Assignment-3

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***Abstract*—This assignment finds the equation of circle passing through different points.**

Download all python codes from

https://github.com/satyam463/matrix-theory-Assignment3/blob/master/assignment3.py

1 Problem Statement

A Circle has its centre on the line $x = 2y$ and passes through the points $\begin{pmatrix}-1\\2\end{pmatrix}, \begin{pmatrix}3\\-2\end{pmatrix}$ Find the coordinates of the centre and the equation of the circle.

2 solution

The equation of circle can be expressed as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

$\mathbf{c}$ is the centre and substituting the points in the equation of circle we get

$$2\begin{pmatrix}-1 & 2\end{pmatrix}\mathbf{c} - f = 5 \tag{2.0.2}$$

$$2\begin{pmatrix}3 & -2\end{pmatrix}\mathbf{c} - f = 13 \tag{2.0.3}$$

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{c} = 0 \tag{2.0.4}$$

can be expressed in matrix form

$$\begin{pmatrix}1 & -2 & 0\\6 & -4 & -1\\-2 & 4 & -1\end{pmatrix}\begin{pmatrix}\mathbf{c}\\f\end{pmatrix} = \begin{pmatrix}0\\13\\5\end{pmatrix} \tag{2.0.5}$$

Row reducing the augmented matrix

$$\begin{pmatrix}1 & -2 & 0 & 0\\6 & -4 & -1 & 13\\-2 & 4 & -1 & 5\end{pmatrix} \xleftrightarrow[R_3\leftarrow R_3+2R_1]{R_2\leftarrow R_2-6R_1} \begin{pmatrix}1 & -2 & 0 & 0\\0 & 8 & -1 & 13\\0 & 0 & -1 & 5\end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_1\leftarrow R_1+R_2]{R_2\leftarrow R_2/4} \begin{pmatrix}1 & 0 & \frac{-1}{4} & \frac{13}{4}\\0 & 2 & \frac{-1}{4} & \frac{13}{4}\\0 & 0 & -1 & 5\end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow[R_1\leftarrow R_1-R_3/4]{R_2\leftarrow R_2-R_3/4} \begin{pmatrix}1 & 0 & 0 & 2\\0 & 2 & 0 & 2\\0 & 0 & -1 & 5\end{pmatrix} \tag{2.0.8}$$

$$\mathbf{c} = \begin{pmatrix}2\\1\end{pmatrix} \tag{2.0.9}$$

$$f = -5 \tag{2.0.10}$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{10} \tag{2.0.11}$$

The required equation of circle is

$$\mathbf{x}^T\mathbf{x} - 2\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} + 5 = 0 \tag{2.0.12}$$

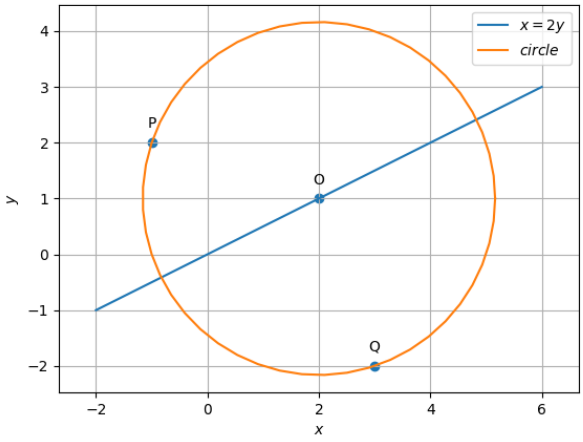

Fig. 0: Circle passing through point P and Q also centre lie on the line x=2y