Basic Electrical and Electronics Engineering

Third Edition

(MU 2014)

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Ravish R Singh
Shree L R Tiwari College of Engineering
Thane

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Dedicated
to
My son, *Aman*
and
daughter, *Aditri*

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Contents

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Preface

Basic Electrical and Electronics Engineering is a core subject studied by all engineering students in the first year of an engineering course. The syllabus for this course is designed to convey a fundamental understanding of the subject and this book is meant to be read as a first text for the same.

AIM

Often, first-year students of engineering face many difficulties while trying to grasp the nuances of this subject through various reference books. Hence, there has always been the need for a book that could cover all the topics of the basic electrical and electronics engineering course in a single volume. This book is written exactly in-sync with the syllabus, common to all engineering branches.

As a rule, numerical problems appear in university examinations on this subject, and the weightage given to problems is more than 40–50%. Keeping up with this trend, this book presents a judicious mix of theory and a plethora of solved and unsolved problems. It is designed to offer an unparalleled learning experience for every reader.

SALIENT FEATURES

- Complete syllabus coverage (FEC 105)
- Self-contained, i.e. explains all the basics comprehensively using several 750 illustrations
- Questions designed on commonly asked examination questions
- Contains a wealth of solved examples (370), demonstrating theory in practice.
- 355 exercise questions provided for practice
- Includes 105 objective-type questions and list of useful formulae for quick recap
- Latest solved question papers available

CHAPTER ORGANISATION

This book is divided into **seven** chapters.

Chapters 1 and 2 comprehensively cover **dc circuits** and basic laws like Ohm's law and Kirchhoff's laws. **Chapter 3** deals with **ac fundamentals** with detailed coverage on root mean square and phasor representations of alternating quantities.

Chapter 4 deals with analysis of **single-phase ac circuits**. Topics like behaviour of *R, L, C,* series *R-L* circuits, series *R-C* circuits, series *R-L-C* circuits, series and parallel ac circuits and resonance are discussed in this chapter. **Chapter 5** discuses **three-phase ac circuits** with lucid coverage on three-phase circuits, star or wye connection, delta or mesh connection and their relation. It also talks about measurement of three-phase power in detail.

Chapter 6 talks about **single-phase transformers** covering their construction, principle of working, the emf equation, losses, ideal and practical transformers, and open-circuit and short-circuit tests.

Chapter 7 introduces semiconductor devices and rectifiers. Centre-tapped and bridge configuration, half-wave and full-wave rectifiers, and CE, CB, CC transistors are discussed in this chapter.

Written in an easy-to-read and student-friendly language, each topic has been thoroughly covered from the examination point of view. A list of **useful formulae** given at the end of important topics, comes as a handy tool for a fast recap. This is followed by **exercises** with **answers** to all problems, **review questions** and **objective-type questions** in each chapter. These are meant to help students in a quick revision of the important topics and enhance their problem-solving skills.

ACKNOWLEDGEMENTS

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Any suggestions for improving the book will be gratefully acknowledged.

RAVISH R SINGH

Publisher's Note

Constructive suggestions and criticism always go a long way in enhancing and endeavour. We request all readers to email us their valuable comments/views/feedback for the betterment of the book at **info.india@mheducation.com**, mentioning the title and author name in the subject line.

Also, please feel free to report any piracy of the book spotted by you.

Roadmap to the Syllabus

Basic Electrical and Electronics Engineering (FEC 105)
(As per the latest syllabus of Mumbai University)

Unit 1: DC Circuits

Kirchhoff's laws, ideal and practical voltage and current source, mesh and nodal analysis (supernode and super mesh excluded), source transformation, star-delta transformation, superposition theorem, Thevenin's theorem, Norton's theorem, maximum power transfer theorem

GO TO

Chapter 1 – Basic Concepts
Chapter 2 – DC Circuits

Unit 2: AC Circuits

Generation of alternating voltage and currents, rms and average value, form factor, crest factor, ac through resistance, inductance and capacitance, *R-L, R-C* and *R-L-C* series and parallel circuits, phasor diagrams, power and power factor, series and parallel resonance, *Q*-factor and bandwidth

GO TO

Chapter 3 – AC Fundamentals
Chapter 4 – Single-Phase AC Circuits

Unit 3: Three-Phase Circuits

Three-phase voltage and current generation, star and delta connections (balanced load), relationship between phase and line currents and voltages, phasor diagram, basic principle of wattmeter, measurements of power by two-wattmeter method

GO TO

Chapter 5 – Three-Phase Circuits

Unit 4: Single-Phase Transformer

Construction, working principle, emf equation, ideal and practical transformer, transformer on no-load and on load, phasor diagrams, equivalent circuit, OC and SC test, efficiency

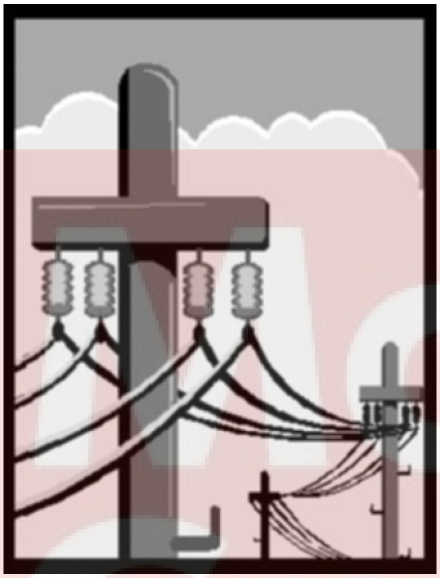
GO TO

Chapter 6 – Single-Phase Transformers

Unit 5: Electronics

Semiconductor diode, diode rectifier with *R* load, half-wave, full-wave, centre-tapped and bridge configurations, rms value and average value of output voltage, ripple factor, rectification efficiency, introduction to *C* and *L* filter (no derivation), CE, CB, CC transistor configuration, CE input-output characteristics

GO TO **Chapter 7 – Electronics**



Chapter 1

Basic Concepts

Chapter Outline

1.1 VOLTAGE

We know that like charges repel each other whereas unlike charges attract each other. To overcome this force of attraction or repulsion, a certain amount of work or energy is required. When the charges are moved, it is said that a potential difference exists and the work or energy per unit charge utilized in this process is known as voltage or potential difference.

$$V = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$$

1.2 CURRENT

There are free electrons available in all conductors. These free electrons move at random in all directions within the structure in the absence of external voltage. If voltage is applied across the conductor, all the free electrons move in one direction depending on the polarity of the applied voltage. This movement of electrons constitutes an electric current. The conventional direction of current flow is opposite to that of electrons.

Current is defined as the rate of flow of electrons in a conductor. It is measured by the number of electrons that flow in unit time.

$$I = \frac{\text{charge}}{\text{time}} = \frac{Q}{t}$$

1.3 SOURCES

A source is a basic network element which supplies energy to the networks. There are two classes of sources, namely,

(i) Independent source (ii) Dependent source

1.3.1 Independent Sources

Output characteristics of independent sources are not dependent on any network variable such as a current or voltage. Its characteristics, however, may be time varying. There are two types of independent sources:

(i) Independent voltage source (ii) Independent current source

Independent Voltage Source An independent voltage source is a two-terminal network element that establishes a specified voltage across its terminals. The value of this voltage at any instant is independent of the value or direction of the current that flows through it. The symbols for such voltage sources are shown in Fig. 1.1.

The terminal voltage may be a constant, or it may be some specified function of time.

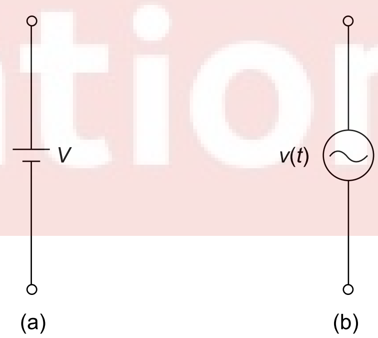


Fig. 1.1 *Independent voltage source*

Independent Current Source An independent current source is a two-terminal network element which produces a specified current. The value and direction of this current at any instant of time is independent of the value or direction of the voltage that appears across the terminals of the source. The symbols for such current sources are shown in Fig. 1.2.

The output current may be a constant or it may be a function of time.

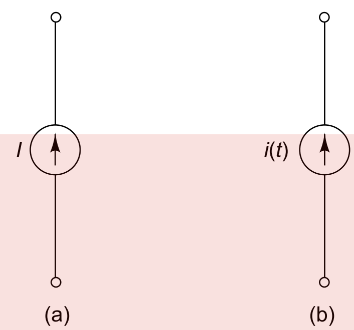


Fig. 1.2 *Independent current source*

1.3.2 Dependent Sources

If the voltage or current of a source depends in turn upon some other voltage or current, it is called as dependent or controlled source. The dependent sources are of four kinds depending on whether the control variable is a voltage or current and the source controlled is a voltage source or current source.

Voltage Controlled Voltage Source (VCVS) A voltage controlled voltage source is a four-terminal network component that establishes a voltage v_{cd} between two points c and d in the circuit that is proportional to a voltage v_{ab} between two points a and b.

The symbol for such a source is shown in Fig. 1.3.

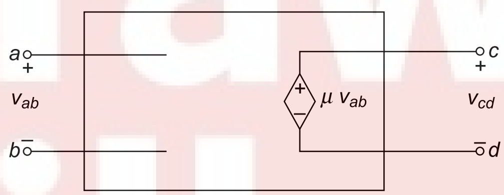


Fig. 1.3 *Voltage controlled voltage sources (VCVS)*

The (+) and (–) sign inside the diamond of the component symbol identify the component as a voltage source.

$$v_{cd} = \mu\, v_{ab}$$

The voltage v_{cd} depends upon the control voltage v_{ab} and the constant μ, a dimensionless constant called voltage gain.

Voltage Controlled Current Source (VCCS) A voltage controlled current source is a four-terminal network component that establishes a current i_{cd} in a branch of the circuit that is proportional to the voltage v_{ab} between two points a and b.

The symbol for such a source is shown in Fig. 1.4.

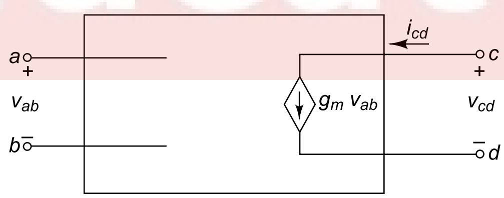


Fig. 1.4 *Voltage controlled current source (VCCS)*

The arrow inside the diamond of the component symbol identifies the component as a current source.

$$i_{cd} = g_m\, v_{ab}$$

The current i_{cd} depends upon the control voltage v_{ab} and the constant g_m, called the transconductance or mutual conductance. Constant g_m has dimension of ampere per volt or siemens (S).

Current Controlled Voltage Source (CCVS) A current controlled voltage source is a four terminal network component that establishes a voltage v_{cd} between two points c and d in the circuit that is proportional to current i_{ab} in some branch of the circuit.

The symbol for such a source is shown in Fig. 1.5.

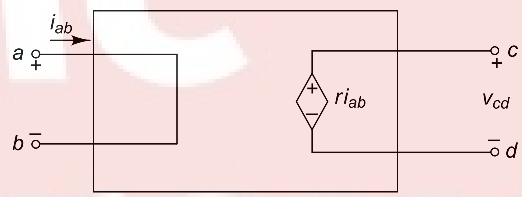


Fig. 1.5 *Current controlled voltage source (CCVS)*

$$v_{cd} = r\, i_{ab}$$

The voltage v_{cd} depends upon the control current i_{ab} and the constant r called the transresistance or mutual resistance. Constant r has dimension of volt per ampere or ohm (Ω).

Current Controlled Current Source (CCCS) A current controlled current source is a four-terminal network component that establishes a current i_{cd} in one branch of a circuit that is proportional to current i_{ab} in some branch of the network.

The symbol for such a source is shown in Fig. 1.6.

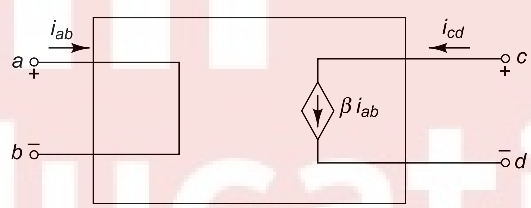


Fig. 1.6 *Current controlled current source (CCCS)*

$$i_{cd} = \beta\, i_{ab}$$

The current i_{cd} depends upon the control current i_{ab} and the dimensionless constant β, called the current gain.

1.4 OHM'S LAW

According to Ohm's law, the potential difference across any two points on a conductor is directly proportional to the current flowing through it, provided the physical conditions, viz., material length, cross-sectional area and temperature of the conductor remain constant.

$$V \propto I$$

$$V = R\,I$$

where R is the resistance between two points of the conductor.

Limitations

1. Ohm's law does not apply to nonmetallic conductors. For example, for silicon carbide, the relationship is given by $V = KI^m$ where K and m are constants and m is less than unity.
2. Ohm's law also does not apply to nonlinear devices such as zener diodes, voltage regulator tubes, etc.
3. Ohm's law is true for metal conductors at constant temperature. If the temperature changes, the law is not applicable.

1.5 RESISTANCE

Resistance is the property of a material due to which it opposes the flow of electric current through it.

Certain materials offer very little opposition to the flow of electric current and are called *conductors*, e.g. metals, acids and salt solutions. Certain materials offer very high resistance to the flow of electric current and are called *insulators*, e.g. mica, glass, rubber, Bakelite, etc.

The SI unit of resistance is ohm and is represented by the symbol Ω. A conductor is said to have a resistance of one ohm if a potential difference of one volt across its terminals causes a current of one ampere to flow through it.

The resistance of a conductor depends on the following factors:

(i) It is directly proportional to its length.
(ii) It is inversely proportional to the area of cross section of the conductor.
(iii) It depends on the nature of the material.
(iv) It also depends on the temperature of the conductor.

Hence,

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A}$$

where l is length of the conductor, A is the cross-sectional area and ρ is a constant known as the **specific resistance**, or **resistivity of the material**.

Specific Resistance The specific resistance, or the resistivity of a material, is the resistance offered by unit length of the material of unit cross-section. If the length is in metres and the area of cross-section in square metres, then the resistivity is expressed in ohm metres (Ω-m).

Table 1.1 shows the resistivities and temperature coefficients of various materials at 20 °C.

Table 1.1

Material	*Resistivity (Ω-m)*	*Temperature coefficient/ °C*
Silver	1.58×10^{-8}	0.0038
Copper	1.72×10^{-8}	0.0039
Gold	2.44×10^{-8}	0.0034
Aluminium	2.82×10^{-8}	0.0039
Calcium	3.36×10^{-8}	0.0041
Tungsten	5.60×10^{-8}	0.0045
Zinc	5.90×10^{-8}	0.0037
Nickel	6.99×10^{-8}	0.006
Iron	1.0×10^{-7}	0.005
Platinum	1.06×10^{-7}	0.00392
Tin	1.09×10^{-7}	0.0045
Lead	2.2×10^{-7}	0.0039
Manganin	4.82×10^{-7}	0.000002
Constantan	4.9×10^{-7}	0.000008
Mercury	9.8×10^{-7}	0.0009
Nichrome	1.10×10^{-6}	0.0004
Carbon	$5–8 \times 10^{-4}$	–0.0005
Germanium	4.6×10^{-1}	–0.048
Silicon	6.40×10^{2}	–0.075
Glass	$10^{10} – 10^{14}$	

Example 1

Calculate the resistance of a copper conductor having a length of 2 km and a cross-section of 22 mm². Assume the resistivity of copper is 1.72×10^{-8} Ω-m.

Solution

$$l = 2 \text{ km} = 2 \times 10^3 \text{ m}$$
$$A = 22 \text{ mm}^2 = 22 \times 10^{-6} \text{ m}^2$$
$$\rho = 1.72 \times 10^{-8}\ \Omega\text{-m}$$

Resistance of copper conductor

$$R = \rho \frac{l}{A}$$

$$= 1.72 \times 10^{-8} \times \frac{2 \times 10^3}{22 \times 10^{-6}}$$

$$= 1.56\ \Omega$$

Example 2

Calculate the resistance of a copper tube with the external diameter of 10 cm, internal diameter of 9 cm, length of 2 m and resistivity of copper as 1.72×10^{-8} Ω-m.

Solution

$$d_1 = 10 \text{ cm} = 0.1 \text{ m}$$

$$d_2 = 9 \text{ cm} = 0.09 \text{ m}$$

$$l = 2 \text{ m}$$

$$\rho = 1.72 \times 10^{-8}\ \Omega\text{-m}$$

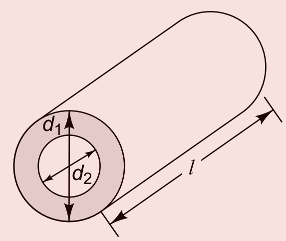


Fig. 1.7

Area of cross section of copper tube

$$A = \frac{\pi}{4} d_1^2 - \frac{\pi}{4} d_2^2$$

$$= \frac{\pi}{4} \times (0.1)^2 - \frac{\pi}{4} \times (0.09)^2$$

$$= 1.49 \times 10^{-3} \text{ m}^2$$

Resistance of copper tube

$$R = \rho \frac{l}{A}$$

$$= 1.72 \times 10^{-8} \times \frac{2}{1.49 \times 10^{-3}}$$

$$= 23.09\ \mu\Omega$$

Example 3

Calculate the resistance of 100 m length of a wire having a uniform cross-sectional area of 0.1 mm², if the wire is made of manganin having a resistivity of 50×10^{-8} Ω-m. If the wire is drawn out to three times its original length, by how many times will the resistance increase?

Solution

$$l_1 = 100 \text{ m}$$

$$A_1 = 0.1 \text{ mm}^2 = 0.1 \times 10^{-6} \text{ m}^2$$

$$\rho_1 = 50 \times 10^{-8}\ \Omega\text{-m}$$

Resistance of the wire

$$R_1 = \rho_1 \frac{l_1}{A_1}$$

$$= 50 \times 10^{-8} \times \frac{100}{0.1 \times 10^{-6}}$$

$$= 500\ \Omega$$

If the wire is drawn out to three times its original length, its volume remains constant.

Volume = $A_2 l_2 = A_1 l_1$

$$\frac{A_1}{A_2} = \frac{l_2}{l_1}$$

$$\rho_2 = \rho_1$$

$$l_2 = 3l_1$$

Now, $$R_2 = \rho_2 \frac{l_2}{l_1}$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{A_1}{A_2} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{l_2}{l_1} = 1 \cdot 3 \cdot 3 = 9$$

$$R_2 = 9R_1$$

Hence, resistance will increase by nine times its original value.

Example 4

A piece of silver wire has a resistance of 3 Ω. What will be the resistance of a manganin wire one-third the length and one-third the diameter, if the resistivity of manganin is 30 times that of silver?

Solution Let R_1, ρ_1, l_1, A_1 and R_2, ρ_2, l_2, A_2 be the resistance, resistivity, length and area of cross-section of silver and manganin wire respectively.

For silver wire, $R_1 = 3\ \Omega$

For manganin wire, $l_2 = \frac{1}{3} l_1,\ d_2 = \frac{1}{3} d_1,\ \rho_2 = 30\rho_1$

$$R_1 = \rho_1 \frac{l_1}{A_1}$$

$$R_2 = \rho_2 \frac{l_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{A_1}{A_2}$$

But $$A_1 = \frac{\pi}{4} d_1^2 \text{ and } A_2 = \frac{\pi}{4} d_2^2$$

where d_1 and d_2 are diameters of the silver and manganin wires respectively.

$$\frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \left(\frac{d_1}{d_2}\right)^2$$

$$= (30)\left(\frac{1}{3}\right)(3)^2$$

$$= 90$$

$$R_2 = 90R_1 = 90(3) = 270\ \Omega$$

Resistance of manganin wire = 270 Ω

Example 5

A 10 m long aluminium wire of 2 mm diameter is connected in parallel to a 6 m long copper wire. A total current of 2 A is passed through the combination and found that current through the aluminium wire is 1.25 A. Calculate the diameter of the copper wire. Specific resistance of copper is 1.6×10^{-6} Ω-cm and that of aluminium is 2.6×10^{-6} Ω-cm.

Solution For aluminium wire, $l_1 = 10$ m, $d_1 = 2$ mm $= 2 \times 10^{-3}$ m, $I_1 = 1.25$ A

$$\rho_1 = 2.6 \times 10^{-6}\ \Omega\text{-cm} = 2.6 \times 10^{-8}\ \Omega\text{-m}$$

For copper wire, $l_2 = 6$ m

$$\rho_2 = 1.6 \times 10^{-6}\ \Omega\text{-cm} = 1.6 \times 10^{-8}\ \Omega\text{-m}$$

$$I_1 = 2\text{ A}$$

An aluminium wire and copper wire are connected in parallel.

For aluminium wire

$$R_1 = \frac{V}{I_1} = \frac{V}{1.25}$$

$$A_1 = \frac{\pi}{4} d_1^2$$

For copper wire

$$I_2 = 2 - 1.25 = 0.75\text{ A}$$

$$R_2 = \frac{V}{I_2} = \frac{V}{0.75}$$

$$A_2 = \frac{\pi}{4} d_2^2$$

$$\frac{R_2}{R_1} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{A_1}{A_2} = \frac{\rho_2}{\rho_1} \cdot \frac{l_2}{l_1} \cdot \frac{d_1^2}{d_2^2}$$

$$\frac{1.25}{0.75} = \frac{1.6 \times 10^{-8}}{2.6 \times 10^{-8}} \cdot \frac{6}{10} \cdot \frac{(2 \times 10^{-3})^2}{d_2^2}$$

$$d_2 = 0.94\text{ mm}$$

Diameter of copper wire = 0.94 mm

1.6 SERIES CIRCUIT

Resistors R_1 and R_2 are said to be connected in series when the same current flows through each resistor.

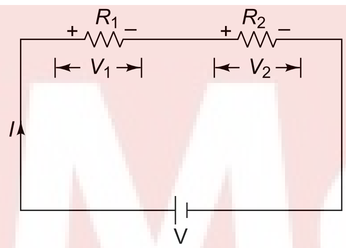


Fig. 1.8 *Series circuit*

Voltage across $R_1 = V_1 = R_1 I$

Voltage across $R_2 = V_2 = R_2 I$

The total voltage applied should be balanced by the sum of voltage drops around the circuit.

$$\begin{aligned} V &= V_1 + V_2 \\ &= R_1 I + R_2 I \\ &= (R_1 + R_2) I \\ &= R_T I \quad \text{where } R_T = R_1 + R_2 \end{aligned}$$

Hence, when a number of resistors are connected in series, the equivalent resistance is the sum of all the individual resistance.

Note

1. Same current flows through each resistor.
2. Voltage drops are additive.
3. Resistances are additive.
4. Power is additive.
5. The applied voltage equals the sum of different voltage drops.

Voltage Division in a Series Circuit

$$I = \frac{V}{R_1 + R_2}$$

Hence, voltage across $R_1 = V_1 = R_1 I$

$$= R_1 \frac{V}{R_1 + R_2} = \frac{R_1}{R_1 + R_2} V$$

Similarly, voltage across $R_2 = V_2 = R_2 I$

$$= R_2 \frac{V}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} V$$

1.7 PARALLEL CIRCUIT

Resistors R_1 and R_2 are said to be connected in parallel when the potential difference across each resistor is same.

$$I_1 = \frac{V}{R_1}$$

$$I_2 = \frac{V}{R_2}$$

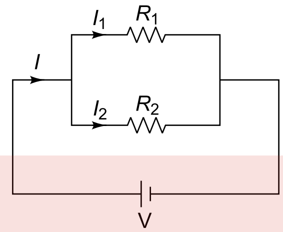


Fig. 1.9 *Parallel circuit*

Since, $$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$I = \frac{V}{R_T} \quad \text{where} \quad \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Hence, when a number of resistors are connected in parallel, the reciprocal of the total resistance is equal to the sum of reciprocals of individual resistances.

Note

1. Same voltage appears across all resistors.
2. Branch currents are additive.
3. Conductances are additive.
4. Power is additive.

Current Division in a Parallel Circuit

Case (i) When two resistances are connected in parallel,

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Also, $$V = R_T I = R_1 I_1 = R_2 I_2$$

Hence, current through $R_1 = I_1 = \dfrac{V}{R_1} = \dfrac{R_T I}{R_1} = \dfrac{R_2}{R_1 + R_2} I$

Similarly, current flowing through $R_2 = I_2 = \dfrac{V}{R_2} = \dfrac{R_T I}{R_2} = \dfrac{R_1}{R_1 + R_2} I$

Case (ii) When three resistances are connected in parallel,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{R_2 R_3 + R_3 R_1 + R_1 R_2}{R_1 R_2 R_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_3 R_1 + R_1 R_2}$$

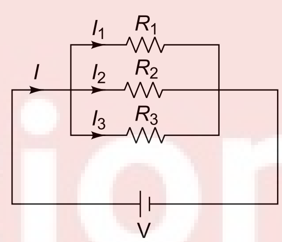


Fig. 1.10 *Parallel circuit*

Also, $$V = R_T I = R_1 I_1 = R_2 I_2 = R_3 I_3$$

Current through $R_1 = I_1 = \dfrac{V}{R_1} = \dfrac{R_T I}{R_1} = \dfrac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$

Current through $R_2 = I_2 = \dfrac{V}{R_2} = \dfrac{R_T I}{R_2} = \dfrac{R_3 R_1}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$

$$\text{Current through } R_3 = I_3 = \frac{V}{R_3} = \frac{R_T I}{R_3} = \frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_3 R_1} I$$

1.8 SHORT AND OPEN CIRCUITS

When two terminals of a circuit are connected by a wire, they are said to be short circuited. A short circuit has following features:

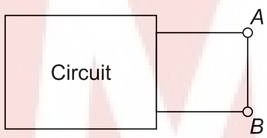


Fig. 1.11 *Short circuit*

(i) It has zero resistance.
(ii) Current through it is very large.
(iii) There is no voltage across it.

When two terminals of a circuit have no direct connection between them, they are said to be open circuited. An open circuit has the following features:

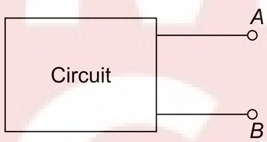


Fig. 1.12 *Open circuit*

(i) It has infinite resistance.
(ii) Current through it is zero.
(iii) The entire voltage appears across it.

1.8.1 Open Circuits and Short Circuits in a Series Circuit

When an open circuit appears in a series circuit, the equivalent resistance becomes infinite and no current flows through the circuit.

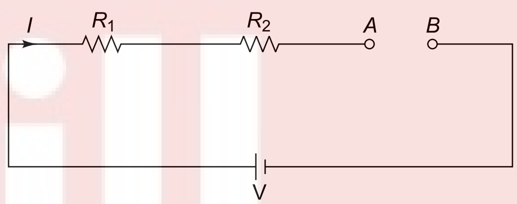


Fig. 1.13 *Open in series circuit*

$$I = \frac{V}{R_1 + R_2 + \infty} = \frac{V}{\infty} = 0$$

$$V_{AB} = V - R_1 I - R_2 I = V$$

When a short circuit appears in a series circuit, as shown in Fig. 1.14, the resistance R_2 becomes zero.

$$I = \frac{V}{R_1 + 0} = \frac{V}{R_1}$$

$$V_{AB} = 0$$

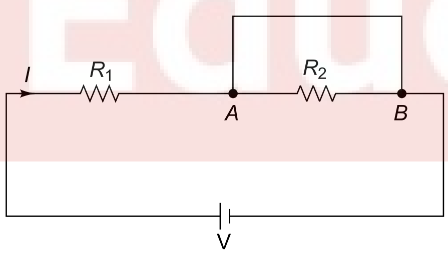


Fig. 1.14 *Short in series circuit*

1.8.2 Open Circuits and Short Circuits in a Parallel Circuit

When an open circuit appears in a parallel circuit, no current flows through that branch. The other branch currents are not affected by the open circuit.

$$I_1 = 0$$

$$I_2 = \frac{V}{R_2}$$

$$V_{AB} = V$$

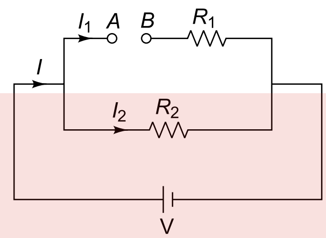


Fig. 1.15 *Open in parallel circuit*

When a short circuit appears in a parallel circuit, the equivalent resistance becomes zero.

$$I_1 = 0$$

$$I_2 = 0$$

$$V_{AB} = 0$$

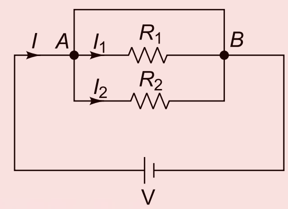


Fig. 1.16 *Short in parallel circuit*

Example 1

Find an equivalent resistance between A and B.

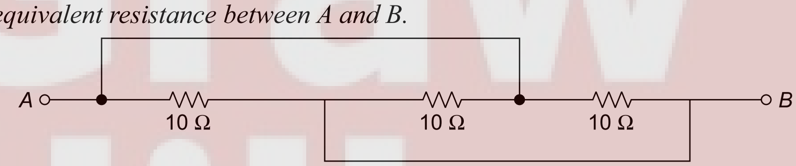


Fig. 1.17

Solution Marking all the junctions and redrawing the network,

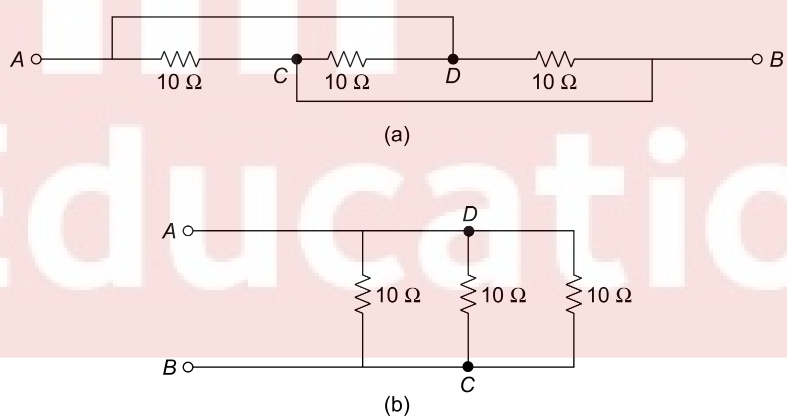


Fig. 1.18

$$R_{AB} = 10 \,||10||\, 10 = 3.33\ \Omega$$

Example 2

Find an equivalent resistance between A and B.

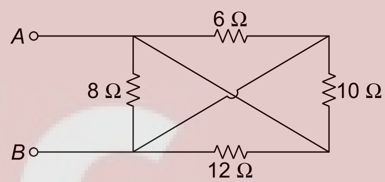


Fig. 1.19

Solution Marking all the junctions and redrawing the network,

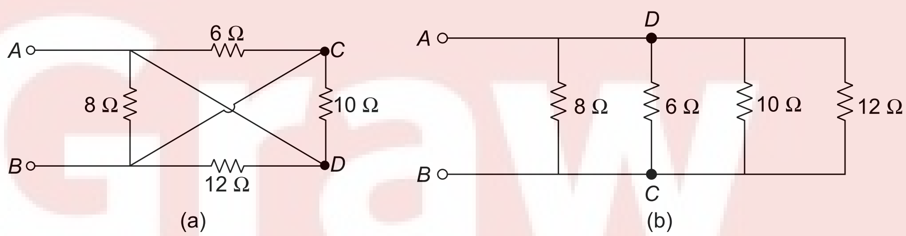


Fig. 1.20

$$R_{AB} = 8 \,||6||10||\, 12 = 2.11\ \Omega$$

Example 3

Find the equivalent resistance between A and B.

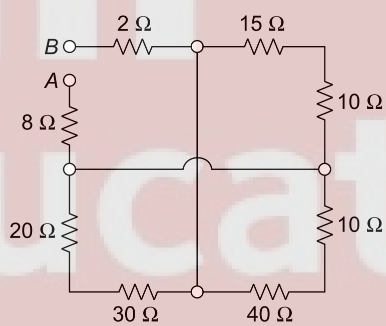


Fig. 1.21

Solution Marking all the junctions and redrawing the network,

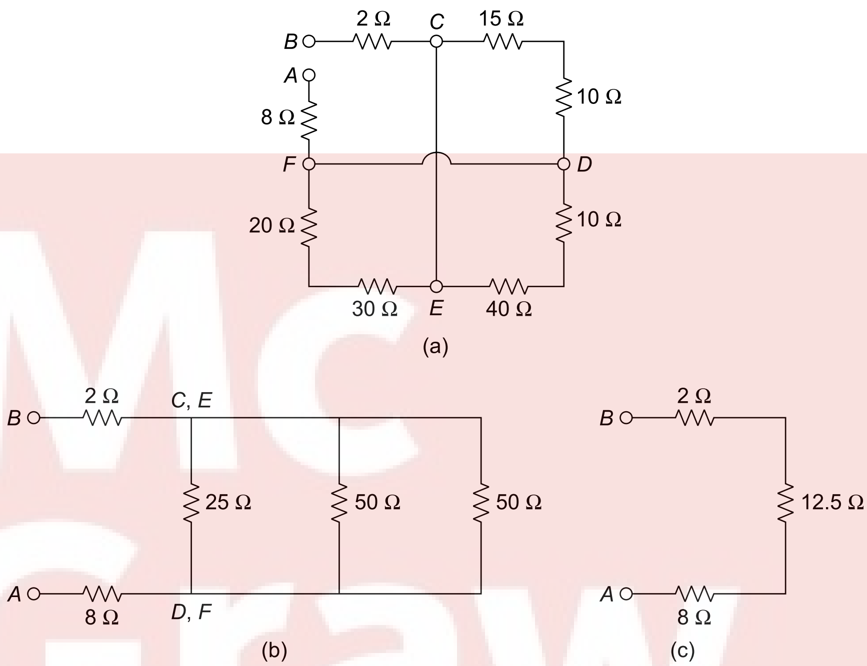


Fig. 1.22

$$R_{AB} = 22.5\ \Omega$$

Example 4

Determine the current delivered by the source.

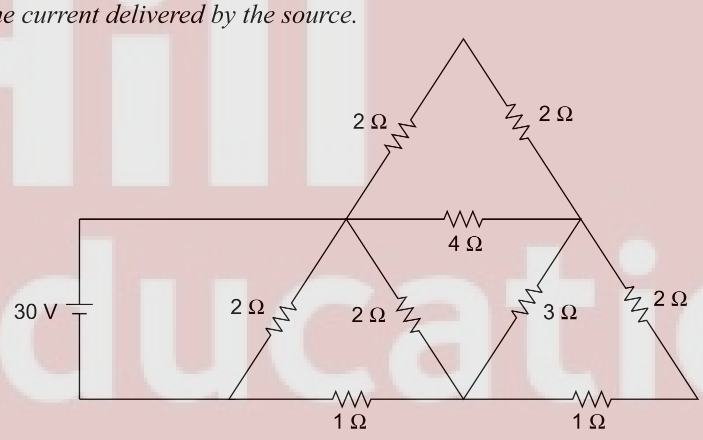


Fig. 1.23

Solution The network can be simplified by series–parallel reduction technique.

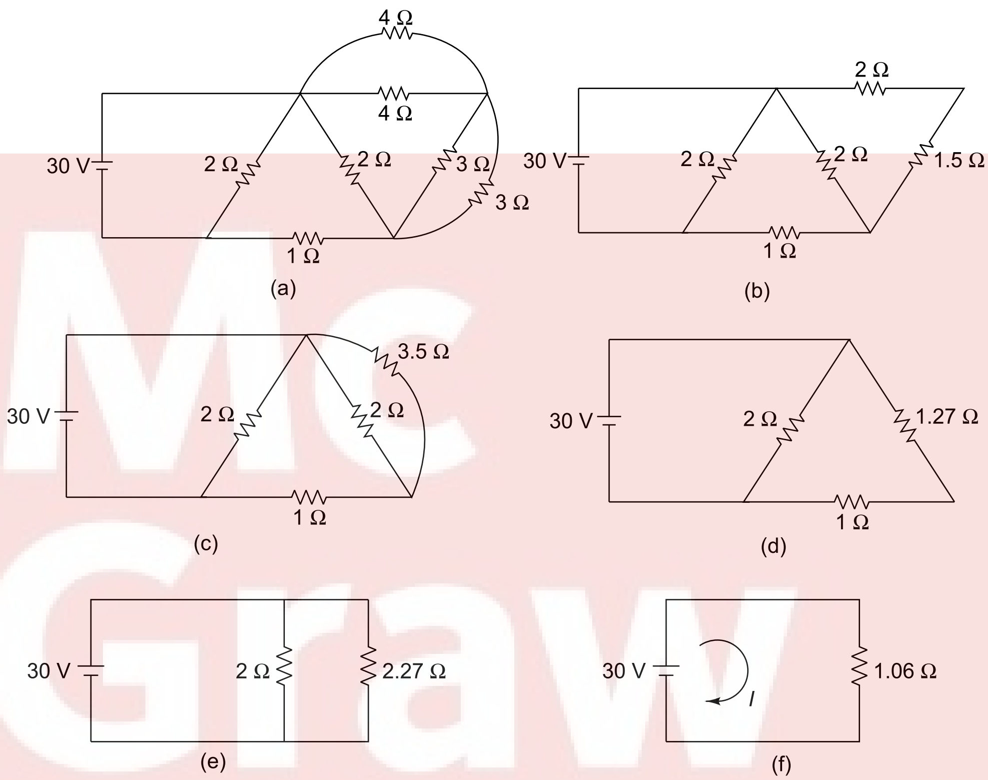


Fig. 1.24

$$I = \frac{30}{1.06} = 28.3 \text{ A}$$

Example 5

Three equal resistors of 30 Ω each are connected in parallel across a 120 V dc supply. What is the current through each of them (i) if one of the resistors burns out, or (ii) if one of the resistors gets shorted?

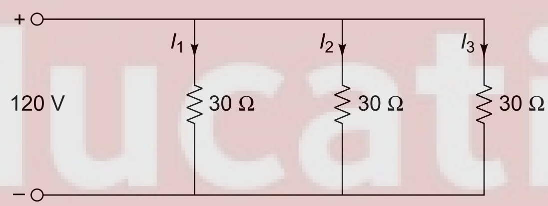


Fig. 1.25

Solution (i) If one of the resistors burns out, it will act as an open circuit.

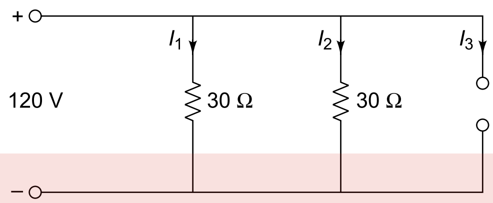


Fig. 1.26

$$I_3 = 0$$

$$I_1 = I_2 = \frac{120}{30} = 4 \text{ A}$$

(ii) If one of the resistors gets shorted, the effective resistance becomes zero.

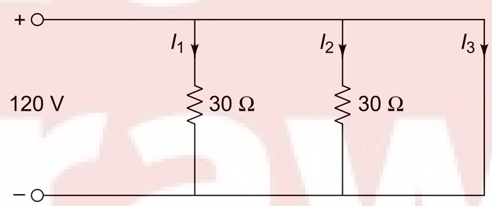


Fig. 1.27

$$I_1 = I_2 = 0$$

Example 6

A lamp rated at 100 V, 75 W is to be connected across a 230 V supply. Find the value of resistance to be connected in series with the lamp. Also find the power loss occuring in the resistor.

Solution

$$V_1 = 100 \text{ V}$$
$$P_1 = 75 \text{ W}$$
$$V = 230 \text{ V}$$

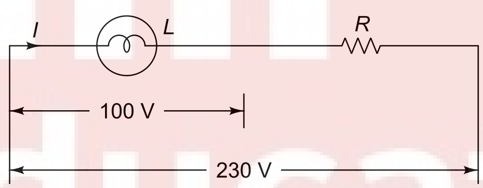


Fig. 1.28

(i) Value of resistance

Rated current of the lamp

$$I = \frac{P_1}{V_1} = \frac{75}{100} = 0.75 \text{ A}$$

Lamp will operate normally on 230 V supply if the current flowing through the lamp remains the rated current, i.e., 0.75 A.

Voltage across resistor R

$$V_2 = 230 - 100 = 130 \text{ V}$$

$$\text{Resistance } R = \frac{130}{0.75} = 173.33\,\Omega$$

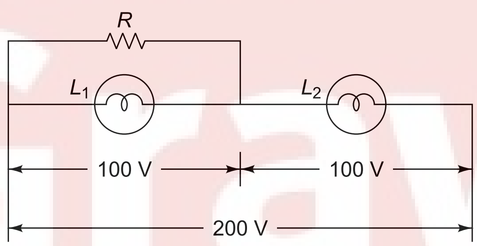
(ii) Power loss occuring in the resistor

$$P_2 = \frac{V_2^2}{R} = \frac{(130)^2}{173.33} = 97.5 \text{ W}$$

Example 7

A 100 V, 60 W lamp is connected in series with a 100 V, 100 W lamp and the combination is connected across 200 V mains. Find the value of the resistance that should be connected in parallel with the first lamp so that each lamp may get the rated current at rated voltage.

Solution



$$V_1 = 100 \text{ V}$$
$$P_1 = 60 \text{ W}$$
$$V_2 = 100 \text{ V}$$
$$P_2 = 100 \text{ W}$$
$$V = 200 \text{ V}$$

Fig. 1.29

Rated current of lamp L_1

$$I_1 = \frac{P_1}{V_1} = \frac{60}{100} = 0.6 \text{ A}$$

Rated current of lamp L_2

$$I_2 = \frac{P_2}{V_2} = \frac{100}{100} = 1 \text{ A}$$

Let R be the value of resistance that should be connected in parallel with the lamp L_1 so that rated current flows through lamp L_1.

Current through resistor R

$$I = 1 - 0.6 = 0.4 \text{ A}$$

$$\text{Resistance} \qquad R = \frac{V_1}{I} = \frac{100}{0.4} = 250\,\Omega$$

Example 8

A 100 V, 60 W lamp is connected in series with a 100 V, 100 W lamp across 200 V supply. What will be the current drawn by the lamps? What will be the power consumed by each lamp and will such a combination work?

Solution $V_1 = 100$ V, $P_1 = 60$ W
$V_2 = 100$ V, $P_2 = 100$ W
$V = 200$ V

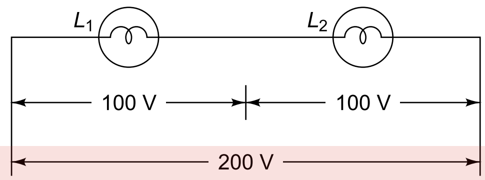


Fig. 1.30

(i) Current drawn by the lamps

Resistance of lamp L_1

$$R_1 = \frac{V_1^2}{P_1} = \frac{(100)^2}{60} = 166.67\ \Omega$$

Resistance of lamp L_2

$$R_2 = \frac{V_2^2}{P_2} = \frac{(100)^2}{100} = 100\ \Omega$$

Current drawn by the lamps

$$I = \frac{V}{R_1 + R_2} = \frac{200}{166.67 + 100} = 0.75\ \text{A}$$

(ii) Power consumed by the lamps

Power consumed by lamp $L_1 = I^2 R_1 = (0.75)^2 \times 166.67 = 93.75$ W

Power consumed by lamp $L_2 = I^2 R_2 = (0.75)^2 \times 100 = 56.25$ W

If a 100 V, 60 W lamp draws a power of 93.75 W, its filament will be overheated and will burn out. Hence, such a combination will not work.

Useful Formulae

1. $V = RI$
2. $R = \rho \dfrac{l}{A}$
3. Resistors R_1 and R_2 in series
   (i) $R_T = R_1 + R_2$
   (ii) $V = V_1 + V_2$
   (iii) $P = P_1 + P_2$
   (iv) $V_1 = \dfrac{R_1}{R_1 + R_2} V$
   (v) $V_2 = \dfrac{R_2}{R_1 + R_2} V$
4. Resistors R_1 and R_2 in parallel
   (i) $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$
   (ii) $I = I_1 + I_2$
   (iii) $P = P_1 + P_2$
   (iv) $I_1 = \dfrac{R_2}{R_1 + R_2} I$
   (v) $I_2 = \dfrac{R_1}{R_1 + R_2} I$

Exercise 1.1

1.1 Calculate the resistance of a 100 m length of wire having a uniform cross-sectional area of 0.1 mm^2 if the wire is made of material having a resistivity of 80×10^{-8} Ω-m. *[800 Ω]*

1.2 Find the resistance of a 2000 km cable at 20 °C, having a diameter of 0.7 cm. Assume specific resistance of copper at 20 °C as $\frac{1}{58}$ per °C for 1 m length and 1 mm^2 cross-section. *[896.01 Ω]*

1.3 A silver wire has a resistance of 2.5 Ω. What will be the resistance of a manganin wire having a diameter half of the silver wire and one-third length? The specific resistance of manganin is 30 times that of silver. *[100 Ω]*

1.4 Calculate the resistance of a 100 m length of wire having a cross-sectional area of 0.02 mm^2 and a resistivity of 40 μΩ-cm. If the wire is drawn out to four times its original length, calculate its new resistance. *[2000 Ω, 32000 Ω]*

1.5 A copper wire of 1 cm diameter had a resistance of 0.15 Ω. It was drawn under pressure so that its diameter was reduced to 50%. What is the new resistance of the wire? *[2.4 Ω]*

1.6 A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in ratio 49 : 24. The former carries 80% more current than the latter and the latter is 47% longer than the former. Determine the ratio of their cross-sectional areas. *[0.4]*

1.7 Two wires, one of aluminium and one of copper, offer the same resistance. The diameter of cross-section of the aluminium wire is double that of the copper wire. If the resistivities of aluminium and copper are 0.028 μΩ-m and 0.0168 μΩ-m respectively, find the ratio of length of the wires. *[0.3]*

1.8 Find the resistance between terminals *A* and *B*.

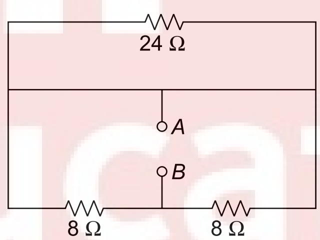


Fig. 1.31

[4 Ω]

1.9 Find the resistance between terminals *A* and *B*.

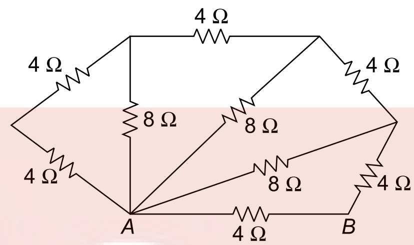


Fig. 1.32

[2.67 Ω]

1.10 Find the equivalent resistance between terminals *A* and *B*.

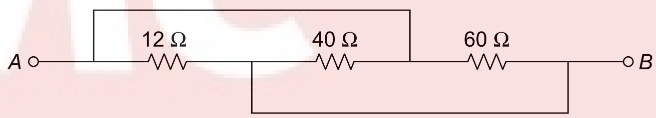


Fig. 1.33

[8 Ω]

1.11 What is the equivalent resistance between terminals *A* and *B* of the networks as shown in Fig. 1.34?

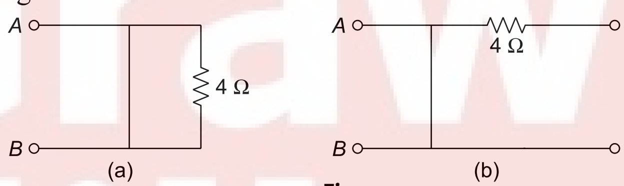


Fig. 1.34

[(a) 0 (b) 0]

1.12 Find the equivalent resistance between terminals *A* and *B*.

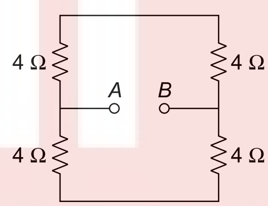


Fig. 1.35

[4 Ω]

1.13 Find the equivalent resistance between terminals *A* and *B*.

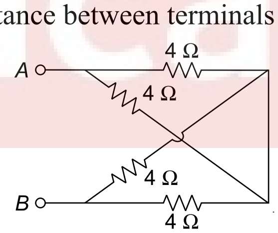


Fig. 1.36

[4 Ω]

1.14 Find the equivalent resistance between terminals *A* and *B*.

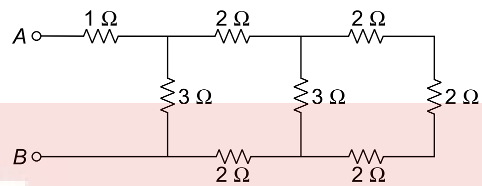


Fig. 1.37

[3 Ω]

1.15 A circuit consists of two parallel resistors, having resistances of 20 Ω and 30 Ω respectively, and is connected in series with a 15 Ω resistor. If the current through 15 Ω resistor is 3 A, find (i) current through 20 Ω and 30 Ω resistors, (ii) the voltage across the whole circuit, and (iii) total power.

[(i) 1.8 A, 1.2 A (ii) 81 V (iii) 243 W]

1.16 A resistor of 10 Ω is connected in series with two resistors each of 15 Ω arranged in parallel. What resistance must be connected across this parallel combination so that the total current taken shall be 1.5 A with 20 V applied? *[6 Ω]*

1.17 A resistor of *R* Ω is connected in series with parallel circuit consisting of two resistors of 8 Ω and 12 Ω respectively. The total power dissipated in the circuit is 70 W when applied voltage is 20 V. Calculate the value of *R*. *[0.914 Ω]*

1.18 A lamp rated 110 V, 60 W is connected with another lamp rated 110 V, 100 W across 220 V mains. Calculate the resistance that should be joined in parallel with the first lamp, so that both the lamps may take their rated power. *[302.5 Ω]*

1.9 EFFECT OF TEMPERATURE ON RESISTANCE

The resistance of a material changes with temperature. For different types of materials, the amount of change in resistance due to change in temperature is different.

The resistance of all pure metals increases linearly with increase in temperature over a limited temperature range. As the temperature increases, the ions inside the metal acquire energy and start oscillating about their mean positions. These vibrating ions collide with the electrons. Hence, resistance increases with increase in temperature.

The resistance of almost all alloys increases with increase in temperature but the rate of change of resistance is less than that of metals. The resistance of certain alloys such as manganin, eureka and constantan show practically no change in resistance for a considerable range of temperature.

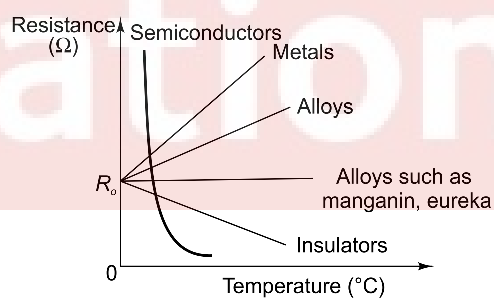


Fig. 1.38 *Effect of temperature on resistance*

In case of electrolytes, insulators and semiconductors, resistance of the material decreases with increase in temperature. As the temperature increases, some of the electrons acquire energy and become free for conduction. Hence, conductivity increases and resistance decreases with increase in temperature.

The change in resistance of a material with rise in temperature can be expressed by means of the temperature coefficient of resistance. Consider a conductor having resistance R_0 at 0 °C and R_t at t °C. In the normal range of temperatures, the increase in resistance, i.e. $(R_t - R_0)$

(i) is directly proportional to the initial resistance R_0,
(ii) is directly proportional to the rise in temperature t °C, and
(iii) also depends upon the nature of the material.

Hence,

$$R_t - R_0 \propto R_0\, t$$

$$R_t - R_0 = \alpha\, R_0\, t$$

$$R_t = R_0\,(1 + \alpha_0 t)$$

where α_0 is a constant and is called the temperature coefficient of resistance at 0 °C. Its value depends upon the nature of the material and temperature.

$$\alpha_0 = \frac{R_t - R_0}{R_0 t}$$

Hence, temperature coefficient of a material may be defined as the increase in resistance per ohm original resistance per °C rise in temperature.

1.10 EFFECT OF TEMPERATURE ON TEMPERATURE COEFFICIENT OF RESISTANCE

Let R_{t_1} and R_{t_2} be the resistances of a conductor at t_1 °C and t_2 °C respectively, and α_{t_1} and α_{t_2} be the corresponding temperature coefficients. Suppose the conductor is heated from one initial temperature t_1 °C to a final temperature t_2 °C.

$$R_{t_2} = R_{t_1}\,[1 + \alpha_{t_1}\,(t_2 - t_1)] \tag{1.1}$$

The same conductor is now cooled from t_2 °C to t_1 °C.

$$R_{t_1} = R_{t_2}\,[1 + \alpha_{t_2}\,(t_1 - t_2)] \tag{1.2}$$

Substituting Eq. (1.2) in Eq. (1.1)

$$R_{t_2} = R_{t_2}\,[1 + \alpha_{t_2}\,(t_1 - t_2)]\,[1 + \alpha_{t_1}\,(t_2 - t_1)]$$

$$1 = [1 + \alpha_{t_2}\,(t_1 - t_2)]\,[1 + \alpha_{t_1}\,(t_2 - t_1)]$$

$$= [1 - \alpha_{t_2}\,(t_2 - t_1)]\,[1 + \alpha_{t_1}\,(t_2 - t_1)]$$

$$\frac{1}{1 + \alpha_{t_1}\,(t_2 - t_1)} = 1 - \alpha_{t_2}\,(t_2 - t_1)$$

$$\alpha_{t_2}\,(t_2 - t_1) = 1 - \frac{1}{1 + \alpha_{t_1}\,(t_2 - t_1)}$$

$$= \frac{1+\alpha_{t_1}(t_2-t_1)-1}{1+\alpha_{t_1}(t_2-t_1)}$$

$$= \frac{\alpha_{t_1}(t_2-t_1)}{1+\alpha_{t_1}(t_2-t_1)}$$

$$\alpha_{t_2} = \frac{\alpha_{t_1}}{1+\alpha_{t_1}(t_2-t_1)}$$

Note: If temperature changes from 0 °C to t °C then

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

1.11 EFFECT OF TEMPERATURE ON RESISTIVITY

The specific resistance or resistivity of a material depends on temperature. The change in temperature affects the resistivity of a material in the same way as it affects the resistance.

The resistivity of metals increases linearly with the increase in temperature. Let ρ_{t_1} and ρ_{t_2} be the resistivity at temperature t_1 °C and t_2 °C respectively. Let m be the slope of the linear part of the curve.

$$m = \frac{\rho_{t_2}-\rho_{t_1}}{t_2-t_1}$$

$$\rho_{t_2} = \rho_{t_1} + m\,(t_2 - t_1)$$

$$= \rho_{t_1}\left[1+\frac{m}{\rho_{t_1}}(t_2-t_1)\right]$$

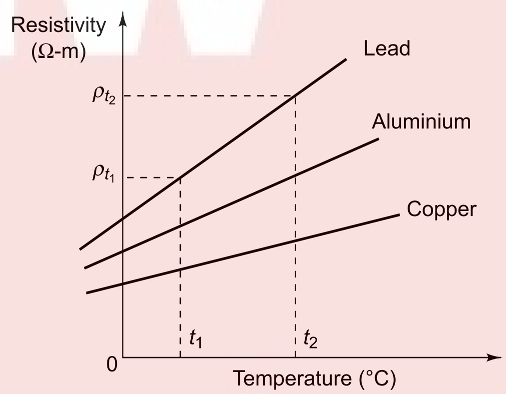


Fig. 1.39 *Effect of temperature on resistivity*

The ratio $\frac{m}{\rho_{t_1}}$ is called the *temperature coefficient of resistivity* at t_1 °C and is almost equal to α_{t_1}.

$$\rho_{t_2} = \rho_{t_1}[1 + \alpha_{t_1}(t_2 - t_1)]$$

Note: If temperature changes from 0 °C to t °C then

$$\rho_t = \rho_0 [1 + \alpha_0 t]$$

Example 1

A specimen of copper wire has a temperature coefficient of $\frac{1}{254.5}$ / °C at 20 °C. Find the temperature coefficients at 0 °C and 60 °C.

Solution $\alpha_{20} = \dfrac{1}{254.5} / °C$

(i) Temperature coefficient of copper wire at 0 °C

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

At t = 20 °C,
$$\alpha_{20} = \frac{\alpha_0}{1+\alpha_0 \times 20}$$

$$\frac{1}{254.5} = \frac{\alpha_0}{1+20\alpha_0}$$

$$\alpha_0 = \frac{1}{234.5} / °C$$

(ii) Temperature coefficient of copper wire at 60 °C

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

At t = 60 °C,
$$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0 \times 60}$$

$$= \frac{1}{\frac{1}{\alpha_0}+60}$$

$$= \frac{1}{234.5+60}$$

$$= \frac{1}{294.5} / °C$$

Example 2

At 0 °C, a specimen of copper wire has its resistance equal to 4 mΩ and its temperature coefficient of resistance equal to $\dfrac{1}{234.5}$ per °C. Find the values of its resistance and temperature coefficient of resistance at 70 °C.

Solution $R_0 = 4 \text{ m}\Omega = 4 \times 10^{-3}\ \Omega$

$$\alpha_0 = \frac{1}{234.5} / °C$$

$$t = 70\ °C$$

(i) Value of resistance at 70 °C

$$R_t = R_0 (1 + \alpha_0 t)$$

$$= 4 \times 10^{-3} \left(1 + \frac{1}{234.5} \times 70\right)$$

$$= 5.19 \times 10^{-3}\ \Omega$$

(ii) Temperature coefficient of resistance at 70 °C

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

At $t = 70$ °C, $$\alpha_{70} = \frac{\alpha_0}{1+\alpha_0 \times 70}$$

$$= \frac{1}{\frac{1}{\alpha_0}+70}$$

$$= \frac{1}{234.5+70}$$

$$= \frac{1}{304.5}\ /°\text{C}$$

$$= 3.28 \times 10^{-3}/\ °\text{C}$$

Example 3

The resistance of a coil of wire increases from 80 Ω at 10 °C to 96.6 Ω at 60 °C. Find (i) the temperature coefficient of the material at 0 °C, (ii) the resistance of the material at 0 °C, (iii) the temperature coefficient of the material at 60 °C, and (iv) the resistance of the material at 40 °C.

Solution $$R_{10} = 80\ \Omega$$

$$R_{60} = 96.6\ \Omega$$

(i) Temperature coefficient of the material at 0 °C

$$R_t = R_0\,(1 + \alpha_0 t)$$

At $t = 10$ °C, $$R_{10} = R_0\,(1 + \alpha_0 \times 10) = 80 \quad (1)$$

At $t = 60$ °C, $$R_{60} = R_0\,(1 + \alpha_0 \times 60) = 96.6 \quad (2)$$

Dividing Eq. (1) by Eq. (2),

$$\frac{1+\alpha_0 \times 10}{1+\alpha_0 \times 60} = \frac{80}{96.6}$$

$$\alpha_0 = 4.33 \times 10^{-3}/\ °\text{C}$$

(ii) Resistance of the material at 0 °C

Substituting α_0 in Eq. (1),

$$R_0\,(1 + 4.33 \times 10^{-2}) = 80$$

$$R_0 = 76.68\ \Omega$$

(iii) Temperature coefficient of the material at 60 °C

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

At $t = 60$ °C, $$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0 \times 60}$$

$$= \frac{4.33\times10^{-3}}{1+4.33\times10^{-3}\times 60}$$

$$= 3.44 \times 10^{-3}/\,°\text{C}$$

(iv) Resistance of the material at 40 °C

$$R_t = R_0\,(1 + \alpha_0\, t)$$

At $t = 40$ °C, $$R_{40} = 76.68\,(1 + 4.33 \times 10^{-3} \times 40)$$

$$= 89.96\ \Omega$$

Example 4

A resistance element having cross-sectional area of 10 mm² and length of 10 metres takes a current of 4 amperes from a 220 V supply at a temperature of 20 °C. Find (i) the resistivity of the material, and (ii) current it will take when temperature rises to 60 °C. Assume $\alpha_{20} = 0.0003/\,°C$.

Solution $$A = 10\text{ mm}^2 = 10 \times 10^{-6}\text{ m}^2$$

$$l = 10\text{ m}$$

$$I_1 = 4\text{ A}$$

$$V = 220\text{ V}$$

$$t_1 = 20\ °\text{C}$$

$$t_2 = 60\ °\text{C}$$

$$\alpha_{20} = 0.0003/\,°\text{C}$$

(i) Resistivity of the material

$$R_{t_1} = \frac{V}{I_1} = \frac{220}{4} = 55\ \Omega$$

$$R_{t_1} = \rho\frac{l}{A}$$

$$55 = \rho\times\frac{10}{10\times10^{-6}}$$

$$\rho = 55 \times 10^{-6}\ \Omega\text{-m}$$

(ii) Current when $t_2 = 60$ °C

$$R_{t_2} = R_{t_1}\,[1 + \alpha_{t_1}\,(t_2 - t_1)]$$

$$= 55\,[1 + 0.0003\,(60 - 20)]$$
$$= 55.66\ \Omega$$
$$I_2 = \frac{V}{R_{t_2}} = \frac{220}{55.66} = 3.95\text{ A}$$

Example 5

A coil has a resistance of 18 Ω when its mean temperature is 20 °C, and of 22 Ω when its temperature is 50 °C. Find its mean temperature rise when its resistance is 24 Ω and the surrounding temperature is 18 °C.

Solution $R_{20} = 18\ \Omega$

$$R_{50} = 22\ \Omega$$

Let α_0 be the temperature coefficient of resistance of coil at 0 °C.

$$R_t = R_0\,(1 + \alpha_0\, t)$$

At $t = 20$ °C, $R_{20} = R_0\,(1 + \alpha_0 \times 20) = 18$ (1)

At $t = 50$ °C, $R_{50} = R_0\,(1 + \alpha_0 \times 50) = 22$ (2)

Dividing Eq. (1) by Eq. (2),

$$\frac{1+\alpha_0 \times 20}{1+\alpha_0 \times 50} = \frac{18}{22}$$
$$\alpha_0 = 0.0087/\ °\text{C}$$

At $t = t_2$, $R_{t_2} = R_0\,(1 + 0.004\, t_2) = 24$ (3)

Dividing Eq. (3) by Eq. (1),

$$\frac{1+0.0087 \times t_2}{1+0.0087 \times 20} = \frac{24}{18}$$
$$t_2 = 65\ °\text{C}$$

Mean temperature rise = 65 – 18 = 47 °C

Example 6

A specimen of copper wire has a specific resistance of 1.7 × 10^{-8} Ω-m at 0 °C and has temperature coefficient of resistance of $\frac{1}{254.5}$ / °C at 20 °C. Find the specific resistance and temperature coefficient of resistance at 60 °C.

Solution $\rho_0 = 1.7 \times 10^{-8}\ \Omega\text{-m}$

$$\alpha_{20} = \frac{1}{254.5} / °\text{C}$$

(i) Temperature coefficient of resistance at 0 °C

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

At t = 20 °C,
$$\alpha_{20} = \frac{\alpha_0}{1+\alpha_0 \times 20}$$

$$\frac{1}{254.5} = \frac{\alpha_0}{1+\alpha_0 \times 20}$$

$$\alpha_0 = 4.26 \times 10^{-3} / °\text{C}$$

(ii) Temperature coefficient of resistance at 60 °C

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

At t = 60 °C,
$$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0 \times 60}$$

$$= \frac{4.26 \times 10^{-3}}{1+4.26 \times 10^{-3} \times 60}$$

$$= 3.39 \times 10^{-3} / °\text{C}$$

(iii) Specific resistance at 60 °C

$$\rho_t = \rho_0 (1 + \alpha_0 t)$$

At t = 60 °C,
$$\rho_{60} = 1.7 \times 10^{-8} (1 + 4.26 \times 10^{-3} \times 60)$$

$$= 2.135 \times 10^{-8}\ \Omega\text{-m}$$

Useful Formulae

1. $R_t = R_0 (1 + \alpha_0 t)$
2. $R_{t_2} = R_{t_1} [1 + \alpha_{t_1} (t_2 - t_1)]$
3. $\alpha_t = \dfrac{\alpha_0}{1+\alpha_0 t}$
4. $\alpha_{t_2} = \dfrac{\alpha_{t_1}}{1+\alpha_{t_1}(t_2 - t_1)}$
5. $\rho_t = \rho_0 (1 + \alpha_0 t)$
6. $\rho_{t_2} = \rho_{t_1} [1 + \alpha_{t_1} (t_2 - t_1)]$

Exercise 1.2

1.1 A coil has a 25 Ω resistance at 40 °C and 45 Ω resistance at 100 °C. Find the resistance and resistance temperature coefficient at 0 °C. *[11.68 Ω, 0.0285/ °C]*

1.2 A coil has a resistance of 15 Ω when its temperature is 20 °C and 20 Ω when its temperature is 50 °C. Find the temperature of the coil when its resistance is 25 Ω. *[80 °C]*

1.3 A potential difference of 250 V is applied to a copper field at a temperature of 15 °C and the current is 5 A. What will be the mean temperature of the coil when the current has fallen to 3.91 A, if the applied voltage is unchanged? *[85 °C]*

1.4 Determine the current flowing at the instant of switching a 60 W lamp on a 230 V supply. The ambient temperature is 25 °C. The filament temperature is 2000°C and the resistance temperature coefficient is 0.005/°C at 0 °C. *[2.55 A]*

1.5 The field winding of a dc motor connected across a 230 V supply takes 1.15 A at room temperature of 20 °C. After working for some hours the current falls to 0.26 A, the supply voltage remaining constant. Calculate the final working temperature of the field winding. Resistance temperature coefficient of copper at 20 °C is $\frac{1}{254.5}$ per °C. *[70.4 °C]*

1.6 The resistance of shunt winding of a dc machine is measured before and after a run of several hours. The average values are 55 Ω and 63 Ω. Calculate the rise in temperature of the winding. Temperature coefficient of resistance of copper is 0.00428 Ω/Ω/ °C. *[36 °C]*

Review Questions

1.1 What are the factors governing the value of resistance? Explain the term resistivity.

1.2 Define temperature coefficient of resistance. How do resistances of different materials vary with temperature?

1.3 Explain the effect of temperature on resistance of the following materials:
(i) pure metals (ii) metal alloys (iii) insulators (iv) semiconductors

1.4 Define temperature coefficient of resistance. State its unit and state if it is true that temperature coefficient of resistance can have (i) zero value (ii) negative value (iii) positive value.

1.5 Explain the effect of temperature on conducting and insulating material with graph.

Objective-Type Questions

Choose the correct alternative in the following questions:

1.1 If the length of a wire of resistance R is uniformly stretched to n times its original value, its new resistance is

(a) nR (b) $\frac{R}{n}$

(c) n^2R (d) $\frac{R}{n^2}$

1.2 Two wires A and B of the same material and length L and $2L$ have radius r and $2r$ respectively. The ratio of their specific resistance will be

(a) 1 : 1 (b) 1 : 2

(c) 1 : 4 (d) 1 : 8

1.3 The current through an electrical conductor is 1 A when the temperature of the conductor is 0 °C and 0.7 A when the temperature is 100 °C. The current when the temperature of the conductor is 1200 °C must be

(a) 0.08 A (b) 0.16 A

(c) 0.32 A (d) 0.64 A

1.4 A length of wire having a resistance of 1Ω is cut into four equal parts and these four parts are bundled together side-by-side to form a wire. The new resistance will be

(a) $\frac{1}{4}$ Ω (b) $\frac{1}{16}$ Ω

(b) 4 Ω (c) 16 Ω

1.5 The hot resistance of the filament of a bulb is higher than the cold resistance because the temperature coefficient of the filament is

(a) negative (b) infinite

(c) zero (d) positive

1.6 A network contains linear resistors and ideal voltage sources. If values of all the resistors are doubled then the voltage across each resistor is

(a) halved (b) doubled

(c) increased by four times (d) not changed

1.7 A 10 V battery with an internal resistance of 1 Ω is connected across a nonlinear load whose V-I characteristic is given by $7\,I = V^2 + 2V$. The current delivered by the battery is

(a) 0 (b) 10 A (c) 5 A (d) 8 A

1.8 All the resistors in Fig. 1.40 are 1 Ω each. The value of I will be

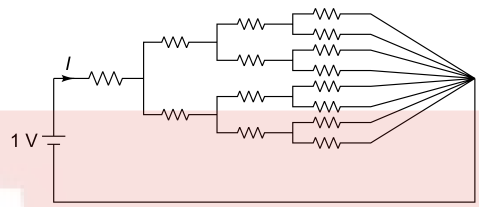


Fig. 1.40

(a) $\frac{1}{15}$ A (b) $\frac{2}{15}$ A (c) $\frac{4}{15}$ A (d) $\frac{8}{15}$ A

1.9 For the circuit shown in Fig. 1.41, the equivalent resistance will be

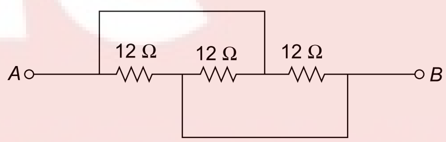


Fig. 1.41

(a) 36 Ω (b) 12 Ω (c) 6 Ω (d) 4 Ω

1.10 Two incandescent light bulbs of 40 W and 60 W rating are connected in series across the mains. Then

(a) the bulbs together consume 100 W (b) the bulbs together consume 50 W
(c) the 60 W bulb glows brighter (d) the 40 W bulb glows brighter

1.11 Twelve 1 Ω resistors are used as edges to form a cube. The resistance between the two diagonally opposite corners of the cube is

(a) $\frac{5}{6}$ Ω (b) 1 Ω (c) $\frac{6}{5}$ Ω (d) $\frac{3}{2}$ Ω

1.12 All resistors in the circuit shown in Fig. 1.42 are of R Ω each. The switch is initially open. When the switch is closed the lamp's intensity

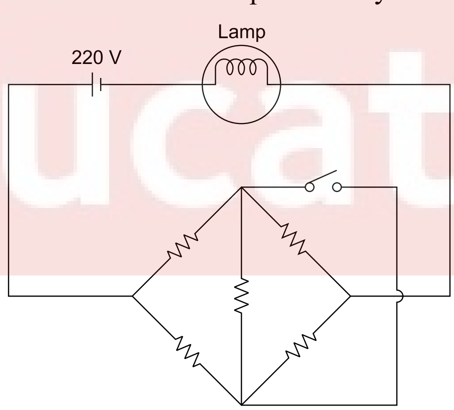


Fig. 1.42

(a) increases (b) decreases
(c) remains the same (d) depends on the value of R

1.13 Two 2 kΩ, 2 W resistors are connected in parallel. Then combined resistance and wattage ratings will be
(a) 4 kΩ, 4 W (b) 1 kΩ, 4 W
(c) 1 kΩ, 2 W (d) 1 kΩ, 1 W

1.14 Two electrical sub-networks N_1 and N_2 are connected through three resistors as shown in Fig. 1.43. The voltages across the 5 Ω resistor and 1 Ω resistor are given to be 10 V and 5 V respectively. Then the voltage across the 15 Ω resistor is

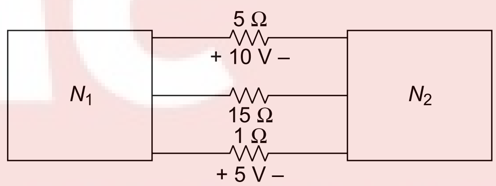
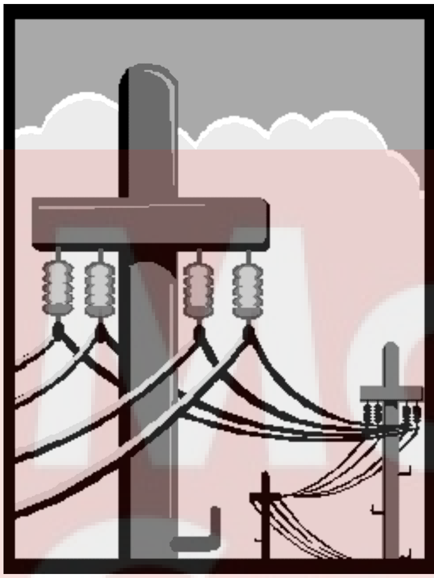


Fig. 1.43

(a) –105 V (b) 105 V (c) –15 V (d) 15 V

Answers to Objective-Type Questions

1.1 (c)	**1.2** (b)	**1.3** (b)	**1.4** (b)	**1.5** (d)	**1.6** (d)
1.7 (c)	**1.8** (d)	**1.9** (d)	**1.10** (d)	**1.11** (a)	**1.12** (c)
1.13 (b)	**1.14** (a)				



Chapter 2

DC Circuits

Chapter Outline

- 2.1 Kirchhoff's Laws
- 2.2 Mesh Analysis
- 2.3 Nodal Analysis
- 2.4 Source Transformation
- 2.5 Star-Delta Transformation
- 2.6 Superposition Theorem
- 2.7 Thevenin's Theorem
- 2.8 Norton's Theorem
- 2.9 Maximum Power Transfer Theorem

2.1 KIRCHHOFF'S LAWS

The entire study of electric circuit analysis is based mainly on Kirchhoff's laws. But before discussing this, it is essential to familiarise ourselves with the following terms:

Node A node is a junction where two or more circuit elements are connected together.

Branch An element or number of elements connected between two nodes constitute a branch.

Loop A loop is any closed part of the circuit.

Mesh A mesh is the most elementary form of a loop and cannot be further divided into other loops. All meshes are loops but all loops are not meshes.

Kirchhoff's Current Law (KCL) The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

Consider five conductors, carrying currents I_1, I_2, I_3, I_4 and I_5 meeting at a point O as shown in Fig. 2.1. Assuming the incoming currents to be positive and outgoing currents negative, we have

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

$$I_1 + I_3 + I_5 = I_2 + I_4$$

Thus, the above law can also be stated as the sum of currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

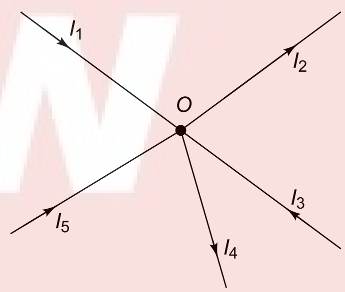


Fig. 2.1 *Kirchhoff's current law*

Kirchhoff's Voltage Law (KVL) The algebraic sum of all the voltages in any closed circuit or mesh or loop is zero.

If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in potential at that point. This means that the sum of emfs and the sum of voltage drops or rises meeting on the way is zero.

Determination of Sign A rise in potential can be assumed to be positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

(i) If we go from the positive terminal of the battery or source to the negative terminal, there is a fall in potential and so the emf should be assigned a negative sign. If we go from the negative terminal of the battery or source to the positive terminal, there is a rise in potential and so the emf should be given a positive sign.

(a) Fall in potential (b) Rise in potential

Fig. 2.2 *Sign convention*

(ii) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in the potential and so the sign of this voltage drop is negative. If we go opposite to the direction of the current flow, there is a rise in potential and hence, this voltage drop should be given a positive sign.

(a) Fall in potential (b) Rise in potential

Fig. 2.3 *Sign convention*

Example 1

The voltage drop across the 15 Ω resistor is 30 V, having the polarity indicated. Find the value of R.

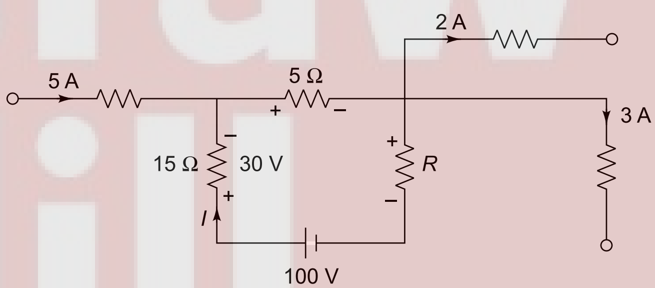


Fig. 2.4

Solution Current through the 15 Ω resistor

$$I = \frac{30}{15} = 2\text{ A}$$

Current through the 5 Ω resistor = 5 + 2 = 7 A

Applying KVL to the closed path,

$$-5\,(7) - R(2) + 100 - 30 = 0$$

$$-35 - 2R + 100 - 30 = 0$$

$$R = 17.5\ \Omega$$

Example 2

Determine the currents I_1, I_2 and I_3.

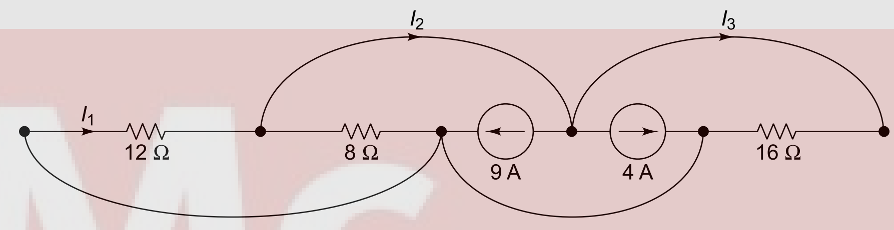


Fig. 2.5

Solution Assigning currents to all the branches,

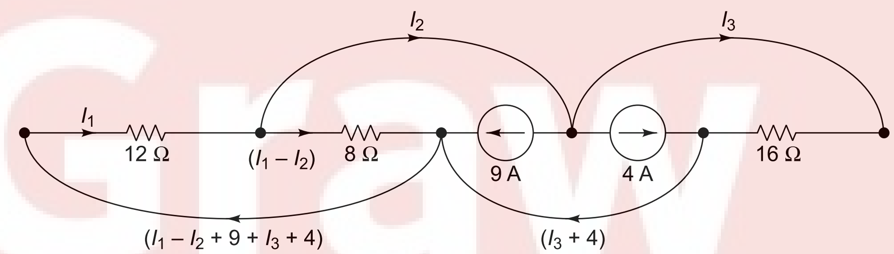


Fig. 2.6

From Fig. 2.6,

$$I_1 = I_1 - I_2 + 9 + I_3 + 4$$

$$I_2 - I_3 = 13 \qquad (1)$$

Also, $$-12I_1 - 8(I_1 - I_2) = 0$$

$$-20I_1 + 8I_2 = 0 \qquad (2)$$

and, $$-12I_1 - 16I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4\text{ A}$$

$$I_2 = 10\text{ A}$$

$$I_3 = -3\text{ A}$$

Example 3

Find currents in all the branches of the network.

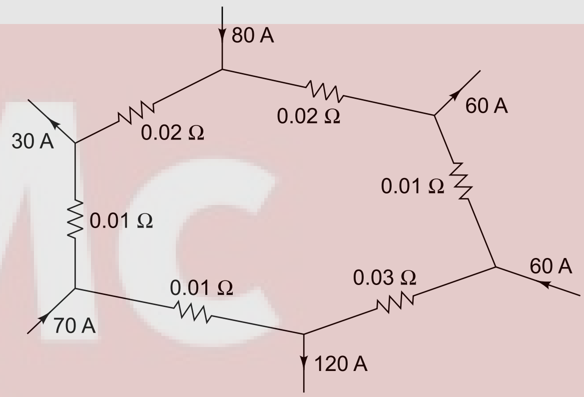


Fig. 2.7

Solution Let $I_{AF} = x$

Then $I_{FE} = x - 30$

$I_{ED} = x + 40$

$I_{DC} = x - 80$

$I_{CB} = x - 20$

$I_{BA} = x - 80$

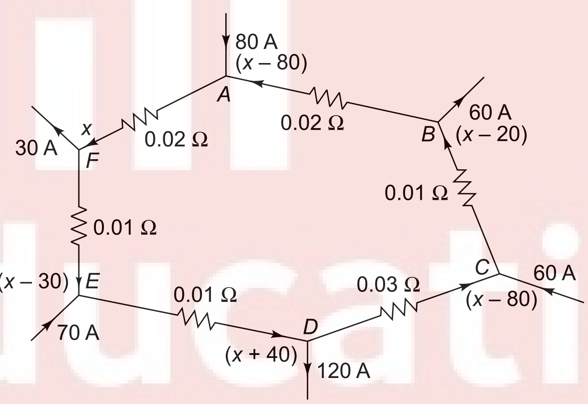


Fig. 2.8

Applying KVL to the closed path *AFEDCBA*,

$$-0.02\,x - 0.01\,(x - 30) - 0.01\,(x + 40) - 0.03\,(x - 80) - 0.01\,(x - 20) - 0.02\,(x - 80) = 0$$

$$x = 41\text{A}$$
$$I_{AF} = 41 \text{ A}$$
$$I_{FE} = 41 - 30 = 11 \text{ A}$$
$$I_{ED} = 41 + 40 = 81 \text{ A}$$
$$I_{DC} = 41 - 80 = -39 \text{ A}$$
$$I_{CD} = 39 \text{ A}$$
$$I_{CB} = 41 - 20 = 21 \text{ A}$$
$$I_{BA} = 41 - 80 = -39 \text{ A}$$
$$I_{AB} = 39 \text{ A}$$

Example 4

Find currents in all the branches of the network.

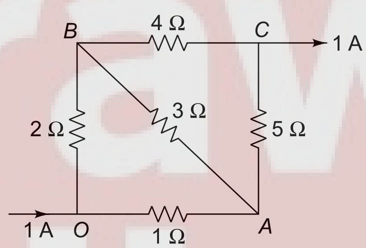


Fig. 2.9

Solution Assigning currents to all the branches,

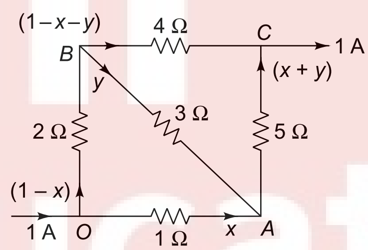


Fig. 2.10

Applying KVL to the closed path *OBAO*,

$$-2(1 - x) - 3y + 1(x) = 0$$
$$3x - 3y = 2 \qquad (1)$$

Applying KVL to the closed path $ABCA$,

$$3y - 4(1 - x - y) + 5(x + y) = 0$$

$$9x + 12y = 4 \qquad (2)$$

Solving Eqs (1) and (2),

$$x = 0.57 \text{ A}$$

$$y = -0.095 \text{ A}$$

$$I_{OA} = 0.57 \text{ A}$$

$$I_{OB} = 1 - 0.57 = 0.43 \text{ A}$$

$$I_{AB} = 0.095 \text{ A}$$

$$I_{AC} = 0.57 - 0.095 = 0.475 \text{ A}$$

$$I_{BC} = 1 - 0.57 + 0.095 = 0.525 \text{ A}$$

Example 5

What is the p.d. between points x and y in the network?

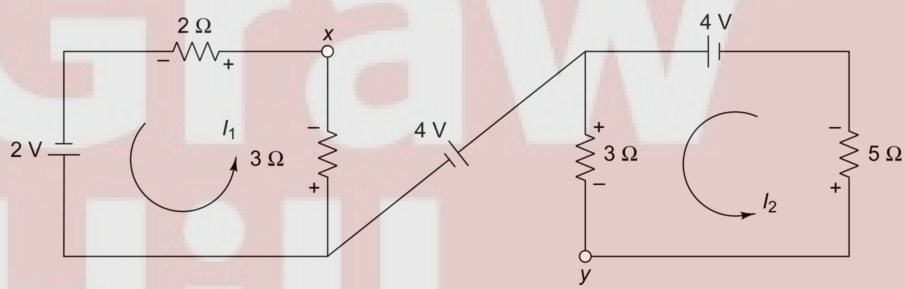


Fig. 2.11

Solution

$$I_1 = \frac{2}{2+3} = 0.4 \text{ A}$$

$$I_2 = \frac{4}{3+5} = 0.5 \text{ A}$$

Potential difference between points x and $y = V_{xy} = V_x - V_y$

Writing KVL equation for the path x to y,

$$V_x + 3I_1 + 4 - 3I_2 - V_y = 0$$

$$V_x + 3(0.4) + 4 - 3(0.5) - V_y = 0$$

$$V_x - V_y = -3.7$$

$$V_{xy} = -3.7 \text{ V}$$

Example 6

Find the voltage between points A and B.

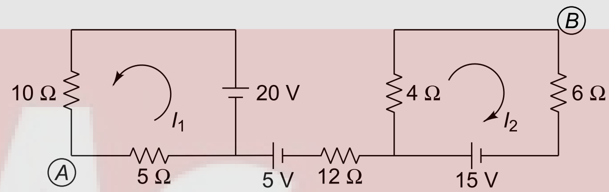


Fig. 2.12

Solution

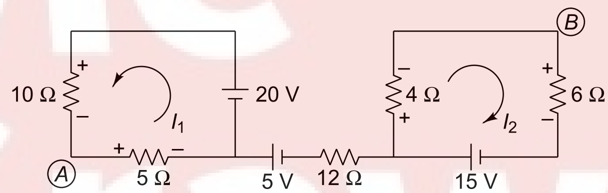


Fig. 2.13

$$I_1 = \frac{20}{10+5} = 1.33 \text{ A}$$

$$I_2 = \frac{15}{4+6} = 1.5 \text{ A}$$

Voltage between points A and $B = V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 5I_1 - 5 - 15 + 6I_2 - V_B = 0$$

$$V_A - 5(1.33) - 5 - 15 + 6(1.5) - V_B = 0$$

$$V_A - V_B = 17.65$$

$$V_{AB} = 17.65 \text{ V}$$

Example 7

Determine the potential difference V_{AB} for the given network.

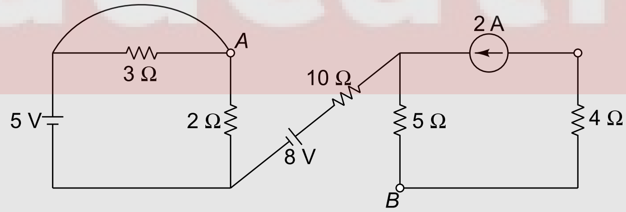


Fig. 2.14

Solution The resistor of 3 Ω is connected across a short circuit. Hence, it gets shorted.

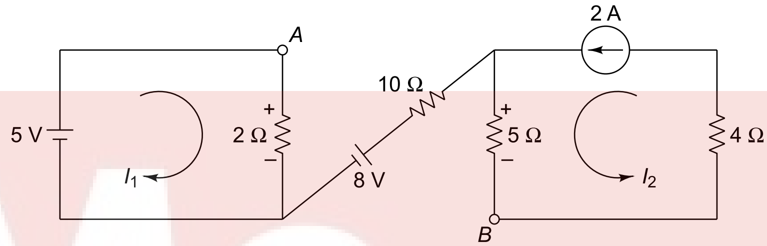


Fig. 2.15

$$I_1 = \frac{5}{2} = 2.5 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Potential difference $\quad V_{AB} = V_A - V_B$

Writing KVL equation for the path A to B,

$$V_A - 2I_1 + 8 - 5I_2 - V_B = 0$$

$$V_A - 2(2.5) + 8 - 5(2) - V_B = 0$$

$$V_A - V_B = 7$$

$$V_{AB} = 7 \text{ V}$$

Example 8

Find the voltage of the point A w.r.t. B.

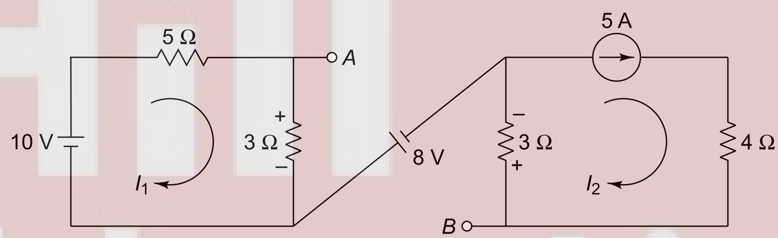


Fig. 2.16

Solution

$$I_1 = \frac{10}{5+3} = 1.25 \text{ A}$$

$$I_2 = 5 \text{ A}$$

Applying KVL to the path from A and B,

$$V_A - 3I_1 - 8 + 3I_2 - V_B = 0$$

$$V_A - 3(1.25) - 8 + 3(5) - V_B = 0$$

$$V_A - V_B = -3.25$$
$$V_{AB} = -3.25 \text{ V}$$

Example 9

What values must R_1 and R_2 have

(a) when $I_1 = 4$ A and $I_2 = 6$ A both charging?

(b) when $I_1 = 2$ A discharging and $I_2 = 20$ A charging?

(c) when $I_1 = 0$?

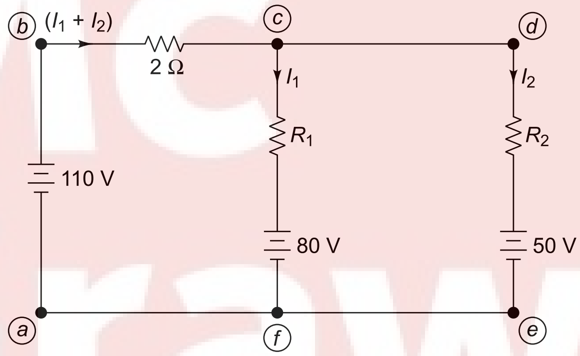


Fig. 2.17

Solution Applying KVL to closed path *abcfa*,

$$110 - 2(I_1 + I_2) - R_1 I_1 - 80 = 0$$
$$110 - 2I_1 - 2I_2 - R_1 I_1 - 80 = 0$$
$$(2 + R_1) I_1 + 2I_2 = 30 \quad (1)$$

Applying KVL to closed path *fcdef*,

$$80 + R_1 I_1 - R_2 I_2 - 50 = 0$$
$$R_1 I_1 - R_2 I_2 = -30 \quad (2)$$

Case (a) $I_1 = 4$ A and $I_2 = 6$ A both charging

i.e. $I_1 = 4$ A and $I_2 = 6$ A

Substituting I_1 and I_2 in Eq. (1),

$$(2 + R_1)\, 4 + 2\,(6) = 30$$
$$R_1 = 2.5\ \Omega$$

Substituting R_1, I_1 and I_2 in Eq. (2),

$$2.5\,(4) - R_2\,(6) = -30$$
$$R_2 = 6.67\ \Omega$$

Case (b) $I_1 = 2$ A discharging and $I_2 = 20$ A charging

i.e. $I_1 = -2$ A and $I_2 = 20$ A

Substituting I_1 and I_2 in Eq. (1),

$$(2 + R_1)(-2) + 2(20) = 30$$
$$R_1 = 3\ \Omega$$

Substituting R_1, I_1 and I_2 in Eq. (2),

$$3(-2) - R_2(20) = -30$$
$$R_2 = 1.2\ \Omega$$

Case (c) $\quad I_1 = 0$

Substituting in Eq. (1),

$$(2 + R_1)(0) + 2I_2 = 30$$
$$I_2 = 15\text{ A}$$

Substituting I_1 and I_2 in Eq. (2),

$$0 - 15R_2 = -30$$
$$R_2 = 2\ \Omega$$

Example 10

Find I_1 and I_2 when (a) $R = 2.3\ \Omega$, (b) $R = 0.5\ \Omega$, and (c) for what values of R is $I_1 = 0$?

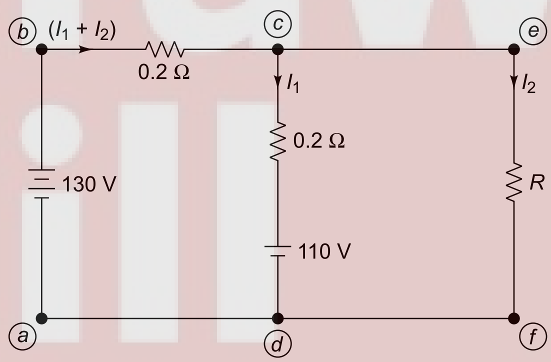


Fig. 2.18

Solution Applying KVL to closed path *abcda*,

$$130 - 0.2(I_1 + I_2) - 0.2I_1 - 110 = 0$$
$$0.4I_1 + 0.2I_2 = 20 \qquad (1)$$

Applying KVL to closed path *dcefd*,

$$110 + 0.2I_1 - RI_2 = 0$$
$$0.2I_1 - RI_2 = -110 \qquad (2)$$

Case (a) $R = 2.3\ \Omega$

Substituting R in Eq. (2),

$$0.2I_1 - 2.3I_2 = -110 \tag{3}$$

Solving Eqs (1) and (3),

$$I_1 = 25\text{ A}$$
$$I_2 = 50\text{ A}$$

Case (b) $R = 0.5\ \Omega$

Substituting R in Eq. (2),

$$0.2I_1 - 0.5I_2 = -110 \tag{4}$$

Solving Eqs (1) and (4),

$$I_1 = -50\text{ A}$$
$$I_2 = 200\text{ A}$$

Case (c) $I_1 = 0$

Substituting I_1 in Eq. (1),

$$0.2I_2 = 20$$
$$I_2 = 100\text{ A}$$

Substituting I_1 and I_2 in Eq. (2),

$$0.2(0) - R\,(100) = -110$$
$$R = 1.1\ \Omega$$

Example 11

Find the value of R.

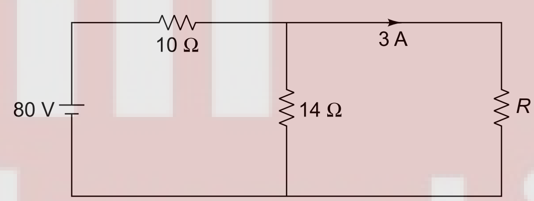


Fig. 2.19

Solution Assigning currents to all the branches,

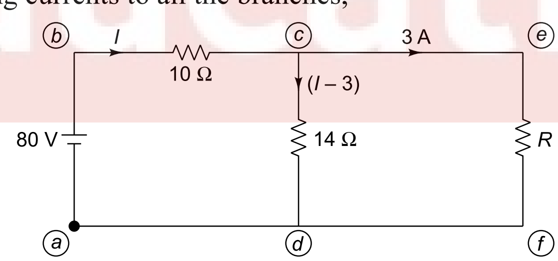


Fig. 2.20

Applying KVL to closed path *abcda*,

$$80 - 10I - 14\,(I - 3) = 0$$
$$I = 5.08 \text{ A}$$

Applying KVL to closed path *dcefd*,

$$14\,(I - 3) - 3R = 0$$
$$14\,(5.08 - 3) - 3R = 0$$
$$R = 9.71\ \Omega$$

Example 12

Determine current drawn by the ammeter shown in Fig. 2.21.

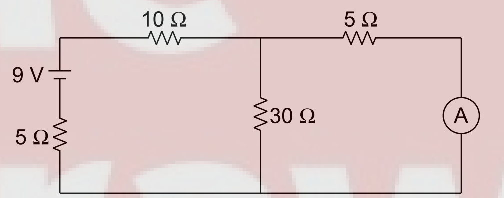


Fig. 2.21

Solution Assigning currents to all the branches,

Applying KVL to closed path *abcda*,

$$-5\,(I_1 + I_2) + 9 - 10\,(I_1 + I_2) - 30I_1 = 0$$
$$45I_1 + 15I_2 = 9 \qquad (1)$$

Applying KVL to closed path *dcefd*,

$$30I_1 - 5I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_2 = 0.4 \text{ A}$$

Current drawn by ammeter = 0.4 A

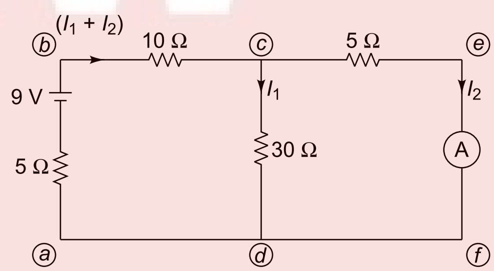


Fig. 2.22

Example 13

Find branch currents in various branches of Fig. 2.23.

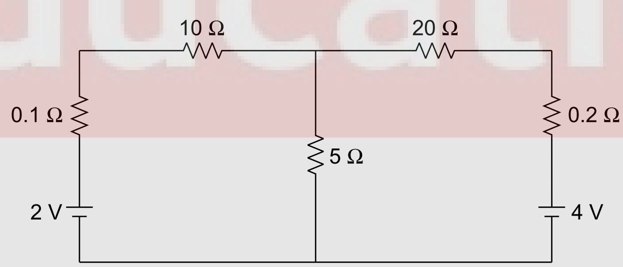


Fig. 2.23

Solution Assigning currents to various branches,

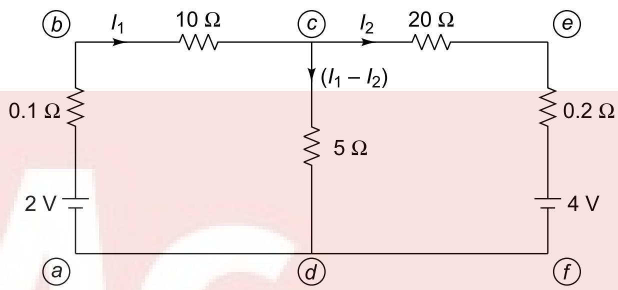


Fig. 2.24

Applying KVL to closed path *abcda*,

$$2 - 0.1I_1 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15.1I_1 - 5I_2 = 2 \qquad (1)$$

Applying KVL to closed path *dcefd*,

$$5(I_1 - I_2) - 20I_2 - 0.2I_2 - 4 = 0$$

$$5I_1 - 25.2I_2 = 4 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 0.086 \text{ A}$$

$$I_2 = -0.142 \text{ A}$$

Example 14

Find the value of R and current flowing through it when the current is zero in the branch OA.

