Assignment-2

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EE20MTECH14015

***Abstract*—This assignment finds the equation of the two straight lines from second degree equation.**

Download all python codes from

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https://github.com/satyam463/matrix−theory/blob/
    master/assignment2.py
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1 Problem Statement

Prove that the following equations represents two straight lines also find their point of intersection and angle between them.

$$y^2 + xy - 2x^2 - 5x - y - 2 = 0 \tag{1.0.1}$$

2 Solution

The general equation of second degree equation is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

and can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix}; \mathbf{u} = \begin{pmatrix} d & e \end{pmatrix} \tag{2.0.3}$$

represents a pair of straight lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u}^T \\ \mathbf{u} & f \end{vmatrix} = 0 \tag{2.0.4}$$

On comparing equation(1.0.1) and equation(2.0.1) we get

$$a = -2, b = \frac{1}{2}, c = 1, d = \frac{-5}{2}, e = \frac{-1}{2}, f = -2 \tag{2.0.5}$$

substituting above value in equation(2.0.4) we get

$$\begin{vmatrix} -2 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 1 & \frac{-1}{2} \\ \frac{-5}{2} & \frac{-1}{2} & -2 \end{vmatrix} \xleftrightarrow[R_1 \to R_1 + R_3]{R_1 \to R_1 - R_2} \begin{vmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 1 & \frac{-1}{2} \\ \frac{-5}{2} & \frac{-1}{2} & -2 \end{vmatrix} = 0 \tag{2.0.6}$$

Hence this represents pair of two straight lines. Now differentiating partially the Equation1.0.1 w.r.t y and x

$$\begin{pmatrix} 1 & 2 \end{pmatrix}\mathbf{x} = 1 \tag{2.0.7}$$

$$\begin{pmatrix} -4 & 1 \end{pmatrix}\mathbf{x} = 5 \tag{2.0.8}$$

consider augmented matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ -4 & 1 & 5 \end{pmatrix} \xleftrightarrow[R_2 \to R_2/9]{R_2 \to R_2 + 4R_1} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \xleftrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.0.9}$$

Therefore point of intersection is $\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Now Angle between two lines can be expressed in terms of normal vector

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ and } \mathbf{n_2} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \tag{2.0.10}$$

Angle between two lines θ can be given by

$$\cos\theta = \frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\| \, \|\mathbf{n_2}\|} = \frac{\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} -4 \\ 1 \end{pmatrix}}{\sqrt{(2)^2 + 1} \times \sqrt{(-4)^2 + 1}} = \frac{-2}{\sqrt{85}} \tag{2.0.11}$$

$$\theta = \cos^{-1}(\frac{-2}{\sqrt{85}}) \tag{2.0.12}$$

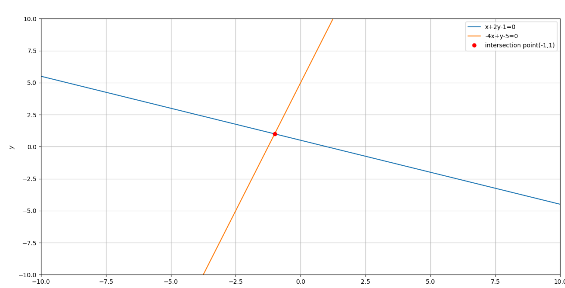

Fig. 0: plot showing intersection of lines