

Assignment-2

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Abstract—This assignment finds the equation of the two straight lines from second degree equation.

Download all python codes from

<https://github.com/satyam463/matrix-theory/blob/master/assignment2.py>

The slopes of the lines are given by the roots of the polynomial

$$cm^2 + 2bm + a = 0 \quad (2.0.8)$$

$$m_1, m_2 = \frac{-\frac{1}{2} \pm \sqrt{\frac{9}{4}}}{1} \quad (2.0.9)$$

$$m_1 = 1, m_2 = -2 \quad (2.0.10)$$

$$\Rightarrow \mathbf{n}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \text{ and } \mathbf{n}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (2.0.11)$$

1 PROBLEM STATEMENT

Prove that the following equations represents two straight lines also find their point of intersection and angle between them.

$$y^2 + xy - 2x^2 - 5x - y - 2 = 0 \quad (1.0.1)$$

2 SOLUTION

$$\mathbf{V} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = \begin{pmatrix} -2 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} \\ -\frac{1}{2} \end{pmatrix} \quad (2.0.2)$$

$$f = -2 \quad (2.0.3)$$

$$\begin{vmatrix} -2 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{1}{2} & -2 \end{vmatrix} \xrightarrow[R_1 \rightarrow R_1 + R_3]{R_1 \rightarrow R_1 - R_2} \begin{vmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{1}{2} & -2 \end{vmatrix} = 0 \quad (2.0.4)$$

Hence it represents the pair of straight lines. Now two intersecting lines are obtained when

$$|V| < 0 \Rightarrow \begin{vmatrix} -2 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{vmatrix} = \frac{-9}{4} < 0 \quad (2.0.5)$$

Let the pair of straight of lines be given by

$$\mathbf{n}_1^T \mathbf{x} = c_1 \quad (2.0.6)$$

$$\mathbf{n}_2^T \mathbf{x} = c_2 \quad (2.0.7)$$

$$(\mathbf{n}_1^T \mathbf{x} - c_1)(\mathbf{n}_2^T \mathbf{x} - c_2) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f \quad (2.0.12)$$

$$c_2 \mathbf{n}_1 + c_1 \mathbf{n}_2 = -2\mathbf{u} \quad (2.0.13)$$

$$c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = -2 \begin{pmatrix} -\frac{5}{2} \\ -\frac{1}{2} \end{pmatrix} \quad (2.0.14)$$

$$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad (2.0.15)$$

Using row reduction we get

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 5 \end{pmatrix} \quad (2.0.16)$$

$$\xrightarrow[R_2 \leftarrow R_2 - 2R_1]{R_2 \leftarrow R_2 / -3} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \quad (2.0.17)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \quad (2.0.18)$$

$$C = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (2.0.19)$$

The convolution of the normal vectors, should satisfy the below condition

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix} * \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ 2b \\ c \end{pmatrix} \quad (2.0.20)$$

The LHS part of equation(2.0.20) can be rewritten using toeplitz matrix as

$$\begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ 2b \\ c \end{pmatrix} \quad (2.0.21)$$

Therefore the equation of lines is given by

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 2 \quad (2.0.22)$$

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = -1 \quad (2.0.23)$$

consider the augmented matrix

$$\begin{pmatrix} -1 & 1 & 2 \\ 2 & 1 & -1 \end{pmatrix} \quad (2.0.24)$$

$$\begin{array}{l} \xleftarrow{R_1 \leftarrow -R_1} \\ \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \end{array} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.25)$$

$$\begin{array}{l} \xleftarrow{R_1 \leftarrow R_1/3} \\ \xrightarrow{R_1 \leftarrow R_1 + R_2} \end{array} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.26)$$

Therefore point of intersection is $\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

Angle between two lines θ can be given by

$$\cos \theta = \frac{\mathbf{n}_1^T \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (2.0.27)$$

$$\cos \theta = \frac{\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}}{\sqrt{(1)^2 + 1} \times \sqrt{(2)^2 + 1}} \quad (2.0.28)$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{10}}\right) \Rightarrow \theta = \tan^{-1}3 \quad (2.0.29)$$

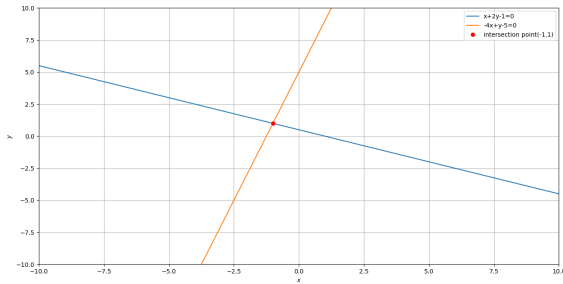


Fig. 0: plot showing intersection of lines