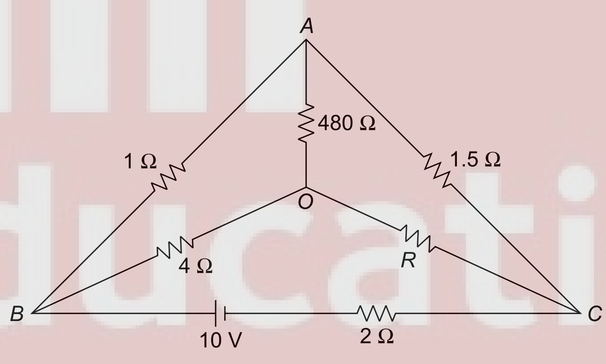


Fig. 2.25

Solution Assigning currents to all branches,

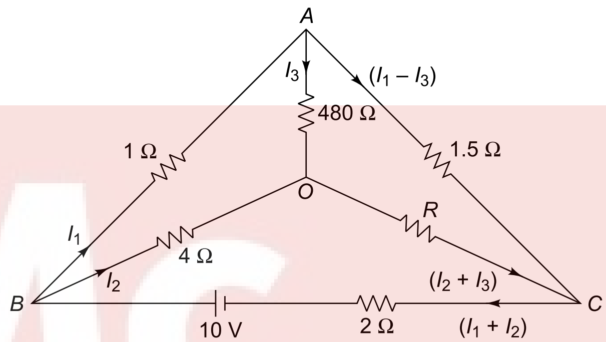


Fig. 2.26

Applying KVL to the closed path $OACO$,

$$480I_3 - 1.5\,(I_1 - I_3) + R\,(I_2 + I_3) = 0$$

But current in the branch OA is zero,

i.e. $\quad I_3 = 0$

$$-1.5I_1 + RI_2 = 0 \tag{1}$$

Applying KVL to the closed path $BOCB$,

$$-4I_2 - R\,(I_2 + I_3) - 2\,(I_1 + I_2) + 10 = 0$$

But $\quad I_3 = 0$

$$-2I_1 - (6 + R)\,I_2 = -10 \tag{2}$$

Applying KVL to the closed path $BOAB$,

$$-4I_2 + 480I_3 + I_1 = 0$$

But $\quad I_3 = 0$

$$-4I_2 + I_1 = 0 \tag{3}$$

$$I_1 = 4I_2$$

Substituting I_1 in Eq. (1) and (2),

$$-6I_2 + RI_2 = 0 \tag{4}$$

and
$$-14I_2 - RI_2 = -10 \tag{5}$$

From Eq. (4) and (5)

$$I_2 = 0.5\text{ A}$$

Substituting I_2 in Eq. (4),

$$-6\,(0.5) + R\,(0.5) = 0$$

$$R = 6\ \Omega$$

Current in branch $\quad OC = I_2 + I_3 = 0.5\text{ A}$

Example 15

Find the current supplied by the battery.

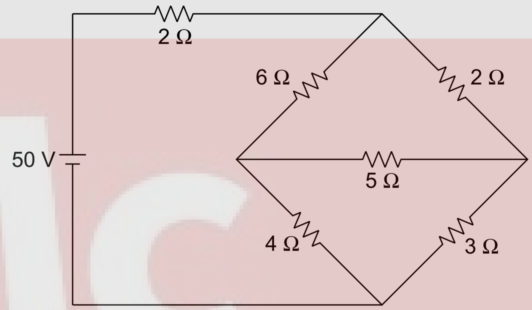


Fig. 2.27

Solution Assigning currents to all the branches,

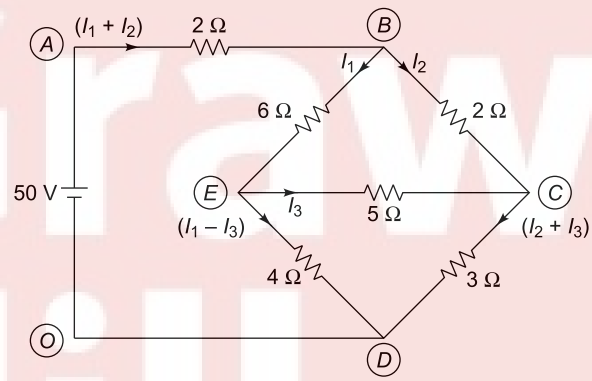


Fig. 2.28

Applying KVL to the closed path *OABEDO*,

$$50 - 2(I_1 + I_2) - 6I_1 - 4(I_1 - I_3) = 0$$

$$12I_1 + 2I_2 - 4I_3 = 50 \quad (1)$$

Applying KVL to the closed path *BCEB*,

$$-2I_2 + 5I_3 + 6I_1 = 0$$

$$6I_1 - 2I_2 + 5I_3 = 0 \quad (2)$$

Applying KVL to the closed path *ECDE*,

$$-5I_3 - 3(I_2 + I_3) + 4(I_1 - I_3) = 0$$

$$4I_1 - 3I_2 - 12I_3 = 0 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2.817 \text{ A}$$

$$I_2 = 6.647 \text{ A}$$

$$I_3 = -0.723 \text{ A}$$

Current supplied by the battery = $I_1 + I_2$

$$= 2.817 + 6.647$$

$$= 9.464 \text{ A}$$

Example 16

Find the current flowing through the 2 Ω resistor.

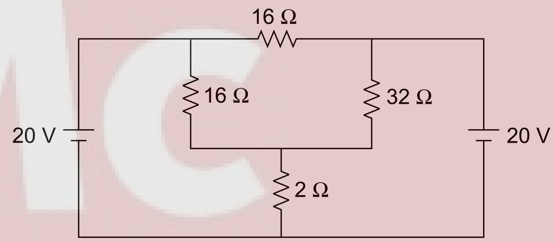


Fig. 2.29

Solution Assigning currents to all the branches,

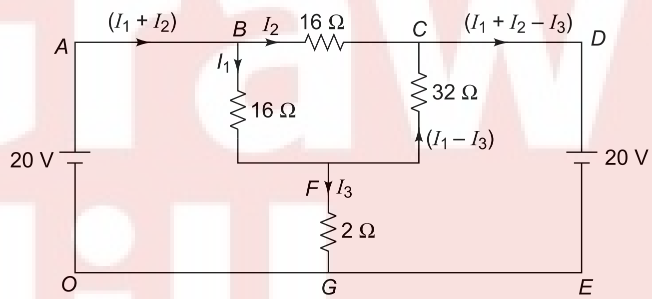


Fig. 2.30

Applying KVL to the closed path *OABFGO,*

$$20 - 16I_1 - 2I_3 = 0$$

$$16I_1 + 2I_3 = 20 \qquad (1)$$

Applying KVL to the closed path *BCFB,*

$$-16I_2 + 32(I_1 - I_3) + 16I_1 = 0$$

$$48I_1 - 16I_2 + 32I_3 = 0 \qquad (2)$$

Applying KVL to the closed path *GFCDEG,*

$$2I_3 - 32(I_1 - I_3) - 20 = 0$$

$$-32I_1 + 34I_3 = 20 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.05 \text{ A}$$

$$I_2 = 6.32 \text{ A}$$

$$I_3 = 1.58 \text{ A}$$

Current through the 2 Ω resistor = I_3 = 1.58 A

Example 17

Find the current through the 4 Ω resistor.

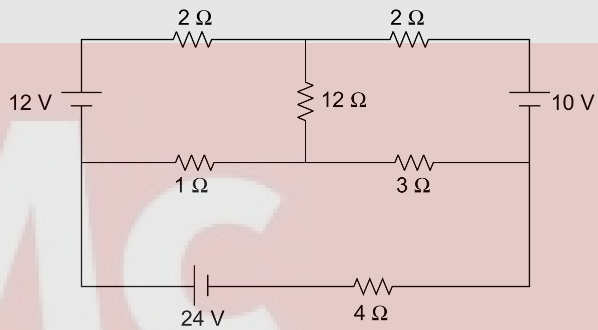


Fig. 2.31

Solution Assigning currents to all the branches,

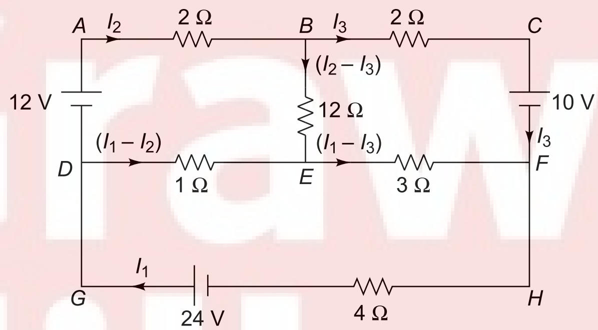


Fig. 2.32

Applying KVL to the closed path *ABEDA*,

$$-2I_2 - 12(I_2 - I_3) + 1(I_1 - I_2) + 12 = 0$$

$$I_1 - 15I_2 + 12I_3 = -12 \qquad (1)$$

Applying KVL to the closed path *BCFEB*,

$$-2I_3 - 10 + 3(I_1 - I_3) + 12(I_2 - I_3) = 0$$

$$3I_1 + 12I_2 - 17I_3 = 10 \qquad (2)$$

Applying KVL to the closed path *DEFHGD*,

$$-1(I_1 - I_2) - 3(I_1 - I_3) - 4I_1 + 24 = 0$$

$$-8I_1 + I_2 + 3I_3 = -24 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4.11 \text{ A}$$

$$I_1 = 2.72 \text{ A}$$

$$I_3 = 2.06$$

Current through the 4 Ω resistor = I_1 = 4.11 A

Example 18

Find the current through the 10 Ω resistor.

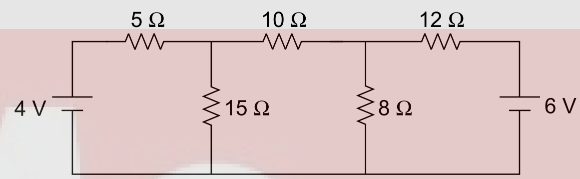


Fig. 2.33

Solution Assigning currents to all the branches,

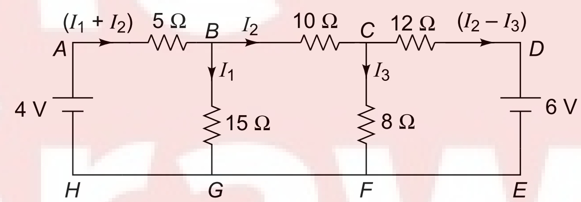


Fig. 2.34

Applying KVL to the closed path *ABGHA*,

$$-5(I_1 + I_2) - 15I_1 + 4 = 0$$

$$-20I_1 - 5I_2 = -4 \qquad (1)$$

Applying KVL to the closed path *BCFGB*,

$$-10I_2 - 8I_3 + 15I_1 = 0$$

$$15I_1 - 10I_2 - 8I_3 = 0 \qquad (2)$$

Applying KVL to the closed path *CDEFC,*

$$-12(I_2 - I_3) - 6 + 8I_3 = 0$$

$$-12I_2 + 20I_3 = 6 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.19 \text{ A}$$

$$I_2 = 0.032 \text{ A}$$

$$I_3 = 0.32 \text{ A}$$

Current through the 10 Ω resistor = I_2 = 0.032 A

Example 19

Determine the current supplied by each battery.

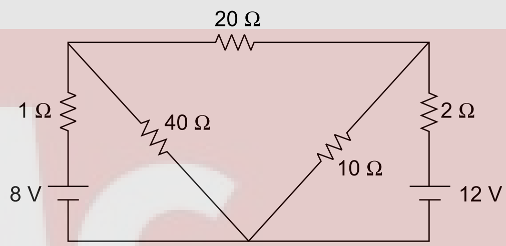


Fig. 2.35

Solution Assigning currents to all the branches,

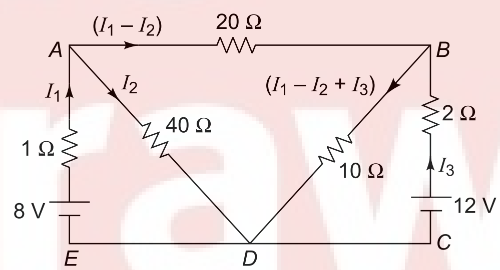


Fig. 2.36

Applying KVL to the closed path *ADEA*,

$$-40I_2 + 8 - 1I_1 = 0$$

$$I_1 + 40I_2 = 8 \qquad (1)$$

Applying KVL to the closed path *ABDA*,

$$-20(I_1 - I_2) - 10(I_1 - I_2 + I_3) + 40I_2 = 0$$

$$-30I_1 + 70I_2 - 10I_3 = 0 \qquad (2)$$

Applying KVL to the closed path *BCDB*,

$$2I_3 - 12 + 10\,(I_1 - I_2 + I_3) = 0$$

$$10I_1 - 10I_2 + 12I_3 = 12 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.1005 \text{ A}$$

$$I_2 = 0.197 \text{ A}$$

$$I_3 = 1.081 \text{ A}$$

Current supplied by the 8 V battery = I_1 = 0.1005 A

Current supplied by the 12 V battery = I_3 = 1.081 A

Example 20

Find the value of the unknown resistance R such that 2 A current flows through it.

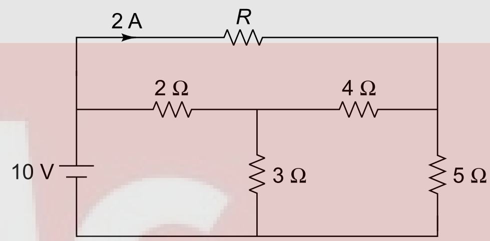


Fig. 2.37

Solution Assigning currents to all the branches,

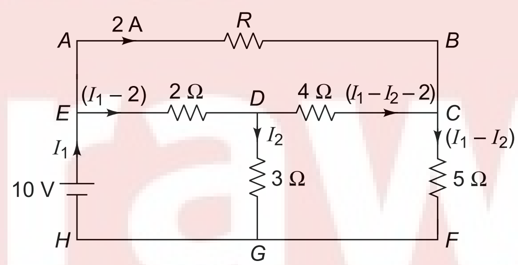


Fig. 2.38

Applying KVL to the closed path *ABCDEA*,

$$-2R + 4(I_1 - I_2 - 2) + 2(I_1 - 2) = 0$$

$$6I_1 - 4I_2 - 2R = 12 \qquad (1)$$

Applying KVL to the closed path *HEDGH*,

$$10 - 2(I_1 - 2) - 3I_2 = 0$$

$$2I_1 + 3I_2 = 14 \qquad (2)$$

Applying KVL to the closed path *GDCFG*,

$$3I_2 - 4(I_1 - I_2 - 2) - 5(I_1 - I_2) = 0$$

$$9I_1 - 12I_2 = 8 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 3.76 \text{ A}$$

$$I_2 = 2.16 \text{ A}$$

$$R = 0.98\ \Omega$$

Unknown resistance $R = 0.98\ \Omega$

Example 21

Find the current delivered by the 12V battery.

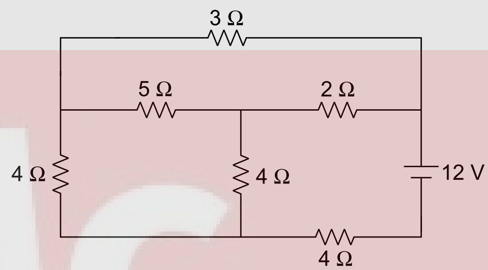


Fig. 2.39

Solution Assigning currents to all the branches,

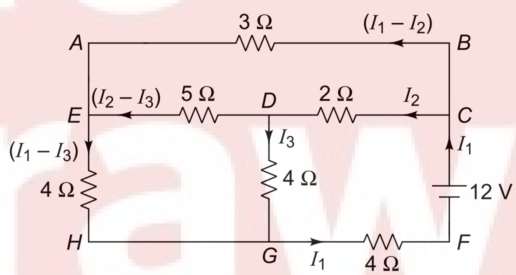


Fig. 2.40

Applying KVL to the closed path *ABCDEA*,

$$3(I_1 - I_2) - 2I_2 - 5(I_2 - I_3) = 0$$

$$3I_1 - 10I_2 + 5I_3 = 0 \qquad (1)$$

Applying KVL to the closed path *HEDGH*,

$$4(I_1 - I_3) + 5(I_2 - I_3) - 4I_3 = 0$$

$$4I_1 + 5I_2 - 13I_3 = 0 \qquad (2)$$

Applying KVL to the closed path *GDCFG*,

$$4I_3 + 2I_2 - 12 + 4I_1 = 0$$

$$4I_1 + 2I_2 + 4I_3 = 12 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.66 \text{ A}$$

$$I_2 = 0.93 \text{ A}$$

$$I_2 = 0.87 \text{ A}$$

Current delivered by the 12 V battery = I_1 = 1.66 A

Exercise 2.1

2.1 Replace the network of sources shown below with

(i) $V_{aa'}$ (ii) $I_{bb'}$

(i)

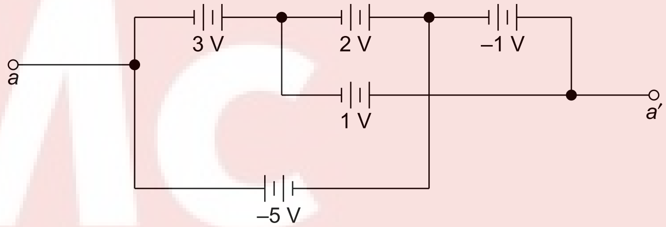


Fig. 2.41

[– 4 V]

(ii)

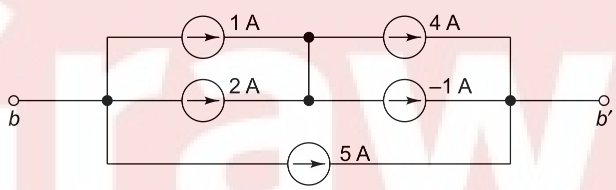


Fig. 2.42

[8 A]

2.2 Find I_x and V_x in the network shown in Fig. 2.43.

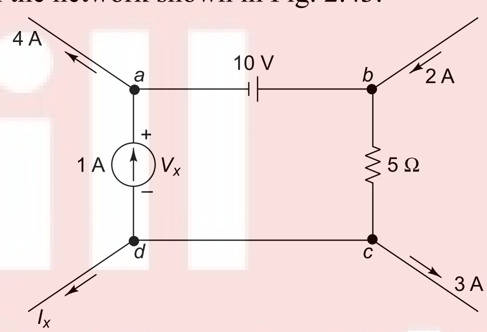


Fig. 2.43

[– 5 A, – 15 V]

2.3 Find V_1 and V_2 in the network shown in Fig. 2.44.

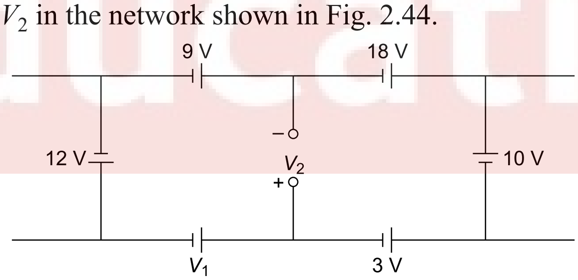


Fig. 2.44

[2 V, 5 V]

2.4 Find the values of unknown currents as shown in Fig. 2.45.

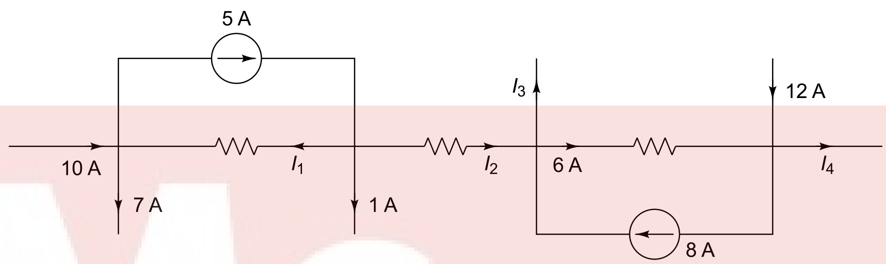


Fig. 2.45

[I_1 = 2 A, I_2 = 2 A, I_3 = 4 A, I_4 = 10 A]

2.5 Find the current in the branch *XY* as shown in Fig. 2.46.

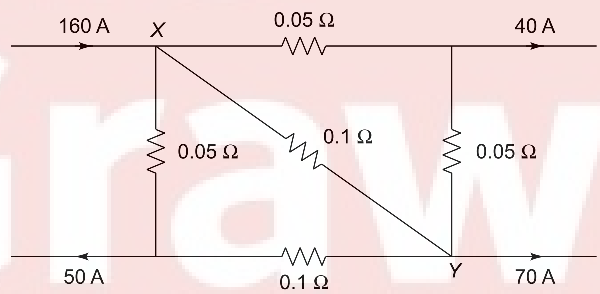


Fig. 2.46

[40 A]

2.6 Find *I* and V_{AB} for the network as shown in Fig. 2.47.

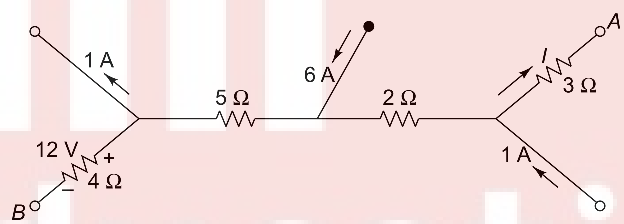


Fig. 2.47

[3 A, 19 V]

2.7 For the network as shown in Fig. 2.48, determine

(i) I_1, I_2 and I_3

(ii) R

(iii) E

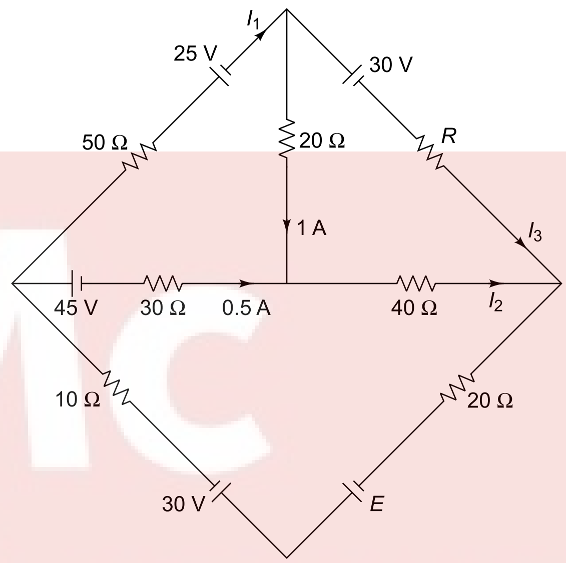


Fig. 2.48

[(i) 0.3 A, 1.5 A, – 0.7 A (ii) 71.43 Ω (iii) 174 V]

2.8 Find the current through the 5 Ω resistor.

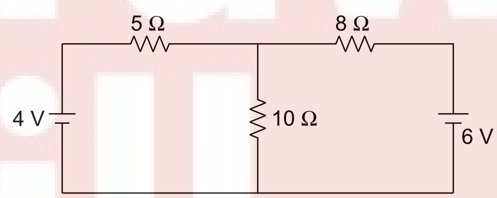


Fig. 2.49

[0.071 A]

2.9 Find the current in the 4 Ω resistor.

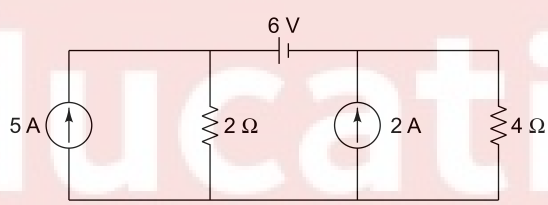


Fig. 2.50

[1.34 A]

2.10 Find the current I_1.

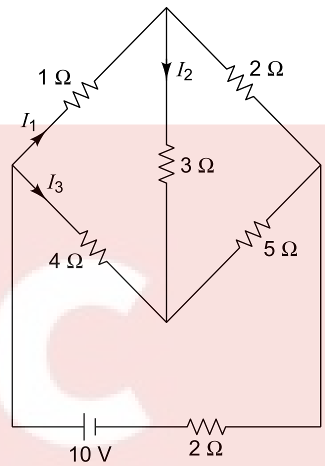


Fig. 2.51

[1.83 A]

2.11 In the network shown in Fig. 2.52, determine the value of E_2 which will reduce the galvanometer current to zero.

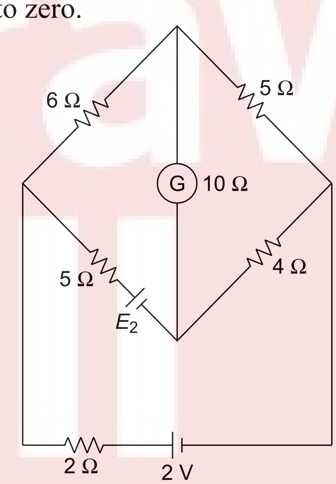


Fig. 2.52

[– 0.0322 V]

2.12 Determine the current supplied by the battery.

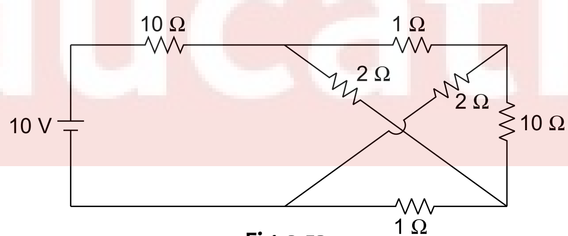


Fig. 2.53

[0.87 A]

2.13 In the network shown in Fig. 2.54, find the voltage between points A and B.

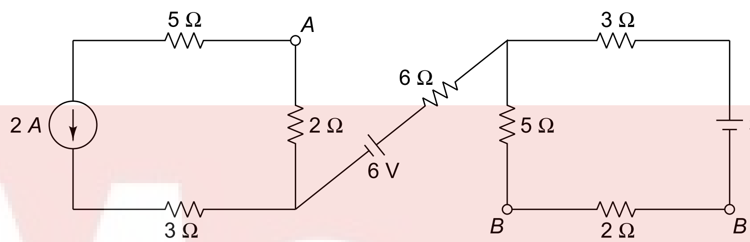


Fig. 2.54

[5 V]

2.14 In the network shown in Fig. 2.55, find the voltage between points A and B.

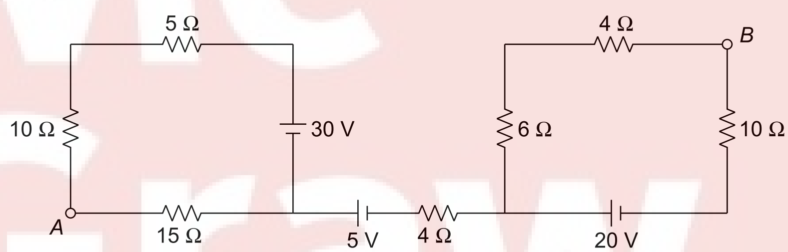


Fig. 2.55

[30 V]

2.15 Using KVL and KCL, find the values of V and I in Fig. 2.56.

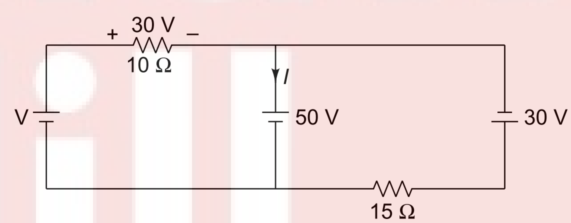


Fig. 2.56

[80 V, – 2.33 A]

2.16 Using KCL, find the values of V_{AB}, I_1, I_2 and I_3 in Fig. 2.57.

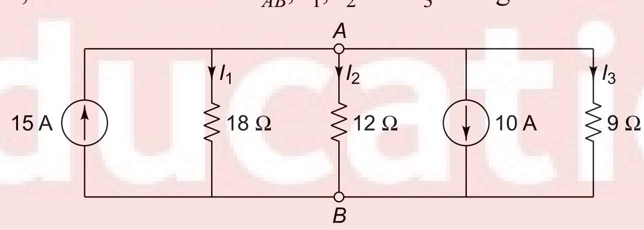


Fig. 2.57

[20 V, 1.11 A, 1.67 A, 2.22 A]

2.2 MESH ANALYSIS

A mesh is defined as a loop which does not contain any other loops within it. Mesh analysis is applicable only for planar networks. A network is said to be planar if it can be drawn on a plane surface without crossovers. In this method, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. If a network has a large number of voltage sources, it is useful to use mesh analysis. Basically, this analysis consists of writing mesh equations by Kirchhoff's voltage law in terms of unknown mesh currents.

2.2.1 Steps to be Followed in Mesh Analysis

1. Identify the mesh, assign a direction to it and assign an unknown current in each mesh.
2. Assign the polarities for voltage across the branches.
3. Apply KVL around the mesh and use Ohm's law to express the branch voltages in terms of unknown mesh currents and the resistance.
4. Solve the simultaneous equations for unknown mesh currents.

Consider the network shown in Fig. 2.58 which has three meshes. Let the mesh currents for the three meshes be I_1, I_2 and I_3 and all the three mesh currents may be assumed to flow in the clockwise direction. The choice of direction for any mesh current is arbitrary.

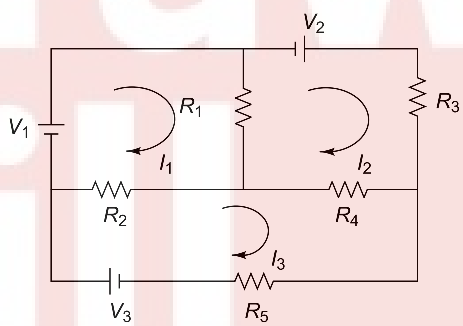


Fig. 2.58 *Mesh analysis*

Applying KVL to Mesh 1,

$$V_1 - R_1 (I_1 - I_2) - R_2 (I_1 - I_3) = 0$$

$$(R_1 + R_2) I_1 - R_1 I_2 - R_2 I_3 = V_1 \tag{2.1}$$

Applying KVL to Mesh 2,

$$V_2 - R_3 I_2 - R_4 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$$

$$-R_1 I_1 + (R_1 + R_3 + R_4) I_2 - R_4 I_3 = V_2 \tag{2.2}$$

Applying KVL to Mesh 3,

$$-R_2(I_3 - I_1) - R_4 (I_3 - I_2) - R_5 I_3 + V_3 = 0$$

$$-R_2 I_1 - R_4 I_2 + (R_2 + R_4 + R_5) I_3 = V_3 \tag{2.3}$$

Writing Eqs (2.1), (2.2) and (2.3) in matrix form,

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 \\ -R_1 & R_1+R_3+R_4 & -R_4 \\ -R_2 & -R_4 & R_2+R_4+R_5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

In general,

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where,

R_{11} = Self-resistance or sum of all the resistances of Mesh 1

$R_{12} = R_{21}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 2

$R_{13} = R_{31}$ = Mutual resistance or sum of all the resistances common to meshes 1 and 3

R_{22} = Self-resistance or sum of all the resistances of Mesh 2

$R_{23} = R_{32}$ = Mutual resistance or sum of all the resistances common to meshes 2 and 3

R_{33} = Self-resistance or sum of all the resistances of Mesh 3

If the directions of the currents passing through the common resistor are the same, the mutual resistance will have a positive sign, and if the direction of the currents passing through common resistor are opposite, then the mutual resistance will have a negative sign. If each mesh currents are assumed to flow in the clockwise direction, then all self-resistances will be always positive and all mutual resistances will always be negative.

The voltages V_1, V_2 and V_3 represent the algebraic sum of all the voltages in meshes 1, 2 and 3 respectively. While going along the current, if we go from negative terminal of the battery to the positive terminal, then its emf is taken as positive. Otherwise, it is taken as negative.

Example 1

Find the current through 5 Ω resistor.

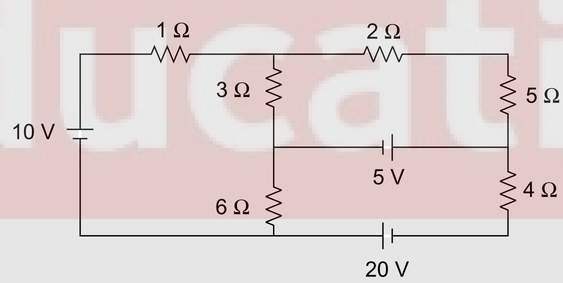


Fig. 2.59

Solution Assigning clockwise currents in three meshes,

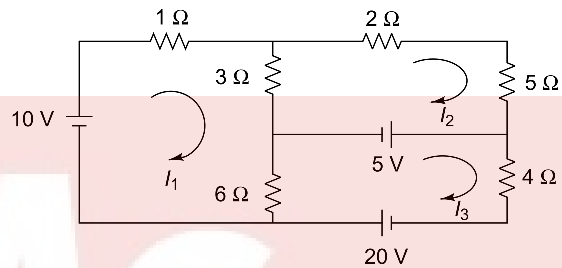


Fig. 2.60

Applying KVL to Mesh 1,

$$10 - 1(I_1) - 3(I_1 - I_2) - 6(I_1 - I_3) = 0$$

$$10I_1 - 3I_2 - 6I_3 = 10 \quad (1)$$

Applying KVL to Mesh 2,

$$-3\,(I_2 - I_1) - 2I_2 - 5I_2 - 5 = 0$$

$$-3I_1 + 10I_2 = -5 \quad (2)$$

Applying KVL to Mesh 3,

$$-6\,(I_3 - I_1) + 5 - 4I_3 + 20 = 0$$

$$-6I_1 + 10I_3 = 25 \quad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 10 & -3 & -6 \\ -3 & 10 & 0 \\ -6 & 0 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 25 \end{bmatrix}$$

We can write matrix equation directly from Fig. 2.60.

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

where R_{11} = Self-resistance of Mesh 1 = 1 + 3 + 6 = 10 Ω

R_{12} = Mutual resistance common to meshes 1 and 2 = –3 Ω

Here, negative sign indicates that the currents through common resistance are in opposite direction.

R_{13} = Mutual resistance common to meshes 1 and 3 = –6 Ω

Similarly,

R_{21} = –3 Ω

R_{22} = 3 + 2 + 5 = 10 Ω

R_{23} = 0

$$R_{31} = -6\ \Omega$$
$$R_{32} = 0$$
$$R_{33} = 6 + 4 = 10\ \Omega$$

For voltage matrix,

$$V_1 = 10\text{ V}$$
$$V_2 = -5\text{ V}$$
$$V_3 = \text{algebraic sum of all the voltages in Mesh 3} = 5 + 20 = 25\text{ V}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 4.27\text{ A}$$
$$I_2 = 0.78\text{ A}$$
$$I_3 = 5.06\text{ A}$$
$$I_{5\Omega} = I_2 = 0.78\text{ A}$$

Example 2

Find the current through the 2 Ω resistor.

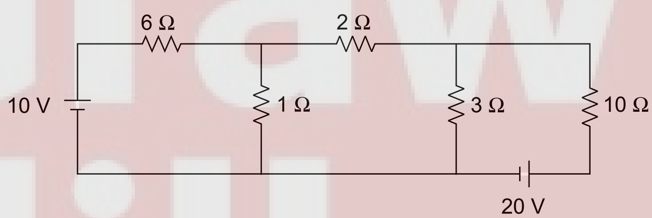


Fig. 2.61

Solution Assigning clockwise currents in the three meshes,

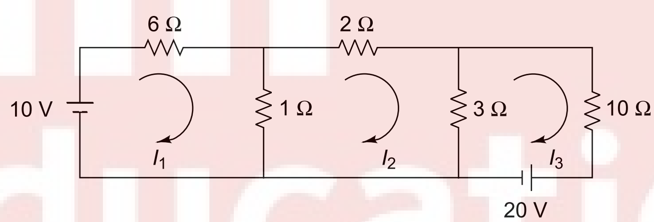


Fig. 2.62

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$
$$7I_1 - I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$
$$-I_1 + 6I_2 - 3I_3 = 0 \qquad (2)$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_2) - 10I_3 - 20 = 0$$

$$-3I_2 + 13I_3 = -20 \qquad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 7 & -1 & 0 \\ -1 & 6 & -3 \\ 0 & -3 & 13 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -20 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 1.34 \text{ A}$$

$$I_2 = -0.62 \text{ A}$$

$$I_3 = -1.68 \text{ A}$$

$$I_{2\Omega} = I_2 = -0.62 \text{ A}$$

Example 3

Determine the current through the 5 Ω resistor.

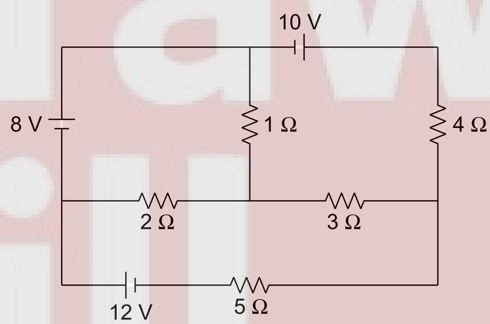


Fig. 2.63

Solution Assigning clockwise currents in the three meshes,

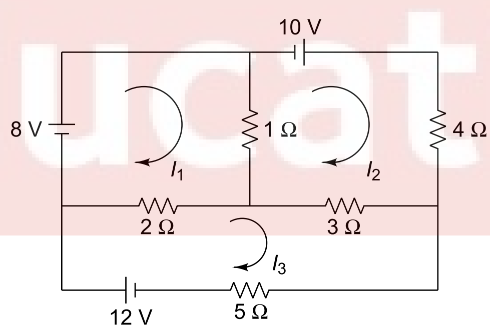


Fig. 2.64

Applying KVL to Mesh 1,

$$8 - 1(I_1 - I_2) - 2(I_1 - I_3) = 0$$

$$3I_1 - I_2 - 2I_3 = 8 \quad (1)$$

Applying KVL to Mesh 2,

$$10 - 4I_2 - 3(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 3I_3 = 10 \quad (2)$$

Applying KVL to Mesh 3,

$$-2(I_3 - I_1) - 3(I_3 - I_2) - 5I_3 + 12 = 0$$

$$-2I_1 - 3I_2 + 10I_3 = 12 \quad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 8 & -3 \\ -2 & -3 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 6.01 \text{ A}$$

$$I_2 = 3.27 \text{ A}$$

$$I_3 = 3.38 \text{ A}$$

$$I_{5\Omega} = I_3 = 3.38 \text{ A}$$

Example 4

Find the current supplied by the battery.

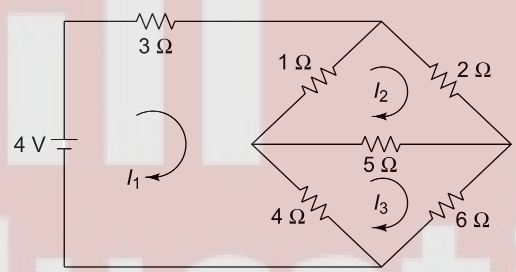


Fig. 2.65

Solution

Applying KVL to Mesh 1,

$$4 - 3I_1 - 1(I_1 - I_2) - 4(I_1 - I_3) = 0$$

$$8I_1 - I_2 - 4I_3 = 4 \quad (1)$$

Applying KVL to Mesh 2,

$$-2I_2 - 5(I_2 - I_3) - 1(I_2 - I_1) = 0$$

$$-I_1 + 8I_2 - 5I_3 = 0 \quad (2)$$

Applying KVL to Mesh 3,

$$-6I_3 - 4(I_3 - I_1) - 5(I_3 - I_2) = 0$$

$$-4I_1 - 5I_2 + 15I_3 = 0 \quad (3)$$

Writing equations in matrix form,

$$\begin{bmatrix} 8 & -1 & -4 \\ -1 & 8 & -5 \\ -4 & -5 & 15 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.66 \text{ A}$$
$$I_2 = 0.24 \text{ A}$$
$$I_3 = 0.26 \text{ A}$$

Current supplied by the battery = I_1 = 0.66 A.

Example 5

Determine the voltage V which cause the current I_1 to be zero.

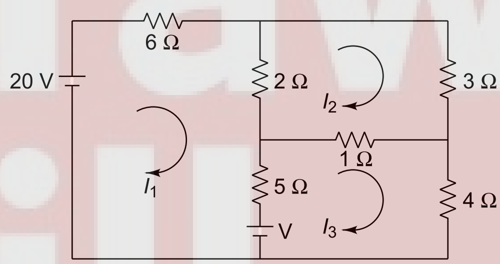


Fig. 2.66

Solution

Applying KVL to Mesh 1,

$$20 - 6I_1 - 2(I_1 - I_2) - 5(I_1 - I_3) - V = 0$$

$$V + 13I_1 - 2I_2 - 5I_3 = 20 \quad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 3I_2 - 1(I_2 - I_3) = 0$$

$$2I_1 - 6I_2 + I_3 = 0 \quad (2)$$

Applying KVL to Mesh 3,

$$-1(I_3 - I_2) - 4I_3 + V - 5(I_3 - I_1) = 0$$

$$V + 5I_1 + I_2 - 10I_3 = 0 \quad (3)$$

Putting $I_1 = 0$ in Eqs (1), (2) and (3),

$$V - 2I_2 - 5I_3 = 20$$
$$-6I_2 + I_3 = 0$$
$$V + I_2 - 10I_3 = 0$$

Writing equations in matrix form,

$$\begin{bmatrix} 1 & -2 & -5 \\ 0 & -6 & 1 \\ 1 & 1 & -10 \end{bmatrix} \begin{bmatrix} V \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

Solving Eqs (1), (2), and (3),

$$V = 43.7 \text{ V}$$

Example 6

Find the current through the 2 Ω resistor.

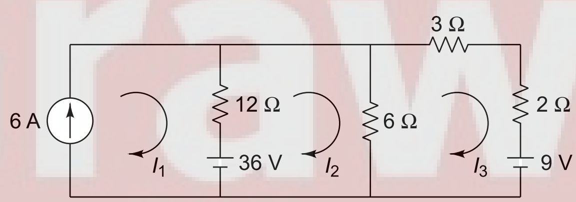


Fig. 2.67

Solution Mesh 1 contains a current source of 6 A. Hence, we can write current equation for Mesh 1. Since direction of the current source and the mesh current I_1 are same,

$$I_1 = 6 \tag{1}$$

Applying KVL to Mesh 2,

$$36 - 12(I_2 - I_1) - 6(I_2 - I_3) = 0$$
$$-12I_1 + 18I_2 - 6I_3 = 36 \tag{2}$$

Applying KVL to Mesh 3,

$$-6(I_3 - I_2) - 3I_3 - 2I_3 - 9 = 0$$
$$-6I_2 + 11I_3 = -9 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_2 = 7 \text{ A}$$
$$I_3 = 3 \text{ A}$$
$$I_{2\Omega} = I_3 = 3 \text{ A}$$

Exercise 2.2

2.1 Find the current through the 10 Ω resistor.

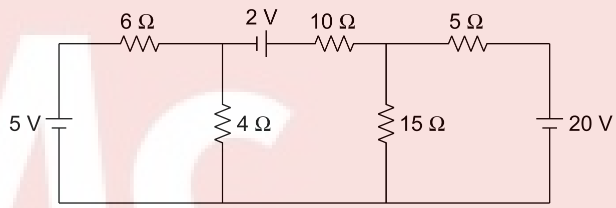


Fig. 2.68

[0.68 A]

2.2 Find the current through the 20 Ω resistor.

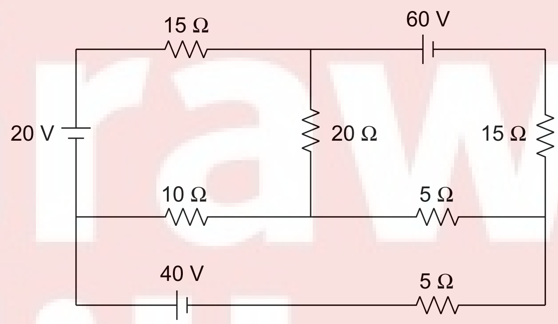


Fig. 2.69

[1.46 A]

2.3 Find the mesh currents.

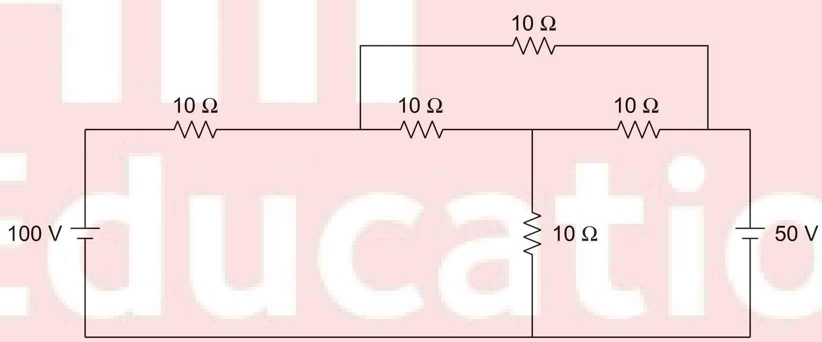


Fig. 2.70

[3.75 A, 0, 1.25 A]

2.4 Calculate the current through the 10 Ω resistor.

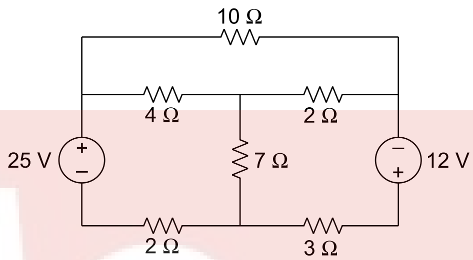


Fig. 2.71

[1.62 A]

2.5 Find the current through 10 Ω resistor.

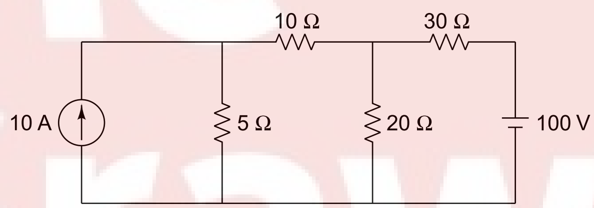


Fig. 2.72

[0.37 A]

2.6 Find the current through the 2 Ω resistor.

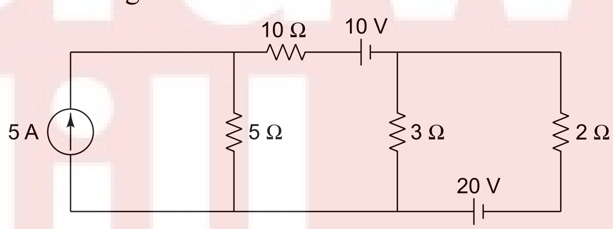


Fig. 2.73

[5 A]

2.7 Find the current through the 20 Ω resistor.

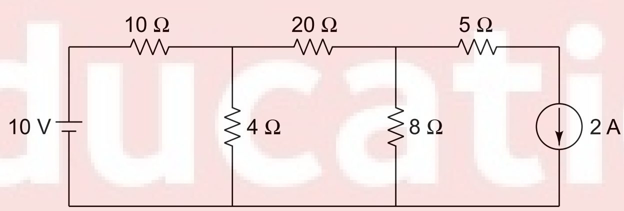


Fig. 2.74

[0.61 A]

2.8 Find the current through the 8 Ω resistor.

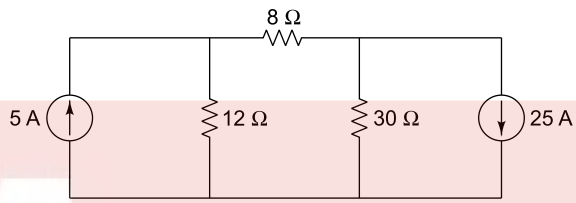


Fig. 2.75

2.3 NODAL ANALYSIS

Nodal analysis is based on Kirchhoff's current law which states that the algebraic sum of currents meeting at a point is zero. Every junction where two or more branches meet is regarded as a node. One of the nodes in the network is taken as *reference node* or *datum node*. If there are n nodes in any network, the number of simultaneous equations to be solved will be $(n - 1)$.

2.3.1 Steps to be followed in Nodal Analysis

1. Assuming that a network has n nodes, assign a reference node and the reference directions, and assign a current and a voltage name for each branch and node respectively.
2. Apply KCL at each node except for the reference node and apply Ohm's law to the branch currents.
3. Solve the simultaneous equations for the unknown node voltages.
4. Using these voltages, find any branch currents required.

Example 1

Calculate the current through 2 Ω resistor for the network shown in Fig. 2.76.

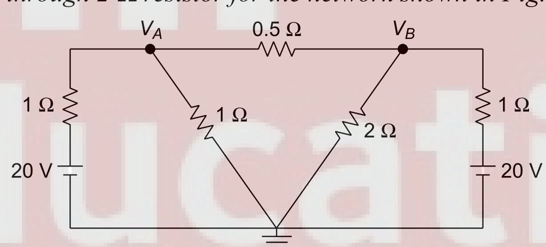


Fig. 2.76

Solution Assume that the currents are moving away from the nodes.

Applying KCL at node A,

$$\frac{V_A - 20}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$\left(\frac{1}{1}+\frac{1}{1}+\frac{1}{0.5}\right)V_A-\frac{1}{0.5}V_B = \frac{20}{1}$$

$$4V_A-2V_B = 20 \tag{1}$$

Applying KCL at node B,

$$\frac{V_B-V_A}{0.5}+\frac{V_B}{2}+\frac{V_B-20}{1} = 0$$

$$-\frac{1}{0.5}V_A+\left(\frac{1}{0.5}+\frac{1}{2}+\frac{1}{1}\right)V_B = \frac{20}{1}$$

$$-2V_A+3.5V_B = 20 \tag{2}$$

Solving Eqs (1) and (2),

$$V_A = 11\text{ V}$$

$$V_B = 12\text{ V}$$

$$\text{Current through } 2\ \Omega \text{ resistor} = \frac{V_B}{2} = \frac{12}{2} = 6\text{ A}$$

Example 2

Find the voltage at nodes 1 and 2.

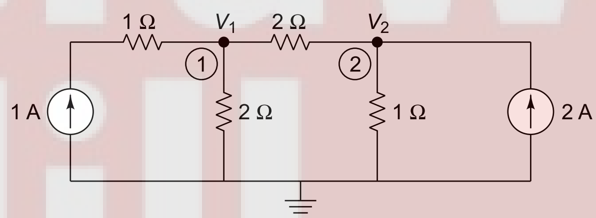


Fig. 2.77

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$1 = \frac{V_1}{2}+\frac{V_1-V_2}{2}$$

$$\left(\frac{1}{2}+\frac{1}{2}\right)V_1-\frac{1}{2}V_2 = 1$$

$$V_1-0.5\,V_2 = 1 \tag{1}$$

Applying KCL at Node 2,

$$2 = \frac{V_2}{1}+\frac{V_2-V_1}{2}$$

$$-\frac{1}{2}V_1+\left(\frac{1}{1}+\frac{1}{2}\right)V_2 = 2$$

$$-0.5\ V_1 + 1.5\ V_2 = 4 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 2\text{ V}$$
$$V_2 = 2\text{ V}$$

Example 3

Find the current in the 100 Ω resistor.

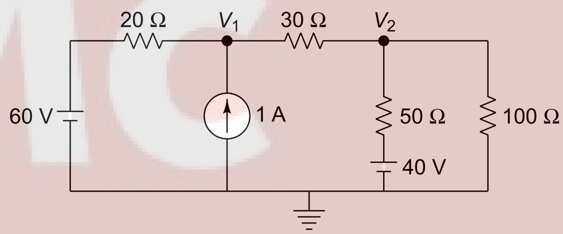


Fig. 2.78

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1 - 60}{20} + \frac{V_1 - V_2}{30} = 1$$

$$\left(\frac{1}{20} + \frac{1}{30}\right)V_1 - \frac{1}{30}V_2 = \frac{60}{20} + 1$$

$$0.083\ V_1 - 0.033\ V_2 = 4 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{30} + \frac{V_2 - 40}{50} + \frac{V_2}{100} = 0$$

$$-\frac{1}{30}V_1 + \left(\frac{1}{30} + \frac{1}{50} + \frac{1}{100}\right)V_2 = \frac{40}{50}$$

$$-0.033\ V_1 + 0.063\ V_2 = 0.8 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 67.25\text{ V}$$
$$V_2 = 48\text{ V}$$

$$\text{Current through the 100 Ω resistor} = \frac{V_2}{100} = \frac{48}{100} = 0.48\text{ A}$$

Example 4

Find V_A and V_B.

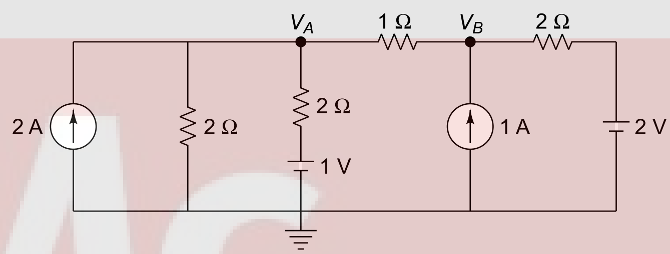


Fig. 2.79

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node A,

$$2 = \frac{V_A}{2} + \frac{V_A - 1}{2} + \frac{V_A - V_B}{1}$$

$$\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{1}\right)V_A - \left(\frac{1}{1}\right)V_B = 2 + \frac{1}{2}$$

$$2V_A - V_B = 2.5 \qquad (1)$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{1} + \frac{V_B - 2}{2} = 1$$

$$-\left(\frac{1}{1}\right)V_A + \left(\frac{1}{1} + \frac{1}{2}\right)V_B = 1 + \frac{2}{2}$$

$$-V_A + 1.5\,V_B = 2 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_A = 2.875 \text{ V}$$

$$V_B = 3.25 \text{ V}$$

Example 5

Find currents I_1, I_2 and I_3.

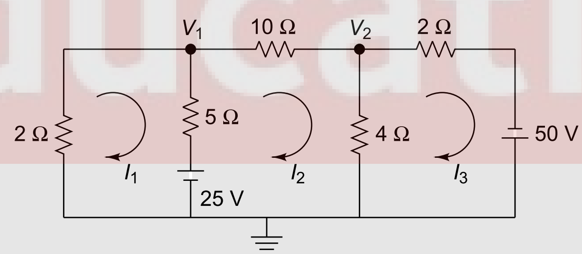


Fig. 2.80

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1}{2}+\frac{V_1-25}{5}+\frac{V_1-V_2}{10}=0$$

$$\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)V_1-\frac{1}{10}V_2=\frac{25}{5}$$

$$0.8\ V_1-0.1\ V_2=5 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{10}+\frac{V_2}{4}+\frac{V_2-(-50)}{2}=0$$

$$-\frac{1}{10}V_1+\left(\frac{1}{10}+\frac{1}{4}+\frac{1}{2}\right)V_2=-\frac{50}{2}$$

$$-0.1\ V_1+0.85\ V_2=-25 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1=2.61\text{ V}$$

$$V_2=-29.1\text{ V}$$

$$I_1=-\frac{V_1}{2}=\frac{-2.61}{2}=-1.31\text{ A}$$

$$I_2=\frac{V_1-V_2}{10}=\frac{2.61-(-29.1)}{10}=3.17\text{ A}$$

$$I_3=\frac{V_2+50}{2}=\frac{-29.1+50}{2}=10.45\text{ A}$$

Example 6

Find currents I_1, I_2 and I_3 and voltages V_a and V_b.

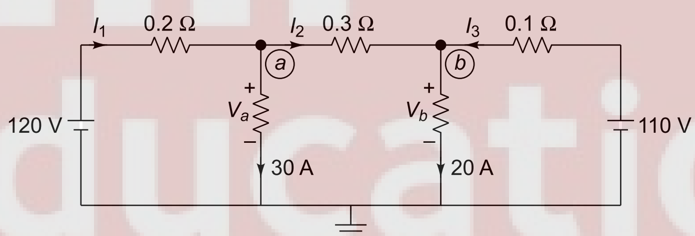


Fig. 2.81

Solution Applying KCL at Node a,

$$I_1=30+I_2$$

$$\frac{120-V_a}{0.2}=30+\frac{V_a-V_b}{0.3}$$

$$36 - 0.3V_a = 1.8 + 0.2V_a - 0.2V_b$$

$$0.5V_a - 0.2V_b = 34.2 \qquad (1)$$

Applying KCL at Node b,

$$I_2 + I_3 = 20$$

$$\frac{V_a - V_b}{0.3} + \frac{110 - V_b}{0.1} = 20$$

$$\frac{0.1V_a - 0.1V_b + 33 - 0.3V_b}{0.03} = 20$$

$$0.1V_a - 0.4V_b = -32.4 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_a = 112 \text{ V}$$

$$V_b = 109 \text{ V}$$

$$I_1 = \frac{120 - V_a}{0.2} = \frac{120 - 112}{0.2} = 40 \text{ A}$$

$$I_2 = \frac{V_a - V_b}{0.3} = \frac{112 - 109}{0.3} = 10 \text{ A}$$

$$I_3 = \frac{110 - V_b}{0.1} = \frac{110 - 109}{0.1} = 10 \text{ A}$$

Example 7

Calculate the current through the 5 Ω resistor.

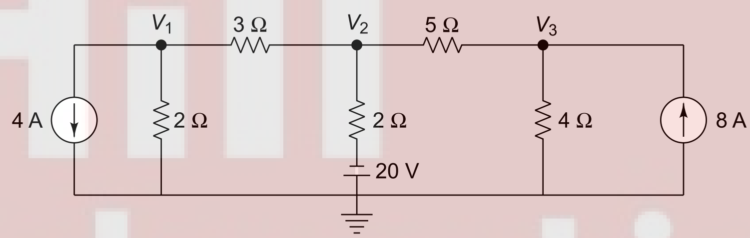


Fig. 2.82

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$4 + \frac{V_1}{2} + \frac{V_1 - V_2}{3} = 0$$

$$\left(\frac{1}{2} + \frac{1}{3}\right)V_1 - \frac{1}{3}V_2 = 4$$

$$0.83\ V_1 - 0.33\ V_2 = -4 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{3} + \frac{V_2 - (-20)}{2} + \frac{V_2 - V_3}{5} = 0$$

$$-\frac{1}{3}V_1 + \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{5}\right)V_2 - \frac{1}{5}V_3 = -\frac{20}{2}$$

$$-0.33\ V_1 + 1.03\ V_2 - 0.2\ V_3 = -10 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3 - V_2}{5} + \frac{V_3}{4} = 8$$

$$-\frac{1}{5}V_2 + \left(\frac{1}{5} + \frac{1}{4}\right)V_3 = 8$$

$$-0.2\ V_2 + 0.45\ V_3 = 8 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = -8.76\text{ V}$$
$$V_2 = -9.92\text{ V}$$
$$V_3 = 13.37\text{ V}$$

Current through the 5 Ω resistor $= \dfrac{V_3 - V_2}{5} = \dfrac{13.37 - (-9.92)}{5} = 4.66\text{ A}$

Example 8

Find the voltage across the 5 Ω resistor.

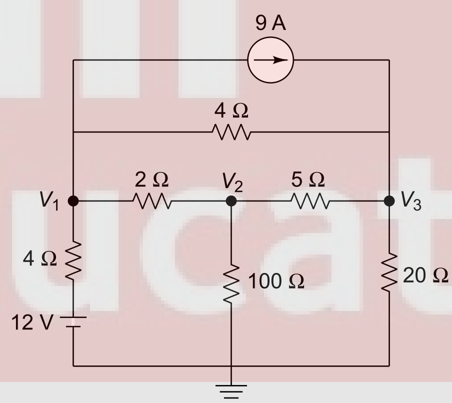


Fig. 2.83

Solution Assume that the currents are moving away from the node.

Applying KCL at Node 1,

$$\frac{V_1-12}{4}+\frac{V_1-V_2}{2}+\frac{V_1-V_3}{4}+9 = 0$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2-\frac{1}{4}V_3=-9+\frac{12}{4}$$

$$V_1-0.5\ V_2-0.25\ V_3 = -6 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{2}+\frac{V_2}{100}+\frac{V_2-V_3}{5} = 0$$

$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=0$$

$$-0.5\ V_1+0.71\ V_2-0.2\ V_3 = 0 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{20}+\frac{V_3-V_1}{4} = 9$$

$$-\frac{1}{4}V_1-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_3 = 9$$

$$-0.25\ V_1-0.2\ V_2+0.5\ V_3 = 9 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 6.35\text{ V}$$

$$V_2 = 11.76\text{ V}$$

$$V_3 = 25.88\text{ V}$$

Voltage across the 5 Ω resistor = $V_3 - V_2$ = 25.88 – 11.76 = 14.12 V

Example 9

Determine the current through the 5 Ω resistor.

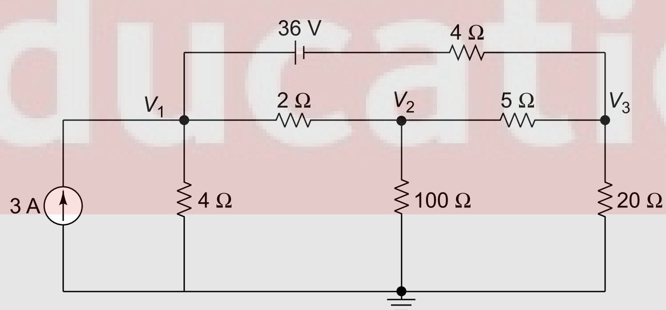


Fig. 2.84

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1}{4}+\frac{V_1-V_2}{2}+\frac{V_1-36-V_3}{4}=3$$

$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{4}\right)V_1-\frac{1}{2}V_2-\frac{1}{4}V_3=3+\frac{36}{4}$$

$$V_1-0.5\,V_2-0.25\,V_3=12 \quad (1)$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{2}+\frac{V_2}{100}+\frac{V_2-V_3}{5}=0$$

$$-\frac{1}{2}V_1+\left(\frac{1}{2}+\frac{1}{100}+\frac{1}{5}\right)V_2-\frac{1}{5}V_3=0$$

$$-0.5V_1+0.71\,V_2-0.2\,V_3=0 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{20}+\frac{V_3-(-36)-V_1}{4}=0$$

$$-\frac{1}{4}V_1-\frac{1}{5}V_2+\left(\frac{1}{5}+\frac{1}{20}+\frac{1}{4}\right)V_3=-9$$

$$-0.25\,V_1-0.2\,V_2+0.5\,V_3=-9 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1=13.41\text{ V}$$

$$V_2=7.06\text{ V}$$

$$V_3=-8.47\text{ V}$$

Current through the 5 Ω resistor $=\dfrac{V_2-V_3}{5}=\dfrac{7.06-(-8.47)}{5}=3.11\text{ A}$

Example 10

Find V_1 and V_2.

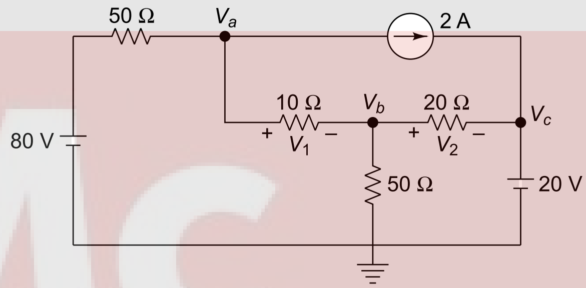


Fig. 2.85

Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node *a*,

$$\frac{V_a - 80}{50} + \frac{V_a - V_b}{10} + 2 = 0$$

$$\left(\frac{1}{50} + \frac{1}{10}\right)V_a - \frac{1}{10}V_b = \frac{80}{50} - 2$$

$$0.12\ V_a - 0.1\ V_b = -0.4 \tag{1}$$

Applying KCL at Node *b*,

$$\frac{V_b - V_a}{10} + \frac{V_b}{50} + \frac{V_b - V_c}{20} = 0$$

$$-\frac{1}{10}V_a + \left(\frac{1}{10} + \frac{1}{50} + \frac{1}{20}\right)V_b - \frac{1}{20}V_c = 0$$

$$-0.1\ V_a + 0.17\ V_b - 0.05\ V_c = 0 \tag{2}$$

Node *c* is directly connected to a voltage source of 20 V. Hence, we can write voltage equation at Node *c*.

$$V_c = 20 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$V_a = 3.08\ \text{V}$$

$$V_b = 7.69\ \text{V}$$

$$V_1 = V_a - V_b = 3.08 - 7.69 = -4.61\ \text{V}$$

$$V_2 = V_b - V_c = 7.69 - 20 = -12.31\ \text{V}$$

Example 11

Find the voltage across the 100 Ω resistor.

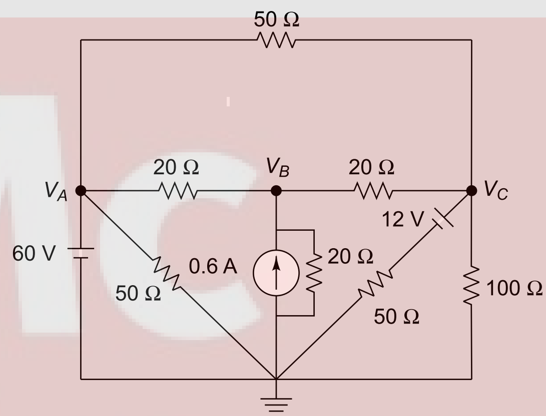


Fig. 2.86

Solution Node A is directly connected to a voltage source of 20 V. Hence, we can write voltage equation at Node A.

$$V_A = 60 \quad (1)$$

Assume that the currents are moving away from the nodes.

Applying KCL at Node B,

$$\frac{V_B - V_A}{20} + \frac{V_B - V_C}{20} + \frac{V_B}{20} = 0.6$$

$$-\frac{1}{20} V_A + \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{20}\right) V_B - \frac{1}{20} V_C = 0.6$$

$$-0.05\ V_A + 0.15\ V_B - 0.05\ V_C = 0.6 \quad (2)$$

Applying KCL at Node C,

$$\frac{V_C - V_A}{50} + \frac{V_C - V_B}{20} + \frac{V_C - 12}{50} + \frac{V_C}{100} = 0$$

$$-\frac{1}{50} V_A - \frac{1}{20} V_B + \left(\frac{1}{50} + \frac{1}{20} + \frac{1}{50} + \frac{1}{100}\right) V_C = \frac{12}{50}$$

$$-0.02\ V_A - 0.05\ V_B + 0.1\ V_C = 0.24 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$V_C = 31.68 \text{ V}$$

$$\text{Voltage across the } 100\ \Omega \text{ resistor} = 31.68 \text{ V}$$

Exercise 2.3

2.1 Find the current I_x.

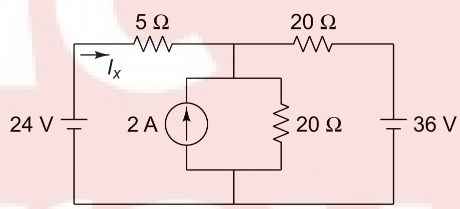


Fig. 2.87

[–0.93 A]

2.2 Find V_A and V_B.

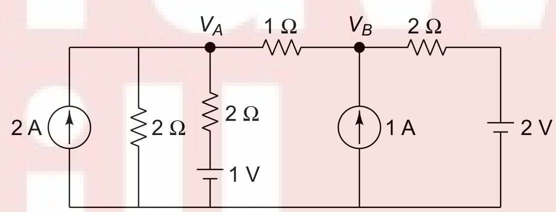


Fig. 2.88

[2.88 V, 3.25 V]

2.3 Find the current through the 6 Ω resistor.

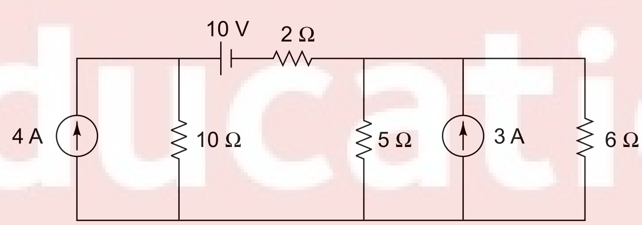


Fig. 2.89

[2.04 A]

2.4 Calculate the current through the 10 Ω resistor.

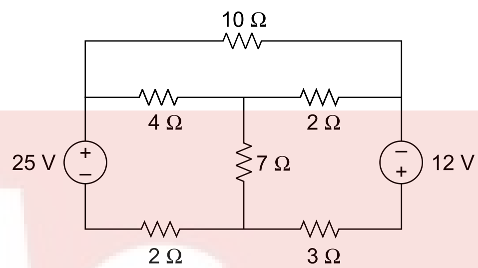


Fig. 2.90

[1.62 A]

2.5 Find the current through the branch *ab*.

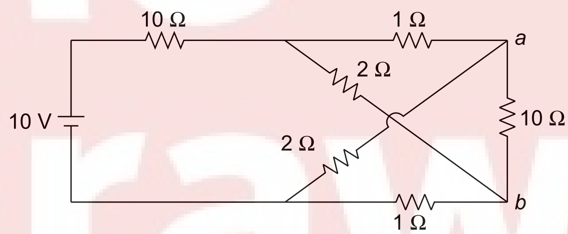


Fig. 2.91

[0.038 A]

2.6 Find the current through the 4 Ω resistor.

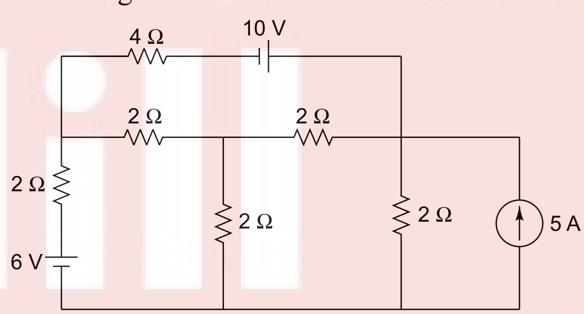


Fig. 2.92

[1.34 A]

2.7 Find the current through the 40 Ω resistor.

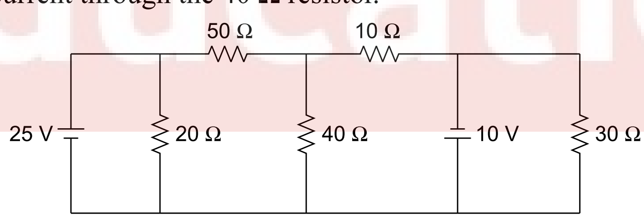


Fig. 2.93

[0.0862 A]

2.8 Find the current through the 10 Ω resistor.

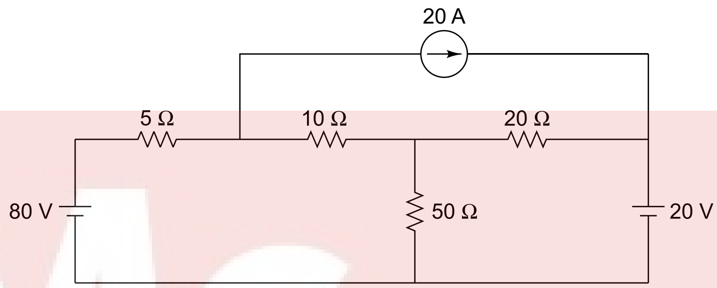


Fig. 2.94

[1.17 A]

2.4 SOURCE TRANSFORMATION

A voltage source with a series resistor can be converted into an equivalent current source with a parallel resistor. Conversely, a current source with a parallel resistor can be converted into a voltage source with a series resistor.

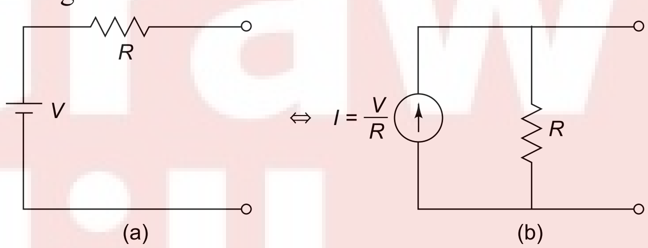


Fig. 2.95 *Source transformation*

Example 1

Replace the given network with a single current source and a resistor.

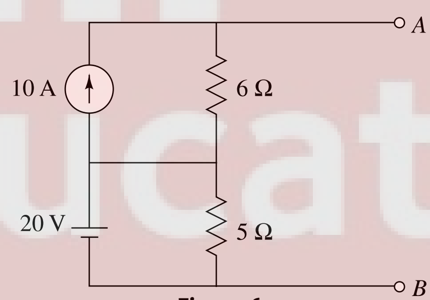


Fig. 2.96

Solution Since the resistor of 5 Ω is connected in parallel with the voltage source of 20 V it becomes redundant. Converting parallel combination of current source and resistor into equivalent voltage source and resistor,

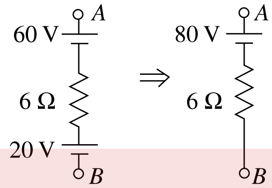


Fig. 2.97

By source transformation,

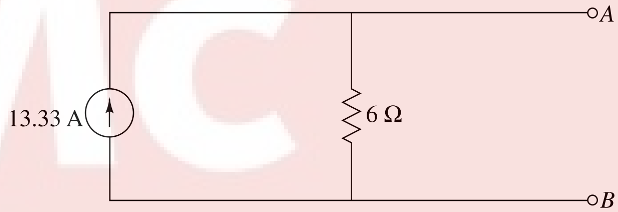


Fig. 2.98

Example 2

Reduce network shown into a single source and a single resistor between terminals A and B.

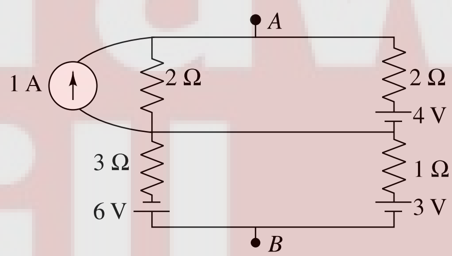


Fig. 2.99

Solution Converting all voltage sources into equivalent current sources,

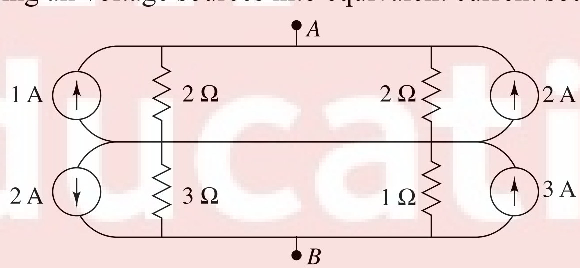


Fig. 2.100

Adding the current sources and simplifying the network,

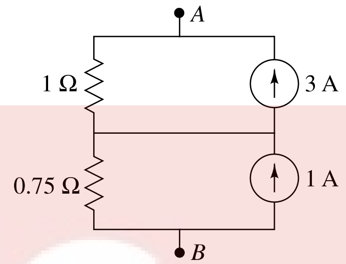


Fig. 2.101

Converting the current sources into equivalent voltage sources,

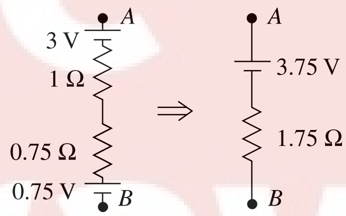


Fig. 2.102

Example 3

Replace the circuit between A and B with a voltage source in series with a single resistor.

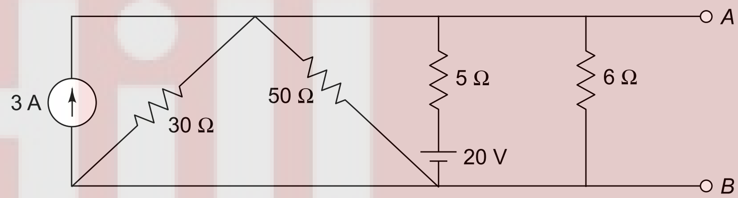


Fig. 2.103

Solution Converting the series combination of voltage source of 20 V and a resistor of 5 Ω into equivalent parallel combination of current source and resistor,

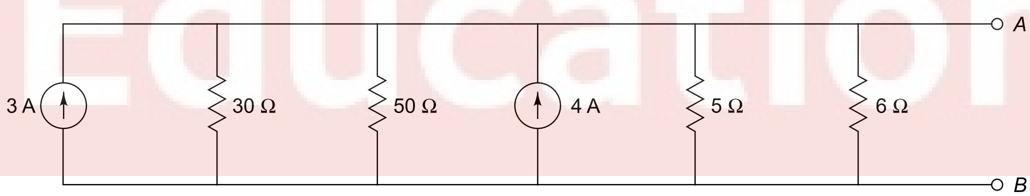


Fig. 2.104

Adding the two current sources and simplifying the circuit,

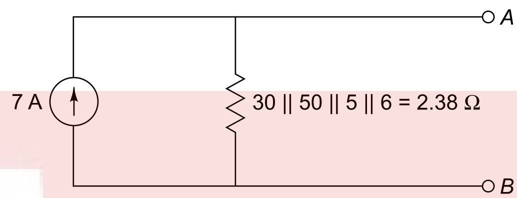


Fig. 2.105

By source transformation,

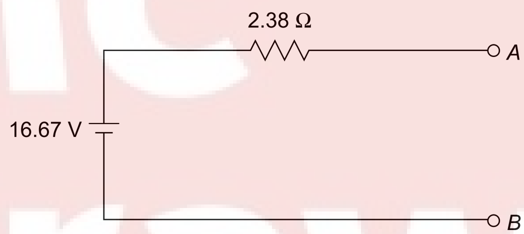


Fig. 2.106

Example 4

Find the power delivered by the 50 V source in the circuit.

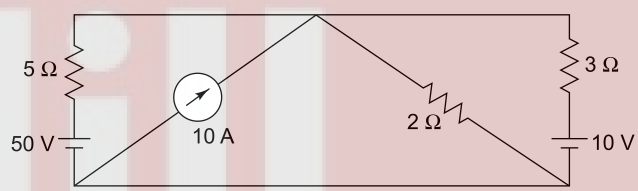


Fig. 2.107

Solution Converting the series combination of voltage source of 10 V and resistor of 3 Ω into equivalent current source and resistor,

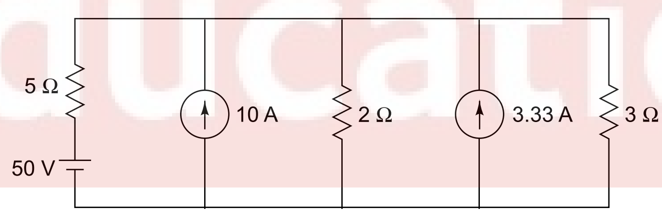


Fig. 2.108

Adding the two current sources and simplifying the circuit,

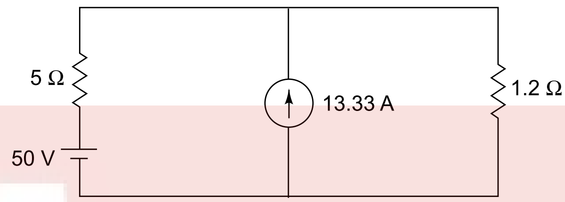


Fig. 2.109

By source transformation,

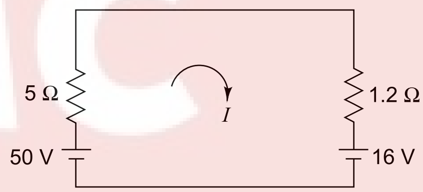


Fig. 2.110

Applying KVL to the circuit,

$$50 - 5I - 1.2I - 16 = 0$$

$$I = 5.48 \text{ A}$$

$$\text{Power delivered by the 50 V source} = 50 \times 5.48 = 274 \text{ W}$$

Example 5

Find the current in the 4 Ω resistor.

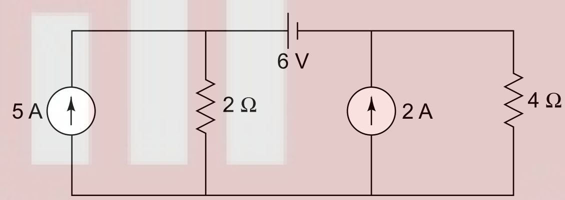


Fig. 2.111

Solution Converting the parallel combination of the current source of 5 A and the resistor of 2 Ω into an equivalent series combination of voltage source and resistor,

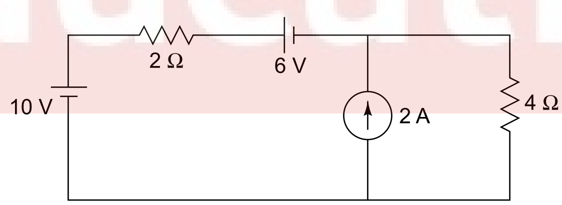


Fig. 2.112

Adding two voltage sources,

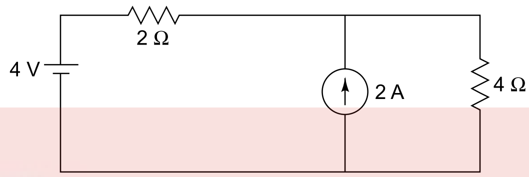


Fig. 2.113

Again by source transformation,

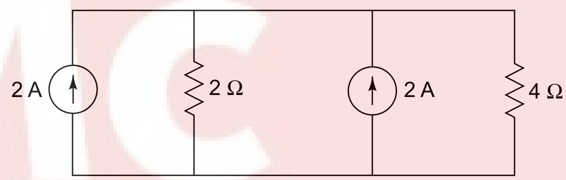


Fig. 2.114

Adding two current sources,

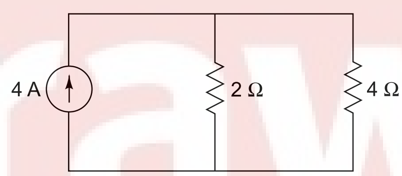


Fig. 2.115

By current-division rule,

$$I_{4\Omega} = 4 \times \frac{2}{2+4} = 1.33 \text{ A}$$

Example 6

Find the voltage across the 4 Ω resistor.

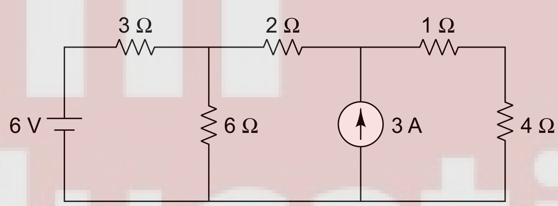


Fig. 2.116

Solution Converting the series combination of the voltage source of 6 V and the resistor of 3 Ω into equivalent current source and resistor,

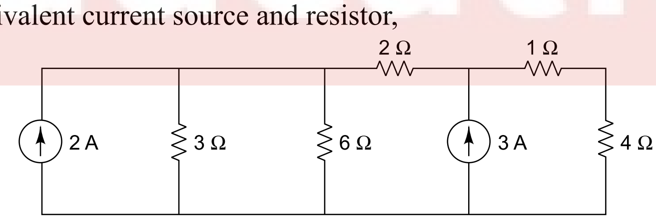


Fig. 2.117

By series–parallel reduction technique,

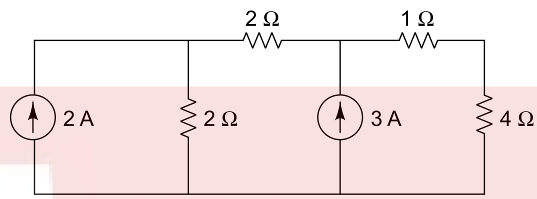


Fig. 2.118

By source transformation,

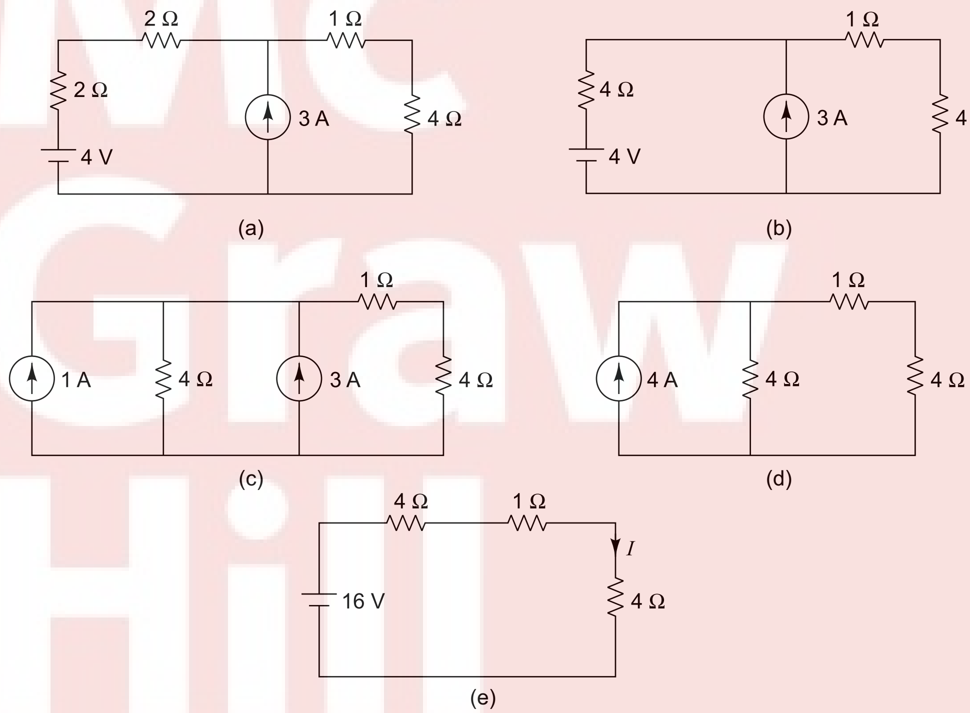


Fig. 2.119

$$I = \frac{16}{4+1+4} = 1.78\text{ A}$$

$$\begin{aligned}\text{Voltage across the } 4\ \Omega \text{ resistor } &= 4I\\ &= 4 \times 1.78\\ &= 7.12\text{ V}\end{aligned}$$

Exercise 2.4

1.1 Replace the given network with a single voltage source and a resistor.

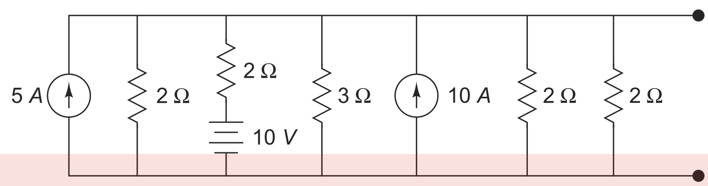


Fig. 2.120

[8.6 V, 0.43 Ω]

1.2 Use source transformation to simplify the network until two elements remain to the left of terminals *a* and *b*.

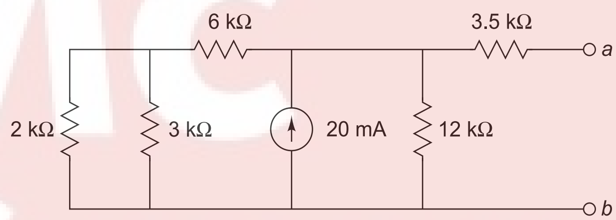


Fig. 2.121

[88.42 V, 7.92 kΩ]

2.5 STAR-DELTA TRANSFORMATION

When a circuit cannot be simplified by normal series–parallel reduction technique, the star-delta transformation can be used.

Figure 2.122(a) shows three resistors R_A, R_B and R_C connected in delta.

Figure 2.122(b) shows three resistors R_1, R_2 and R_3 connected in star.

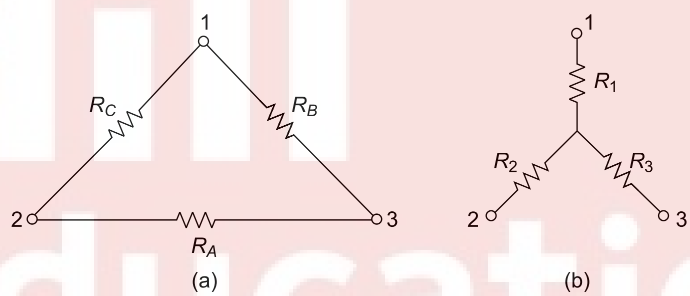


Fig. 2.122 *Delta and star networks*

These two networks will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both the arrangements.

2.5.1 Delta to Star Transformation

Referring to delta network shown in Fig. 2.122(a),
the resistance between terminals 1 and 2 = $R_C \parallel (R_A + R_B)$

$$= \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \tag{2.4}$$

Referring to the star network shown in Fig. 2.122(b),

the resistance between terminals 1 and 2 = $R_1 + R_2$ (2.5)

Since the two networks are electrically equivalent,

$$R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \tag{2.6}$$

Similarly, $$R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + R_B + R_C} \tag{2.7}$$

and $$R_3 + R_1 = \frac{R_B(R_A + R_C)}{R_A + R_B + R_C} \tag{2.8}$$

Subtracting Eq. (2.7) from Eq. (2.6),

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C} \tag{2.9}$$

Adding Eq. (2.9) and Eq. (2.8),

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \tag{2.10}$$

Similarly, $$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \tag{2.11}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \tag{2.12}$$

Thus, star resistor connected to a terminal is equal to the product of the two delta resistors connected to the same terminal divided by the sum of the delta resistors.

2.5.2 Star to Delta Transformation

Multiplying the above equations,

$$R_1 R_2 = \frac{R_A R_B R_C^2}{(R_A + R_B + R_C)^2} \tag{2.13}$$

$$R_2 R_3 = \frac{R_A^2 R_B R_C}{(R_A + R_B + R_C)^2} \tag{2.14}$$

$$R_3 R_1 = \frac{R_A R_B^2 R_C}{(R_A + R_B + R_C)^2} \tag{2.15}$$

Adding Eqs (2.13), (2.14) and (2.15),

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_A R_B R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^2}$$

$$= \frac{R_A R_B R_C}{R_A + R_B + R_C}$$

$$= R_A R_1$$

$$= R_B R_2$$

$$= R_C R_3$$

Hence,

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$= R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$= R_1 + R_3 + \frac{R_3 R_1}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$= R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Thus, delta resistor between the two terminals is the sum of two star resistors connected to the same terminals plus the product of the two resistors divided by the remaining third star resistor.

Note: When three equal resistors are connected in delta, the equivalent star resistance is given by

$$R_Y = \frac{R_\Delta R_\Delta}{R_\Delta + R_\Delta + R_\Delta} = \frac{R_\Delta}{3}$$

or

$$R_\Delta = 3R_Y$$

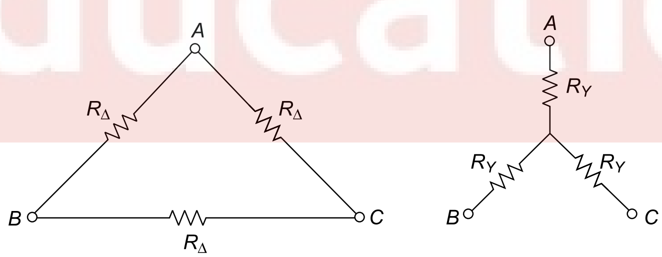


Fig. 2.123

Example 1

Find an equivalent resistance between A and B.

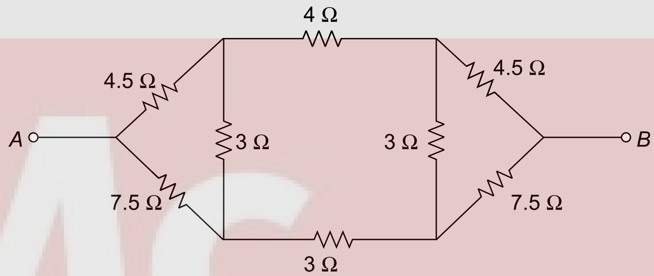


Fig. 2.124

Solution Converting the two delta networks formed by resistors of 4.5 Ω, 3 Ω and 7.5 Ω into equivalent star networks,

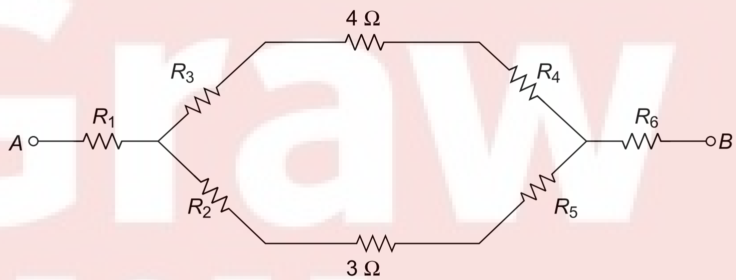


Fig. 2.125

$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\,\Omega$$

$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\,\Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\,\Omega$$

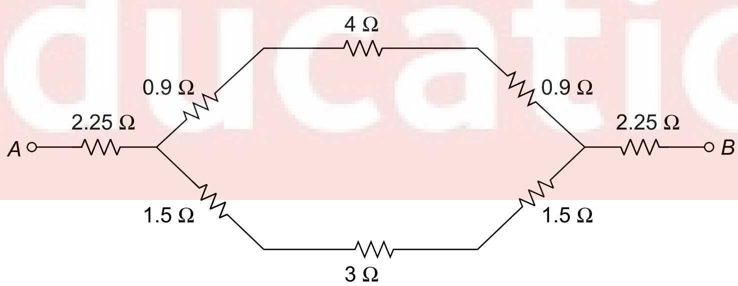


Fig. 2.126

Simplifying the network,

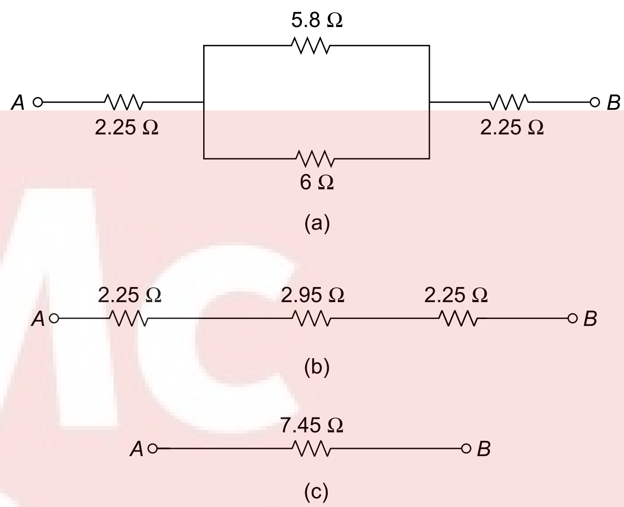


Fig. 2.127

$$R_{AB} = 7.45\ \Omega$$

Example 2

Find an equivalent resistance between A and B.

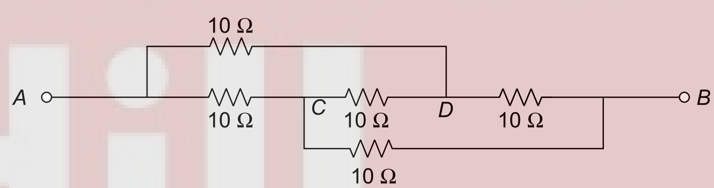


Fig. 2.128

Solution Redrawing the network,

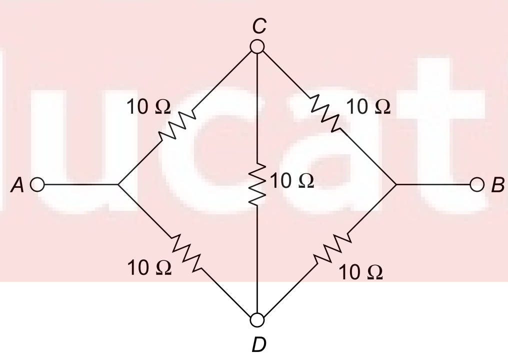


Fig. 2.129

Converting the delta network formed by three resistors of 10 Ω into an equivalent star network,

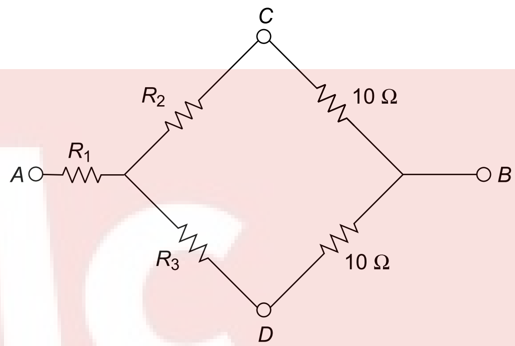


Fig. 2.130

$$R_1 = R_2 = R_3 = \frac{10 \times 10}{10+10+10} = \frac{10}{3}\ \Omega$$

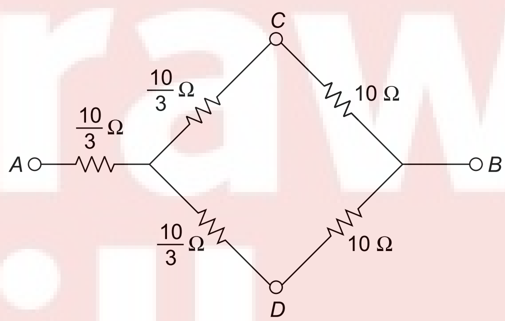


Fig. 2.131

Simplifying the network,

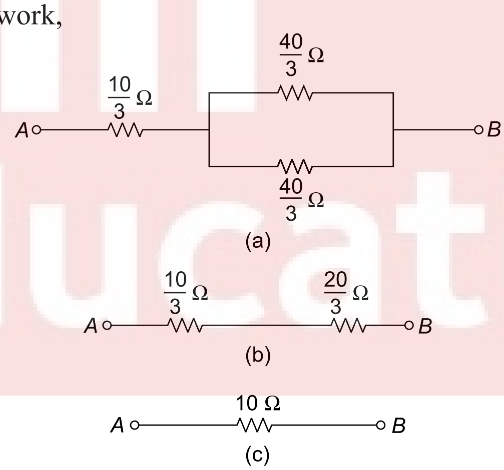


Fig. 2.132

$$R_{AB} = 10\ \Omega$$

Example 3

Find an equivalent resistance between A and B.

