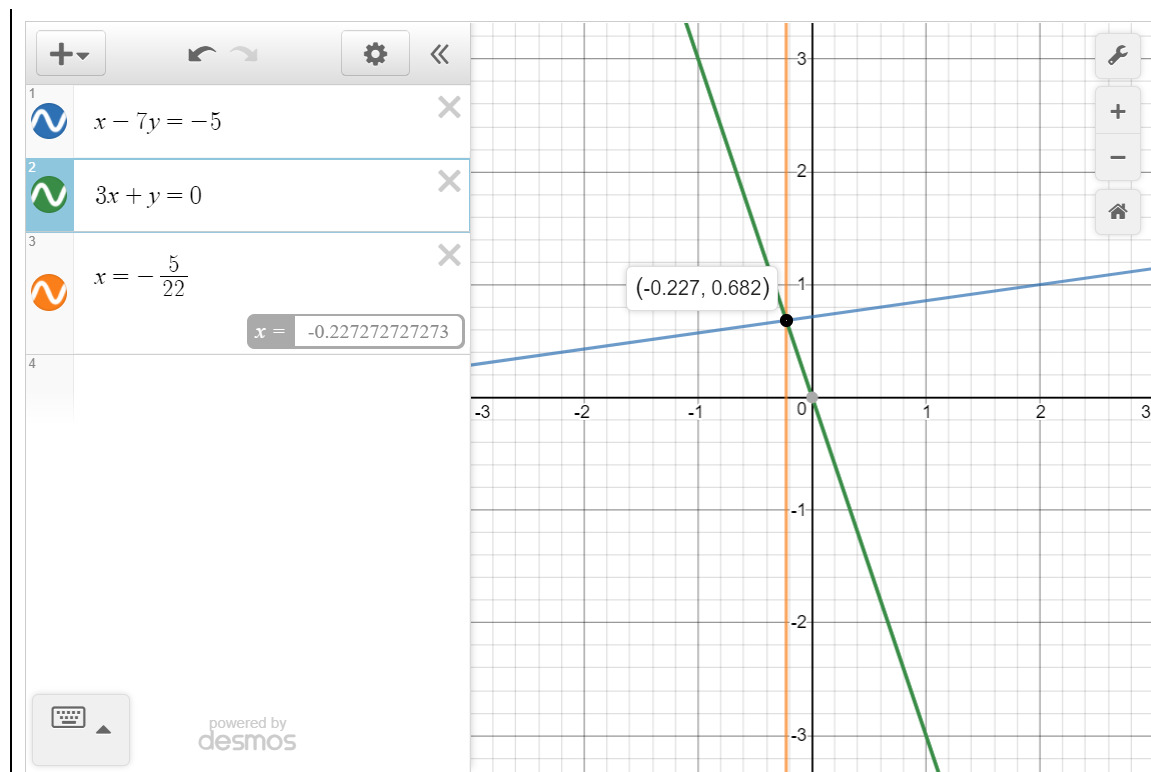$$\vec{x} = \begin{pmatrix} \frac{-5}{22} \end{pmatrix} \tag{9}$$

Figure 1: graphical representation of systems of lines