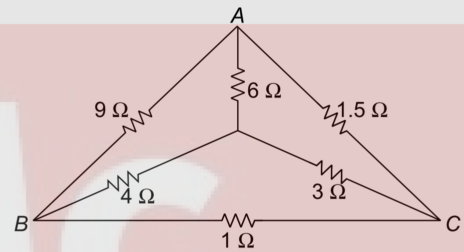


Fig. 2.133

Solution Converting the star network formed by resistors of 3 Ω, 4 Ω and 6 Ω into an equivalent delta network,

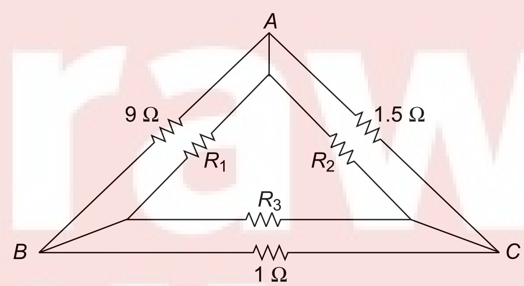


Fig. 2.134

$$R_1 = 6 + 4 + \frac{6 \times 4}{3} = 18\ \Omega$$

$$R_2 = 6 + 3 + \frac{6 \times 3}{4} = 13.5\ \Omega$$

$$R_3 = 4 + 3 + \frac{4 \times 3}{6} = 9\ \Omega$$

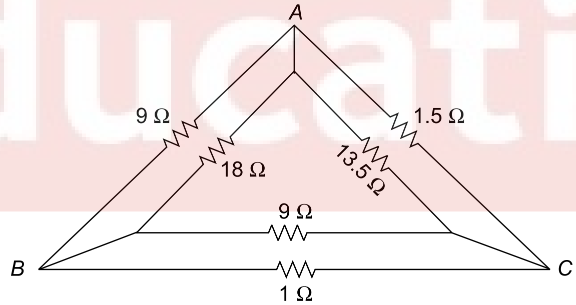


Fig. 2.135

Simplifying the network,

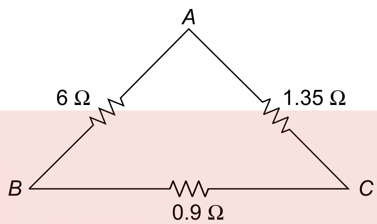


Fig. 2.136

$$R_{AB} = 6 \parallel (1.35 + 0.9)$$
$$= 6 \parallel 2.25$$
$$= 1.64\ \Omega$$

Example 4

Find an equivalent resistance between A and N by solving outer delta ABC.

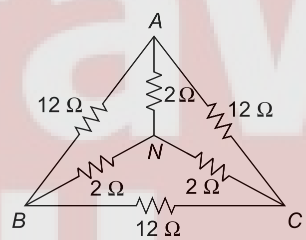


Fig. 2.137

Solution Converting outer delta ABC into a star network,

$$R_Y = \frac{12\times12}{12+12+12} = 4\ \Omega$$

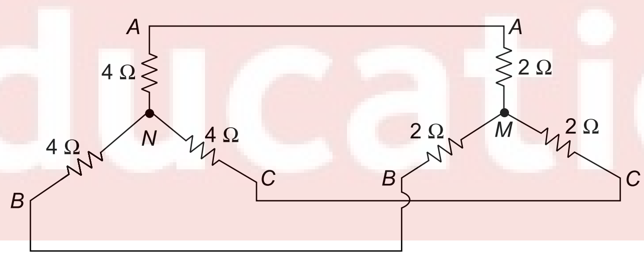


Fig. 2.138

Simplifying the network,

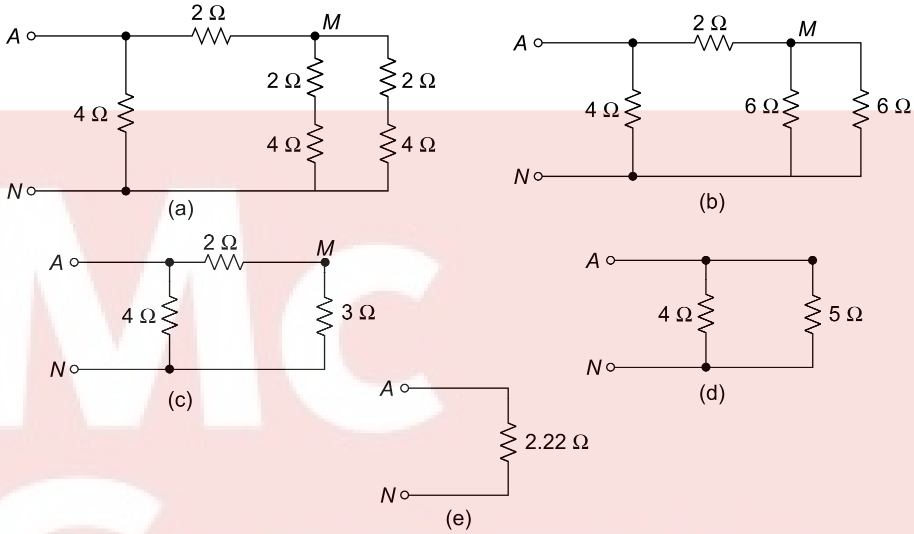


Fig. 2.139

$$R_{AN} = 2.22\ \Omega$$

Example 5

Find an equivalent resistance between A and B.

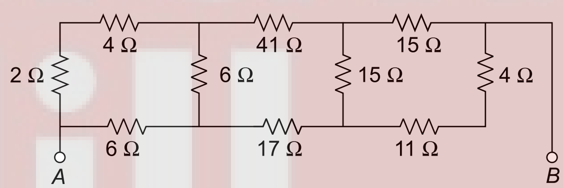


Fig. 2.140

Solution The resistors of 2 Ω and 4 Ω and the resistors of 4 Ω and 11 Ω are connected in series.

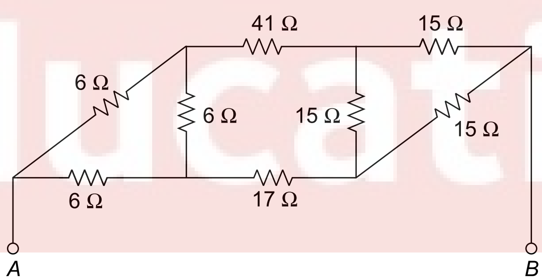


Fig. 2.141

Converting the two outer delta networks into equivalent star networks,

$$R_{Y_1} = \frac{6\times 6}{6+6+6} = 2\ \Omega$$

$$R_{Y_2} = \frac{15\times 15}{15+15+15} = 5\ \Omega$$

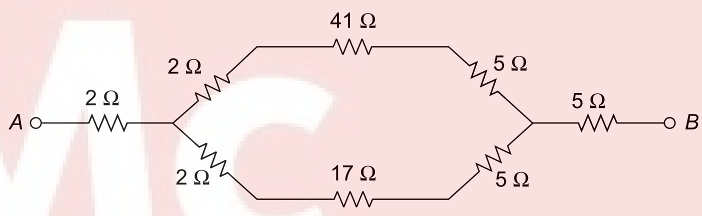


Fig. 2.142

Simplifying the network,

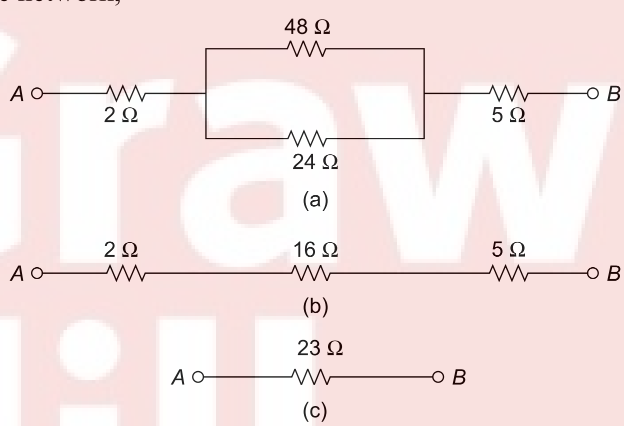


Fig. 2.143

$$R_{AB} = 23\ \Omega$$

Example 6

Find an equivalent resistance between A and B.

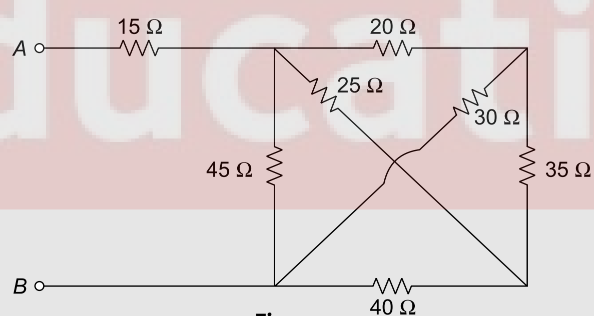


Fig. 2.144

Solution Drawing the resistor of 30 Ω from outside,

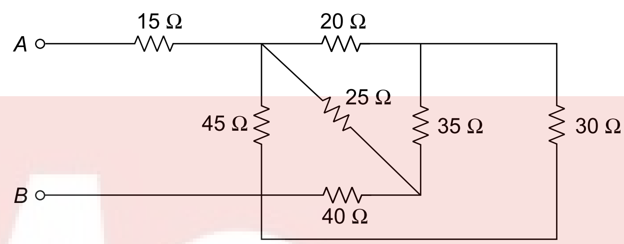


Fig. 2.145

Converting the delta network formed by resistors of 20 Ω, 25 Ω and 35 Ω into an equivalent star network,

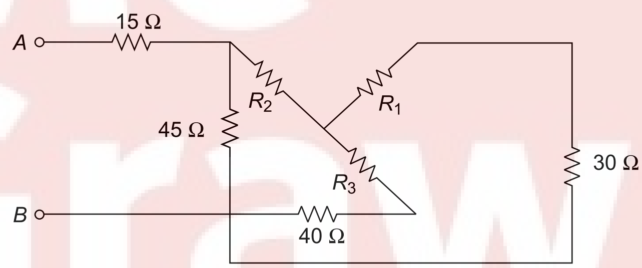


Fig. 2.146

$$R_1 = \frac{20\times 35}{20+35+25} = 8.75\ \Omega$$

$$R_2 = \frac{20\times 25}{20+35+25} = 6.25\ \Omega$$

$$R_3 = \frac{35\times 25}{20+35+25} = 10.94\ \Omega$$

Redrawing the network,

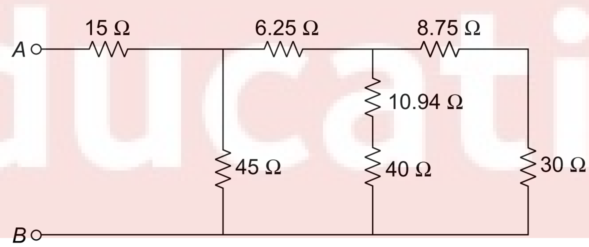


Fig. 2.147

Simplifying the network,

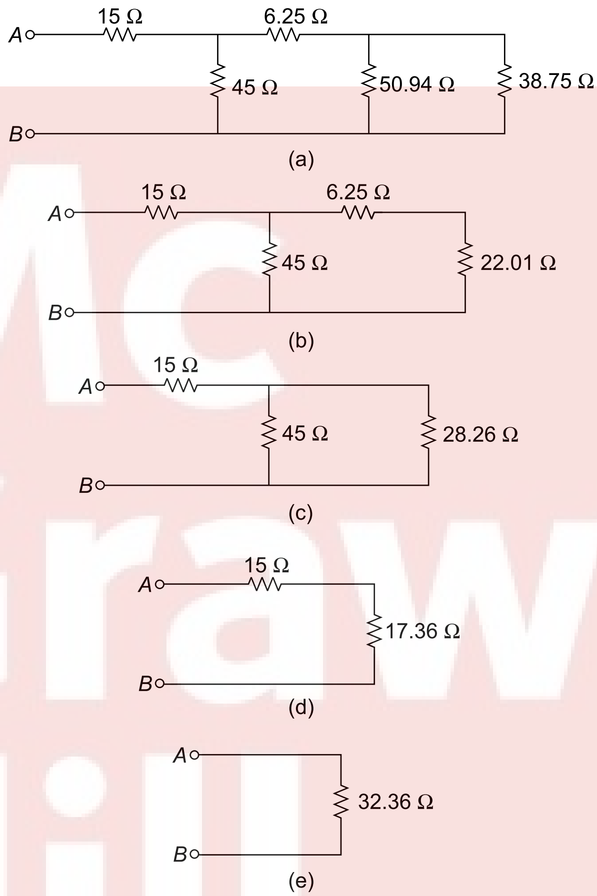


Fig. 2.148

$R_{AB} = 32.36\ \Omega$

Example 7

Find an equivalent resistance between A and B.

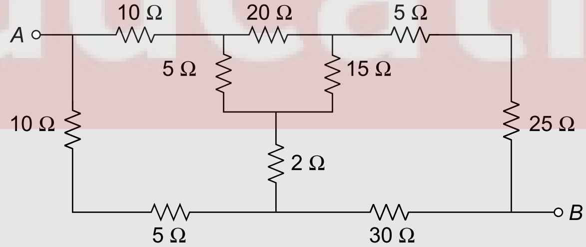


Fig. 2.149

Solution The resistors of 5 Ω and 25 Ω and the resistors of 10 Ω and 5 Ω are connected in series.

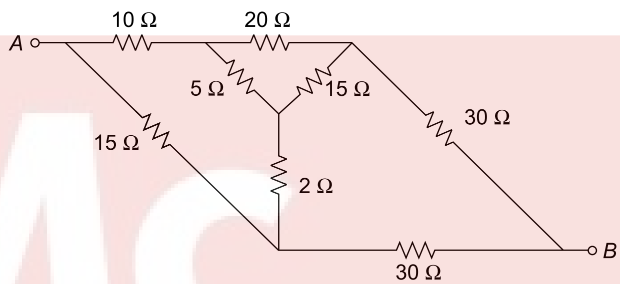


Fig. 2.150

Converting the delta network formed by the resistors of 20 Ω, 5 Ω and 15 Ω into an equivalent star network,

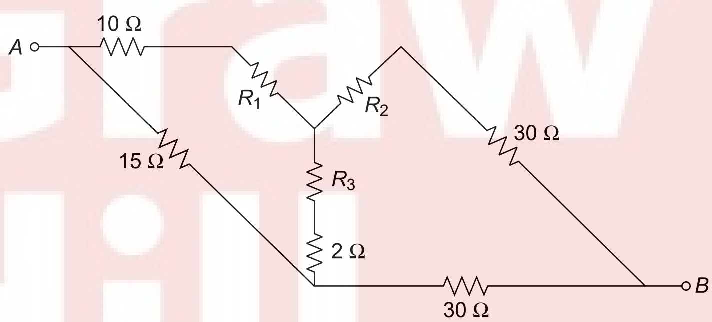


Fig. 2.151

$$R_1 = \frac{20 \times 5}{20+5+15} = 2.5\ \Omega$$

$$R_2 = \frac{20 \times 15}{20+5+15} = 7.5\ \Omega$$

$$R_3 = \frac{5 \times 15}{20+5+15} = 1.875\ \Omega$$

Redrawing the network,

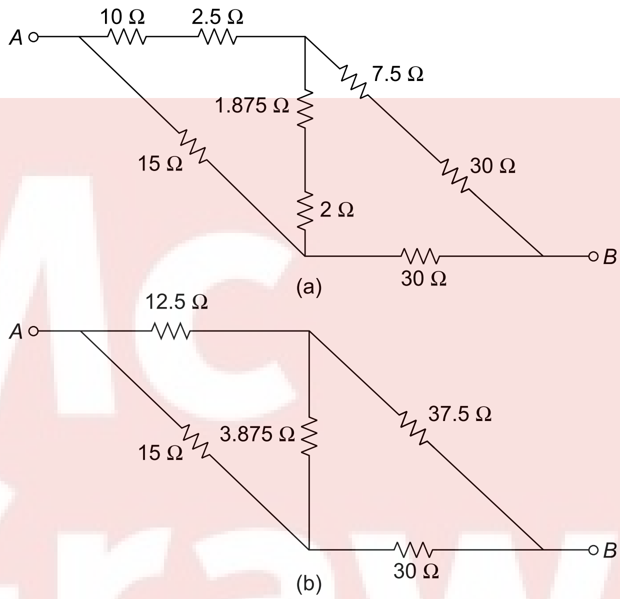


Fig. 2.152

Converting the delta network formed by the resistors of 3.875 Ω, 37.5 Ω and 30 Ω into an equivalent star network,

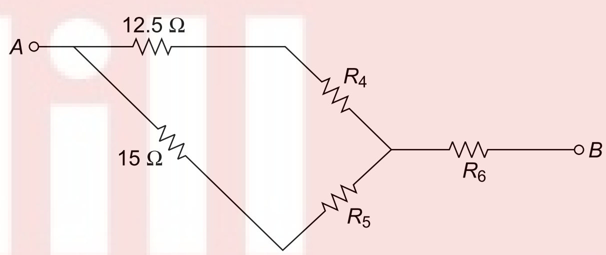


Fig. 2.153

$$R_4 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = 2.04\ \Omega$$

$$R_5 = \frac{3.875 \times 30}{3.875 + 37.5 + 30} = 1.63\ \Omega$$

$$R_6 = \frac{37.5 \times 30}{3.875 + 37.5 + 30} = 15.76\ \Omega$$

Simplifying the network,

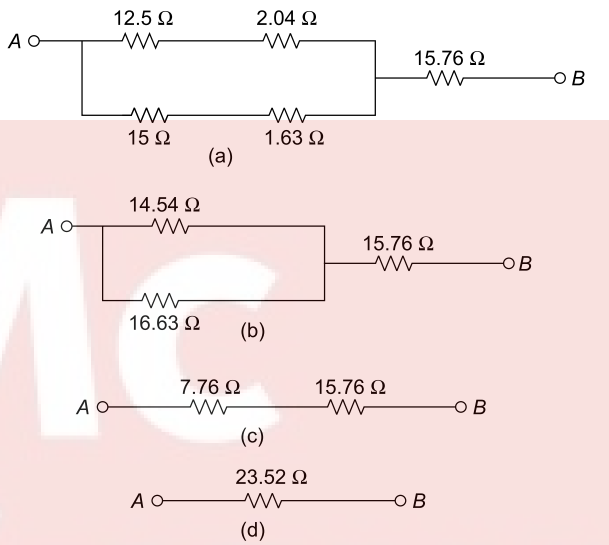


Fig. 2.154

$$R_{AB} = 23.52\ \Omega$$

Example 8

Find an equivalent resistance between A and B.

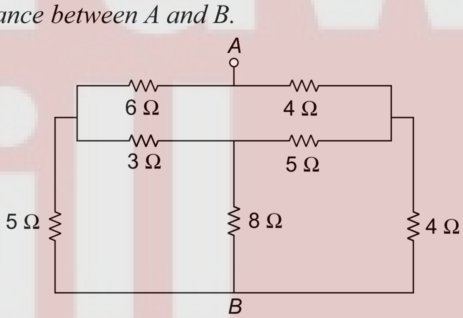


Fig. 2.155

Solution

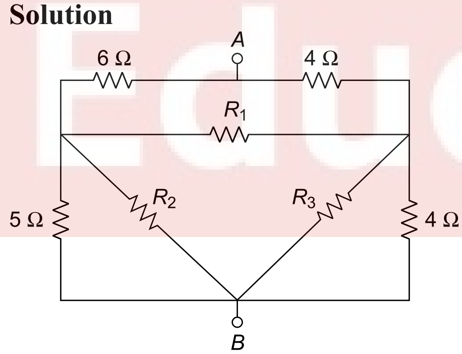


Fig. 2.156

Converting the star network formed by the resistors of 3 Ω, 5 Ω and 8 Ω into an equivalent delta network,

$$R_1 = 3 + 5 + \frac{3\times 5}{8} = 9.875\ \Omega$$

$$R_2 = 3 + 8 + \frac{3\times 8}{5} = 15.8\ \Omega$$

$$R_3 = 5 + 8 + \frac{5\times 8}{3} = 26.33\ \Omega$$

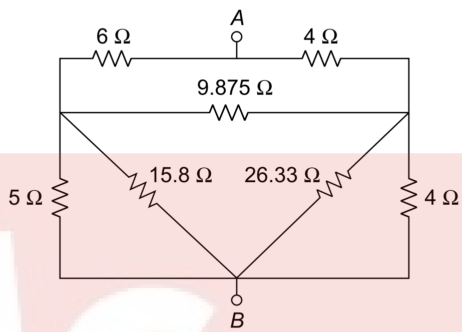


Fig. 2.157

The resistors of 15.8 Ω and 5 Ω and the resistors of 26.33 Ω and 4 Ω are connected in parallel.

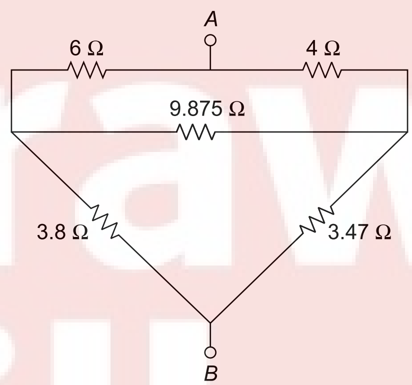


Fig. 2.158

Converting the delta network into a star network,

$$R_4 = \frac{3.8 \times 9.875}{3.8 + 9.875 + 3.47} = 2.19\ \Omega$$

$$R_5 = \frac{3.8 \times 3.47}{3.8 + 9.875 + 3.47} = 0.77\ \Omega$$

$$R_6 = \frac{3.47 \times 9.875}{3.8 + 9.875 + 3.47} = 2\ \Omega$$

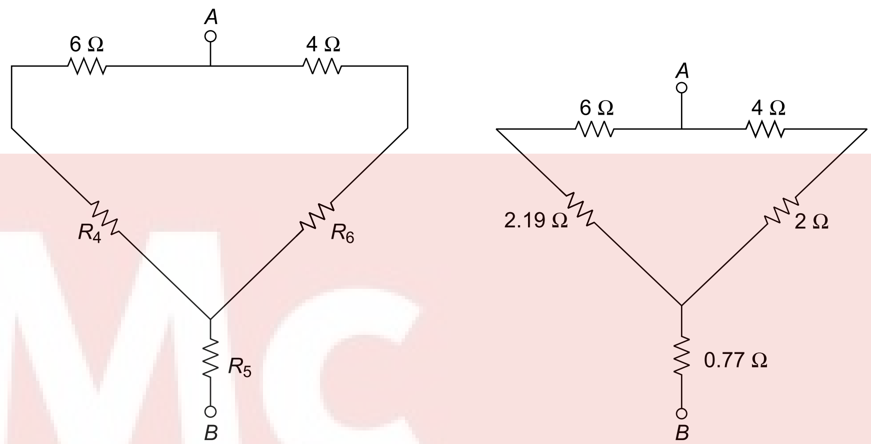


Fig. 2.159

Simplifying the network,

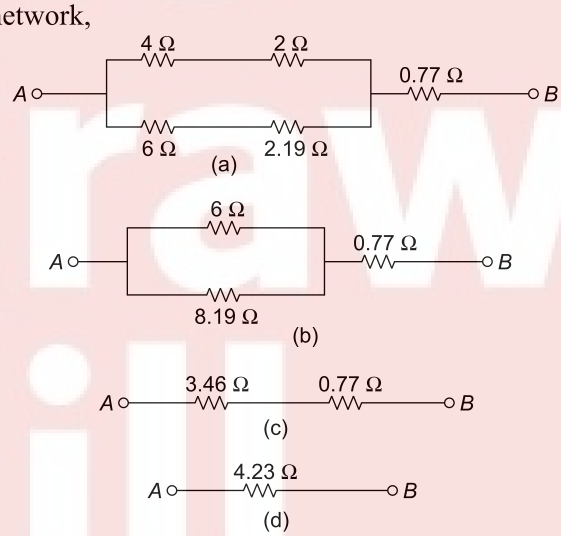


Fig. 2.160

$$R_{AB} = 4.23\ \Omega$$

Example 9

Find an equivalent resistance between A and B.

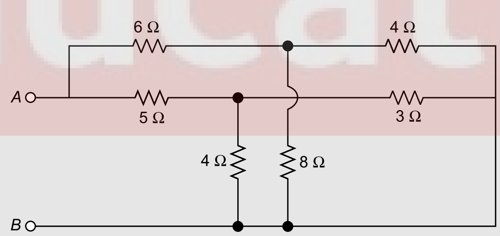


Fig. 2.161

Solution Converting the star network formed by the resistors of 3 Ω, 4 Ω and 5 Ω into an equivalent delta network,

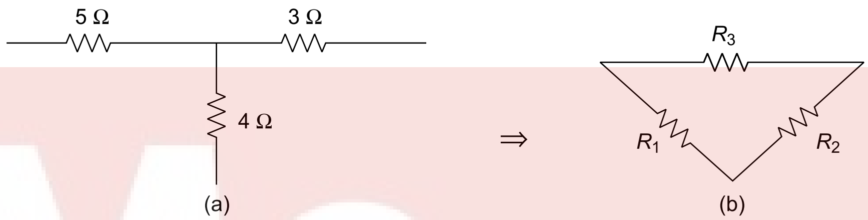


Fig. 2.162

$$R_1 = 5+4+\frac{5\times 4}{3} = 15.67\ \Omega$$

$$R_2 = 3+4+\frac{3\times 4}{5} = 9.4\ \Omega$$

$$R_3 = 5+3+\frac{5\times 3}{4} = 11.75\ \Omega$$

Similarly, converting the star network formed by the resistors of 4 Ω, 6 Ω and 8 Ω into an equivalent delta network,

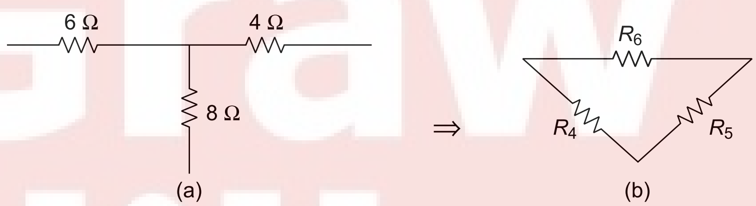


Fig. 2.163

$$R_4 = 6+8+\frac{6\times 8}{4} = 26\ \Omega$$

$$R_5 = 4+8+\frac{4\times 8}{6} = 17.33\ \Omega$$

$$R_6 = 6+4+\frac{6\times 4}{8} = 13\ \Omega$$

These two delta networks are connected in parallel between points A and B.

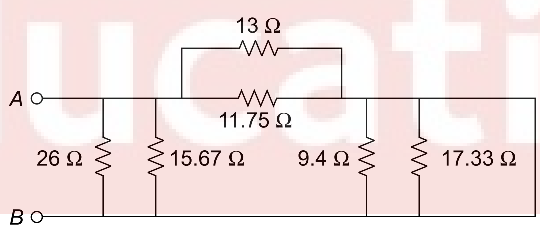


Fig. 2.164

The resistors of 9.4 Ω and 17.33 Ω are in parallel with a short. Hence, the equivalent resistance of this combination becomes zero.

Simplifying the parallel networks,

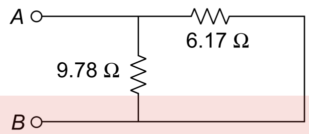


Fig. 2.165

$$R_{AB} = 6.17 \parallel 9.78 = 3.78\ \Omega$$

Example 10

Determine the current supplied by the battery.

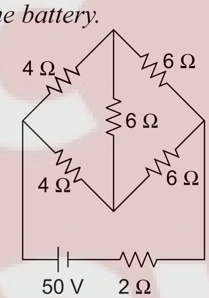


Fig. 2.166

Solution Converting the delta network formed by resistors of 6 Ω, 6 Ω and 6 Ω into an equivalent star network,

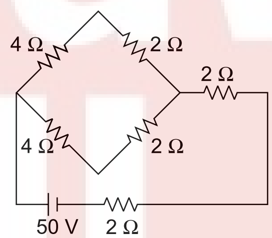


Fig. 2.167

Simplifying the network,

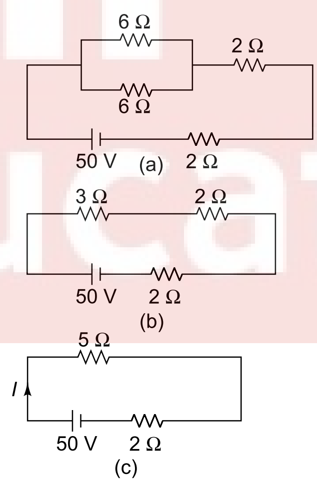


Fig. 2.168

$$I = \frac{50}{5+2} = 7.14 \text{ A}$$

Example 11

Calculate the current flowing through the 10 Ω resistor.

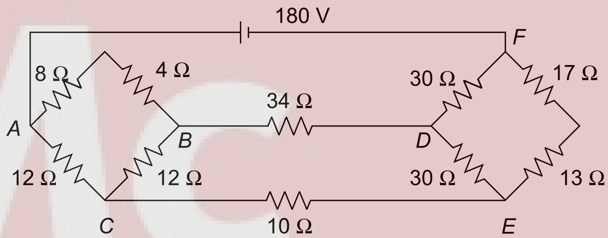


Fig. 2.169

Solution Between terminals *A* and *B* resistors of 8 Ω and 4 Ω are connected in series. Similarly, between terminals *F* and *E*, resistors of 17 Ω and 13 Ω are connected in series.

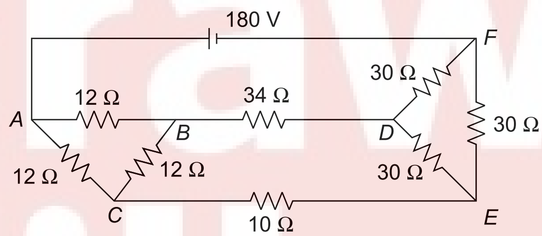


Fig. 2.170

Converting delta *ABC* and *DEF* into an equivalent star network,

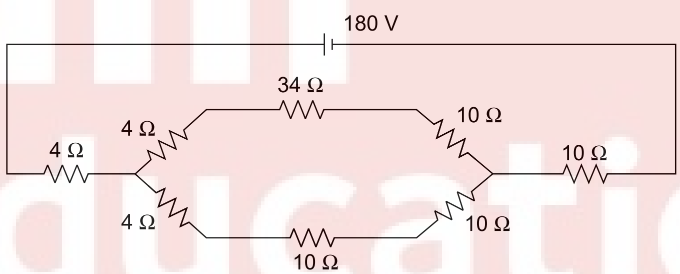


Fig. 2.171

Simplifying the network,

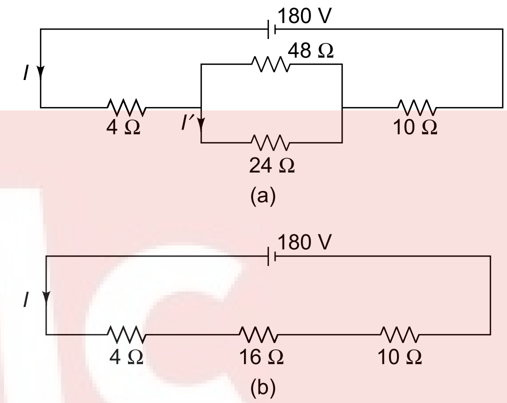


Fig. 2.172

$$I = \frac{180}{4+16+10} = 6\text{ A}$$

By current-division rule,

$$I' = I_{24\,\Omega} = I_{10\,\Omega} = 6 \times \frac{48}{24+48} = 4\text{ A}$$

Example 12

Find the current supplied by the battery.

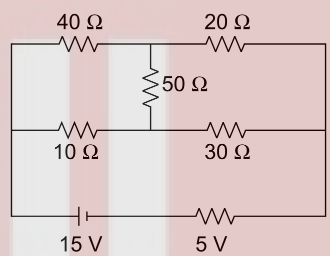


Fig. 2.173

Solution Converting the star network formed by resistors of 40 Ω, 20 Ω and 50 Ω into an equivalent delta network,

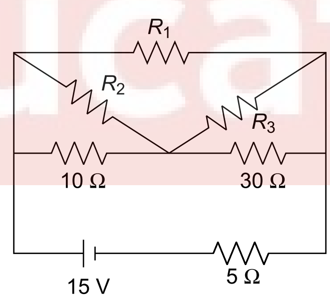


Fig. 2.174

$$R_1 = 40 + 20 + \frac{40 \times 20}{50} = 76\ \Omega$$

$$R_2 = 40 + 50 + \frac{40 \times 50}{20} = 190\ \Omega$$

$$R_3 = 20 + 50 + \frac{20 \times 50}{40} = 95\ \Omega$$

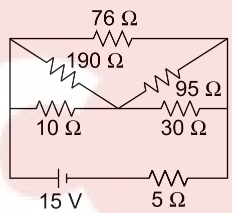


Fig. 2.175

The resistors of 190 Ω and 10 Ω and the resistors of 95 Ω and 30 Ω are connected in parallel.

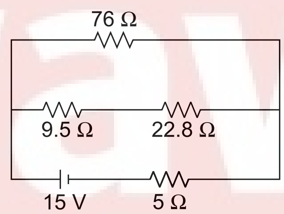


Fig. 2.176

Simplifying the network,

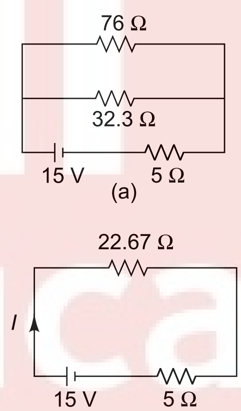
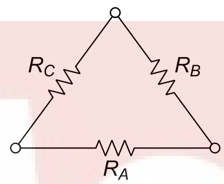


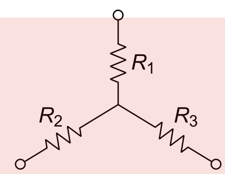
Fig. 2.177

$$I = \frac{15}{22.67 + 5} = 0.542\ \text{A}$$

Useful Formulae



Delta



Star

Delta to star transformation

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Star to delta transformation

$$R_A = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_B = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Exercise 2.5

2.1 Find the equivalent resistance between A and B.

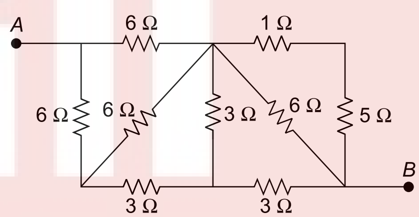


Fig. 2.178

[5 Ω]

2.2 Find the equivalent resistance between A and B.

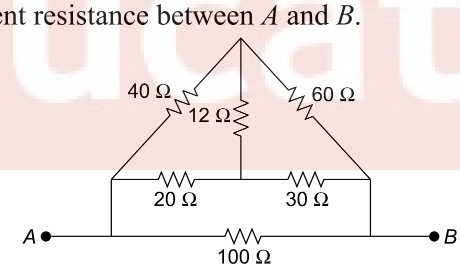


Fig. 2.179

[25 Ω]

2.3 Find the equivalent resistance between A and B.

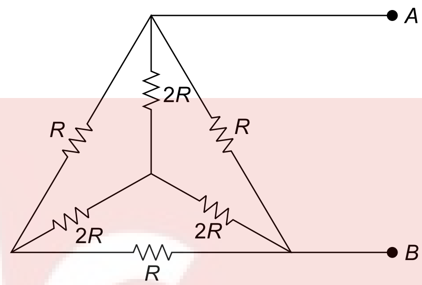


Fig. 2.180

$\left[\frac{4}{7}R\right]$

2.4 Find the equivalent resistance between A and B.

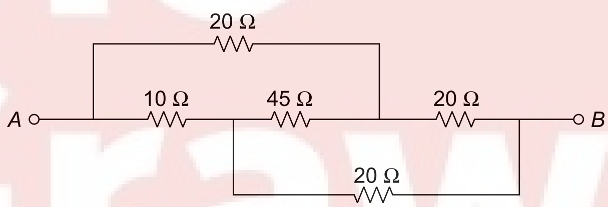


Fig. 2.181

[17 Ω]

2.5 Find R_{AB} by solving the outer delta (X–B–Y) only.

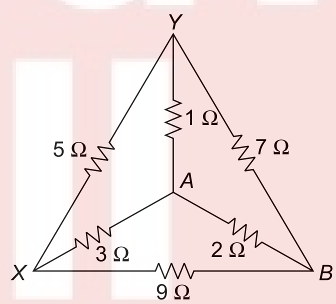


Fig. 2.182

[1.41 Ω]

2.6 Find the equivalent resistance between A and B.

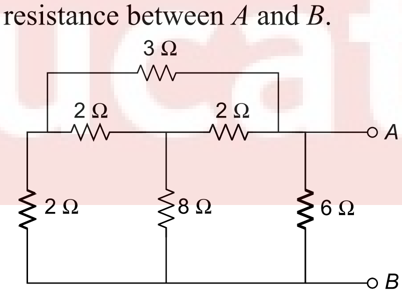


Fig. 2.183

[2.625 Ω]

2.7 Find the equivalent resistance between the terminals *A* and *B*.

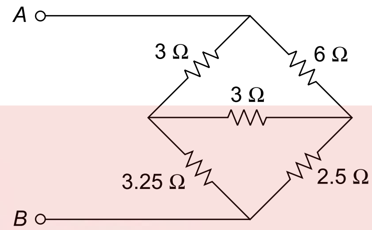


Fig. 2.184

[3.5 Ω]

2.8 Find the equivalent resistance between terminals *A* and *B*.

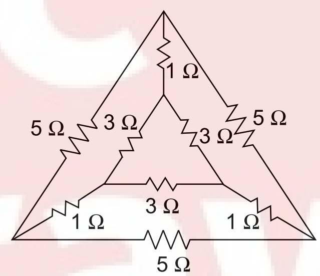


Fig. 2.185

[1.82 Ω]

2.9 Find the equivalent resistance between the terminals *A* and *B*.

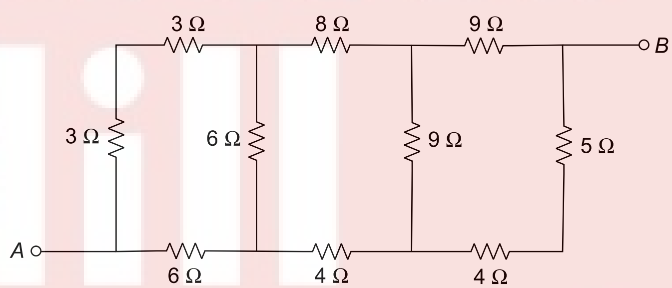


Fig. 2.186

[10.32 Ω]

2.10 Find the equivalent resistance between *A* and *B*.

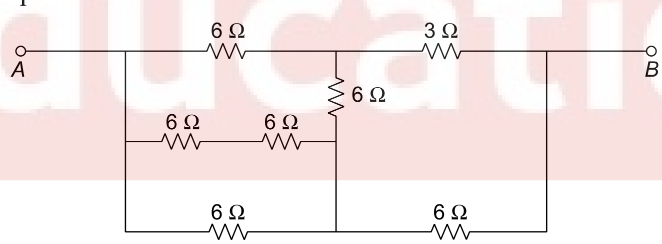


Fig. 2.187

[4.59 Ω]

2.11 Find the equivalent resistance between A and B.

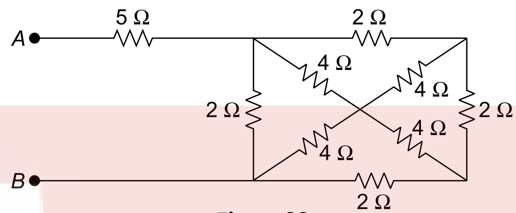


Fig. 2.188

[6.24 Ω]

2.12 Determine the current I.

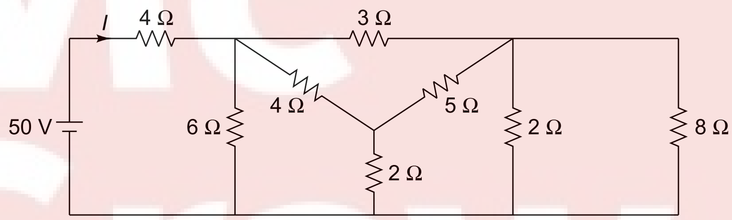


Fig. 2.189

[8.59 A]

2.13 Find the voltage between terminals A and B.

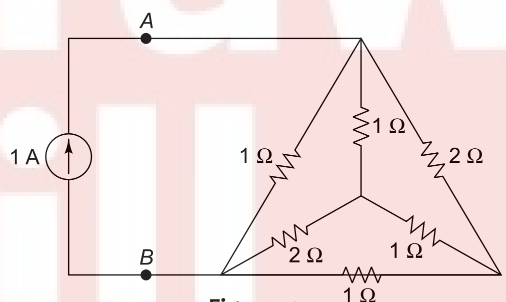


Fig. 2.190

[0.56 V]

2.14 Determine the power supplied to the network.

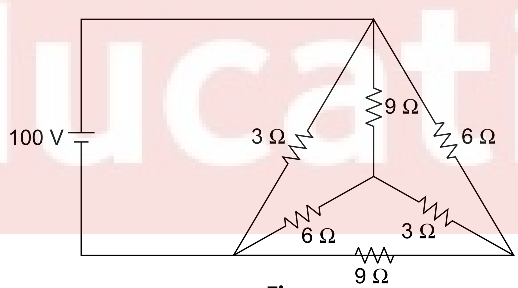


Fig. 2.191

[4705.88 Ω]

2.15 Find the current I.

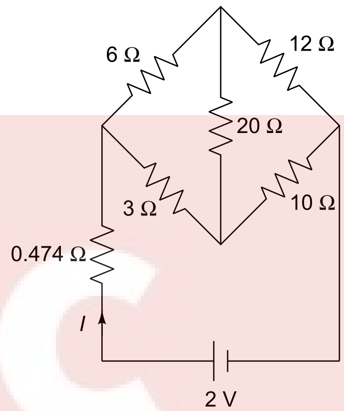


Fig. 2.192

[0.25 A]

2.16 Determine the current through the 10 Ω resistor.

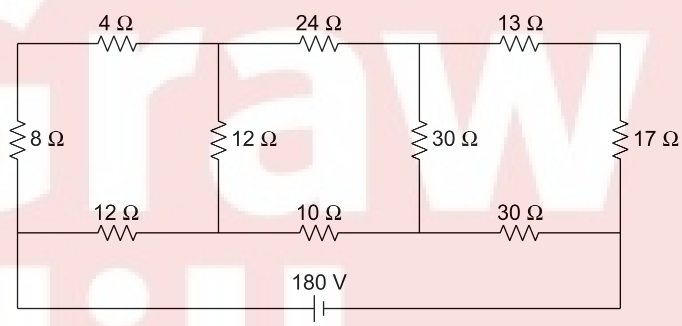


Fig. 2.193

[3.84 A]

2.6 SUPERPOSITION THEOREM

It states that '*In a linear network containing more than one independent sources, the resultant current in any element is the algebraic sum of the currents that would be produced by each independent source acting alone, all the other independent sources being represented meanwhile by their respective internal resistances.*'

The independent voltage sources are represented by their internal resistances if given or simply with zero resistances, i.e., short circuits if internal resistances are not mentioned.

The independent current sources are represented by infinite resistances, i.e., open circuits.

A linear network is one whose parameters are constant, i.e., they do not change with voltage and current.

Explanation Consider the circuit shown in Fig. 2.194. Suppose we have to find current I_4 flowing through R_4.

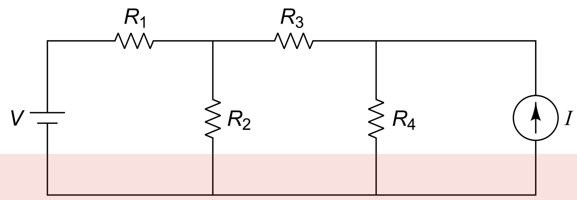


Fig. 2.194 *Superposition theorem*

2.6.1 Steps to be followed in Superposition Theorem

1. Find the current I_4' flowing through R_4 due to independent voltage source 'V', representing independent current source with infinite resistance, i.e., open circuit.

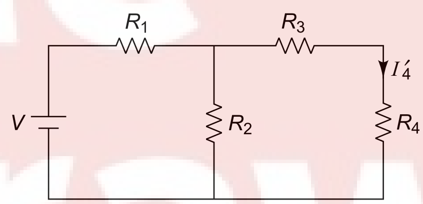


Fig. 2.195 *Step 1*

2. Find the current I_4'' flowing through R_4 due to independent current source 'I', representing the independent voltage source with zero resistance or short circuit.

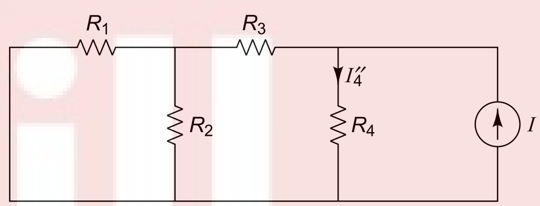


Fig. 2.196 *Step 2*

3. Find the resultant current I_4 through R_4 by the superposition theorem.

$$I_4 = I_4' + I_4''$$

Example 1

Find the current through the 2 Ω resistor.

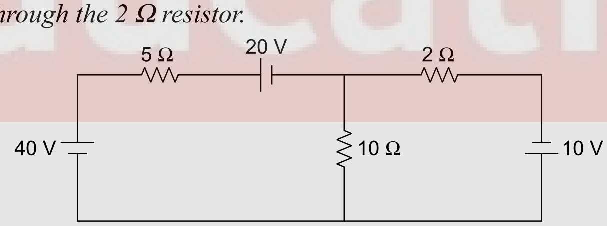


Fig. 2.197

Solution *Step I: When the 40 V source is acting alone*

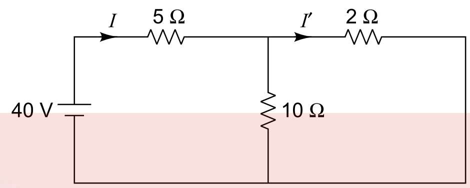


Fig. 2.198

By series–parallel reduction technique,

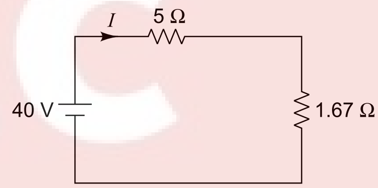


Fig. 2.199

$$I = \frac{40}{5+1.67} = 6 \text{ A}$$

From Fig. 2.198, by current-division rule,

$$I' = 6 \times \frac{10}{10+2} = 5 \text{ A} (\rightarrow)$$

Step II: When the 20 V source is acting alone

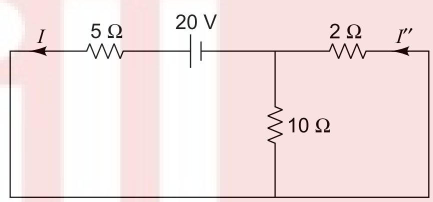


Fig. 2.200

By series–parallel reduction technique,

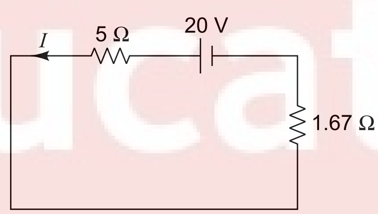


Fig. 2.201

$$I = \frac{20}{5+1.67} = 3 \text{ A}$$

From Fig. 2.200, by current-division rule,

$$I'' = 3 \times \frac{10}{10+2} = 2.5 \text{ A } (\leftarrow) = -2.5\text{A } (\rightarrow)$$

Step III: *When the 10 V source is acting alone*

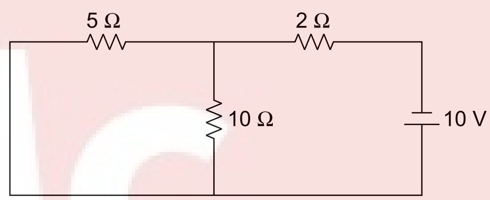


Fig. 2.202

By series–parallel reduction technique,

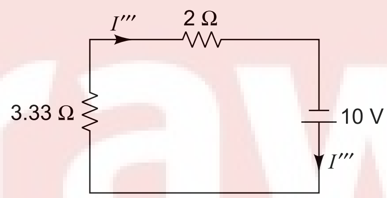


Fig. 2.203

$$I''' = \frac{10}{3.33+2} = 1.88 \text{ A } (\rightarrow)$$

Step IV: By superposition theorem

$$\begin{aligned} I &= I' + I'' + I''' \\ &= 5 - 2.5 + 1.88 \\ &= 4.38 \text{ A } (\rightarrow) \end{aligned}$$

Example 2

Find the current through the 1 Ω resistor.

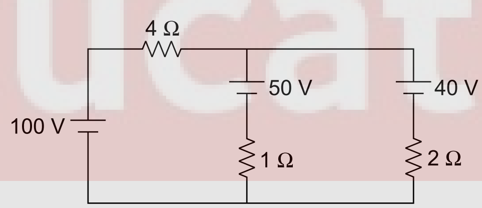


Fig. 2.204

Solution *Step I: When the 100 V source is acting alone*

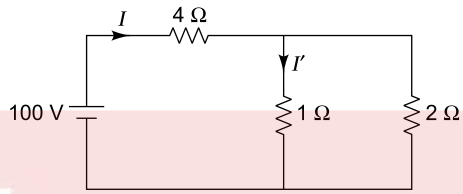


Fig. 2.205

By series–parallel reduction technique

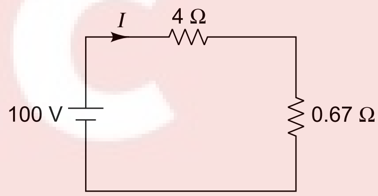


Fig. 2.206

$$I = \frac{100}{4+0.67} = 21.41 \text{ A}$$

From Fig. 2.205, by current-division rule,

$$I' = 21.41 \times \frac{2}{1+2} = 14.27 \text{ A } (\downarrow)$$

Step II: When the 50 V source is acting alone

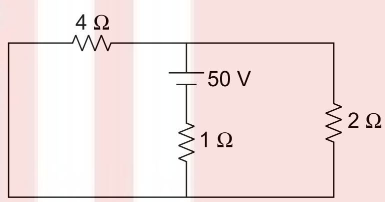


Fig. 2.207

By series–parallel reduction technique,

Fig. 2.208

$$I'' = \frac{50}{1+1.33} = 21.46 \text{ A } (\uparrow) = -21.46 \text{ A } (\downarrow)$$

Step III: When the 40 V source is acting alone

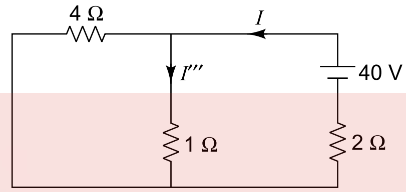


Fig. 2.209

By series–parallel reduction technique,

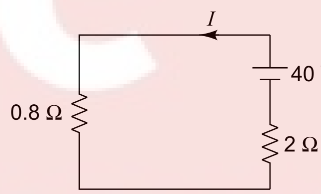


Fig. 2.210

$$I = \frac{40}{0.8+2} = 14.29 \text{ A}$$

From Fig. 2.209, by current-division rule,

$$I''' = 14.29 \times \frac{4}{4+1} = 11.43 \text{ A } (\downarrow)$$

Step IV: By superposition theorem

$$\begin{aligned} I &= I' + I'' + I''' \\ &= 14.27 - 21.46 + 11.43 \\ &= 4.24 \text{ A } (\downarrow) \end{aligned}$$

Example 3

Find the current through the 8 Ω resistor.

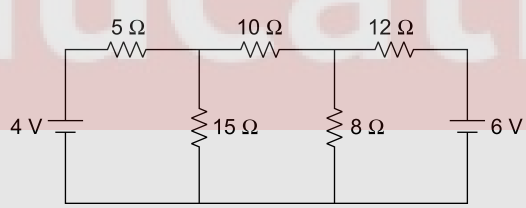


Fig. 2.211

Solution *Step I: When the 4 V source is acting alone*

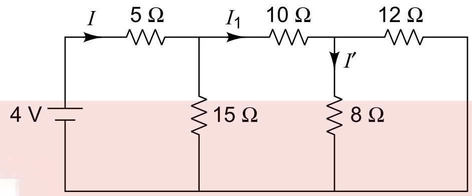
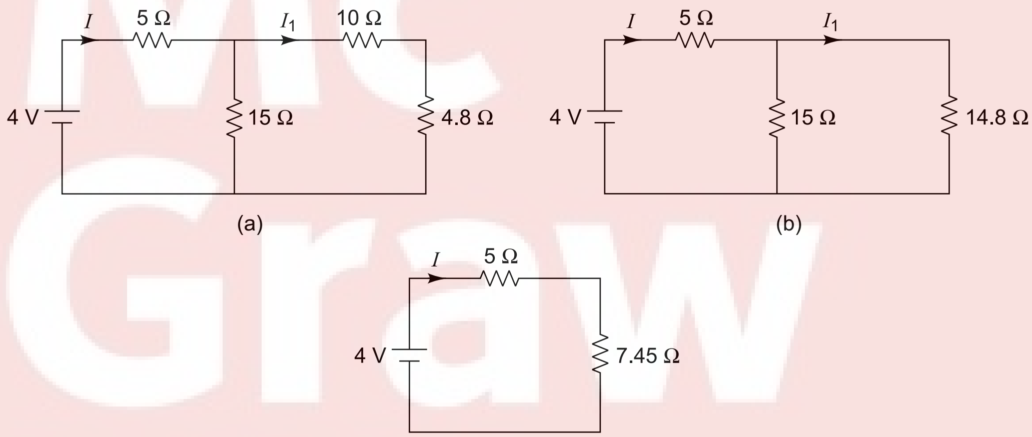


Fig. 2.212

By series–parallel reduction technique,



(a) (b) (c)

Fig. 2.213

$$I = \frac{4}{5+7.45} = 0.32\ \text{A}$$

From Fig. 2.213(b), by current-division rule,

$$I_1 = 0.32 \times \frac{15}{15+14.8} = 0.16\ \text{A}$$

From Fig. 2.212, by current-division rule,

$$I' = 0.16 \times \frac{12}{12+8} = 0.096\ \text{A}\ (\downarrow)$$

Step II: When the 6 V source is acting alone

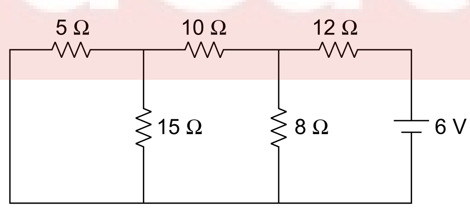


Fig. 2.214

By series–parallel reduction technique,

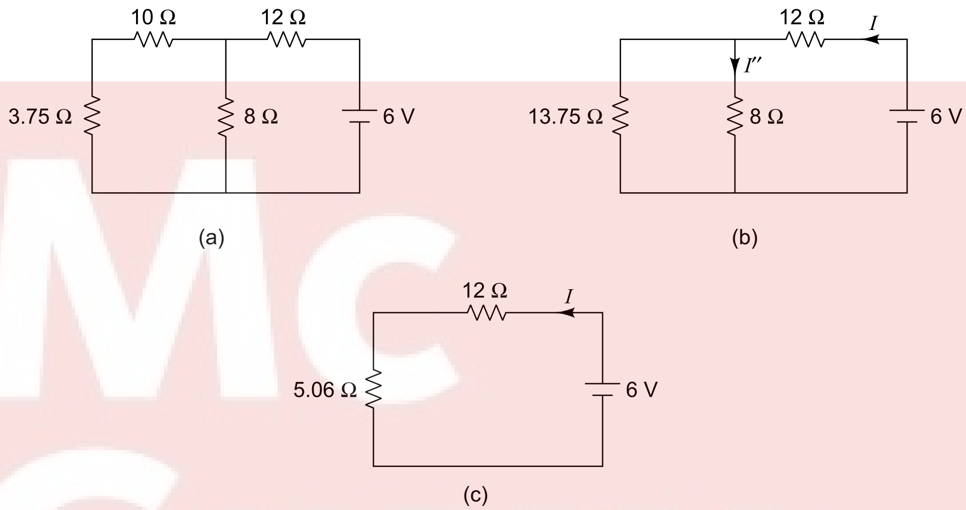


Fig. 2.215

$$I = \frac{6}{12 + 5.06} = 0.35 \text{ A}$$

From Fig. 2.215(b), by current division rule,

$$I'' = 0.35 \times \frac{13.75}{13.75 + 8} = 0.22 \text{ A} (\downarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= 0.096 + 0.22 \\ &= 0.316 \text{ A} (\downarrow) \end{aligned}$$

Example 4

Find the current through the 4 Ω resistor.

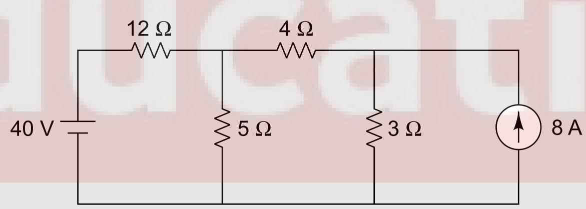


Fig. 2.216

Solution *Step I: When the 40 V source is acting alone*

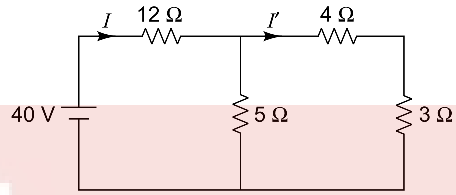


Fig. 2.217

By series–parallel reduction technique,

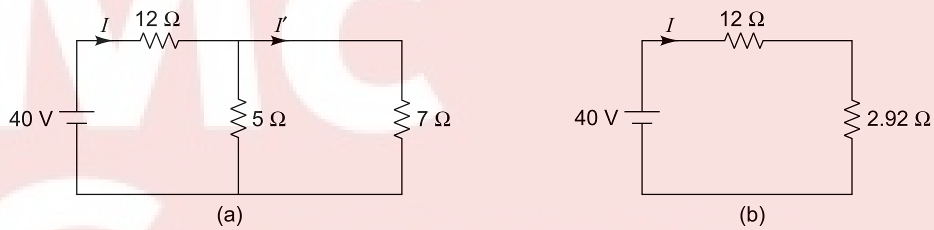


Fig. 2.218

$$I = \frac{40}{12 + 2.92} = 2.68 \text{ A}$$

From Fig. 2.218(a), by current-division rule,

$$I' = 2.68 \times \frac{5}{5+7} = 1.12 \text{ A} (\rightarrow) = -1.12 \text{ A} (\leftarrow)$$

Step II: When the 8 A source is acting alone

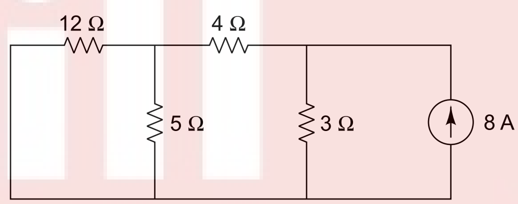


Fig. 2.219

By series–parallel reduction technique,

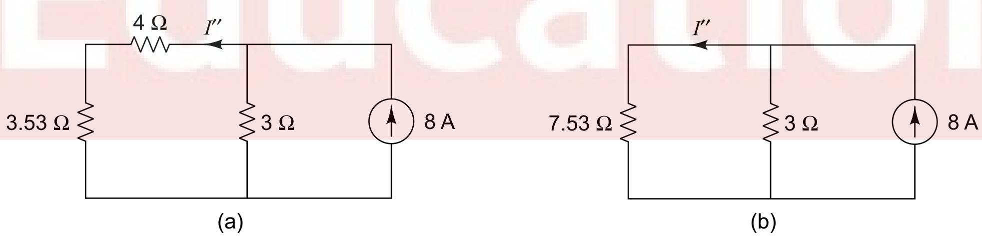


Fig. 2.220

From Fig. 2.220(b), by current-division rule,

$$I'' = 8 \times \frac{3}{7.53 + 3} = 2.28 \text{ A } (\leftarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= -1.12 + 2.28 \\ &= 1.16 \text{ A } (\leftarrow) \end{aligned}$$

Example 5

Find the current in the 10 Ω resistor.

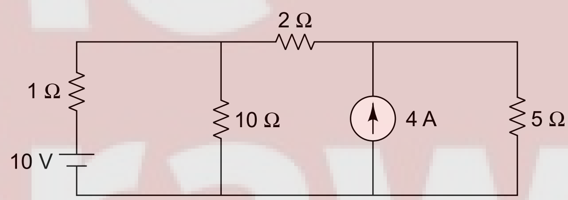


Fig. 2.221

Solution *Step I: When the 10 V source is acting alone*

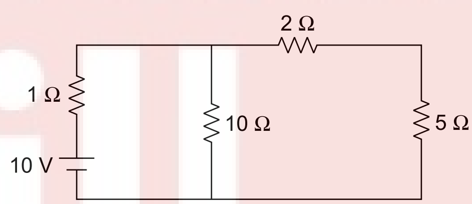


Fig. 2.222

By series–parallel reduction technique,

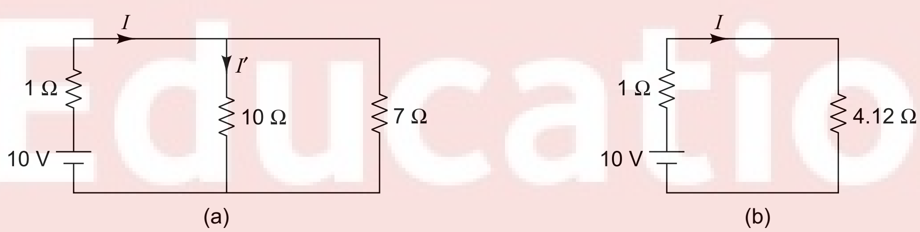


Fig. 2.223

$$I = \frac{10}{1 + 4.12} = 1.95 \text{ A}$$

From Fig. 2.223(a), by current-division rule,

$$I' = 1.95 \times \frac{7}{7+10} = 0.8 \text{ A } (\downarrow)$$

Step II: When the 4 A source is acting alone

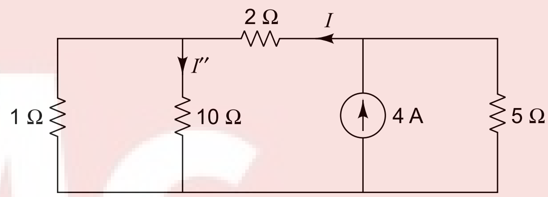


Fig. 2.224

By series–parallel reduction technique,

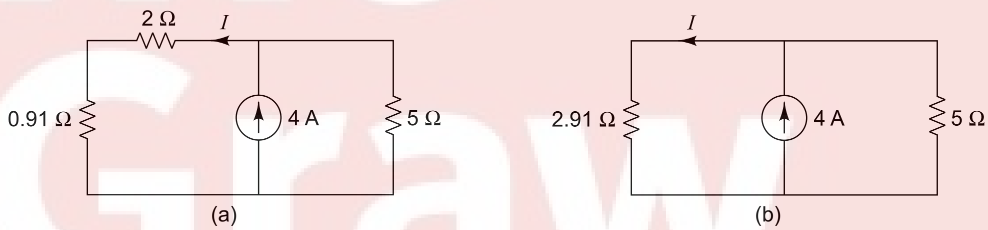


Fig. 2.225

$$I = 4 \times \frac{5}{2.91+5} = 2.53 \text{ A}$$

From Fig. 2.224, by current-division rule,

$$I'' = 2.53 \times \frac{1}{1+10} = 0.23 \text{ A } (\downarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= 0.8 + 0.23 \\ &= 1.03 \text{ A } (\downarrow) \end{aligned}$$

Example 6

Find the current through the 8 Ω resistor.

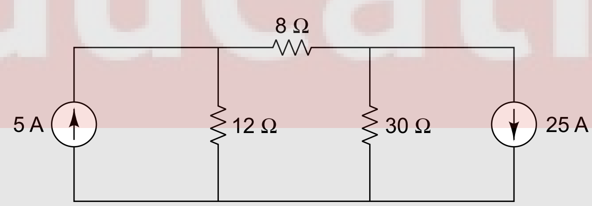


Fig. 2.226

Solution *Step I: When the 5A source is acting alone*

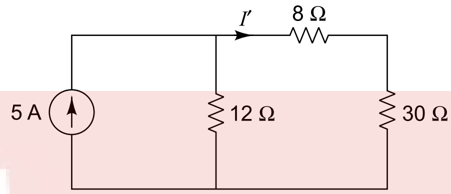


Fig. 2.227

By current-division rule,

$$I' = 5 \times \frac{12}{12+8+30} = 1.2 \text{ A } (\rightarrow)$$

Step II: When the 25 A source is acting alone

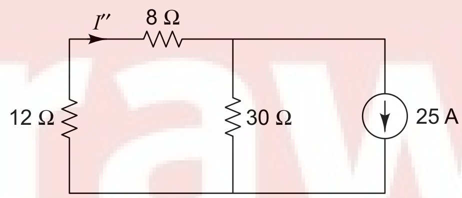


Fig. 2.228

By current-division rule,

$$I'' = 25 \times \frac{30}{30+12+8} = 15 \text{ A } (\rightarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= 1.2 + 15 \\ &= 16.2 \text{ A } (\rightarrow) \end{aligned}$$

Example 7

Find the current through the 4 Ω resistor.

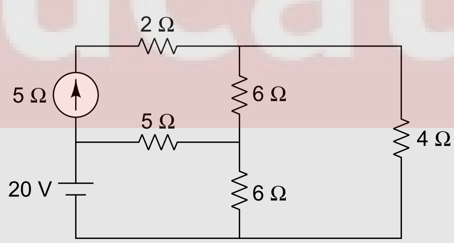


Fig. 2.229

Solution *Step I: When the 5 A source is acting alone*

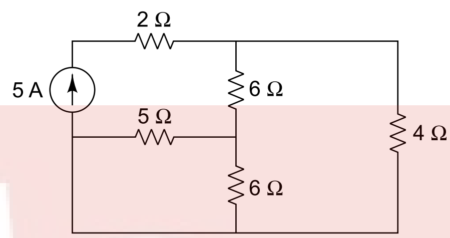


Fig. 2.230

By series–parallel reduction technique,

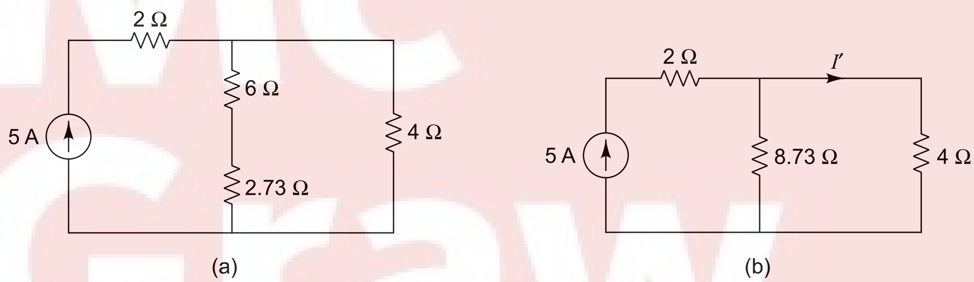


Fig. 2.231

From Fig. 2.231(b), by current-division rule,

$$I' = 5 \times \frac{8.73}{8.73 + 4} = 3.43 \text{ A} (\downarrow)$$

Step II: When the 20 V source is acting alone

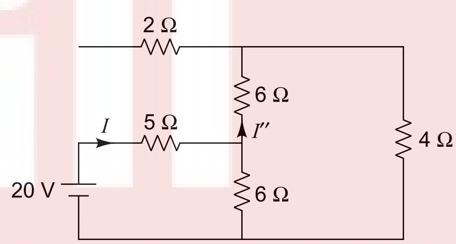


Fig. 2.232

By series–parallel reduction technique.

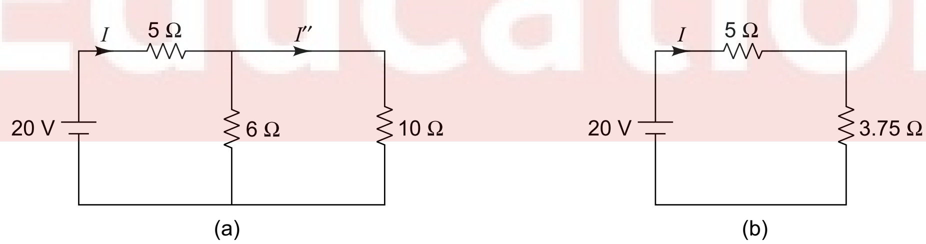


Fig. 2.233

$$I = \frac{20}{5+3.75} = 2.29 \text{ A}$$

From Fig. 2.233(a), by current-division rule,

$$I'' = 2.29 \times \frac{6}{6+10} = 0.86 \text{ A } (\downarrow)$$

Step III: By superposition theorem

$$I = I' + I''$$
$$= 3.43 + 0.86$$
$$= 4.29 \text{ A } (\downarrow)$$

Example 8

Find the current through the 3 Ω resistor.

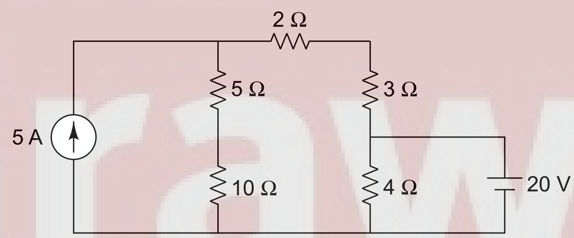


Fig. 2.234

Solution *Step I: When the 5 A source is acting alone*

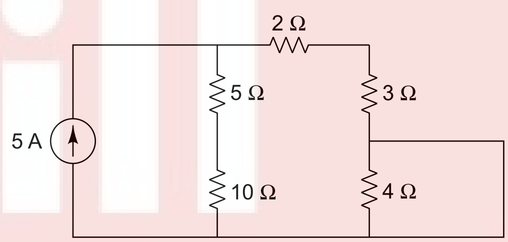


Fig. 2.235

By series–parallel reduction technique,

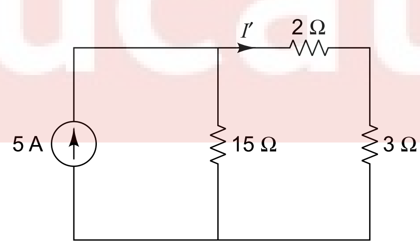


Fig. 2.236

By current-division rule,

$$I' = 5 \times \frac{15}{15+2+3} = 3.75 \text{ A} (\downarrow)$$

Step II: When the 20 V source is acting alone

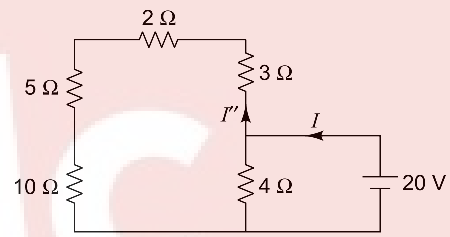


Fig. 2.237

By series–parallel reduction technique,

Fig. 2.238

$$I = \frac{20}{3.33} = 6 \text{ A}$$

From Fig. 2.238(a), by current-division rule,

$$I'' = 6 \times \frac{4}{20+4} = 1 \text{ A} (\uparrow) = -1 \text{ A} (\downarrow)$$

Step III: By superposition theorem

$$\begin{aligned} I &= I' + I'' \\ &= 3.75 - 1 \\ &= 2.75 \text{ A} (\downarrow) \end{aligned}$$

Example 9

Find the current in the 1 Ω resistor.

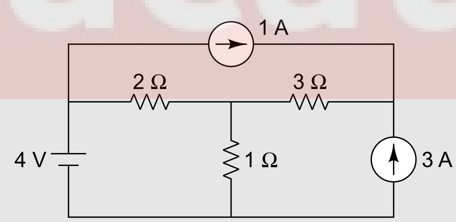


Fig. 2.239

Step I: When the 4 V source is acting alone

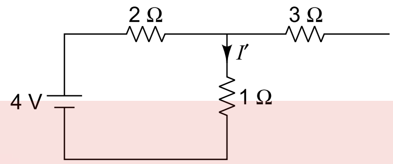


Fig. 2.240

By current-division rule,

$$I' = \frac{4}{2+1} = 1.33 \text{ A } (\downarrow)$$

Step II: When the 3 A source is acting alone

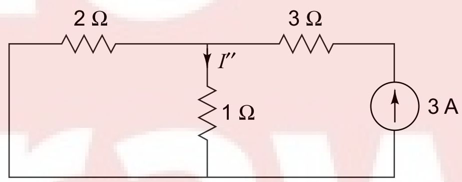


Fig. 2.241

By current-division rule,

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A } (\downarrow)$$

Step III: When the 1 A source is acting alone

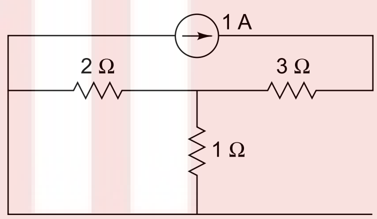


Fig. 2.242

Redrawing the circuit,

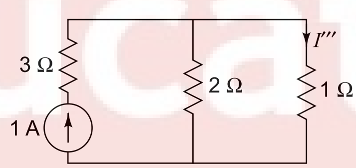


Fig. 2.243

By current-division rule,

$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$
$$= 1.33 + 2 + 0.66$$
$$= 4\text{ A} (\downarrow)$$

Example 10

Find the voltage V_{AB}.

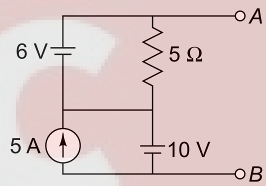


Fig. 2.244

Step I: When the 6 V source is acting alone

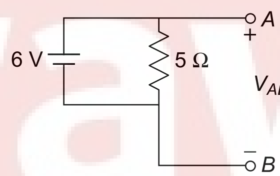


Fig. 2.245

$$V_{AB}' = 6\text{ V}$$

Step II: When the 10 V source is acting alone

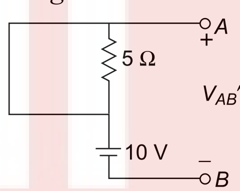


Fig. 2.246

Since the resistor of 5 Ω is shorted, the voltage across it is zero.

$$V_{AB}'' = 10\text{ V}$$

Step III: When the 5 A source is acting alone

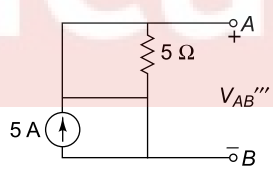


Fig. 2.247

Due to short circuit in both the parts,

$$V_{AB}''' = 0 \text{ V}$$

Step IV: By superposition theorem,

$$\begin{aligned} V_{AB} &= V_{AB}' + V_{AB}'' + V_{AB}''' \\ &= 6 + 10 + 0 \\ &= 16 \text{ V} \end{aligned}$$

Example 11

Find the current through the 5 Ω resistor.

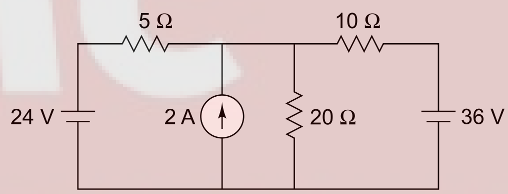


Fig. 2.248

Step I: When the 24 V source is acting alone

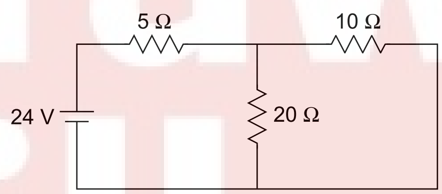


Fig. 2.249

By series–parallel reduction technique,

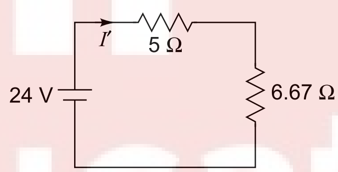


Fig. 2.250

$$I' = \frac{24}{5 + 6.67} = 2.06 \text{ A } (\rightarrow) = -2.06 \text{ A } (\leftarrow)$$

Step II: When the 2 A source is acting alone

By series-parallel reduction technique,

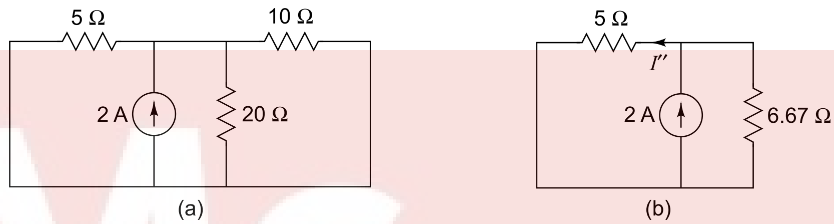


Fig. 2.251

From Fig. 2.251(b), by current-division rule,

$$I'' = 2 \times \frac{6.67}{5 + 6.67} = 1.14 \text{ A } (\leftarrow)$$

Step III: When the 36 V source is acting alone

By series-parallel reduction technique,

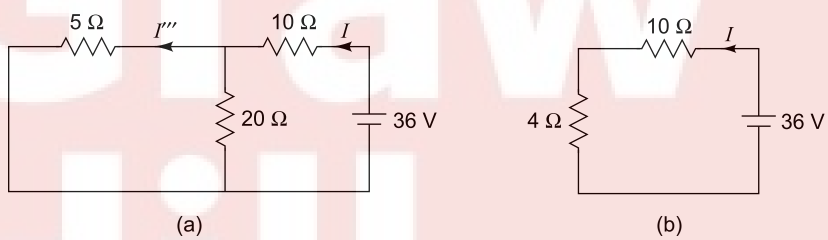


Fig. 2.252

$$I = \frac{36}{10 + 4} = 2.57 \text{ A}$$

From Fig. 2.252(a), by current-division rule,

$$I''' = 2.57 \times \frac{20}{20 + 5} = 2.06 \text{ A } (\leftarrow)$$

Step IV: By superposition theorem,

$$\begin{aligned} I &= I' + I'' + I''' \\ &= -2.06 + 1.14 + 2.06 \\ &= 1.14 \text{ A } (\leftarrow) \end{aligned}$$

Example 12

Find the current through the 4 Ω resistor.

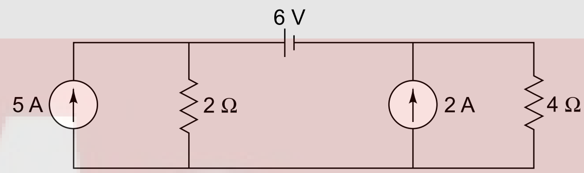


Fig. 2.253

Step I: When the 5 A source is acting alone

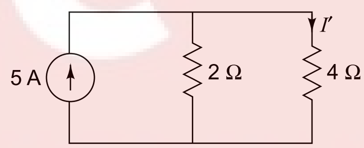


Fig. 2.254

By current-division rule,

$$I' = 5 \times \frac{2}{2+4} = 1.67 \text{ A } (\downarrow)$$

Step II: When the 2 A source is acting alone

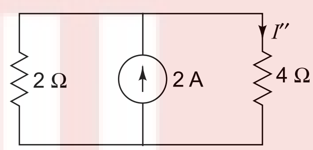


Fig. 2.255

By current-division rule,

$$I'' = 2 \times \frac{2}{2+4} = 0.67 \text{ A } (\downarrow)$$

Step III: When the 6 V source is acting alone

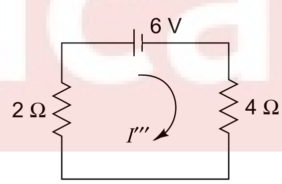


Fig. 2.256

Applying KVL to the loop,

$$-2I''' - 6 - 4I''' = 0$$

$$I''' = -1 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I'''$$

$$= 1.67 + 0.67 - 1$$

$$= 1.34 \text{ A } (\downarrow)$$

Exercise 2.6

2.1 Find the current through the 1 Ω resistor.

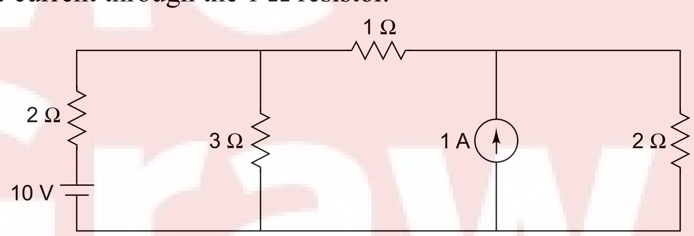


Fig. 2.257

[0.95 A]

2.2 Find the current through the 10 Ω resistor.

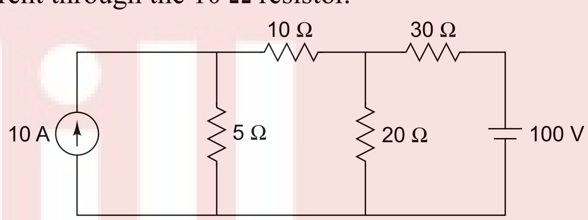


Fig. 2.258

[0.37 A]

2.3 Calculate the current through the 10 Ω resistor.

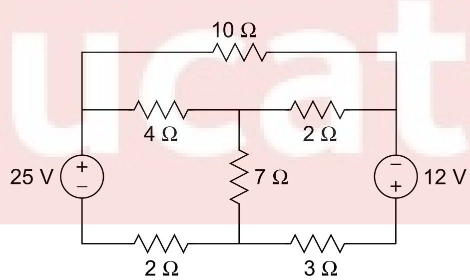


Fig. 2.259

[1.62 A]

2.4 Find the current in the 2 Ω resistor. Also, find voltage across the current source.

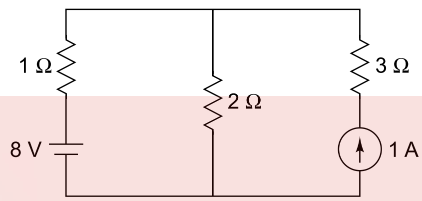


Fig. 2.260

[3 A, 9 V]

2.5 Find the current I_x.

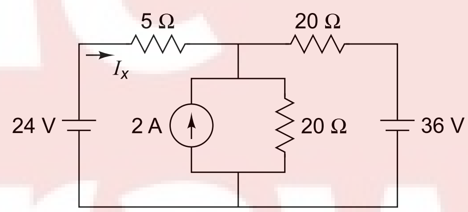


Fig. 2.261

[–0.93 A]

2.7 THEVENIN'S THEOREM

It states that *'Any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the voltage across the two terminals with load, if any, removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source being replaced by its internal resistance (or if it is not given with zero resistance, i.e., short circuit) and constant current source replaced by infinite resistance, i.e., open circuit.'*

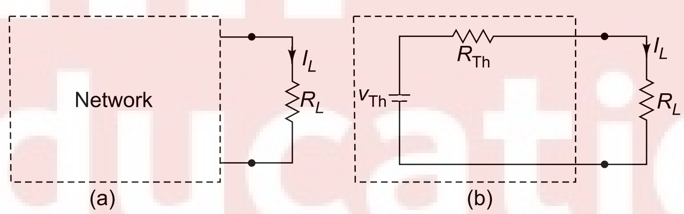


Fig. 2.262 *Thevenin's theorem*

Explanation The above method of determining the load current through a given load resistance can be explained with the help of the following circuit.

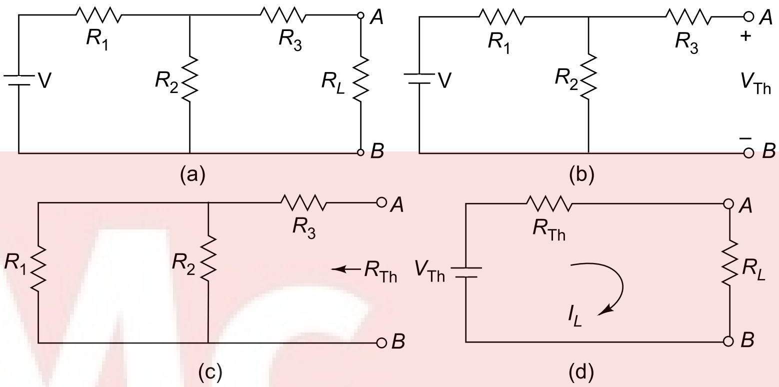


Fig. 2.263 *Steps in Thevenin's theorem*

2.7.1 Steps to be followed in Thevenin's Theorem

1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with the voltage sources and current sources replaced by internal resistances.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

Example 1

Find the current through the 2 Ω resistor.

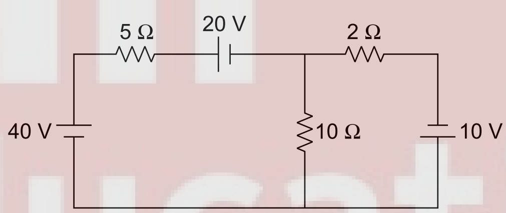


Fig. 2.264

Solution *Step I : Calculation of V_{Th}*

Removing the 2 Ω resistor from the network,

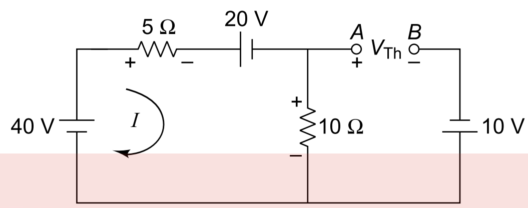


Fig. 2.265

Applying KVL to the mesh,

$$\begin{aligned} 40 - 5I - 20 - 10I &= 0 \\ 15I &= 20 \\ I &= 1.33 \text{ A} \end{aligned}$$

Writing V_{Th} equation,

$$\begin{aligned} 10I - V_{\text{Th}} + 10 &= 0 \\ V_{\text{Th}} &= 10I + 10 \\ &= 10\,(1.33) + 10 \\ &= 23.33 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

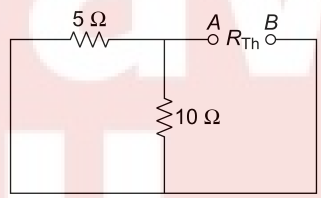


Fig. 2.266

$$R_{\text{Th}} = 5 \parallel 10 = 3.33\ \Omega$$

Step III: Calculation of I_L

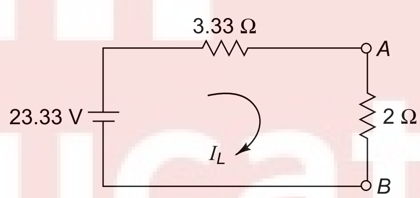


Fig. 2.267

$$I_L = \frac{23.33}{3.33 + 2} = 4.38 \text{ A}$$

Example 2

Find the current through the 8 Ω resistor.

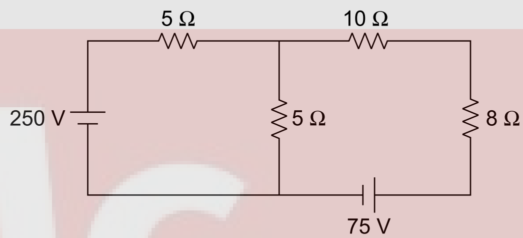


Fig. 2.268

Solution *Step I: Calculation of V_{Th}*

Removing the 8 Ω resistor from the network,

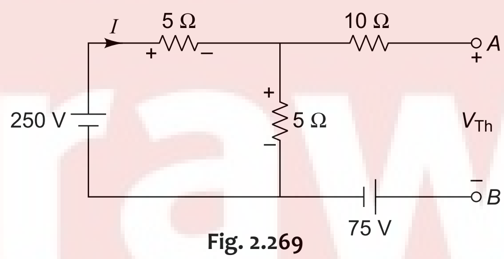


Fig. 2.269

$$I = \frac{250}{5+5} = 25 \text{ A}$$

Writing V_{Th} equation,

$$250 - 5I - V_{Th} - 75 = 0$$

$$V_{Th} = 175 - 5I$$

$$= 175 - 5\,(25)$$

$$= 50 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

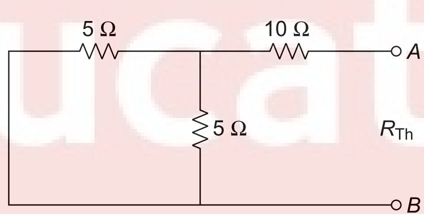


Fig. 2.270

$$R_{Th} = (5 \parallel 5) + 10 = 12.5 \text{ } \Omega$$

Step III: Calculation of I_L

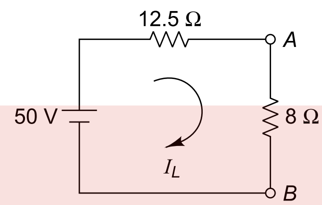


Fig. 2.271

$$I_L = \frac{50}{12.5+8} = 2.44 \text{ A}$$

Example 3

Find the current through the 2 Ω resistor connected between terminals A and B.

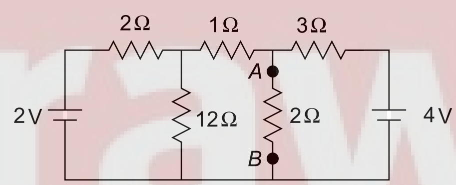


Fig. 2.272

Solution

Step I: Calculation of V_{Th}

Removing the 2 Ω resistor connected between terminals *A* and *B*,

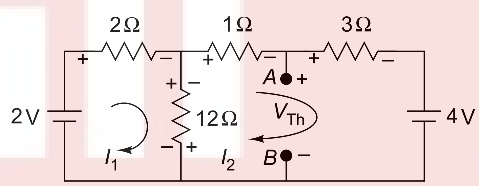


Fig. 2.273

Applying KVL to Mesh 1,

$$2 - 2I_1 - 12(I_1 - I_2) = 0$$

$$14I_1 - 12I_2 = 2 \quad (1)$$

Applying KVL to Mesh 2,

$$-12(I_2 - I_1) - 1I_2 - 3I_2 - 4 = 0$$

$$-12I_1 + 16I_2 = -4 \quad (2)$$

Solving Eqs (1) and (2),

$$I_2 = -0.4 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 3I_2 - 4 = 0$$
$$V_{\text{Th}} = 4 + 3I_2$$
$$= 4 + 3\,(-0.4)$$
$$= 2.8 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing all voltage sources by short circuits,

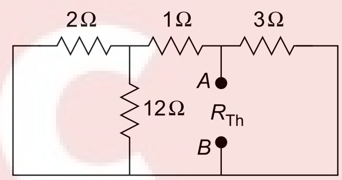


Fig. 2.274

$$R_{\text{Th}} = [|(2\|\ 12) + 1] \ \|\ 3 = 1.43\ \Omega$$

Step III: Calculation of I_L

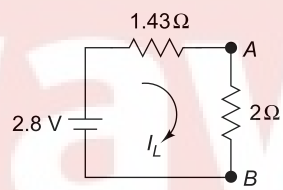


Fig. 2.275

$$I_L = \frac{2.8}{1.43 + 2} = 0.82 \text{ A}$$

Example 4

Find the current through the 10 Ω resistor.

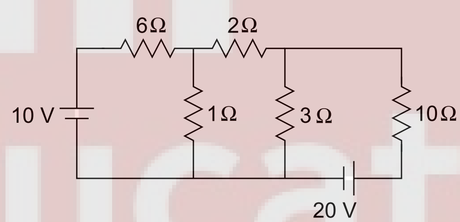


Fig. 2.276

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

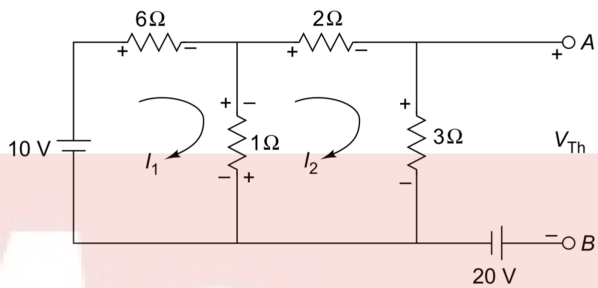


Fig. 2.277

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1(I_2 - I_1) - 2I_2 - 3I_2 = 0$$

$$I_1 - 6I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_2 = 0.24 \text{ A}$$

Writing V_{Th} equation,

$$3I_2 - V_{Th} - 20 = 0$$

$$V_{Th} = 3I_2 - 20$$

$$= 3\,(0.24) - 20$$

$$= -19.28 \text{ V}$$

$$= 19.28 \text{ V (terminal } B \text{ is positive w.r.t } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

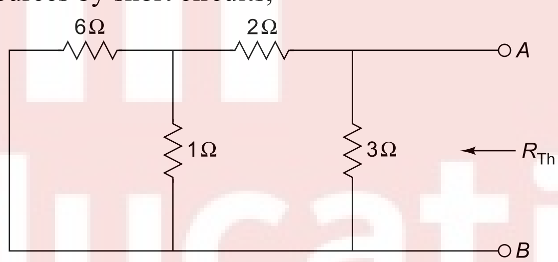


Fig. 2.278

$$R_{Th} = [(6 \| 1) + 2] \| 3 = 1.47\ \Omega$$

Step III: Calculation of I_L

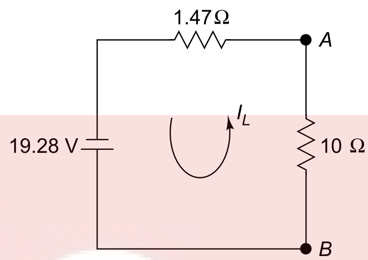


Fig. 2.279

$$I_L = \frac{19.28}{10+1.47} = 1.68 \text{ A } (\uparrow)$$

Example 5

Find the current through the 10 Ω resistor.

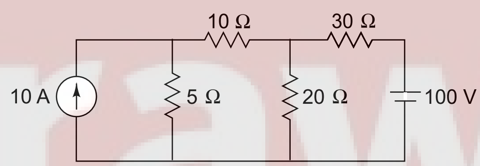


Fig. 2.280

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

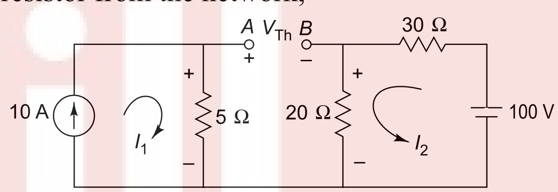


Fig. 2.281

For Mesh 1,

$$I_1 = 10$$

Applying KVL to Mesh 2,

$$100 - 30I_2 - 20I_2 = 0$$
$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$5I_1 - V_{\text{Th}} - 20I_2 = 0$$
$$V_{\text{Th}} = 5I_1 - 20I_2$$
$$= 5(10) - 20(2)$$
$$= 10 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source by an open circuit and the voltage source by a short circuit,

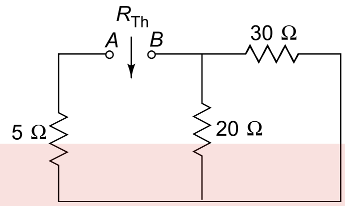


Fig. 2.282

$$R_{Th} = 5 + (20 \,||\, 30) = 17\ \Omega$$

Step III: Calculation of I_L

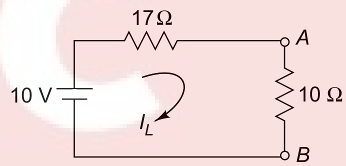


Fig. 2.283

$$I_L = \frac{10}{17+10} = 0.37\ \text{A}$$

Example 6

Find the current through the 40 Ω resistor.

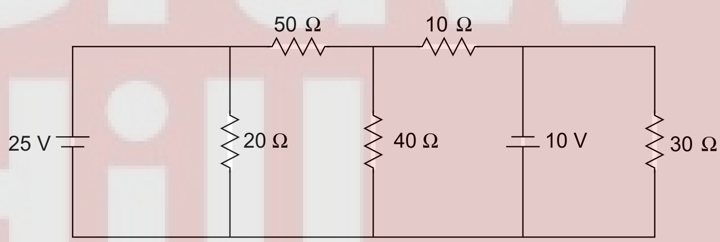


Fig. 2.284

Solution

Step I : Calculation of V_{Th}

Removing the 40 Ω resistor from the network,

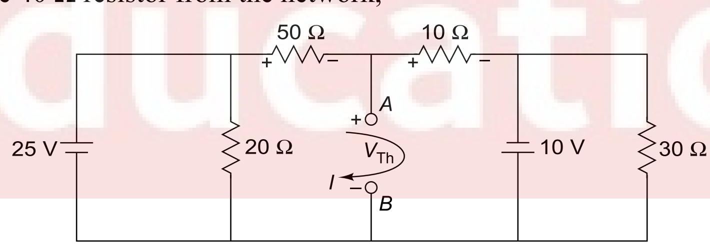


Fig. 2.285

Since the 20 Ω resistor is connected across the 25 V source, the resistor becomes redundant.

$$V_{20\ \Omega} = 25\ \text{V}$$

Applying KVL to the mesh,

$$25 - 50I - 10I + 10 = 0$$

$$I = 0.58 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} - 10I + 10 = 0$$

$$V_{Th} = 10\,(I) - 10$$
$$= 10\,(0.58) - 10$$
$$= -4.2 \text{ V}$$
$$= 4.2 \text{ V (terminal } B \text{ is positive w.r.t. A)}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits,

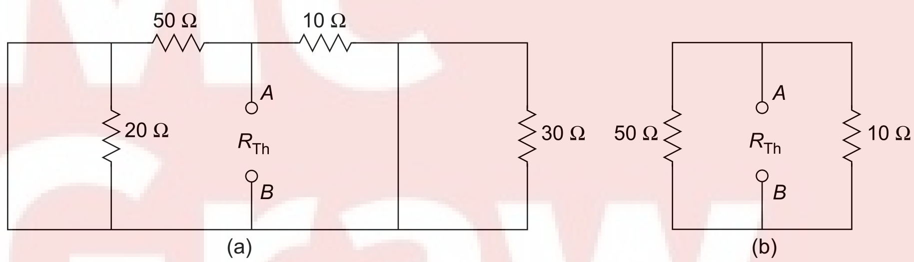


Fig. 2.286

$$R_{Th} = 50 \parallel 10 = 8.33\ \Omega$$

Step III: Calculation of I_L

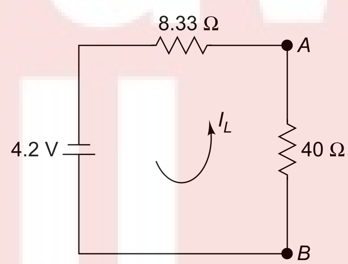


Fig. 2.287

$$I_L = \frac{4.2}{8.33 + 40} = 0.09 \text{ A } (\uparrow)$$

Example 7

Find the current through the 10 Ω resistor.

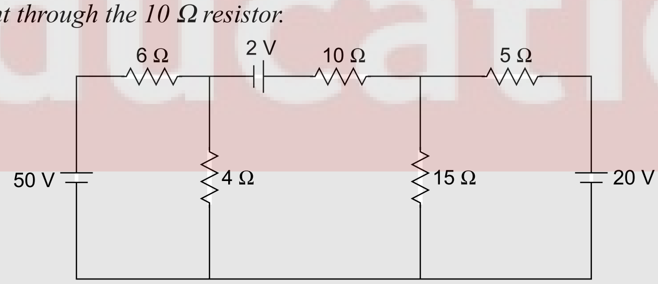


Fig. 2.288

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

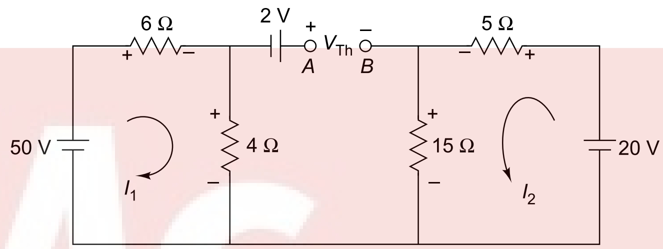


Fig. 2.289

$$I_1 = \frac{50}{10} = 5 \text{ A}$$

$$I_2 = \frac{20}{20} = 1 \text{ A}$$

Writing V_{Th} equation,

$$4I_1 + 2 - V_{\text{Th}} - 15I_2 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 4I_1 + 2 - 15I_2 \\ &= 4(5) + 2 - 15(1) \\ &= 7\ V \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

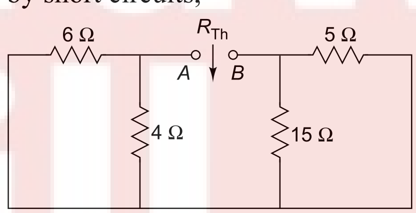


Fig. 2.290

$$R_{\text{Th}} = (6 \parallel 4) + (5 \parallel 15) = 6.15\ \Omega$$

Step III: Calculation of I_L

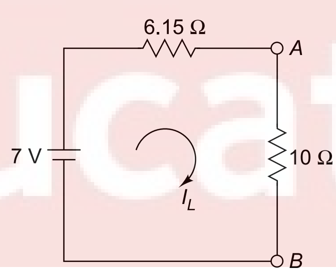


Fig. 2.291

$$I_L = \frac{7}{6.15 + 10} = 0.43 \text{ A}$$

Example 8

Determine the current through the 24 Ω resistor.

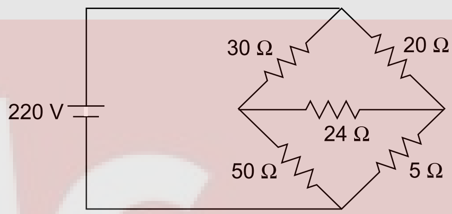


Fig. 2.292

Solution

Step I: Calculation of V_{Th}

Removing the 24 Ω resistor from the network,

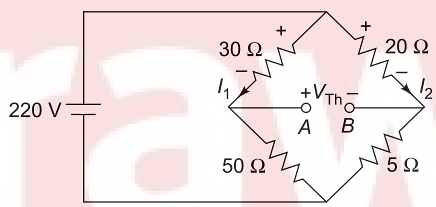


Fig. 2.293

$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} + 30I_1 - 20I_2 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 20I_2 - 30I_1 \\ &= 20\,(8.8) - 30\,(2.75) \\ &= 93.5 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

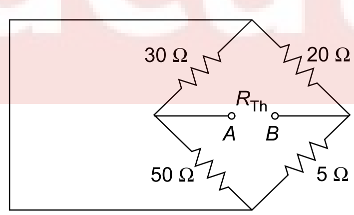


Fig. 2.294

Redrawing the circuit,

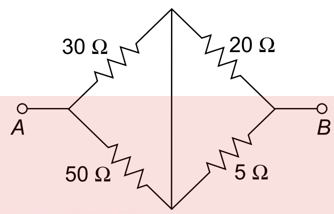


Fig. 2.295

$$R_{\text{Th}} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75\ \Omega$$

Step III: Calculation of I_L

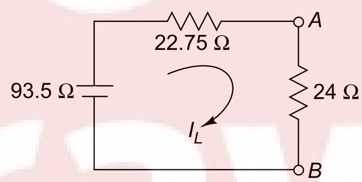


Fig. 2.296

$$I_L = \frac{93.5}{22.75 + 24} = 2\text{ A}$$

Example 9

Find the current through the 3 Ω resistor.

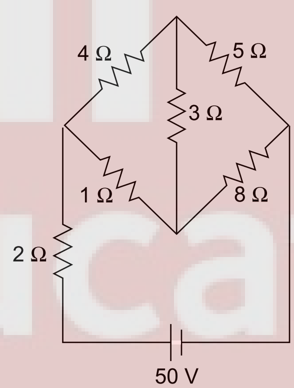


Fig. 2.297

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

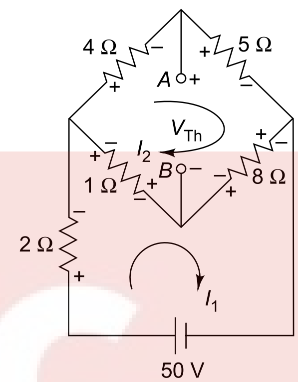


Fig. 2.298

Applying KVL to Mesh 1,

$$50 - 2I_1 - 1(I_1 - I_2) - 8(I_1 - I_2) = 0$$

$$11I_1 - 9I_2 = 50 \qquad (1)$$

Applying KVL to Mesh 2,

$$-4I_2 - 5I_2 - 8(I_2 - I_1) - 1(I_2 - I_1) = 0$$

$$-9I_1 + 18I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 7.69 \text{ A}$$

$$I_2 = 3.85 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} - 5I_2 - 8(I_2 - I_1) = 0$$

$$\begin{aligned} V_{Th} &= 5I_2 + 8\,(I_2 - I_1) \\ &= 5(3.85) + 8(3.85 - 7.69) \\ &= -11.47 \text{ V} \\ &= 11.47 \text{ V (the terminal } B \text{ is positive w.r.t. } A) \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

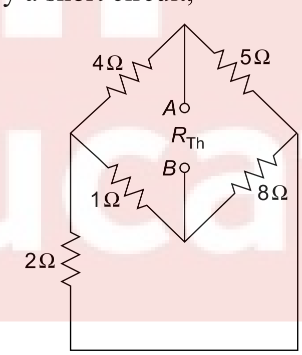


Fig. 2.299

Redrawing the network,

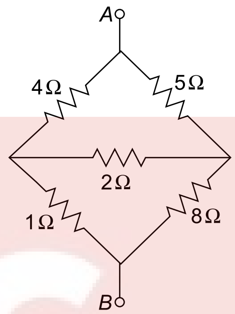


Fig. 2.300

Converting the upper delta into equivalent star network,

$$R_1 = \frac{4 \times 2}{4+2+5} = 0.73\ \Omega$$

$$R_2 = \frac{4 \times 5}{4+2+5} = 1.82\ \Omega$$

$$R_3 = \frac{5 \times 2}{4+2+5} = 0.91\ \Omega$$

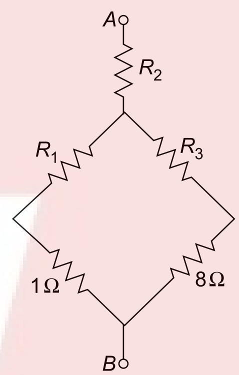


Fig. 2.301

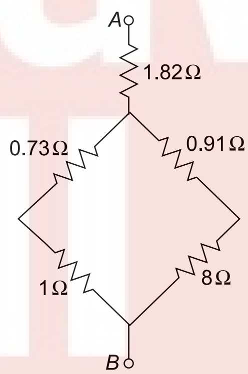


Fig. 2.302

Simplifying the network,

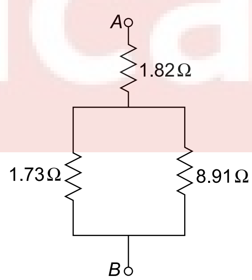


Fig. 2.303

$$R_{\text{Th}} = 1.82 + (1.73\,||\,8.91) = 3.27\ \Omega$$

Step III: Calculation of I_L

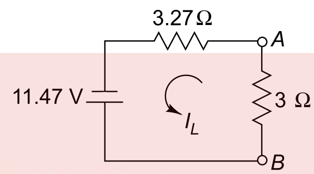


Fig. 2.304

$$I_L = \frac{11.47}{3.27 + 3} = 1.83\ \text{A}\ (\uparrow)$$

Example 10

Find the current through the 20 Ω resistor.

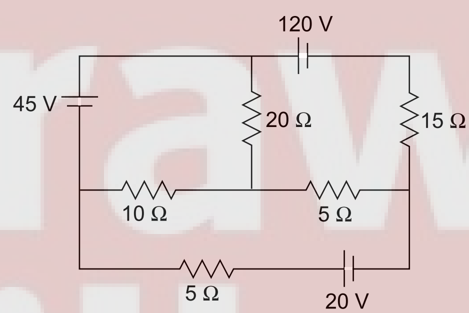


Fig. 2.305

Solution

Step I: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

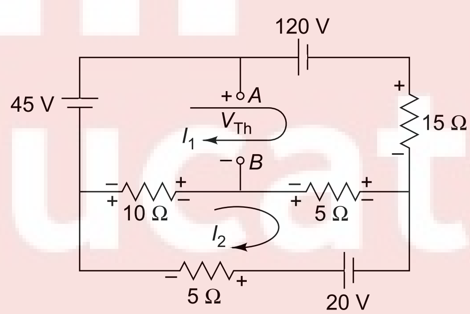


Fig. 2.306

Applying KVL to Mesh 1,

$$45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0$$

$$30I_1 - 15I_2 = -75 \qquad (1)$$

Applying KVL to Mesh 2,

$$20 - 5I_2 - 10(I_2 - I_1) - 5(I_2 - I_1) = 0$$

$$-15I_1 + 20I_2 = 20 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -3.2 \text{ A}$$
$$I_2 = -1.4 \text{ A}$$

Writing V_{Th} equation,

$$45 - V_{\text{Th}} - 10\,(I_1 - I_2) = 0$$

$$V_{Th} = 45 - 10\,(I_1 - I_2)$$
$$= 45 - 10\,[-3.2 - (-1.4)]$$
$$= 63 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

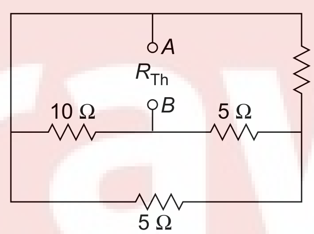


Fig. 2.307

Converting the delta formed by resistors of 10 Ω, 5 Ω and 5 Ω into an equivalent star network,

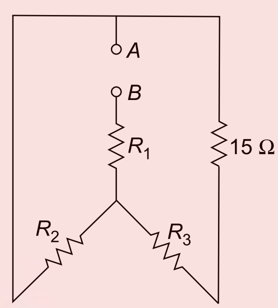


Fig. 2.308

$$R_1 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_2 = \frac{10 \times 5}{20} = 2.5\ \Omega$$

$$R_3 = \frac{5 \times 5}{20} = 1.25\ \Omega$$

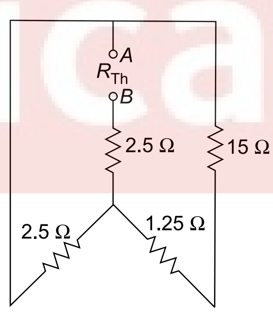


Fig. 2.309

Simplifying the network,

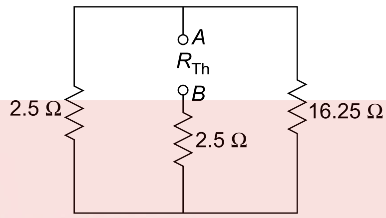


Fig. 2.310

$$R_{Th} = (16.25 \,||\, 2.5) + 2.5 = 4.67\ \Omega$$

Step III: Calculation of I_L

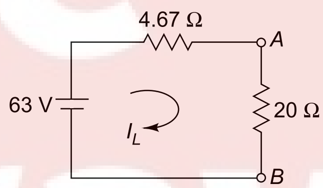


Fig. 2.311

$$I_L = \frac{63}{4.67 + 20} = 2.55\ \text{A}$$

Example 11

Find the current through the 3 Ω resistor.

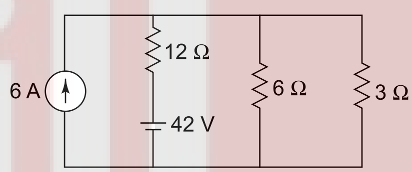


Fig. 2.312

Solution

Step I: Calculation of V_{Th}

Removing the 3 Ω resistor from the network,

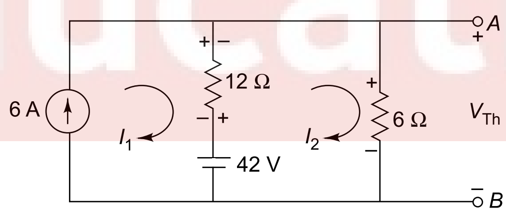


Fig. 2.313

Writing equation for Mesh 1,

$$I_1 = 6 \quad (1)$$

Applying KVL to Mesh 2,

$$42 - 12(I_2 - I_1) - 6\,I_2 = 0$$

$$-12\,I_1 + 18\,I_2 = 42 \quad (2)$$

Solving Eqs (1) and (2),

$$I_2 = 6.33 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 6\,I_2 = 38 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by short circuit and current source by open circuit,

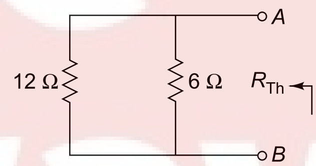


Fig. 2.314

$$R_{\text{Th}} = 6 \,||\, 12 = 4\ \Omega$$

Step III: Calculation of I_L

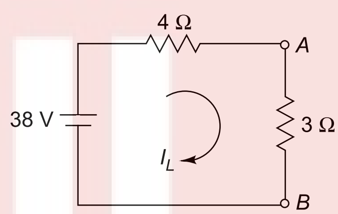


Fig. 2.315

$$I_L = \frac{38}{4+3} = 5.43 \text{ A}$$

Example 12

Find the current through the 30 Ω resistor.

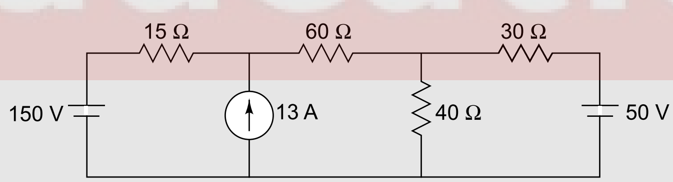


Fig. 2.316

Solution

Step I: Calculation of V_{Th}

Removing the 30 Ω resistor from the network,

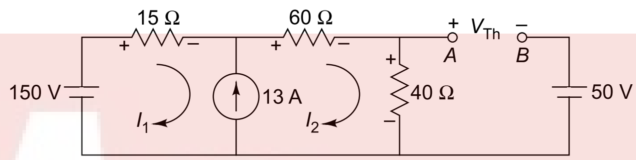


Fig. 2.317

Meshes 1 and 2 form a loop.

Writing current equation for loop,

$$I_2 - I_1 = 13 \qquad (1)$$

Writing voltage equation for loop,

$$150 - 15I_1 - 60I_2 - 40I_2 = 0$$

$$15I_1 + 100I_2 = 150 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -10 \text{ A}$$

$$I_2 = 3 \text{ A}$$

Writing V_{Th} equation,

$$40I_2 - V_{\text{Th}} - 50 = 0$$

$$V_{\text{Th}} = 40I_2 - 50$$

$$= 40(3) - 50$$

$$= 70 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

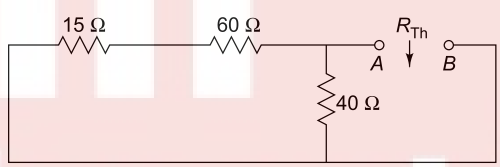


Fig. 2.318

$$R_{\text{Th}} = 75 \parallel 40 = 26.09 \ \Omega$$

Step III: Calculation of I_L

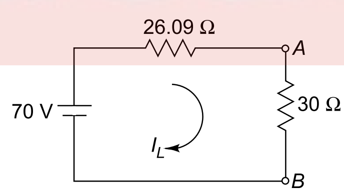


Fig. 2.319

$$I_L = \frac{70}{26.09+30} = 1.25 \text{ A}$$

Example 13

Find the current through the 20 Ω resistor.

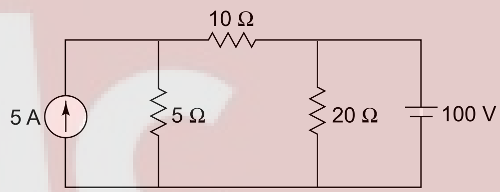


Fig. 2.320

Solution

Step I: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

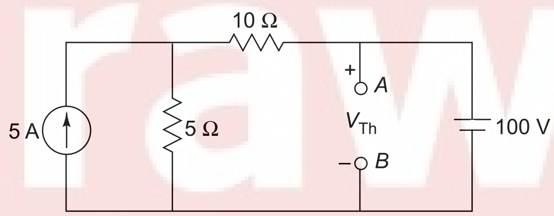


Fig. 2.321

From Fig. 2.321,

$$V_{\text{Th}} = 100 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

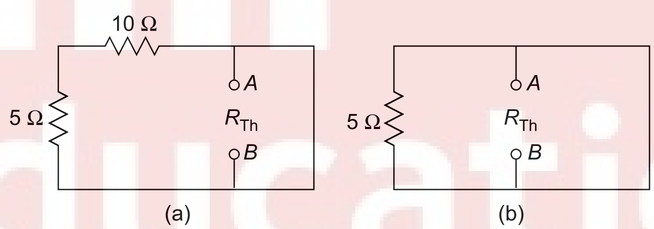


Fig. 2.322

$$R_{\text{Th}} = 0$$

Step III: Calculation of I_L

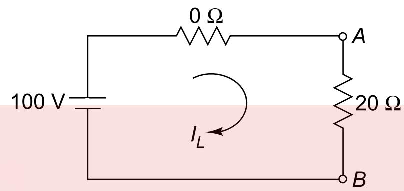


Fig. 2.323

$$I_L = \frac{100}{20} = 5 \text{ A}$$

Example 14

Find the current through the 20 Ω resistor.

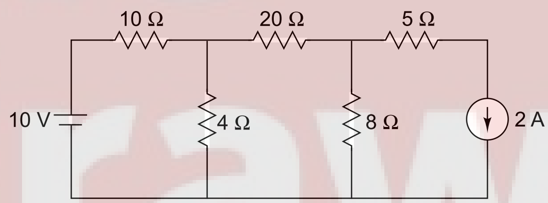


Fig. 2.324

Solution

Step 1: Calculation of V_{Th}

Removing the 20 Ω resistor from the network,

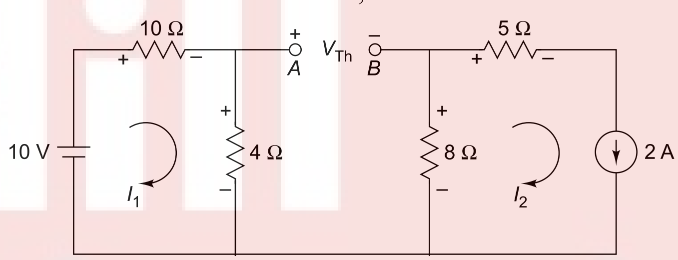


Fig. 2.325

$$I_1 = \frac{10}{10+4} = 0.71 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Writing the V_{Th} equation,

$$4\,I_1 - V_{\text{Th}} + 8\,I_2 = 0$$

$$V_{\text{Th}} = 4\,(0.71) + 8\,(2)$$

$$= 18.84 \text{ V}$$

Step II : Calculation of R_{Th}

Replacing the voltage source by short circuit and current source by an open circuit,

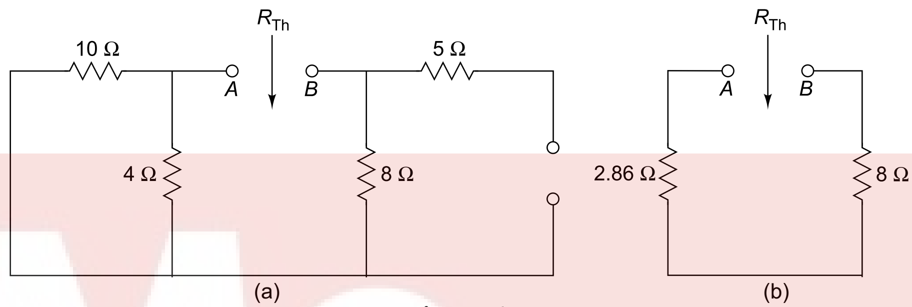


Fig. 2.326

$$R_{Th} = 10.86\ \Omega$$

Step III : Calculation of I_L

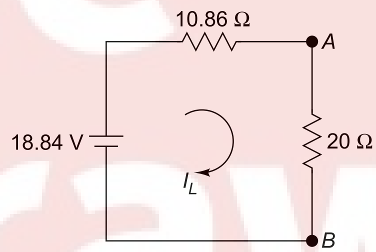


Fig. 2.327

$$I_L = \frac{18.84}{10.86 + 20} = 0.61\ \text{A}$$

Example 15

Find the current through the 5 Ω resistor.

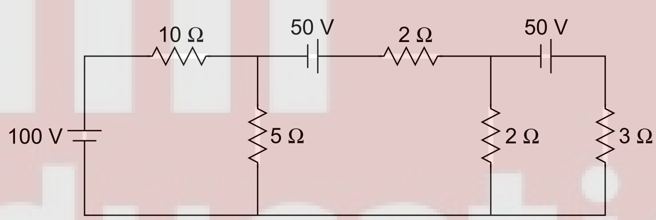


Fig. 2.328

Solution

Step I: Calculation of V_{Th}

Removing the 5 Ω resistor from the network,

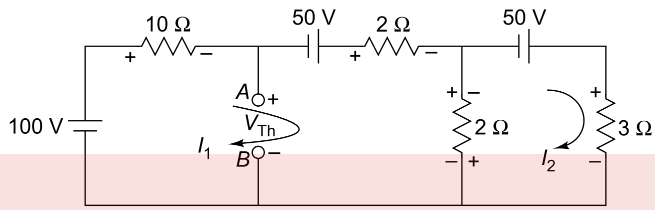


Fig. 2.329

Applying KVL to Mesh 1,

$$100 - 10\,I_1 + 50 - 2\,I_1 - 2\,(I_1 - I_2) = 0$$

$$14\,I_2 - 2\,I_2 = 150 \qquad (1)$$

Applying KVL to Mesh 2,

$$-2\,(I_2 - I_1) + 50 - 3\,I_2 = 0$$

$$-2\,I_1 + 5\,I_2 = 50 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 12.88 \text{ A}$$

$$I_2 = 15.15 \text{ A}$$

Writing the V_{Th} equation,

$$100 - 10\,I_1 - V_{Th} = 0$$

$$V_{Th} = 100 - 10\,(12.88)$$

$$= -28.8 \text{ V}$$

$$= 28.8 \text{ V (terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

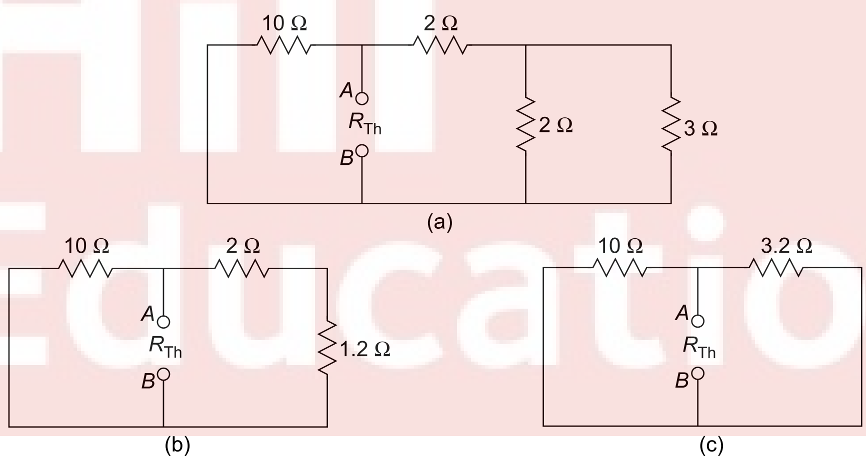


Fig. 2.330

$$R_{Th} = 10 \,\|\, 3.2 = 2.42\ \Omega$$

Step III: Calculation of I_L

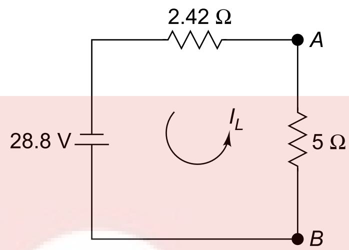


Fig. 2.331

$$I_L = \frac{28.8}{2.42+5} = 3.88 \text{ A} (\uparrow)$$

Example 16

Find the current through the 10 Ω resistor.

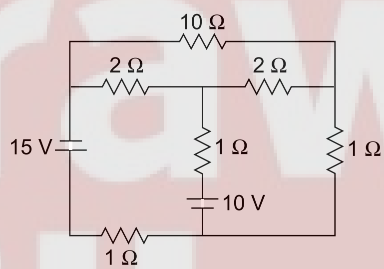


Fig. 2.332

Solution

Step I: Calculation of V_{Th}

Removing the 10 Ω resistor from the network,

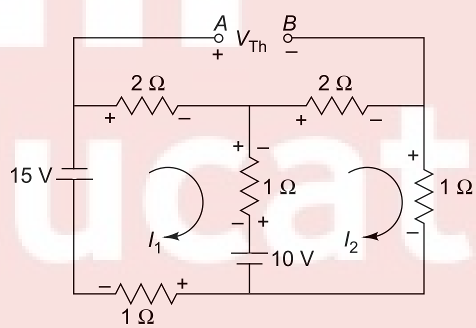


Fig. 2.333

Applying KVL to Mesh 1,

$-15 - 2I_1 - 1\,(I_1 - I_2) - 10 - 1I_1 = 0$

$$4I_1 - I_2 = -25 \qquad (1)$$

Applying KVL to Mesh 2,

$$10 - 1(I_2 - I_1) - 2I_2 - I_2 = 0$$

$$-I_1 + 4I_2 = 10 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -6 \text{ A}$$

$$I_2 = 1 \text{ A}$$

Writing V_{Th} equation,

$$-V_{Th} + 2I_2 + 2I_1 = 0$$

$$V_{Th} = 2I_1 + 2I_2$$

$$= 2(-6) + 2(1)$$

$$= -10 \text{ V}$$

$$= 10 \text{ V (the terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

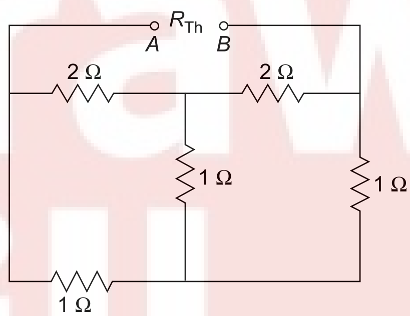


Fig. 2.334

Converting the star network formed by resistors of 2 Ω, 2 Ω and 1 Ω into an equivalent delta network.

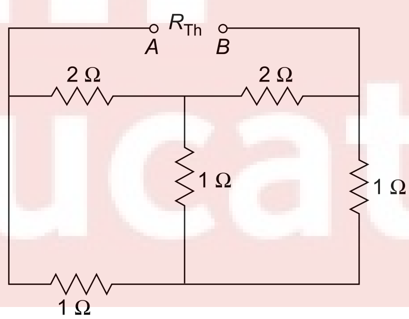


Fig. 2.335

$$R_1 = 2 + 2 + \frac{2 \times 2}{1} = 8\ \Omega$$

$$R_2 = 2 + 1 + \frac{2 \times 1}{2} = 4\ \Omega$$

$$R_3 = 2 + 1 + \frac{2 \times 1}{2} = 4\ \Omega$$

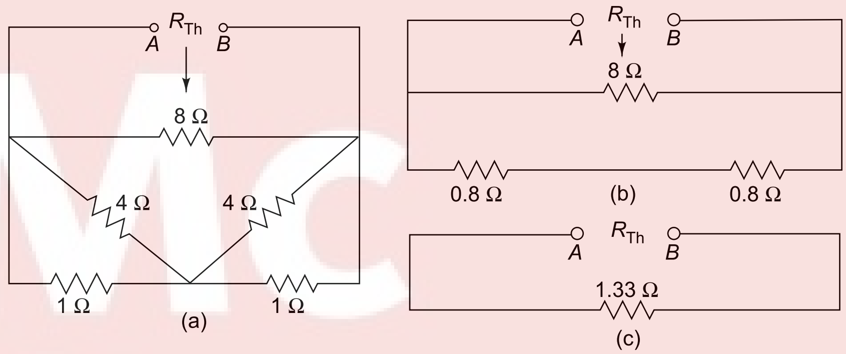


Fig. 2.336

$$R_{\text{Th}} = 1.33\ \Omega$$

Step III: Calculation of I_L

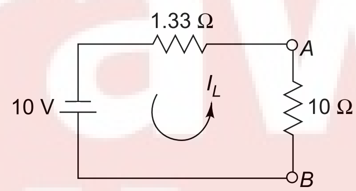


Fig. 2.337

$$I_L = \frac{10}{1.33 + 10} = 0.88\ \text{A}\ (\uparrow)$$

Example 17

Find the current through the 1 Ω resistor.

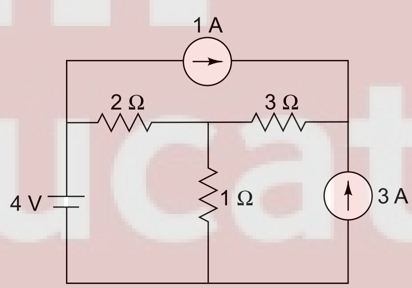


Fig. 2.338

Solution

Step I: Calculation of V_{Th}

Removing the 1 Ω resistor from the network,

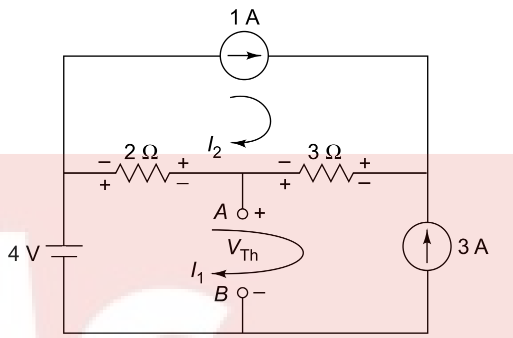


Fig. 2.339

Writing the current equation for meshes 1 and 2,

$$I_1 = -3$$
$$I_2 = 1$$

Writing V_{Th} equation,

$$4 - 2\,(I_1 - I_2) - V_{Th} = 0$$
$$V_{Th} = 4 - 2(-3 - 1)$$
$$= 4 - 2(-4)$$
$$= 12 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

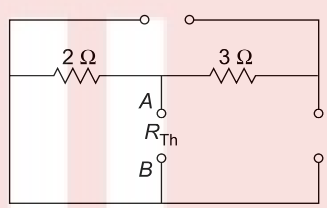


Fig. 2.340

$$R_{Th} = 2\ \Omega$$

Step III: Calculation of I_L

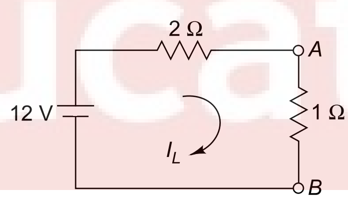


Fig. 2.341

$$I_L = \frac{12}{2+1} = 4 \text{ A}$$

Example 18

Find the current through the 3 Ω resistor.

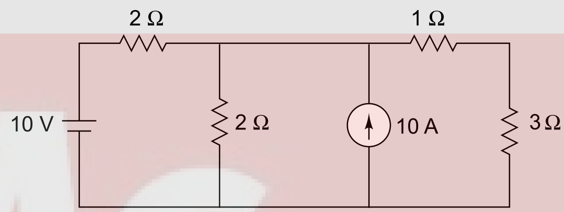


Fig. 2.342

Solution *Step I: Calculation of V_{Th}*

Removing the 3 Ω resistor from the network,

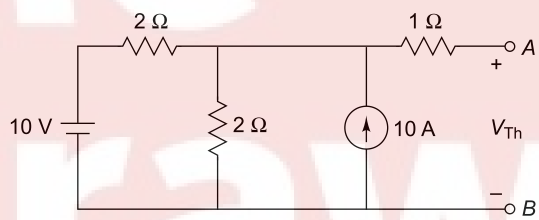


Fig. 2.343

By source transformation,

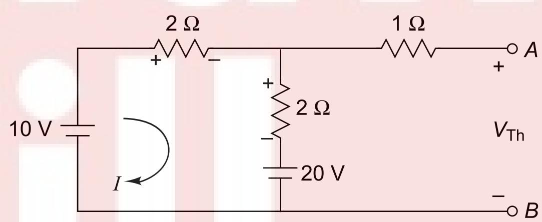


Fig. 2.344

Applying KVL to the mesh,

$$10 - 2I - 2I - 20 = 0$$
$$4I = -10$$
$$I = -2.5 \text{ A}$$

Writing V_{Th} equation,

$$10 - 2I - V_{Th} = 0$$
$$V_{Th} = 10 - 2I$$
$$= 10 - 2(-2.5)$$
$$= 15 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit and current source by an open circuit,

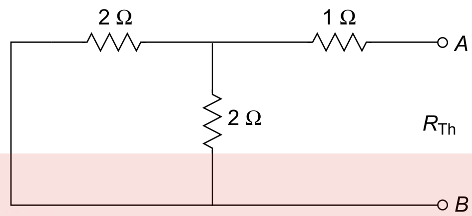


Fig. 2.345

$$R_{Th} = (2 \parallel 2) + 1 = 1 + 1 = 2\ \Omega$$

Step III: Calculation of I_L

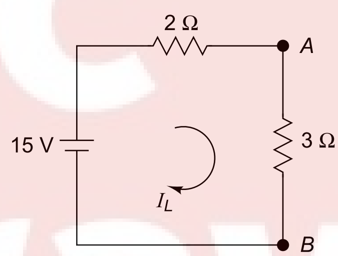


Fig. 2.346

$$I_L = \frac{15}{2+3} = 3\ \text{A}$$

Exercise 2.7

2.1 Find the current through the 6 Ω resistor.

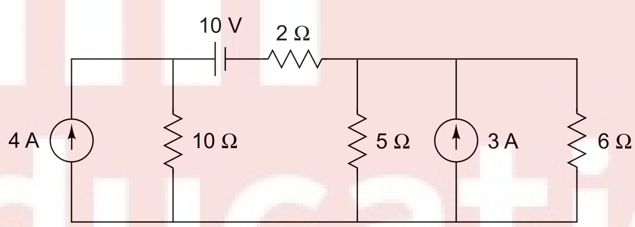


Fig. 2.347

[2.04 A]

2.2 Find the current through the 2 Ω resistor connected between terminals *A* and *B*.

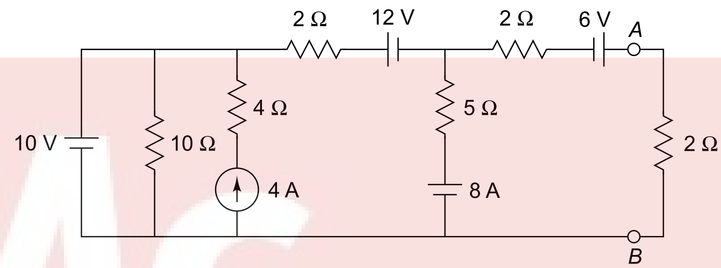


Fig. 2.348

[1.26 A]

2.3 Find the current through the 5 Ω resistor.

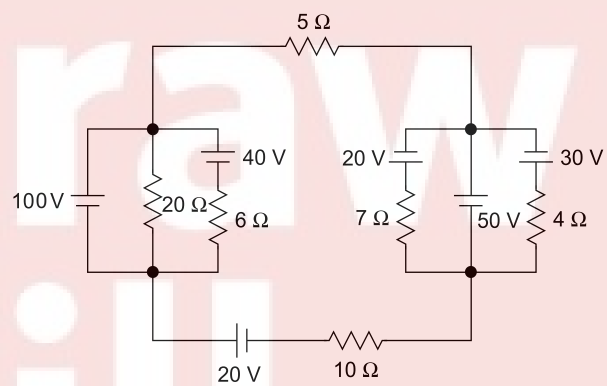


Fig. 2.349

[4.67 A]

2.4 Find the current through the 20 Ω resistor.

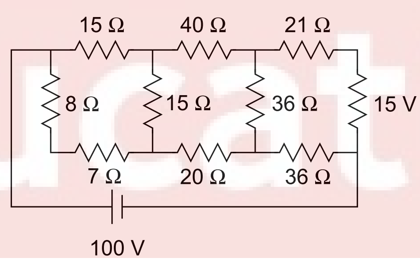


Fig. 2.350

[1.54 A]

2.5 Calculate the current through the 10 Ω resistor.

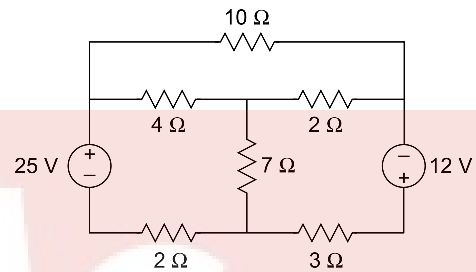


Fig. 2.351

[1.62 A]

2.6 Find the current through the 2 Ω resistor.

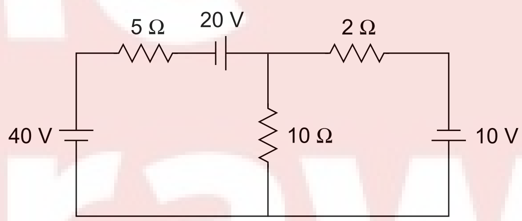


Fig. 2.352

[9.375 A]

2.7 Find the current through the 5 Ω resistor.

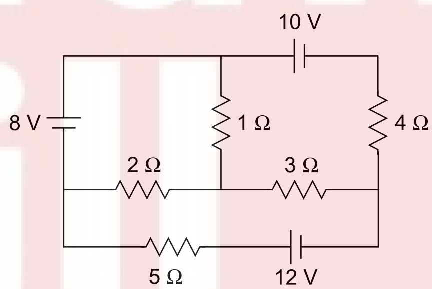


Fig. 2.353

[2 A]

2.8 NORTON'S THEOREM

It states that *'Any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.' The constant current is equal to the current which would flow in a short circuit placed across the terminals. The parallel resistance is the resistance of the network when viewed from these open-circuited terminals after all voltage and current sources have been removed and replaced by internal resistances*.

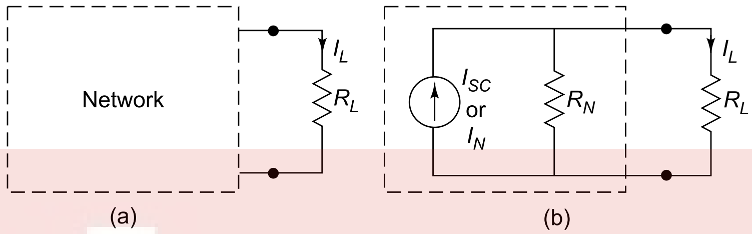


Fig. 2.354 *Norton's theorem*

Explanation The method of determining the load current through a given load resistance can be explained with the help of the following circuit.

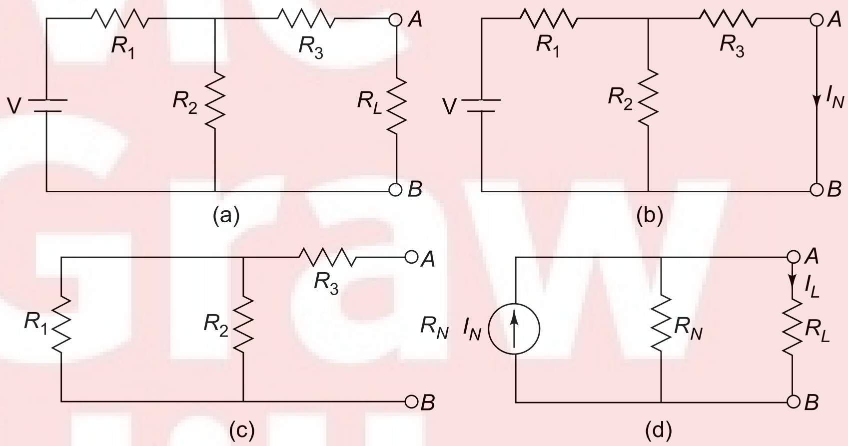


Fig. 2.355 *Steps in Norton's theorem*

2.8.1 Steps to be followed in Norton's Theorem

1. Remove the load resistance R_L and put a short circuit across the terminals.
2. Find the short-circuit current I_{sc} or I_N.
3. Find the resistance R_N as seen from points A and B by replacing the voltage sources and current sources by internal resistances.
4. Replace the network by a current source I_N in parallel with resistance R_N.
5. Find current through R_N by current–division rule,

$$I_L = \frac{I_N R_N}{R_N + R_L}$$

Example 1

Find the current through the 10 Ω resistor.

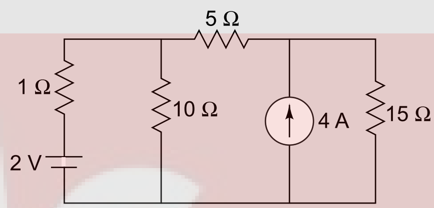


Fig. 2.356

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short cirrcuit,

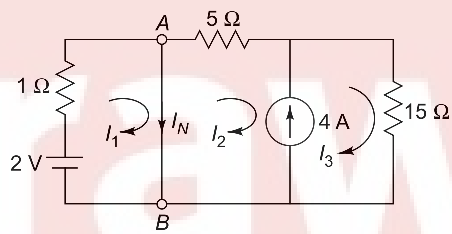


Fig. 2.357

Applying KVL to Mesh 1,

$$2 - 1I_1 = 0 \tag{1}$$

$$I_1 = 2$$

Meshes 2 and 3 will form a loop.

Writing current equation for the loop,

$$I_3 - I_2 = 4 \tag{2}$$

Applying KVL to the loop,

$$-5I_2 - 15I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = 2\text{ A}$$

$$I_2 = -3\text{ A}$$

$$I_3 = 1\text{ A}$$

$$I_N = I_1 - I_2 = 2 - (-3) = 5\text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and current source by an open circuit,

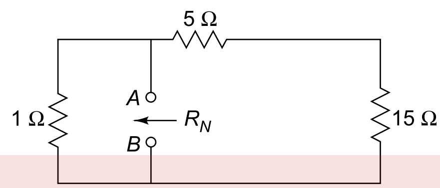


Fig. 2.358

$$R_N = 1 \parallel (5 + 15) = 0.95\ \Omega$$

Step III: Calculation of I_L

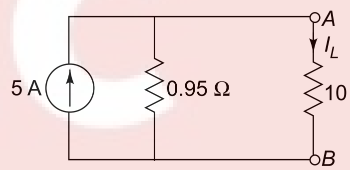


Fig. 2.359

$$I_L = 5 \times \frac{0.95}{10 + 0.95} = 0.43\text{ A}$$

Example 2

Find the current through the 10 Ω resistor.

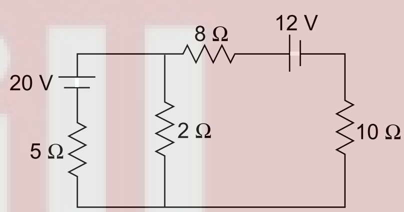


Fig. 2.360

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circircuit,

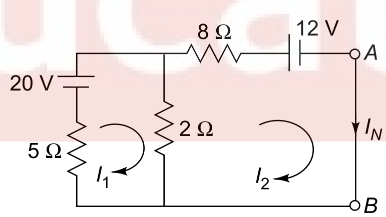


Fig. 2.361

Applying KVL to Mesh 1,

$$-5I_1 + 20 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = 20 \quad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 8I_2 - 12 = 0$$

$$-2I_1 + 10I_2 = -12 \quad (2)$$

Solving Eqs. (1) and (2),

$$I_2 = -0.67 \text{ A}$$

$$I_N = I_2 = -0.67 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

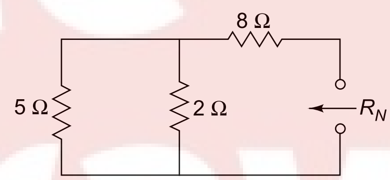


Fig. 2.362

$$R_N = (5 \,||\, 2) + 8 = 9.43 \ \Omega$$

Step III: Calculation of I_L

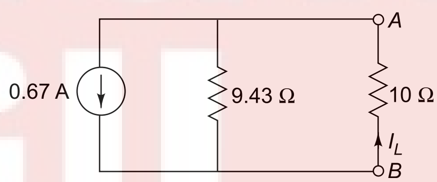


Fig. 2.363

$$I_L = 0.67 \times \frac{9.43}{9.43 + 10} = 0.33 \text{ A} (\uparrow)$$

Example 3

Find the current in the 10 Ω resistor.

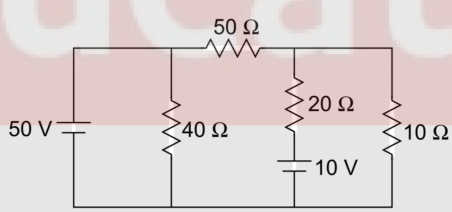


Fig. 2.364

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

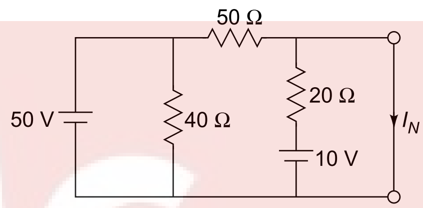


Fig. 2.365

The resistance of 40 Ω becomes redundant as it is connected across the 50 V source.

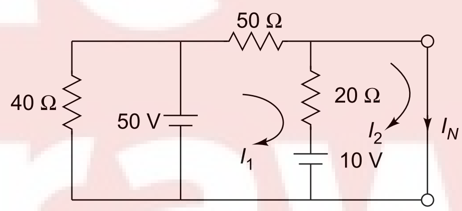


Fig. 2.366

Applying KVL to Mesh 1,

$$50 - 50\,I_1 - 20\,(I_1 - I_2) - 10 = 0$$

$$70\,I_1 - 20\,I_2 = 40 \qquad (1)$$

Applying KVL to Mesh 2,

$$10 - 20\,(I_2 - I_1) = 0$$

$$-20\,I_1 + 20\,I_2 = 0 \qquad (2)$$

Solving Eqs. (1) and (2),

$$I_1 = 1\text{ A}$$

$$I_2 = 1.5\text{ A}$$

$$I_N = I_2 = 1.5\text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits, resistor of 40 Ω gets shorted.

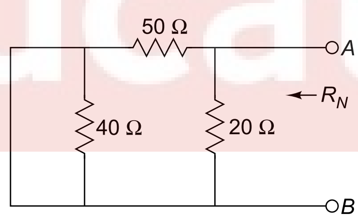


Fig. 2.367

$$R_N = 50 \| 20 = 14.28\ \Omega$$

Step III: Calculation of I_L

$$I_L = 1.5 \times \frac{14.28}{14.28+10} = 0.88 \text{ A}$$

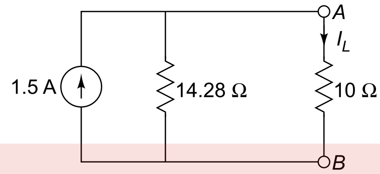


Fig. 2.368

Example 4

Find the current through the 10 Ω resistor in Fig. 2.223.

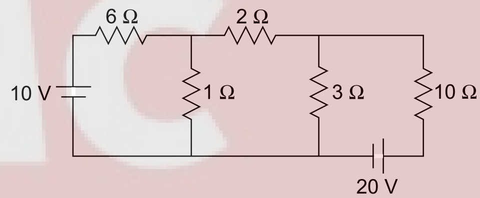


Fig. 2.369

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

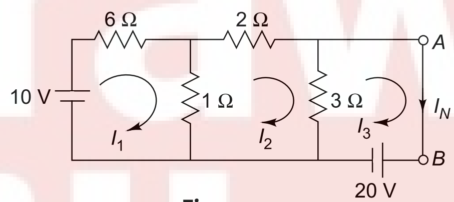


Fig. 2.370

Applying KVL to Mesh 1,

$$10 - 6I_1 - 1\,(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1\,(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \qquad (2)$$

Applying KVL to Mesh 3,

$$-3\,(I_3 - I_2) - 20 = 0$$

$$3I_2 - 3I_3 = 20 \qquad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_3 = -13.17 \text{ A}$$

$$I_N = I_3 = -13.17 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

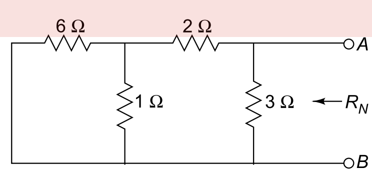


Fig. 2.371

$$R_N = [(6 \parallel 1) + 2] \parallel 3 = 1.46\ \Omega$$

Step III: Calculation of I_L

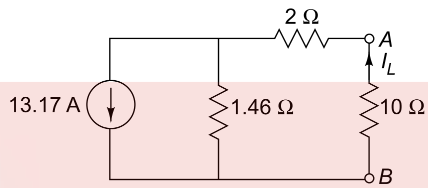


Fig. 2.372

$$I_L = 13.17 \times \frac{1.46}{1.46 + 10} = 1.68\text{ A } (\uparrow)$$

Example 5

Find the current through the 10 Ω resistor.

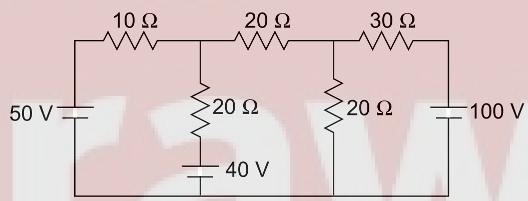


Fig. 2.373

Solution

Step I: Calculation of I_N

Replacing the 10 Ω resistor by a short circuit,

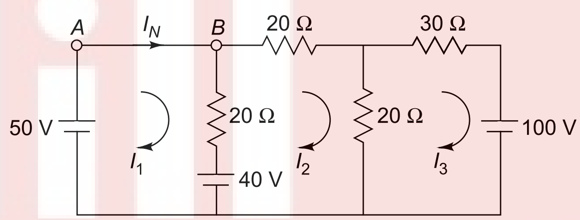


Fig. 2.374

Applying KVL to Mesh 1,

$$50 - 20\,(I_1 - I_2) - 40 = 0$$

$$20I_1 - 20I_2 = 10 \tag{1}$$

Applying KVL to Mesh 2,

$$40 - 20\,(I_2 - I_1) - 20I_2 - 20\,(I_2 - I_3) = 0$$

$$-20I_1 + 60I_2 - 20I_3 = 40 \tag{2}$$

Applying KVL to Mesh 3,

$$-20\,(I_3 - I_2) - 30I_3 - 100 = 0$$

$$-20I_2 + 50I_3 = -100 \tag{3}$$

Solving Eqs. (1), (2) and (3),

$$I_1 = 0.81\text{ A}$$

$$I_N = I_1 = 0.81 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

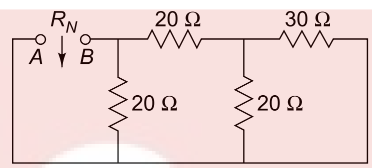


Fig. 2.375

$$R_N = [(20 \parallel 30) + 20] \parallel 20 = 12.3\ \Omega$$

Step III: Calculation of I_L

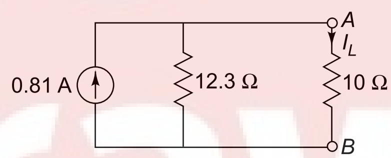


Fig. 2.376

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10} = 0.45 \text{ A}$$

Example 6

Obtain Norton's equivalent network as seen by R_L.

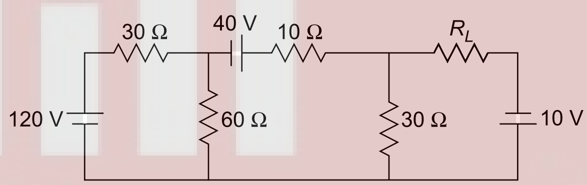


Fig. 2.377

Solution

Step I: Calculation of I_N

Replacing the resistor R_L by a short circuit,

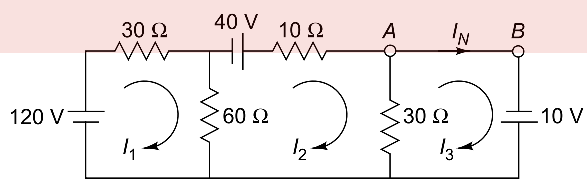


Fig. 2.378

Applying KVL to Mesh 1,

$$120 - 30I_1 - 60\,(I_1 - I_2) = 0$$

$$90I_1 - 60I_2 = 120 \quad (1)$$

Applying KVL to Mesh 2,

$$-60\,(I_2 - I_1) + 40 - 10I_2 - 30\,(I_2 - I_3) = 0$$

$$-60I_1 + 100I_2 - 30I_3 = 40 \quad (2)$$

Applying KVL to Mesh 3,

$$-30\,(I_3 - I_2) + 10 = 0$$

$$30I_2 - 30I_3 = -10 \quad (3)$$

Solving Eqs (1), (2) and (3),

$$I_3 = 4.67 \text{ A}$$

$$I_N = I_3 = 4.67 \text{ A}$$

Step II: Calculation of R_N

Replacing voltage sources by short circuits,

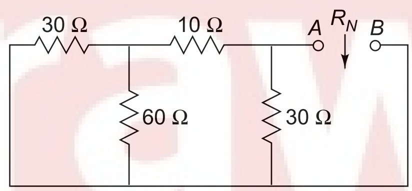


Fig. 2.379

$$R_N = [(30 \parallel 60) + 10] \parallel 30 = 15\ \Omega$$

Step III: Norton's equivalent network

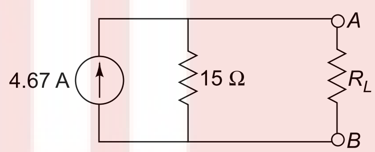


Fig. 2.380

Example 7

Find the current through the 8 Ω resistor.

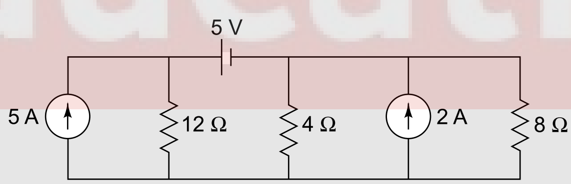


Fig. 2.381

Solution

Step I: Calculation of I_N

Replacing the 8 Ω resistor by a short circuit,

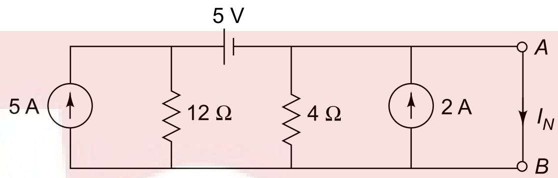


Fig. 2.382

The resistor of the 4 Ω gets shorted as it is in parallel with the short circuit. Simplifying the network by source transformation,

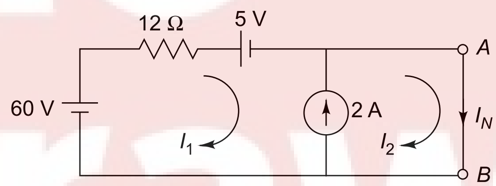


Fig. 2.383

Meshes 1 and 2 will form a loop.

Writing current equation for the loop,

$$I_2 - I_1 = 2 \tag{1}$$

Applying KVL to the loop,

$$60 - 12I_1 - 5 = 0$$

$$12I_1 = 55 \tag{2}$$

Solving Eqs (1) and (2),

$$I_1 = 4.58 \text{ A}$$

$$I_2 = 6.58 \text{ A}$$

$$I_N = I_2 = 6.58 \text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

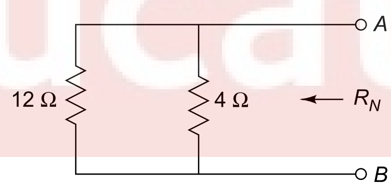


Fig. 2.384

$$R_N = 12 \parallel 4 = 3 \text{ Ω}$$

Step III: Calculation of I_L

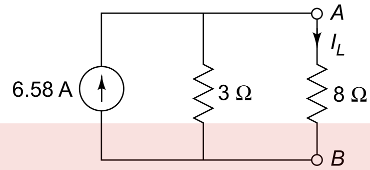


Fig. 2.385

$$I_L = 6.58 \times \frac{3}{3+8} = 1.79 \text{ A}$$

Example 8

Find current through the 1 Ω resistor.

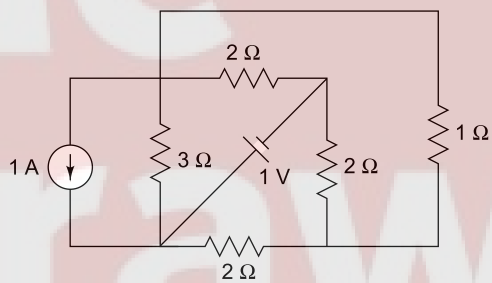


Fig. 2.386

Solution

Step I: Calculation of I_N

Replacing the 1 Ω resistor by a short circuit,

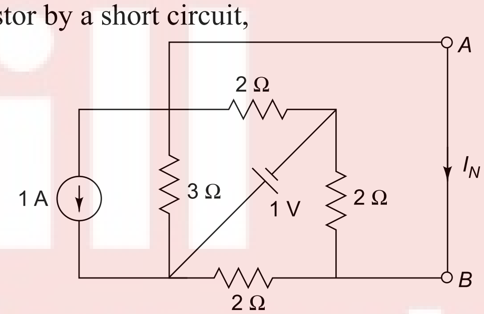


Fig. 2.387

By source transformation,

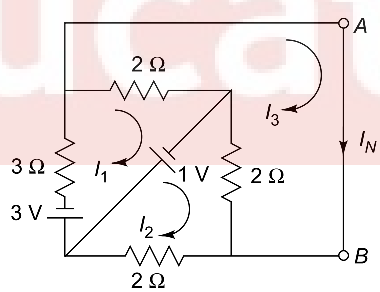


Fig. 2.388

Applying KVL to Mesh 1,

$$-3 - 3I_1 - 2(I_1 - I_3) + 1 = 0$$

$$5I_1 - 2I_3 = -2 \qquad (1)$$

Applying KVL to Mesh 2,

$$-1 - 2\,(I_2 - I_3) - 2\,I_2 = 0$$

$$4I_2 - 2I_3 = -1 \qquad (2)$$

Applying KVL to Mesh 3,

$$-2\,(I_3 - I_1) - 2\,(I_3 - I_2) = 0$$

$$-2I_1 - 2I_2 + 4I_3 = 0 \qquad (3)$$

Solving Eqs. (1), (2) and (3),

$$I_1 = -0.64\text{ A}$$

$$I_2 = -0.55\text{ A}$$

$$I_3 = -0.59\text{ A}$$

$$I_N = I_3 = -0.59\text{ A}$$

Step II: Calculation of R_N

Replacing the voltage source by a short circuit and the current source by an open circuit,

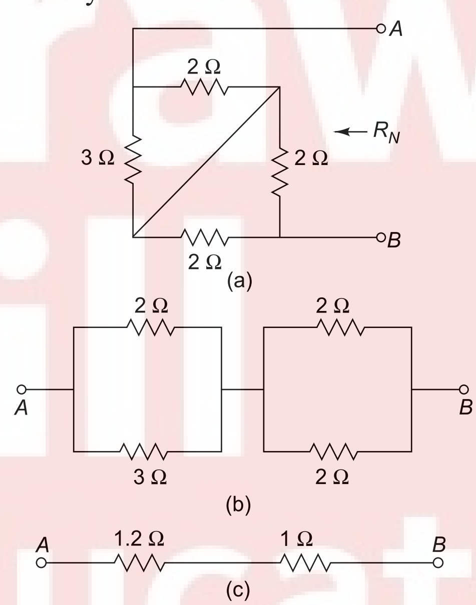


Fig. 2.389

$$R_N = 2.2\ \Omega$$

Step III: Calculation of I_L

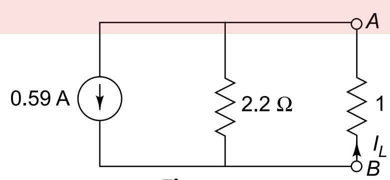


Fig. 2.390

$$I_L = 0.59 \times \frac{2.2}{2.2+1} = 0.41 \text{ A}$$

Exercise 2.8

2.1 Find the current through the 10 Ω resistor.

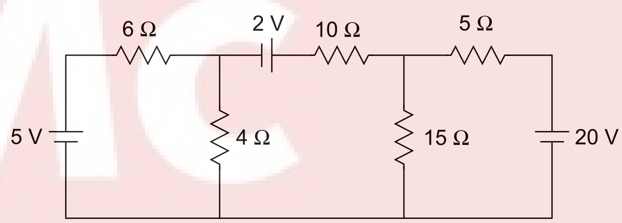


Fig. 2.391

[0.68 A]

2.2 Find the current through the 20 Ω resistor.

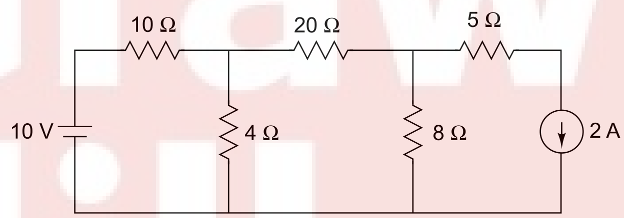


Fig. 2.392

[0.61 A]

2.3 Find the current through the 2 Ω resistor.

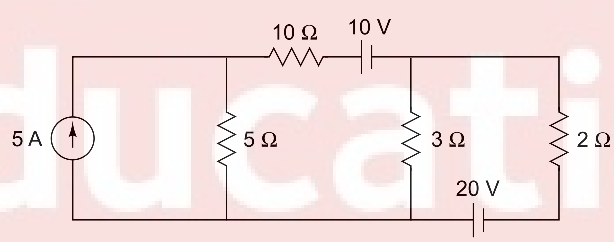


Fig. 2.393

[5 A]

2.4 Find the current through the 5 Ω resistor.

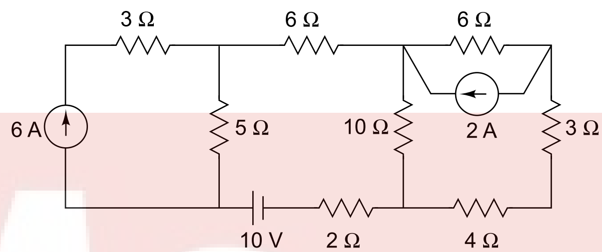


Fig. 2.394

[4.13 A]

2.5 Find the current through the 15 Ω resistor.

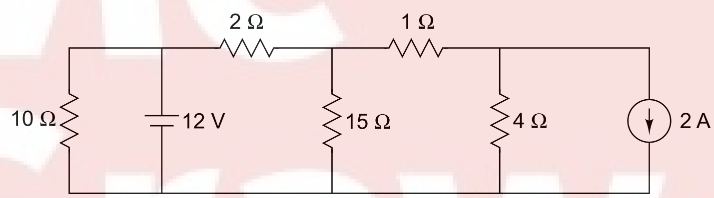


Fig. 2.395

[0.382 A]

2.6 Find Norton's equivalent network.

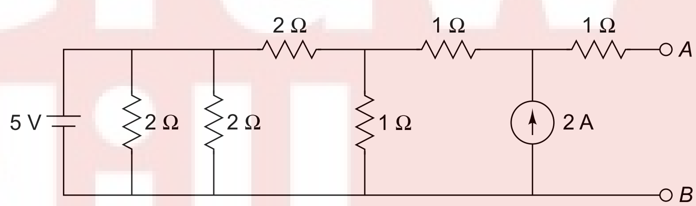


Fig. 2.396

[1.8 A, 1.67 Ω]

2.7 Find Norton's equivalent circuit for the portion of network shown in Fig. 2.251 to the left of *ab*. Hence obtain the current in the 10 Ω resistor.

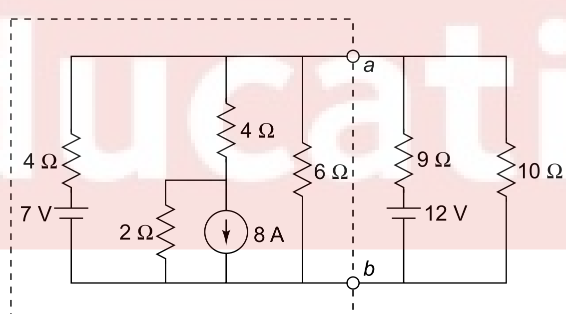


Fig. 2.397

[0.053 A]

2.9 MAXIMUM POWER TRANSFER THEOREM

It states that *'the maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.'*

$$I = \frac{V}{R_S + R_L}$$

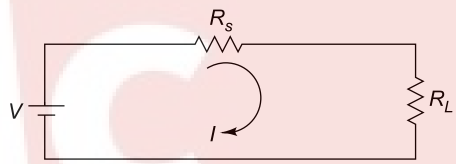


Fig. 2.398 *Maximum power transfer theorem*

Power delivered to the load $R_L = P = I^2 R_L = \dfrac{V^2 R_L}{(R_S + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_S + R_L)^2} R_L$$

$$= \frac{V^2[(R_S + R_L)^2 - (2R_L)(R_S + R_L)]}{(R_S + R_L)^4}$$

$$(R_S + R_L)^2 - 2\,R_L\,(R_S + R_L) = 0$$

$$R_S^2 + R_L^2 + 2R_S\,R_L - 2R_L\,R_S - 2R_L^2 = 0$$

$$R_L = R_S$$

Hence, the maximum power will be transferred to the load when load resistance is equal to the source resistance.

2.9.1 Steps to be followed in Maximum Power Transfer Theorem

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from points A and B with voltage sources and current sources replaced by internal resistances.

4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{\text{Th}}$$

5. Find the maximum power.

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{V_{\text{Th}}}{2R_{\text{Th}}}$$

$$P_{\max} = I_L^2 R_L = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}^2} \times R_{\text{Th}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}$$

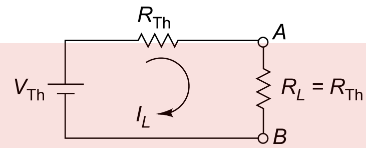


Fig. 2.399 *Equivalent circuit*

Example 1

Find the value of resistance R_L for maximum power transfer calculate maximum power.

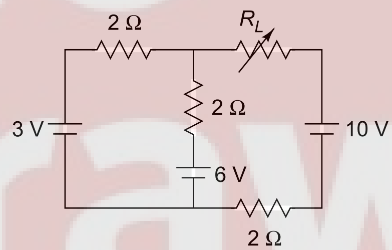


Fig. 2.400

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

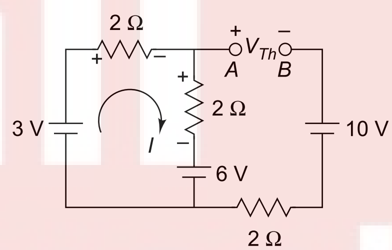


Fig. 2.401

Applying KVL to the mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75 \text{ A}$$

Writing V_{Th} equation,

$$6 + 2I - V_{\text{Th}} - 10 = 0$$

$$V_{\text{Th}} = 6 + 2I - 10$$

$$= 6 + 2(-0.75) - 10$$

$$= -5.5 \text{ V}$$
$$= 5.5 \text{ V (terminal } B \text{ is positive w.r.t } A)$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits,

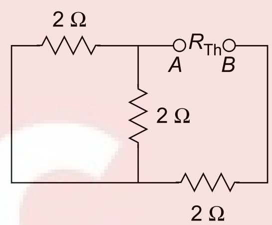


Fig. 2.402

$$R_{\text{Th}} = (2 \parallel 2) + 2 = 3\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 3\ \Omega$$

Step IV: Calculation of P_{max}

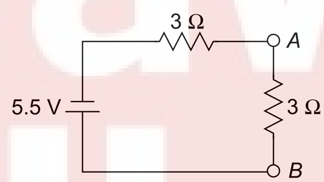


Fig. 2.403

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(5.5)^2}{4 \times 3} = 2.52 \text{ W}$$

Example 2

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

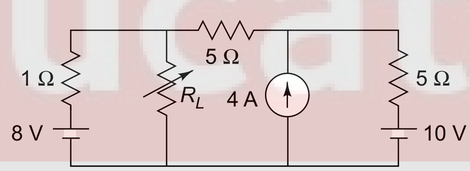


Fig. 2.404

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

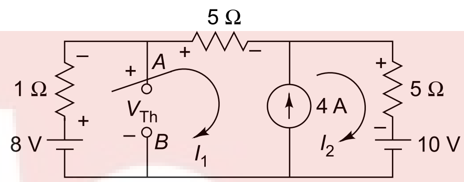


Fig. 2.405

Meshes 1 and 2 will form a loop.

Writing current equation for the loop,

$$I_2 - I_1 = 4 \tag{1}$$

Applying KVL to the loop,

$$8 - 1I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \tag{2}$$

Solving Eqs. (1) and (2),

$$I_1 = -2\text{ A}$$

$$I_2 = 2\text{ A}$$

Writing V_{Th} equation,

$$8 - 1I_1 - V_{\text{Th}} = 0$$

$$V_{\text{Th}} = 8 - I_1$$

$$= 8 - (-2)$$

$$= 10\text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and current source by an open circuit,

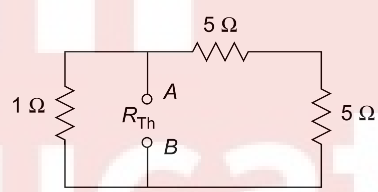


Fig. 2.406

$$R_{\text{Th}} = 10 \parallel 1 = 0.91\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 0.91\ \Omega$$

Step IV: Calculation of P_{max}

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(10)^2}{4 \times 0.91} = 27.47 \text{ W}$$

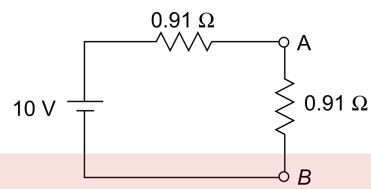


Fig. 2.407

Example 3

Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

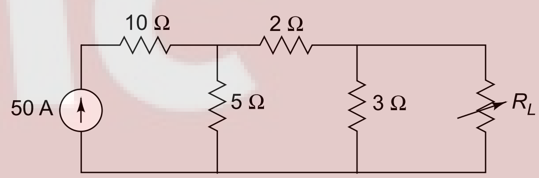


Fig. 2.408

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

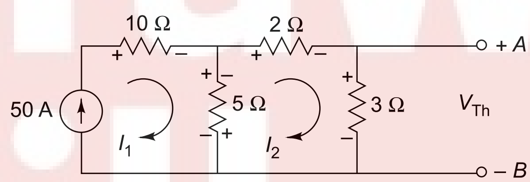


Fig. 2.409

For Mesh 1,

$$I_1 = 50$$

Applying KVL to Mesh 2,

$$-5\,(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$5I_1 - 10I_2 = 0$$
$$I_1 = 2I_2$$
$$I_2 = 25 \text{ A}$$
$$V_{\text{Th}} = 3I_2 = 3(25) = 75 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source by an open circuit,

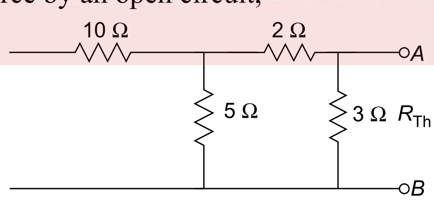


Fig. 2.410

$$R_{Th} = 7 \parallel 3 = 2.1\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 2.1\ \Omega$$

Step IV: Calculation of P_{max}

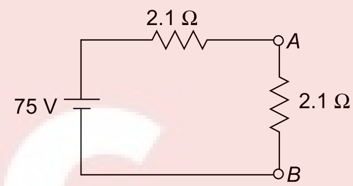


Fig. 2.411

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(75)^2}{4 \times 2.1} = 669.64\ \text{W}$$

Example 4

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

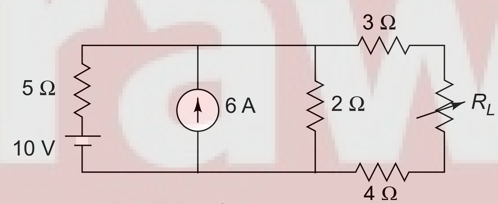


Fig. 2.412

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

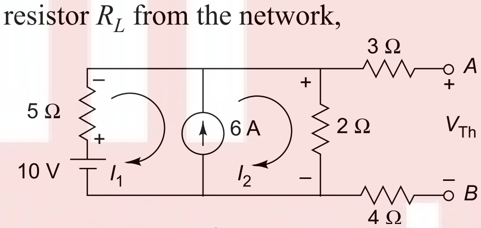


Fig. 2.413

Meshes 1 and 2 will form a loop.

Writing the current equation for the loop,

$$I_2 - I_1 = 6 \qquad (1)$$

Applying KVL to the loop,

$$10 - 5I_1 - 2I_2 = 0$$

$$5I_1 + 2I_2 = 10 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -0.29\ \text{A}$$

$$I_2 = 5.71\ \text{A}$$

Writing V_{Th} equation,

$$V_{Th} = 2I_2 = 11.42 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

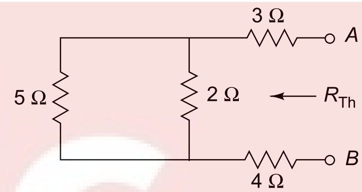


Fig. 2.414

$$R_{Th} = (5 \,||\, 2) + 3 + 4 = 8.43 \text{ Ω}$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{Th} = 8.43 \text{ Ω}$$

Step IV: Calculation of P_{max}

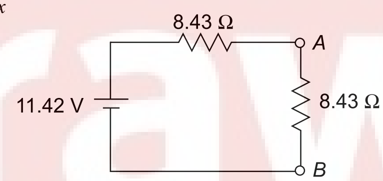


Fig. 2.415

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(11.42)^2}{4 \times 8.43} = 3.87 \text{ W}$$

Example 5

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

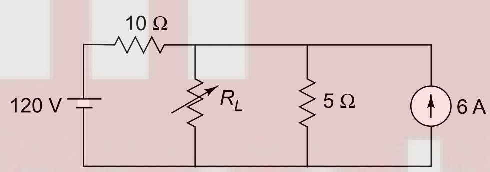


Fig. 2.416

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

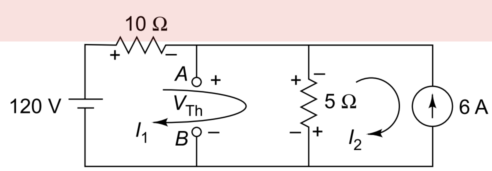


Fig. 2.417

Applying KVL to Mesh 1,

$$120 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15I_1 - 5I_2 = 120 \qquad (1)$$

Writing current equation for Mesh 2,

$$I_2 = -6 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 6\text{ A}$$

Writing V_{Th} equation,

$$120 - 10I_1 - V_{Th} = 0$$

$$V_{Th} = 120 - 10\,(6)$$

$$= 60\text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

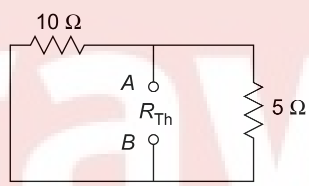


Fig. 2.418

$$R_{Th} = 10 \,||\, 5 = 3.33\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.33\ \Omega$$

Step IV: Calculation of P_{max}

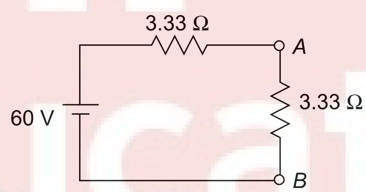


Fig. 2.419

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 3.33} = 270.27\text{ W}$$

Example 6

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

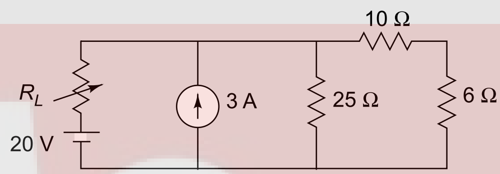


Fig. 2.420

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

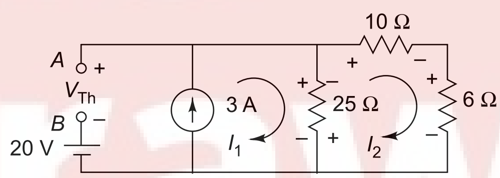


Fig. 2.421

For Mesh 1,

$$I_1 = 3 \tag{1}$$

Applying KVL to Mesh 2,

$$-25(I_2 - I_1) - 10I_2 - 6I_2 = 0$$

$$-25I_1 + 41I_2 = 0 \tag{2}$$

Solving Eqs (1) and (2),

$$I_2 = 1.83 \text{ A}$$

Writing V_{Th} equation,

$$20 + V_{\text{Th}} - 10I_2 - 6I_2 = 0$$

$$V_{\text{Th}} = -20 + 10\,(1.83) + 6\,(1.83)$$

$$= 9.28 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current source by an open circuit,

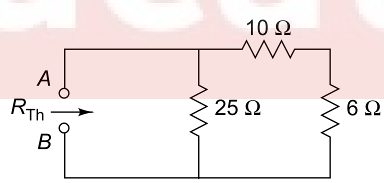


Fig. 2.422

$$R_{\text{Th}} = 25 \parallel 16 = 9.76\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{Th} = 9.76\ \Omega$$

Step IV: Calculation of P_{max}

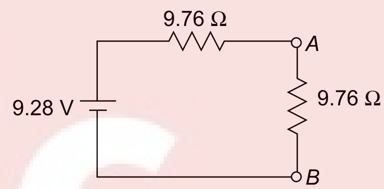


Fig. 2.423

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(9.28)^2}{4 \times 9.76} = 2.21\ \text{W}$$

Example 7

Find the value of resistance R_L for maximum power transfer and calculate maximum power.

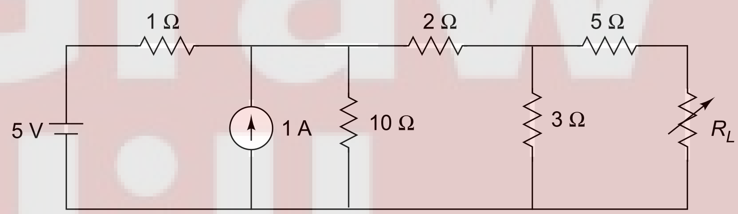


Fig. 2.424

Solution

Step I : Calculation of V_{Th}

Removing the variable resistor R_L from the network,

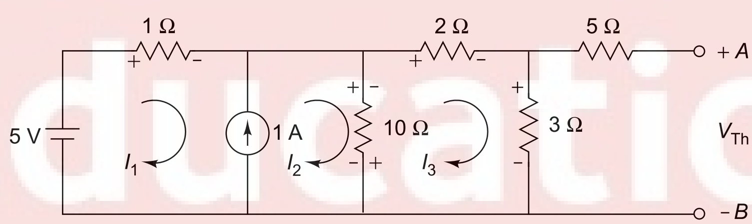


Fig. 2.425

Meshes 1 and 2 will form a loop.

Writing the current equation for the loop,

$$I_2 - I_1 = 1 \tag{1}$$

Writing the voltage equation for the loop,

$$5 - 1I_1 - 10\,(I_2 - I_3) = 0$$

$$I_1 + 10\,I_2 - 10\,I_3 = 5 \qquad (2)$$

Applying KVL to Mesh 3,

$$-10\,(I_3 - I_2) - 2\,I_3 - 3\,I_3 = 0$$

$$-10\,I_2 + 15\,I_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$I_1 = 0.38 \text{ A}$$

$$I_2 = 1.38 \text{ A}$$

$$I_3 = 0.92 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} = 3\,I_3 = 2.76 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit and current source by an open circuit,

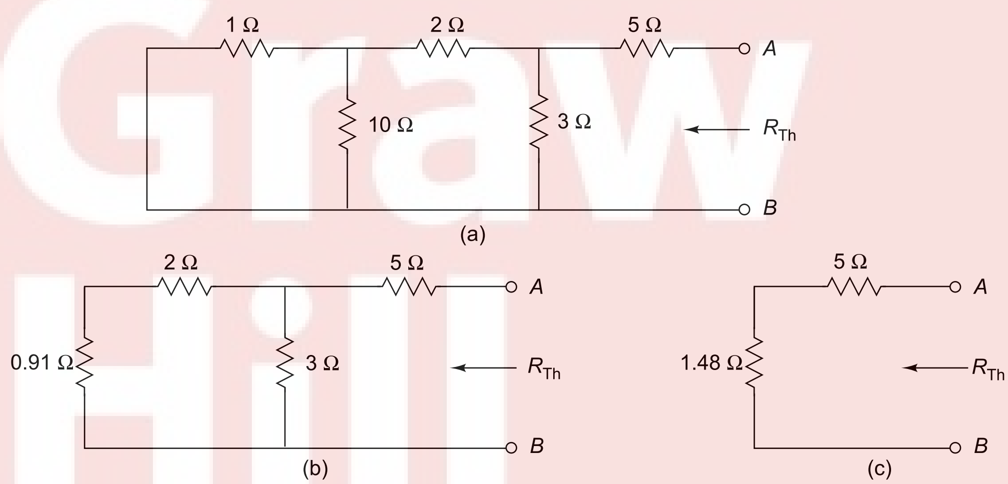


Fig. 2.426

$$R_{\text{Th}} = 6.48\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 6.48\ \Omega$$

Step IV: Calculation of P_{max}

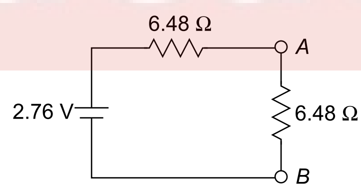


Fig. 2.427

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(2.76)^2}{4 \times 6.48} = 0.29 \text{ W}$$

Example 8

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

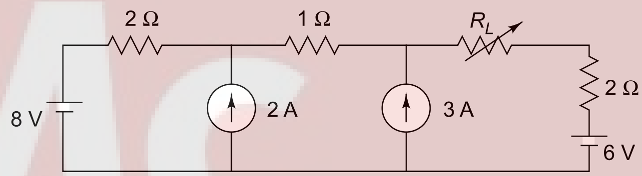


Fig. 2.428

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

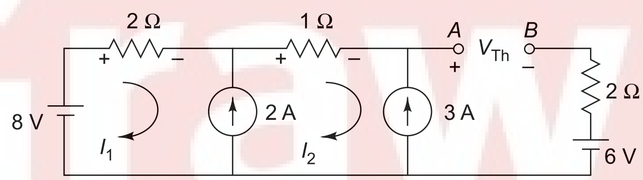


Fig. 2.429

From Fig. 2.429,

$$I_2 - I_1 = 2 \quad (1)$$

$$I_2 = -3 \text{ A} \quad (2)$$

Solving Eqs (1) and (2),

$$I_1 = -5 \text{ A}$$

Writing V_{Th} equation,

$$8 - 2I_1 - 1I_2 - V_{\text{Th}} - 6 = 0$$

$$V_{\text{Th}} = 8 - 2(-5) - (-3) - 6$$

$$= 15 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

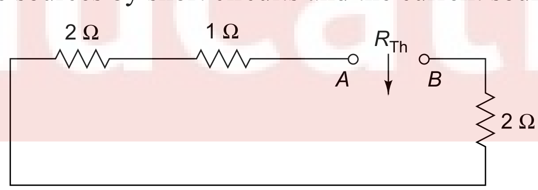


Fig. 2.430

$$R_{\text{Th}} = 5\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 5\ \Omega$$

Step IV: Calculation of P_{max}

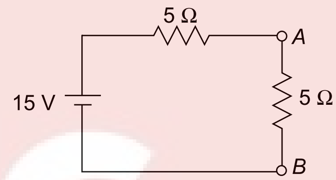


Fig. 2.431

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(15)^2}{4 \times 5} = 11.25\ \text{W}$$

Example 9

Find the value of resistance the R_L for maximum power transfer and calculate the maximum power.

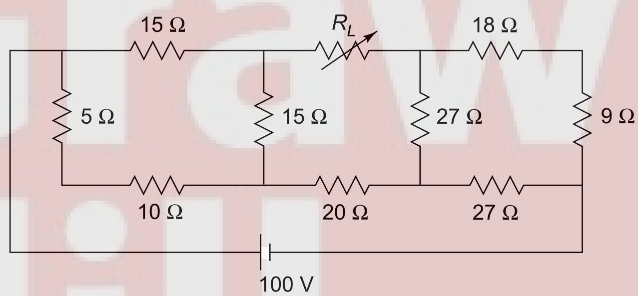


Fig. 2.432

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

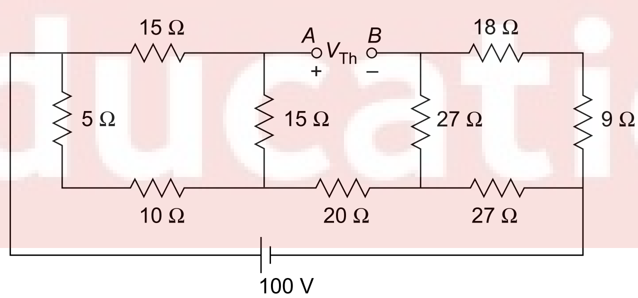


Fig. 2.433

By star-delta transformation,

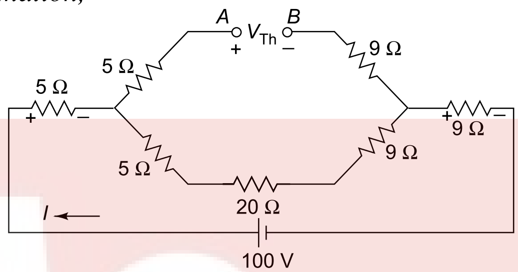


Fig. 2.434

$$I = \frac{100}{5+5+20+9+9} = 2.08 \text{ A}$$

Writing V_{Th} equation,

$$100 - 5I - V_{Th} - 9I = 0$$

$$V_{Th} = 100 - 14I$$

$$= 100 - 14(2.08)$$

$$= 70.88 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit,

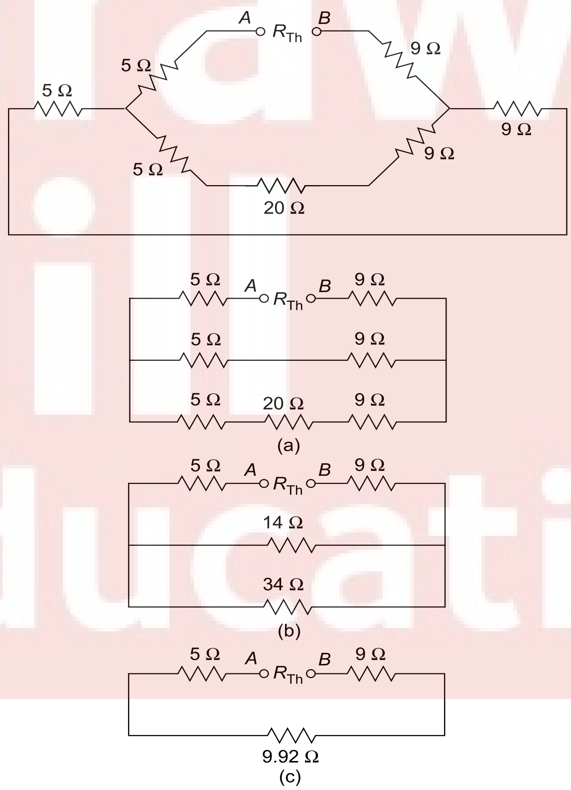


Fig. 2.435

$$R_{Th} = 23.92\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 23.92\ \Omega$$

Step IV: Calculation of P_{max}

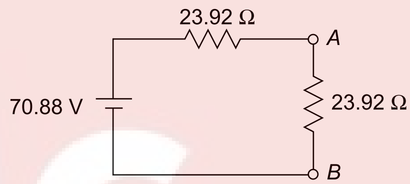


Fig. 2.436

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(70.88)^2}{4 \times 23.92} = 52.51\ \text{W}$$

Example 10

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

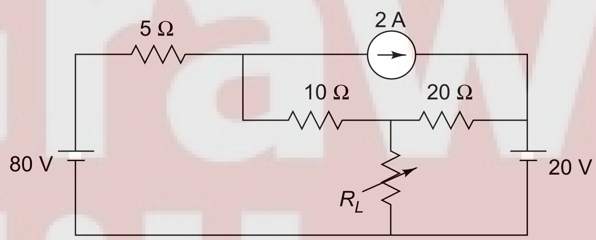


Fig. 2.437

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

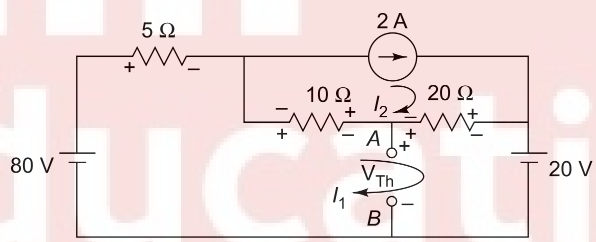


Fig. 2.438

Applying KVL to Mesh 1,

$$80 - 5I_1 - 10(I_1 - I_2) - 20(I_1 - I_2) - 20 = 0$$

$$35I_1 - 30I_2 = 60 \tag{1}$$

Writing the current equation for Mesh 2,

$$I_2 = 2 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 3.43 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} - 20\,(I_1 - I_2) - 20 = 0$$

$$V_{\text{Th}} = 20(3.43 - 2) + 20$$

$$= 48.6 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits and the current source by an open circuit,

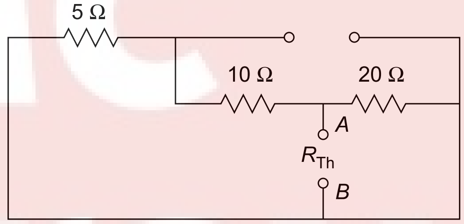


Fig. 2.439

$$R_{\text{Th}} = 15 \,||\, 20 = 8.57\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 8.57\ \Omega$$

Step IV: Calculation of P_{max}

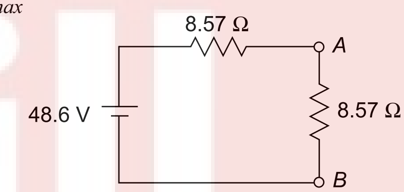


Fig. 2.440

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(48.6)^2}{4 \times 8.57} = 68.9 \text{ W}$$

Example 11

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

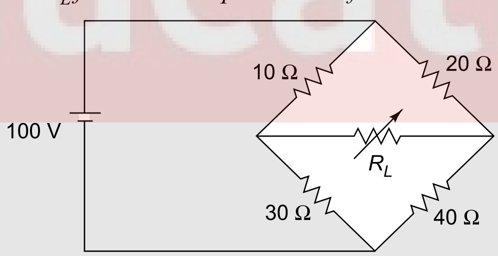


Fig. 2.441

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

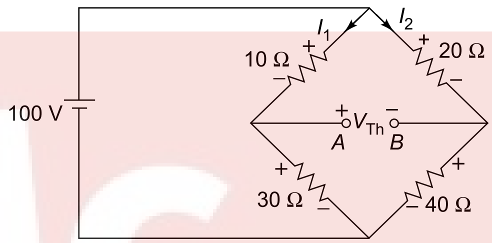


Fig. 2.442

$$I_1 = \frac{100}{10+30} = 2.5 \text{ A}$$

$$I_2 = \frac{100}{20+40} = 1.66 \text{ A}$$

Writing V_{Th} equation,

$$V_{\text{Th}} + 10I_1 - 20I_2 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 20I_2 - 10I_1 \\ &= 20(1.66) - 10(2.5) \\ &= 8.2 \text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by short circuit,

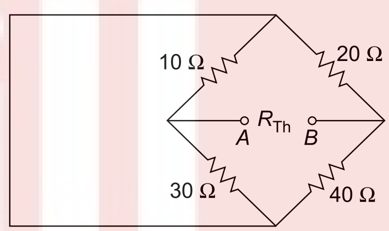


Fig. 2.443

Redrawing the network,

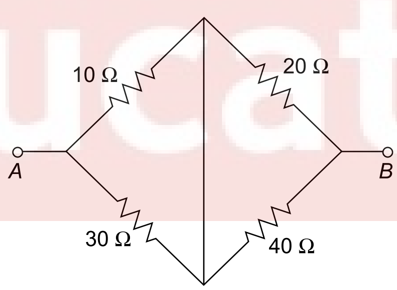


Fig. 2.444

$$R_{\text{Th}} = (10 \,||\, 30) + (20 \,||\, 40) = 20.83\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 20.83\ \Omega$$

Step IV: Calculation of P_{max}

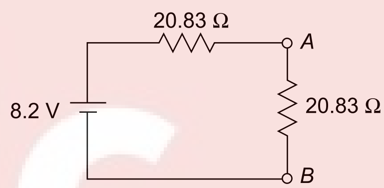


Fig. 2.445

$$P_{\text{max}} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(8.2)^2}{4 \times 20.83} = 0.81\ \text{W}$$

Example 12

Find the value of resistance R_L for maximum power transfer and calculate the maximum power.

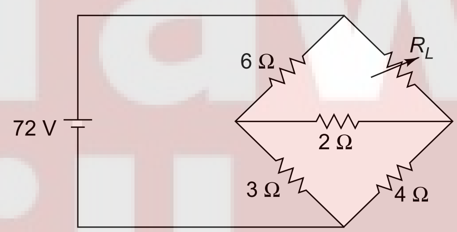


Fig. 2.446

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

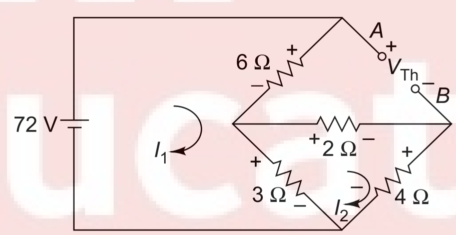


Fig. 2.447

Applying KVL to Mesh 1,

$$72 - 6I_1 - 3\,(I_1 - I_2) = 0$$

$$9I_1 - 3I_2 = 72 \qquad (1)$$

Applying KVL to Mesh 2,

$$-3\,(I_2 - I_1) - 2I_2 - 4I_2 = 0$$

$$-3I_1 + 9I_2 = 0 \qquad (2)$$

Solving Eqs (1) and (2),

$$I_1 = 9\text{ A}$$

$$I_2 = 3\text{ A}$$

Writing V_{Th} equation

$$V_{\text{Th}} - 6I_1 - 2I_2 = 0$$

$$\begin{aligned} V_{\text{Th}} &= 6I_1 + 2I_2 \\ &= 6\,(9) + 2\,(3) \\ &= 60\text{ V} \end{aligned}$$

Step II: Calculation of R_{Th}

Replacing voltage source by a short circuit,

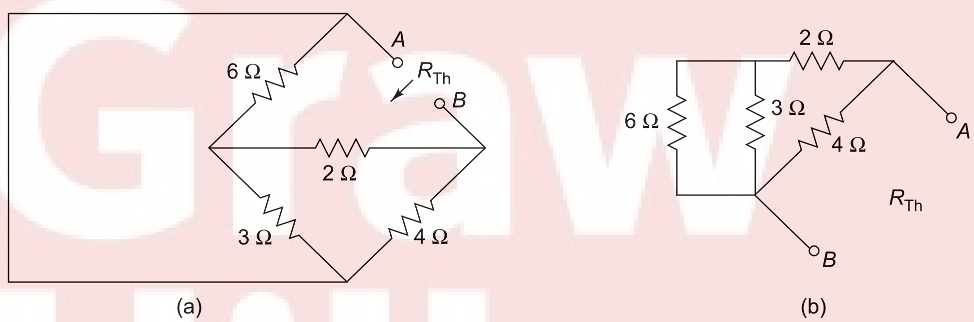


Fig. 2.448

$$R_{\text{Th}} = [(6 \,||\, 3) + 2] \,||\, 4 = 2\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer

$$R_L = R_{\text{Th}} = 2\ \Omega$$

Step IV: Calculation of P_{max}

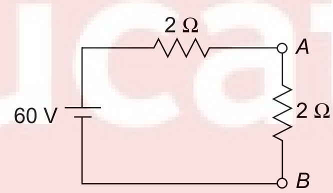


Fig. 2.449

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} = \frac{(60)^2}{4 \times 2} = 450\text{ W}$$

Example 13

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate maximum power.

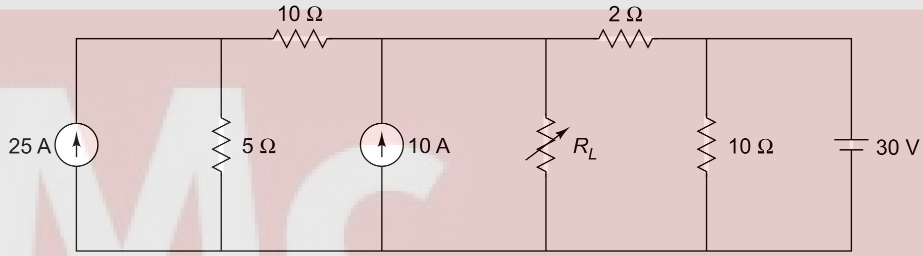


Fig. 2.450

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

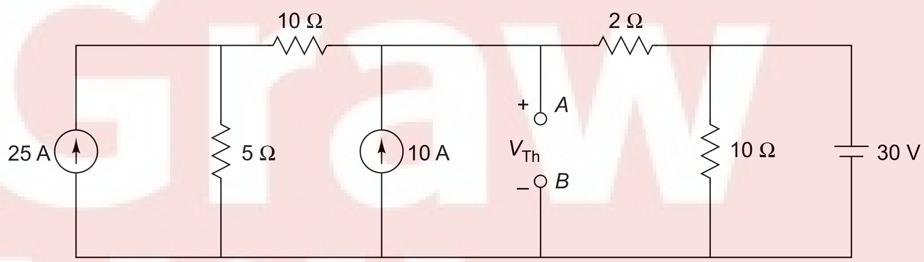


Fig. 2.451

By source transformation, the current source of 25 A and the 5 Ω resistor is converted into an equivalent voltage source of 125 V and a series resistor of 5 Ω. Also the voltage source of 30 V is connected across the 10 Ω resistor. Hence, the 10 Ω resistor becomes redundant.

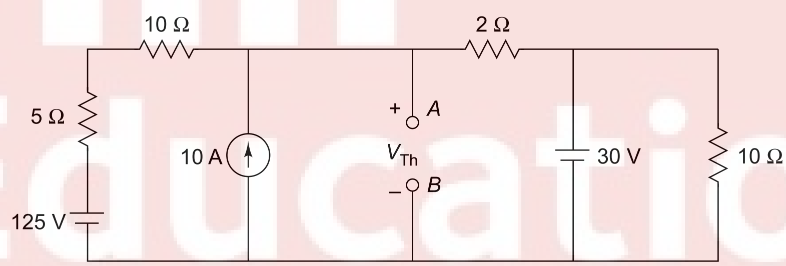


Fig. 2.452

Applying KCL at node,

$$\frac{V_{\text{Th}} - 125}{15} - 10 + \frac{V_{\text{Th}} - 30}{2} = 0$$

$$V_{\text{Th}} = 58.81 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage source by a short circuit and the current sources by open circuits,

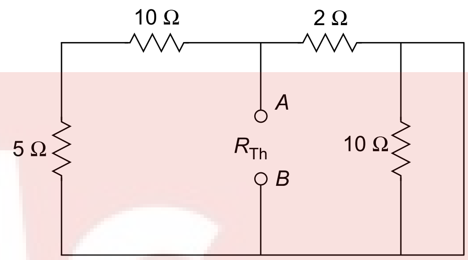


Fig. 2.453

Simplifying the network,

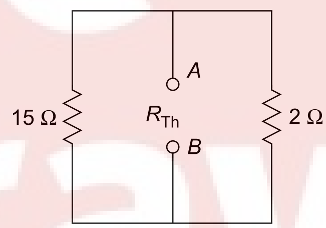


Fig. 2.454

$$R_{Th} = 15 \parallel 2 = 1.76\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 1.76\ \Omega$$

Step IV: Calculation of P_{max}

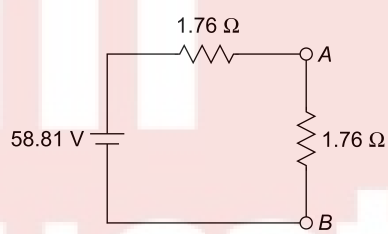


Fig. 2.455

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(58.81)^2}{4 \times 1.76} = 491.28\ \text{W}$$

Example 14

For the circuit shown, find the value of the resistance R_L for maximum power transfer and calculate maximum power.

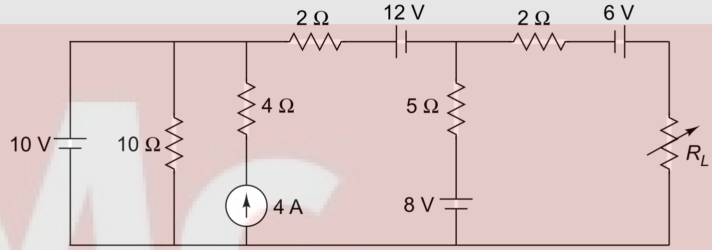


Fig. 2.456

Solution

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the network,

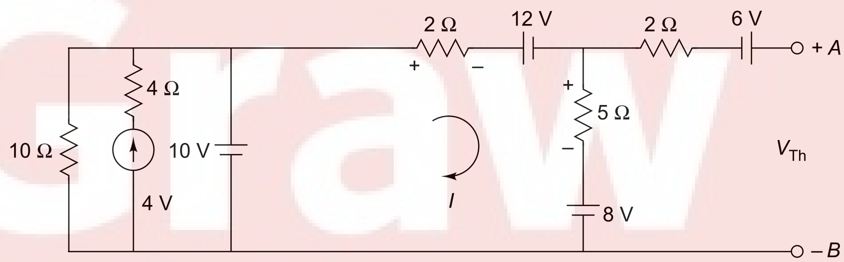


Fig. 2.457

Applying KVL to the outer path,

$$10 - 2I - 12 - 5I - 8 = 0$$

$$I = -\frac{10}{7} = -1.43 \text{ A}$$

Writing V_{Th} equation,

$$8 + 5I + 6 - V_{Th} = 0$$

$$V_{Th} = 8 + 6 + 5I$$

$$= 8 + 6 + 5(-1.43)$$

$$= 6.85 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing voltage sources by short circuits and current source by an open circuit,

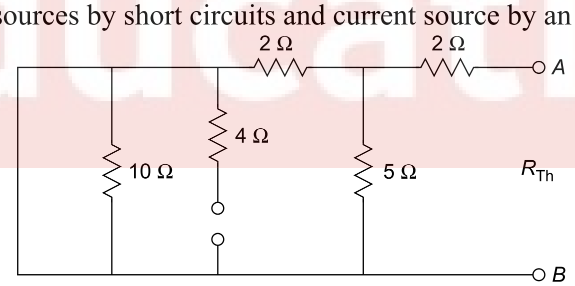


Fig. 2.458

$$R_{Th} = (2 \parallel 5) + 2$$
$$= 3.43\ \Omega$$

Step III: Value of R_L

For maximum power transfer

$$R_L = R_{Th} = 3.43\ \Omega$$

Step IV: Calculation of P_{max}

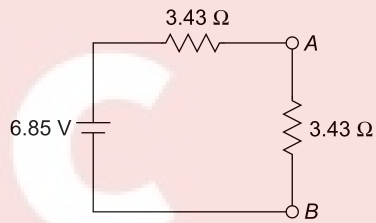


Fig. 2.459

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(6.85)^2}{4 \times 3.43} = 3.42\ \text{W}$$

Exercise 2.9

2.1 Find the value of the resistance R_L for maximum power transfer and calculate maximum power.

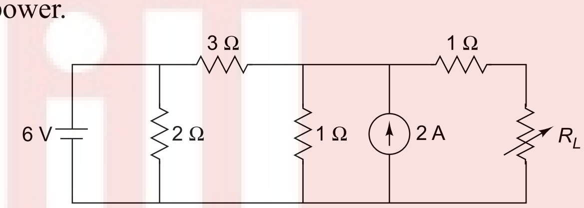


Fig. 2.460

[1.75 Ω, 1.29 W]

2.2 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

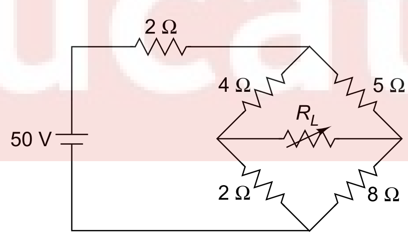


Fig. 2.461

[4.51 Ω, 4.95 W]

2.3 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

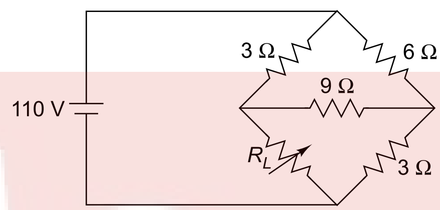


Fig. 2.462

[2.36 Ω, 940 W]

2.4 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

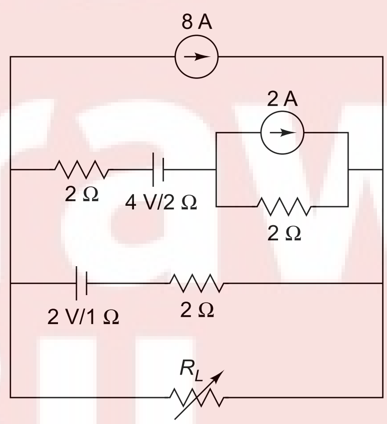


Fig. 2.463

[2.18 Ω, 29.35 W]

2.5 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

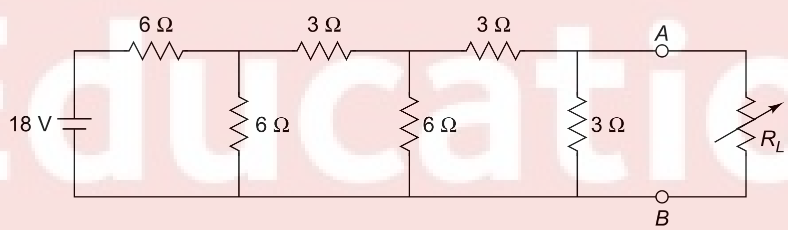


Fig. 2.464

[2 Ω, 0.281 W]

2.6 Find the value of the resistance R_L for maximum power transfer and calculate the maximum power.

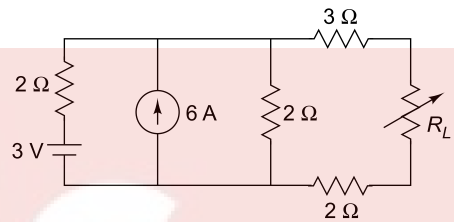


Fig. 2.465

[6 Ω, 2.52 W]

2.7 Find the value of the resistance R_L for maximum power transfer and calculate maximum power.

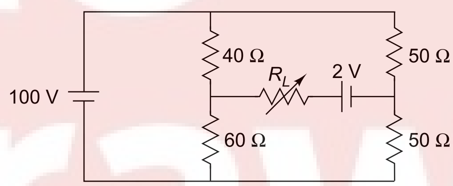


Fig. 2.466

[49 Ω, 0.32 W]

Review Questions

2.1 State and explain Kirchhoff's voltage and current law.
2.2 Define delta network and star network and derive the formulae to convert a delta network into its equivalent star network.
2.3 Derive the formulae to convert star connected network into its equivalent delta connected network.
2.4 State and explain superposition theorem.
2.5 State and explain Thevenin's theorem.
2.6 State and explain Norton's theorem.
2.7 State and prove maximum power transfer theorem.

Objective-Type Questions

Choose the correct alternative in the following questions:

2.1 The nodal method of circuit analysis is based on
(a) KVL and Ohm's law (b) KCL and Ohm's law
(c) KCL and KVL (d) KCL, KVL and Ohm's law

2.2 A network contains only an independent current source and resistors. If the values of all resistors are doubled, the value of the node voltages will

(a) become half (b) remain unchanged
(c) become double (d) none of these

2.3 Superposition theorem is not applicable to networks containing

(a) nonlinear elements (b) dependent voltage source
(c) dependent current source (d) transformers

2.4 The value of R required for maximum power transfer in the network shown in Fig. 2.467 is

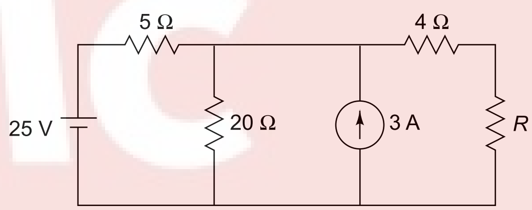


Fig. 2.467

(a) 2 Ω (b) 4 Ω (c) 8 Ω (d) 16 Ω

2.5 The maximum power that can be transferred to the load R_L from the voltage source in Fig. 2.468 is

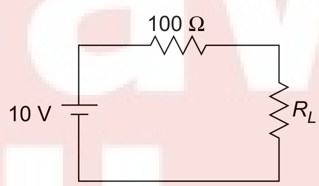


Fig. 2.468

(a) 1 W (b) 10 W (c) 0.25 W (d) 0.5 W

2.6 The value of R_L for maximum power transfer is

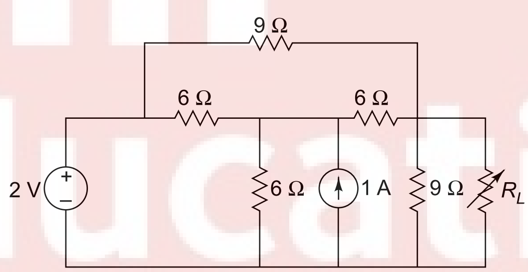


Fig. 2.469

(a) 3 Ω (b) 1.125 Ω (c) 4.1785 Ω (d) none of these

2.7 The Thevenin impedance across the terminal *AB* of Fig. 2.470 is

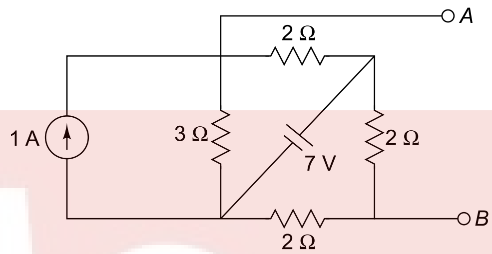


Fig. 2.470

(a) 2.2 Ω (b) $\frac{20}{9}$ Ω (c) 9 Ω (d) $\frac{11}{5}$ Ω

2.8 The Thevenin equivalent of the network shown in Fig. 2.471(a) is 10 V in series with a resistance of 2 Ω. If now, a resistance of 3 Ω is connected across *AB* as shown in Fig. 2.471(b), the Thevenin equivalent of the modified network across *AB* will be

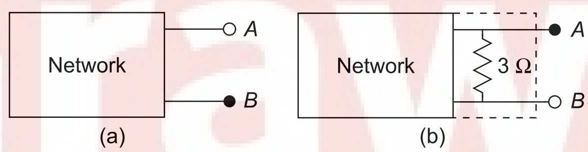


Fig. 2.471

(a) 10 V in series with 1.2 Ω resistance
(b) 6 V in series with 1.2 Ω resistance
(c) 10 V in series with 5 Ω resistance
(d) 6 V in series with 5 Ω resistance

2.9 The current I_4 in the circuit of Fig. 2.472 is equal to

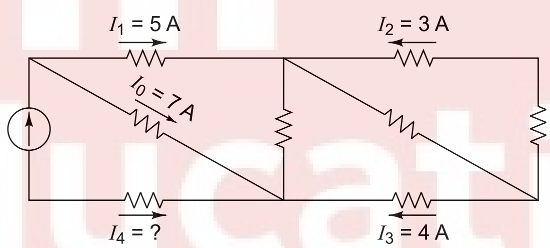


Fig. 2.472

(a) 12 A (b) –12 A (c) 4 A (d) none

2.10 The voltage V in Fig. 2.473 is equal to

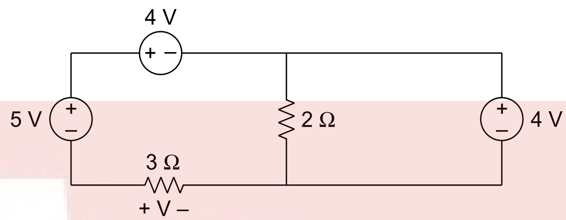


Fig. 2.473

(a) 3 V (b) –3 V (c) 5 V (d) none

2.11 The voltage V in Fig. 2.474 is equal to

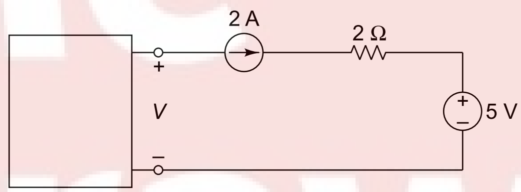


Fig. 2.474

(a) 9 V (b) 5 V (c) 1 V (d) none

2.12 The voltage V in Fig. 2.475 is

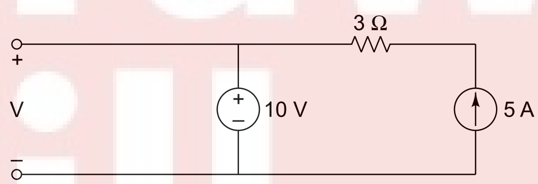


Fig. 2.475

(a) 10 V (b) 15 V (c) 5 V (d) none

2.13 In the circuit of Fig. 2.476, the value of the voltage source E is

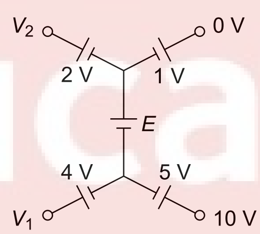


Fig. 2.476

(a) –16 V (b) 4 V (c) –6 V (d) 16 V

2.14 The voltage V_0 in Fig 2.477 is

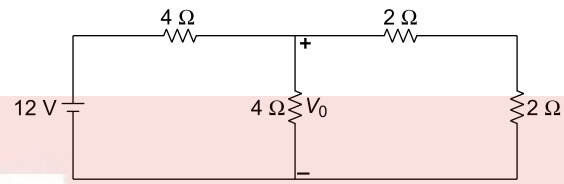


Fig. 2.477

(a) 2 V (b) $\frac{4}{3}$ V (c) 4 V (d) 8 V

2.15 If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1\ R$ in the bridge circuit shown in Fig. 2.478, then the reading in the ideal voltmeter connected between *a* and *b* is

Fig. 2.478

(a) 0.238 V (b) 0.138 V (c) – 0.238 V (d) 1 V

2.16 The voltage across terminals *a* and *b* in Fig. 2.479 is

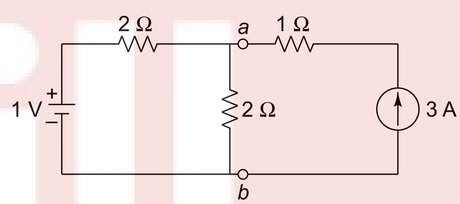


Fig. 2.479

(a) 0.5 V (b) 3 V (c) 3.5 V (d) 4 V

2.17 A delta-connected network with its wye-equivalent is shown in Fig. 2.480. The resistors R_1, R_2 and R_3 (in ohms) are respectively

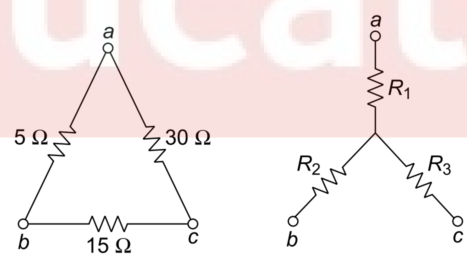


Fig. 2.480

(a) 1.5, 3 and 9 (b) 3, 9 and 1.5
(c) 9, 3 and 1.5 (d) 3, 1.5 and 9

2.18 If each branch of a delta circuit has resistance $\sqrt{3}R$, then each branch of the equivatent wye circuit has resistance

(a) $\dfrac{R}{\sqrt{3}}$ (b) 3R (c) $3\sqrt{3}\,R$ (d) $\dfrac{R}{3}$

2.19 The voltage V_0 in the Fig. 2.481 is

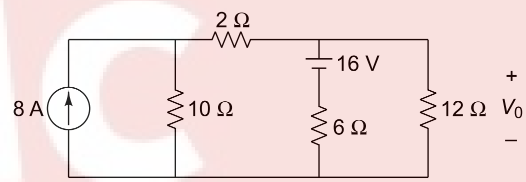


Fig. 2.481

(a) 48 V (b) 24 V (c) 36 V (d) 28 V

2.20 If $V = 4$ in Fig. 2.482 the value of I_S is given by

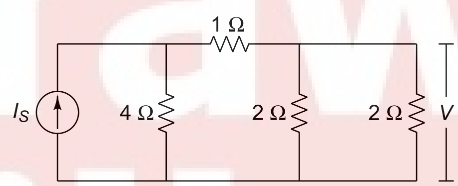


Fig. 2.482

(a) 6 A (b) 2.5 A (c) 12 A (d) none of these

2.21 The value of V_x, V_y and V_z in Fig. 2.483 shown are

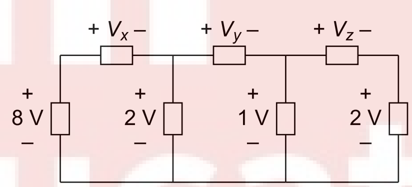


Fig. 2.483

(a) –6, 3, –3 (b) –6, –3, 1 (c) 6, 3, 3 (d) 6, 1, 3

2.22 Viewed from the terminal *AB*, the following circuit can be reduced to an equivalent circuit of a single voltage source in series with a single resistor with the following parameters:

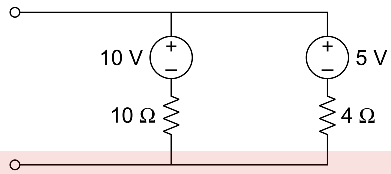


Fig. 2.484

(a) 5 volt source in series with a 10 Ω resistor
(b) 1 volt source in series with a 2.4 Ω resistor
(c) 15 volt source in series with a 2.4 Ω resistor
(d) 1 volt source in series with a 10 Ω resistor

2.23 Consider the star network shown in Fig. 2.485. The resistance between terminals *A* and *B* with *C* open is 6 Ω, between terminals *B* and *C* with *A* open is 11 Ω and between terminals *C* and *A* with *B* open is

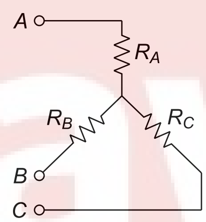


Fig. 2.485

(a) $R_A = 4\ \Omega$, $R_B = 2\ \Omega$, $R_C = 5\ \Omega$
(b) $R_A = 2\ \Omega$, $R_B = 4\ \Omega$, $R_C = 7\ \Omega$
(c) $R_A = 3\ \Omega$, $R_B = 3\ \Omega$, $R_C = 4\ \Omega$
(d) $R_A = 5\ \Omega$, $R_B = 1\ \Omega$, $R_C = 10\ \Omega$

2.24 In Fig. 2.486, the value of *R* is

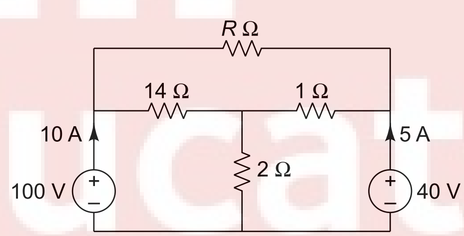
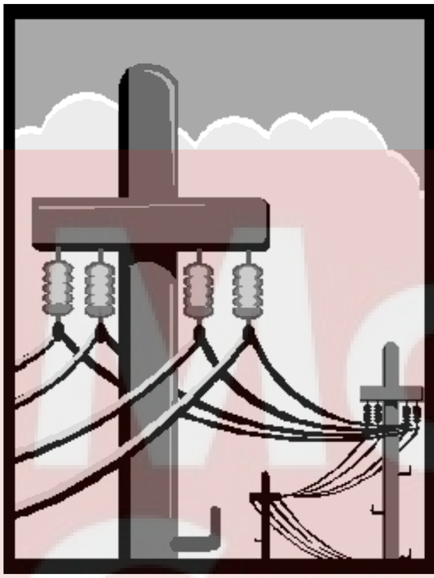


Fig. 2.486

(a) 10 Ω (b) 18 Ω (c) 24 Ω (d) 12 Ω

Answers to Objective-Type Questions

2.1 (b)	**2.2** (c)	**2.3** (a)	**2.4** (c)	**2.5** (c)	**2.6** (a)
2.7 (a)	**2.8** (b)	**2.9** (b)	**2.10** (a)	**2.11** (d)	**2.12** (a)
2.13 (a)	**2.14** (c)	**2.15** (c)	**2.16** (c)	**2.17** (d)	**2.18** (a)
2.19 (d)	**2.20** (a)	**2.21** (d)	**2.22** (b)	**2.23** (b)	**2.24** (d)



Chapter 3

AC Fundamentals

Chapter Outline

3.1 GENERATION OF ALTERNATING VOLTAGES

An alternating voltage can be generated either by rotating a coil in a stationary magnetic field or by a rotating a magnetic field within a stationary coil. In both the cases, the magnetic field is cut by the conductors or coils and an emf is induced in the coil according to Faraday's laws of electromagnetic induction. The magnitude of the induced emf depends upon the number of turns of the coil, the strength of the magnetic field and the speed at which the coil or magnetic field rotates.

Consider a rectangular coil of N turns of area A m^2 and rotating in anti-clockwise direction with angular velocity of ω radians per second in a uniform magnetic field as shown in Fig. 3.1(a).

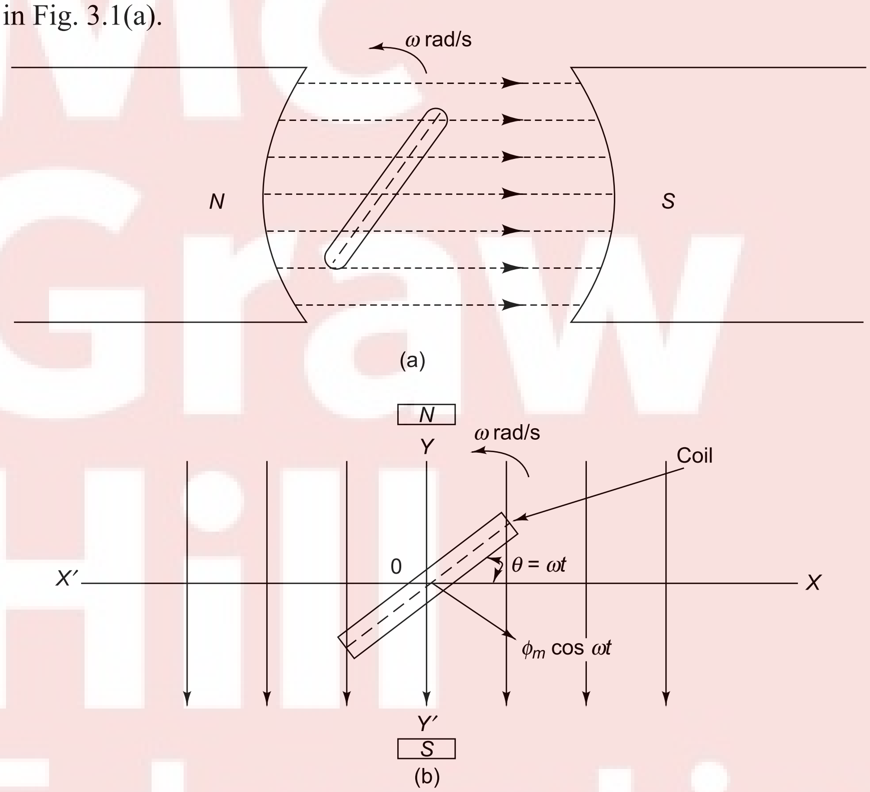


Fig. 3.1 *Generation of alternating voltage*

Let ϕ_m be the maximum flux cutting the coil when its axis coincides with the XX' axis (reference position of the coil). Thus when the coil is along XX', the flux linking with it is maximum, i.e., ϕ_m. When the coil is along YY', i.e., parallel to the lines of flux, the flux linking with it is zero.

The coil rotates through an angle $\theta = \omega t$ at any instant t.

At this instant, the flux linking with the coil is

$$\phi = \phi_m \cos \omega t$$

According to Faraday's laws of electromagnetic induction,

$$e = -N\frac{d\phi}{dt}$$

$$= -N\frac{d}{dt}(\phi_m \cos \omega t)$$

$$= N\,\phi_m\,\omega \sin \omega t$$

$$= E_m \sin \omega t$$

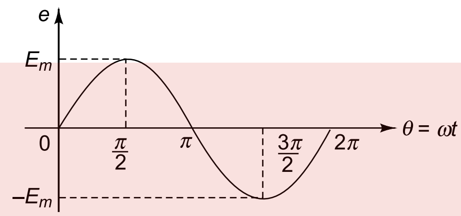


Fig. 3.2 *Sinusoidal waveform*

where $\quad E_m = N\,\phi_m \omega$

$\quad = $ maximum value of induced emf

When $\quad \omega t = 0, \quad \sin \omega t = 0, \quad e = 0$

When $\quad \omega t = \frac{\pi}{2}, \quad \sin \frac{\pi}{2} = 1 \quad e = E_m$

If the induced emf is plotted against time, a sinusoidal waveform is obtained.

3.2 TERMS RELATED TO ALTERNATING QUANTITIES

Waveform A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Figure 3.3 shows a few waveforms.

Cycle One complete set of positive and negative values of an alternating quantity is termed a cycle.

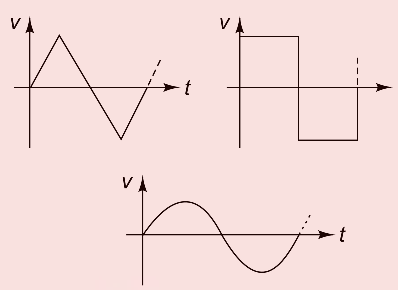


Fig. 3.3 *Alternating waveforms*

Frequency The number of cycles per second of an alternating quantity is known as its frequency. It is denoted by f and is measured in hertz (Hz) or cycles per second (c/s).

Time Period The time taken by an alternating quantity to complete one cycle is called its time period. It is denoted by T and is measured in seconds.

$$T = \frac{1}{f}$$

Amplitude The maximum positive or negative value of an alternating quantity is called the amplitude.

Phase The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.

Phase Difference This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude.

A leading alternating quantity is one which reaches its maximum or zero value earlier compared to the other quantity.

A lagging alternating quantity is one which attains its maximum or zero value later than the other quantity.

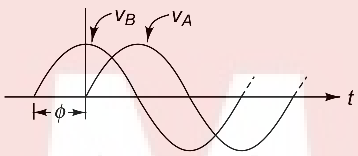


Fig. 3.4 *Phase difference*

A plus (+) sign, when used in connection with the phase difference, denotes 'lead' whereas a minus (–) sign denotes 'lag'.

$$v_A = V_m \sin \omega t$$

$$v_B = V_m \sin (\omega t + \phi)$$

Here, the quantity B leads A by a phase angle ϕ.

3.3 ROOT MEAN SQUARE (RMS) OR EFFECTIVE VALUE

Normally, the current is measured by the amount of work it will do or the amount of heat it will produce. Hence, rms or effective value of alternating current is defined as that value of steady current (direct current) which will do the same amount of work in the same time or would produce the same heating effect as when the alternating current is applied for the same time.

Figure 3.5 shows the positive half cycle of a non-sinusoidal alternating current waveform. The waveform is divided in m equal intervals with the instantaneous currents, these intervals being i_1, i_2, ..., i_m. This waveform is applied to a circuit consisting of a resistance of R ohms. Then work done in different intervals will be $\left(i_1^2 R \times \frac{t}{m}\right)$, $\left(i_2^2 R \times \frac{t}{m}\right)$, ..., $\left(i_m^2 R \times \frac{t}{m}\right)$ joules.

Thus, the total work done in t seconds on applying an alternating current waveform to a resistance $R = \dfrac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times Rt$ joules

Let I be the value of the direct current that while flowing through the same resistance does the same amount of work in the same time t. Then

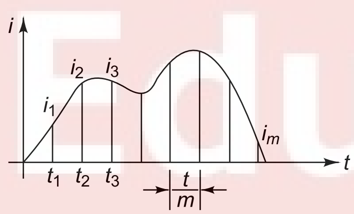


Fig. 3.5 *Mid-ordinate method*

$$I^2 Rt = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times Rt$$

$$I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}$$

Hence, rms value of the alternating current is given by

$$I_{\text{rms}} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}} = \sqrt{\text{Mean value of}(i)^2}$$

The rms value of any current $i(t)$ over the specified interval t_1 to t_2 is expressed mathematically as

$$I_{\text{rms}} = \sqrt{\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i^2(t)\,dt}$$

The rms value of an alternating current is of considerable importance in practice because the ammeters and voltmeters record the rms value of alternating current and voltage respectively.

3.3.1 RMS Value of Sinusoidal Waveform

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

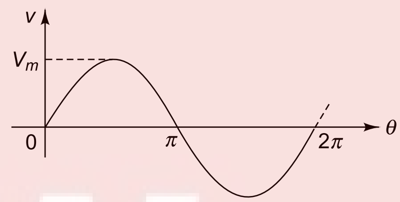


Fig. 3.6 *Sinusoidal waveform*

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]}$$

$$= \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\,V_m$$

Crest or Peak or Amplitude Factor It is defined as the ratio of maximum value to rms value of the given quantity.

$$\text{Peak factor } (k_p) = \frac{\text{Maximum value}}{\text{rms value}}$$

3.4 AVERAGE VALUE

The average value of an alternating quantity is defined as the arithmetic mean of all the values over one complete cycle.

In case of a symmetrical alternating waveform (whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in such a case, the average value is obtained over half the cycle only.

Referring to Fig. 3.5, the average value of the current is given by

$$I_{\text{avg}} = \frac{i_1 + i_2 + \ldots + i_m}{m}$$

The average value of any current $i(t)$ over the specified interval t_1 to t_2 is expressed mathematically as

$$I_{\text{avg}} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i(t)\,dt$$

3.4.1 Average Value of Sinusoidal Waveform

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

Since this is a symmetrical waveform, the average value is calculated over half the cycle.

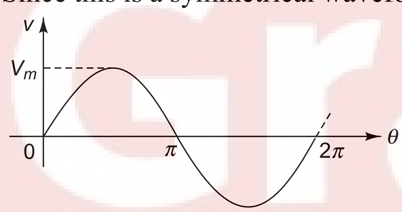


Fig. 3.7 *Sinusoidal waveform*

$$V_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1]$$

$$= \frac{2V_m}{\pi} = 0.637\, V_m$$

Form Factor It is defined as the ratio of rms value to the average value of the given quantity.

$$\text{Form factor } (k_f) = \frac{\text{rms value}}{\text{Average value}}$$

Example 1

An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of the current is 40 Hz. Find the maximum value of the alternating current.

Solution

$$i = 15 \text{ A}$$
$$t = 3.375 \text{ ms}$$
$$f = 40 \text{ Hz}$$
$$i = I_m \sin 2\pi f t$$
$$15 = I_m \sin (2 \times 180 \times 40 \times 3.375 \times 10^{-3}) \qquad \text{(angle in degrees)}$$
$$15 = I_m \times 0.75$$
$$I_m = 20 \text{ A}$$

Example 2

An alternating current of 50 c/s frequency has a maximum value of 100 A. (i) Calculate its value $\frac{1}{600}$ second and after the instant the current is zero. (ii) In how many seconds after the zero value will the current attain the value of 86.6 A?

Solution

$$f = 50 \text{ c/s}$$
$$I_m = 100 \text{ A}$$

(i) Value of current $\frac{1}{600}$ second after the instant the current is zero

$$i = I_m \sin 2\pi f t$$
$$= 100 \sin \left(2 \times 180 \times 50 \times \frac{1}{600}\right) \qquad \text{(angle in degrees)}$$
$$= 100 \sin (30^\circ) = 50 \text{ A}$$

(ii) Time at which current will attain the value of 86.6 A after the zero value

$$i = I_m \sin 2\pi f t$$
$$86.6 = 100 \sin (2 \times 180 \times 50 \times t) \qquad \text{(angle in degrees)}$$
$$\sin (18000\, t) = 0.866$$
$$18000\, t = 60^\circ$$
$$t = \frac{1}{300} \text{ second}$$

Example 3

An alternating current varying sinusoidally with a frequency of 50 c/s has an rms value of 20 A. Write down the equation for the instantaneous value and find this value at (i) 0.0025 s, and (ii) 0.0125 s after passing through zero and increasing positively. (iii) At what time, measured from zero, will the value of the instantaneous current be 14.14 A?

Solution $f = 50$ c/s

$$I_{\text{rms}} = 20 \text{ A}$$

$$I_m = I_{\text{rms}} \times \sqrt{2} = 20\sqrt{2} = 28.28 \text{ A}$$

Equation of current, $i = I_m \sin 2\pi ft$

$$= 28.28 \sin (100\pi \times t)$$

$$= 28.28 \sin (100 \times 180 \times t) \quad \text{(angle in degrees)}$$

(i) Value of current at $t = 0.0025$ second

$$i = 28.28 \sin (100 \times 180 \times 0.0025) \quad \text{(angle in degrees)}$$

$$= 28.28 \sin (45^\circ) = 20 \text{ A}$$

(ii) Value of current at $t = 0.0125$ second

$$i = 28.28 \sin (100 \times 180 \times 0.0125) \quad \text{(angle in degrees)}$$

$$= 28.28 \sin (225^\circ) = -20 \text{ A}$$

(iii) Time at which value of instantaneous current will be 14.14 A

$$i = 28.28 \sin 100\pi t$$

$$14.14 = 28.28 \sin 18000\, t \quad \text{(angle in degrees)}$$

$$\sin 18000\, t = 0.5$$

$$18000\, t = 30^\circ$$

$$t = 1.66 \text{ ms}$$

Example 4

An alternating current of 60 Hz frequency has a maximum value of 110 A. Calculate (i) time required to reach 90 A after the instant current is zero and increasing positively, and (ii) its value $\frac{1}{600}$ second after the instant current is zero and its value decreasing thereafter.

Solution $f = 60$ Hz

$$I_m = 110 \text{ A}$$

(i) Time required to reach 90 A after the instant current is zero and increasing positively.

$$i = I_m \sin 2\pi ft$$

$$90 = 110 \sin (2 \times 180 \times 60 \times t) \quad \text{(angle in degrees)}$$

$$\sin 21600\, t = 0.818$$

$$21600\, t = 54.88^\circ$$

$$t = 2.54 \text{ ms}$$

(ii) Value of current $\frac{1}{600}$ second after the instant current is zero and decreasing thereafter.

From Fig. 3.8,

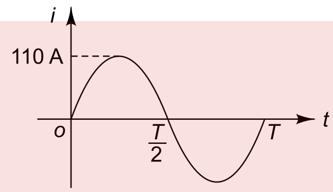


Fig. 3.8

$$t = \frac{T}{2} + \frac{1}{600}$$
$$= \frac{1}{2f} + \frac{1}{600}$$
$$= \frac{1}{2 \times 60} + \frac{1}{600}$$
$$= 0.01 \text{ s}$$
$$i = I_m \sin 2\pi ft$$
$$= 110 \sin (2 \times 180 \times 60 \times 0.01) \qquad \text{(angle in degrees)}$$
$$= -64.66 \text{ A}$$

Example 5

A sinusoidal wave of 50 Hz frequency has its maximum value of 9.2 A. What will be its value at (i) 0.002 s after the wave passes through zero in the positive direction, and (ii) 0.0045 s after the wave passes through the positive maximum.

Solution

$$f = 50 \text{ Hz}$$
$$I_m = 9.2 \text{ A}$$

(i) Value of current at 0.002 s after the wave passes through zero in the positive direction

$$i = I_m \sin 2\pi ft$$
$$= 9.2 \sin (2 \times 180 \times 50 \times 0.002) \qquad \text{(angle in degrees)}$$
$$= 5.41 \text{ A}$$

(ii) Value of current 0.0045 s after the wave passes through the positive maximum

From Fig. 3.9,

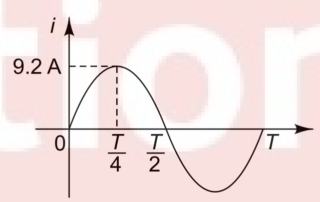


Fig. 3.9

$$t = \frac{T}{4} + 0.0045$$
$$= \frac{1}{4f} + 0.0045$$
$$= \frac{1}{4 \times 50} + 0.0045$$
$$= 9.5 \text{ ms}$$
$$i = I_m \sin 2\pi ft$$

$$= 9.2 \sin (2 \times 180 \times 50 \times 9.5 \times 10^{-3}) \qquad \text{(angle in degrees)}$$
$$= 1.44 \text{ A}$$

Example 6

An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of current of 20 A. At what time measured from negative maximum value will the instantaneous current be $10\sqrt{2}$ A?

Solution

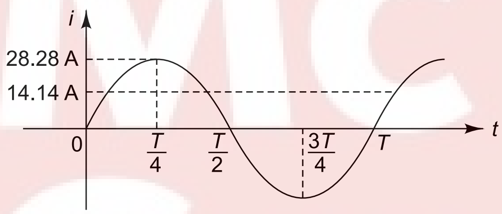


Fig. 3.10

$$f = 50 \text{ Hz}$$
$$I_{\text{rms}} = 20 \text{ A}$$

(i) Time at which instantaneous current will be $10\sqrt{2}$ A

$$i = 10\sqrt{2} \text{ A} = 14.14 \text{ A}$$
$$I_m = I_{\text{rms}} \times \sqrt{2}$$
$$= 20\sqrt{2} = 28.28 \text{ A}$$

$$i = I_m \sin 2\pi ft$$
$$14.14 = 28.28 \sin (2 \times 180 \times 50 \times t) \qquad \text{(angle in degrees)}$$
$$0.5 = \sin (18000\, t)$$
$$18000\, t = 30°$$
$$t = 1.67 \text{ ms}$$

(ii) Time, measured from negative maximum value, at which instantaneous current will be $10\sqrt{2}$ A

$$t = \frac{T}{4} + 1.67 \times 10^{-3}$$
$$= \frac{1}{4f} + 1.67 \times 10^{-3}$$
$$= \frac{1}{4 \times 50} + 1.67 \times 10^{-3}$$
$$= 6.67 \text{ ms}$$

Example 7

An alternating current is given by $i = 14.14 \sin 377\, t$. Find (i) rms value of the current, (ii) frequency, (iii) instantaneous value of the current when $t = 3$ ms, and (iv) time taken by the current to reach 10 A for first time after passing through zero.

Solution $\quad i = 14.14 \sin 377\, t$

(i) The rms value of the current

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} = 10 \text{ A}$$

(ii) Frequency

$$2\pi f = 377$$

$$f = \frac{377}{2\pi} = 60 \text{ Hz}$$

(iii) Instantaneous value of the current when $t = 3$ ms

$$i = 14.14 \sin (377 \times 3 \times 10^{-3}) \qquad \text{(angle in radians)}$$
$$= 12.79 \text{ A}$$

(iv) Time taken by the current to reach 10 A for the first time after passing through zero

$$i = 14.14 \sin 377\, t \qquad \text{(angle in radians)}$$
$$10 = 14.14 \sin 377\, t$$
$$\sin 377\, t = 0.707$$
$$377\, t = 0.79 \text{ radian}$$
$$t = 2.084 \text{ ms}$$

Example 8

An alternating current varying sinusoidally at 50 Hz has its rms value of 10 A. Write down an equation for the instantaneous value of the current. Find the value of the current at (i) 0.0025 second after passing through the positive maximum value, and (ii) 0.0075 second after passing through zero value and increasing negatively.

Solution

$$f = 50 \text{ Hz}$$
$$I_{\text{rms}} = 10 \text{ A}$$

(i) Equation for instantaneous value of the current

$$I_m = I_{\text{rms}} \times \sqrt{2} = 10\sqrt{2} = 14.14 \text{ A}$$
$$i = I_m \sin 2\pi ft$$
$$= 14.14 \sin (2 \times 180 \times 50 \times t) \qquad \text{(angle in degrees)}$$
$$= 14.14 \sin (18000\, t)$$

(ii) Value of the current at 0.0025 s after passing through the positive maximum value

From Fig. 3.11(a),

$$t = \frac{T}{4} + 0.0025$$
$$= \frac{1}{4f} + 0.0025$$
$$= \frac{1}{4 \times 50} + 0.0025$$
$$= 7.5 \text{ ms}$$

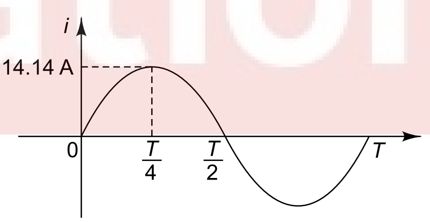


Fig. 3.11(a)

$$i = 14.14 \sin(18000 \times 7.5 \times 10^{-3}) \quad \text{(angle in degrees)}$$
$$= 10 \text{ A}$$

(ii) Value of the current 0.0075 s after passing through zero value and increasing negatively

From Fig. 3.11(b),

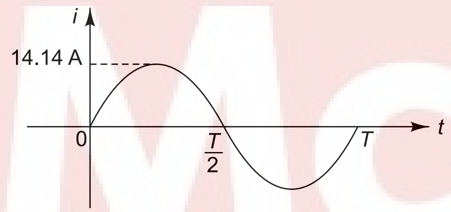


Fig. 3.11(b)

$$t = \frac{T}{2} + 0.0075$$
$$= \frac{1}{2f} + 0.0075$$
$$= \frac{1}{2 \times 50} + 0.0075$$
$$= 17.5 \text{ ms}$$

$$i = 14.14 \sin(18000 \times 17.5 \times 10^{-3}) \quad \text{(angle in degrees)}$$
$$= -10 \text{ A}$$

Example 9

Draw a neat sketch in each case of the waveform and write expressions of instantaneous value for the following:

(i) Sinusoidal current of amplitude 10 A, 50 Hz passing through its zero value at $\omega t = \frac{\pi}{3}$ and increasing positively

(ii) Sinusoidal current of amplitude 8 A, 50 Hz passing through its zero value at $\omega t = -\frac{\pi}{6}$ and increasing positively.

Solution (i) The current waveform is lagging in nature

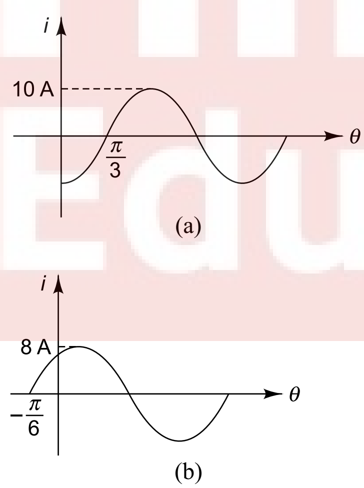


Fig. 3.12

$$i = I_m \sin(2\pi ft - \phi)$$
$$= 10 \sin\left(2\pi \times 50 \times t - \frac{\pi}{3}\right)$$
$$= 10 \sin\left(100\pi t - \frac{\pi}{3}\right)$$

(ii) The current waveform is leading in nature

$$i = I_m \sin(2\pi ft + \phi)$$
$$= 8 \sin\left(2\pi \times 50 \times t + \frac{\pi}{6}\right)$$
$$= 8 \sin\left(100\pi t + \frac{\pi}{6}\right)$$

Example 10

The instantaneous current is given by $I = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$*. Find its effective value, periodic time and the instant at which it reaches its positive maximum value. Sketch the waveform from t = 0 over one complete cycle.*

Solution
$$i = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right)$$

(i) Effective value

$$I_{\text{eff}} = I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = \frac{7.071}{\sqrt{2}} = 5 \text{ A}$$

(ii) Periodic time

$$2\pi f = 157.08$$
$$f = 25 \text{ Hz}$$
$$T = \frac{1}{f} = \frac{1}{25} = 0.04 \text{ s}$$

(iii) Instant at which the current reaches its positive maximum value, i.e., $i = 7.071$ A

$$7.071 = 7.071 \sin\left(157.08t - \frac{\pi}{4}\right) \quad \text{(angle in radians)}$$
$$1 = \sin(157.08t - 0.785)$$
$$1.5708 = 157.08t - 0.785$$
$$t = 0.015 \text{ s}$$

(iv) The waveform is shown in Fig. 3.13.

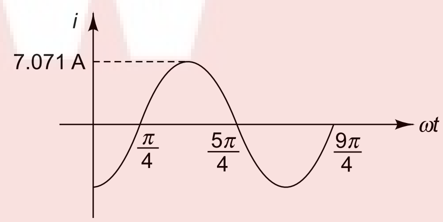


Fig. 3.13

Example 11

A 60 Hz sinusoidal current has an instantaneous value of 7.07 A at t = 0 and rms value of $10\sqrt{2}$ *A. Assuming the current wave to enter positive half at t = 0, determine (i) expression for instantaneous current, (ii) magnitude of the current at t = 0.0125 second, and (iii) magnitude of the current at t = 0.025 second after t = 0.*

Solution
$$f = 60 \text{ Hz}$$
$$i(0) = 7.07 \text{ A}$$
$$I_{\text{rms}} = 10\sqrt{2} \text{ A}$$

(i) Expression for instantaneous current

$$I_m = I_{\text{rms}} \times \sqrt{2} = 10\sqrt{2} \times \sqrt{2} = 20 \text{ A}$$

Since $i = 7.07$ A at $t = 0$, the current is leading in nature.

At $\quad t = 0$

$$i = I_m \sin(2\pi ft + \phi)$$
$$7.07 = 20 \sin(2\pi \times 60 \times 0 + \phi)$$
$$\phi = 20.7°$$
$$i = 20 \sin(120\pi t + 20.7°)$$

(ii) Magnitude of current at $t = 0.0125$ second

$$i = 20 \sin(120 \times 180 \times 0.0125 + 20.7°) \qquad \text{(angle in degrees)}$$
$$= -18.7 \text{ A}$$

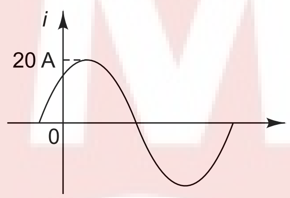


Fig. 3.14

(iii) Magnitude of current at $t = 0.025$ second after $t = 0$

Time corresponding to a phase shift of 20.7°

$$= \frac{20.7°}{360°} \times T$$
$$= \frac{20.7°}{360°} \times \frac{1}{f} = \frac{20.7°}{360°} \times \frac{1}{60} = 0.958 \text{ ms}$$

Time 0.025 second after $t = 0$

$$t = 0.958 \times 10^{-3} + 0.025$$
$$= 25.958 \text{ ms}$$
$$i = 20 \sin(120 \times 180 \times 25.958 \times 10^{-3} + 20.7°) \qquad \text{(angle in degrees)}$$
$$= -13.22 \text{ A}$$

Example 12

A 50 Hz sinusoital voltage applied to a single-phase circuit has an rms value of 200 V. Its value at t = 0 is $(\sqrt{2} \times 200)$ V positive. The current drawn by the circuit is 5 A (rms) and lags behind the voltage by one-sixth of a cycle. Write the expressions for the instantaneous values of voltage and current. Sketch their waveforms and find their values at t = 0.0125 second.

Solution

$$f = 50 \text{ Hz}$$
$$V_{\text{rms}} = 200 \text{ V}$$
$$I_{\text{rms}} = 5 \text{ A}$$
$$v(0) = \sqrt{2} \times 200 = 282.84 \text{ V}$$

(i) Instantaneous value of voltage

$$V_m = V_{\text{rms}} \times \sqrt{2} = 200\sqrt{2} = 282.84 \text{ V}$$
$$v = V_m \sin(2\pi ft + \phi)$$

At $t = 0$,

$$282.84 = 282.84 \sin(0 + \phi)$$
$$\sin\phi = 1$$

$$\phi = 90°$$
$$v = 282.84 \sin (2\pi \times 50 \times t + 90°)$$
$$= 282.84 \sin (100\ \pi t + 90°)$$

(ii) Instantaneous value of current

The current lags behind the voltage by one-sixth of a cycle.

$$\phi = \frac{1}{6} \times 360 = 60°$$
$$I_m = I_{\text{rms}} \times \sqrt{2} = 5\sqrt{2} = 7.07\text{ A}$$
$$i = I_m \sin (2\pi ft + 90° - 60°)$$
$$= 7.07 \sin (2\pi \times 50 \times t + 30°)$$
$$= 7.07 \sin (100\pi t + 30°)$$

(iii) Voltage and current waveforms are shown in Fig. 3.15.

(iv) Value of voltage at $t = 0.0125$ s

$$v = 282.84 \sin (100 \times 180 \times 0.0125 + 90°) \quad \text{(angle in degrees)}$$
$$= 200\text{ V}$$

(v) Value of current at $t = 0.0125$ s

$$i = 7.07 \sin (100 \times 180 \times 0.0125 + 30°) \quad \text{(angle in degrees)}$$
$$= -6.83\text{ A}$$

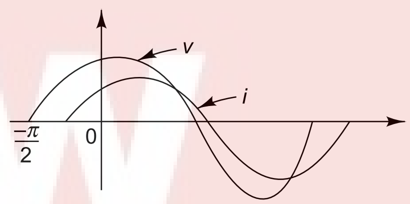


Fig. 3.15

Example 13

At the instant t = 0, the instantaneous value of a 50 Hz sinusoidal current is 5 A and increases in magnitude further. Its rms value is 10 A. (i) Write the expression for its instantaneous value. (ii) Find the current at t = 0.01 s and t = 0.015 s. (iii) Sketch the waveform indicating these values.

Solution $f = 50$ Hz

$$i(0) = 5\text{ A}$$
$$I_{\text{rms}} = 10\text{ A}$$

(i) Expression for instantaneous value of current

$$I_m = I_{\text{rms}} \times \sqrt{2} = 10\sqrt{2} = 14.14\text{ A}$$

Since $i = 5$ A at $t = 0$, the current is leading in nature

$$i = I_m \sin (2\pi ft + \phi)$$

At $t = 0$

$$5 = 14.14 \sin (2 \times 180 \times 50 \times 0 + \phi) \quad \text{(angle in degrees)}$$
$$5 = 14.14 \sin \phi$$
$$\phi = 20.7°$$

$i = 14.14 \sin(100\pi\, t + 20.7°)$

(ii) Current at $t = 0.01$ s

$i = 14.14 \sin(100 \times 180 \times 0.01 + 20.7°)$ (angle in degrees)

$= -5$ A

(iii) Current at $t = 0.015$ s

$i = 14.14 \sin(100 \times 180 \times 0.015 + 20.7°)$ (angle in degrees)

$= -13.23$ A

(iv) Waveform

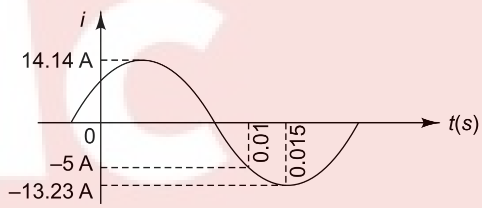


Fig. 3.16

Example 14

In a certain circuit supplied from 50 Hz mains, the potential difference has a maximum value of 500 V and the current has a maximum value of 10 A. At the instant t = 0, the instantaneous values of potential difference and current are 400 V and 4 A respectively, both increasing in the positive direction. State expressions for instantaneous values of potential difference and current at time t. Calculate the instantaneous values at time t = 0.015 second. Find the phase angle between potential difference and current.

Solution

$$f = 50 \text{ Hz}$$
$$V_m = 500 \text{ V}$$
$$I_m = 10 \text{ A}$$
$$v(0) = 400 \text{ V}$$
$$i(0) = 4 \text{ A}$$

(i) Expression for instantaneous values of potential difference and current.

Since $v = 400$ V and $i = 4$ A at $t = 0$, the voltage and current waveforms are leading in nature.

(a) $v = V_m \sin(2\pi f t + \phi_1)$

At $t = 0$

$400 = 500 \sin(2\pi \times 50 \times 0 + \phi_1)$

$\phi_1 = 53.13°$

$\therefore$ $v = 500 \sin(100\pi t + 53.13°)$

(b) $i = I_m \sin(2\pi f t + \phi_2)$

At $t = 0$

$4 = 10 \sin(2\pi \times 50 \times 0 + \phi_2)$

$\phi_2 = 23.58°$

$$\therefore \qquad i = 10 \sin (100\pi t + 23.58°)$$

(ii) Instantaneous values of potential difference and current at $t = 0.015$ second

(a) $$v = 500 \sin (100 \times 180 \times 0.015 + 53.13°) \quad \text{(angle in degrees)}$$
$$= -300 \text{ V}$$

(b) $$i = 10 \sin (100 \times 180 \times 0.015 + 23.58°) \quad \text{(angle in degrees)}$$
$$= -9.17 \text{ A}$$

(iii) Phase angle between potential difference and current

$$\phi = \phi_1 - \phi_2 = 53.13° - 23.58° = 29.55°$$

Example 15

Find the following parameters of a voltage $v = 200 \sin 314\, t$:
(i) frequency, (ii) form factor, and (iii) crest factor.

Solution $$v = 200 \sin 314\, t$$

(i) Frequency

$$v = V_m \sin 2\pi f t$$
$$2\pi f = 314$$
$$f = \frac{314}{2\pi} = 50 \text{ Hz}$$

For a sinusoidal waveform,

$$V_{\text{avg}} = \frac{2V_m}{\pi}$$
$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

(ii) Form factor

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = 1.11$$

(iii) Crest factor

$$k_p = \frac{V_m}{V_{\text{rms}}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = 1.414$$

Example 16

A non-sinusoidal voltage has a form factor of 1.2 and peak factor of 1.5. If the average value of the voltage is 10 V, calculate (i) rms value, and (ii) maximum value.

Solution $$k_f = 1.2$$
$$k_p = 1.5$$
$$V_{\text{avg}} = 10$$

(i) rms value

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}}$$

$$1.2 = \frac{V_{\text{rms}}}{10}$$

$$V_{\text{rms}} = 12 \text{ V}$$

(ii) Maximum value

$$k_p = \frac{V_m}{V_{\text{rms}}}$$

$$1.5 = \frac{V_m}{12}$$

$$V_m = 18 \text{ V}$$

Example 17

The waveform of a voltage has a form factor of 1.15 and a peak factor of 1.5. If the maximum value of the voltage is 4500 V, calculate the average value and rms value of the voltage.

Solution

$$k_f = 1.15$$

$$k_p = 1.5$$

$$V_m = 4500 \text{ V}$$

(i) rms value of the voltage

$$k_p = \frac{V_m}{V_{\text{rms}}}$$

$$1.5 = \frac{4500}{V_{\text{rms}}}$$

$$V_{\text{rms}} = 3000 \text{ V}$$

(ii) Average value of the voltage

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}}$$

$$1.15 = \frac{3000}{V_{\text{avg}}}$$

$$V_{\text{avg}} = 2608.7 \text{ V}$$

Example 18

A 50 Hz sinusoidal current has a peak factor of 1.4 and a form factor of 1.1. Its average value is 20 A. The instantaneous value of the current is 15 A at t = 0. Write the equation of the current and draw its waveform.

Solution

$$f = 50 \text{ Hz}$$
$$k_p = 1.4$$
$$k_f = 1.1$$
$$I_{\text{avg}} = 20 \text{ A}$$
$$i(0) = 15 \text{ A}$$

(i) Equation of current

$$k_f = \frac{I_{\text{rms}}}{I_{\text{avg}}}$$
$$1.1 = \frac{I_{\text{rms}}}{20}$$
$$I_{\text{rms}} = 22 \text{ A}$$
$$k_p = \frac{I_m}{I_{\text{rms}}}$$
$$1.4 = \frac{I_m}{22}$$
$$I_m = 30.8 \text{ A}$$

Since $i = 15$ A at $t = 0$, the current is leading in nature

$$i = I_m \sin(2\pi f t + \phi)$$

At $t = 0$

$$15 = 30.8 \sin(2\pi \times 50 \times 0 + \phi)$$
$$\phi = 29.14°$$
$$\therefore \quad i = 30.8 \sin(100\pi t + 29.14°)$$

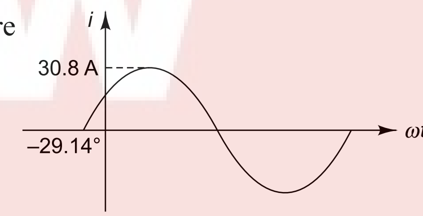


Fig. 3.17

(ii) The waveform is shown is Fig. 3.17.

Example 19

Find the average value and rms value of the waveform shown in Fig. 3.18.

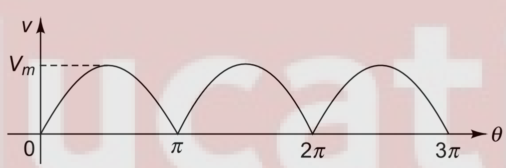


Fig. 3.18

Solution $\quad v = V_m \sin\theta \qquad 0 < \theta < \pi$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta \; d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1]$$

$$= \frac{2V_m}{\pi} = 0.637\ V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\; d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta \; d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta \; d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2}-\frac{\sin 2\pi}{4}-0+\frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\ V_m$$

Example 20

Find the average and rms values of the waveform shown in Fig. 3.19.

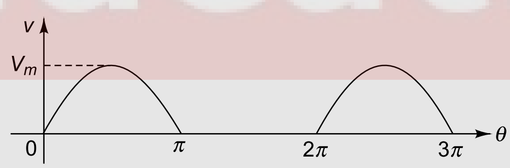


Fig. 3.19

Solution $v = V_m \sin\theta \qquad 0 < \theta < \pi$

$= 0 \qquad \pi < \theta < 2\pi$

(i) Average value of the waveform

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\,d\theta$$

$$= \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{V_m}{2\pi}[1+1] = \frac{V_m}{\pi} = 0.318\,V_m$$

(ii) rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2} = 0.5\,V_m$$

Example 21

Find the average value and rms value of the waveform shown in Fig. 3.20.

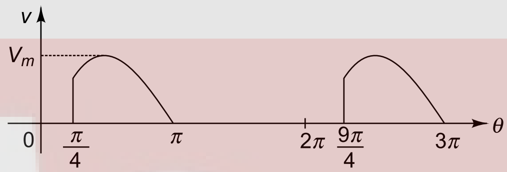


Fig. 3.20

Solution

$$v = 0 \qquad 0 < \theta < \pi/4$$
$$= V_m \sin \theta \qquad \pi/4 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta$$
$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m \sin\theta\, d\theta$$
$$= \frac{V_m}{2\pi}[-\cos\theta]_{\pi/4}^{\pi}$$
$$= \frac{V_m}{2\pi}\,[1 + 0.707] = 0.272\, V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$
$$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta}$$
$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \sin^2\theta\, d\theta}$$
$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$
$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\pi}{8} + \frac{\sin \pi/2}{4}\right]}$$

$$= \sqrt{0.227 V_m^2} = 0.476\ V_m$$

Example 22

A full-wave rectified wave is clipped at 70.7% of its maximum value as shown in Fig. 3.21. Find its average and rms values.

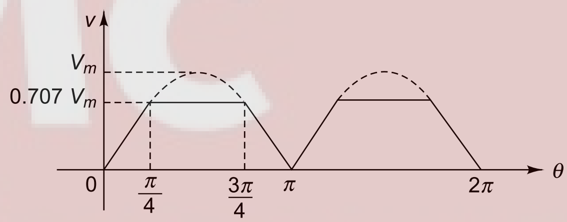


Fig. 3.21

Solution

$$\begin{aligned} v &= V_m \sin\theta && 0 < \theta < \pi/4 \\ &= 0.707\ V_m && \pi/4 < \theta < 3\pi/4 \\ &= V_m \sin\theta && 3\pi/4 < \theta < \pi \end{aligned}$$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\theta\ d\theta + \int_{\pi/4}^{3\pi/4} 0.707 V_m\, d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\ d\theta\right]$$

$$= \frac{V_m}{\pi}\left\{[-\cos\theta]_0^{\pi/4} + 0.707[\theta]_{\pi/4}^{3\pi/4} + [-\cos\theta]_{3\pi/4}^{\pi}\right\}$$

$$= \frac{V_m}{\pi}(0.293 + 1.11 + 0.293) = 0.54\ V_m$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/4} V_m^2 \sin^2\theta\ d\theta + \int_{\pi/4}^{3\pi/4} (0.707 V_m)^2\, d\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\ d\theta\right]}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left\{\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi/4}+0.499[\theta]_{\pi/4}^{3\pi/4}+\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_{3\pi/4}^{\pi}\right\}}$$

$$= \sqrt{0.341 V_m^2} = 0.584\ V_m$$

Example 23

Find the rms value of the waveform shown in Fig. 3.22.

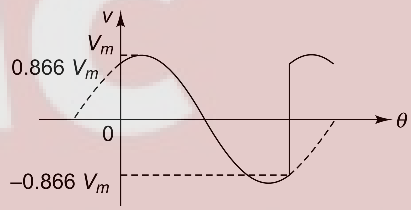


Fig. 3.22

Solution The equation of the waveform is given by $v = V_m \sin(\theta + \phi)$ where ϕ is the phase difference.

When $\theta = 0$, $v = 0.866\ V_m$.

$$0.866\ V_m = V_m \sin(0 + \phi)$$

$$\phi = \sin^{-1}(0.866) = \frac{\pi}{3}$$

$$v = V_m \sin\left(\theta + \frac{\pi}{3}\right)$$

The time period of a complete sine wave is always 2π. Since some part of the waveform is chopped from both the sides,

$$\text{Time period} = 2\pi - \frac{\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{4\pi/3}\int_0^{4\pi/3} V_m^2 \sin^2\left(\theta+\frac{\pi}{3}\right) d\theta}$$

$$= \sqrt{\frac{3}{4\pi}\int_0^{4\pi/3} V_m^2 \sin^2\left(\theta+\frac{\pi}{3}\right) d\theta}$$

$$= \sqrt{\frac{3V_m^2}{4\pi}\int_0^{4\pi/3}\left[\frac{1-\cos 2(\theta+\pi/3)}{2}\right] d\theta}$$

$$= \sqrt{\frac{3V_m^2}{4\pi}\left[\frac{\theta}{2} - \frac{\sin 2(\theta + \pi/3)}{4}\right]_0^{4\pi/3}}$$

$$= \sqrt{0.6031\,V_m^2} = 0.776\ V_m$$

Example 24

Find the average and rms values of the waveform shown in Fig. 3.23.

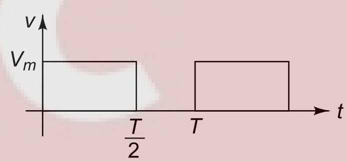


Fig. 3.23

Solution $v = V_m \qquad 0 < t < T/2$

$= 0 \qquad T/2 < t < T$

(i) Average value of the waveform

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/2} V_m\,dt + \int_{T/2}^{T} 0\,dt\right]$$

$$= \frac{1}{T}\int_0^{T/2} V_m\,dt$$

$$= \frac{V_m}{T}[t]_0^{T/2}$$

$$= \frac{V_m}{T}\cdot\frac{T}{2} = 0.5\ V_m$$

(ii) rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2\,dt}$$

$$= \sqrt{\frac{V_m^2}{T}[t]_0^{T/2}}$$

$$= \sqrt{\frac{V_m^2}{T}.\frac{T}{2}} = \sqrt{\frac{V_m^2}{2}} = 0.707\, V_m$$

Example 25

Find the average and rms values of the waveform shown in Fig. 3.24.

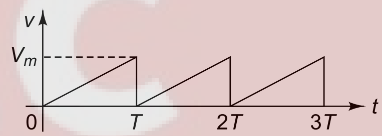


Fig. 3.24

Solution $v = \frac{V_m}{T} t \qquad 0 < t < T$

(i) Average value of the waveform

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\, dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m}{T} t\; dt$$

$$= \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{V_m}{T^2}\cdot\frac{T^2}{2} = 0.5\, V_m$$

(ii) rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \frac{V_m^2}{T^2}.t^2\, dt}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{T^3}{3}\right]}$$

$$= \sqrt{\frac{V_m^2}{3}} = 0.577\ V_m$$

Example 26

Find the average and rms values of the waveform shown in Fig. 3.25.

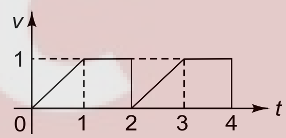


Fig. 3.25

Solution $v = t \qquad 0 < t < 1$

$= 1 \qquad 1 < t < 2$

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 1\,dt\right]$$

$$= \frac{1}{2}\left\{\left[\frac{t^2}{2}\right]_0^1 + [t]_1^2\right\}$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0 + 2 - 1\right]$$

$$= \frac{3}{4} = 0.75\ \text{V}$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (1)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{2}\left\{\left[\frac{t^3}{3}\right]_0^1 + [t]_1^2\right\}}$$

$$= \sqrt{\frac{1}{2}\left[\frac{1}{3} - 0 + 2 - 1\right]}$$

$$= \sqrt{\frac{4}{6}} = 0.816 \text{ V}$$

Example 27

Find the average and rms values of the waveform shown in Fig. 3.26.

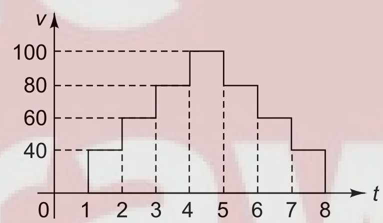


Fig. 3.26

Solution

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{0+40+60+80+100+80+60+40}{8} = 57.5 \text{ V}$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{0^2+(40)^2+(60)^2+(80)^2+(100)^2+(80)^2+(60)^2+(40)^2}{8}} = 64.42 \text{ V}$$

Example 28

Find the average and rms values of the waveform shown in Fig. 3.27.

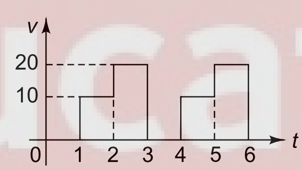


Fig. 3.27

Solution

(i) Average value of the waveform

$$V_{\text{avg}} = \frac{0+10+20}{3} = 10 \text{ V}$$

(ii) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{0^2 + (10)^2 + (20)^2}{3}} = 12.9 \text{ V}$$

Example 29

Find the effective value of the resultant current which carries simultaneously a direct current of 10 A and a sinusoidally alternating current with a peak value of 10 A.

Solution

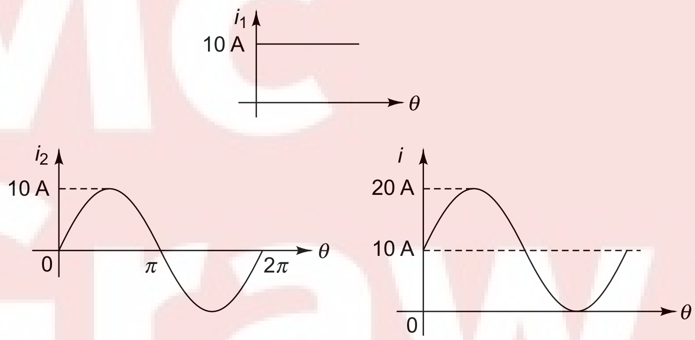


Fig. 3.28

$$i = i_1 + i_2 = 10 + 10 \sin \theta$$

$$I_{\text{eff}} = I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\theta)^2\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (100 + 200\sin\theta + 100\sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} (1 + 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} \left[1 + 2\sin\theta + \left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\left[\theta - 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi - 2\cos 2\pi + \frac{2\pi}{2} - \frac{\sin 4\pi}{4} - 0 + 2\cos 0 - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi - 2 + \frac{2\pi}{2} + 2\right]}$$

$$= \sqrt{\frac{100}{2\pi} \times 3\pi} = \sqrt{150} = 12.25 \text{ A}$$

Example 30

Find the effective value of a resultant current in a wire which carries simultaneously a direct current of i_1 = 10 A and alternating current given by $i_2 = 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$.

Solution

$$i_1 = 10 \text{ A}$$

$$i_2 = 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$$

$$i = i_1 + i_2$$

$$= 10 + 12 \sin \omega t + 6 \sin\left(3\omega t - \frac{\pi}{6}\right) + 4 \sin\left(5\omega t + \frac{\pi}{3}\right)$$

$$= 10 + 12 \sin \theta + 6 \sin (3\theta - 30°) + 4 \sin (5\theta + 60°)$$

$$I_{\text{eff}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\theta) d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [10 + 12\sin\theta + 6\sin(3\theta - 30°) + 4\sin(5\theta + 60°)]^2 d\theta}$$

$$= 14.07 \text{ A}$$

Example 31

Find the relative heating effects of two current waves of equal peak value, one sinusoidal and the other, rectangular in shape.

Solution

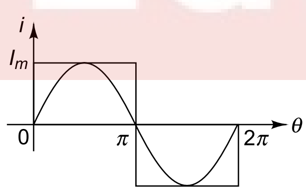


Fig. 3.29

rms value of the rectangular wave = I_m

rms value of the sinusoidal current wave = $\frac{I_m}{\sqrt{2}}$

Heating effect due to the rectangular current wave = $(I_m)^2 RT$

Heating effect due to the sinusoidal current wave $= \left(\dfrac{I_m}{\sqrt{2}}\right)^2 RT = \dfrac{(I_m)^2}{2}\ RT$

$$\text{Relative heating effects} = \frac{(I_m)^2}{2} RT : (I_m)^2\, RT$$

$$= \frac{1}{2} : 1 = 1 : 2$$

Useful Formulae

Average Value and rms value

$$F_{\text{avg}} = \frac{1}{T}\int_0^T f(t)dt \qquad k_P = \frac{\text{max. value}}{\text{rms value}}$$

$$F_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt} \qquad k_f = \frac{\text{rms value}}{\text{avg. value}}$$

Exercise 3.1

3.1 An alternating current varying sinusoidally with a frequency of 50 Hz has an rms value of 20 A. Write down the equation for the instantaneous value and find this value (i) 0.0025 second, and (ii) 0.0125 second after passing through a positive maximum value. At what time, measured from a positive maximum value, will the instantaneous current be 14.14 A? *[$i = 28.28 \sin 100\,\pi\, t$, 20 A, –20 A, $\frac{1}{300}$ s]*

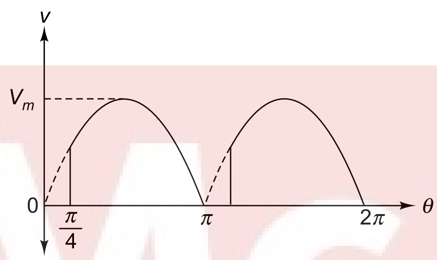
3.2 A certain waveform has a form factor of 1.2 and a peak factor of 1.5. If the maximum value is 100, find the rms value and average value. *[66.67 V, 55.67 V]*

3.3 Find the root mean square value, the average value and the form factor of the resultant current in a wire that carries simultaneously a direct current of 5 A and a sinusoidal alternating current with an amplitude of 5 A. *[6.12 A, 5 A, 1.224]*

3.4 Find the relative heating effects of two in-phase current waveforms of equal peak values and time periods, but one sinusoidal and the other triangular. *[3:2]*

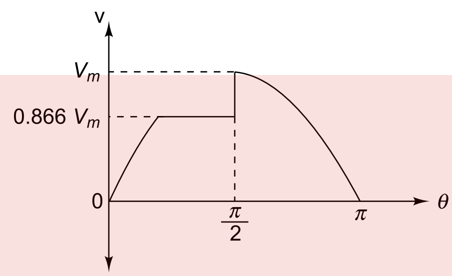
3.5 Prove that if a dc current of I amperes is superimposed in a conductor by an ac current of maximum value I amperes, the root mean square value of its resultant is $\sqrt{\dfrac{3}{2}}\, I$.

3.6 Find the average values and rms values of the waveforms shown in Fig. 3.30 to Fig. 3.33.



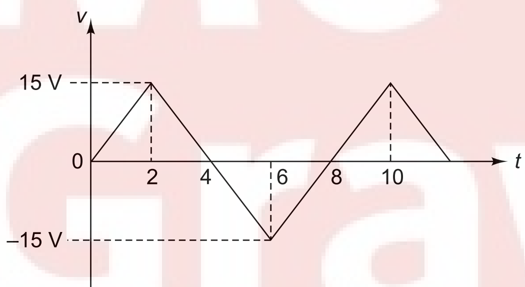
[0.543 V_m, 0.674 V_m]

Fig. 3.30



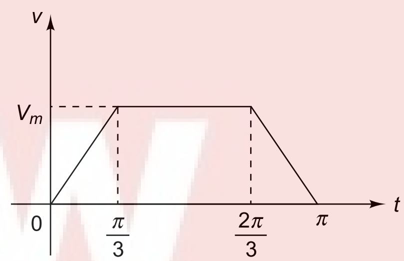
[0.622 V_m, 0.687 V_m]

Fig. 3.31



[7.5 V, 8.666 V]

Fig. 3.32



[0.67 V_m, 0.745 V_m]

Fig. 3.33

3.7 Find the rms value of the periodic waveform with time period T given in Fig. 3.34.

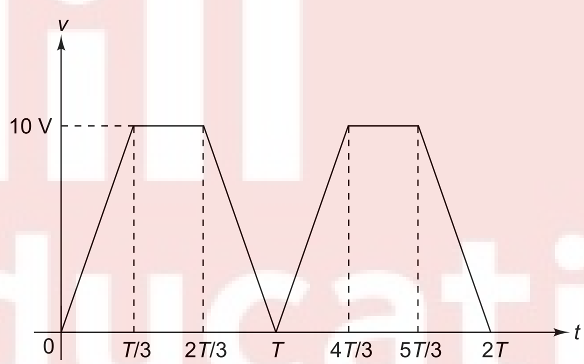


Fig. 3.34

[7.45 V]

3.8 A voltage wave has the variation shown in Fig. 3.35.
(i) Find the average and effective values of the voltage.
(ii) If this voltage is applied to a 10 Ω resistance, find the dissipated power.

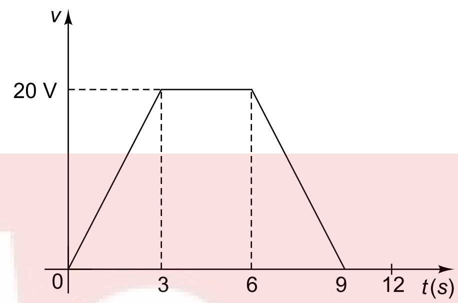


Fig. 3.35

[10 V, 12.91 V, 16.67 W]

3.5 PHASOR REPRESENTATIONS OF ALTERNATING QUANTITIES

The alternating quantities are represented by phasors. A phasor is a line of definite length rotating in an anticlockwise direction at a constant angular velocity ω. The length of a phasor is equal to the maximum value of the alternating quantity, and the angular velocity is equal to the angular velocity of alternating quantity.

As shown in Fig. 3.36(a), consider a phasor $OP = I_m$, where I_m is the maximum value of the alternating current. Let this phasor rotate in an anticlockwise direction at a uniform angular velocity of ω radians/second. The projection of the phasor OP on the Y-axis at any instant gives the instantaneous value of that alternating current.

$$\begin{aligned} OM &= OP \sin \omega t \\ &= I_m \sin \omega t = i \end{aligned}$$

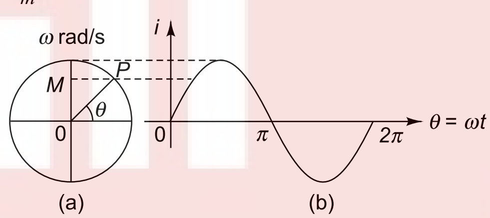


Fig. 3.36 *Representation of alternating quantities in terms of phasors*

Thus, if we plot the projections of the phasor on the Y-axis versus its angular position point by point, a sinusoidal alternating current waveform is obtained.

Phasor Diagram using rms Values Sinusoidal alternating currents and voltages can be represented by phasors. Electrical measuring instruments like ammeters and voltmeters are calibrated to read the rms values of ac quantities. Hence, instead of using maximum values, it is more convenient to draw phasor diagrams using rms values of alternating quantities. However, such a phasor diagram will not generate a sine wave of proper amplitude unless the length of the phasor is multiplied by $\sqrt{2}$.

Example 1

Two currents i_1 and i_2 are given by the expressions $i_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right)$ and $i_2 = 8 \sin\left(\omega t - \frac{\pi}{3}\right)$. Find (i) $i_1 + i_2$, and (ii) $i_1 - i_2$. Express the answers in the form $i = I_m \sin(\omega t \pm \phi)$.

Solution

$$i_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$i_2 = 8 \sin\left(\omega t - \frac{\pi}{3}\right)$$

(i) Let phasors $\bar{I}_1$ and $\bar{I}_2$ represent the alternating currents i_1 and i_2 respectively in terms of their maximum values.

(a) Analytical method:

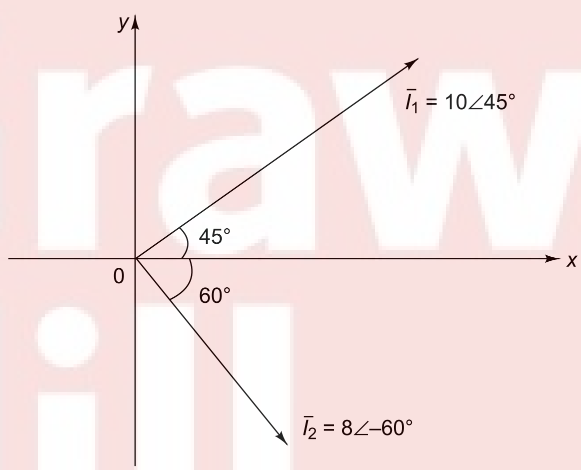


Fig. 3.37

Resolving $\bar{I}_1$ and $\bar{I}_2$ into x- and y-components,

$$\Sigma x = 10 \cos(45°) + 8 \cos(-60°) = 11.07$$

$$\Sigma y = 10 \sin(45°) + 8 \sin(-60°) = 0.14$$

$$\text{Magnitude of } (\bar{I}_1 + \bar{I}_2) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(11.07)^2 + (0.14)^2}$$

$$= 11.07 \text{ A}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{0.14}{11.07}\right) = 0.72°$$

$$i = i_1 + i_2 = 11.07\ (\omega t + 0.72°)$$

(b) Graphical method: The phasor sum $\bar{I}_1 + \bar{I}_2$ is obtained by adding phasors $\bar{I}_1$ and $\bar{I}_2$ by the parallelogram law.

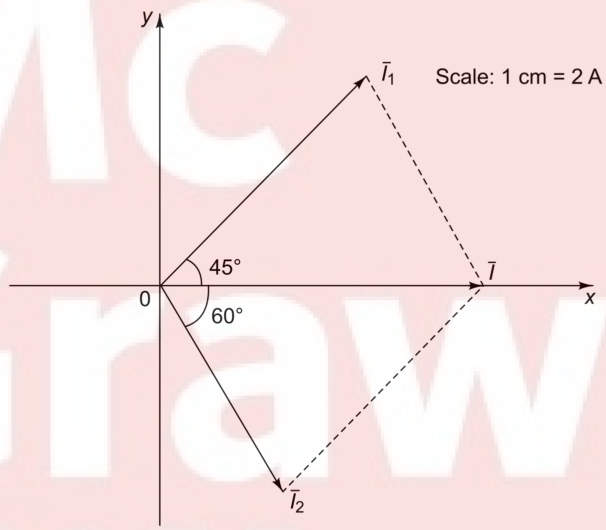


Fig. 3.38

(ii) (a) Analytical method:

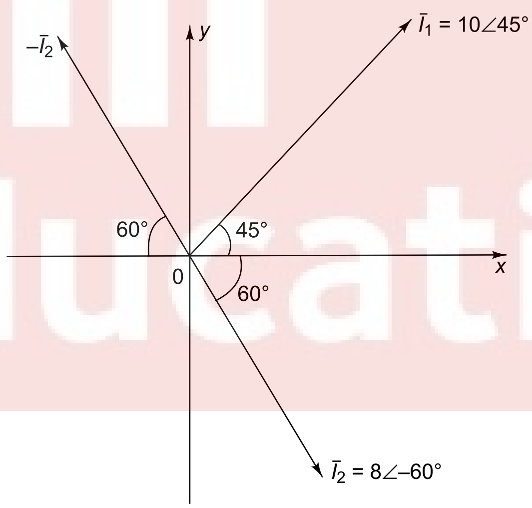


Fig. 3.39

Resolving $\bar{I}_1$ and $-\bar{I}_2$ into x- and y-components,

$$\Sigma x = 10 \cos(45°) - 8 \cos(-60°) = 3.07$$

$$\Sigma y = 10 \sin(45°) - 8 \sin(-60°) = 14$$

$$\text{Magnitude of } (\bar{I}_1 - \bar{I}_2) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(3.07)^2 + (14)^2}$$

$$= 14.33 \text{ A}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{14}{3.07}\right)$$

$$= 77.63°$$

$$i = i_1 - i_2 = 14.33 \sin(\omega t + 77.63°)$$

(b) Graphical method: The phasor sum $\bar{I}_1 - \bar{I}_2$ is obtained by adding phasors $\bar{I}_1$ and $-\bar{I}_2$ by the parallelogram law.

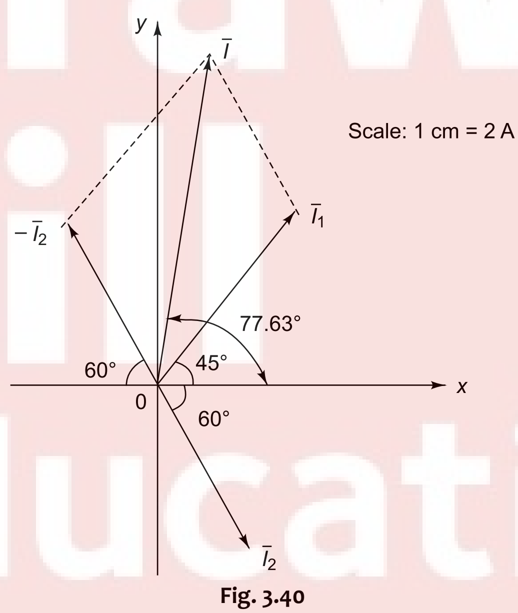


Fig. 3.40

Example 2

Three voltages are represented by $v_1 = 10 \sin \omega t$, $v_2 = 20 \sin\left(\omega t - \frac{\pi}{6}\right)$ and $v_3 = 30 \sin\left(\omega t + \frac{\pi}{4}\right)$. Find the magnitude and phase angle of the resultant voltage.

Solution $v_1 = 10 \sin \omega t$

$$v_2 = 20 \sin \left(\omega t - \frac{\pi}{6}\right)$$

$$v_3 = 30 \sin \left(\omega t + \frac{\pi}{4}\right)$$

Let phasors $\bar{V}_1$, $\bar{V}_2$ and $\bar{V}_3$ represent the alternating voltages v_1, v_2 and v_3 respectively in terms of their maximum values.

(a) Analytical method:

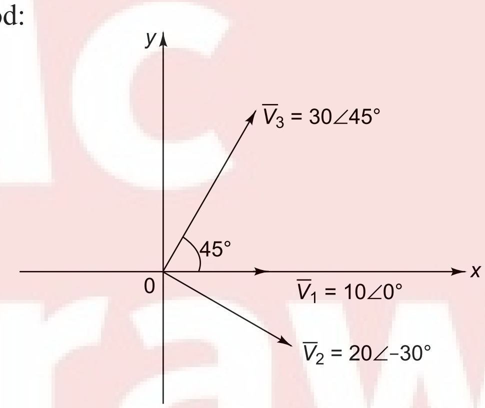


Fig. 3.41

Resolving $\bar{V}_1$, $\bar{V}_2$ and $\bar{V}_3$ into *x*- and *y*-components,

$$\Sigma x = 10 + 20 \cos(-30°) + 30 \cos(45°) = 48.53$$

$$\Sigma y = 20 \sin(-30°) + 30 \cos(45°) = 11.21$$

$$\text{Magnitude of } (\bar{V}_1 + \bar{V}_2 + \bar{V}_3) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(48.53)^2 + (11.21)^2}$$

$$= 49.81 \text{ V}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{11.21}{48.53}\right)$$

$$= 13°$$

$$v = 49.81 \sin(\omega t + 13°)$$

(b) Graphical method: The phasor sum $\bar{V}_1 + \bar{V}_2 + \bar{V}_3$ is obtained by first adding phasors $\bar{V}_1$ and $\bar{V}_2$ by the parallelogram law and then adding $\bar{V}_3$ to the resultant of $\bar{V}_1$ and $\bar{V}_2$ by the parallelogram law.

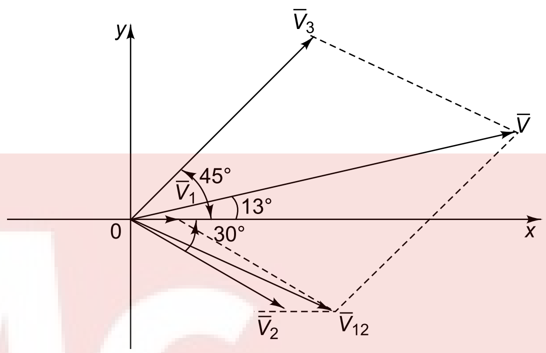


Fig. 3.42

Example 3

The instantaneous voltages across each of the four coils connected in series are given by

$$v_1 = 100 \sin \omega t, \quad v_2 = 250 \cos \omega t, \quad v_3 = 150 \sin\left(\omega t + \frac{\pi}{6}\right), \quad v_4 = 200 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Determine the resultant voltage by analytical method.

Solution

$$v_1 = 100 \sin \omega t$$

$$v_2 = 250 \cos \omega t = 250 \sin (\omega t + 90°)$$

$$v_3 = 150 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_4 = 200 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Let phasors $\bar{V}_1$, $\bar{V}_2$, $\bar{V}_3$ and $\bar{V}_4$ represent the instantaneous voltages v_1, v_2, v_3 and v_4 respectively in terms of their maximum values.

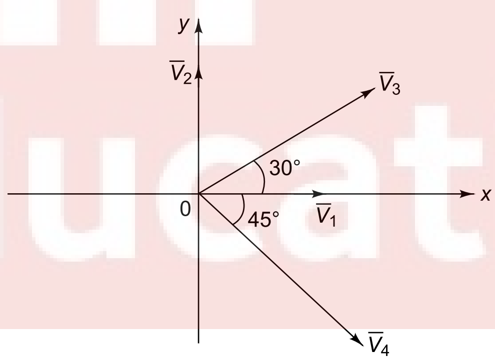


Fig. 3.43

Resolving $\bar{V}_1$, $\bar{V}_2$, $\bar{V}_3$ and $\bar{V}_4$ into *x*- and *y*-components,

$$\Sigma x = 100 + 250 \cos (90°) + 150 \cos (30°) + 200 \cos (-45°) = 371.33$$

$$\Sigma y = 250 \sin (90°) + 150 \sin (30°) + 200 \sin (-45°) = 183.58$$

$$\text{Magnitude of } (\bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(371.33)^2 + (183.58)^2}$$

$$= 414.23 \text{ V}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{183.58}{371.33}\right) = 26.31°$$

$$v = v_1 + v_2 + v_3 + v_4 = 414.23 \sin (\omega t + 26.31°)$$

3.6 MATHEMATICAL REPRESENTATIONS OF PHASORS

A phasor can be represented in four forms.

(i) Rectangular form

$$\bar{V} = X \pm jY$$

$$\text{Magnitude of phasor, } V = \sqrt{X^2 + Y^2}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{Y}{X}\right)$$

(ii) Trigonometric form

$$\bar{V} = V(\cos \phi \pm j \sin \phi)$$

(iii) Exponential form

$$\bar{V} = V e^{\pm j\phi}$$

(iv) Polar form

$$\bar{V} = V \angle \pm \phi$$

Significance of Operator j The operator *j* is used in rectangular form. It is used to indicate anticlockwise rotation of a phasor through 90°. Mathematically,

$$j = \sqrt{-1}$$

Whenever a phasor is multiplied by *j*, the phasor is rotated once in the anticlockwise direction through 90°. The power of *j* represents the number of times the phasor should be rotated through 90° in the anticlockwise direction.

Example 1

Two sinusoidal currents are given as

$$i_1 = 10\sqrt{2}\ \sin \omega t,\ i_2 = 20\sqrt{2}\ \sin(\omega t + 60°).$$

Find the expression for the sum of these currents.

Solution

$$i_1 = 10\sqrt{2}\ \sin \omega t$$

$$i_2 = 20\sqrt{2}\ \sin(\omega t + 60°)$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{10\sqrt{2}}{\sqrt{2}}\angle 0° = 10\angle 0°$$

$$\bar{I}_2 = \frac{20\sqrt{2}}{\sqrt{2}}\angle 60° = 20\angle 60°$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

$$= 10\angle 0° + 20\angle 60° = 26.46\angle 40.89°$$

$$i = 26.46\sqrt{2}\ \sin(\omega t + 40.89°)$$

$$= 37.42\ \sin(\omega t + 40.89°)$$

Example 2

The following three sinusoidal currents flow into the junction $i_1 = 3\sqrt{2}\sin \omega t$, $i_2 = 5\sqrt{2}\sin(\omega t + 30°)$ and $i_3 = 6\sqrt{2}\sin(\omega t - 120°)$. Find the expression for the resultant current which leaves the junction.

Solution

$$i_1 = 3\sqrt{2}\ \sin \omega t$$

$$i_2 = 5\sqrt{2}\ \sin(\omega t + 30°)$$

$$i_3 = 6\sqrt{2}\ \sin(\omega t - 120°)$$

Writing currents i_1, i_2 and i_3 in the phasor form,

$$\bar{I}_1 = \frac{3\sqrt{2}}{\sqrt{2}}\angle 0° = 3\angle 0°$$

$$\bar{I}_2 = \frac{5\sqrt{2}}{\sqrt{2}}\angle 30° = 5\angle 30°$$

$$\bar{I}_3 = \frac{6\sqrt{2}}{\sqrt{2}}\angle -120° = 6\angle -120°$$

The resultant current which leaves the junction is given by

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 3\angle 0° + 5\angle 30° + 6\angle -120°$$
$$= 5.1\angle -31.9°$$
$$i = 5.1\sqrt{2}\sin(\omega t - 31.9°)$$
$$= 7.21\sin(\omega t - 31.9°)$$

Example 3

In a circuit, four currents as indicated below, are meeting at a point. Find the resultant current.

$i_1 = 5\sin\omega t,\qquad i_2 = 10\sin(\omega t - 30°)$

$i_3 = 5\cos(\omega t - 30°)\qquad i_4 = -10\sin(\omega t + 45°)$

Solution

$$i_1 = 5\sin\omega t$$
$$i_2 = 10\sin(\omega t - 30°)$$
$$i_3 = 5\cos(\omega t - 30°) = 5\sin(\omega t + 60°)$$
$$i_4 = -10\sin(\omega t + 45°) = 10\sin(\omega t + 225°)$$

Writing currents i_1, i_2, i_3 and i_4 in the phasor form,

$$\bar{I}_1 = \frac{5}{\sqrt{2}}\angle 0° = 3.54\angle 0°$$
$$\bar{I}_2 = \frac{10}{\sqrt{2}}\angle -30° = 7.07\angle -30°$$
$$\bar{I}_3 = \frac{5}{\sqrt{2}}\angle 60° = 3.54\angle 60°$$
$$\bar{I}_4 = \frac{10}{\sqrt{2}}\angle 225° = 7.07\angle 225°$$

Resultant current $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 + \bar{I}_4$

$$= 3.54\angle 0° + 7.07\angle -30° + 3.54\angle 60° + 7.07\angle 225°$$
$$= 8.44\angle -40.36°$$
$$i = 8.44\sqrt{2}\sin(\omega t - 40.36°)$$
$$= 11.94\sin(\omega t - 40.36°)$$

Example 4

Find the resultant voltage and its equation for the given voltages.

$$e_1 = 20\sin\omega t,\qquad e_2 = 30\sin\left(\omega t - \frac{\pi}{4}\right),\qquad e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right)$$

Solution

$$e_1 = 20 \sin \omega t$$

$$e_2 = 30 \sin \left(\omega t - \frac{\pi}{4}\right) = 30 \sin (\omega t - 45°)$$

$$e_3 = 40 \cos \left(\omega t + \frac{\pi}{6}\right) = 40 \sin (\omega t + 120°)$$

Writing voltages e_1, e_2 and e_3 in the phasor form,

$$\bar{E}_1 = \frac{20}{\sqrt{2}} \angle 0° = 14.14\angle 0°$$

$$\bar{E}_2 = \frac{30}{\sqrt{2}} \angle -45° = 21.21\angle -45°$$

$$\bar{E}_3 = \frac{40}{\sqrt{2}} \angle 120° = 28.28\angle 120$$

Resultant voltage

$$\begin{aligned}\bar{E} &= \bar{E}_1 + \bar{E}_2 + \bar{E}_3 \\ &= 14.14\angle 0° + 21.21\angle -45° + 28.28\angle 120° \\ &= 17.75\angle 32.33°\end{aligned}$$

$$\begin{aligned}e &= 17.75\sqrt{2} \sin (\omega t + 32.33°) \\ &= 25.1 \sin (\omega t + 32.33°)\end{aligned}$$

Example 5

Obtain the sum of the three voltages.

$$v_1 = 147.3 \cos (\omega t + 98.1°)$$

$$v_2 = 294.6 \cos (\omega t - 45°)$$

$$v_3 = 88.4 \sin (\omega t + 135°)$$

Solution

$$v_1 = 147.3 \cos (\omega t + 98.1°) = 147.3 \sin (\omega t + 188.1°)$$

$$v_2 = 294.6 \cos (\omega t - 45°) = 294.6 \sin (\omega t + 45°)$$

$$v_3 = 88.4 \sin (\omega t + 135°)$$

Writing voltages v_1, v_2 and v_3 in the phasor form,

$$\bar{V}_1 = \frac{147.3}{\sqrt{2}} \angle 188.1° = 104.16\angle 188.1°$$

$$\bar{V}_2 = \frac{294.6}{\sqrt{2}} \angle 45° = 208.31\angle 45°$$

$$\bar{V}_3 = \frac{88.4}{\sqrt{2}} \angle 135° = 62.51\angle 135°$$

Resultant voltage $\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3$

$= 104.16\angle 188.1° + 208.31\angle 45° + 62.51\angle 135°$

$= 176.82\angle 90°$

$v = 176.82\sqrt{2}\sin(\omega t + 90°)$

$= 250.06 \sin(\omega t + 90°)$

Example 6

Find vectorially the resultant of the following four voltages.

$$e_1 = 25 \sin \omega t, \qquad e_2 = 30 \sin\left(\omega t + \frac{\pi}{6}\right),$$

$$e_3 = 30 \cos \omega t, \qquad e_4 = 20 \sin\left(\omega t - \frac{\pi}{6}\right)$$

Obtain the answer in similar form.

Solution $e_1 = 25 \sin \omega t$

$$e_2 = 30 \sin\left(\omega t + \frac{\pi}{6}\right) = 30 \sin(\omega t + 30°)$$

$$e_3 = 30 \cos \omega t = 30 \sin(\omega t + 90°)$$

$$e_4 = 20 \sin\left(\omega t - \frac{\pi}{6}\right) = 20 \sin(\omega t - 30°)$$

Writing voltages e_1, e_2, e_3 and e_4 in the phasor form,

$$\bar{E}_1 = \frac{25}{\sqrt{2}}\angle 0° = 17.68\angle 0°$$

$$\bar{E}_2 = \frac{30}{\sqrt{2}}\angle 30° = 21.21\angle 30°$$

$$\bar{E}_3 = \frac{30}{\sqrt{2}}\angle 90° = 21.21\angle 90°$$

$$\bar{E}_4 = \frac{20}{\sqrt{2}}\angle -30° = 14.14\angle -30°$$

Resultant voltage $\bar{E} = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \bar{E}_4$

$= 17.68\angle 0° + 21.21\angle 30° + 21.21\angle 90° + 14.14\angle -30°$

$= 54.26\angle 27.13°$

$e = 54.26\sqrt{2}\sin(\omega t + 27.13°)$

$= 76.74 \sin(\omega t + 27.13°)$

Example 7

Two currents are represented by $i_1 = 15 \sin\left(\omega t + \frac{\pi}{3}\right)$ *and* $i_2 = 25 \sin\left(\omega t + \frac{\pi}{4}\right)$. *These currents are fed into a common conductor. Find the total current in the form* $i = I_m \sin(\omega t + \phi)$. *If the conductor has a resistance of 10 Ω, what will be the energy loss in 24 hours?*

Solution

$$i_1 = 15 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$i_2 = 25 \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$R = 10\ \Omega$$

$$t = 24 \text{ hours} = 86400 \text{ seconds}$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{15}{\sqrt{2}} \angle 60° = 10.61\angle 60°$$

$$\bar{I}_2 = \frac{25}{\sqrt{2}} \angle 45° = 17.68\angle 45°$$

Total current $\bar{I} = \bar{I}_1 + \bar{I}_2 = 10.61\angle 60° + 17.68\angle 45°$

$$= 28.06\ \angle 50.62°$$

$$i = 28.06\sqrt{2} \sin(\omega t + 50.62°)$$

$$= 39.68 \sin(\omega t + 50.62°)$$

Energy loss in 24 hours, $E = I^2Rt$ where I is the rms value of the current

$$E = (28.06)^2 \times 10 \times 86400 = 6.8 \times 10^8 \text{ J}$$

Example 8

The voltage drops across four series-connected impedances are given:

$$v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right) \qquad v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right),$$

$$v_3 = 100 \cos\left(\omega t + \frac{\pi}{4}\right), \qquad v_4 = V_{4m} \sin(\omega t + \phi_4)$$

Calculate the values of V_{4m} *and* ϕ_4 *if the voltage applied across the series circuit is* $140 \sin\left(\omega t + \frac{3\pi}{5}\right)$.

Solution $v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right) = 60 \sin(\omega t + 30°)$

$$v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right) = 75 \sin(\omega t - 150°)$$

$$v_3 = 100 \cos\left(\omega t + \frac{\pi}{4}\right) = 100 \sin(\omega t + 135°)$$

$$v = 140 \sin\left(\omega t + \frac{3\pi}{5}\right) = 140 \sin(\omega t + 108°)$$

Writing voltages v_1, v_2, v_3 and v in the phasor form,

$$\bar{V}_1 = \frac{60}{\sqrt{2}}\angle 30° = 42.43\angle 30°$$

$$\bar{V}_2 = \frac{75}{\sqrt{2}}\angle -150° = 53.03\angle -150°$$

$$\bar{V}_3 = \frac{100}{\sqrt{2}}\angle 135° = 70.71\angle 135°$$

$$\bar{V} = \frac{140}{\sqrt{2}}\angle 108° = 98.99\angle 108°$$

For series-connected impedances,

$$\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4$$

$$\bar{V}_4 = \bar{V} - \bar{V}_1 - \bar{V}_2 - \bar{V}_3$$

$$= 98.99\angle 108° - 42.43\angle 30° - 53.03\angle -150° - 70.71\angle 135°$$

$$= 57.13\angle 59.96°$$

$$v_4 = 57.13\sqrt{2}\sin(\omega t + 59.96°)$$

$$= 80.79 \sin(\omega t + 59.96°)$$

$$V_{4m} = 80.79 \text{ V}$$

$$\phi_4 = 59.96°$$

Example 9

A circuit consists of three parallel branches. The branch currents are given as $i_1 = 10 \sin \omega t$, $i_2 = 20 \sin(\omega t + 60°)$ and $i_3 = 7.5 \sin(\omega t - 30°)$. Find the resultant current and express it in the form $i = I_m \sin(\omega t + \phi)$. If the supply frequency is 50 Hz, calculate the resultant current when (i) $t = 0$ (ii) $t = 0.001$ s.

Solution

$$i_1 = 10 \sin \omega t$$

$$i_2 = 20 \sin(\omega t + 60°)$$

$$i_3 = 7.5 \sin(\omega t - 30°)$$

Writing currents i_1, i_2 and i_3 in the phasor form,

$$\bar{I}_1 = \frac{10}{\sqrt{2}}\angle 0^\circ = 7.07\angle 0^\circ$$

$$\bar{I}_2 = \frac{20}{\sqrt{2}}\angle 60^\circ = 14.14\angle 60^\circ$$

$$\bar{I}_3 = \frac{7.5}{\sqrt{2}}\angle -30^\circ = 5.3\angle -30^\circ$$

Resultant current $\bar{I}_1 = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

$$= 7.07\angle 0^\circ + 14.14\angle 60^\circ + 5.3\angle -30^\circ$$

$$= 21.04\angle 27.13^\circ$$

$$i = 21.04\sqrt{2}\,\sin(\omega t + 27.13^\circ)$$

$$= 29.76\sin(\omega t + 27.13^\circ)$$

(i) Resultant current at $t = 0$

$$i = 29.76\sin(0 + 27.13^\circ)$$

$$= 13.57\text{ A}$$

(ii) Resultant current at $t = 0.001$ s

$$i = 29.76\sin(2\pi f t + 27.13^\circ)$$

$$= 29.76\sin(2\times 180\times 50\times 0.001 + 27.13^\circ) \qquad \text{(angle in degrees)}$$

$$= 21.09\text{ A}$$

Example 10

Two currents, $\bar{I}_1 = 10\angle 50^\circ$ A and $\bar{I}_2 = 5\angle -100^\circ$ A, flow in a single-phase ac circuit. Estimate (i) $\bar{I}_1 + \bar{I}_2$ (ii) $\bar{I}_1 . \bar{I}_2$ (ii) $\frac{\bar{I}_1}{\bar{I}_2}$.

Solution $\bar{I}_1 = 10\angle 50^\circ$ A

$\bar{I}_2 = 5\angle -100^\circ$ A

(i) $\bar{I}_1 + \bar{I}_2 = 10\angle 50^\circ + 5\angle -100^\circ = 6.2\angle 26.21^\circ$ A

(ii) $\bar{I}_1 . \bar{I}_2 = (10\angle 50^\circ)(5\angle -100^\circ) = 50\angle -50^\circ$ A

(ii) $$\frac{\bar{I}_1}{\bar{I}_2} = \frac{10\angle 50^\circ}{5\angle -100^\circ} = 2\angle 150^\circ\text{ A}$$

Example 11

Two voltages having rms values of 50 V and 75 V have a phase difference of 60°. Find the resultant sum of these two voltages.

Solution $V_1 = 50$ V

$V_2 = 75$ V

$\phi = 60°$

Let $\bar{V}_1 = 50 \angle 0°$ V

$\bar{V}_2 = 75 \angle -60°$ V

Resultant voltage $\bar{V} = \bar{V}_1 + \bar{V}_2$

$= 50\angle 0° + 75\angle -60° = 108.97 \angle -36.58°$ V

Example 12

Two single-phase alternators supply 300 A and 400 A respectively at a phase difference of 20° to a common load. Find the resultant current and its phase relation to its component.

Solution $I_1 = 300$ A

$I_2 = 400$ A

$\phi = 20°$

Let $\bar{I}_1 = 300 \angle 0°$ A

$\bar{I}_2 = 400 \angle -20°$ A

Resultant current $\bar{I} = \bar{I}_1 + \bar{I}_2$

$= 300\angle 0° + 400\angle -20° = 689.59 \angle -11.44°$ A

Example 13

Two voltage sources have equal emfs and a phase difference α. When they are connected in series, the voltage is 200 V. When one source is reversed, the voltage is 15 V. Find their emfs and phase angle α.

Solution $\bar{E}_1 = E \angle 0°$

$\bar{E}_2 = E \angle \alpha°$

$E_1 = E_2 = E$

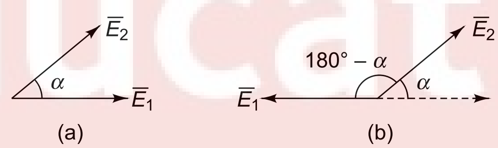


Fig. 3.44

When two sources are connected in series,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\alpha} = 200$$

$$\sqrt{E^2 + E^2 + 2E^2 \cos\alpha} = 200$$

$$2E^2 + 2E^2 \cos \alpha = 40000 \qquad (1)$$

When one source is reversed,

$$\sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha} = 15$$

$$\sqrt{E^2 + E^2 - 2E^2 \cos\alpha} = 15$$

$$2E^2 - 2E^2 \cos \alpha = 225 \qquad (2)$$

Adding Eqs (1) and (2),

$$4E^2 = 40225$$

$$E^2 = 10056.25$$

$$E = 100.28 \text{ V}$$

$$2E^2 + 2E^2 \cos \alpha = 40000$$

$$20112.5 + 20112.5 \cos \alpha = 40000$$

$$\cos \alpha = 0.988$$

$$\alpha = 8.58°$$

Example 14

Two sinusoidal sources of emf have rms values of E_1 and E_2 and a phase difference of α. When connected in series, the resultant voltage is 41.1 V. When one of the sources is reversed, the resultant emf is 17.52 V. When phase displacement is made zero, the resultant emf is 42.5 V. Calculate E_1, E_2 and α.

Solution

$$\bar{E}_1 = E_1 \angle 0°$$

$$\bar{E}_2 = E_2 \angle \alpha°$$

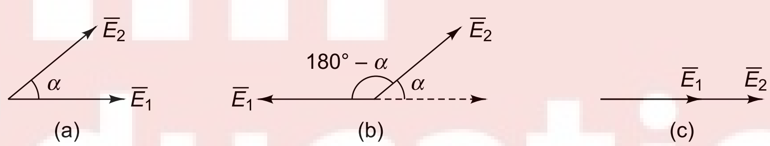


Fig. 3.45

When two sources are connected in series,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\alpha} = 41.1$$

$$E_1^2 + E_2^2 + 2E_1E_2 \cos \alpha = 1689.21 \qquad (1)$$

When one of the sources is reversed,

$$\sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha} = 17.52$$

$$E_1^2 + E_2^2 - 2E_1E_2 \cos \alpha = 306.95 \qquad (2)$$

When phase displacement is made zero,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos 0°} = 42.5$$

$$E_1 + E_2 = 42.5 \qquad (3)$$

Adding Eqs (1) and (2),

$$2(E_1^2 + E_2^2) = 1996.16$$

$$E_1^2 + E_2^2 = 998.08$$

$$(42.5 - E_2)^2 + E_2^2 = 998.08$$

$$1806.25 - 85E_2 + E_2^2 + E_2^2 = 998.08$$

$$E_2^2 - 42.5E_2 + 404.09 = 0 \qquad (4)$$

Solving Eq. (4),

$$E_2 = 28.14 \text{ V} \quad \text{or} \quad E_2 = 14.36 \text{ V}$$

$$E_1 = 14.36 \text{ V} \quad \text{or} \quad E_1 = 28.14 \text{ V}$$

Subtracting Eq. (2) from Eq. (1),

$$4E_1E_2 \cos \alpha = 1382.26$$

$$4 \times 14.37 \times 28.14 \cos \alpha = 1382.26$$

$$\cos \alpha = 0.855$$

$$\alpha = 31.24°$$

Exercise 3.2

3.1 Find the resultants of the following voltages:

(a) $v_1 = 60 \cos \theta$, $v_2 = 40 \sin \left(\theta - \frac{\pi}{3}\right)$, $v_3 = 15 \sin \theta$ *[43.22 sin (θ + 35.92°)]*

(b) $e_1 = 50 \sin \omega t$, $e_2 = 100 \sin (\omega t + 120°)$, $e_3 = 120 \sin (\omega t - 30°)$ *[107.27 sin (ωt + 14.35°)]*

(c) $v_1 = 4\sqrt{2} \sin (\omega t + 135°)$, $v_2 = -4\sqrt{3} \sin (\omega t + 60°)$, $v_3 = 4 \cos (\omega t - 150°)$ *[7.73 sin (ωt – 135°)]*

(d) $e_1 = 25 \sin \omega t$, $e_2 = 10 \sin \left(\omega t + \frac{\pi}{6}\right)$, $e_3 = 30 \cos \omega t$, $e_4 = 20 \sin \left(\omega t - \frac{\pi}{4}\right)$ *[52.14 sin (ωt + 23.57°)]*

(e) $v_1 = 10 \sin \omega t$, $v_2 = -10 \cos \omega t$ *[14.14 sin (ωt – 45°)]*

3.2 Three alternating currents $i_1 = 141 \sin\left(\omega t + \dfrac{\pi}{4}\right)$, $i_2 = 30 \sin\left(\omega t + \dfrac{\pi}{2}\right)$ and $i_3 = 20 \sin\left(\omega t + \dfrac{\pi}{6}\right)$ are fed into a common conductor. Find graphically or otherwise the equation of the resultant current and its rms value.

[167.5 sin (ωt + 45.6°), 118.54]

3.3 Four wires, p, q, r, s are connected to a common point. The currents in lines p, q and r are $6 \sin\left(\omega t + \dfrac{\pi}{3}\right)$, $5 \cos\left(\omega t + \dfrac{\pi}{3}\right)$ and $3 \cos\left(\omega t + \dfrac{\pi}{3}\right)$ respectively. Find the current in the wire s. *[9.99 sin (ωt – 66.84°)]*

3.4 A sinusoidal voltage $V_m \sin \omega t$ is applied to three parallel branches yielding branch currents as follows:

$i_1 = 14.14 \sin(\omega t - 45°)$ $\quad$ $i_2 = 28.3 \cos(\omega t - 60°)$ $\quad$ $i_3 = 7.07 \sin(\omega t + 60°)$

Find the complete expression for the source current. Draw the phasor diagram in terms of effective values. Use the voltage as reference. *[39.4 sin (ωt + 15.1°)]*

3.5 A sinusoidal voltage $V_m \sin \omega t$ is applied to three parallel branches. Two of the branch currents are given by

$i_1 = 14.14 \sin(\omega t - 37°)$ $\quad$ $i_2 = 28.28 \cos(\omega t - 143°)$

The source current is found to be $i = 63.8 \sin(\omega t + 12.8°)$.

(i) Find the effective value of the current in the third branch.

(ii) Write the complete time expression for the instantaneous value of the current in part (i).

(iii) Draw the phasor diagram showing the source current and the three branch currents. Use voltage as the reference phasor.

[39.95A, 56.51 sin (ωt + 53.12°)]

3.6 A voltage wave $e(t) = 170 \sin 120\, t$ produces a net current of $14.14 \sin 120\, t + 7.07 \cos(120\, t + 30°)$.

(i) Express the effective value of the current as a single phasor quantity.

(ii) Draw the phasor diagram. Show the components of the current as well as the resultant.

(iii) Determine the power delivered by the source. *[8.66 ∠ 30°A, 901.7 W]*

3.7 The instantaneous values of two alternating voltages are represented by $v_1 = 60 \sin \theta$ and $v_2 = 40 \sin(\theta - \pi/3)$. Derive an expression for the instantaneous value of

(i) the sum, and

(ii) the difference of these voltages.

[87.17 sin (θ – 23.41°), 52.91 sin (θ + 40.89°)]

Review Questions

3.1 Define the following terms related to alternating quantities:

(i) Waveform (ii) Cycle (iii) Periodic time
(iv) rms value (v) Average value (vi) Form factor
(vii) Peak factor (viii) Amplitude (ix) Frequency

3.2 Explain the concepts of phase and phase difference in alternating quantities.

3.3 Derive an expression for the average value of a sinusoidally varying current in terms of peak value.

3.4 Find the rms value of a full-wave sinusoidal current whose maximum value is I_m.

3.5 Derive an expression for the rms value of a sinusoidally varying quantity in terms of its peak value.

3.6 Define phasors and explain how phasors can be used to represent a sinusoidal quantity.

3.7 What is the physical significance of the operator '*j*'.

3.8 What are the different mathematical representations of phasors?

Objective-Type Questions

Choose the correct alternative in the following questions:

3.1 A current is said to be alternating when it changes in

(a) magnitude only (b) direction only
(c) both magnitude and direction (d) neither magnitude nor direction

3.2 An alternating current of 50 Hz frequency and 100 A maximum value is given as

(a) $i = 200 \sin 628\, t$ (b) $i = 100 \sin 314\, t$
(c) $i = 100\sqrt{2} \sin 314\, t$ (d) $i = 100\sqrt{2} \sin 157\, t$

3.3 An alternating current of 50 Hz frequency has a maximum value of 100 A. Its value $\dfrac{1}{600}$ second after the instant the current is zero will be

(a) 25 A (b) 12.5 A (c) 50 A (d) 75 A

3.4 A sinusoidal voltage varies from zero to a maximum of 250 V. The voltage at the instant of 60° of the cycle will be

(a) 150 V (b) 216.5 V (c) 125 V (d) 108.25 V

3.5 An alternating current is given by the expression $i = 200\sin\left(314t + \frac{\pi}{3}\right)$ amperes. The maximum value and frequency of the current are

(a) 200 A, 50 Hz (b) $100\sqrt{2}$, 50 Hz
(c) 200 A, 100 Hz (d) 200 A, 25 Hz

3.6 The average value of the current $i = 200 \sin t$ from $t = 0$ to $t = \frac{\pi}{2}$ is

(a) $400\,\pi$ (b) $\frac{400}{\pi}$ (c) $\frac{1}{400}$ (d) $\frac{\pi}{400}$

3.7 The rms value of a half-wave rectified current is 50 A. Its rms value for full wave rectification would be

(a) 100 A (b) 70.7 A (c) $\frac{50}{\pi}$ A (d) $\frac{100}{\pi}$ A

3.8 When the two quantities are in quadrature, the phase angle between them will be

(a) 45° (b) 90° (c) 135° (d) 60°

3.9 The phase difference between the two waveforms can be compared when they

(a) have the same frequency (b) have the same peak value
(c) have the same effective value (d) are sinusoidal

3.10 If two sinusoids of the same frequency but of different amplitude and phase difference are added, the resultant is a

(a) sinusoid of the same frequency
(b) sinusoid of double the original frequency
(c) sinusoid of half the original frequency
(d) non-sinusoid

3.11 Two alternating currents represented as $i_1 = 4 \sin \omega t$ and $i_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ are fed into a common conductor. The rms value of the resultant current is

(a) 9.62 A (b) 8.83 A (c) 12.48 A (d) 13.6 A

3.12 A constant current of 2.8 A exists in a resistor. The rms value of the current is

(a) 2.8 A (b) 2 A (c) 1.4 A (d) undefined

3.13 The current through a resistor has a waveform as shown in Fig. 3.46. The reading shown by a moving coil ammeter will be

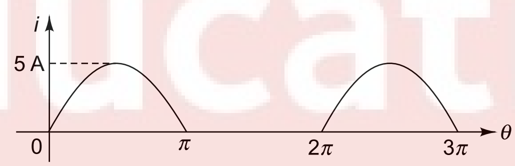


Fig. 3.46

(a) $\frac{5}{\sqrt{2}}$ A (b) $\frac{2.5}{\sqrt{2}}$ A (c) $\frac{5}{\pi}$ A (d) 0

3.14 Which of the following statements about the two alternating currents shown in Fig 3.47 is correct?

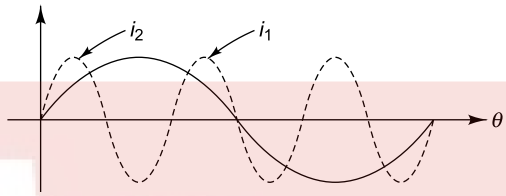


Fig. 3.47

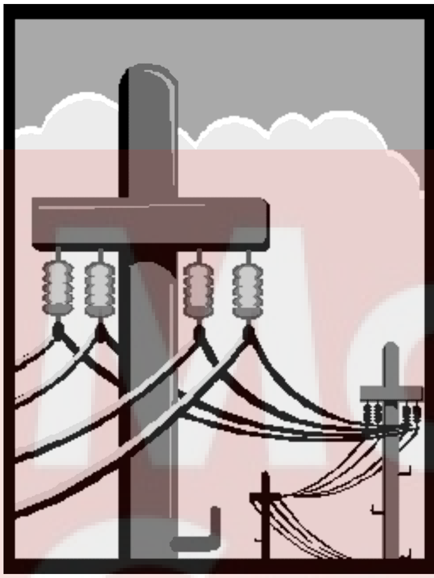
(a) The peak values of i_1 and i_2 are different
(b) The rms values of i_1 and i_2 are different
(c) The time period of current i_1 is more than that of the current i_2
(d) The frequency of current i_1 is more than that of the current i_2.

3.15 The alternating voltage $e = 200 \sin 314\, t$ is applied to a device which offers an ohmic resistance of 20 Ω to the flow of current in one direction while entirely preventing the flow in the opposite direction. The overage value of the current will be

(a) 5 A (b) 3.18 A (c) 1.57 A (d) 1.10 A

Answers to Objective-Type Questions

3.1 (c)	**3.2** (b)	**3.3** (c)	**3.4** (b)	**3.5** (a)	**3.6** (b)
3.7 (b)	**3.8** (b)	**3.9** (a)	**3.10** (a)	**3.11** (c)	**3.12** (a)
3.13 (c)	**3.14** (c)	**3.15** (b)			



Chapter 4

Single-Phase AC Circuits

Chapter Outline

4.1 BEHAVIOUR OF A PURE RESISTOR IN AN AC CIRCUIT

Consider a pure resistor R connected across an alternating voltage source v as shown in Fig. 4.1. Let the alternating voltage be $v = V_m \sin \omega t$.

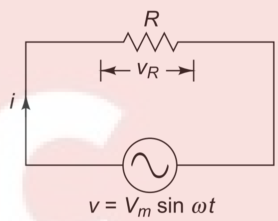


Fig. 4.1 *Purely resistive circuit*

Current The alternating current i is given by

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \ldots \left(I_m = \frac{V_m}{R} \right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current is in phase with the voltage in a purely resistive circuit.

Waveforms

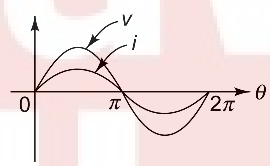


Fig. 4.2 *Waveforms*

Phasor Diagram

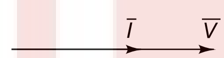


Fig. 4.3 *Phasor diagram*

Impedance It is the resistance offered to the flow of current in an ac circuit. In a purely resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

Phase Difference Since the voltage and current are in phase with each other, the phase difference is zero.

$$\phi = 0°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

Power factor = $\cos \phi = \cos (0°) = 1$

Power Instantaneous power p is given by

$$
\begin{aligned}
p &= vi \\
&= V_m \sin \omega t \, I_m \sin \omega t \\
&= V_m I_m \sin^2 \omega t \\
&= \frac{V_m I_m}{2}(1 - \cos 2\omega t) \\
&= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t
\end{aligned}
$$

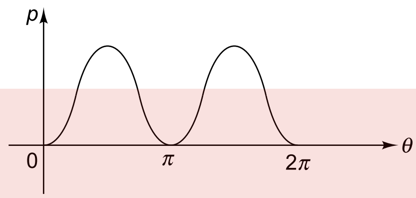


Fig. 4.4 *Power waveform*

The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$. The frequency of the fluctuating power is twice the applied voltage frequency and its average value over one complete cycle is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

Thus, power in a purely resistive circuit is equal to the product of rms values of voltage and current.

4.2 BEHAVIOUR OF A PURE INDUCTOR IN AN AC CIRCUIT

Consider a pure inductor L connected across an alternating voltage v as shown in Fig. 4.5. Let the alternating voltage be $v = V_m \sin \omega t$.

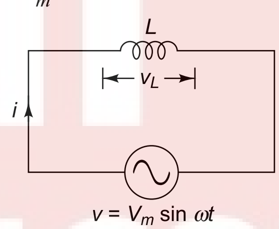


Fig. 4.5 *Purely inductive circuit*

Current The alternating current i is given by

$$
\begin{aligned}
i &= \frac{1}{L} \int v \, dt \\
&= \frac{1}{L} \int V_m \sin \omega t \, dt \\
&= \frac{V_m}{\omega L} (-\cos \omega t)
\end{aligned}
$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \cdots \left(I_m = \frac{V_m}{\omega L}\right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current lags behind the voltage by 90° in a purely inductive circuit.

Waveforms

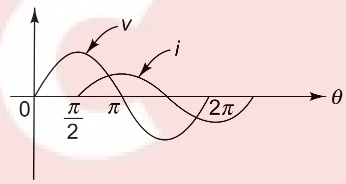


Fig. 4.6 *Waveforms*

Phasor Diagram

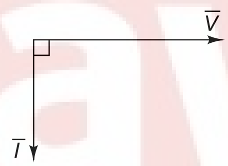


Fig. 4.7 *Phasor diagram*

Impedance In a purely inductive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_L = 0$

Thus, an inductor acts as a short circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous powers p is given by

$$p = vi$$

$$= V_m \sin\omega t \, I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin\omega t \cos\omega t$$

$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely inductive circuit is zero.

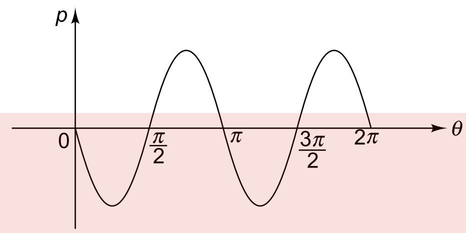


Fig. 4.8 *Power waveform*

4.3 BEHAVIOUR OF A PURE CAPACITOR IN AN AC CIRCUIT

Consider a pure capacitor C connected across an alternating voltage v as shown in Fig. 4.9. Let the alternating voltage be $v = V_m \sin \omega t$.

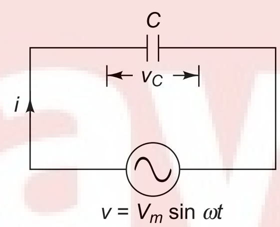


Fig. 4.9 *Purely capacitive circuit*

Current The alternating current i is given by

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}(V_m \sin \omega t)$$

$$= \omega C V_m \cos \omega t$$

$$= \omega C V_m \sin(\omega t + 90°)$$

$$= I_m \sin(\omega t + 90°) \quad \ldots(I_m = \omega C V_m)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

Waveforms

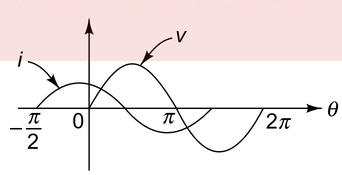


Fig. 4.10 *Waveforms*

Phasor Diagram

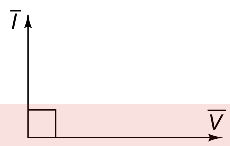


Fig. 4.11 *Phasor diagram*

Impedance In a purely capacitive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_C and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_C = \infty$

Thus, the capacitor acts as an open circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

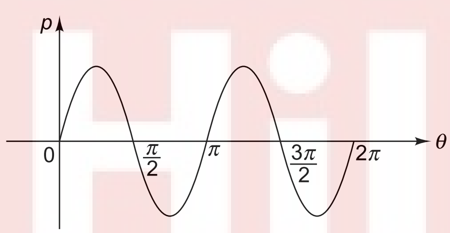


Fig. 4.12 *Power waveform*

Power Instantaneous power p is given by

$$\begin{aligned} p &= vi \\ &= V_m \sin\omega t \, I_m \sin(\omega t + 90) \\ &= V_m I_m \sin\omega t \cos\omega t \\ &= \frac{V_m I_m}{2} \sin 2\omega t \end{aligned}$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely capacitive circuit is zero.

Example 1

An ac circuit consists of a pure resistance of 10 ohms and is connected across an ac supply of 230 V, 50 Hz. Calculate (i) current, (ii) power consumed, (iii) power factor, and (iv) write down the equations for voltage and current.

Solution

$$R = 10\ \Omega$$
$$V = 230\ \text{V}$$
$$f = 50\ \text{Hz}$$

(i) Current

$$I = \frac{V}{R} = \frac{230}{10} = 23 \text{ A}$$

(ii) Power consumed

$$P = VI = 230 \times 23 = 5290 \text{ W}$$

(iii) Power factor

Since the voltage and current are in phase with each other, $\phi = 0°$

$$\text{pf} = \cos \phi = \cos (0°) = 1$$

(iv) Voltage and current equations

$$V_m = \sqrt{2}\, V = \sqrt{2} \times 230 = 325.27 \text{ V}$$

$$I_m = \sqrt{2}\, I = \sqrt{2} \times 23 = 32.53 \text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s}$$

$$v = V_m \sin \omega t = 325.27 \sin 314.16\, t$$

$$i = I_m \sin \omega t = 32.53 \sin 314.16\, t$$

Example 2

An inductive coil having negligible resistance and 0.1 henry inductance is connected across a 200 V, 50 Hz supply. Find (i) inductive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution

$$L = 0.1 \text{ H}$$

$$V = 200 \text{ V}$$

$$f = 50 \text{ Hz}$$

(i) Inductive reactance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_L} = \frac{200}{31.42} = 6.37 \text{ A}$$

(iii) Power

Since the current lags behind the voltage by 90° in purely inductive circuit, $\phi = 90°$

$$P = VI \cos \phi = 200 \times 6.37 \times \cos (90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos \phi = \cos (90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\, V = \sqrt{2} \times 200 = 282.84 \text{ V}$$

$$I_m = \sqrt{2}\, I = \sqrt{2} \times 6.37 = 9 \text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s}$$

$$v = V_m \sin \omega t = 282.84 \sin 314.16\, t$$

$$i = I_m \sin \left(\omega t - \frac{\pi}{2}\right) = 9 \sin \left(314.16\, t - \frac{\pi}{2}\right)$$

Example 3

The voltage and current through circuit elements are

$$v = 100 \sin(314\,t + 45°) \text{ volts}$$
$$i = 10 \sin(314\,t + 315°) \text{ amperes}$$

(i) Identify the circuit elements. (ii) Find the value of the elements. (iii) Obtain an expression for power.

Solution

$$v = 100 \sin(314\,t + 45°)$$
$$i = 10 \sin(314\,t + 315°)$$
$$= 10 \sin(314\,t + 315° - 360°)$$
$$= 10 \sin(314\,t - 45°)$$

(i) Identification of elements
From voltage and current equations, it is clear that the current i lags behind the voltage by 90°. Hence, the circuit element is an inductor.

(ii) Value of elements

$$X_L = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100}{10} = 10\ \Omega$$
$$X_L = \omega L$$
$$10 = 314\,L$$
$$L = 31.8 \text{ mH}$$

(iii) Expression for power

$$p = -\frac{V_m I_m}{2} \sin 2\omega t = -\frac{100 \times 10}{2} \sin(2 \times 314 t) = -500 \sin 628\,t$$

Example 4

A capacitor has a capacitance of 30 microfarads which is connected across a 230 V, 50 Hz supply. Find (i) capacitive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution

$$C = 30\ \mu\text{F}$$
$$V = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$

(i) Capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.1\ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_C} = \frac{230}{106.1} = 2.17 \text{ A}$$

(iii) Power
Since the current leads the voltage by 90° in purely capacitive circuit, $\phi = 90°$

$$P = VI \cos\phi = 230 \times 2.17 \times \cos(90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27 \text{ V}$$
$$I_m = \sqrt{2}\,I = \sqrt{2} \times 2.17 = 3.07 \text{ A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16 \text{ rad/s}$$
$$v = V_m \sin \omega t = 325.27 \sin 314.16\, t$$
$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = 3.07 \sin\left(314.16\, t + \frac{\pi}{2}\right)$$

4.4 SERIES *R-L* CIRCUIT

Figure 4.13 shows a pure resistor R connected in series with a pure inductor L across an alternating voltage v.

Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90°.

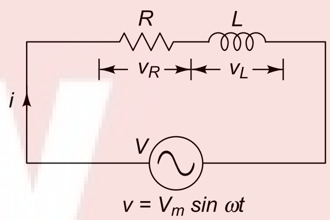
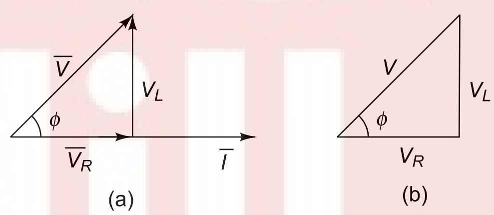


Fig. 4.13 *Series R-L circuit*



$$\cos\phi = \frac{V_R}{V}$$

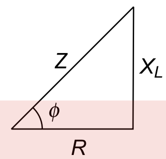
Fig. 4.14 *(a) Phasor diagram (b) Voltage triangle*

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_L = R\bar{I} + jX_L\,\bar{I}$$
$$= (R + jX_L)\,\bar{I}$$
$$\frac{\bar{V}}{\bar{I}} = R + jX_L = \bar{Z}$$
$$\bar{Z} = Z\angle\phi$$
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$
$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the *complex impedance* of the *R-L* circuit.

Impedance Triangle



$$\cos\phi = \frac{R}{Z}$$

Fig. 4.15 *Impedance triangle*

Current From the phasor diagram, it is clear that the current I lags behind the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t - \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Waveforms

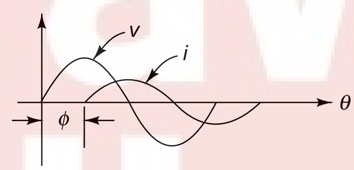


Fig. 4.16 *Waveforms*

Power Instantaneous power p is given by

$$\begin{aligned} p &= v\, i \\ &= V_m \sin \omega t\, I_m \sin(\omega t - \phi) \\ &= V_m I_m \sin \omega t \sin(\omega t - \phi) \\ &= V_m I_m \left[\frac{\cos\phi - \cos(2\omega t - \phi)}{2}\right] \\ &= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi) \end{aligned}$$

Thus, power consists of a constant part $\frac{V_m I_m}{2}\cos\phi$ and a fluctuating part $\frac{V_m I_m}{2}\cos(2\omega t - \phi)$. The frequency of the fluctuating part is twice the applied voltage frequency and its average value over one complete cyce is zero.

$$\text{Average power } P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi$$

Thus, power is dependent upon the in-phase component of the current. The average power is also called *active power* and is measured in watts.

We know that a pure inductor and capacitor consume no power because all the power received from the source in a half cycle is returned to the source in the next half cycle. This circulating power is called *reactive power*. It is a product of the voltage and reactive component of the current, i.e., $I \sin\phi$ and is measured in VAR (volt–ampere-reactive).

Reactive power $Q = VI \sin\phi$.

The product of voltage and current is known as *apparent power* (S) and is measured in volt–ampere (VA).

$$S = \sqrt{P^2 + Q^2}$$

Power Triangle In terms of circuit components,

$$\cos\phi = \frac{R}{Z}$$

and $V = ZI$

$$P = VI\cos\phi = ZII\frac{R}{Z} = I^2R$$

$$Q = VI\sin\phi = ZII\frac{X_L}{Z} = I^2X_L$$

$$S = VI = ZII = I^2Z$$

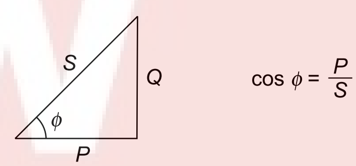


Fig. 4.17 *Power triangle*

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle, $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an R-L series circuit, the power factor is lagging in nature.

Example 1

An alternating voltage of 80 + j60 V is applied to a circuit and the current flowing is 4 – j2 A. Find the (i) impedance, (ii) phase angle, (iii) power factor, and (iv) power consumed.

Solution $\bar{V} = 80 + j60$ V

$\bar{I} = 4 - j2$ A

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{80 + j60}{4 - j2} = \frac{100\angle 36.87°}{4.47\angle -26.56°} = 22.37\ \angle 63.43°\ \Omega$$

$Z = 22.37\ \Omega$

(ii) Phase angle

$\phi = 63.43°$

(iii) Power factor

pf = cos ϕ = cos (63.43°) = 0.447 (lagging)

(iv) Power consumed

$P = VI \cos\phi = 100 \times 4.47 \times 0.447 = 199.81$ W

Example 2

The voltage and current in a circuit are given by $\bar{V}$ = 150 ∠30° V and $\bar{I}$ = 2 ∠–15° A. If the circuit works on a 50 Hz supply, determines impedance, resistance, reactance, power factor and power loss considering the circuit as a simple series circuit.

Solution $\bar{V}$ = 150 ∠30° V

$\bar{I}$ = 2∠–15° A

f = 50 Hz

(i) Impedance

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{150\angle 30°}{2\angle -15°} = 75\ \angle 45°\ \Omega = 53.03 + j53.03\ \Omega$$

$Z = 75\ \Omega$

(ii) Resistance

$R = 53.03\ \Omega$

(iii) Reactance

$X = 53.03\ \Omega$

(iv) Power factor

$\phi = 45°$

pf = cos ϕ = cos (45°) = 0.707 (lagging)

(v) Power loss

$P = VI \cos\phi = 150 \times 2 \times 0.707 = 212.1$ W

Example 3

An rms voltage of 100 ∠0° V is applied to a series combination of Z_1 and Z_2 when Z_1 = 20 ∠30° Ω. The effective voltage drop across Z_1 is known to be 40 ∠–30° V. Find the reactive component of Z_2.

Solution

$$\bar{V} = 100\ \angle 0°\ \text{V}$$

$$\bar{Z}_1 = 20\ \angle 30°\ \Omega$$

$$\bar{V}_1 = 40\ \angle -30°\ \text{V}$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{40\angle -30^\circ}{20\angle 30^\circ} = 2\angle -60°\ \text{A}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{100\angle 0°}{2\angle -60°} = 50\ \angle 60° = 25 + j43.3\ \Omega$$

$$\bar{Z}_1 = 20\ \angle 30° = 17.32 + j10\ \Omega$$

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$

$$\bar{Z}_2 = \bar{Z} - \bar{Z}_1 = 25 + j43.3 - 17.32 - j10 = 7.68 + j33.3\ \Omega$$

Reactive component of $\bar{Z}_2 = 33.3\ \Omega$

Example 4

A voltage v(t) = 177 sin (314t + 10°) is applied to a circuit. It causes a steady-state current to flow, which is described by i(t) = 14.14 sin (314t – 20°). Determine the power factor and average power delivered to the circuit.

Solution

$$v(t) = 177 \sin (314t + 10°)$$

$$i(t) = 14.14 \sin (314t - 20°)$$

(i) Power factor

Current $i(t)$ lags behind voltage $v(t)$ by 30°.

$$\phi = 30°$$

$$\text{pf} = \cos \phi = \cos (30°) = 0.866 \text{ (lagging)}$$

(ii) Average power

$$P = VI \cos \phi = \frac{177}{\sqrt{2}} \times \frac{14.14}{\sqrt{2}} \times 0.866 = 1083.7\ \text{W}$$

Example 5

When a sinusoidal voltage of 120 V (rms) is applied to a series R-L circuit, it is found that there occurs a power dissipation of 1200 W and a current flow given by i(t) = 28.3 sin (314t – ϕ). Find the circuit resistance and inductance.

Solution

$$V = 120\ \text{V}$$

$$P = 1200\ \text{W}$$

$$i(t) = 28.3 \sin (314t - \phi)$$

(i) Resistance

$$I = \frac{28.3}{\sqrt{2}} = 20.01\ \text{A}$$

$$P = VI \cos \phi$$
$$1200 = 120 \times 20.01 \times \cos \phi$$
$$\cos \phi = 0.499$$
$$\phi = 60.02°$$
$$Z = \frac{V}{I} = \frac{120}{20.01} = 6\ \Omega$$
$$\bar{Z} = Z \angle \phi = 6 \angle 60.02° = 3 + j5.2\ \Omega$$
$$R = 3\ \Omega$$

(ii) Inductance

$$X_L = 5.2\ \Omega$$
$$X_L = \omega L$$
$$5.2 = 314 \times L$$
$$L = 0.0165\ \text{H}$$

Example 6

In a series circuit containing resistance and inductance, the current and voltage are expressed as $i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$ and $v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$. (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance and power factor? (iii) What is the average power drawn by the circuit?

Solution

$$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$$
$$v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$$

(i) Impedance

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{20}{5} = 4\ \Omega$$

(ii) Power factor, resistance and inductance

Current $i(t)$ lags behind voltage $v(t)$ by an angle $\phi = 150° - 120° = 30°$

$$\text{pf} = \cos \phi = \cos (30°) = 0.866 \text{ (lagging)}$$
$$\bar{Z} = 4 \angle 30° = 3.464 + j2\ \Omega$$
$$R = 3.464\ \Omega$$
$$X_L = 2\ \Omega$$
$$X_L = \omega L$$
$$2 = 314 \times L$$
$$L = 6.37\ \text{mH}$$

(iii) Average power

$$P = VI\cos\phi = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3 \text{ W}$$

Example 7

A series circuit consists of a non-inductive resistance of 6 Ω and an inductive reactance of 10 Ω. When connected to a single-phase ac supply, it draws a current i(t) = 27.89 sin (628t – 45°). Calculate (i) the voltage applied to the series circuit in the form $V_m \sin(\omega t \pm \phi)$, (ii) inductance, and (iii) power drawn by the circuit.

Solution

$$R = 6\ \Omega$$
$$X_L = 10\ \Omega$$
$$i(t) = 27.89 \sin(628t - 45°)$$

(i) Voltage applied to the series circuit

$$\bar{Z} = R + jX_L = 6 + j10 = 11.66\ \angle 59.04°\ \Omega$$
$$\bar{I} = \frac{27.89}{\sqrt{2}} \angle -45° = 19.72\ \angle -45°\ \text{A}$$
$$\bar{V} = \bar{Z}\ \bar{I} = (11.66\ \angle 59.04°)\ (19.72\ \angle -45°) = 229.95\ \angle 14.04°\ \text{V}$$
$$v = 229.95\sqrt{2}\sin(\omega t + 14.04°) = 325.2\sin(\omega t + 14.04°)$$

(ii) Inductance

$$X_L = \omega L$$
$$10 = 628 \times L$$
$$L = 15.9 \text{ mH}$$

(iii) Power drawn by the circuit

$$P = VI\cos\phi = 229.95 \times 19.72 \times \cos(59.04°) = 2332.78 \text{ W}$$

Example 8

When an inductive coil is connected to a dc supply at 240 V, the current in it is 16 A. When the same coil is connected to an ac supply at 240 V, 50 Hz, the current is 12.27 A. Calculate (i) resistance, (ii) impedance, (iii) reactance, and (iv) inductance of the coil.

Solution For dc: $V = 240$ V, $I = 16$ A

For ac: $V = 240$ V, $I = 12.27$ A

(i) Resistance

When an inductive coil is connected to a dc supply,

$$f = 0$$
$$X_L = 2\pi f L = 0$$

The coil behaves like a pure resistor.

$$R = \frac{V}{I} = \frac{240}{16} = 15\ \Omega$$

(ii) Impedance

When the coil is connected to an ac supply,

$$Z = \frac{V}{I} = \frac{240}{12.27} = 19.56\ \Omega$$

(iii) Reactance

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(19.56)^2 - (15)^2} = 12.55\ \Omega$$

(iv) Inductance

$$X_L = 2\pi f L$$

$$12.55 = 2\pi \times 50 \times L$$

$$L = 0.04\ \text{H}$$

Example 9

An inductive coil draws 10 A current and consumes 1 kW power from a 200 V, 50 Hz ac supply. Determine (i) impedance in Cartesian and polar forms, (ii) power factor, and (iii) reactive and apparent power.

Solution

$$I = 10\ \text{A}$$

$$P = 1\ \text{kW}$$

$$V = 200\ \text{V}$$

$$f = 50\ \text{Hz}$$

(i) Impedance in Cartesian and polar forms

$$Z = \frac{V}{I} = \frac{200}{10} = 20\ \Omega$$

$$P = VI\cos\phi$$

$$1000 = 200 \times 10 \times \cos\phi$$

$$\cos\phi = 0.5$$

$$\phi = 60°$$

Expressing $\bar{Z}$ in polar form,

$$\bar{Z} = Z\angle\phi° = 20\angle 60°\ \Omega$$

Expressing $\bar{Z}$ in Cartesian form,

$$\bar{Z} = 10 + j17.32\ \Omega$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(60°) = 0.5\ \text{(lagging)}$$

(iii) Reactive and apparent power

$$Q = VI \sin \phi = 200 \times 10 \times \sin(60°) = 1.73 \text{ kVAR}$$

$$S = VI = 200 \times 10 = 2 \text{ kVA}$$

Example 10

A coil connected across a 250 V, 50 Hz supply takes a current of 10 A at 0.8 lagging power factor. What will be the power taken by the choke coil when connected across a 200 V, 25 Hz supply? Also calculate resistance and inductance of the coil.

Solution

$$V_1 = 250 \text{ V}$$
$$f_1 = 50 \text{ Hz}$$
$$I_1 = 10 \text{ A}$$
$$\text{pf} = 0.8 \text{ (lagging)}$$
$$V_2 = 200 \text{ V}$$
$$f_2 = 25 \text{ Hz}$$

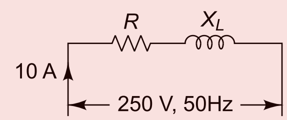


Fig. 4.18

(i) Resistance and inductance of the coil

$$Z_1 = \frac{V_1}{I_1} = \frac{250}{10} = 25\ \Omega$$

$$\phi_1 = \cos^{-1}(0.8) = 36.87°$$

$$\bar{Z}_1 = Z_1 \angle \phi_1 = 25 \angle 36.87° = 20 + j15\ \Omega$$

$$R = 20\ \Omega$$

$$X_{L_1} = 15\ \Omega$$

$$X_{L_1} = 2\pi f_1 L$$

$$15 = 2\pi \times 50 \times L$$

$$L = 0.0477 \text{ H}$$

(ii) Power taken by the choke coil when connected across a 200 V, 50 Hz supply.

$$X_{L_2} = 2\pi f_2 L = 2\pi \times 25 \times 0.0477 = 7.49\ \Omega$$

$$\bar{Z}_2 = R + jX_{L_2} = 20 + j\,7.49 = 21.36 \angle 20.53°\ \Omega$$

$$\bar{Z}_2 = 21.36\ \Omega$$

$$\phi_2 = 20.53°$$

$$I_2 = \frac{V_2}{Z_2} = \frac{200}{21.36} = 9.36 \text{ A}$$

$$P = V_2 I_2 \cos \phi_2 = 200 \times 9.36 \times \cos(20.53°) = 1.753 \text{ kW}$$

Example 11

A load of 22 kW operates at 0.8 lagging power factor when connected to a 420 V, single-phase, 50 Hz source. Find (i) current in the load, (ii) power factor angle, (iii) impedance, (iv) resistance of load, (v) reactance of load, (vi) voltage and current equations.

Solution

$$P = 22 \text{ kW}$$
$$\text{pf} = 0.8 \text{ (lagging)}$$
$$V = 420 \text{ V}$$
$$f = 50 \text{ Hz}$$

(i) Current in the load

$$P = VI\cos\phi$$
$$22 \times 10^3 = 420 \times I \times 0.8$$
$$I = 65.48 \text{ A}$$

(ii) Power-factor angle

$$\phi = \cos^{-1}(0.8) = 36.87°$$

(iii) Impedance

$$Z = \frac{V}{I} = \frac{420}{65.48} = 6.41\ \Omega$$

(iv) Resistance of load

$$R = Z\cos\phi = 6.41 \times 0.8 = 5.13\ \Omega$$

(v) Reactance of load

$$X_L = Z\sin\phi = 6.41 \times \sin(36.87°) = 3.85\ \Omega$$

(vi) Voltage and current equations

$$v = V_m \sin 2\pi ft = 420\sqrt{2}\,\sin(2\pi \times 50)\,t = 593.97 \sin 100\pi t$$

The current lags behind the voltage by 36.87°

$$i = I_m \sin(2\pi ft - \phi) = 65.48\sqrt{2}\,\sin(2\pi \times 50\,t - 36.87°)$$
$$= 92.6 \sin(100\pi t - 36.87°)$$

Example 12

A choke coil is connected in series with a fixed resistor. A 240 V, 50 Hz supply is applied and a current of 2.5 A flows. If the voltage drops across the coil and fixed resistor are 140 V and 160 V respectively, calculate the value of the fixed resistance, the resistance and inductance of the coil, and power drawn by the coil.

Solution

$$V = 240 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$I = 2.5 \text{ A}$$
$$V_{\text{coil}} = 140 \text{ V}$$
$$V_R = 160 \text{ V}$$

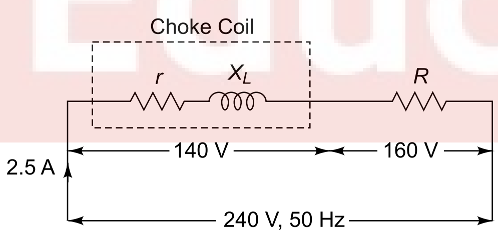


Fig. 4.19

(i) Resistance of fixed resistor

$$R = \frac{V_R}{I} = \frac{160}{2.5} = 64\ \Omega$$

(ii) Resistance and inductance of the coil

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{140}{2.5} = 56\ \Omega$$

$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = 56$$

$$r^2 + X_L^2 = 3136 \qquad (1)$$

$$Z = \frac{V}{I} = \frac{240}{2.5} = 96\ \Omega$$

$$\bar{Z} = (R + r) + jX_L$$

$$Z = \sqrt{(R+r)^2 + X_L^2}$$

$$96 = \sqrt{(64+r)^2 + X_L^2}$$

$$(64 + r)^2 + X_L^2 = 9216 \qquad (2)$$

Subtracting Eq. (2) from Eq. (1),

$$(64 + r)^2 - r^2 = 6080$$

$$4096 + 128\,r + r^2 - r^2 = 6080$$

$$128\,r = 1984$$

$$r = 15.5\ \Omega$$

Substituting the value of r in Eq. (1),

$$(15.5)^2 + X_L^2 = 3136$$

$$X_L^2 = 2895.75$$

$$X_L = 53.81\ \Omega$$

$$X_L = 2\pi f L$$

$$53.81 = 2\pi \times 50 \times L$$

$$L = 0.17\ \text{H}$$

(iii) Power drawn by the coil

$$P_{\text{coil}} = I^2 r = (2.5)^2 \times 15.5 = 96.875\ \text{W}$$

Example 13

A 100 Ω resistor is connected in series with a choke coil. When a 400 V, 50 Hz supply is applied to this combination, the voltages across the resistance and the choke coil are 200 V and 300 V respectively. Find the power consumed by the choke coil. Also, calculate the power factor of the choke coil and the power factor of the circuit.

Solution $R = 100\ \Omega$

$V = 400\ \text{V}$

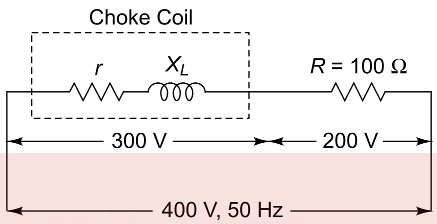


Fig. 4.20

$$f = 50 \text{ Hz}$$
$$V_R = 200 \text{ V}$$
$$V_{coil} = 300 \text{ V}$$

(i) Power consumed by choke coil

$$I = \frac{V_R}{R} = \frac{200}{100} = 2 \text{ A}$$

$$Z_{coil} = \frac{V_{coil}}{I} = \frac{300}{2} = 150\ \Omega$$

$$\sqrt{r^2 + X_L^2} = 150$$

$$r^2 + X_L^2 = 22500 \qquad (1)$$

$$Z = \frac{V}{I} = \frac{400}{2} = 200\ \Omega$$

$$\bar{Z} = (R + r) + jX_L$$

$$Z = \sqrt{(R+r)^2 + X_L^2} = 200$$

$$(100 + r)^2 + X_L^2 = 40000 \qquad (2)$$

Subtracting Eq. (1) from Eq. (2),

$$(100 + r)^2 - r^2 = 17500$$
$$10000 + 200r + r^2 - r^2 = 17500$$
$$200r = 7500$$
$$r = 37.5\ \Omega$$

Substituting the value of r in Eq. (1),

$$(37.5)^2 + X_L^2 = 22500$$
$$X_L^2 = 21093.75$$
$$X_L = 145.24\ \Omega$$
$$P_{coil} = I^2 r = (2)^2 \times 37.5 = 50 \text{ W}$$

(ii) Power factor of the choke coil

$$\text{pf}_{coil} = \frac{r}{Z_{coil}} = \frac{37.5}{150} = 0.25 \text{ (lagging)}$$

(iii) Power factor of the circuit

$$\text{pf}_{circuit} = \frac{R+r}{Z} = \frac{100+37.5}{200} = 0.6875 \text{ (lagging)}$$

Example 14

A resistor of 25 Ω is connected in series with a choke coil. The series combination when connected across a 250 V, 50 Hz supply, draws a current of 4 A which lags behind the voltage by 65°. Calculate (i) resistance and inductance of the coil, (ii) total power, (iii) power consumed by resistance, and (iv) power consumed by choke coil.

Solution

$$R = 25\ \Omega$$
$$V = 250\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$I = 4\ \text{A}$$
$$\phi = 65°$$

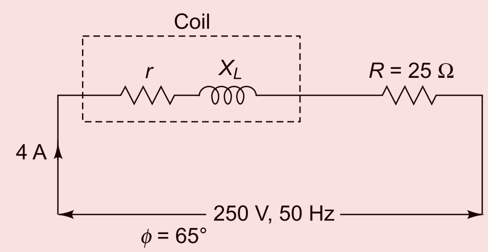


Fig. 4.21

(i) Resistance and inductance of the coil

$$Z = \frac{V}{I} = \frac{250}{4} = 62.5\ \Omega$$

$$\bar{Z} = Z\angle\phi = 62.5\ \angle 65° = 26.41 + j56.64\ \Omega$$

But
$$\bar{Z} = (R + r) + jX_L$$
$$X_L = 56.64\ \Omega$$
$$R + r = 26.41$$
$$r = 26.41 - 25 = 1.41\ \Omega$$
$$X_L = 2\pi f L$$
$$56.64 = 2\pi \times 50 \times L$$
$$L = 0.18\ \text{H}$$

(ii) Total power

$$P = I^2 (R + r) = (4)^2 \times 26.41 = 422.56\ \text{W}$$

(iii) Power consumed by resistance

$$P_R = I^2R = (4)^2 \times 25 = 400\ \text{W}$$

(iv) Power consumed by choke coil

$$P_{\text{coil}} = I^2r = (4)^2 \times 1.41 = 22.56\ \text{W}$$

Example 15

When a resistor and a coil in series are connected to a 240 V supply, a current of 3 A flows, lagging 37° behind the supply voltage. The voltage across the coil is 171 volts. Find the resistance and reactance of the coil, and the resistance of the resistor.

Solution

$$V = 240\ \text{V}$$
$$I = 3\ \text{A}$$

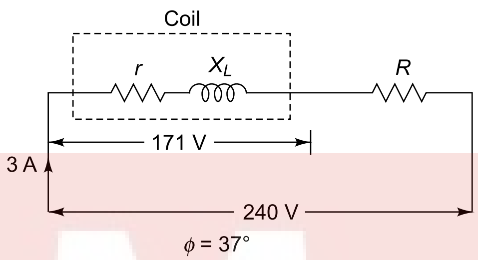


Fig. 4.22

$$\phi = 37°$$

$$V_{coil} = 171 \text{ V}$$

(i) Resistance and reactance of the coil

$$Z_{coil} = \frac{V_{coil}}{I} = \frac{171}{3} = 57 \ \Omega$$

$$\sqrt{r^2 + X_L^2} = 57$$

$$r^2 + X_L^2 = 3249$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80 \ \Omega$$

$$\bar{Z} = Z\angle\phi = 80 \angle 37° = 63.89 + j48.15 \ \Omega$$

But
$$\bar{Z} = (R + r) + jX_L$$

$$X_L = 48.15 \ \Omega$$

$$r^2 + X_L^2 = 3249$$

$$r^2 + (48.15)^2 = 3249$$

$$r^2 = 931.04$$

$$r = 30.51 \ \Omega$$

(ii) Resistance of the resistor

$$R + r = 63.89$$

$$R + 30.51 = 63.89$$

$$R = 33.38 \ \Omega$$

Example 16

A choke coil and a resistor are connected in series across a 230 V, 50 Hz ac supply. The circuit draws a current of 2 A at 0.866 lagging pf. The voltage drop across the resistor is 100 V. Calculate the power factor of the choke coil.

Solution

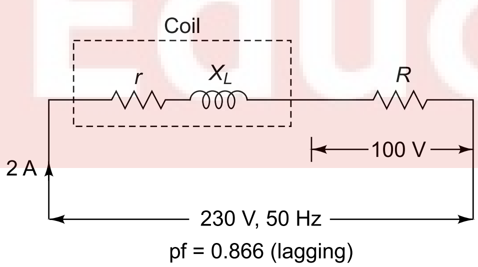


Fig. 4.23

$$V = 230 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$I = 2 \text{ A}$$

$$\text{pf} = 0.866 \text{ (lagging)}$$

$$V_R = 100 \text{ V}$$

$$R = \frac{V_R}{I} = \frac{100}{2} = 50 \ \Omega$$

$$Z = \frac{V}{I} = \frac{230}{2} = 115\ \Omega$$

$$\text{pf} = 0.866 \text{ (lagging)}$$

$$\phi = \cos^{-1}(0.866) = 30°$$

$$\bar{Z} = Z\angle\phi = 115\angle 30° = 99.59 + j57.5\ \Omega$$

$$R + r = 99.59$$

$$50 + r = 99.59$$

$$r = 49.59\ \Omega$$

$$X_L = 57.5\ \Omega$$

$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = \sqrt{(49.59)^2 + (57.5)^2} = 75.93\ \Omega$$

$$\text{pf}_{\text{coil}} = \frac{r}{Z_{\text{coil}}} = \frac{49.59}{75.93} = 0.653 \text{ (lagging)}$$

Example 17

A circuit consists of a pure resistor and coil in series. Power dissipated in the resistor and in the coil are 1000 W and 250 W respectively. The voltage drops across the resistor and the coil are 200 V and 300 V respectively. Determine (i) value of pure resistance, (ii) resistance and reactance of the coil, (iii) combined resistance of the circuit, (iv) combined impedance, and (v) supply voltage.

Solution

$$P_R = 1000 \text{ W}$$

$$P_{\text{coil}} = 250 \text{ W}$$

$$V_R = 200 \text{ V}$$

$$V_{\text{coil}} = 300 \text{ V}$$

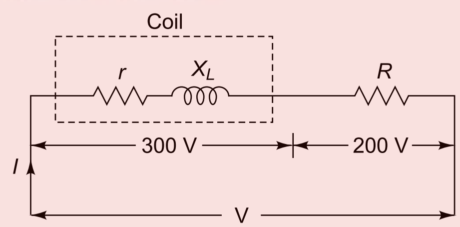


Fig. 4.24

(i) Value of pure resistance

$$P_R = \frac{V_R^2}{R}$$

$$1000 = \frac{(200)^2}{R}$$

$$R = 40\ \Omega$$

(ii) Resistance and reactance of the coil

$$V_R = RI$$

$$200 = 40I$$

$$I = 5 \text{ A}$$

$$P_{\text{coil}} = I^2 r$$

$$250 = (5)^2 \times r$$
$$r = 10\ \Omega$$
$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{300}{5} = 60\ \Omega$$
$$X_L = \sqrt{Z_{\text{coil}}^2 - r^2} = \sqrt{(60)^2 - (10)^2} = 59.2\ \Omega$$

(iii) Combined resistance of the circuit

$$R_T = R + r = 40 + 10 = 50\ \Omega$$

(iv) Combined impedance

$$Z = \sqrt{(R+r)^2 + X_L^2} = \sqrt{(50)^2 + (59.2)^2} = 77.5\ \Omega$$

(v) Supply voltage

$$V = Z\,I = 77.5 \times 5 = 387.5\ \text{V}$$

Example 18

A coil A takes 2 A at a power factor of 0.8 lagging with an applied p.d. of 10 V. A second coil B takes 2 A with a power factor of 0.7 lagging with an applied voltage of 5 V. What voltage will be required to produce a total current of 2 A with coils A and B in series? Find the power factor in this case.

Solution

Coil A: $I_A = 2$ A, $\text{pf}_A = 0.8$ (lagging), $V_A = 10$ V

Coil B: $I_B = 2$ A, $\text{pf}_B = 0.7$ (lagging), $V_B = 5$ V

$I = 2$ A

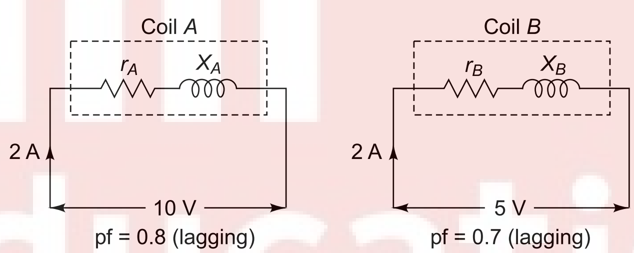


Fig. 4.25

For Coil A, $\phi_A = \cos^{-1}(0.8) = 36.87°$

$$Z_A = \frac{V_A}{I_A} = \frac{10}{2} = 5\ \Omega$$
$$\bar{Z}_A = Z_A \angle\phi_A = 5\ \angle 36.87° = 4 + j3\ \Omega$$
$$r_A = 4\ \Omega$$
$$X_A = 3\ \Omega$$

For Coil B, $\phi_B = \cos^{-1}(0.7) = 45.57°$

$$Z_B = \frac{V_B}{I_B} = \frac{5}{2} = 2.5\ \Omega$$

$$\bar{Z}_B = Z_B \angle \phi_B = 2.5\ \angle 45.57° = 1.75 + j1.78\ \Omega$$

$$r_B = 1.75\ \Omega$$

$$X_B = 1.78\ \Omega$$

When coils A and B are connected in series,

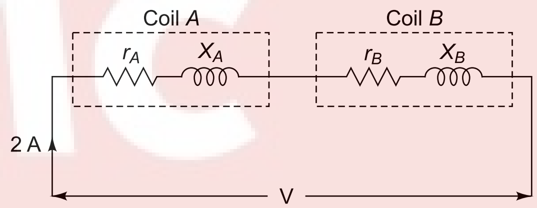


Fig. 4.26

$$\bar{Z} = r_A + jX_A + r_B + jX_B = 4 + j3 + 1.75 + j1.78$$

$$= 5.75 + j4.78 = 7.48\ \angle 39.74°\ \Omega$$

$$Z = 7.48\ \Omega$$

$$\phi = 39.74°$$

$$V = ZI = 7.48 \times 2 = 14.96\ \text{V}$$

$$\text{pf} = \cos\phi = \cos(39.74°) = 0.77\ \text{(lagging)}$$

Example 19

When a voltage of 100 V is applied to a coil A, the current taken is 8 A and the power is 120 W. When applied to a coil B, the current is 10 A and the power is 500 W. What current and power will be taken when 100 V is applied to the two coils connected in series?

Solution

Coil A: $V_A = 100$ V, $I_A = 8$ A, $P_A = 120$ W

Coil B: $V_B = 100$ V, $I_B = 10$ A, $P_B = 500$ W

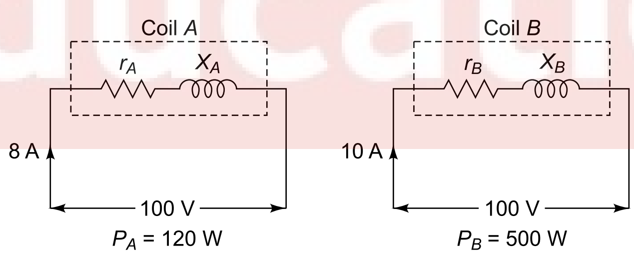


Fig. 4.27

For Coil A,
$$Z_A = \frac{V_A}{I_A} = \frac{100}{8} = 12.5\ \Omega$$
$$P_A = I_A^2 r_A$$
$$120 = (8)^2 \times r_A$$
$$r_A = 1.875\ \Omega$$
$$X_A = \sqrt{Z_A^2 - r_A^2} = \sqrt{(12.5)^2 - (1.875)^2} = 12.36\ \Omega$$

For Coil B,
$$Z_B = \frac{V_B}{I_B} = \frac{100}{10} = 10\ \Omega$$
$$P_B = I_B^2 r_B$$
$$500 = (10)^2 \times r_B$$
$$r_B = 5\ \Omega$$
$$X_B = \sqrt{Z_B^2 - r_B^2} = \sqrt{(10)^2 - (5)^2} = 8.66\ \Omega$$

When coils A and B are connected in series,

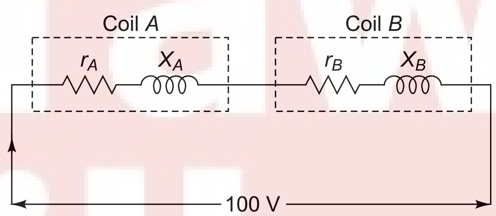


Fig. 4.28

$$\bar{Z} = r_A + jX_A + r_B + jX_B$$
$$= 1.875 + j12.36 + 5 + j8.66$$
$$= 6.875 + j21.02$$
$$= 22.11 \angle 71.89°\ \Omega$$
$$Z = 22.11\ \Omega$$
$$\phi = 71.89°$$
$$I = \frac{V}{Z} = \frac{100}{22.11} = 4.52\ \text{A}$$
$$P = I^2 (r_A + r_B) = (4.52)^2 \times (6.875) = 140.64\ \text{W}$$

Example 20

In a particular circuit, a voltage of 10 V at 25 Hz produces 100 mA, while the same voltage at 75 Hz produces 60 mA. Find the values of components of the circuit.

Solution $V_1 = 10$ V, $f_1 = 25$ Hz, $I_1 = 100$ mA

$V_2 = 10$ V, $f_2 = 75$ Hz, $I_2 = 60$ mA

Case (i) $V_1 = 10$ V, $f_1 = 25$ Hz, $I_1 = 100$ mA

$$Z_1 = \frac{V_1}{I_1} = \frac{10}{100 \times 10^{-3}} = 100\ \Omega$$

Case (ii) $V_2 = 10$ V, $f_1 = 75$ Hz, $I_2 = 60$ mA

$$Z_2 = \frac{V_2}{I_2} = \frac{10}{60 \times 10^{-3}} = 166.67\ \Omega$$

As frequency increases, impedance of the circuit increases. In a series *R-L* circuit, inductive reactance X_L increases with frequency. Hence, impedance increases.

Hence, the circuit consists of a resistance R and an inductance L.

$$Z_1 = \sqrt{R^2 + X_{L_1}^2} = \sqrt{R^2 + (2\pi \times 25 \times L)^2} = 100\ \Omega$$

$$R^2 + (50\pi L)^2 = 10000 \qquad (1)$$

$$Z_2 = \sqrt{R^2 + X_{L_2}^2} = \sqrt{R^2 + (2\pi \times 75 \times L)^2} = 166.67\ \Omega$$

$$R^2 + (150\pi L)^2 = 27778.89 \qquad (2)$$

Solving Eqs (1) and (2),

$$R = 88.1\ \Omega$$

$$L = 0.3\ \text{H}$$

Example 21

When 1 A is passed through three coils A, B and C in series, the voltages across them are 6 V, 3 V and 8 V respectively on a dc supply and 7 V, 5 V and 10 V respectively on an ac supply. Find the power factor and the power dissipated in each coil and the power factor of the whole circuit.

Solution $I = 1$ A

On dc supply, $V_A = 6$ V, $V_B = 3$ V, $V_C = 8$ V

On ac supply, $V_A = 7$ V, $V_B = 5$ V, $V_C = 10$ V

For dc supply, $f = 0$

$$X_L = 2\pi f L = 0$$

The coils behave like pure resistors.

$$R_A = \frac{V_A}{I} = \frac{6}{1} = 6\ \Omega$$

$$R_B = \frac{V_B}{I} = \frac{3}{1} = 3\ \Omega$$

$$R_C = \frac{V_C}{I} = \frac{8}{1} = 8\ \Omega$$

For ac supply,
$$Z_A = \frac{V_A}{I} = \frac{7}{1} = 7\ \Omega$$

$$Z_B = \frac{V_B}{I} = \frac{5}{1} = 5\ \Omega$$

$$Z_C = \frac{V_C}{I} = \frac{10}{1} = 10\ \Omega$$

$$X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{(7)^2 - (6)^2} = 3.6\ \Omega$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{(5)^2 - (3)^2} = 4\ \Omega$$

$$X_C = \sqrt{Z_C^2 - R_C^2} = \sqrt{(10)^2 - (8)^2} = 6\ \Omega$$

(i) Power factor of Coil A

$$\text{pf}_A = \frac{R_A}{Z_A} = \frac{6}{7} = 0.857 \text{ (lagging)}$$

(ii) Power factor of Coil B

$$\text{pf}_B = \frac{R_B}{Z_B} = \frac{3}{5} = 0.6 \text{ (lagging)}$$

(iii) Power factor of Coil C

$$\text{pf}_C = \frac{R_C}{Z_C} = \frac{8}{10} = 0.8 \text{ (lagging)}$$

(iv) Power dissipated in Coil A

$$P_A = I^2 R_A = (1)^2 \times 6 = 6\ \text{W}$$

(v) Power dissipated in Coil B

$$P_B = I^2 R_B = (1)^2 \times 3 = 3\ \text{W}$$

(vi) Power dissipated in Coil C

$$P_C = I^2 R_C = (1)^2 \times 8 = 8\ \text{W}$$

(vii) Power factor of the whole circuit

$$\begin{aligned}\bar{Z} &= R_A + jX_A + R_B + jX_B + R_C + jX_C \\ &= 6 + j3.6 + 3 + j4 + 8 + j6 \\ &= 17 + j13.6 = 21.77\ \angle 38.68^\circ\ \Omega\end{aligned}$$

$$\text{pf}_T = \cos(38.68^\circ) = 0.78 \text{ (lagging)}$$

Example 22

An air-cored coil takes 5 A of current and consumes 600 W of power when connected across a 200 V, 50 Hz ac supply. Calculate the value of the current drawn by the coil if the supply frequency increases to 60 Hz.

Solution $I = 5\text{ A}$

$P = 600\text{ W}$

$V = 200\text{ V}$

For $f = 50\text{ Hz}$

$$Z = \frac{V}{I} = \frac{200}{5} = 40\ \Omega$$

$$P = I^2 r$$

$$600 = (5)^2 \times r$$

$$r = 24\ \Omega$$

$$X_L = \sqrt{Z^2 - r^2} = \sqrt{(40)^2 - (24)^2} = 32\ \Omega$$

$$X_L = 2\pi f L$$

$$32 = 2\pi \times 50 \times L$$

$$L = 0.1019\text{ H}$$

For $f = 60\text{ Hz}$

$$X_L = 2\pi \times 60 \times 0.1019 = 38.4\ \Omega$$

$$r = 24\ \Omega$$

$$Z = \sqrt{r^2 + X_L^2} = \sqrt{(24)^2 + (38.4)^2} = 45.28\ \Omega$$

$$I = \frac{V}{Z} = \frac{200}{45.28} = 4.417\text{ A}$$

Example 23

When an iron-cored choking coil is connected to a 12 V dc supply, it draws a current of 2.5 A and when it is connected to a 230 V, 50 Hz supply, it draws a 2 A current and consumes 50 W of power. Determine for this value of current (i) power loss in the iron core, (ii) inductance of the coil, (iii) power factor, and (iv) value of the series resistance which is equivalent to the effect of iron loss.

Solution For dc $V = 12\text{ V}$, $I = 2.5\text{ A}$

For ac $V = 230\text{ V}$, $I = 2\text{ A}$, $P = 50\text{ W}$

In an iron-cored coil, there are two types of losses.

(i) Losses in core known as core or iron loss

(ii) Losses in winding known as copper loss

$$P = I^2 R + P_i$$

$$\frac{P}{I^2} = R + \frac{P_i}{I^2}$$

$$R_T = R + \frac{P_i}{I^2}$$

where R is the resistance of the coil and $\frac{P_i}{I^2}$ is the resistance which is equivalent to the effect of iron loss.

For dc supply, $f = 0$

$$X_L = 0$$

$$R = \frac{12}{2.5} = 4.8\ \Omega$$

For ac supply, $Z = \frac{230}{2} = 115\ \Omega$

(i) Iron loss

$$P_i = P - I^2R = 50 - (2)^2 \times 4.8 = 30.8 \text{ W}$$

$$R_T = \frac{P}{I^2} = \frac{50}{(2)^2} = 12.5\ \Omega$$

$$X_L = \sqrt{Z^2 - R_T^2} = \sqrt{(115)^2 - (12.5)^2} = 114.3\ \Omega$$

(ii) Inductance

$$X_L = 2\pi f L$$

$$114.3 = 2\pi \times 50 \times L$$

$$L = 0.363\ H$$

(iii) Power factor

$$\text{pf} = \frac{R_T}{Z} = \frac{12.5}{115} = 0.108 \text{ (lagging)}$$

(iv) The series resistance equivalent to the effect of iron loss

$$R_i = \frac{P_i}{I^2} = \frac{30.8}{(2)^2} = 7.7\ \Omega$$

Example 24

An iron-cored coil takes 4 A at a power factor of 0.5 when connected to a 200 V, 50 Hz supply. When the iron core is removed and the voltage is reduced to 40 V, the current rises to 5 A at a pf of 0.8. Find the iron loss in the core and inductance in each case.

Solution With iron core $I = 4$ A, pf $= 0.5$, $V = 200$ V

Without iron core $I = 5$ A, pf $= 0.8$, $V = 40$ V

(i) Inductance of the coil

(a) When the iron core is removed,

$$Z = \frac{V}{I} = \frac{40}{5} = 8\ \Omega$$

$$\text{pf} = \frac{R}{Z}$$

$$0.8 = \frac{R}{8}$$

$$R = 6.4\ \Omega$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(8)^2 - (6.4)^2} = 4.8\ \Omega$$

$$X_L = 2\pi f L$$

$$4.8 = 2\pi \times 50 \times L$$

$$L = 0.0153\ \text{H}$$

(b) With iron core,

$$Z = \frac{V}{I} = \frac{200}{4} = 50\ \Omega$$

$$\text{pf} = \frac{R_T}{Z}$$

$$0.5 = \frac{R_T}{50}$$

$$R_T = 25\ \Omega$$

$$X_L = \sqrt{Z^2 - R_T^2} = \sqrt{(50)^2 - (25)^2} = 43.3\ \Omega$$

$$X_L = 2\pi f L$$

$$43.3 = 2\pi \times 50 \times L$$

$$L = 0.1378\ \text{H}$$

(ii) Iron loss

$$P_i = P - I^2R = VI\cos\phi - I^2R = 200 \times 4 \times 0.5 - (4)^2 \times 6.4 = 297.6\ \text{W}$$

4.5 SERIES R-C CIRCUIT

Figure 4.29 shows a pure resistor R connected in series with a pure capacitor C across an alternating voltage v.

Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$ whereas voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

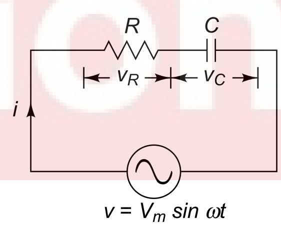


Fig. 4.29 *Series R-C circuit*

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

Phasor Diagram Since the same current flows through R and C, the curent I is taken as a reference phasor.

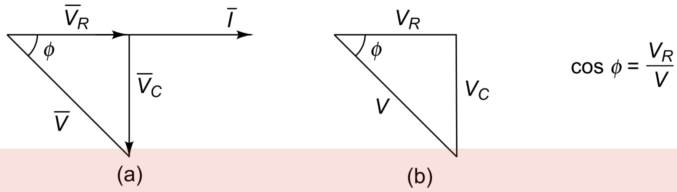


Fig. 4.30 *(a) Phasor diagram (b) Voltage triangle*

Impedance

$$\overline{V} = \overline{V}_R + \overline{V}_C$$
$$= R\overline{I} - jX_C\,\overline{I}$$
$$= (R - jX_C)\,\overline{I}$$
$$\frac{\overline{V}}{\overline{I}} = R - jX_C = \overline{Z}$$
$$\overline{Z} = Z\angle -\phi$$
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

The quantity $\overline{Z}$ is called the *complex impedance* of the *R*-*C* circuit.

Impedance Triangle

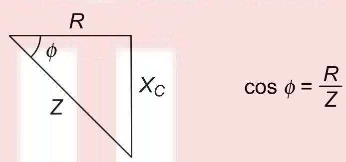


Fig. 4.31 *Impedance triangle*

Current From the phasor diagram, it is clear that the current I leads the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t + \phi)$$

where
$$I_m = \frac{V_m}{Z}$$

and
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Waveforms

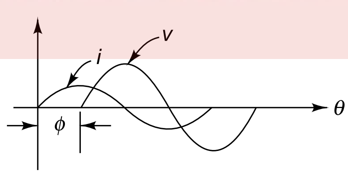


Fig. 4.32 *Waveforms*

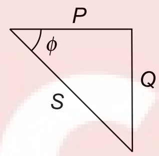
Power

Active power $P = VI \cos \phi = I^2R$

Reactive power $Q = VI \sin \phi = I^2X_C$

Apparent power $S = VI = I^2Z$

Power Triangle



$$\cos \phi = \frac{P}{S}$$

Fig. 4.33 *Power triangle*

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos \phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of an R-C series circuit, the power factor is leading in nature.

Example 1

The voltage applied to a circuit is e = 100 sin (ωt + 30°) and the current flowing in the circuit is i = 15 sin (ωt + 60°). Determine impedance, resistance, reactance, power factor and power.

Solution

$$e = 100 \sin(\omega t + 30°)$$

$$i = 15 \sin(\omega t + 60°)$$

(i) Impedance

$$\bar{E} = \frac{100}{\sqrt{2}} \angle 30° \text{ V}$$

$$\bar{I} = \frac{15}{\sqrt{2}} \angle 60° \text{ A}$$

$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{\dfrac{100}{\sqrt{2}} \angle 30°}{\dfrac{15}{\sqrt{2}} \angle 60°} = 6.67 \angle{-30°} = 5.77 - j3.33 = R - j\,X_C$$

$$Z = 6.67\ \Omega$$

(ii) Resistance

$$R = 5.77\ \Omega$$

(iii) Reactance

$$X_C = 3.33\ \Omega$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (leading)}$$

(v) Power

$$P = EI\cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5 \text{ W}$$

Example 2

A series circuit consumes 2000 W at 0.5 leading power factor, when connected to 230 V, 50 Hz ac supply. Calculate (i) current, (ii) kVA, and (iii) kVAR.

Solution

$$P = 2000 \text{ W}$$

$$\text{pf} = 0.5 \text{ (leading)}$$

$$V = 230 \text{ V}$$

(i) Current

$$P = VI\cos\phi$$

$$2000 = 230 \times I \times 0.5$$

$$I = 17.39 \text{ A}$$

(ii) Apparent power

$$S = VI = \frac{P}{\cos\phi} = \frac{2000}{0.5} = 4 \text{ kVA}$$

(iii) Reactive power

$$\phi = \cos^{-1}(0.5) = 60°$$

$$Q = VI\sin\phi = 230 \times 17.39 \times \sin(60°) = 3.464 \text{ kVAR}$$

Example 3

A resistor R in series with a capacitor C is connected to a 240 V, 50 Hz ac supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and maximum stored energy in C.

Solution

$$V = 240 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$P = 300 \text{ W}$$

$$V_R = 100 \text{ V}$$

(i) Value of C

$$P = \frac{V_R^2}{R}$$

$$300 = \frac{(100)^2}{R}$$

$$R = 33.33\ \Omega$$

$$P = I^2R$$

$$300 = I^2 \times 33.33$$

$$I = 3\text{ A}$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(80)^2 - (33.33)^2} = 72.72\ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$72.72 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 43.77\ \mu\text{F}$$

(ii) Maximum charge

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17\text{ V}$$

$$V_{C\max} = 218.17 \times \sqrt{2} = 308.54\text{ V}$$

$$Q_{\max} = CV_{C\max} = 43.77 \times 10^{-6} \times 308.54 = 0.0135\text{ C}$$

(iii) Maximum stored energy

$$E_{\max} = \frac{1}{2}C(V_{C\max})^2 = \frac{1}{2} \times 43.77 \times 10^{-6} \times (308.54)^2 = 2.08\text{ J}$$

Example 4

A capacitor of 35 μF is connected in series with a variable resistor. The circuit is connected across 50 Hz mains. Find the value of the resistor for a condition when the voltage across the capacitor is half the supply voltage.

Solution

$$C = 35\ \mu\text{F}$$

$$f = 50\text{ Hz}$$

$$V_C = \frac{1}{2}V$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 35 \times 10^{-6}} = 90.946\ \Omega$$

$$V_C = \frac{1}{2}\text{V}$$

$$X_C I = \frac{1}{2} Z I$$

$$X_C = \frac{1}{2} \text{Z}$$

$$Z = 2X_C$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$(2X_C)^2 = R^2 + X_C^2$$

$$R^2 = 3X_C^2 = 3 \times (90.946)^2 = 24813.35$$

$$R = 157.5\ \Omega$$

Example 5

A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a capacitor. The current is 2.2 A. The power loss in the resistor is 96.8 W. Calculate the resistance and capacitance.

Solution

$$V = 125\text{ V}$$

$$f = 50\text{ Hz}$$

$$I = 2.2\text{ A}$$

$$P = 96.8\text{ W}$$

(i) Resistance

$$Z = \frac{V}{I} = \frac{125}{2.2} = 56.82\text{ A}$$

$$P = I^2 R$$

$$96.8 = (2.2)^2 \times R$$

$$R = 20\ \Omega$$

(ii) Capacitance

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2} = 53.18\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$53.18 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 59.85\ \mu\text{F}$$

Example 6

A resistor and a capacitor are connected across a 250 V supply. When the supply frequency is 50 Hz, the current drawn is 5 A. When the frequency is increased to 60 Hz, it draws 5.8 A. Find the values of R and C and power drawn in the second case.

Solution

$$V = 250 \text{ V}$$
$$f_1 = 50 \text{ Hz}$$
$$I_1 = 5 \text{ A}$$
$$f_2 = 60 \text{ Hz}$$
$$I_2 = 5.8 \text{ A}$$

(i) Values of R and C

For $f_1 = 50$ Hz,

$$Z_1 = \frac{V}{I_1} = \frac{250}{5} = 50\ \Omega$$

$$Z_1 = \sqrt{R^2 + \left(\frac{1}{2\pi f_1 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{100\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{100\pi C}\right)^2 = 2500 \qquad (1)$$

For $f_2 = 60$ Hz,

$$Z_2 = \frac{V}{I_2} = \frac{250}{5.8} = 43.1\ \Omega$$

$$Z_2 = \sqrt{R^2 + \left(\frac{1}{2\pi f_2 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{120\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{120\pi C}\right)^2 = 1857.9\ \Omega \qquad (2)$$

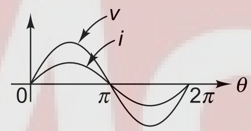
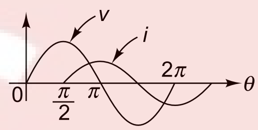
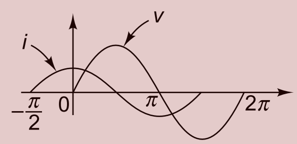
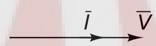
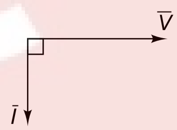
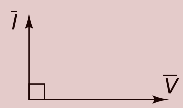
Solving Eqs (1) and (2),

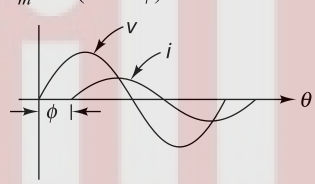
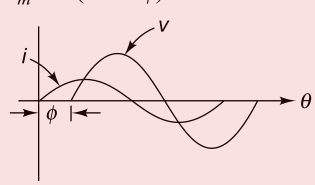
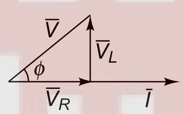
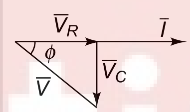
$$R = 19.96\ \Omega$$
$$C = 69.4\ \mu\text{F}$$

(ii) Power drawn in the second case

$$P_2 = I_2^2 R = (5.8)^2 \times 19.96 = 671.45 \text{ W}$$

Useful Formulae

	R	L	C
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin \omega t$	$I_m \sin (\omega t - 90°)$	$I_m \sin (\omega t + 90°)$
Waveform			
Phasor Diagram			
Impedance	R	$j\omega L$	$\frac{1}{j\omega C} = -j\frac{1}{\omega C}$
Phase Difference	0°	90°	90°
Power Factor	1	0	0
Power	VI	0	0

	Series R-L Circuit	*Series R-C Circuit*
Voltage	$V_m \sin \omega t$	$V_m \sin \omega t$
Current	$I_m \sin (\omega t - \phi)$	$I_m \sin (\omega t + \phi)$
Waveform		
Phasor Diagram		
Impedance	$R + jX_L, Z \angle\phi$	$R - jX_C, Z \angle -\phi$
Phase Difference	$0° < \phi < 90°$	$0° < \phi < 90°$
Power Factor	$\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$	$\frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$
Power	$P = VI \cos \phi = I^2 R$ $Q = VI \sin \phi = I^2 X_L$ $S = VI = I^2 Z$	$P = VI \cos \phi = I^2 R$ $Q = VI \sin \phi = I^2 X_C$ $S = VI = I^2 Z$

Exercise 4.1

4.1 A 250 V, 50 Hz voltage is applied across a circuit consisting of a pure resistance of 20 Ω. Determine (i) the current flowing through the circuit, and (ii) power absorbed by the circuit. Give the expressions for the voltage and current.

[(i) 12.5 A (ii) 3.125 kW (iii) $v = 353.6 \sin 314\, t$, $i = 17.68 \sin 314\, t$]

4.2 A purely inductive circuit allows a current of 20 A to flow through a 230 V, 50 Hz supply. Find (i) inductive reactance, (ii) inductance of the coil, (iii) power absorbed, and (iv) equations for voltage and current.

$$\left[\text{(i) } 11.5\ \Omega \text{ (ii) } 36.62\ \text{mH (iii) } 0 \text{ (iv) } v = 325.27 \sin 314\, t,\ i = 28.28 \sin\left(314\, t - \frac{\pi}{2}\right)\right]$$

4.3 A capacitor connected to a 230 V, 50 Hz supply draws a 15 A current. What current will it draw when the capacitance and frequency are both reduced to half?

[3.75 A]

4.4 Calculate the impedance of the circuit shown in Fig. 4.34.

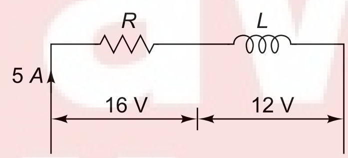


Fig. 4.34

[4 Ω]

4.5 An ac circuit has the following voltage and current: $v = 325 \sin 314\, t$, $i = 65 \sin (314\, t - 1.57)$. Find (i) frequency, (ii) rms value of voltage and current, (iii) impedance, and (iv) power factor.

[50 Hz, 229.81 V, 45.96 A, 5 Ω 0.99 (lagging)]

4.6 Two sources of an electromotive force represented respectively by $200 \sin \omega t$ and $200 \sin (\omega t + \pi/6)$ are in series. Express the resultant in vector notation with reference to $200 \sin \omega t$ and calculate the rms current and power supplied to a circuit of $8 + j6\ \Omega$ impedance. *[$273.2 \angle 15°$ V, $27.32 \angle -21.87°$ A, 5971 W]*

4.7 The voltage applied to a series circuit consisting of two pure elements is given by $v = 180 \sin \omega t$ and the resulting current is given by $i = 2.5 \sin (\omega t - 45°)$. Find the average power taken by the circuit and the values of the elements.

[159 W, 50.91 Ω, 50.91 Ω]

4.8 Find an expression for the current and calculate the power when a voltage represented by $v = 283 \sin 100\, \pi t$ is applied to a coil having $R = 50\ \Omega$ and $L = 0.159$ H. *[$4 \sin (100\, \pi t - \pi/4)$, 400 W]*

4.9 A voltage $V = (150 + j180)$ V is applied across an impedance and the current is found to be $I = (5 - j4)$ A. Determine (i) scalar impedance, (ii) reactance, and (iii) power consumed. *[36.6 Ω, 36.6 Ω, 30 W]*

4.10 The current in a series circuit of $R = 5$ Ω and $L = 30$ mH lags the applied voltage by 72°. Determine the source frequency and the impedance Z. *[81.63 Hz, 16.18 Ω]*

4.11 Current flowing through an inductive circuit is 15 sin ($\omega t + \pi/4$). When the voltage applied across it is 30 cos ωt, find the pf of the circuit. *[0.707 (lagging)]*

4.12 Voltage and current in an ac circuit are given by

$v = 200 \sin 377\, t \qquad i = 8 \sin (377\, t - \pi/6)$

Determine true power, reactive power and apparent power drawn by the circuit. *[692.82 W, 400 VAR, 800 VA]*

4.13 A current of 5 A flows through a non-inductive resistor in series with a choke coil when supplied at 250 V, 50 Hz. If the voltage drops across the coil and non-inductive resistor are 200 V and 125 V respectively, calculate the resistance and inductance of the impedance coil, value of non-inductive resistor and power drawn by the coil. Draw the vector diagram. *[5.5 Ω, 0.126 H, 25 Ω, 137.5 W]*

4.14 Two coils A and B are connected in series across a 200 V, 50 Hz ac supply. The power input to the circuit is 2 kW and 1.15 kVAR. If the resistance and reactance of the coil A are 5 Ω and 8 Ω respectively, calculate resistance and reactance of the coil B. Also, calculate the active power consumed by coils A and B. *[10.03 Ω, 0.642 Ω, 665. 3 W, 1334.7 W]*

4.15 A choking coil and a pure resistor are connected in series across a supply of 230 V, 50 Hz. The voltage drop across the resistor is 100 V and that across the chocking coil is 150 V. Find graphically the voltage drop across the inductance and resistance of the choking coil. Hence, find their values if the current is 1 A. *[109.98 V, 102 V, 0.366 H, 102 Ω]*

4.16 For Fig. 4.35, find R and L.

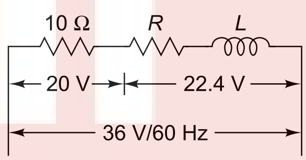


Fig. 4.35

[4.928 Ω, 0.0266 H]

4.17 Voltage and current for a circuit with two pure elements in series are expressed as follows:

$$v(t) = 170 \sin \left(6280\, t + \frac{\pi}{3}\right) \text{ volts}$$

$$i(t) = 8.5 \sin \left(6280\, t + \frac{\pi}{2}\right) \text{ amperes}$$

Sketch the two waveforms. Determine (i) frequency, (ii) power stating its nature, and (iii) values of the elements. *[(ii) 0.866 (leading) (iii) 17.32 Ω, 16 μF]*

4.18 A load consisting of a capacitor in series with a resistor has an impedance of 50 Ω and a pf of 0.707 leading. The load is connected in series with a 40 Ω resistor across an ac supply and the resulting current is of 3 A. Determine the supply voltage and overall phase angle. *[249.69 V, 25.135°]*

4.19 A capacitive load takes 10 kVA and 5 kVAR, when connected to a 200 V, 50 Hz ac supply. Calculate (i) resistance, (ii) capacitance, (iii) active power, and (iv) pf. *[3.464 Ω, 1.59 × 10^{-3} F, 8.66 kW, 0.866 (leading)]*

4.20 A resistor of 100 Ω is connected in series with a 50 μF capacitor to a 50 Hz, 200 V supply. Find:
(i) impedance (ii) current (iii) power factor (iv) phase angle
(v) voltage across the resistor and across the capacitor
[118.6 Ω, 1.69 A, 0.845 (leading), 32.48°, 168.712 V, 107.42 V]

4.6 SERIES *R-L-C* CIRCUIT

Figure 4.36 shows a pure resistor *R*, pure inductor *L* and pure capacitor *C* connected in series across an alternating voltage *v*.

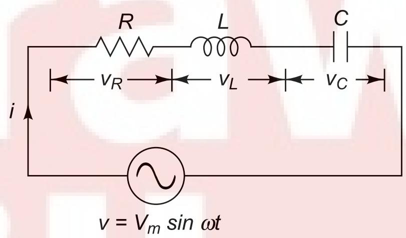


Fig. 4.36 *Series R-L-C circuit*

Let V and I be the rms values of the applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$, the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90° and the voltage $\bar{V}_C$ lags behind the current $\bar{I}$ by 90°.

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Phasor Diagram Since the same current flows through *R*, *L* and *C*, the current *I* is taken as a reference phasor.

Case (i) $X_L > X_C$

The reactance X will be inductive in nature and the circuit will behave like an *R-L* circuit.

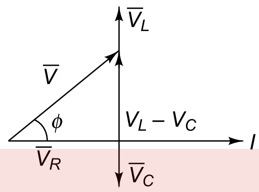


Fig. 4.37 *Phasor diagram*

Case (ii) $X_C > X_L$

The reactance X will be capacitive in nature and the circuit will behave like an R-C circuit.

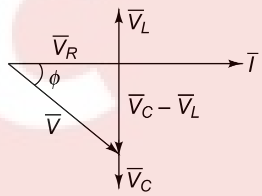


Fig. 4.38 *Phasor diagram*

Impedance

$$\overline{V} = \overline{V}_R + \overline{V}_L + \overline{V}_C = R\overline{I} + jX_L I - jX_C\overline{I} = [R + j(X_L - X_C)]\,\overline{I}$$

$$\frac{\overline{V}}{\overline{I}} = R + j(X_L - X_C) = \overline{Z}$$

$$\overline{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance Triangles

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

Fig. 4.39 *Impedance triangles*

Current Equation If the applied voltage is given by $v = V_m \sin \omega t$ then current equation will be

$$i = I_m \sin(\omega t \pm \phi)$$

'–' sign is used when $X_L > X_C$.

'+' sign is used when $X_C > X_L$.

Waveforms

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

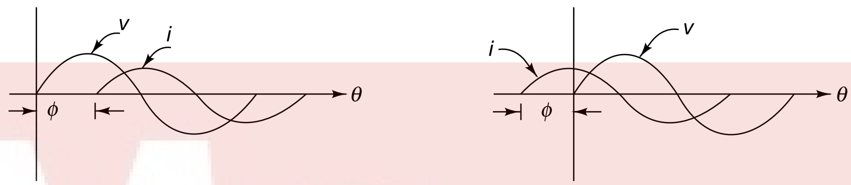


Fig. 4.40 *Waveforms*

Power

Average power $P = VI\cos\phi = I^2R$

Reactive power $Q = VI\sin\phi = I^2X$

Apparent power $S = VI = I^2Z$

Power Triangles

Case (i) $X_L > X_C$ *Case (ii)* $X_C > X_L$

Fig. 4.41 *Power triangles*

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$\text{pf} = \cos\phi$$

$$\text{pf} = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

Example 1

A resistor of 20 Ω, inductor of 0.05 H and a capacitor of 50 μF are connected in series. A supply voltage 230 V, 50 Hz is connected across the series combination. Calculate the following: (i) impedance, (ii) current drawn by the circuit, (iii) phase difference and power factor, and (iv) active and reactive power consumed by the circuit.

Solution

$R = 20\ \Omega$

$L = 0.05$ H

$C = 50\ \mu$F

$V = 230$ V

$f = 50$ Hz

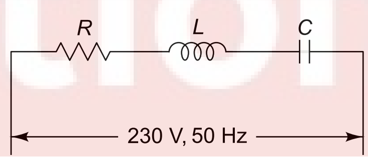


Fig. 4.42

(i) Impedance

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\bar{Z} = R + jX_L - jX_C$$

$$= 20 + j15.71 - j63.66$$

$$= 51.95 \angle -67.36°\ \Omega$$

$$Z = 51.95\ \Omega$$

(ii) Phase difference

$$\phi = 67.36°$$

(iii) Current

$$I = \frac{V}{Z} = \frac{230}{51.95} = 4.43\text{ A}$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(67.36°) = 0.385 \text{ (leading)}$$

(v) Active power

$$P = VI\cos\phi = 230 \times 4.43 \times 0.385 = 392.28\text{ W}$$

(vi) Reactive power

$$Q = VI\sin\phi = 230 \times 4.43 \times \sin(67.36°) = 940.39\text{ VAR}$$

Example 2

A circuit consists of a pure inductor, a pure resistor and a capacitor connected in series. When the circuit is supplied with 100 V, 50 Hz supply, the voltages across inductor and resistor are 240 V and 90 V respectively. If the circuit takes a 10 A leading current, calculate (i) value of inductance, resistance and capacitance, (ii) power factor of the circuit, and (iii) voltage across the capacitor.

Solution

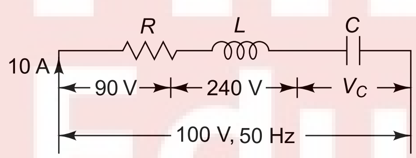


Fig. 4.43

$$V = 100\text{ V}$$
$$f = 50\text{ Hz}$$
$$V_L = 240\text{ V}$$
$$V_R = 90\text{ V}$$
$$I = 10\text{ A}$$

(i) Value of inductance, resistance and capacitance

$$R = \frac{V_R}{I} = \frac{90}{10} = 9\ \Omega$$

$$X_L = \frac{V_L}{I} = \frac{240}{10} = 24\ \Omega$$

$$Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

$$\bar{Z} = R + j\,X_L - j\,X_C = R - j(X_C - X_L)$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$10 = \sqrt{(9)^2 + (X_C - 24)^2}$$

$$X_C = 28.36\ \Omega$$

$$X_L = 2\pi f L$$

$$24 = 2\pi \times 50 \times L$$

$$L = 0.076\ \text{H}$$

$$X_C = \frac{1}{2\pi f C}$$

$$28.36 = \frac{1}{2\pi \times 50 \times C}$$

$$\text{C} = 112.24\ \mu\text{F}$$

(ii) Power factor of the circuit

$$\text{pf} = \frac{R}{Z} = \frac{9}{10} = 0.9\ \text{(leading)}$$

(iii) Voltage across the capacitor

$$V_C = X_C\, I = 28.36 \times 10 = 283.6\ \text{V}$$

Example 3

Two impedances $Z_1 = 40\angle 30°\ \Omega$ and $Z_2 = 30\ \angle 60°\ \Omega$ are connected in series across a single-phase 230 V, 50 Hz supply. Calculate the (i) current drawn, (ii) pf, and (iii) power consumed by the circuit.

Solution

$$\bar{Z}_1 = 40\ \angle 30°\ \Omega$$

$$\bar{Z}_2 = 30\ \angle 60°\ \Omega$$

$$V = 230\ \text{V}$$

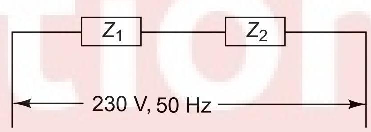


Fig. 4.44

(i) Current drawn

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 40\ \angle 30° + 30\ \angle 60° = 67.66\ \angle 42.81°\ \Omega$$

$$I = \frac{V}{Z} = \frac{230}{67.66} = 3.4\ \text{A}$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(42.81°) = 0.734 \text{ (lagging)}$$

(iii) Power consumed

$$P = VI\cos\phi = 230 \times 3.4 \times 0.734 = 573.99 \text{ W}$$

Example 4

A circuit takes a current of 3 A at a power factor of 0.6 lagging when connected to 115 V, 50 Hz supply. Another circuit takes a current of 5 A at a power factor of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230 V, 50 Hz supply, calculate (i) current, (ii) power consumed, and (iii) power factor.

Solution Circuit 1: $I_1 = 3$ A, $\text{pf}_1 = 0.6$ (lagging), $V_1 = 115$ V

Circuit 2: $I_2 = 5$ A, $\text{pf}_2 = 0.707$ (leading), $V_2 = 115$ V

$V = 230$ V

For Circuit 1, $\phi_1 = \cos^{-1}(0.6) = 53.13°$

$$Z_1 = \frac{V_1}{I_1} = \frac{115}{3} = 38.33\ \Omega$$

$$\bar{Z}_1 = Z_1\angle\phi_1 = 38.33\angle 53.13°\ \Omega$$

For Circuit 2, $\phi_2 = \cos^{-1}(0.707) = 45°$

$$Z_2 = \frac{V_2}{I_2} = \frac{115}{5} = 23\ \Omega$$

$$\bar{Z}_2 = Z_2\angle\phi_2 = 23\angle -45°\ \Omega$$

When the two circuits are connected in series,

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 38.33\angle 53.13° + 23\angle -45° = 41.82\angle 20.14°\ \Omega$$

(i) Current

$$I = \frac{V}{Z} = \frac{230}{41.82} = 5.5 \text{ A}$$

(ii) Power consumed

$$P = VI\cos\phi = 230 \times 5.5 \times \cos(20.14°) = 1.187 \text{ kW}$$

(iii) Power factor

$$\text{pf} = \cos\phi = \cos(20.14°) = 0.939 \text{ (lagging)}$$

Example 5

Two impedances Z_1 and Z_2, having the same numerical value, are connected in series. If Z_1 has a pf of 0.866 lagging and Z_2 has a pf of 0.8 leading, calculate the pf of the series combination.

Solution $\text{pf}_1 = 0.866$ (lagging)

$\text{pf}_2 = 0.8$ (leading)

$$
\begin{aligned}
Z_1 &= Z_2 = Z \\
\phi_1 &= \cos^{-1}(0.866) = 30° \\
\phi_2 &= \cos^{-1}(0.8) = 36.87° \\
\bar{Z}_1 &= Z\angle\phi_1 = Z\angle 30° = 0.866\,Z + j0.5\,Z\ \Omega \\
\bar{Z}_2 &= Z\angle{-\phi_2} = Z\angle{-36.87°} = 0.8\,Z - j0.6\,Z\ \Omega
\end{aligned}
$$

For a series combination,

$$
\begin{aligned}
\bar{Z} &= \bar{Z}_1 + \bar{Z}_2 = 0.866\,Z + j0.5\,Z + 0.8\,Z - j0.6\,Z \\
&= 1.666\,Z - j0.1\,Z = Z\,(1.666 - j0.1) = 1.668\,Z\angle{-3.43°}\ \Omega \\
\text{pf} &= \cos(3.43°) = 0.9982
\end{aligned}
$$

Example 6

A coil of 3 Ω resistance and an inductance of 0.22 H is connected in series with an imperfect capacitor. When such a series circuit is connected across a 200 V, 50 Hz supply, it has been observed that their combined impedance is (3.8 + j6.4) Ω. Calculate the resistance and capacitance of the imperfect capacitor.

Solution

$$
\begin{aligned}
r &= 3\ \Omega \\
L &= 0.22\ \text{H} \\
V &= 200\ \text{V} \\
f &= 50\ \text{Hz} \\
Z &= 3.8 + j\,6.4\ \Omega
\end{aligned}
$$

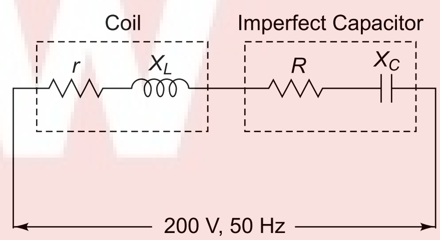


Fig. 4.45

(i) Resistance of the imperfect capacitor

$$
\begin{aligned}
\bar{Z} &= 3.8 + j6.4\ \Omega \\
X_L &= 2\pi f L = 2\pi \times 50 \times 0.22 = 69.12\ \Omega \\
\bar{Z} &= 3 + j69.12 + R - jX_C = (3 + R) + j\,(69.12 - X_C) \\
3 + R &= 3.8 \\
R &= 0.8\ \Omega
\end{aligned}
$$

(ii) Capacitance of the imperfect capacitor

$$
\begin{aligned}
69.12 - X_C &= 6.4 \\
X_C &= 62.72\ \Omega \\
X_C &= \frac{1}{2\pi f C} \\
62.72 &= \frac{1}{2\pi \times 50 \times C} \\
C &= 50.75\ \mu\text{F}
\end{aligned}
$$

Example 7

An R-L-C series circuit has a current which lags the applied voltage by 45°. The voltage across the inductance has a maximum value equal to twice the maximum value of voltage across the capacitor. Voltage across the inductance is 300 sin (1000t) and R = 20 Ω. Find the value of inductance and capacitance.

Solution

$$\phi = 45°$$

$$v_L = 300 \sin (1000t)$$

$$R = 20\ \Omega$$

(i) Value of inductance

$$V_{L(\max)} = 2V_{C(\max)}$$

$$V_L = 2V_C$$

$$I X_L = 2I X_C$$

$$X_L = 2X_C$$

$$\cos \phi = \frac{R}{Z}$$

$$\cos (45°) = \frac{20}{Z}$$

$$Z = 28.28\ \Omega$$

For a series *R-L-C* circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$28.28 = \sqrt{(20)^2 + (2X_C - X_C)^2}$$

$$= \sqrt{400 + X_C^2}$$

$$X_C = 20\ \Omega$$

$$X_L = 2X_C = 40\ \Omega$$

$$X_L = \omega L$$

$$40 = 1000 \times L$$

$$L = 0.04\ \text{H}$$

(ii) Value of capacitance

$$X_C = \frac{1}{\omega C}$$

$$20 = \frac{1}{1000 \times C}$$

$$C = 50\ \mu\text{F}$$

Example 8

A coil having a power factor of 0.5 is in series with a 79.57 μF capacitor and when connected across a 50 Hz supply, the p.d. across the coil is equal to the p.d. across the capacitor. Find the resistance and inductance of the coil.

Solution

$$\text{pf}_{\text{coil}} = 0.5$$
$$C = 79.57\ \mu\text{F}$$
$$f = 50\ \text{Hz}$$
$$V_{\text{coil}} = V_C$$

(i) Resistance of the coil

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 79.57 \times 10^{-6}} = 40\ \Omega$$
$$V_{\text{coil}} = V_C$$
$$I Z_{\text{coil}} = I X_C$$
$$Z_{\text{coil}} = X_C = 40\ \Omega$$
$$\text{pf}_{\text{coil}} = \cos\phi = \frac{R}{Z_{\text{coil}}}$$
$$0.5 = \frac{R}{40}$$
$$R = 20\ \Omega$$

(ii) Inductance of the coil

$$X_L = \sqrt{Z_{\text{coil}}^2 - R^2} = \sqrt{(40)^2 - (20)^2} = 34.64\ \Omega$$
$$X_L = 2\pi f L$$
$$34.64 = 2\pi \times 50 \times L$$
$$L = 0.11\ \text{H}$$

Example 9

A 250 V, 50 Hz voltage is applied to a coil having a resistance of 5 Ω and an inductance of 9.55 H in series with a capacitor C. If the voltage across the coil is 300 V, find the value of C.

Solution

$$V = 250\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$R = 5\ \Omega$$
$$L = 9.55\ \text{H}$$
$$V_{\text{coil}} = 300\ \text{V}$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 9.55 = 3000\ \Omega$$

$$Z_{\text{coil}} = \sqrt{R^2 + X_L^2} = \sqrt{(5)^2 + (3000)^2} = 3000\ \Omega$$

$$I = \frac{V_{\text{coil}}}{Z_{\text{coil}}} = \frac{300}{3000} = 0.1\ \text{A}$$

$$Z = \frac{V}{I} = \frac{250}{0.1} = 2500\ \Omega$$

When $X_L > X_C$,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$2500 = \sqrt{(5)^2 + (3000 - X_C)^2}$$

$$X_C = 500$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 500} = 6.37\ \mu\text{F}$$

When $X_C > X_L$,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$2500 = \sqrt{(5)^2 + (X_C - 300)^2}$$

$$X_C = 5500$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 5500} = 0.58\ \mu\text{F}$$

Example 10

Draw the phasor diagram for the series circuit shown in Fig. 4.46 when the current in the circuit is 2 A. Find the values of V_1 and V_2 and show these voltages on the phasor diagram.

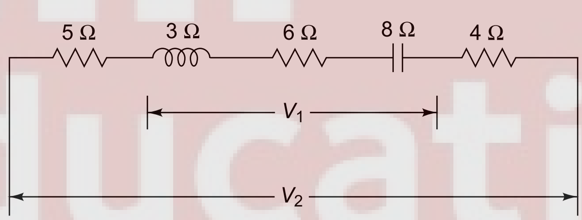


Fig. 4.46

Solution

$$\overline{Z}_1 = j3 + 6 - j8 = 6 - j5 = 7.81\ \angle{-39.8°}\ \Omega$$

$$\overline{Z}_2 = 5 + j3 + 6 - j8 + 4 = 15 - j5 = 15.81\ \angle{-18.43°}\ \Omega$$

$$I = 2\ \text{A}$$

(i) Values of V_1 and V_2

Let $\bar{I} = 2\angle 0°$ A

$$\bar{V}_1 = \bar{Z}_1 \bar{I} = (7.81\angle -39.8°)(2\angle 0°) = 15.62\angle -39.8° \text{ V}$$

$$\bar{V}_2 = \bar{Z}_2 \bar{I} = (15.81\angle -18.43°)(2\angle 0°) = 31.62\angle -18.43° \text{ V}$$

(ii) Phasor diagram

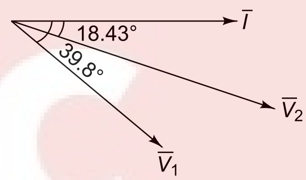


Fig. 4.47

Example 11

Draw a vector diagram for the circuit shown in Fig. 4.48 indicating terminal voltages V_1 and V_2 and the current. Find the value of (i) current, (ii) V_1 and V_2, and (iii) power factor.

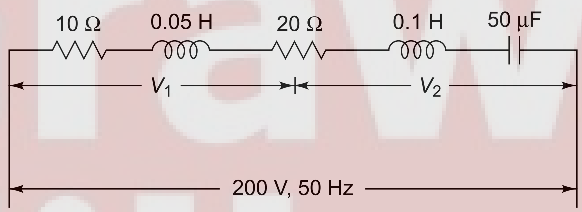


Fig. 4.48

Solution

(i) Current

$$X_{L_1} = 2\pi f L = 2\pi \times 50 \times 0.05 = 15.71\ \Omega$$

$$X_{L_2} = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\ \Omega$$

$$\bar{Z} = 10 + j15.71 + 20 + j31.42 - j63.66$$

$$= 30 - j16.53 = 34.25\angle -28.85°\ \Omega$$

$$I = \frac{V}{Z} = \frac{200}{34.25} = 5.84 \text{ A}$$

(ii) V_1 and V_2

Let $\bar{I} = 5.84\angle 0°$A

$$\bar{Z}_1 = 10 + j15.71 = 18.62\angle 57.52\ \Omega$$

$$\overline{V}_1 = \overline{Z}_1 \overline{I} = (18.62 \angle 57.52°)\ (5.84 \angle 0°) = 108.74 \angle 57.52° \text{ V}$$

$$\overline{Z}_2 = 20 + j31.42 - j63.66 = 20 - j32.24 = 37.94 \angle -58.19° \ \Omega$$

$$\overline{V}_2 = \overline{Z}_2 \overline{I} = (37.94 \angle -58.19°)\ (5.84 \angle 0°) = 221.57 \angle -58.19° \text{ V}$$

(iii) Power factor

$$\text{pf} = \cos \phi = \cos (28.85°) = 0.875 \text{ (leading)}$$

(iv) Vector diagram

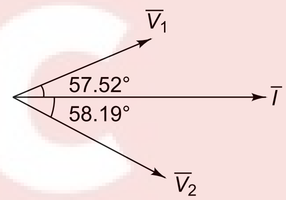


Fig. 4.49

Example 12

Find the values of R and C so that $V_x = 3V_y$, V_x and V_y are in quadrature.

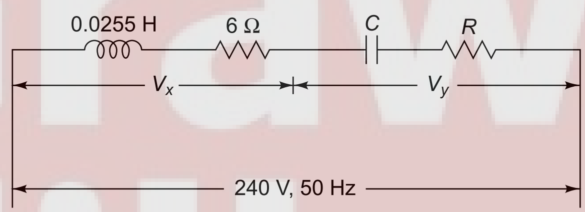


Fig. 4.50

Solution

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.0255 = 8 \ \Omega$$

$$\overline{Z}_x = 6 + j8 = 10 \angle 53.13° \ \Omega$$

$$V_x = 3V_y$$

$$I Z_x = 3I Z_y$$

$$Z_x = 3Z_y$$

V_x and V_y are in quadrature, i.e., phase angle between V_x and V_y is 90°. Hence, the angle between Z_x and Z_y will be 90°. The impedance Z_y is capacitive in nature.

$$\overline{Z}_y = Z_y \angle -\phi$$

$$\overline{Z}_y = \frac{10}{3} \angle (53.13 - 90)° = 3.33 \angle -36.87° = 2.66 - j2 \ \Omega$$

$$R = 2.66 \ \Omega$$

$$X_C = 2 \ \Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$2 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 1.59 \text{ mF}$$

Example 13

A pure resistor R, a choke coil and a pure capacitor of 15.91 μF are connected in series across a supply of V volts and carries a current of 0.25 A. The voltage across the choke coil is 40 V, the voltage across the capacitor is 50 V and the voltage across the resistor is 20 V. The voltage across the combination of R and the choke coil is 45 V. Calculate (i) supply voltage, (ii) frequency, and (iii) power loss in the choke coil.

Solution

$$C = 15.91\ \mu\text{F}$$

$$I = 0.25 \text{ A}$$

$$V_{\text{coil}} = 40 \text{ V}$$

$$V_C = 50 \text{ V}$$

$$V_R = 20 \text{ V}$$

$$V_{R\text{-coil}} = 45 \text{ V}$$

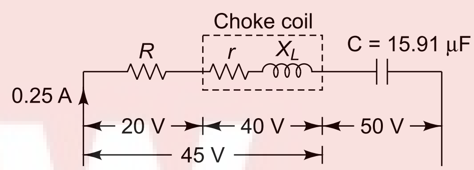


Fig. 4.51

(i) Supply voltage

$$R = \frac{V_R}{I} = \frac{20}{0.25} = 80\ \Omega$$

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{40}{0.25} = 160\ \Omega$$

$$X_c = \frac{V_C}{I} = \frac{50}{0.25} = 200\ \Omega$$

$$Z_{R\text{-coil}} = \frac{V_{R\text{-coil}}}{I} = \frac{45}{0.25} = 180\ \Omega$$

$$Z_{\text{coil}} = \sqrt{r^2 + X_L^2} = 160$$

$$r^2 + X_L^2 = 25600 \qquad (1)$$

$$\bar{Z}_{R\text{-coil}} = (R + r) + j\,X_L$$

$$Z_{R\text{-coil}} = \sqrt{(R + r)^2 + X_L^2}$$

$$180 = \sqrt{(80 + r)^2 + X_L^2}$$

$$(80 + r)^2 + X_L^2 = 32400 \qquad (2)$$

Subtracting Eq. (1) from Eq. (2),

$$(80 + r)^2 - r^2 = 6800$$

$$6400 + 160\,r + r^2 - r^2 = 6800$$

$$160\, r = 400$$
$$r = 2.5\ \Omega$$

Substituting the value of r in Eq. (1),

$$(2.5)^2 + X_L^2 = 25600$$
$$X_L^2 = 25593.75$$
$$X_L = 159.98\ \Omega$$
$$Z = R + r + j\,X_L - j\,X_C = 80 + 2.5 + j\,159.98 - j\,200$$
$$= 82.5 - j\,40.02 = 91.69\ \angle{-25.88°}\ \Omega$$
$$V = Z\,I = 91.69 \times 0.25 = 22.92\ \text{V}$$

(ii) Frequency

$$X_c = \frac{1}{2\pi fC}$$
$$200 = \frac{1}{2\pi f \times 15.91 \times 10^{-6}}$$
$$f = 50.02\ \text{Hz}$$

(iii) Power loss in the choke coil

$$P_{coil} = I^2 r = (0.25)^2 \times 2.5 = 0.156\ \text{W}$$

Example 14

Two impedances, one inductive and the other capacitive, are connected in series across the voltage of 120 ∠30° V and a frequency of 50 Hz. The current flowing in the circuit is 3∠–15°A. If one of the impedances is (10 + j 48.3) Ω, find the other. Also calculate the values of L and C in the impedances.

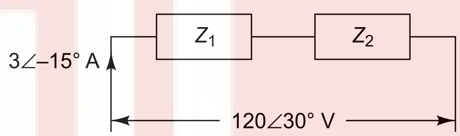


Fig. 4.52

Solution

$$V = 120\ \angle 30°\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$I = 3\ \angle{-15°}\ \text{A}$$
$$\bar{Z}_1 = 10 + j\,48.3\ \Omega$$

(i) Impedance Z_2

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{120\angle 30°}{3\angle -15°} = 40\ \angle 45° = 28.28 + j\,28.28\ \Omega$$
$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$
$$\bar{Z}_2 = \bar{Z} - \bar{Z}_1 = 28.28 + j\,28.28 - 10 - j\,48.3 = 18.28 - j\,20.02\ \Omega$$

(ii) Value of L

$$\bar{Z}_1 = 10 + j\,48.3 = R_1 + j\,X_L$$
$$X_L = 48.3$$
$$X_L = 2\pi f L$$
$$48.3 = 2\pi \times 50 \times L$$
$$L = 0.1537 \text{ H}$$

(iii) Value of C

$$\bar{Z}_2 = 18.28 - j\,20.02 = R_2 - j\,X_C$$
$$X_C = 20.02$$
$$X_C = \frac{1}{2\pi f C}$$
$$20.02 = \frac{1}{2\pi \times 50 \times C}$$
$$C = 159\ \mu\text{F}$$

Exercise 4.2

4.1 In an *RLC* series circuit, the voltage across the resistor, inductor and capacitor are 10 V, 15 V and 10 V respectively. What is the supply voltage? *[11.18 V]*

4.2 A resistor R, inductor L = 0.01 H and capacitor C are connected in series. When a voltage of 400 sin (3000 t – 10°) is applied to a series combination, the current flowing is $10\sqrt{2}$ cos (3000 t – 55°) A. Find R and C. *[20 Ω, 6.66 μF]*

4.3 A resistor of 20 Ω, an inductor of 0.2 H and a capacitor of 100 μF are connected in series across a 220 V, 50 Hz supply. Determine
(i) impedance (ii) current flowing through the circuit
(iii) voltage drop across each element (iv) power consumed
[36.9 Ω, 5.96 A, 119.2 V, 374.52 V, 189.64 V, 710.43 W]

4.4 A series *R-L-C* circuit having a resistance of 8 Ω, an inductance of 80 mH and a capacitance of 100 μF is connected across a 150 V, 50 Hz supply. Calculate (i) current, (ii) power factor, and (iii) voltage across the inductance and capacitance. *[(i) 14.35 A (ii) 0.766 (leading) (iii) 360.47 V, 457.04 V]*

4.5 An emf of 100 V, 50 Hz is applied to an impedance $Z_1 = 8 + j6$ Ω. An impedance Z_2 is added in series with Z_1. The current becomes half of the original and leads it by 20°. Determine Z_2. *[(11.14 – j0.2) Ω]*

4.6 In a series *RLC* circuit, the inductor is imperfect and the capacitor has no leakage but has a lead resistance. When the circuit is connected to a 50 Hz supply, the voltage drop across the resistor and the inductor is 200 V. The voltage drop across the resistor and the capacitor is 250 V. The voltage drop across the resistor alone is 150 V. The voltage drop across the capacitor and inductor are 125 V and 100 V respectively. If the current flowing through the circuit is 1 A, find graphically

(i) capacitance (ii) lead resistance of the capacitor

(iii) inductance (iv) resistance of the inductor

(v) applied voltage

[33.5 μF, 81.25 Ω, 0.3 H, 25 Ω, 256.26 V]

4.7 Two impedances of $10 \angle 30°\ \Omega$ and $20 \angle -45°\ \Omega$ are connected in series. Calculate the power factor of the series combination. *[0.9281 (lagging)]*

4.8 Two impedances Z_1 and Z_2 are connected in series across a 230 V, 50 Hz ac supply. The total current drawn by the series combination is 2.3 A. The pf of Z_1 is 0.8 lagging. The voltage drop across Z_1 is twice the voltage drop across Z_2 and it is 90° out of phase with it. Determine the value of Z_2. *[44.719 Ω]*

4.9 In the arrangement shown in Fig. 4.53, C = 20 microfarads and the current flowing through the circuit is 0.345 A. If the voltages are as indicated, find the applied voltage, frequency and loss in the iron-cored inductor. Draw the phasor diagram.

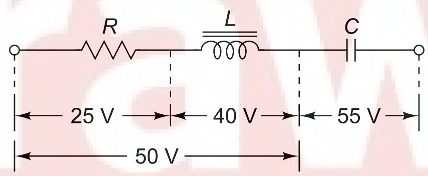


Fig. 4.53

[34.2 V, 50 Hz, 1.89 W]

4.7 PARALLEL AC CIRCUITS

In parallel circuits, resistor, inductor and capacitor or any combination of these elements are connected across same supply. Hence the voltage is same across each branch of the parallel ac circuit. The total current supplied to the circuit is equal to the phasor sum of the branch currents.

For the parallel ac circuit shown in Fig. 4.54,

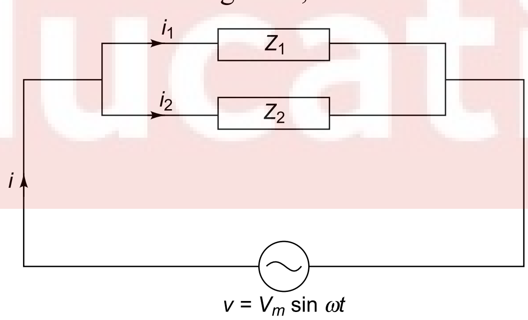


Fig. 4.54 *Parallel ac circuit*

$$\frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2}$$

$$\overline{Y} = \overline{Y}_1 + \overline{Y}_2$$

where Y represents the admittance of the circuit and is defined as the reciprocal of impedance. The real part of admittance is called conductance (G) and the imaginary part is called susceptance (B), and these are measured in mhos (℧) or siemens (S).

If $\overline{Z}_1 = R + jX_L$, and $\overline{Z}_2 = -jX_C$

then,

$$\frac{1}{\overline{Z}} = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + j\frac{1}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right)$$

$$= G + jB$$

where,

$$G = \frac{R}{R^2 + X_L^2}$$

$$B = \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}$$

The current in the parallel ac circuit can be found as the phasor sum of the branch currents,

i.e., $\overline{I} = \overline{I}_1 + \overline{I}_2$

Note: 1. For a series *R-L* circuit,

$$\overline{Z} = R + jX_L$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$= \frac{R}{R^2 + X_L^2} - j\frac{X_L}{R^2 + X_L^2} = G - jB_L$$

where $G = \dfrac{R}{R^2 + X_L^2}$ and $B_L = \dfrac{X_L}{R^2 + X_L^2}$

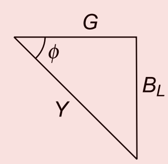


Fig. 4.55 *Admittance triangle*

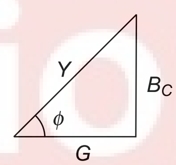


Fig. 4.56 *Admittance triangle*

2. For a series *R-C* circuit,

$$\overline{Z} = R - jX_C$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2} = \frac{R}{R^2 + X_C^2} + j\frac{X_C}{R^2 + X_C^2}$$

$$= G + jB_C$$

where $G = \dfrac{R}{R^2 + X_C^2}$ and $B_C = \dfrac{X_C}{R^2 + X_C^2}$

Example 1

A coil having a resistance of 50 Ω and an inductance of 0.02 H is connected in parallel with a capacitor of 25 μF across a single-phase 200 V, 50 Hz supply. Calculate the current in coil and capacitance. Calculate also the total current drawn, total pf and total power consumed by the circuit.

Solution

$$R = 50\ \Omega$$
$$L = 0.02\ \text{H}$$
$$C = 25\ \mu\text{F}$$
$$V = 200\ \text{V}$$
$$f = 50\ \text{Hz}$$

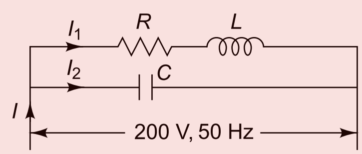


Fig. 4.57

(i) Current in coil and capacitance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 25 \times 10^{-6}} = 127.32\ \Omega$$

$$\bar{Z}_1 = R + jX_L = 50 + j\,6.28 = 50.39\ \angle 7.16°\ \Omega$$

$$\bar{Z}_2 = -jX_C = -j127.32 = 127.32\ \angle -90°\ \Omega$$

$$\bar{V} = 200\ \angle 0° = 200\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{50.39\angle 7.16°} = 3.97\ \angle -7.16°\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{127.32\angle -90°} = 1.57\ \angle 90°\ \text{A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 3.97\ \angle -7.16° + 1.57\ \angle 90° = 4.08\ \angle 15.27°\ \text{A}$$

(iii) Total pf

$$\text{pf} = \cos\phi = \cos(15.27°) = 0.965\ \text{(lagging)}$$

(iv) Total power consumed

$$P = VI\cos\phi = 200 \times 4.08 \times 0.965 = 787.44\ \text{W}$$

Example 2

Two impedances $\bar{Z}_1 = 30\ \angle 45°\ \Omega$ and $\bar{Z}_2 = 45\angle 30°\ \Omega$ are connected in parallel across a single-phase 230 V, 50 Hz supply. Calculate (i) current drawn by each branch, (ii) total current, and (iii) overall power factor.

Also draw the phasor diagram indicating the current drawn by each branch and the total current, taking the supply voltage as reference.

Solution $\bar{Z}_1 = 30 \angle 45° \ \Omega$

$\bar{Z}_2 = 45 \angle 30° \ \Omega$

$V = 230 \text{ V}$

(i) Current drawn by each branch

Let $\bar{V} = 230 \angle 0° = 230 \text{ V}$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{230}{30\angle 45°} = 7.67 \angle -45° \text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{230}{45\angle 30°} = 5.11 \angle -30° \text{ A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 7.67 \angle -45° + 5.11 \angle -30° = 12.67 \angle -39.01° \text{ A}$$

(iii) Overall power factor

$$\text{pf} = \cos\phi = \cos(39.01°) = 0.777 \text{ (lagging)}$$

(iv) Phasor diagram

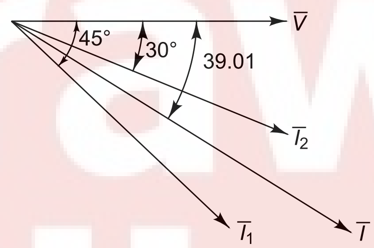


Fig. 4.58

Example 3

Two circuits, the impedances of which are given by $Z_1 = (10 + j15)$ ohms and $Z_2 = (6 - j8)$ ohms, are connected in parallel across an ac supply. If the total current supplied is 15 A, what is the power taken by each branch?

Solution $\bar{Z}_1 = (10 + j15) \ \Omega$

$\bar{Z}_2 = (6 - j8) \ \Omega$

$I = 15 \text{ A}$

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(10 + j15)(6 - j8)}{10 + j15 + 6 - j8} = 10.32 \angle -20.45° \ \Omega$$

$$V = ZI = 10.32 \times 15 = 154.8 \text{ V}$$

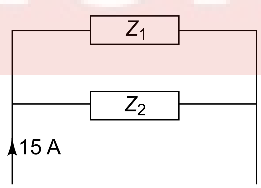


Fig. 4.59

Let $\bar{V} = 154.8 \angle 0° = 154.8 \text{ V}$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{154.8}{10 + j15} = 8.59 \angle -56.31° \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{154.8}{6 - j8} = 15.48\ \angle 53.13°\text{A}$$

$$P_1 = I_1^2 R_1 = (8.59)^2 \times 10 = 737.88\text{ W}$$

$$P_2 = I_2^2 R_2 = (15.48)^2 \times 6 = 1437.78\text{ W}$$

Example 4

A circuit consists of a 25 Ω resistor, 64 mH inductor and 80 μF capacitor connected in parallel across a 110 V, 50 Hz single-phase supply. Calculate the individual currents drawn by each element, the total current drawn from the supply and the overall power factor of the circuit. Draw the phasor diagram.

Solution

$$R = 25\ \Omega$$
$$L = 64\text{ mH}$$
$$C = 80\ \mu\text{F}$$
$$V = 110\text{ V}$$
$$f = 50\text{ Hz}$$

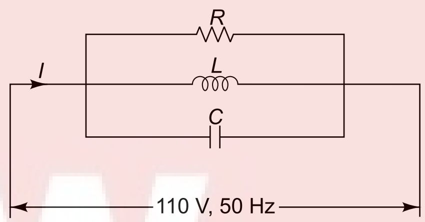


Fig. 4.60

(i) Individual currents

$$X_L = 2\pi f L = 2\pi \times 50 \times 64 \times 10^{-3} = 20.11\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.79\ \Omega$$

$$\bar{Z}_1 = R = 25\ \Omega$$

$$\bar{Z}_2 = jX_L = j\,20.11\ \Omega$$

$$\bar{Z}_3 = -jX_C = -j\,39.79\ \Omega$$

Let $\bar{V} = 110\ \angle 0° = 110\text{ V}$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{110}{25} = 4.4\ \angle 0°\text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{110}{j20.11} = 5.47\ \angle -90°\text{ A}$$

$$\bar{I}_3 = \frac{\bar{V}}{\bar{Z}_3} = \frac{110}{-j39.79} = 2.76\ \angle 90°\text{ A}$$

(ii) Total current

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = 4.4\ \angle 0° + 5.47\ \angle -90° + 2.76\ \angle 90°$$
$$= 5.17\ \angle -31.63°\text{ A}$$

(iii) Overall power factor

$$\text{pf} = \cos\phi = \cos(31.63°) = 0.851\text{ (lagging)}$$

(iv) Phasor diagram

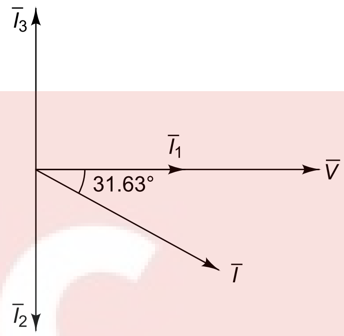


Fig. 4.61

Example 5

An ac circuit connected across a 200 V, 50 Hz, supply has two parallel branches A and B. Branch A draws a current of 4 A at 0.8 lagging power factor, while the total current drawn by the parallel combination is 5 A at unity power factor. Find (i) current and power factor of Branch B, and (ii) admittances of branches A and B, and their parallel combination both in polar and rectangular forms.

Solution

$$V = 200\text{ V},$$
$$f = 50\text{ Hz}$$
$$I_A = 4\text{ A}$$
$$\text{pf}_A = 0.8\text{ (lagging)}$$
$$I = 5\text{ A}$$
$$\text{pf} = 1$$

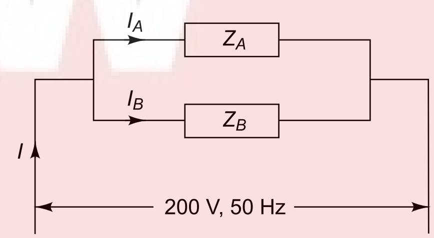


Fig. 4.62

(i) Current and power factor of Branch *B*

$$\phi_A = \cos^{-1}(0.8) = 36.87°$$
$$\phi = \cos^{-1}(1) = 0°$$
$$\bar{I}_A = I_A \angle -\phi_A = 4\angle -36.87°\text{ A}$$
$$\bar{I} = I\angle\phi = 5\angle 0°\text{ A}$$
$$\bar{I} = \bar{I}_A + \bar{I}_B$$
$$\bar{I}_B = \bar{I} - \bar{I}_A = 5\angle 0° - 4\angle -36.87° = 3\angle 53.13°\text{ A}$$
$$\text{pf}_B = \cos(53.13°) = 0.6\text{ (leading)}$$

(ii) Admittances of branches *A* and *B* and their parallel combination

Let $\bar{V} = 200\angle 0° = 200\text{ V}$

$$\bar{Y}_A = \frac{\bar{I}_A}{\bar{V}} = \frac{4\angle -36.87°}{200\angle 0°} = 0.02\angle -36.87°\ \mho$$

$$\bar{Y}_B = \frac{\bar{I}_B}{\bar{V}} = \frac{3\angle 53.13°}{200\angle 0°} = 0.015\ \angle 53.13°\ \mho$$

$$\bar{Y} = \bar{Y}_A + \bar{Y}_B = 0.02\ \angle{-36.87°} + 0.015\ \angle 53.13°\ \mho = 0.025\ \angle 0°\ \mho$$

Example 6

Two circuits A and B are connected in parallel to a 115 V, 50 Hz supply. The total current taken by the combination is 10 A at unity power factor. Circuit A consists of a 10 Ω resistor and 200 μF capacitor connected in series. Circuit B consists of a resistor and an inductor in series. Determine (i) current, (ii) power factor, (iii) impedance, (iv) resistance, and (v) reactance of the circuit B.

Solution

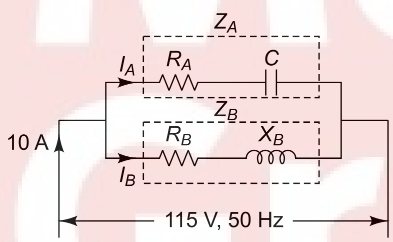


Fig. 4.63

$$V = 115 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$I = 10 \text{ A}$$
$$\text{pf} = 1$$
$$R_A = 10\ \Omega$$
$$C = 200\ \mu\text{F}$$

(i) Current I_B

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\ \Omega$$

$$\bar{Z}_A = 10 - j\,15.92 = 18.8\ \angle{-57.87°}\ \Omega$$

$$\bar{I}_A = \frac{\bar{V}}{\bar{Z}_A} = \frac{115}{18.8\angle -57.87°} = 6.12\ \angle 57.87°\ \text{A}$$

$$\bar{I} = 10\ \angle 0°\ \text{A}$$

$$\bar{I}_B = \bar{I} - \bar{I}_A = 10\ \angle 0° - 6.12\ \angle 57.87° = 8.5\angle{-37.54°}\ \text{A}$$

(ii) Power factor of Circuit *B*

$$\text{pf}_B = \cos(37.54°) = 0.79 \text{ (lagging)}$$

(iii) Impedance of Circuit *B*

$$Z_B = \frac{\bar{V}}{\bar{I}_B} = \frac{115}{8.5\angle -37.54} = 13.53\ \angle 37.54°\ \Omega = 10.73 + j\,8.24\ \Omega$$

(iv) Resistance of Circuit *B*

$$R_B = 10.73\ \Omega$$

(v) Reactance of Circuit *B*

$$X_B = 8.24\ \Omega$$

Example 7

Two circuits, the impedances of which are given by $Z_1 = (6 + j8)\ \Omega$ and $Z_2 = (8 - j6)\ \Omega$, are connected in parallel. If the applied voltage to the combination is 100 V, find (i) current and pf of each branch, (ii) overall current and pf of the combination, and (iii) power consumed by each impedance.

Solution

$$\bar{Z}_1 = 6 + j8\ \Omega$$

$$\bar{Z}_2 = 8 - j6\ \Omega$$

$$V = 100\ \text{V}$$

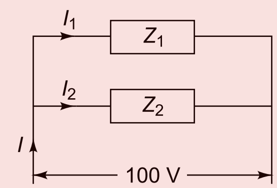


Fig. 4.64

(i) Current and pf of each branch

$$\bar{V} = 100\ \angle 0° = 100\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100}{6 + j8} = 10\ \angle -53.13°\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_1} = \frac{100}{8 - j6} = 10\ \angle 36.9°\ \text{A}$$

$$\cos\phi_1 = \cos(53.13°) = 0.6\ \text{(lagging)}$$

$$\cos\phi_2 = \cos(36.9°) = 0.8\ \text{(leading)}$$

(ii) Overall current and pf of the combination

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10\ \angle -53.13° + 10\ \angle 36.9° = 14.14\ \angle -8.13°\ \text{A}$$

$$\text{pf} = \cos\phi = \cos(8.13°) = 0.989\ \text{(lagging)}$$

(iii) Power consumed by each impedance

$$P_1 = I_1^2 R_1 = (10)^2 \times (6) = 600\ \text{W}$$

$$P_2 = I^2 R_2 = (10)^2 \times (8) = 800\ \text{W}$$

Example 8

Two impedances $R_1 - j\,X_{C_1}$ and $R_2 + jX_{L_2}$ are connected in parallel across a supply voltage $v = 100\sqrt{2}\ \sin 314\,t$. The current flowing through two impedances are $i_1 = 10\sqrt{2}\ \sin\left(314t + \frac{\pi}{4}\right)$ and $i_2 = 10\sqrt{2}\ \sin\left(314t - \frac{\pi}{4}\right)$ respectively. Find the equation for instantaneous value of total current drawn from the supply. Also find values of R_1, R_2, X_{C_1} and X_{L_2}.

Solution

$$\bar{Z}_1 = R_1 - j\,X_{C_1}$$

$$\bar{Z}_2 = R_2 + j\,X_{L_2}$$

$$v = 100\sqrt{2}\ \sin 314\,t$$

$$i_1 = 10\sqrt{2}\ \sin\left(314t + \frac{\pi}{4}\right)$$

$$i_2 = 10\sqrt{2}\ \sin\left(314t - \frac{\pi}{4}\right)$$

(i) Instantaneous value of total current

Writing v, i_1 and i_2 in polar form,

$$\bar{V} = 100\ \angle 0°\ \text{V}$$

$$\bar{I}_1 = 10\ \angle 45°\ \text{A}$$

$$\bar{I}_2 = 10\ \angle{-45°}\ \text{A}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 10\angle 45° + 10\angle{-45°} = 14.14\angle 0°\ \text{A}$$

$$i = I_m \sin 2\pi ft = 14.14\sqrt{2}\ \sin 314\,t = 20 \sin 314\,t$$

(ii) Values of R_1, R_2, X_{C_1} and X_{L_2}

$$\bar{Z}_1 = \frac{\bar{V}}{\bar{I}_1} = \frac{100\angle 0°}{10\angle 45°} = 10\ \angle{-45°} = 7.07 - j7.07\ \Omega$$

$$R_1 = 7.07\ \Omega$$

$$X_{C_1} = 7.07\ \Omega$$

$$\bar{Z}_2 = \frac{\bar{V}}{\bar{I}_2} = \frac{100\angle 0°}{10\angle -45°} = 10\ \angle 45° = 7.07 + j7.07\ \Omega$$

$$R_2 = 7.07\ \Omega$$

$$X_{L_2} = 7.07\ \Omega$$

Example 9

An impedance of (7 + j5) Ω is connected in parallel with another impedance of (10 – j8) Ω across a 230 V, 50 Hz supply. Calculate (i) admittance, conductance and susceptance of the combined circuit, and (ii) total current and power factor.

Solution

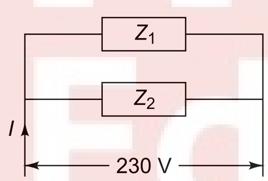


Fig. 4.65

$$\bar{Z}_1 = (7 + j5)\ \Omega$$

$$\bar{Z}_2 = (10 - j8)\ \Omega$$

$$V = 230\ \text{V}$$

(i) Admittance, conductance and susceptance of the combined circuit

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{7 + j5} = 0.12\angle{-35.54°}\ ℧$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10 - j8} = 0.08\angle 38.66°\ ℧$$

$$\overline{Y} = \overline{Y}_1 + \overline{Y}_2$$
$$= 0.12\angle{-35.54°} + 0.08\angle 38.66°$$
$$= 0.16\angle{-7.04°}\ \mho$$
$$= 0.16 - j0.02\ \mho$$
$$Y = 0.16\ \mho$$
$$G = 0.16\ \mho$$
$$B = 0.02\ \mho$$

(ii) Total current and power factor

$$\overline{I} = \overline{V}\,\overline{Y} = (230\angle 0°)(0.16\angle{-7.04°}) = 36.8\angle{-7.04°}\text{ A}$$
$$I = 36.8\text{ A}$$
$$\text{pf} = \cos\phi = \cos(7.04°) = 0.99\text{ (lagging)}$$

Example 10

Two impedances Z_1 and Z_2 are connected in parallel across a 200 V, 50 Hz ac supply. The current drawn by the impedance Z_1 is 4 A at 0.8 lagging pf. The total current drawn from the supply is 5 A at unity pf. Calculate the impedance Z_2.

Solution

$$V = 200\text{ V}$$
$$I_1 = 4\text{ A at 0.8 lagging pf}$$
$$I = 5\text{ A at unity pf}$$
$$\overline{I}_1 = 4\angle{-\cos^{-1}(0.8)} = 4\angle{-36.87°}\text{ A}$$
$$\overline{I} = 5\angle\cos^{-1}(1) = 5\angle 0°\text{ A}$$
$$\overline{I} = \overline{I}_1 + \overline{I}_2$$
$$\overline{I}_2 = \overline{I} - \overline{I}_1 = 5\angle 0° - 4\angle{-36.87°} = 3\angle 53.13°\text{ A}$$
$$\overline{Z}_2 = \frac{\overline{V}}{\overline{I}_2} = \frac{200\angle 0°}{3\angle 53.13°} = 66.67\angle{-53.13°}\ \Omega$$

Example 11

Compute Z_{eq} and Y_{eq} for the circuit is shown in Fig. 4.66.

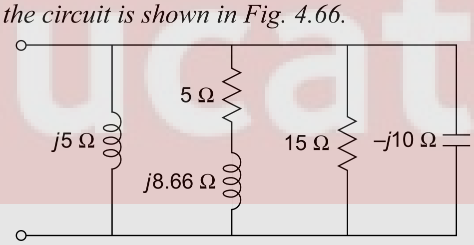


Fig. 4.66

Solution

$$\overline{Z}_1 = j5\ \Omega$$

$$\overline{Z}_2 = 5 + j8.66\ \Omega$$
$$\overline{Z}_3 = 15\ \Omega$$
$$\overline{Z}_4 = -j10\ \Omega$$
$$\overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3 + \overline{Y}_4$$
$$= \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3} + \frac{1}{\overline{Z}_4}$$
$$= \frac{1}{j5} + \frac{1}{5 + j8.66} + \frac{1}{15} + \frac{1}{-j10}$$
$$= 0.22\ \angle{-57.99°}\ \mho$$
$$\overline{Z}_{eq} = \frac{1}{\overline{Y}_{eq}} = \frac{1}{0.22\angle{-57.99°}} = 4.54\ \angle 57.99°\ \Omega$$

Example 12

Find currents I_1 and I_2 in Fig. 4.67.

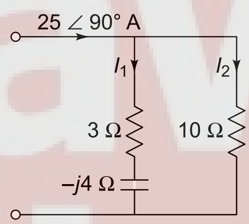


Fig. 4.67

Solution

$$\overline{Z}_1 = 3 - j4\ \Omega$$
$$\overline{Z}_2 = 10\ \Omega$$
$$\overline{I} = 25\ \angle 90°\ \text{A}$$

By current-division rule,

$$\overline{I}_1 = \overline{I}\,\frac{\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} = (25\ \angle 90°)\frac{10}{3 - j4 + 10} = 18.38\ \angle 107.1°\ \text{A}$$
$$\overline{I}_2 = \overline{I} - \overline{I}_1 = 25\angle 90° - 18.38\ \angle 107.1° = 9.19\ \angle 54°\ \text{A}$$

Example 13

Three impedances of 25 ∠53.1° Ω, 5 ∠–53.1° Ω and 10 ∠36.9° Ω are connected in parallel. The combination is in series with another impedance of 14.14 ∠45° Ω. Calculate the equivalent impedance of the circuit.

Solution

$$\overline{Z}_1 = 25\ \angle 53.1°\ \Omega$$
$$\overline{Z}_2 = 5\ \angle{-53.1}\ \Omega$$
$$\overline{Z}_3 = 10\ \angle 36.9°\ \Omega$$

$$\overline{Z}_4 = 14.14\ \angle 45°\ \Omega$$

$$\overline{Y} = \frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3} = \frac{1}{25\angle 53.1°} + \frac{1}{5\angle -53.1°} + \frac{1}{10\angle 36.9°}$$

$$= 0.23\ \angle 16.86°\ \mho$$

$$\overline{Z} = 4.27\ \angle -16.86°\ \Omega$$

$$\overline{Z}_{eq} = \overline{Z} + \overline{Z}_4 = 4.27\ \angle -16.86° + 14.14\ \angle 45° = 16.58\ \angle 31.87°\ \Omega$$

Example 14

A voltage of 200 ∠25° V is applied to a circuit composed of two parallel branches. If the branch currents are 10 ∠40° A and 20 ∠–30° A, determine the kVA, kVAR and kW in each branch. Also, calculate the pf of the combined load.

Solution

$$\overline{V} = 200\ \angle 25°\ \text{V}$$

$$\overline{I}_1 = 10\ \angle 40°\ \text{A}$$

$$\overline{I}_2 = 20\ \angle -30°\ \text{A}$$

Phase difference between V and I_1

$$\phi_1 = 40° - 25° = 15°$$

Phase difference between V and I_2

$$\phi_2 = 25° - (-30°) = 55°$$

$$\cos\phi_1 = \cos(15°) = 0.97 \text{ (leading)}$$

$$\cos\phi_2 = \cos(55°) = 0.57 \text{ (lagging)}$$

(i) kVA, kVAR and kW for the branch current of 10 ∠40° A

$$P_1 = VI_1 \cos\phi_1 = 200 \times 10 \times 0.97 = 1.94 \text{ kW}$$

$$Q_1 = VI_1 \sin\phi_1 = 200 \times 10 \times \sin(15°) = 0.52 \text{ kVAR}$$

$$S_1 = VI_1 = 200 \times 10 = 2 \text{ kVA}$$

(ii) kVA, kVAR and kW for the branch current of 20 ∠–30° A

$$P_2 = VI_2 \cos\phi_2 = 200 \times 20 \times 0.57 = 2.28 \text{ kW}$$

$$Q_2 = VI_2 \sin\phi_2 = 200 \times 20 \times \sin(55°) = 3.28 \text{ kVAR}$$

$$S_2 = VI_2 = 200 \times 20 = 4 \text{ kVA}$$

(iii) Power factor of the combined load

$$\overline{I}_1 = 10\ \angle 40°\ \text{A}$$

$$\overline{I}_2 = 20\ \angle -30°\ \text{A}$$

$$\overline{I} = \overline{I}_1 + \overline{I}_2 = 10\ \angle 40° + 20\ \angle -30° = 25.24\ \angle -8.14°\ \text{A}$$

$$\phi = 25° - (-8.14°) = 33.14°$$

$$\text{pf} = \cos(33.14°) = 0.84 \text{ (lagging)}$$

Example 15

The load taken from a supply consists of a (i) lamp load of 10 kW at unity power factor, (ii) motor load of 80 kVA at 0.8 power factor lagging, and (iii) motor load of 40 kVA at 0.7 power factor lagging. Calculate the total load taken from the supply in kW and in kVA and the power factor of the combined load.

Solution Lamp load: $P_1 = 10$ kW, $\text{pf}_1 = 1$

Motor load: $S_2 = 80$ kVA, $\text{pf}_2 = 0.8$ (lagging)

Motor load: $S_3 = 40$ kVA, $\text{pf}_3 = 0.7$ (lagging)

(i) Total load in kW

For motor loads,

$$P_2 = S_2 \times \text{pf}_2 = 80 \times 0.8 = 64 \text{ kW}$$

$$P_3 = S_3 \times \text{pf}_3 = 40 \times 0.7 = 28 \text{ kW}$$

$$P = P_1 + P_2 + P_3 = 10 + 64 + 28 = 102 \text{ kW}$$

(ii) Total load in kVA

For lamp load,

$$S_1 = \frac{P_1}{\text{pf}_1} = \frac{10}{1} = 10 \text{ kVA}$$

$$S = S_1 + S_2 + S_3 = 10 + 80 + 40 = 130 \text{ kVA}$$

(iii) Power factor of the combined load

$$\text{pf} = \frac{P}{S} = \frac{102}{130} = 0.785 \text{ (lagging)}$$

Example 16

Two circuits have the same numerical value of impedance. The pf of one is 0.8 lagging and that of the other is 0.6 lagging. What is the pf of combination if they are connected in parallel?

Solution $\text{pf}_1 = 0.8$ (lagging)

$\text{pf}_2 = 0.6$ (lagging)

$$\bar{Z}_1 = Z \angle\cos^{-1}(0.8) = Z \angle 36.87° \ \Omega$$

$$\bar{Z}_2 = Z \angle\cos^{-1}(0.6) = Z \angle 53.13° \ \Omega$$

For parallel combination,

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(Z\angle 36.87°)(Z\angle 53.13°)}{Z\angle 36.87° + Z\angle 53.13°}$$

$$= \frac{Z^2\angle 90^\circ}{Z(1.4 + j1.4)}$$

$$= \frac{Z^2\angle 90^\circ}{1.98Z\angle 45^\circ}$$

$$= 0.505\, Z\,\angle 45^\circ\ \Omega$$

$$\text{pf} = \cos(45^\circ) = 0.707$$

Example 17

When a 240 V, 50 Hz supply is fed to a 15 Ω resistor in parallel with an inductor, the total current is 22.1 A. What value must the frequency have for the total current to be 34 A?

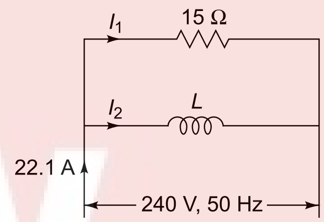


Fig. 4.68

Solution $V = 240$ V

$R = 15\ \Omega$

$I = 22.1$ A

Let $\bar{V} = 240\ \angle 0^\circ$ V

$$\bar{I}_1 = \frac{\bar{V}}{R} = \frac{240\angle 0^\circ}{15\angle 0^\circ} = 16\ \angle 0^\circ = 16 \text{ A}$$

$$\bar{I}_2 = \frac{\bar{V}}{jX_L} = \frac{240\angle 0^\circ}{X_L\angle 90^\circ} = \frac{240}{X_L}\ \angle -90^\circ = -j\,\frac{240}{X_L}\text{ A}$$

$$\bar{I} = 16 - j\,\frac{240}{X_L}$$

$$\sqrt{(16)^2 + \left(\frac{240}{X_L}\right)^2} = 22.1$$

$$256 + \frac{57600}{X_L^2} = 488.41$$

$$X_L = 15.74\ \Omega$$

$$X_L = 2\pi f L$$

$$15.74 = 2\pi \times 50 \times L$$

$$L = 0.05 \text{ H}$$

Let the new frequency be f for the total current of 34 A.

$$\sqrt{(16)^2 + \left(\frac{240}{2\pi f \times 0.05}\right)^2} = 34$$

$$256 + \frac{57600}{0.0987 f^2} = 1156$$

$$f = 25.47 \text{ Hz}$$

Example 18

Determine the current in the circuit of Fig. 4.69. Also, find the power consumed as well as pf.

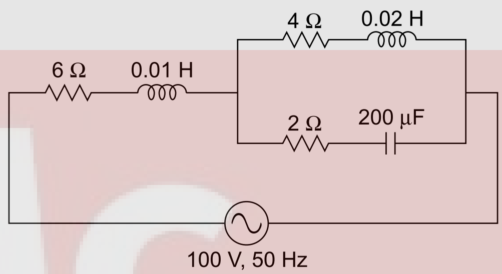


Fig. 4.69

Solution

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.01 = 3.14\ \Omega$$

$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\ \Omega$$

$$\overline{Z}_1 = 6 + j3.14\ \Omega$$

$$\overline{Z}_2 = 4 + j6.28\ \Omega$$

$$\overline{Z}_3 = 2 - j15.92\ \Omega$$

$$\overline{Z} = \overline{Z}_1 + \frac{\overline{Z}_2\,\overline{Z}_3}{\overline{Z}_2 + \overline{Z}_3}$$

$$= (6 + j3.14) + \frac{(4 + j6.28)(2 - j15.92)}{(4 + j6.28) + (2 - j15.92)} = 17.27\ \angle 30.75°\ \Omega$$

$$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{100\angle 0°}{17.27\angle 30.75°} = 5.79\ \angle -30.75°\ \text{A}$$

$$P = VI\cos\phi = 100 \times 5.79 \times \cos(30.75°) = 497.94\ \text{W}$$

$$\text{pf} = \cos\phi = \cos(30.75°) = 0.86\ \text{(lagging)}$$

Example 19

Two impedances $\overline{Z}_A = (4 + j3)\ \Omega$ and $\overline{Z}_B = (10 - j7)\ \Omega$ are connected in parallel and impedance $\overline{Z}_C = (6 + j5)\ \Omega$ is connected in series with parallel combination of $\overline{Z}_A$ and $\overline{Z}_B$. If the voltage applied across the circuit is 200 V at 59 Hz, calculate (i) currents flowing in Z_A, Z_B and Z_C, and (ii) total power factor of the circuit.

Solution

$$\overline{Z}_A = (4 + j3)\ \Omega$$

$$\overline{Z}_B = (10 - j7)\ \Omega$$

$$\overline{Z}_C = (6 + j5)\ \Omega$$

$$V = 200\ \text{V}$$

$$f = 59\ \text{Hz}$$

(i) Currents flowing in Z_A, Z_B and Z_C

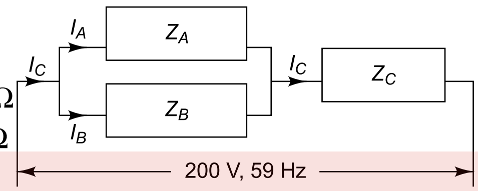


Fig. 4.70

$$\bar{Z}_A = 4 + j3 = 5\ \angle 36.87°\ \Omega$$

$$\bar{Z}_B = 10 - j7 = 12.21\ \angle -35°\ \Omega$$

$$\bar{Z}_C = 6 + j5 = 7.81\ \angle 39.81°\ \Omega$$

$$\bar{Z} = \frac{\bar{Z}_A \bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} + \bar{Z}_C$$

$$= \frac{(5\angle 36.87°)(12.21\angle -35°)}{5\angle 36.87° + 12.21\angle -35°} + 7.81\ \angle 39.81°$$

$$= 11.8\ \angle 32.17°\ \Omega$$

$$\bar{I}_C = \frac{\bar{V}}{\bar{Z}} = \frac{200\angle 0°}{11.8\angle 32.17°} = 16.95\ \angle -32.17°\ \text{A}$$

$$\bar{I}_A = \bar{I}_C\ \frac{\bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} = (16.95\angle -32.17°)\ \frac{(12.21\angle -35°)}{5\angle 36.87 + 12.21\angle -35°}$$

$$= 14.21\ \angle -51.21°\ \text{A}$$

$$\bar{I}_B = \bar{I}_C - \bar{I}_A = 16.95\ \angle -32.17° - 14.21\ \angle -51.21° = 5.82\ \angle 20.64\ \text{A}$$

(ii) Total power factor of the circuit

$$\text{pf} = \cos(32.17°) = 0.85\ \text{(lagging)}$$

Example 20

In a series–parallel circuit, the parallel branches A and B are in series with Branch C. The impedances are $Z_A = (4 + j3)\ \Omega$, $Z_B = \left(4 - j\frac{16}{3}\right)\ \Omega$ and $Z_C = (2 + j8)\ \Omega$. If the current $I_C = (25 + j0)$ A, determine the branch currents, voltages and the total voltage. Hence, calculate active and reactive powers for each branch and the whole circuit.

Solution

$$\bar{Z}_A = (4 + j3)\ \Omega$$

$$\bar{Z}_B = \left(4 - j\frac{16}{3}\right)\Omega$$

$$\bar{Z}_C = (2 + j8)\ \Omega$$

$$\bar{I}_C = (25 + j0) = 25\angle\ 0°\ \text{A}$$

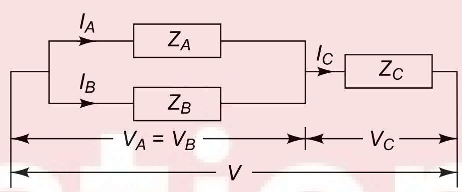


Fig. 4.71

(i) Branch currents

By current-division rule,

$$\bar{I}_A = \bar{I}_C\ \frac{\bar{Z}_B}{\bar{Z}_A + \bar{Z}_B} = (25 + j0)\frac{\left(4 - j\frac{16}{3}\right)}{4 + j3 + 4 - j\frac{16}{3}} = 20\ \angle -36.87°\ \text{A}$$

$$\bar{I}_B = \bar{I}_C - \bar{I}_A = 25\ \angle 0° - 20\ \angle -36.87° = 15\ \angle 53.13°\ \text{A}$$

(ii) Voltages and total voltage

$$\bar{V}_A = \bar{V}_B = \bar{Z}_A I_A = (4 + j3)\,(20 \angle{-36.87°}) = 100 \angle{-1.02°}\text{ V}$$
$$\bar{V}_C = \bar{Z}_C \bar{I}_C = (2 + j8)\,(25 \angle 0°) = 206.16 \angle 75.96°\text{V}$$
$$\bar{V} = \bar{V}_A + \bar{V}_C = 100 \angle{-1.02°} + 206.16 \angle 75.16° = 249.7 \angle 52.27°\text{ V}$$

(iii) Active and reactive powers for each branch and the whole circuit

$$P_A = I_A^2 R_A = (20)^2 \times 4 = 1600\text{ W}$$
$$P_B = I_B^2 R_B = (15)^2 \times 4 = 900\text{ W}$$
$$P_C = I_C^2 R_C = (25)^2 \times 2 = 1250\text{ W}$$
$$P = P_A + P_B + P_C = 1600 + 900 + 1250 = 3750\text{ W}$$
$$Q_A = I_A^2 X_A = (20)^2 \times 3 = 1200\text{ VAR}$$
$$Q_B = I_B^2 X_B = (15)^2 \times \frac{16}{3} = 1200\text{ VR}$$
$$Q_C = I_C^2 X_C = (25)^2 \times 8 = 5000\text{ VAR}$$
$$Q = Q_A + Q_B + Q_C = 1200 + 1200 + 5000 = 7400\text{ VAR}$$

Example 21

Find the applied voltage V_{AB} so that a 10 A current may flow through the capacitor in Fig. 4.72.

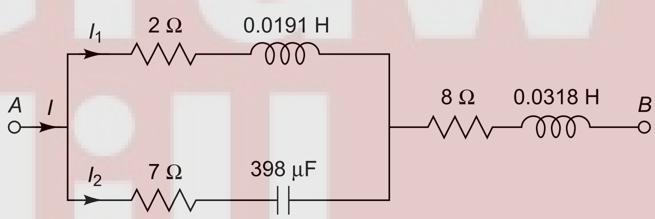


Fig. 4.72

Solution

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 0.0191 = 6\ \Omega$$
$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 0.0318 = 10\ \Omega$$
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 398 \times 10^{-6}} = 8\ \Omega$$
$$\bar{Z}_1 = 2 + j6 = 6.32 \angle 71.56°\ \Omega$$
$$\bar{Z}_2 = 7 - j8 = 10.63 \angle{-48.8°}$$
$$\bar{Z}_3 = 8 + j10 = 12.8 \angle 51.34°\ \Omega$$
$$\bar{Z} = \frac{\bar{Z}_1\,\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} + \bar{Z}_3 = \frac{(2 + j6)(7 - j8)}{(2 + j6 + 7 - j8)} + (8 + j10) = 19.91 \angle 45.53°\ \Omega$$

Let

$$\bar{I}_2 = 10 \angle 0°\text{ A}$$
$$\bar{V}_2 = \bar{Z}_2 \bar{I}_2 = (10.63 \angle{-48.8°})\,(10 \angle 0°) = 106.3 \angle{-48.8°}\text{ V}$$

$$\bar{V}_1 = \bar{V}_2 = 106.3\ \angle{-48.8°}\text{ V}$$

$$\bar{I}_1 = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{106.3\angle-48.8°}{6.32\angle 71.56°} = 16.82\ \angle{-120.36°}\text{ A}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 16.82\ \angle{-120.36°} + 10\angle 0° = 14.58\ \angle{-84.09°}\text{ A}$$

$$\bar{V}_{AB} = \bar{Z}\,\bar{I} = (19.91\ \angle 45.53°)(14.58\ \angle{-84.09°}) = 290.28\ \angle{-38.56°}\text{ V}$$

Example 22

If a voltage of 150 V applied between terminals A and B produces a current of 32 A for the circuit shown in Fig. 4.73, calculate the value of the resistance R and pf of the circuit.

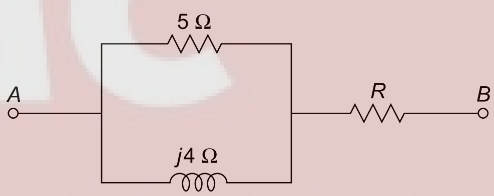


Fig. 4.73

Solution

$$V = 150\text{ V}$$

$$\bar{I} = 32\text{ A}$$

$$Z = \frac{150}{32} = 4.687\ \Omega$$

$$\bar{Z} = \frac{(5)(j4)}{5+j4} + R = \frac{(5)(4\angle 90°)}{64\angle 38.66°} + R$$

$$= 3.125\ \angle 51.34° + R = 1.95 + j2.44 + R$$

$$\sqrt{(1.95+R)^2 + (2.44)^2} = 4.687$$

$$(1.95 + R)^2 + (2.44)^2 = (4.687)^2$$

$$(1.95 + R)^2 = (4.687)^2 - (244)^2$$

$$1.95 + R = 4$$

$$R = 2.05\ \Omega$$

$$\text{pf} = \frac{\text{Total resistance}}{\text{Total impedance}} = \frac{1.95+2.05}{4.687} = 0.853\text{ (lagging)}$$

Example 23

An impedance of R + jX ohms is connected in parallel with another impedance of –j5 ohms. The combination is then connected in series with a pure resistance of 2 Ω. When connected across a 100 V, 50 Hz ac supply, the total current drawn by the circuit is 20 A and the total power consumed by the circuit is 2 kW. Calculate (i) currents through parallel branches, and (ii) R and L.

Solution $P = 2\text{ kW}$

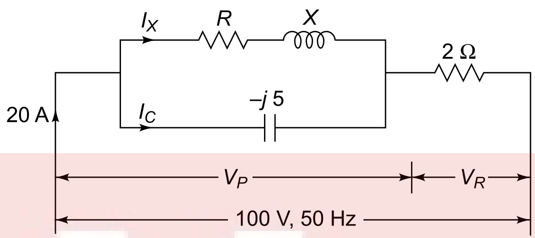


Fig. 4.74

$$V = 100 \text{ V}$$
$$I = 20 \text{ A}$$

(i) Currents through parallel branches

$$P = VI\cos\phi$$
$$2000 = 100 \times 20 \times \cos\phi$$
$$\cos\phi = 1$$
$$\phi = 0°$$

$$\bar{V} = 100\ \angle 0° \text{ V}$$
$$\bar{I} = 20\angle 0° \text{A}$$
$$\bar{V}_R = 2 \times 20°\ \angle 0° = 40\ \angle 0° \text{ V}$$
$$\bar{V}_P = \bar{V} - \bar{V}_R = 100\ \angle 0° - 40\angle 0° = 60\ \angle 0° \text{ V}$$
$$\bar{I}_C = \frac{\bar{V}_P}{\bar{Z}_C} = \frac{60\angle 0°}{5\angle -90°} = 12\ \angle 90° \text{ A}$$
$$\bar{I}_X = \bar{I} - \bar{I}_C = 20\angle 0° - 12\ \angle 90° = 23.32\ \angle -30.96° \text{ A}$$

(ii) Resistance and inductance

$$\bar{Z}_X = \frac{60\angle 0°}{23.32\angle -30.96°} = 2.57\ \angle 30.96°\ \Omega = 2.2 + j1.32\ \Omega$$
$$R = 2.2\ \Omega$$
$$X_L = 1.32\ \Omega$$
$$X_L = 2\pi f L$$
$$1.32 = 2\pi \times 50 \times L$$
$$L = 4.2 \text{ mH}$$

Example 24

The circuit of Fig. 4.75 takes 12 A at a lagging power factor and dissipates 1800 W. The reading of the voltmeter is 200 V. Find R_1, X_1 and X_2.

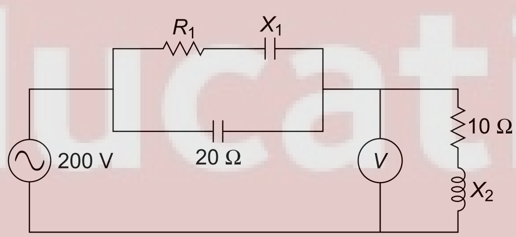


Fig. 4.75

Solution

$$I = 12 \text{ A}$$
$$P = 1800 \text{ W}$$

$$\text{Let} \qquad \bar{I} = 12\,\angle 0°\text{A}$$

$$Z_2 = \frac{200}{12} = 16.67\ \Omega$$

$$Z_2 = \sqrt{R_2^2 + X_2^2}$$

$$16.67 = \sqrt{(10)^2 + X_2^2}$$

$$(16.67)^2 = 10^2 + X_2^2$$

$$277.88 = 100 + X_2^2$$

$$X_2 = 13.33\ \Omega$$

$$\bar{V}_2 = (12\,\angle 0°)\,(10 + j13.33) = (12\,\angle 0°)\,(16.67\,\angle 53.13°) = 200\,\angle 53.13°\ \text{V}$$

$$P = VI\cos\phi$$

$$1800 = 200 \times 12 \times \cos\phi$$

$$\cos\phi = 0.75$$

$$\phi = 41.41°$$

Applied voltage $\bar{V}_{\text{req}} = 200\,\angle 41.41°$ V

Voltage across parallel branches $= 200\,\angle 41.41° - 200\,\angle 53.13° = 40.84\,\angle -42.73°$ V

Current through capacitor $= \dfrac{40.84\angle -42.73°}{20\angle -90°} = 2.04\,\angle 47.27°$ A

Current through R_1 and X_1 $= 12\,\angle 0° - 2.04\angle 47.27° = 10.72\angle{-8.03°}$ A

$$\bar{Z}_1 = \frac{40.84\angle -42.73°}{10.72\angle -8.03°} = 3.81\,\angle{-34.7°}\ \Omega = 3.13 - j2.17\ \Omega$$

$$R_1 = 3.13\ \Omega$$

$$X_1 = 2.17\ \Omega$$

Example 25

For the circuit shown in Fig. 4.76, calculate (i) total admittance, total conductance and total susceptance, (ii) total current and total pf, and (iii) value of pure capacitance to be connected in parallel with the above combination to make the total pf unity.

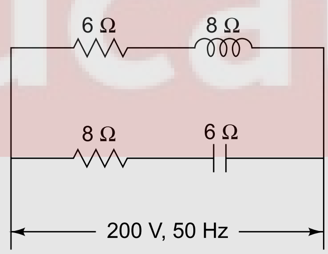


Fig. 4.76

Solution

(i) Total admittance, total conductance and total susceptance

$$\bar{Z}_1 = 6 + j8 = 10 \angle 53.13° \ \Omega$$

$$\bar{Z}_2 = 8 - j6 = 10 \angle -36.87° \ \Omega$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{10\angle 53.13°} = 0.1 \angle -53.13° \ \mho$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10\angle -36.87°} = 0.1\angle 36.87° \ \mho$$

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2$$

$$= 0.1 \angle -53.13° + 0.1 \angle 36.87° = 0.14 - j0.02 \ \mho = 0.14 \angle -8.13° \ \mho$$

$$Y = 0.14 \ \mho$$

$$G = 0.14 \ \mho$$

$$B = 0.02 \ \mho$$

(ii) Total current and total pf

Let $\bar{V} = 200 \angle 0° \text{ V}$

$$I = \bar{V}\,\bar{Y} = (200 \angle 0°)\,(0.14 \angle -8.13°) = 28 \angle -8.13° \text{ A}$$

$$\text{pf} = \cos(8.13°) = 0.989 \text{ (lagging)}$$

(iii) Value of pure capacitance to make total pf unity

Since the current lags behind voltage, the circuit is inductive in nature. In order to make the total pf unity, a pure capacitor is connected in parallel so that pf becomes unity and imaginary part of $\bar{Y}_{req}$ becomes zero.

$$\bar{Y}_{req} = Y_1 + Y_2 + Y_3$$

$$\bar{Y}_{req} = 0.14 - j0.02 + j0.02 = 0.14$$

$$Y_3 = Y_C = \frac{1}{X_C} = 0.02$$

$$X_C = 50 \ \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 50} = 63.66 \ \mu\text{F}$$

Exercise 4.3

4.1 Two impedances of $14 + j5\ \Omega$ and $18 + j10\ \Omega$ are connected in parallel across a 200 V, 50 Hz supply. Determine (i) admittance of each branch and the entire circuit, (ii) current in each branch and total current, (iii) power and power factor of each branch, (iv) total power factor, and (v) draw phasor diagram.

[0.067 $\angle -19.6°$ ℧, 0.048 $\angle -29.05°$ ℧, 0.115 $\angle -23.75°$ ℧, 13.45 $\angle -19.65°$ A, 9.6 $\angle -29.05°$ A, 22.91 $\angle -23.75°$ A, 2532.64 W, 1658.8 W, 0.94 (lagging), 0.874 (lagging), 0.916 (lagging)]

4.2 In a parallel *RC* circuit shown in Fig. 4.77, $i_R = 15 \cos (5000\,t - 30°)$ amperes. Obtain the current in capacitance.

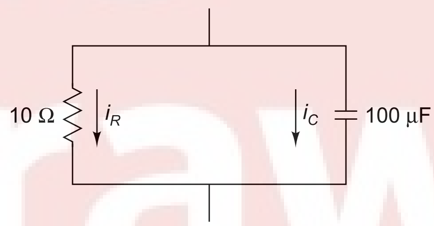


Fig. 4.77

[74.95 sin (5000 t + 150°)]

4.3 A voltage applied across a three-branch circuit is shown by $v = 100 \sin (5000\,t + \pi/4)$ in Fig. 4.78. Find the rms value of the current in the inductor.

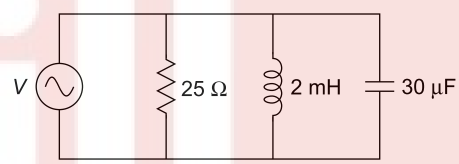


Fig. 4.78

[7.07 A]

4.4 A circuit comprises of a conductance *G* in parallel with a susceptance *B*. Calculate the admittance $G + jB$ if the impedance is $10 + j5\Omega$. *[0.08 S, 0.04 S]*

4.5 If an admittance of a circuit is $(8 + j6)$ ℧ and the circuit current is $2 \angle 30°$A, find the pf of the circuit. Also, calculate the apparent power in the circuit.

[0.8 (leading), 0.2 VA]

4.6 A resistor of 50 Ω, an inductor of 0.15 H and a capacitor of 100 μF are connected in parallel across a 100 V, 50 Hz ac supply. Calculate (i) current in each circuit, (ii) resultant current. Draw individual phasor diagrams and the overall phasor diagram. *[2 $\angle 0°$ A, 2.12 $\angle -90°$ A, 3.14 $\angle 90°$ A, 2.25 $\angle 27.02°$ A]*

4.7 The power dissipated in the coil A is 300 W and in the coil B is 400 W. Each coil takes a current of 5 A when connected to a 110 V, 50 Hz supply. Find the current drawn when the coils are connected in parallel. *[9.93 ∠–50.11° A]*

4.8 Find the total impedance, supply current and pf of the entire circuit.

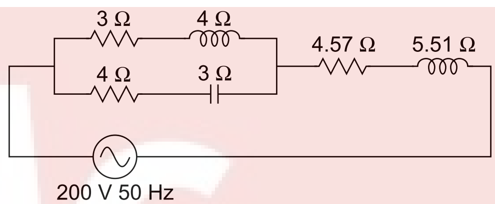


Fig. 4.79

[10.06 ∠ 36.68° Ω, 19.88 ∠ –36.68° A, 0.8 (lagging)]

4.9 Determine kVA, kVAR and kW consumed by the two impedances $\overline{Z}_1 = (20 + j37.7)\ \Omega$ and $\overline{Z}_2 = (50 + j0)\ \Omega$, when connected in parallel across a 230 V, 50 Hz supply. *[1.971 kVA, 1.095 kVAR, 1.64 kW]*

4.10 In the parallel circuit shown in Fig. 4.80, the power in the 3 Ω resistor is 666 W and the total volt-amperes taken by the circuit is 3370 VA. The power factor of the whole circuit is 0.937 leading. Find *Z*.

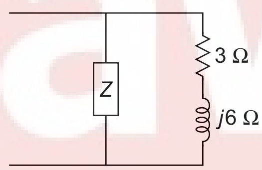


Fig. 4.80

[(2 – j2) Ω]

4.11 A coil is connected across a non-inductive resistor of 120 Ω. When a 240 V, 50 Hz supply is applied to this circuit, the coil draws a current of 5 A and the total current is 6 A. Determine the power and the power factor of (i) the coil, and (ii) the whole circuit. *[420 W, 0.35 (lagging), 900 W, 0.625 (lagging)]*

4.12 A coil takes a current of 10 A and dissipates 1410 W when connected to 220 V, 50 Hz supply. If another coil is connected in parallel with it, the total current taken from the supply is 20 A at a power factor of 0.868. Determine the current and the overall power factor when the coils are connected in series across the same supply. *[5.55 A, 0.8499 (lagging)]*

4.13 Draw an impedance triangle between terminals *AB* in Fig. 4.81, labelling its sides with appropriate values and units.

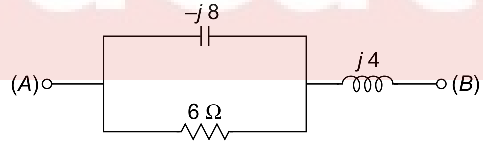


Fig. 4.81

[R = 3.84 Ω, X = 1.12 Ω (inductive), Z = 4 Ω]

4.14 Draw the impedance triangle between terminals *AB* in Fig. 4.82 for the following conditions:

(i) $X_C = 8\ \Omega$, $R = 6\ \Omega$, $X_L = 4\ \Omega$ (ii) $X_C = 4\ \Omega$, $R = 6\ \Omega$, $X_L = 6\ \Omega$

Label its side with appropriate values and units.

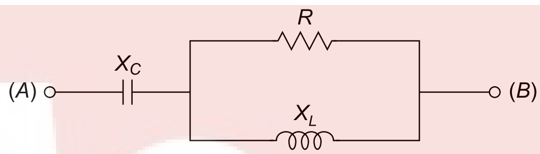


Fig. 4.82

[R = 1.8461 Ω, X = 5.2308 Ω (capacitive), Z = 5.547 Ω, R = 3 Ω, X = 1 Ω (capacitive), Z = 3.1623 Ω]

4.15 Draw an admittance triangle between terminals *AB* in Fig. 4.83, labelling its sides with appropriate values and units in case of

(i) $X_L = 4\ \Omega$ and $X_C = 8\ \Omega$ (ii) $X_L = 10\ \Omega$ and $X_C = 5\ \Omega$

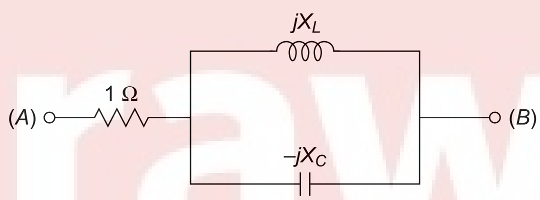


Fig. 4.83

[G = 0.015 ℧, B_L = 0.123 ℧, Y = 0.124 ℧, G = 0.0098 ℧, B_C = 0.099 ℧, Y = 0.1 ℧]

4.16 An inductive impedance *BC* is connected in series with a parallel combination *AB* consisting of a capacitor and a non-inductive resistor. The circuit constants are so adjusted that the current in the parallel branches *AB* are equal, and that the voltage across *AB* is equal to and in quadrature with the voltage across *BC*. When a voltage of 200 V is applied to *AC*, the total power absorbed is 1200 W. Calculate the circuit constants and draw a vector diagram.

[AB = R = 33.32 Ω in parallel with X_C = 33.32 Ω, BC = (16.66 + j16.66) Ω]

4.17 A 100 Ω resistor, shunted by a 0.4 H inductor is in series with a capacitor *C*. A voltage of 250 V at 50 Hz is applied to the circuit. Find

(i) the value of *C* to give unity power factor
(ii) the total current
(iii) current in the inductive branch *[65.4 µF, 4.08 A, 2.55 A]*

4.18 A non-inductive 10 Ω resistor is in series with a coil of 1.3 Ω resistance and 0.018 H inductance. If a voltage of maximum value of 100 V at a frequency of 100 Hz is applied to this circuit, what will be the voltage across the resistor? *[62.54 V]*

4.19 Two impedances $\bar{Z} = 10 - j15\ \Omega$ and $\bar{Z} = 4 + j8\ \Omega$ are connected in parallel. The supply voltage is 100 V, 25 Hz. Calculate (i) the admittance, conductance and susceptance of the combined circuit, and (ii) total current drawn and pf.

[0.097 ℧, 0.081 ℧, 0.054 ℧, 9.7 A, 0.83 (lagging)]

4.20 A voltage of $200 \angle 53.13°$ V is applied across two impedances in parallel. The values of the impedances are $(12 + j16)\ \Omega$ and $(10 - j20)\ \Omega$. Determine kVA, kVAR and kW in each branch and the pf of the whole circuit.

[2 kVA, 1.2 kW, 1.6 kVAR, 1.788 kVA, 0.8 kW, 1.6 kVAR, unity pf]

4.21 For the circuit shown in Fig. 4.84, evaluate the current through and voltage across each element.

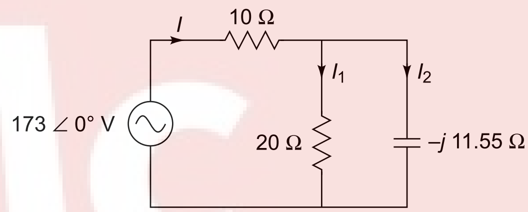


Fig. 4.84

[$\bar{I}_1 = 5 \angle -30°$ A, $\bar{I}_2 = 8.66 \angle 60°$ A, $\bar{V}_1 = 100 \angle -30°$ V, $\bar{V}_2 = 100 \angle -30°$ V, $V_{10\ \Omega} = 100 \angle 30°$ V]

4.22 Two impedances Z_1 and Z_2 are connected in parallel. The first branch takes a leading current of 16 A and has a resistance of 5 Ω, while the second branch takes a lagging current at a pf of 0.8. The total power supplied is 5 kW, the applied voltage being $(100 + j200)$ V. Determine branch currents and total current.

[$16 \angle 132.46°$ A, $20.8 \angle 26.57°$ A, 22.49 A]

4.23 For the parallel branch shown in Fig. 4.85, find the value of R_2 when the overall power factor is 0.92 lag.

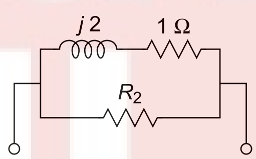


Fig. 4.85

[1.35 Ω]

4.24 In a series–parallel circuit, two parallel branches A and B are in series with C. The impedances are $Z_A = (10 + j8)\ \Omega$, $Z_B = (9 - j6)\ \Omega$ and $Z_C = (3 + j2)\ \Omega$. If the voltage across Z_C is $100 \angle 0°$ V, determine the values I_A and I_B.

[$15.7 \angle -73.39°$ A, $18.59 \angle -1.04°$ A]

4.25 A capacitor is placed in parallel with two inductive loads. The current through the first inductor is 20 A at 30° lag and the current through the second is 40 A at 60° lag. What must be the current in the capacitor so that the current in the external circuit is of unity power factor? *[$44.64 \angle 90°$A]*

4.26 A circuit of 15 Ω resistance and 12 Ω inductive reactance is connected in parallel with another circuit consisting of a resistor of 25 Ω in series with a capacitive reactance of 17 Ω. This combination is energized from a 200 V, 40 Hz mains. Find the branch currents, total current and power factor of the circuit. It is desired to raise the power factor of this circuit to unity by connecting a capacitor in parallel. Determine the value of the capacitance of the capacitor.

[10.42 ∠– 38.56° A, 6.61 ∠34.21° A, 13.95 ∠– 11.56° A, 0.98 (lagging) 54.9 μF]

4.27 A resistor of 30 Ω and a capacitor of unknown value are connected in parallel across a 110 V, 50 Hz supply. The combination draws a current of 5 A form the supply. Find the value of the unknown capacitance of the capacitor. This combination is again connected across a 110 V supply of unknown frequency. It is observed that the total current drawn from the mains falls to 4 A. Determine the frequency of the supply. *[98.58 μF, 23.68 Hz]*

4.28 Two reactive circuits have an impedance of 20 Ω each. One of them has a lagging power factor of 0.8 and the other has a leading power factor of 0.6. Find (i) voltage necessary to send a current of 10 A through the two in series, and (ii) current drawn from 200 V supply if the two are connected in parallel. Draw a phasor diagram in each case. *[282.8 V, 14.14 A]*

4.29 Inductor loads of 0.8 kW and 1.2 kW at lagging power factors of 0.8 and 0.6 respectively are connected across a 200 V, 50 Hz supply. Find the total current, power factor and the value of the capacitor to be put in parallel to both to raise the overall power factor to 0.9 lagging. *[14.87 A, 0.673 (lagging), 98 μF]*

4.8 SERIES RESONANCE

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series *R-L-C* circuit as shown in Fig. 4.86. The impedance of the circuit is

$$\bar{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

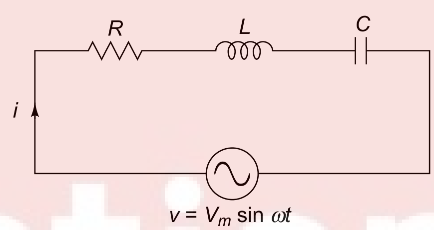


Fig. 4.86 *Series circuit*

At resonance, Z must be resistive. Therefore, the condition for resonance is

$$\omega L - \frac{1}{j\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Power Factor

$$\text{Power factor} = \cos\phi = \frac{R}{Z}$$

At resonance $\quad Z = R$

$$\text{Power factor} = \frac{R}{R} = 1$$

Current Since impedance is minimum, the current is maximum at resonance. Thus, the circuit accepts more current and as such, an *R-L-C* circuit under resonance is called an *acceptor circuit*.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

Voltage At resonance,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

Thus, potential difference across inductor equal to potential difference across capacitor being equal and opposite cancel each other. Also, since I_0 is maximum, V_{L_0} and V_{C_0} will also be maximum. Thus, voltage magnification takes place during resonance. Hence, it is also referred to as voltage magnification circuit.

Phasor Diagram

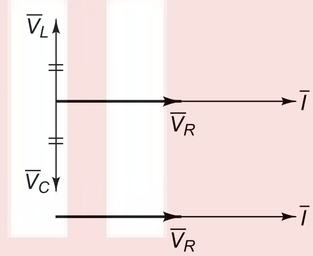


Fig. 4.87 *Phasor diagram*

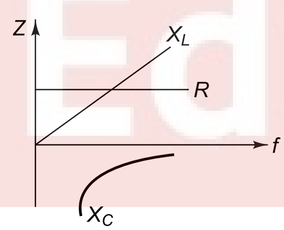


Fig. 4.88 *Behaviour of R, L and C with change in frequency*

Behaviour of R, L and C with Change in Frequency Resistance remains constant with the change in frequencies. Inductive reactance X_L is directly proportional to frequency f. It can be drawn as a straight line passing through the origin. Capacitive reactance X_C is inversely proportional to the frequency f. It can be drawn as a rectangular hyperbola in the fourth quadrant.

Total impedance $\overline{Z} = R + j\,(X_L - X_C)$

(a) When $f < f_0$, impedance is capacitive and decreases up to f_0. The power factor is leading in nature.

(b) At $f = f_0$, impedance is resistive. The power factor is unity.

(c) When $f > f_0$, impedance is inductive and goes on increasing beyond f_0. The power factor is lagging in nature.

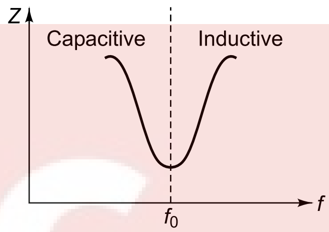


Fig. 4.89 *Impedance*

Bandwidth For the series *R-L-C* circuit, bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at resonance. From the shape of the resonance curve, it is clear that there are two frequencies for which the power delivered to R is half the power at resonance. For this reason, these frequencies are referred as those corresponding to the half-power points. The magnitude of the current at each half-power point is the same.

Hence, $$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

where the subscript 1 denotes the lower half point and the subscript 2, the higher half point. It follows then that

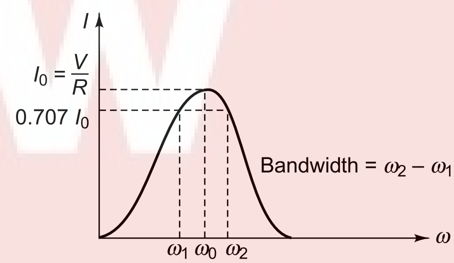


Fig. 4.90 *Resonance curve*

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

Accordingly, the bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of the current is equal to or greater than 0.707 of the current at resonance. In Fig. 4.90, the bandwidth is $\omega_2 - \omega_1$.

Expression for Bandwidth Generally, at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad (4.1)$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But $$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \tag{4.2}$$

From Eqs (4.1) and (4.2),

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

$$\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}R}$$

Squaring both the sides,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{R}{L}\omega - \frac{1}{LC} = 0$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low values of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\frac{1}{LC}$.

Then ω is given by, $\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$

The resonant frequency for this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm \frac{R}{2L} + \omega_0 \qquad \text{(considering only positive sign of } \omega_0\text{)}$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

and $$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

and $$f_2 = f_0 + \frac{R}{4\pi L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

or $$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

Quality Factor It is a measure of voltage magnification in the series resonant circuit. It is also a measure of selectivity or sharpness of the series resonant circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}}$$

$$= \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substituting values of V_{L_0} and V,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R}$$

$$= \frac{X_{L_0}}{R}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Example 1

A series R-L-C circuit has the following parameter values: R = 10 Ω, L = 0.01 H, C = 100 μF. Compute the resonant frequency, bandwidth, and lower and upper frequencies of the bandwidth.

Solution $R = 10\ \Omega$

$L = 0.01$ H

$C = 100\ \mu$F

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}} = 159.15\,\text{Hz}$$

(ii) Bandwidth

$$BW = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.01} = 159.15\,\text{Hz}$$

(iii) Lower frequency of bandwidth

$$f_1 = f_0 - \frac{BW}{2} = 159.15 - \frac{159.15}{2} = 79.58\text{ Hz}$$

(iv) Upper frequency of bandwidth

$$f_2 = f_0 + \frac{BW}{2} = 159.15 + \frac{159.15}{2} = 238.73\text{ Hz}$$

Example 2

An R-L-C series circuit with a resistance of 10 Ω, inductance of 0.2 H and a capacitance of 40 μF is supplied with a 100 V supply at variable frequency. Find the following w.r.t. the series resonant circuit:

(i) frequency at which resonance takes place
(ii) current
(iii) power
(iv) power factor
(v) voltage across R-L-C at that time
(vi) quality factor
(vii) half-power points
(viii) resonance and phasor diagrams

Solution $R = 10\ \Omega$

$L = 0.2$ H

$C = 40\ \mu$F

$V = 100$ V

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 40 \times 10^{-6}}} = 56.3\,\text{Hz}$$

(ii) Current

$$I_0 = \frac{V}{R} = \frac{100}{10} = 10\,\text{A}$$

(iii) Power

$$P_0 = I_0^2 R = (10)^2 \times 10 = 1000 \text{ W}$$

(iv) Power factor

$$\text{pf} = 1$$

(v) Voltage across R, L, C

$$V_{R_0} = R\,I_0 = 10 \times 10 = 100 \text{ V}$$

$$V_{L_0} = X_{L_0} I_0 = 2\pi f_0 L I_0 = 2\pi \times 56.3 \times 0.2 \times 10 = 707.5 \text{ V}$$

$$V_{C_0} = X_{C_0} I_0 = \frac{1}{2\pi f_0 C} I_0 = \frac{1}{2\pi \times 56.3 \times 40 \times 10^{-6}} \times 10 = 707.5 \text{ V}$$

(vi) Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.2}{40 \times 10^{-6}}} = 7.07$$

(vii) Half-power points

$$f_1 = f_0 - \frac{R}{4\pi L} = 56.3 - \frac{10}{4\pi \times 0.2} = 52.32 \text{ Hz}$$

$$f_2 = f_0 + \frac{R}{4\pi L} = 56.3 + \frac{10}{4\pi \times 0.2} = 60.3 \text{ Hz}$$

(viii) Resonance and phasor diagram

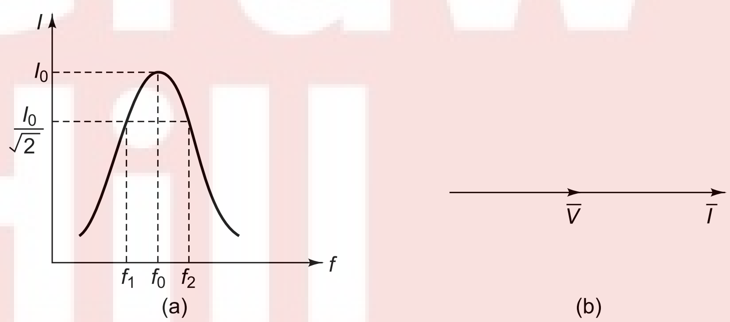


Fig. 4.91

Example 3

A series R-L-C circuit is connected to a 200 V ac supply. The current drawn by the circuit at resonance is 20 A. The voltage drop across the capacitor is 5000 V at series resonance. Calculate resistance and inductance if capacitance is 4 μF. Also, calculate the resonant frequency.

Solution

$$V = 200 \text{ V}$$

$$I_0 = 20 \text{ A}$$

$$V_{C_0} = 5000 \text{ V}$$

$$C = 4 \text{ μF}$$

(i) Resistance

$$R = \frac{V}{I_0} = \frac{200}{20} = 10\ \Omega$$

(ii) Resonant frequency

$$X_{C_0} = \frac{V_{C_0}}{I_0} = \frac{5000}{20} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times f_0 \times 4 \times 10^{-6}}$$

$$f_0 = 159.15\ \text{Hz}$$

(iii) Inductance

At resonance $X_{C_0} = X_{L_0} = 250\ \Omega$

$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \times 159.15 \times L$$

$$L = 0.25\ \text{H}$$

Example 4

A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 230 V, 50 Hz supply, the maximum current obtained by varying the inductance is 2 A. The voltage across the capacitor is 500 V. Calculate the resistance, inductance and capacitance of the circuit.

Solution

$$V = 230\ \text{V}$$

$$f_0 = 50\ \text{Hz}$$

$$I_0 = 2\ \text{A}$$

$$V_{C_0} = 500\ \text{V}$$

(i) Resistance

$$R = \frac{V}{I_0} = \frac{230}{2} = 115\ \Omega$$

(ii) Capacitance

$$X_{C_0} = \frac{V_{C_0}}{I_0} = \frac{500}{2} = 250\ \Omega$$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$250 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 12.73\ \mu\text{F}$$

(iii) Inductance

At resonance $X_{C_0} = X_{L_0} = 250\ \Omega$

$$X_{L_0} = 2\pi f_0 L$$

$$250 = 2\pi \times 50 \times L$$

$$L = 0.795 \text{ H}$$

Example 5

A coil of 2 Ω resistance and 0.01 H inductance is connected in series with a capacitor across 200 V mains. What must be the capacitance in order that maximum current occurs at a frequency of 50 Hz? Find also the current and voltage across the capacitor.

Solution

$$R = 2\ \Omega$$

$$L = 0.01 \text{ H}$$

$$V = 200 \text{ V}$$

$$f_0 = 50 \text{ Hz}$$

(i) Capacitance

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$50 = \frac{1}{2\pi\sqrt{0.01 \times C}}$$

$$C = 1013.2\ \mu\text{F}$$

(ii) Current

$$I_0 = \frac{V}{R} = \frac{200}{2} = 100 \text{ A}$$

(iii) Voltage across capacitor

$$V_{C_0} = I_0\, X_{C_0} = I_0 \frac{1}{2\pi f_0 C} = 100 \times \frac{1}{2\pi \times 50 \times 1013.2 \times 10^{-6}} = 314.16 \text{ V}$$

Example 6

A voltage v(t) = 10 sin ωt is applied to a series R-L-C circuit. At the resonant frequency of the circuit, the voltage across the capacitor is found to be 500 V. The bandwidth of the circuit is known to be 400 rad/s and the impedance of the circuit at resonance is 100 Ω. Determine inductance and capacitance resonant frequency, upper and lower cut-off frequencies.

Solution

$$v(t) = 10 \sin \omega t$$

$$V_{C_0} = 500 \text{ V}$$

$$BW = 400 \text{ rad/s}$$

$$R = 100\ \Omega$$

(i) Inductance and capacitance

$$V = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ V}$$

$$I_0 = \frac{V}{R} = \frac{7.07}{100} = 0.0707 \text{ A}$$

$$BW = \frac{R}{L}$$

$$400 = \frac{100}{L}$$

$$L = 0.25 \text{ H}$$

$$Q_0 = \frac{V_{C0}}{V} = \frac{500}{7.07} = 70.72$$

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$70.72 = \frac{1}{100}\sqrt{\frac{0.25}{C}}$$

$$C = 4.99 \text{ nF}$$

(ii) Resonant frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0.25 \times 4.99 \times 10^{-9}}} = 4506.09 \text{ Hz}$$

(iii) Lower cut-off frequency

$$f_1 = f_0 - \frac{R}{4\pi L} = 4506.09 - \frac{100}{4\pi \times 0.25} = 4474.26 \text{ Hz}$$

(iv) Upper cut-off frequency

$$f_2 = f_0 + \frac{R}{4\pi L} = 4506.09 + \frac{100}{4\pi \times 0.25} = 4537.92 \text{ Hz}$$

Example 7

A series resonant circuit has an impedance of 500 Ω at resonant frequency. Cut-off frequencies are 10 kHz and 100 Hz. Determine (i) resonant frequency, (ii) value of L, C, and (iii) quality factor at resonant frequency.

Solution

$$R = 500\ \Omega$$

$$f_1 = 100 \text{ Hz}$$

$$f_2 = 10 \text{ kHz}$$

(i) Resonant frequency

$$BW = f_2 - f_1 = 10000 - 100 = 9900 \text{ Hz}$$

$$f_1 = f_0 - \frac{R}{4\pi L} \qquad (1)$$

$$f_2 = f_0 + \frac{R}{4\pi L} \qquad (2)$$

Adding Eqs (1) and (2),

$$f_1 + f_2 = 2f_0$$

$$f_0 = \frac{f_1 + f_2}{2} = \frac{100 + 10000}{2} = 5050 \text{ Hz}$$

(ii) Values of L and C

$$BW = \frac{R}{2\pi L}$$

$$9900 = \frac{500}{2\pi L}$$

$$L = 8.038 \text{ mH}$$

$$X_{L_0} = 2\pi f_0 L = 2\pi \times 5050 \times 8.038 \times 10^{-3} = 255.05\ \Omega$$

At resonance $\quad X_{L_0} = X_{C_0} = 255.05\ \Omega$

$$X_{C_0} = \frac{1}{2\pi f_0 C}$$

$$255.05 = \frac{1}{2\pi \times 5050 \times C}$$

$$C = 0.12\ \mu\text{F}$$

(iii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{500}\sqrt{\frac{8.038 \times 10^{-3}}{0.12 \times 10^{-6}}} = 0.5176$$

Example 8

Impedance of a circuit is observed to be capacitive and decreasing from 1 Hz to 100 Hz. Beyond 100 Hz, the impedance starts increasing. Find the values of circuit elements if the power drawn by this circuit is 100 W at 100 Hz, when the current is 1 A. The power factor of the circuit at 70 Hz is 0.707.

Solution $\quad f_0 = 100 \text{ Hz}$

$$P_0 = 100 \text{ W}$$

$$I_0 = 1 \text{ A}$$

$$(\text{pf})_{70 \text{ Hz}} = 0.707$$

The impedance of the circuit is capacitive and decreasing from 1 Hz to 100 Hz. Beyond 100 Hz, the impedance starts increasing.

$$f_0 = 100 \text{ Hz}$$

$$P_0 = I_0^2 R$$

$$100 = (1)^2 \times R$$

$$R = 100\ \Omega$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$100 = \frac{1}{2\pi\sqrt{LC}}$$

$$LC = 2.53 \times 10^{-6} \qquad (1)$$

Power factor at 70 Hz is 0.707.

$$\frac{R}{Z} = 0.707$$

$$\frac{100}{Z} = 0.707$$

$$Z = 141.44\ \Omega$$

$$\text{Impedance at 70 Hz} = Z_{70} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$141.44 = \sqrt{(100)^2 + \left(\frac{1}{2\pi \times 70 \times C} - 2\pi \times 70 \times L\right)^2}$$

$$\frac{2.27 \times 10^{-3}}{C} - 439.82\,L = 100.02 \qquad (2)$$

Solving Eqs (1) and (2),

$$L = 0.2187 \text{ H}$$

$$C = 11.58\ \mu\text{F}$$

Example 9

A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set to 500 pF, the current has its maximum value while it is reduced to one-half when the capacitance is 600 pF. Find resistance, inductance and Q-factor of inductor.

Solution

$$f_0 = 1 \text{ MHz}$$

$$C_1 = 500 \text{ pF}$$

$$C_2 = 600 \text{ pF}$$

(i) Resistance and inductance of inductor

At resonance $\quad C = 500$ pF

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$10^6 = \frac{1}{2\pi\sqrt{L \times 500 \times 10^{-12}}}$$

$$L = 0.05 \text{ mH}$$

$$X_L = 2\pi f_0 L = 2\pi \times 10^6 \times 0.05 \times 10^{-3} = 314.16\ \Omega$$

When capacitance is 600 pF, the current reduces to one-half of the current at resonance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}} = 265.26\ \Omega$$

$$I = \frac{1}{2} I_0$$

$$\frac{V}{Z} = \frac{1}{2}\frac{V}{R}$$

$$Z = 2R$$

$$\sqrt{R^2 + (X_L - X_C)^2} = 2R$$

$$R^2 + (314.16 - 265.26)^2 = 4R^2$$

$$3R^2 = 2391.21$$

$$R = 28.23\ \Omega$$

(ii) Quality factor

$$Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{28.23}\sqrt{\frac{0.05 \times 10^{-3}}{500 \times 10^{-12}}} = 11.2$$

4.9 PARALLEL RESONANCE

Consider a parallel circuit consisting of a coil and a capacitor as shown in Fig. 4.92. The impedances of two branches are

$$\bar{Z}_1 = R + jX_L$$

$$\bar{Z}_2 = -jX_C$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

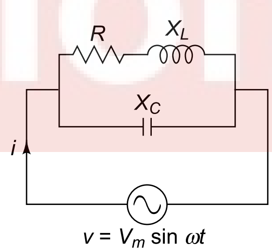


Fig. 4.92 *Parallel circuit*

Admittance of the circuit $\overline{Y} = \overline{Y}_1 + \overline{Y}_2$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left(\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Therefore, the condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$\omega_0 L \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where f_0 is called the resonant frequency of the circuit.

If R is very small as compared to L then

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

Dynamic Impedance of a Parallel Circuit At resonance, the circuit is purely resistive. The real part of admittance is $\frac{R}{R^2 + X_L^2}$. Hence, the dynamic impedance at resonance is given by

$$Z_D = \frac{R^2 + X_L^2}{R}$$

At resonance,

$$R^2 + X_L^2 = X_L X_C = \frac{L}{C}$$

$$Z_D = \frac{L}{CR}$$

Current Since impedance is maximum at resonance, the current is minimum at resonance.

$$I_0 = \frac{V}{Z_D} = \frac{V}{\frac{L}{CR}} = \frac{VCR}{L}$$

Phasor Diagram At resonance, power factor of the circuit is unity and the total current drawn by the circuit is in phase with the voltage. This will happen only when the current I_C is equal to the reactive component of the current in the inductive branch, i.e., $I_C = I_L \sin\phi$

Hence, at resonance

$$I_C = I_L \sin\phi$$

and

$$I = I_L \cos\phi$$

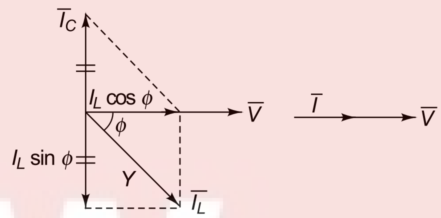


Fig. 4.93 *Phasor diagram*

Behaviour of Conductance G, Inductive Susceptance B_L and Capacitive Susceptance with Change in Frequency Conductance remains constant with the change in frequencies.

Inductive susceptance B_L is

$$B_L = \frac{1}{jX_L} = -j\frac{1}{X_L} = -j\frac{1}{2\pi f L}$$

It is inversely proportional to the frequency. Thus, it decreases with the increase in the frequency. Hence, it can be drawn as a rectangular hyperbola in the fourth quadrant.

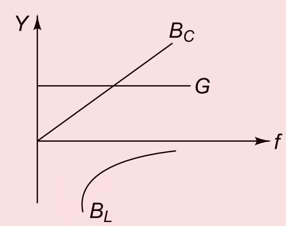


Fig. 4.94 *Behaviour of G, B_L and B_C with change in frequency*

Capacitive susceptance B_C is

$$B_C = \frac{1}{-jX_C} = j\frac{1}{X_C} = j2\pi f C$$

It is directly proportional to the frequency. It can be drawn as a straight line passing through the origin.

(a) When $f < f_0$, inductive susceptance predominates. Hence, the current lags behind the voltage and the power factor is lagging in nature.

(b) When $f = f_0$, net susceptance is zero. Hence, the admittance is minimum and impedance is maximum. At f_0, the current is in phase with the voltage and the power factor is unity.

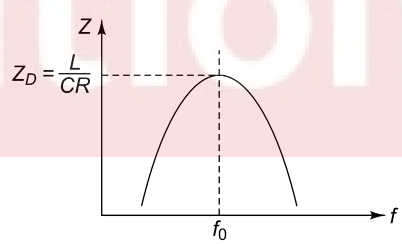


Fig. 4.95 *Impedance*

(c) When $f > f_0$, capacitive susceptance predominates. Hence, the current leads the voltage and power factor is leading in nature.

Bandwidth The bandwidth of a parallel resonant circuit is defined in the same way as that for a series resonant circuit.

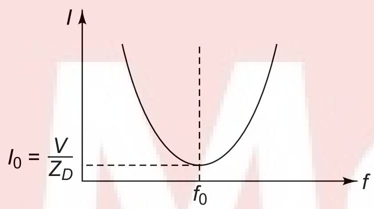


Fig. 4.96 *Resonance curve*

Quality Factor It is a measure of current magnification in a parallel resonant circuit.

$$Q_0 = \frac{\text{Current through inductor or capacitor}}{\text{Current at resonance}}$$

$$= \frac{I_{C_0}}{I_0}$$

Substituting values of I_{C_0} and I_0,

$$Q_0 = \frac{\dfrac{V}{X_{C_0}}}{\dfrac{VCR}{L}} = \frac{\dfrac{1}{X_{C_0}}}{\dfrac{CR}{L}} = \frac{\omega_0 C}{\dfrac{CR}{L}} = \frac{\omega_0 L}{R}$$

Neglecting the resistance R, the resonant frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\left(\dfrac{1}{\sqrt{LC}}\right)L}{R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Comparison of Series and Parallel Resonant Circuits

Parameter	*Series Circuit*	*Parallel Circuit*
Current at resonance	$I = \frac{V}{R}$ and is maximum	$I = \frac{VCR}{L}$ and is minimum
Impedance at resonance	$Z = R$ and is minimum	$Z = \frac{L}{CR}$ and is maximum
Power factor at resonance	Unity	Unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q-factor	$Q = \frac{2\pi f_0 L}{R}$	$Q = \frac{2\pi f_0 L}{R}$
It magnifies	Voltage across L and C	Current through L and C

Example 1

Derive the expression for resonant frequency for the parallel circuit shown in Fig. 4.97. Also calculate the impedance and current at resonance.

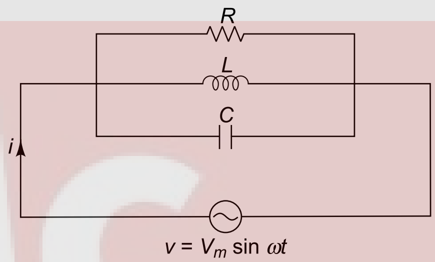


Fig. 4.97

Solution

$$\bar{Z}_1 = R$$

$$\bar{Z}_2 = jX_L$$

$$\bar{Z}_3 = -jX_C$$

(i) Resonant frequency of the circuit

For parallel circuit

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

$$\bar{Y} = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}$$

$$= \frac{1}{R} - j\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Hence, the condition for resonance is

$$\frac{1}{X_L} - \frac{1}{X_C} = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the resonant frequency of the circuit.

(ii) Impedence at resonance

At resonance, the circuit is purely resistive. Hence, the imaginary part of $\overline{Y}$ is zero.

$$Y_D = \frac{1}{R}$$

$$Z_D = R$$

(iii) Current at resonance

$$I_0 = \frac{V}{Z_D} = \frac{V}{R}$$

Example 2

Derive the expression for the resonant frequency of the parallel circuit shown in Fig. 4.98.

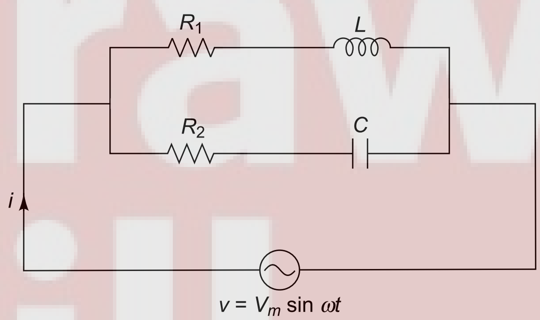


Fig. 4.98

Solution

$$\overline{Z}_1 = R_1 + jX_L$$

$$\overline{Z}_2 = R_2 - jX_C$$

For parallel circuit,

$$\frac{1}{\overline{Z}} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2}$$

$$\overline{Y} = \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C}$$

$$= \frac{R_1 - jX_L}{R_1^2 + X_L^2} + \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2} - j\left[\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right]$$

At resonance, the circuit is purely resistive. Hence, the condition for resonance is

$$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega_0 L}{R_1^2 + \omega_0^2 L^2} = \frac{\dfrac{1}{\omega_0 C}}{R_2^2 + \dfrac{1}{\omega_0^2 C^2}}$$

$$\frac{\omega_0^2 LC}{R_1^2 + \omega_0^2 L^2} = \frac{\omega_0^2 C^2}{R_2^2 \omega_0^2 C^2 + 1}$$

$$LC\,(R_2^2\,\omega_0^2\,C^2 + 1) = C^2\,(R_1^2 + \omega_0^2 L^2)$$

$$\omega_0^2 R_2^2\, LC^3 + LC = C^2 R_1^2 + \omega_0^2 L^2 C^2$$

$$\omega_0^2 R_2^2\, LC^3 - \omega_0^2 L^2 C^2 = C^2 R_1^2 - LC$$

$$\omega_0^2 LC^2 (CR_2^2 - L) = C(CR_1^2 - L)$$

$$\omega_0^2 LC\,(CR_2^2 - L) = CR_1^2 - L$$

$$\omega_0^2 = \frac{CR_1^2 - L}{LC(CR_2^2 - L)}$$

$$\omega_0 = \sqrt{\frac{CR_1^2 - L}{LC(CR_2^2 - L)}} = \frac{1}{\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

where f_0 is called the resonant frequency of the circuit.

Example 3

A coil having an inductance of L henries and a resistance of 12 Ω is connected in parallel with a variable capacitor. At $\omega = 2.3 \times 10^6$ rad/s, resonance is achieved and at this instant, capacitance C = 0.021 μF. Find the inductance of the coil.

Solution

$$R = 12\ \Omega$$

$$\omega_0 = 2.3 \times 10^6 \text{ rad/s}$$

$$C = 0.021\ \mu\text{F}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$2.3 \times 10^6 = \sqrt{\frac{1}{L \times 0.021 \times 10^{-6}} - \frac{(12)^2}{L^2}}$$

$$L = 89.7\ \mu\text{H}$$

Example 4

A coil of 20 Ω resistance has an inductance of 0.2 H and is connected in parallel with a condenser of 100 μF capacitance. Calculate the frequency at which this circuit will behave as a non-inductive resistance. Find also the value of dynamic resistance.

Solution

$$R = 20\ \Omega$$
$$L = 0.2\ \text{H}$$
$$C = 100\ \mu\text{F}$$

(i) Resonant frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 100 \times 10^{-6}} - \left(\frac{20}{0.2}\right)^2} = 31.83\ \text{Hz}$$

(ii) Dynamic resistance

$$Z_D = \frac{L}{CR} = \frac{0.2}{100 \times 10^{-6} \times 20} = 100\ \Omega$$

Example 5

A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit. Calculate (i) the value of capacitance at resonance, (ii) Q factor of the circuit, (iii) dynamic impedance of the circuit, and (iv) total circuit current.

Solution

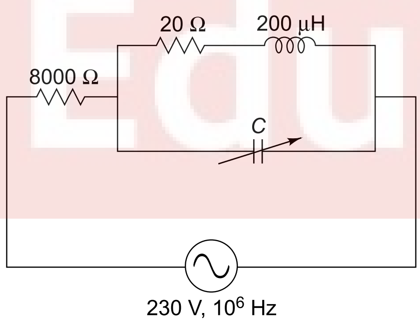


Fig. 4.99

$$R = 20\ \Omega$$
$$L = 200\ \mu\text{H}$$
$$f_0 = 10^6\ \text{Hz}$$
$$V = 230\ \text{V}$$
$$R_S = 8000\ \Omega$$

(i) Value of capacitance

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200\times10^{-6}\times C} - \frac{(20)^2}{(200\times10^{-6})^2}}$$

$$C = 126.65 \text{ pF}$$

(ii) Quality factor

$$Q_0 = \frac{2\pi f_0 L}{R} = \frac{2\pi\times10^6\times200\times10^{-6}}{20} = 62.83$$

(iii) Dynamic impedance

$$Z_D = \frac{L}{CR} = \frac{200\times10^{-6}}{126.65\times10^{-12}\times20} = 78958\ \Omega$$

(iv) Current

Total impedance of the circuit at resonance = $Z_D + R_S = 78958 + 8000 = 86958\ \Omega$

$$I = \frac{230}{86958} = 2.65 \text{ mA}$$

Example 6

A coil of 400 Ω resistance and 318 μH inductance is connected in parallel with a capacitor and the circuit resonates at 1 MHz. If a second capacitor of 23.5 pF capacitance is connected in parallel with the first capacitor, find the frequency at which the circuit resonates.

Solution

$$R = 400\ \Omega$$

$$L = 318\ \mu\text{H}$$

$$f_0 = 1 \text{ MHz}$$

$$C_2 = 23.5 \text{ pF}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1} - \frac{R^2}{L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{318\times10^{-6}\times C_1} - \frac{(400)^2}{(318\times10^{-6})^2}}$$

$$C_1 = 76.59 \text{ pF}$$

When capacitor of 23.5 pF is connected in parallel with $C_1 = 76.59$ pF

$$C_T = C_1 + C_2 = 76.59 + 23.5 = 100.09 \text{ pF}$$

The new resonant frequency $f_0' = \frac{1}{2\pi}\sqrt{\frac{1}{LC_T} - \frac{R^2}{L^2}}$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{318\times10^{-6}\times100.09\times10^{-12}} - \frac{(400)^2}{(318\times10^{-6})^2}}$$

$$= 869.34 \text{ kHz}$$

Example 7

A coil having a resistance and inductance of 15 Ω and 8 mH respectively is connected in parallel with another coil having a resistance and inductance of 4 Ω and 18 mH. If this parallel combination is to be replaced by a single coil, calculate the value of resistance and inductance of that coil. What value of capacitance should be connected in parallel with this coil in order to get unity power factor? Assume operating frequency to be 50 Hz.

Solution

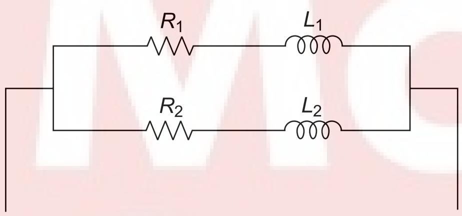


Fig. 4.100

$$R_1 = 15\ \Omega$$
$$L_1 = 8\ \text{mH}$$
$$R_2 = 4\ \Omega$$
$$L_2 = 18\ \text{mH}$$
$$f = 50\ \text{Hz}$$

(i) Value of resistance and inductance of the coil

$$X_{L_1} = 2\pi f L_1 = 2\pi \times 50 \times 8 \times 10^{-3} = 2.51\ \Omega$$

$$X_{L_2} = 2\pi f L_2 = 2\pi \times 50 \times 18 \times 10^{-8} = 5.65\ \Omega$$

$$\bar{Z}_1 = R_1 + jX_{L_1} = 15 + j2.51 = 15.21\ \angle 9.5°\ \Omega$$

$$\bar{Z}_2 = R_2 + jX_{L_2} = 4 + j5.65 = 6.92\ \angle 54.7°\ \Omega$$

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} = \frac{(15.21\angle 9.5°)(6.92\angle 54.7°)}{15.21\angle 9.5° + 6.92\angle 54.7°} = 5.09\ \angle 40.96°\ \Omega$$

$$= 3.84 + j3.34\ \Omega$$

$$R = 3.84\ \Omega$$

$$X_L = 3.34\ \Omega$$

$$X_L = 2\pi f L$$

$$3.34 = 2\pi \times 50 \times L$$

$$L = 10.63\ \text{mH}$$

(ii) Value of capacitance

When a capacitance C is connected in parallel with this coil, power factor becomes unity. Hence, the circuit behaves like a parallel resonant circuit at 50 Hz.

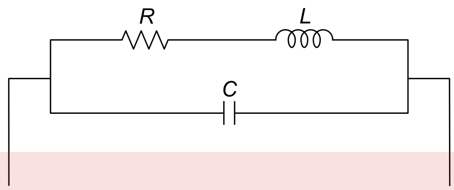


Fig. 4.101

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{10.63\times10^{-3}\times C} - \frac{(3.84)^2}{(10.63\times10^{-3})^2}}$$

$$C = 410.46\ \mu\text{F}$$

Example 8

A series circuit consisting of a 12 Ω resistor, 0.3 H inductor and a variable capacitor is connected across a 100 V, 50 Hz ac supply. The capacitance value is adjusted to obtain maximum current. Find, the capacitance value and the power drawn by the circuit under the condition. Now, the supply frequency is raised to 60 Hz, the voltage remaining same at 100 V. Find the value of the capacitance C_1 to be connected across the above series circuit so that current drawn from the supply is minimum.

Solution

$$R = 12\ \Omega$$
$$L = 0.3\ \text{H}$$
$$V = 100\ \text{V}$$
$$f = 50\ \text{Hz}$$

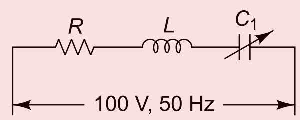


Fig. 4.102(a)

(i) Value of capacitance C_1

The resonance occurs at $f = 50$ Hz

$$f_0 = \frac{1}{2\pi\sqrt{LC_1}}$$

$$50 = \frac{1}{2\pi\sqrt{0.3\times C_1}}$$

$$C_1 = 33.77\ \mu\text{F}$$

(ii) Value of capacitance C_2

The resonance occurs at 60 Hz.

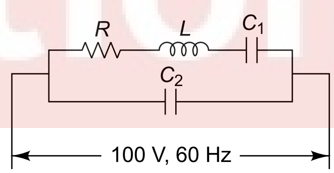


Fig. 4.102(b)

$$X_L = 2\pi f_0 L = 2\pi \times 60 \times 0.3 = 113.1\ \Omega$$

$$X_{C_1} = \frac{1}{2\pi f_0 C_1} = \frac{1}{2\pi \times 60 \times 33.77 \times 10^{-6}} = 78.55\ \Omega$$

$$\bar{Z}_1 = 12 + j\,113.1 - j\,78.55 = 36.57\ \angle 70.85\ \Omega$$

$$\bar{Y}_1 = \frac{1}{Z_1} = \frac{1}{36.57\angle 70.85^\circ} = 0.027\ \angle 70.85^\circ\ \mho = 8.86 \times 10^{-3} - j\,0.0255\ \mho$$

$$\bar{Y}_{\text{req}} = \bar{Y}_1 + \bar{Y}_2 = 8.86 \times 10^{-3} - j\,0.0255 + \bar{Y}_2$$

At resonance, the imaginary part of $\bar{Y}_{\text{req}}$ becomes zero.

$$\therefore \quad \bar{Y}_2 = j\,0.0255\ \mho$$

$$\bar{Y}_2 = \frac{1}{X_{C_2}} = 0.0255\ \mho$$

$$X_{C_2} = 39.22\ \Omega$$

$$X_{C_2} = \frac{1}{2\pi f_0 C_2}$$

$$39.22 = \frac{1}{2\pi \times 60 \times C_2}$$

$$C_2 = 67.63\ \mu\text{F}$$

Useful Formulae

Series Resonance

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$I_0 = \frac{V}{R}$$

$$Z_0 = R$$

$$\text{pf} = 1$$

$$V_{L_0} = V_{C_0}$$

$$V_{L_0} = I_0\, X_{L_0}$$

$$V_{C_0} = I_0\, X_{C_0}$$

$$X_L = X_C$$

$$BW = f_2 - f_1 = \frac{R}{2\pi L}\ \text{(Hz)}$$

$$= \omega_2 - \omega_1 = \frac{R}{L}\ \text{(rad/s)}$$

$$f_1 = f_0 - \frac{R}{4\pi L}$$

$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonance (Coil in parallel with capacitor)

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$Z_D = \frac{L}{CR}$$

$$I_0 = \frac{VCR}{L}$$

Exercise 4.4

4.1 A series *RLC* circuit has the following parameter values:
$R = 10\ \Omega$, $L = 0.014$ H, $C = 100\ \mu$F. Compute the following:
(i) resonant frequency in rad/s
(ii) quality factor of the circuit
(iii) bandwidth
(iv) lower and upper frequency points of the bandwidth

[845.2 rad/s, 1.185, 714.3 rad/s 487.9 rad/s, 1202.1 rad/s]

4.2 A series *RLC* circuit as $R = 500\ \Omega$, $L = 60$ mH and $C = 0.053\ \mu$F. Estimate
(i) circuit impedance at 50 Hz
(ii) resonance frequency in rad/s
(iii) quality factor of the circuit
(iv) bandwidth
(v) current at resonance frequency with an applied voltage of 30 V

[0.0041 Ω, 17733 rad/s, 2.128, 8333.33 rad/s 60 mA]

4.3 A choking coil of 10 Ω resistance and 0.1 H inductance is connected in series with a capacitor of 200 μF capacitance across a supply voltage of 230 V, 50 Hz ac. Find the following w.r.t. the resonant circuit.
(i) Neat circuit diagram indicating all the relevant parameters
(ii) At what frequency will the circuit resonate
(iii) Value of reactances at resonance condition
(iv) Impedance of the coil at resonance condition
(v) Voltage across the coil and voltage across the capacitance
(vi) Half-power frequencies f_1 and f_2
(vii) *Q* factor of the circuit
(viii) Power and power factor

[35.59 Hz, 22.36 Ω, 10 Ω, 744.28 V, 514.28 V, 27.63 Hz, 43.55 Hz, 2.24, 5.29 kW, 1]

4.4 A coil of 40 Ω resistance and 0.75 H inductance forms part of a series circuit for which the resonant frequency is 55 Hz. If the supply is 250 V, 50 Hz, find

(i) the line current (ii) the power factor
(iii) the voltage across the coil

[3.85 A, 0.616 (leading), 920.07 V]

4.5 An *RLC* series circuit of 10 Ω resistance should be designed to have a bandwidth of 100 Hz. Determine the values of *L* and *C* so that the circuit resonates at 250 Hz. *[0.0159 H, 25 μF]*

4.6 A resistor and capacitor are connected in series with a variable inductor. When the circuit is connected to a 220 V, 50 Hz supply, the maximum current obtained by varying the inductance is 0.314 A. The voltage across the capacitor is 800 V. Calculate the resistance, inductance and capacitance of the circuit.

[700.63 Ω, 1.25 μF, 8.10 H]

4.7 A coil of 2 Ω resistance and 0.01 H inductance is connected in series with a capacitor across 230 V mains. What must be the capacitance, in order that maximum current occurs at a frequency of 50 Hz? Find also the current and the voltage across the capacitor. *[1 mF, 115 A, 361.28 V]*

4.8 A choking coil of 10-ohm resistance and 0.1 H inductance is connected in series with a capacitor of 200 μF capacitance. Calculate the current, the coil voltage and the capacitor voltage. The supply voltage is 230 V at 50 Hz. At what frequency will the circuit resonate? Calculate the voltages at resonant frequency across the coil and capacitor. For this, assume that supply voltage is 230 V of variable frequency.

[12.47 A, 411.17, 198.47 V, 35.6 Hz, 563.3 V, 514.2 V]

4.9 An inductive coil having an inductance of 0.04 H and a resistance of 25 Ω has been connected in series with another inductive coil of 0.2 H inductance and 15 Ω resistance. The whole circuit has been energized from 230 V, 50 Hz mains. Calculate power dissipation in each coil and power factor of the whole circuit. Draw the phasor diagram. Suggest a suitable capacitor for the above circuit to resonate at 50 Hz. *[181.57 W, 108.95 W, 0.468 (lagging), 42.22 μF]*

4.10 A voltage $v(t) = 10\sqrt{2}\,\sin\omega t$ is applied to a series *RLC* circuit. At resonant frequency, voltage across the capacitor is found to be 500 V. The bandwidth of the circuit is known to be 400 rad/s and impedance at resonance is 10 Ω. Determine resonant frequency, upper and lower cut-off frequencies, *L* and *C*.

[3.183 kHz, 3.151 kHz, 3.214 kHz, 0.025 H, 0.1 μF]

4.11 An inductive coil having a resistance of 20 Ω and an inductance of 0.2 H is connected in parallel with a 20 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with the supply voltage. Also, find the value of this current. Draw the phasor diagram. *[19.49 Hz, 4.6 A]*

Review Questions

4.1 Show that current through pure inductor lags behind the applied sinusoidal voltage by 90°. Also show that pure inductance does not consume any power. Draw voltage, current and power waveforms.

4.2 Explain behaviour of a pure capacitor when connected across a single-phase ac supply.

4.3 Show that the average power absorbed by a capacitor is zero.

4.4 Draw a power triangle and name the sides with units and give their formulas.

4.5 If the impedance of a circuit is $z \angle -\phi$, what type of circuit is this? Which element should be added in series to bring the circuit under resonance condition? Explain with a phasor diagram.

4.6 Show the Z-triangle for an R-L-C series circuit, when inductive reactance is dominant over capacitive reactance.

4.7 Derive the condition for resonance in a series circuit.

4.8 Derive the relation for bandwidth of a series R-L-C circuit.

4.9 Define and find the expression for quality factor of a series R-L-C circuit.

4.10 Derive the condition for resonance in parallel circuit.

4.11 Mention the conditions under which a parallel R-L-C circuit is in electrical resonance.

4.12 Discuss graphical representation of series resonance and parallel resonance.

4.13 Compare series and parallel resonance.

Objective-Type Questions

Choose the correct alternative in the following questions:

4.1 A 10 mH inductor carries a sinusoidal current of 1 A rms at a frequency of 50 Hz. The average power dissipated by the inductor is

(a) 0 (b) 0.25 W (c) 0.5 W (d) 1 W

4.2 The power factor of an ordinary bulb is

(a) zero (b) unity
(c) slightly more than unity (d) slightly less than unity

4.3 When a sinusoidal voltage is applied across R-L series circuit having $R = X_L$, the phase angle will be

(a) 90° (b) 45° lag (c) 45° lead (d) 90° lead

4.4 An ac source having voltage $\bar{e} = 110 \sin\left(\omega t + \frac{\pi}{3}\right)$ is connected in an ac circuit. If the current drawn from the circuit varies as $i = 5 \sin\left(\omega t - \frac{\pi}{3}\right)$, the impedance of the circuit will be

(a) 22 Ω (b) 16 Ω (c) 30.8 Ω (d) none of these

4.5 An ac source of 200 V rms supplies active power of 600 W and reactive power of 800 VAR. The rms current drawn from the source is

(a) 10 A (b) 5 A (c) 3.75 A (d) 2.5 A

4.6 The voltage phasor of a circuit is 10 $\angle 15°$ V and the current phasor is 2 $\angle -45°$ A. The active and reactive powers in the circuit are

(a) 10 W and 17.32 VAR (b) 5 W and 8.66 VAR
(c) 20 W and 60 VAR (d) $20\sqrt{2}$ and $10\sqrt{2}$ VAR

4.7 In a two element series circuit, the applied voltage and resultant current are respectively $v(t) = 50 + 50 \sin (5 \times 10^3\, t)$ and $i(t) = 11.2 \sin (5 \times 10^3\, t + 63.4°)$. The nature of the elements would be

(a) *R*-*L* (b) *R*-*C*
(c) *L*-*C* (d) neither *R*, nor *L*, nor *C*

4.8 A boiler at home is switched on to the ac mains supplying power at 230 V, 50 Hz. The frequency of instantaneous power consumed is

(a) 0 (b) 50 Hz (c) 100 Hz (d) 150 Hz

4.9 In a series *R*-*L*-*C* circuit, $V_R = 3$ V, $V_L = 14$ V, $V_C = 10$ V. The input voltage to the circuit is

(a) 10 V (b) 5 V (c) 27 V (d) 24 V

4.10 A_1, A_2 and A_3 are ideal ammeters, shown in Fig. 4.103. If A_1 reads 5 A, A_2 reads 12 A, then A_3 should read

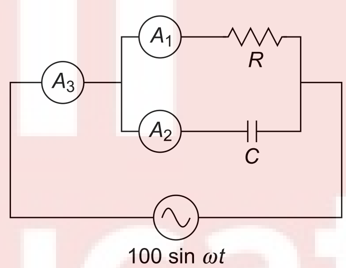


Fig. 4.103

(a) 7 A (b) 12 A (c) 13 A (d) 17 A

4.11 A series *R*-*L*-*C* circuit consisting of $R = 10\ \Omega$, $X_L = 20\ \Omega$ and $X_C = 20\ \Omega$ is connected across an ac supply of 200 V rms. The rms voltage across the capacitor is

(a) 200 $\angle -90°$ V (b) 200 $\angle 90°$ V
(c) 400 $\angle -90°$ V (d) 400 $\angle 90°$ V

4.12 In Fig. 4.104, A_1, A_2 and A_3 are ideal ammeters. If A_1 and A_3 read 5 A and 13 A respectively, the reading of A_2 will be

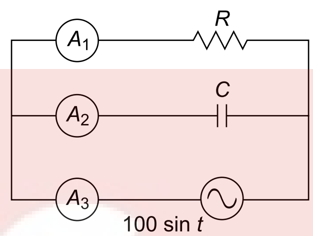


Fig. 4.104

(a) 8 A (b) 12 A (c) 18 A (d) 10 A

4.13 In Fig. 4.105, $i(t)$ under steady state is

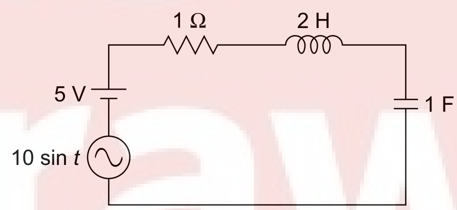


Fig. 4.105

(a) zero (b) 5
(c) 7.07 sin t (d) 7.07 sin $(t - 45°)$

4.14 The circuit shown with $R = \dfrac{1}{3}\ \Omega$, $L = \dfrac{1}{4}$ H, $C = 3$ F in Fig. 4.106 has input voltage $v(t) = \sin 2t$. The resulting current $i(t)$ is

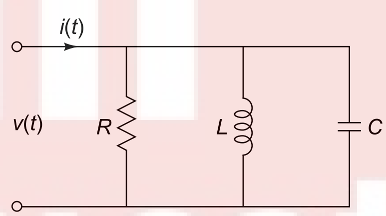


Fig. 4.106

(a) 5 sin $(2t + 53.1°)$ (b) 5 sin $(2t - 53.1°)$
(c) 25 sin $(2t + 53.1°)$ (d) 25 sin $(2t - 53.1°)$

4.15 A series R-L-C circuit will have unity power factor if operated at a frequency of

(a) $\dfrac{1}{LC}$ (b) $\dfrac{1}{\omega\sqrt{LC}}$ (c) $\dfrac{1}{\omega^2\sqrt{LC}}$ (d) $\dfrac{1}{2\pi\sqrt{LC}}$

4.16 For a series *R-L-C* resonant circuit, the total reactance at the lower half power frequency is

(a) $\sqrt{2}R\angle 45°$ (b) $\sqrt{2}R\angle -45°$ (c) R (d) $-R$

4.17 In a series *R-L-C* high *Q* circuit, the current peaks at a frequency

(a) equal to the resonant frequency (b) greater than the resonant frequency
(c) less than the resonant frequency (d) none of the above

4.18 In a series *R-L-C* circuit, $R = 2$ kΩ, $L = 1$ H, $C = \frac{1}{400}$ μF . The resonant frequency is

(a) 2×10^4 Hz (b) $\frac{1}{\pi} \times 10^4$ Hz (c) 10^4 Hz (d) $2\pi \times 10^4$ Hz

4.19 For a series resonant circuit at low frequency, circuit impedance is __________ and at high frequency circuit impedance is __________ .

(a) capacitive, inductive (b) inductive, capacitive
(c) resistive, inductive (d) capacitive, resistive

4.20 A circuit with a resistor, inductor and capacitor in series is resonant of f_0 Hz. If all the component values are now doubled, the new resonant frequency is

(a) $2f_0$ (b) f_0 (c) $f_0/4$ (d) $f_0/2$

4.21 In a series *R-L-C* circuit at resonance, the magnitude of the voltage developed across the capacitor

(a) is always zero
(b) can never be greater than the input voltage
(c) can be greater than the input voltage, however it is 90° out of phase with the input voltage
(d) can be greater than the input voltage and is in phase with the input voltage

4.22 The power in a series *R-L-C* circuit will be half of that at resonance when the magnitude of the current is equal to

(a) $\frac{V}{2R}$ (b) $\frac{V}{\sqrt{3}R}$ (c) $\frac{V}{\sqrt{2}R}$ (d) $\frac{\sqrt{2}V}{R}$

4.23 A series *R-L-C* circuit has a resonance frequency of 1 kHz and a qualify factor *Q* of 100. If each of *R*, *L* and *C* is doubled from its original value, the new *Q* of the circuit is

(a) 25 (b) 50 (c) 100 (d) 200

4.24 A series *R-L-C* circuit has a *Q* of 100 and an impedance of $(100 + j0)$ Ω at its resonant angular frequency of 10^7 rad/s. The values of *R* and *L* are

(a) 100 Ω, 10^3 H (b) 100 Ω, 10^{-3} H
(c) 10 Ω, 10 H (d) 10 Ω, 0.1 H

4.25 The following circuit in Fig. 4.107 resonates at

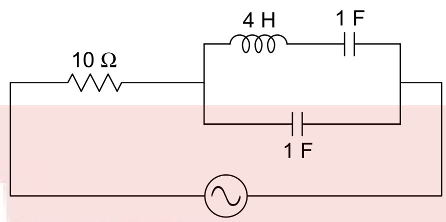


Fig. 4.107

(a) all frequencies (b) 0.5 rad/s
(c) 0.5 rad/s (d) 1 rad/s

4.26 At resonance, the parallel circuit shown in Fig. 4.108 behaves like

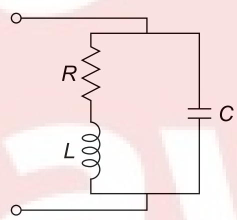
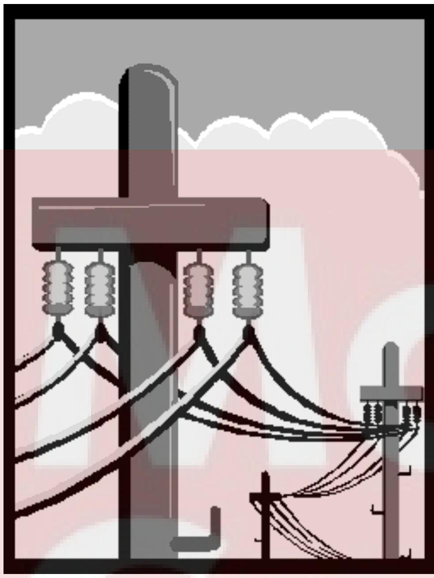


Fig. 4.108

(a) an open circuit
(b) a short circuit
(c) a pure resistor of value R
(d) a pure resistor of value much higher than R

Answers to Objective-Type Questions

4.1 (a)	**4.2** (d)	**4.3** (b)	**4.4** (a)
4.5 (b)	**4.6** (a)	**4.7** (b)	**4.8** (c)
4.9 (b)	**4.10** (c)	**4.11** (c)	**4.12** (b)
4.13 (d)	**4.14** (a)	**4.15** (d)	**4.16** (d)
4.17 (a)	**4.18** (b)	**4.19** (a)	**4.20** (d)
4.21 (c)	**4.22** (c)	**4.23** (b)	**4.24** (b)
4.25 (b)	**4.26** (d)		



Chapter 5

Three-Phase Circuits

Chapter Outline

5.1 THREE-PHASE SYSTEM

A system which generates a single alternating voltage and current is termed a *single-phase system*. It utilises only one winding. A *polyphase system* utilises more than one winding. It will produce as many induced voltages as the number of windings.

A three-phase system consists of three separate but identical windings that are displaced by 120 electrical degrees from each other. When these three windings are rotated in an anticlockwise direction with constant angular velocity in a uniform magnetic field, the emfs are induced in each winding which have the same magnitude and frequency but are displaced 120° from one another.

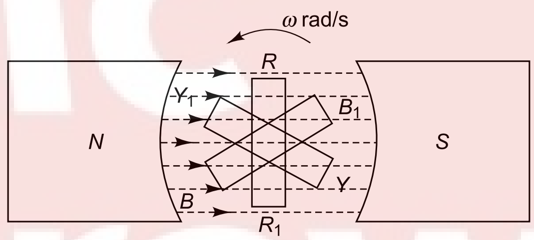


Fig. 5.1 *Three-phase system*

The instantaneous values of generated voltage in windings RR_1, YY_1 and BB_1 are given by

$$e_R = E_m \sin \theta$$
$$e_Y = E_m \sin (\theta - 120°)$$
$$e_B = E_m \sin (\theta - 240°)$$

where E_m is the maximum value of the induced voltage in each winding. The waveforms of these three voltages are shown in Fig. 5.2.

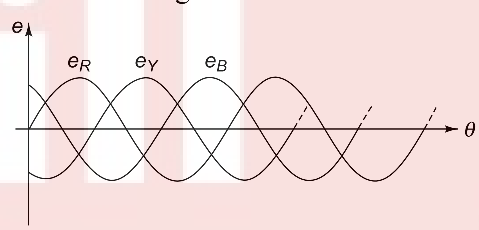


Fig. 5.2 *Voltage waveforms*

Figure 5.3 shows the phasor diagram of these three induced voltages.

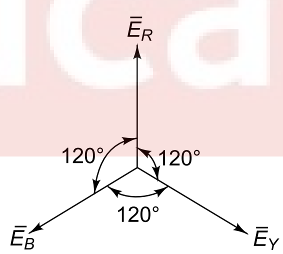


Fig. 5.3 *Phasor diagram*

5.2 ADVANTAGES OF A THREE-PHASE SYSTEM

1. In a single-phase system, the instantaneous power is fluctuating in nature. However, in a three-phase system, it is constant at all times.
2. The output of a three-phase system is greater than that of a single-phase system.
3. Transmission and distribution of a three-phase system is cheaper than that of a single-phase system.
4. Three-phase motors are more efficient and have higher power factors than single-phase motors of the same frequency.
5. Three-phase motors are self-starting whereas single-phase motors are not self-starting.

5.3 SOME DEFINITIONS

Phase Sequence The sequence in which the voltages in the three phases reach the maximum positive value is called the *phase sequence* or *phase order*. From the phasor diagram of a three-phase system, it is clear that the voltage in the coil R attains maximum positive value first, next in the coil Y and then in the coil B. Hence, the phase sequence is R-Y-B.

Phase Voltage The voltage induced in each winding is called the *phase voltage*.

Phase Current The current flowing through each winding is called the *phase current*.

Line Voltage The voltage available between any pair of terminals or lines is called the *line voltage*.

Line Current The current flowing through each line is called the *line current*.

Symmetrical or Balanced System A three-phase system is said to be balanced if the

(a) voltages in the three phases are equal in magnitude and differ in phase from one another by 120°, and
(b) currents in the three phases are equal in magnitude and differ in phase from one another by 120°.

Balanced Load The load is said to be balanced if loads connected across the three phases are identical, i.e., all the loads have the same magnitude and power factor.

5.4 INTERCONNECTION OF THREE PHASES

In a three-phase system, there are three windings. Each winding has two terminals, viz., ‘start’ and ‘finish’. If a separate load is connected across each winding as shown in Fig. 5.4, six conductors are required to transmit and distribute power. This will make the system complicated and expensive.

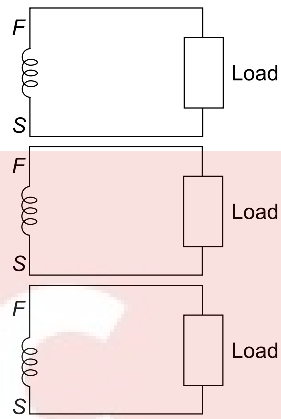


Fig. 5.4 *Non-interlinked three-phase system*

In order to reduce the number of conductors, the three windings are connected in the following two ways:

1. Star, or Wye, connection
2. Delta, or Mesh, connection

5.5 STAR OR WYE CONNECTION

In this method, similar terminals (start or finish) of the three windings are joined together as shown in Fig. 5.5. The common point is called *star* or *neutral point.*

Figure 5.6 shows a three-phase system in star connection.

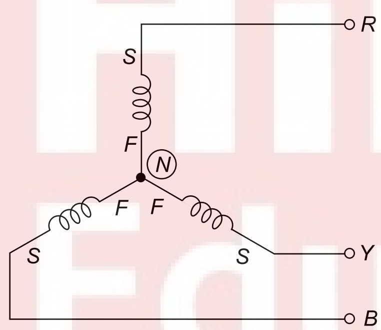


Fig. 5.5 *Three-phase star connection*

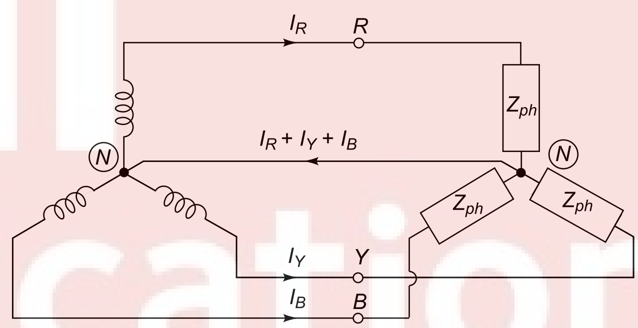


Fig. 5.6 *Three-phase, four-wire system*

This system is called a three-phase, four-wire system. If three identical loads are connected to each phase, the current flowing through the neutral wire is the sum of the three currents I_R, I_Y and I_B. Since the impedances are identical, the three currents are equal in magnitude but differ in phase from one another by 120°.

$$i_R = I_m \sin \theta$$

$$i_Y = I_m \sin(\theta - 120°)$$

$$i_B = I_m \sin(\theta - 240°)$$

$$i_R + i_Y + i_B = I_m \sin\theta + I_m \sin(\theta - 120°) + I_m \sin(\theta - 240°) = 0$$

Therefore, the neutral wire can be removed without any way affecting the voltages or currents in the circuit as shown in Fig. 5.7. This constitutes a three-phase, three-wire system. If the load is not balanced, the neutral wire carries some current.

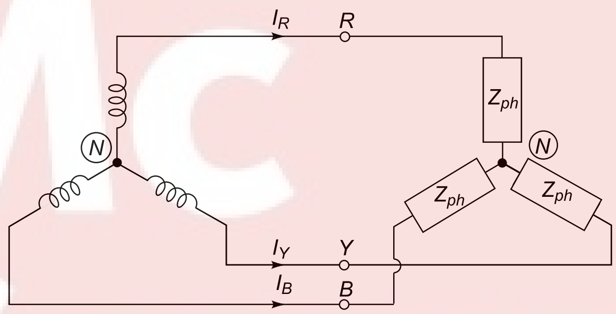


Fig. 5.7 *Three-phase, three-wire system*

5.6 DELTA OR MESH CONNECTION

In this method, dissimilar terminals of the three windings are joined together, i.e., the 'finish' terminal of one winding is connected to the 'start' terminal of the other winding, and so on, as shown in Fig. 5.8. This system is also called a three-phase, three-wire system.

For a balanced system, the sum of the three phase voltages round the closed mesh is zero. The three emfs are equal in magnitude but differ in phase from one another by 120°.

$$e_R = E_m \sin\theta$$

$$e_Y = E_m \sin(\theta - 120°)$$

$$e_B = E_m \sin(\theta - 240°)$$

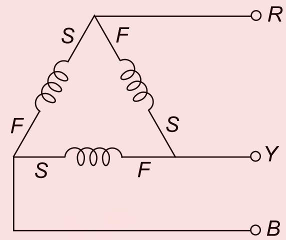


Fig. 5.8 *Three-phase delta connection*

$$e_R + e_Y + e_B = E_m \sin\theta + E_m \sin(\theta - 120°) + E_m \sin(\theta - 240°) = 0$$

5.7 VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED STAR-CONNECTED LOAD

5.7.1 Relation between Line Voltage and Phase Voltage

Since the system is balanced, the three-phase voltages V_{RN}, V_{YN} and V_{BN} are equal in magnitude and differ in phase from one another by 120°.

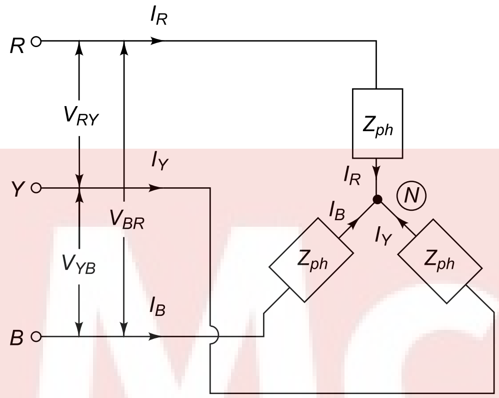


Fig. 5.9 *Star connection*

Let $\quad V_{RN} = V_{YN} = V_{BN} = V_{ph}$

where V_{ph} indicates the rms value of phase voltage.

$$\bar{V}_{RN} = V_{ph}\angle 0°$$
$$\bar{V}_{YN} = V_{ph}\angle -120°$$
$$\bar{V}_{BN} = V_{ph}\angle -240°$$

Let $\quad V_{RY} = V_{YB} = V_{BR} = V_L$

where V_L indicates the rms value of line voltage.

Applying Kirchhoff's voltage law,

$$\bar{V}_{RY} = \bar{V}_{RN} + \bar{V}_{NY} = \bar{V}_{RN} - \bar{V}_{YN}$$
$$= V_{ph}\angle 0° - V_{ph}\angle -120°$$
$$= (V_{ph} + j0) - (-0.5\, V_{ph} - j0.866\, V_{ph})$$
$$= 1.5\, V_{ph} + j0.866\, V_{ph}$$
$$= \sqrt{3}\, V_{ph}\angle 30°$$

Similarly,

$$\bar{V}_{YB} = \bar{V}_{YN} + \bar{V}_{NB} = \sqrt{3}\, V_{ph}\angle 30°$$
$$\bar{V}_{BR} = \bar{V}_{BN} + \bar{V}_{NR} = \sqrt{3}\, V_{ph}\angle 30°$$

Thus, in a star-connected, three-phase system, $V_L = \sqrt{3}\, V_{ph}$ and line voltages lead respective phase voltages by 30°.

5.7.2 Relation between Line Current and Phase Current

From Fig. 5.9, it is clear that line current is equal to the phase current.

$$I_L = I_{ph}$$

5.7.3 Phasor Diagram (Lagging Power Factor)

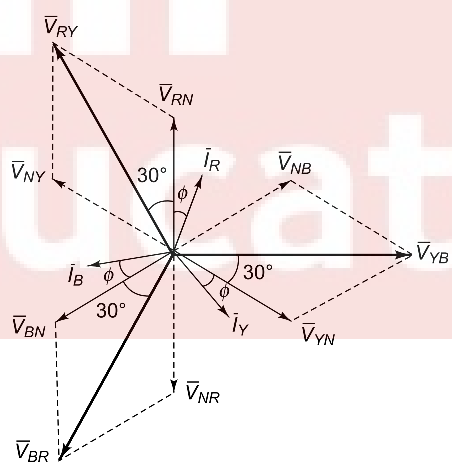


Fig. 5.10 *Phasor diagram*

5.7.4 Power

The total power in a three-phase system is the sum of powers in the three phases. For a balanced load, the power consumed in each load phase is the same.

Total active power $P = 3 \times$ power in each phase $= 3\, V_{ph}\, I_{ph} \cos \phi$

In a star-connected, three-phase system,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi = \sqrt{3}\, V_L\, I_L \cos \phi$$

where ϕ is the phase difference between phase voltage and corresponding phase current.

Similarly, total reactive power $Q = 3\, V_{ph}\, I_{ph} \sin \phi = \sqrt{3}\; V_L\, I_L \sin \phi$

Total apparent power $S = 3 V_{ph}\, I_{ph} = \sqrt{3}\; V_L\, I_L$

5.8 VOLTAGE, CURRENT AND POWER RELATIONS IN A BALANCED DELTA-CONNECTED LOAD

5.8.1 Relation between Line Voltage and Phase Voltage

From Fig. 5.11, it is clear that line voltage is equal to phase voltage.

$$V_L = V_{ph}$$

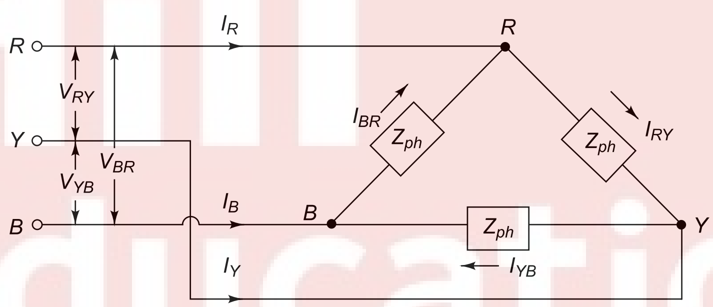


Fig. 5.11 *Delta connection*

5.8.2 Relation between Line Current and Phase Current

Since the system is balanced, the three-phase currents I_{RY}, I_{YB} and I_{BR} are equal in magnitude but differ in phase from one another by 120°.

Let $\quad I_{RY} = I_{YB} = I_{BR} = I_{ph}$

where I_{ph} indicates rms value of the phase current.

$$\bar{I}_{RY} = I_{ph} \angle 0°$$

$$\bar{I}_{YB} = I_{ph} \angle -120°$$

$$\bar{I}_{BR} = I_{ph} \angle -240°$$

Let $\quad I_R = I_Y = I_B = I_L$

where I_L indicates rms value of the line current.

Applying Kirchhoff's current law,

$$\bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\begin{aligned}\bar{I}_R &= \bar{I}_{RY} - \bar{I}_{BR} = I_{ph} \angle 0° - I_{ph} \angle -240° \\ &= (I_{ph} + j0) - (-0.5\, I_{ph} + j0.866\, I_{ph}) \\ &= 1.5\, I_{ph} - j0.866\, I_{ph} \\ &= \sqrt{3}\; I_{ph} \angle -30°\end{aligned}$$

Similarly, $\quad \bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = \sqrt{3}\; I_{ph} \angle -30°$

$$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = \sqrt{3}\; I_{ph} \angle -30°$$

Thus, in a delta-connected, three-phase system, $I_L = \sqrt{3}\; I_{ph}$ and line currents are 30° behind the respective phase currents.

5.8.3 Phasor Diagram (Lagging Power Factor)

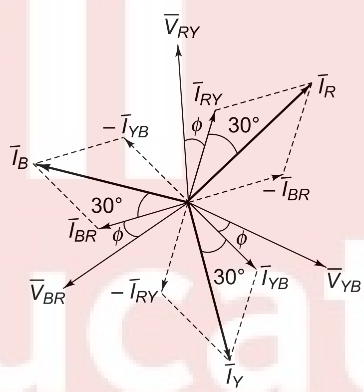


Fig. 5.12 *Phasor diagram*

5.8.4 Power

$$P = 3\, V_{ph}\, I_{ph} \cos \phi$$

In a delta-connected, three-phase system,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi = \sqrt{3}\ V_L I_L \cos\phi$$

Total reactive power $Q = 3\ V_{ph} I_{ph} \sin\phi = \sqrt{3}\ V_L I_L \sin\phi$

Total apparent power $S = 3\ V_{ph} I_{ph} = \sqrt{3}\ V_L I_L$

5.9 BALANCED Y/Δ AND Δ/Y CONVERSIONS

Any balanced star-connected system can be converted into the equivalent delta-connected system and vice versa.

For a balanced star-connected load,

$$\text{Line voltage} = V_L$$

$$\text{Line current} = I_L$$

$$\text{Impedance/phase} = Z_Y$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = I_L$$

$$Z_Y = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\sqrt{3} I_L}$$

For an equivalent delta-connected system, the line voltages and currents must have the same values as in the star-connected system, i.e.,

$$\text{Line voltage} = V_L$$

$$\text{Line current} = I_L$$

$$\text{Impedance/phase} = Z_\Delta$$

$$V_{ph} = V_L$$

$$I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$Z_\Delta = \frac{V_{ph}}{I_{ph}} = \frac{V_L}{\frac{I_L}{\sqrt{3}}} = \sqrt{3}\frac{V_L}{I_L} = 3Z_Y$$

$$Z_Y = \frac{1}{3} Z_\Delta$$

Thus, when three equal phase impedances are connected in delta, the equivalent star impedance is one third of the delta impedance.

5.10 RELATION BETWEEN POWER IN DELTA AND STAR SYSTEMS

Let a balanced load be connected in star having impedance per phase as Z_{ph}.

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

$$I_L = I_{ph} = \frac{V_L}{\sqrt{3}Z_{ph}}$$

Now $$P_Y = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times V_L \times \frac{V_L}{\sqrt{3}Z_{ph}} \times \cos\phi$$

$$= \frac{V_L^2}{Z_{ph}} \cos\phi$$

For a delta-connected load,

$$V_{ph} = V_L$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3}\frac{V_L}{Z_{ph}}$$

Now $$P_\Delta = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$= \sqrt{3} \times V_L \times \sqrt{3}\, \frac{V_L}{Z_{ph}} \times \cos\phi$$

$$= 3\, \frac{V_L^2}{Z_{ph}} \cos\phi$$

$$= 3P_Y$$

$$P_Y = \frac{1}{3} P_\Delta$$

Thus, power consumed by a balanced star-connected load is one-third of that in the case of a delta-connected load.

5.11 COMPARISON BETWEEN STAR AND DELTA CONNECTIONS

Star Connection	*Delta Connection*
1. $V_L = \sqrt{3}\ V_{ph}$	1. $V_L = V_{ph}$
2. $I_L = I_{ph}$	2. $I_L = \sqrt{3}\ I_{ph}$
3. Line voltage leads the respective phase voltage by 30°.	3. Line current lags behind the respective phase current by 30°.
4. Power in star connection is one-third of power in delta connection.	4. Power in delta connection is 3 times of the power in star connection.
5. Three-phase, three-wire and three-phase, four-wire systems are possible.	5. Only three-phase, three-wire system is possible.
6. The phasor sum of all the phase currents is zero.	6. The phasor sum of all the phase voltages is zero.

Example 1

Three equal impedances, each of 8 + j10 ohms, are connected in star. This is further connected to a 440 V, 50 Hz, three-phase supply. Calculate (i) phase voltage, (ii) phase angle, (iii) phase current, (iv) line current, (v) active power, and (vi) reactive power.

Solution

$$\bar{Z}_{ph} = 8 + j10\ \Omega$$
$$V_L = 440\ \text{V}$$
$$f = 50\ \text{Hz}$$

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03\ \text{V}$$

(ii) Phase angle

$$\bar{Z}_{ph} = 8 + j10 = 12.81\ \angle 51.34°\ \Omega$$
$$Z_{ph} = 12.81\ \Omega$$
$$\phi = 51.34°$$

(iii) Phase current

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{12.81} = 19.83\ \text{A}$$

(iv) Line current

$$I_L = I_{ph} = 19.83\ \text{A}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 19.83 \times \cos(51.34°) = 9.44 \text{ kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 19.83 \times \sin(51.34°) = 11.81 \text{ kVAR}$$

Example 2

A balanced delta-connected load of impedance (8 – j6) ohms per phase is connected to a three-phase, 230 V, 50 Hz supply. Calculate (i) power factor, (ii) line current, and (iii) reactive power.

Solution

$$\bar{Z}_{ph} = 8 - j6\ \Omega$$
$$V_L = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a delta-connected load,

(i) Power factor

$$\bar{Z}_{ph} = 8 - j6 = 10 \angle -36.87°\ \Omega$$
$$Z_{ph} = 10\ \Omega$$
$$\phi = 36.87°$$
$$pf = \cos\phi = \cos(36.87°) = 0.8 \text{ (leading)}$$

(ii) Line current

$$V_{ph} = V_L = 230 \text{ V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230}{10} = 23 \text{ A}$$
$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 23 = 39.84 \text{ A}$$

(iii) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 39.84 \times \sin(36.87°) = 9.52 \text{ kVAR}$$

Example 3

Three coils, each having a resistance and an inductance of 8 Ω and 0.02 H respectively, are connected in star across a three-phase, 230 V, 50 Hz supply. Find the (i) power factor, (ii) line current, (iii) power, (iv) reactive volt-amperes, and (v) total volt-amperes.

Solution

$$R = 8\ \Omega$$
$$L = 0.02 \text{ H}$$
$$V_L = 230 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) Power factor

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$
$$\bar{Z}_{ph} = R + jX_L$$
$$= 8 + j6.28 = 10.17\ \angle 38.13°\ \Omega$$
$$Z_{ph} = 10.17\ \Omega$$
$$\phi = 38.13°$$
$$pf = \cos\phi = \cos(38.13°) = 0.786 \text{ (lagging)}$$

(ii) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79\ \text{V}$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{10.17} = 13.05\ \text{A}$$
$$I_L = I_{ph} = 13.05\ \text{A}$$

(iii) Power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 230 \times 13.05 \times 0.786 = 4.088\ \text{kW}$$

(iv) Reactive volt-amperes

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 230 \times 13.05 \times \sin(38.13°) = 3.21\ \text{kVAR}$$

(v) Total volt-ampere

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 230 \times 13.05 = 5.198\ \text{kVA}$$

Example 4

Three coils, each having a resistance of 8 Ω and an inductance of 0.02 H, are connected in delta to a three-phase, 400 V, 50 Hz supply. Calculate the (i) line current, and (ii) power absorbed.

Solution

$$R = 8\ \Omega$$
$$L = 0.02\ \text{H}$$
$$V_L = 400\ \text{V}$$
$$f = 50\ \text{Hz}$$

For a delta-connected load,

(i) Line current

$$V_L = V_{ph} = 400\ \text{V}$$
$$X_L = 2\pi fL = 2\pi \times 50 \times 0.02 = 6.28\ \Omega$$
$$\bar{Z}_{ph} = R + jX_L = 8 + j6.28 = 10.17\ \angle 38.13°\ \Omega$$
$$Z_{ph} = 10.17\ \Omega$$

$$\phi = 38.13°$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10.17} = 39.33 \text{ A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 39.33 = 68.12 \text{ A}$$

(ii) Power absorbed

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 68.12 \times \cos(38.13°) = 37.12 \text{ kW}$$

Example 5

The three equal impedances of each of 10 ∠60° Ω, are connected in star across a three-phase, 400 V, 50 Hz supply. Calculate the (i) line voltage and phase voltage, (ii) power factor and active power consumed, (iii) If the same three impedances are connected in delta to the same source of supply, what is the active power consumed?

Solution

$$\bar{Z}_{ph} = 10\ \angle 60°\ \Omega$$

$$V_L = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) Line voltage and phase voltage

$$V_L = 400 \text{ V}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

(ii) Power factor and active power consumed

$$\phi = 60°$$

$$pf = \cos\phi = \cos(60°) = 0.5 \text{ (lagging)}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{10} = 23.094 \text{ A}$$

$$I_L = I_{ph} = 23.094 \text{ A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 23.094 \times 0.5 = 8 \text{ kW}$$

(iii) Active power consumed for delta-connected load

$$V_L = 400 \text{ V}$$

$$Z_{ph} = 10\ \Omega$$

$$V_{ph} = V_L = 400 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40 \text{ A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 40 = 69.28 \text{ A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 69.28 \times \cos(60°) = 24 \text{ kW}$$

Example 6

Three similar coils A, B, and C are available. Each coil has a 9 Ω resistance and a 12 Ω reactance. They are connected in delta to a three-phase, 440 V, 50 Hz supply. Calculate for this load, the (i) phase current, (ii) line current, (iii) power factor, (iv) total kVA, (v) active power, and (vi) reactive power. If these coils are connected in star across the same supply, calculate all the above quantities.

Solution

$$R = 9\ \Omega$$
$$X_L = 12\ \Omega$$
$$V_L = 440\ \text{V}$$
$$f = 50\ \text{Hz}$$

For a delta-connected load,

(i) Phase current

$$V_L = V_{ph} = 440\ \text{V}$$
$$\bar{Z}_{ph} = 9 + j12 = 15\ \angle 53.13°\ \Omega$$
$$Z_{ph} = 15\ \Omega$$
$$\phi = 53.13°$$
$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{15} = 29.33\ \text{A}$$

(ii) Line current

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 29.33 = 50.8\ \text{A}$$

(iii) Power factor

$$pf = \cos\phi = \cos(53.13°) = 0.6\ \text{(lagging)}$$

(iv) Total kVA

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 440 \times 50.8 = 38.71\ \text{kVA}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 50.8 \times 0.6 = 23.23\ \text{kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 50.8 \times \sin(53.13°) = 30.97\ \text{kVAR}$$

If these coils are connected in star across the same supply,

(i) Phase current

$$V_L = 440\ \text{V}$$
$$Z_{ph} = 15\ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{15} = 16.94 \text{ A}$$

(ii) Line current

$$I_L = I_{ph} = 16.94 \text{ A}$$

(iii) Power factor

$$pf = 0.6 \text{ (lagging)}$$

(iv) Total kVA

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 440 \times 16.94 = 12.91 \text{ kVA}$$

(v) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 440 \times 16.94 \times 0.6 = 7.74 \text{ kW}$$

(vi) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 440 \times 16.94 \times \sin(53.13°) = 12.33 \text{ kVAR}$$

Example 7

A 415 V, 50 Hz, three-phase voltage is applied to three star-connected identical impedances. Each impedance consists of a resistance of 15 Ω, a capacitance of 177 μF and an inductance of 0.1 henry in series. Find the (i) power factor, (ii) phase current, (iii) line current, (iv) active power, (v) reactive power, and (vi) total VA. Draw a neat phasor diagram. If the same impedances are connected in delta, find the (i) line current, and (ii) power consumed.

Solution $V_L = 415$ V

$f = 50$ Hz

$R = 15\ \Omega$

$C = 177\ \mu$F

$L = 0.1$ H

For a star-connected load,

(i) Power factor

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 177 \times 10^{-6}} = 17.98\ \Omega$$

$$\begin{aligned}\bar{Z}_{ph} &= R + jX_L - jX_C \\ &= 15 + j31.42 - j17.98 \\ &= 15 + j13.44 \\ &= 20.14\ \angle 41.86°\ \Omega\end{aligned}$$

$$Z_{ph} = 20.14\ \Omega$$

$$\phi = 41.86°$$

$$pf = \cos\phi = \cos(41.86°) = 0.744 \text{ (lagging)}$$

(ii) Phase current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{20.14} = 11.9 \text{ A}$$

(iii) Line current

$$I_L = I_{ph} = 11.9 \text{ A}$$

(iv) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 415 \times 11.9 \times 0.744 = 6.36 \text{ kW}$$

(v) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 415 \times 11.9 \times \sin(41.86°) = 5.71 \text{ kVAR}$$

(vi) Total VA

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 415 \times 11.9 = 8.55 \text{ kVA}$$

Phasor Diagram

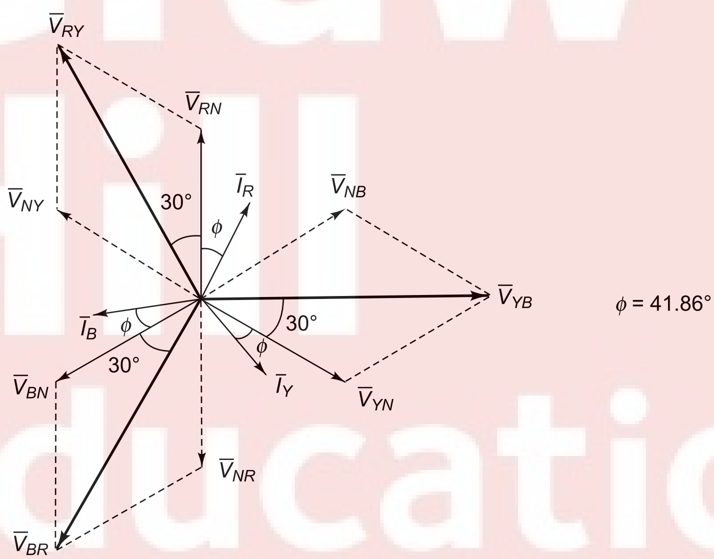


Fig. 5.13

If the same impedances are connected in delta,

(i) Line current

$$V_L = V_{ph} = 415 \text{ V}$$

$$Z_{ph} = 20.14\ \Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{20.14} = 20.61\ \text{A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 20.61 = 35.69\ \text{A}$$

(ii) Power consumed

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 415 \times 35.69 \times 0.744 = 19.09\ \text{kW}$$

Example 8

Each phase of a delta-connected load consists of a 50 mH inductor in series with a parallel combination of a 50 Ω resistor and a 50 μF capacitor. The load is connected to a three-phase, 550 V, 800 rad/s ac supply. Find the (i) power factor, (ii) phase current, (iii) line current, (iv) power drawn, (v) reactive power, and (vi) kVA rating of the load.

Solution

$$L = 50\ \text{mH}$$
$$R = 50\ \Omega$$
$$C = 50\ \mu\text{F}$$
$$V_L = 550\ \text{V}$$
$$\omega = 800\ \text{rad/s}$$

For a delta-connected load,

(i) Power factor

$$X_L = \omega L = 800 \times 50 \times 10^{-3} = 40\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{800 \times 50 \times 10^{-6}} = 25\ \Omega$$

$$\bar{Z}_{ph} = jX_L + \frac{R(-jX_C)}{R - jX_C}$$

$$= j40 + \frac{50(-j25)}{50 - j25}$$

$$= 10 + j20 = 22.36\ \angle 63.43^\circ\ \Omega$$

$$Z_{ph} = 22.36\ \Omega$$

$$\phi = 63.43^\circ$$

$$pf = \cos\phi = \cos(63.43^\circ) = 0.447\ \text{(lagging)}$$

(ii) Phase current

$$V_L = V_{ph} = 550\ \text{V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{550}{22.36} = 24.6\ \text{A}$$

(iii) Line current

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3}\ \times 24.6 = 42.61\ \text{A}$$

(iv) Power drawn

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3}\ \times 550 \times 42.61 \times 0.447 = 18.14\ \text{kW}$$

(v) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3}\ \times 550 \times 42.61 \times \sin(63.43°) = 36.3\ \text{kVAR}$$

(vi) kVA rating of the load

$$S = \sqrt{3}\ V_L I_L = \sqrt{3}\ \times 550 \times 42.61 = 40.59\ \text{kVA}$$

Example 9

A balanced star-connected load is supplied from a symmetrical three-phase 400 volts, 50 Hz system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find the (i) phase voltage, (ii) resistance and reactance per phase, (iii) load inductance per phase, and (iv) total power consumed.

Solution

$$V_L = 400\ \text{V}$$
$$f = 50\ \text{Hz}$$
$$I_{ph} = 30\ \text{A}$$
$$\phi = 30°$$

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\ \text{V}$$

(ii) Resistance and reactance per phase

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{30} = 7.7\,\Omega$$

$$Z_{ph} = Z_{ph}\angle\phi = 7.7\angle 30° = (6.67 + j\,3.85)\ \Omega$$

$$R_{ph} = 6.67\ \Omega$$

$$X_{ph} = 3.85\ \Omega$$

(iii) Load inductance per phase

$$X_{ph} = 2\pi f L_{ph}$$

$$3.85 = 2\pi \times 50 \times L_{ph}$$

$$L_{ph} = 0.01225\ \text{H}$$

(iv) Total power consumed

$$P = 3V_{ph} I_{ph} \cos\phi = 3 \times 230.94 \times 30 \times \cos(30°) = 18\ \text{kW}$$

Example 10

A symmetrical three-phase 400 V system supplies a basic load of 0.8 lagging power factor and is connected in star. If the line current is 34.64 A, find the (i) impedance, (ii) resistance and reactance per phase, (iii) total power, and (iv) total reactive voltamperes.

Solution $V_L = 400$ V

$pf = 0.8$ (lagging)

$I_L = 34.64$ A

For a star-connected load,

(i) Impedance

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = I_L = 34.64 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{34.64} = 6.67\ \Omega$$

(ii) Resistance and reactance per phase

$$pf = \cos\phi = 0.8$$

$$\phi = \cos^{-1}(0.8) = 36.87°$$

$$Z_{ph} = Z_{ph}\angle\phi = 6.67\ \angle 36.87° = (5.33 + j\,4)\ \Omega$$

$$R_{ph} = 5.33\ \Omega$$

$$X_{ph} = 4\ \Omega$$

(iii) Total power

$$P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 34.64 \times 0.8 = 19.19 \text{ kW}$$

(iv) Total reactive volt-amperes

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 400 \times 34.64 \times \sin(36.87°) = 14.4 \text{ kVAR}$$

Example 11

A balanced star-connected load is supplied by a 415 V, 50 Hz three-phase system. Current in each phase is 20 A and lags 30° behind its phase voltage. Find the (i) phase voltage, (ii) power, and (iii) circuit parameters. Also, find power consumed when the same load is connected in delta across the same supply.

Solution $V_L = 415$ V

$f = 50$ Hz

$I_{ph} = 20$ A

$\phi = 30°$

For a star-connected load,

(i) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

(ii) Power

$$I_L = I_{ph} = 20 \text{ A}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 415 \times 20 \times \cos(30°) = 12.45 \text{ kW}$$

(iii) Circuit parameters

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{239.6}{20} = 11.98\ \Omega$$

$$\bar{Z}_{ph} = Z_{ph} \angle\phi = 11.98 \angle 30° = (10.37 + j6)\ \Omega$$

$$R_{ph} = 10.37\ \Omega$$

$$X_{ph} = 6\ \Omega$$

$$X_{ph} = 2\pi f L_{ph}$$

$$6 = 2\pi \times 50 \times L_{ph}$$

$$L_{ph} = 19.1 \text{ mH}$$

(iv) Power consumed by same delta load across the same supply

$$P_\Delta = 3P_Y = 3 \times 12.45 \times 10^3 = 37.35 \text{ kW}$$

Example 12

Three identical coils connected in delta to a 440 V, three-phase supply take a total power of 50 kW and a line current of 90 A. Find the (i) phase current, (ii) power factor, and (iii) apparent power taken by the coils.

Solution

$$V_L = 440 \text{ V}$$

$$P = 50 \text{ kW}$$

$$I_L = 90 \text{ A}$$

For a delta-connected load,

(i) Phase current

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{90}{\sqrt{3}} = 51.96 \text{ A}$$

(ii) Power factor

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$50 \times 10^3 = \sqrt{3} \times 440 \times 90 \times \cos\phi$$

$$pf = \cos\phi = 0.73 \text{ (lagging)}$$

(iii) Apparent power

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 440 \times 90 = 68.59 \text{ kVA}$$

Example 13

Three similar choke coils are connected in star to a three-phase supply. If the line current is 15 A, the total power consumed is 11 kW and the volt-ampere input is 15 kVA, find the line and phase voltages, the VAR input and the reactance and resistance of each coil. If these coils are now connected in delta to the same supply, calculate phase and line currents, active and reactive power.

Solution $I_L = 15$ A

$$P = 11 \text{ kW}$$

$$S = 15 \text{ kVA}$$

For a star-connected load,

(i) Line voltage

$$S = \sqrt{3}\ V_L I_L$$

$$15 \times 10^3 = \sqrt{3} \times V_L \times 15$$

$$V_L = 577.35 \text{ V}$$

(ii) Phase voltage

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{577.35}{\sqrt{3}} = 333.33 \text{ V}$$

(iii) VAR input

$$\cos\phi = \frac{P}{S} = \frac{11 \times 10^3}{15 \times 10^3} = 0.733$$

$$\phi = 42.86°$$

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 577.35 \times 15 \times \sin(42.86°) = 10.2 \text{ kVAR}$$

(iv) Reactance and resistance of coil

$$I_{ph} = I_L = 15 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{333.33}{15} = 22.22\ \Omega$$

$$R = Z_{ph}\cos\phi = 22.22 \times 0.733 = 16.29\ \Omega$$

$$X_L = Z_{ph}\sin\phi = 22.22 \times \sin(42.86°) = 15.11\ \Omega$$

If these coils are now connected in delta,

(i) Phase current

$$V_{ph} = V_L = 577.35 \text{ V}$$

$$Z_{ph} = 22.22\ \Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{577.35}{22.22} = 25.98 \text{ A}$$

(ii) Line current

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 25.98 = 45 \text{ A}$$

(iii) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 577.35 \times 45 \times 0.733 = 32.98 \text{ kW}$$

(iv) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 577.35 \times 45 \times \sin(42.86°) = 30.61 \text{ kVAR}$$

Example 14

A three-phase, star-connected source feeds 1500 kW at 0.85 power factor lag to a balanced mesh-connected load. Calculate the current, its active and reactive components in each phase of the source and the load. The line voltage is 2.2 kV.

Solution $P = 1500$ kW

$pf = 0.85$ (lagging)

$V_L = 2.2$ kV

For a mesh or delta-connected load,

(i) Line current

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$1500 \times 10^3 = \sqrt{3} \times 2.2 \times 10^3 \times I_L \times 0.85$$

$$I_L = 463.12 \text{ A}$$

(ii) Active component of current in each phase of the load

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{463.12}{\sqrt{3}} = 267.38 \text{ A}$$

$$I_{ph} \cos\phi = 267.38 \times 0.85 = 227.27 \text{ A}$$

(iii) Reactive component of current in each phase of the load

$$I_{ph} \sin\phi = 267.38 \times \sin(\cos^{-1} 0.85)$$
$$= 267.38 \times 0.526 = 140.85 \text{ A}$$

For a star-connected source, the phase current in the source will be the same as the line current drawn by the load.

(iv) Active component of this current in each phase of the source

$$I_L \cos\phi = 463.12 \times 0.85 = 393.65 \text{ A}$$

(v) Reactive component of this current in each phase of the source

$$I_L \sin\phi = 463.12 \times 0.526 = 243.6 \text{ A}$$

Example 15

A three-phase, 208-volt generator supplies a total of 1800 W at a line current of 10 A when three identical impedances are arranged in a Wye connection across the line terminals of the generator. Compute the resistive and reactive components of each phase impedance.

Solution

$$V_L = 208 \text{ V}$$
$$P = 1800 \text{ W}$$
$$I_L = 10 \text{ A}$$

For a Wye-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120.09 \text{ V}$$
$$I_{ph} = I_L = 10 \text{ A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{120.09}{10} = 12\ \Omega$$
$$P = \sqrt{3}\ V_L I_L \cos\phi$$
$$1800 = \sqrt{3} \times 208 \times 10 \times \cos\phi$$
$$\cos\phi = 0.5$$
$$\phi = 60°$$
$$R_{ph} = Z_{ph}\cos\phi = 12 \times 0.5 = 6\ \Omega$$
$$X_{ph} = Z_{ph}\sin\phi = 12 \times \sin(60°) = 10.39\ \Omega$$

Example 16

A balanced, three-phase, star-connected load of 100 kW takes a leading current of 80 A, when connected across a three-phase, 1100 V, 50 Hz supply. Find the circuit constants of the load per phase.

Solution

$$P = 100 \text{ kW}$$
$$I_L = 80 \text{ A}$$
$$V_L = 1100 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1100}{\sqrt{3}} = 635.08 \text{ V}$$
$$I_{ph} = I_L = 80 \text{ A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{635.08}{80} = 7.94\ \Omega$$

$$P = \sqrt{3}\, V_L\, I_L \cos \phi$$

$$100 \times 10^3 = \sqrt{3} \times 1100 \times 80 \times \cos \phi$$

$$\cos \phi = 0.656 \text{ (leading)}$$

$$\phi = 49°$$

$$R_{ph} = Z_{ph} \cos \phi = 7.94 \times 0.656 = 5.21\ \Omega$$

$$X_{ph} = Z_{ph} \sin \phi = 7.94 \times \sin (49°) = 6\ \Omega$$

This reactance will be capacitive in nature as the current is leading.

$$X_C = \frac{1}{2\pi fC}$$

$$6 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 530.52\ \mu\text{F}$$

Example 17

Three identical impedances are connected in delta to a three-phase supply of 400 V. The line current is 34.65 A, and the total power taken from the supply is 14.4 kW. Calculate the resistance and reactance values of each impedance.

Solution $V_L = 400$ V

$$I_L = 34.65 \text{ A}$$

$$P = 14.4 \text{ kW}$$

For a delta-connected load,

$$V_L = V_{ph} = 400 \text{ V}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{34.65}{\sqrt{3}} = 20 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{20} = 20\ \Omega$$

$$P = \sqrt{3}\, V_L\, I_L \cos \phi$$

$$14.4 \times 10^3 = \sqrt{3} \times 400 \times 34.65 \times \cos \phi$$

$$\cos \phi = 0.6$$

$$\phi = 53.13°$$

$$R_{ph} = Z_{ph} \cos \phi = 20 \times 0.6 = 12\ \Omega$$

$$X_{ph} = Z_{ph} \sin \phi = 20 \times \sin (53.13°) = 16\ \Omega$$

Example 18

A balanced, three-phase load connected in delta draws a power of 10.44 kW at 200 V at a power factor of 0.5 lead. Find the values of the circuit elements and the reactive volt-amperes drawn.

Solution $P = 10.44 \text{ kW}$

$V_L = 200 \text{ V}$

$pf = 0.5 \text{ (leading)}$

For a delta-connected load,

(i) Values of circuit elements

$$V_L = V_{ph} = 200 \text{ V}$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$10.44 \times 10^3 = \sqrt{3} \times 200 \times I_L \times 0.5$$

$$I_L = 60.28 \text{ A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{60.28}{\sqrt{3}} = 34.8 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{200}{34.8} = 5.75\ \Omega$$

$$R_{ph} = Z_{ph} \cos\phi = 5.75 \times 0.5 = 2.875\ \Omega$$

$$X_{ph} = Z_{ph} \sin\phi = 5.75 \times \sin(\cos^{-1} 0.5) = 5.747 \times 0.866 = 4.98\ \Omega$$

(ii) Reactive volt-amperes drawn

$$Q = \sqrt{3}\, V_L\, I_L \sin\phi = \sqrt{3} \times 200 \times 60.28 \times 0.866 = 18.08 \text{ kVAR}$$

Example 19

Each leg of a balanced, delta-connected load consists of a 7 Ω resistance in series with a 4 Ω inductive reactance. The line-to-line voltages are

$$E_{ab} = 2360 \angle 0° \text{ V}$$

$$E_{bc} = 2360 \angle -120° \text{ V}$$

$$E_{ca} = 2360 \angle 120° \text{ V}$$

Determine (i) phase current I_{ab}, I_{bc} and I_{ca} (both magnitude and phase)

(ii) each line current and its associated phase angle

(iii) the load power factor

Solution $R = 7\ \Omega$

$X_L = 4\ \Omega$

$V_L = 2360 \text{ V}$

For a delta-connected load,

(i) Phase current

$$V_{ph} = V_L = 2360 \text{ V}$$

$$\bar{Z}_{ph} = 7 + j4 = 8.06 \angle 29.74° \ \Omega$$

$$\bar{I}_{ab} = \frac{\bar{E}_{ab}}{\bar{Z}_{ph}} = \frac{2360\angle 0°}{8.06\angle 29.74°} = 292.8 \angle -29.74° \text{ A}$$

$$\bar{I}_{bc} = \frac{\bar{E}_{bc}}{\bar{Z}_{ph}} = \frac{2360\angle -120°}{8.06\angle 29.74°} = 292.8 \angle -149.71° \text{ A}$$

$$\bar{I}_{ca} = \frac{\bar{E}_{ca}}{\bar{Z}_{ph}} = \frac{2360\angle 120°}{8.06\angle 29.74°} = 292.8 \angle 90.26° \text{ A}$$

(ii) Line current

In a delta-connected, three-phase system, line currents lag behind respective phase currents by 30°.

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 292.8 = 507.14 \text{ A}$$

$$\bar{I}_{La} = 507.14 \angle -59.71° \text{ A}$$

$$\bar{I}_{Lb} = 507.14 \angle -179.71° \text{ A}$$

$$\bar{I}_{Lc} = 507.14 \angle 60.26° \text{ A}$$

(iii) Load power factor

$$pf = \cos(29.74°) = 0.868 \text{ (lagging)}$$

Example 20

A three-phase, 200 kW, 50 Hz, delta-connected induction motor is supplied from a three-phase, 440 V, 50 Hz supply system. The efficiency and power factor of the three-phase induction motor are 91% and 0.86 respectively. Calculate (i) line currents, (ii) currents in each phase of the motor, (iii) active, and (iv) reactive components of phase current.

Solution

$$P_o = 200 \text{ kW}$$

$$V_L = 440 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$\eta = 91\%$$

$$pf = 0.86$$

For a delta-connected load (induction motor),

(i) Line current

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_o}{P_i}$$

$$0.91 = \frac{200 \times 10^3}{P_i}$$

$$P_i = 219.78 \text{ kW}$$

$$P_i = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$219.78 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.86$$

$$I_L = 335.3 \text{ A}$$

(ii) Currents in each phase of motor

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{335.3}{\sqrt{3}} = 193.6 \text{ A}$$

(iii) Active component of phase current

$$I_{ph} \cos\phi = 193.6 \times 0.86 = 166.5 \text{ A}$$

(iv) Reactive component of phase current

$$I_{ph} \sin\phi = 193.6 \times \sin(\cos^{-1} 0.86) = 193.6 \times 0.51 = 98.7 \text{ A}$$

Example 21

A three-phase, 400 V, star-connected alternator supplies a three-phase, 112 kW, mesh-connected induction motor of efficiency and power factor 0.88 and 0.86 respectively. Find the (i) current in each motor phase, (ii) current in each alternator phase, and (iii) active and reactive components of current in each case.

Solution

$$V_L = 400 \text{ V}$$

$$P_o = 112 \text{ kW}$$

$$\eta = 0.88$$

$$pf = 0.86$$

For a mesh-connected load (induction motor),

(i) Current in each motor phase

$$V_{ph} = V_L = 400 \text{ V}$$

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{P_o}{P_i}$$

$$0.88 = \frac{112 \times 10^3}{P_i}$$

$$P_i = 127.27 \text{ kW}$$

$$P_i = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$127.27 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.86$$

$$I_L = 213.6 \text{ A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{213.6}{\sqrt{3}} = 123.32 \text{ A}$$

Current in a star-connected alternator phase will be same as the line current drawn by the motor.

(ii) Current in each alternator phase

$$I_L = 213.6 \text{ A}$$

(iii) Active component of current in each phase of motor

$$I_{ph} \cos \phi = 123.32 \times 0.86 = 105.06 \text{ A}$$

Reactive component of current in each phase of the motor

$$I_{ph} \sin \phi = 123.32 \times \sin (\cos^{-1} 0.86) = 123.32 \times 0.51 = 62.89 \text{ A}$$

(iv) Active component of current in each alternator phase

$$I_L \cos \phi = 213.6 \times 0.86 = 183.7 \text{ A}$$

Reactive component of current in each alternator phase

$$I_L \sin \phi = 213.6 \times \sin (\cos^{-1} 0.86) = 213.6 \times 0.51 = 108.94 \text{ A}$$

Example 22

Three similar resistors are connected in star across 400 V, three-phase lines. The line current is 5 A. Calculate the value of each resistor. To what value should the line voltage be changed to obtain the same line current with the resistors connected in delta?

Solution $V_L = 400$ V

$$I_L = 5 \text{ A}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = I_L = 5 \text{ A}$$

$$Z_{ph} = R_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{5} = 46.19\ \Omega$$

For a delta-connected load,

$$I_L = 5 \text{ A}$$

$$R_{ph} = 46.19\ \Omega$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{5}{\sqrt{3}} \text{ A}$$

$$V_{ph} = I_{ph} R_{ph} = \frac{5}{\sqrt{3}} \times 46.19 = 133.33 \text{ V}$$

$$V_L = 133.33 \text{ V}$$

Voltage needed is one-third of the star value.

Example 23

Three 100 Ω, non-inductive resistors are connected in (a) star, and (b) delta across a 400 V, 50 Hz, three-phase supply. Calculate the power taken from the supply in each case. If one of the resistors is open circuited, what would be the value of total power taken from the mains in each of the two cases?

Solution

$$V_L = 400 \text{ V}$$

$$Z_{ph} = 100\ \Omega$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{100} = 2.31 \text{ A}$$

$$I_L = I_{ph} = 2.31 \text{ A}$$

$$\cos\phi = 1 \qquad \text{(For pure resistor, pf = 1)}$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 2.31 \times 1 = 1600.41 \text{ W}$$

For a delta-connected load,

$$V_{ph} = V_L = 400 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{100} = 4 \text{ A}$$

$$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 4 = 6.93 \text{ A}$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 6.93 \times 1 = 4801.24 \text{ W}$$

When one of the resistors is open circuited

(i) Star connection The circuit consists of two 100 Ω resistors in series across a 400 V supply.

$$\text{Currents in lines } A \text{ and } C = \frac{400}{200} = 2 \text{ A}$$

$$\text{Power taken from the mains} = 400 \times 2 = 800 \text{ W}$$

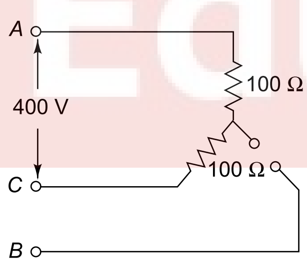


Fig. 5.14(a) *Star connection*

Hence, when one of the resistors is open circuited, the power consumption is reduced by half.

(ii) Delta connection In this case, currents in A and C remain as usual 120° out of phase with each other.

$$\text{Current in each phase} = \frac{400}{100} = 4 \text{ A}$$

Power taken from the mains $= 2 \times 4 \times 400 = 3200$ W

Hence, when one of the resistors is open circuited, the power consumption is reduced by one-third.

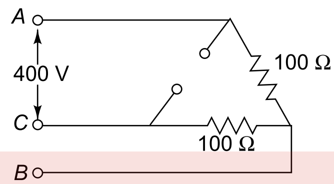


Fig. 5.14(b) *Delta connection*

Example 24

Three identical impedances of 10 ∠30° Ω each are connected in star and another set of three identical impedances of 18 ∠60° Ω are connected in delta. If both the sets of impedances are connected across a balanced, three-phase 400 V supply, find the line current, total volt-amperes, active power and reactive power.

Solution

$$\bar{Z}_Y = 10 \angle 30° \ \Omega$$
$$\bar{Z}_\Delta = 18 \angle 60° \ \Omega$$
$$V_L = 400 \text{ V}$$

Three identical delta impedances can be converted into equivalent star impedances.

$$\bar{Z}'_Y = \frac{\bar{Z}_\Delta}{3} = \frac{18\angle 60°}{3} = 6\angle 60° \ \Omega$$

Now two star-connected impedances of 10 ∠30° Ω and 6 ∠60° Ω are connected in parallel across a three-phase supply.

$$\bar{Z}_{eq} = \frac{(10\angle 30°)(6\angle 60°)}{10\angle 30° + 6\angle 60°} = 3.87 \angle 48.83° \ \Omega$$

For a star-connected load,

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_{ph}}{Z_{eq}} = \frac{230.94}{3.87} = 59.67 \text{ A}$$

$$I_L = I_{ph} = 59.67 \text{ A}$$

(ii) Total volt-amperes

$$S = \sqrt{3}\ V_L I_L = \sqrt{3} \times 400 \times 59.67 = 41.34 \text{ kVA}$$

(iii) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 400 \times 59.67 \times \cos(48.83°) = 27.21 \text{ kW}$$

(iv) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 400 \times 59.67 \times \sin(48.83°) = 31.12 \text{ kVAR}$$

Example 25

Three star-connected impedances $Z_Y = (20 + j37.7)\ \Omega$ per phase are connected in parallel with three delta-connected impedances $Z_\Delta = (30 - j159.3)\ \Omega$ per phase. The line voltage is 398 V. Find the line current, pf, active and reactive power taken by the combination.

Solution

$$\bar{Z}_Y = 20 + j37.7 = 42.68\ \angle 62.05°\ \Omega$$
$$\bar{Z}_\Delta = 30 - j159.3 = 162.1\ \angle -79.3°\ \Omega$$
$$V_L = 398\text{ V}$$

Three identical delta-connected impedances can be converted into equivalent star impedances.

$$\bar{Z}'_Y = \frac{162.1\angle -79.3°}{3} = 54.03\ \angle -79.3°\ \Omega$$

Now two star-connected impedances of 42.68 $\angle 62.05°\ \Omega$ and 54.03 $\angle -79.3°\ \Omega$ are connected in parallel across the three-phase supply.

$$\bar{Z}_{\text{eq}} = \frac{(42.68\angle 62.05°)(54.03\angle -79.3°)}{42.68\angle 62.05° + 54.03\angle -79.3°)} = 68.33\angle 9.88°\ \Omega$$

For a star-connected load,

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{398}{\sqrt{3}} = 229.79\text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_{ph}}{Z_{eq}} = \frac{229.79}{68.33} = 3.36\text{ A}$$

$$I_L = I_{ph} = 3.36\text{ A}$$

(ii) Power factor

$$pf = \cos\phi = \cos(9.88°) = 0.99\text{ (lagging)}$$

(iii) Active power

$$P = \sqrt{3}\ V_L I_L \cos\phi = \sqrt{3} \times 398 \times 3.36 \times 0.99 = 2.29\text{ kW}$$

(iv) Reactive power

$$Q = \sqrt{3}\ V_L I_L \sin\phi = \sqrt{3} \times 398 \times 3.36 \times \sin(9.88°) = 397.43\text{ VAR}$$

Example 26

Three coils, each having a resistance of 20 Ω and a reactance of 15 Ω, are connected in star to a 400 V, three-phase, 50 Hz supply. Calculate (i) line current, (ii) power supplied, and (iii) power factor. If three capacitors, each of same capacitance, are connected in delta to the same supply so as to form a parallel circuit with the above coils, calculate the capacitance of each capacitor to obtain a resultant power factor of 0.95 lagging.

Solution

$$R_{ph} = 20\ \Omega$$
$$X_{ph} = 15\ \Omega$$
$$V_L = 400\text{ V}$$

For a star-connected load,

(i) Line current

$$\bar{Z}_{ph} = R_{ph} + jX_{ph} = 20 + j15 = 25\ \angle 36.87^\circ\ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{25} = 9.24\text{ A}$$

$$I_L = I_{ph} = 9.24\text{ A}$$

(ii) Power supplied

$$P_1 = \sqrt{3}\ V_L\ I_L \cos\phi_1 = \sqrt{3} \times 400 \times 9.24 \times \cos(36.87^\circ) = 5.12\text{ kW}$$

(iii) Power factor

$$pf = \cos\phi_1 = \cos(36.87^\circ) = 0.8 \text{ (lagging)}$$

(iv) Value of capacitance of each capacitor

$$Q_1 = \sqrt{3}\ V_L\ I_L \sin\phi_1 = \sqrt{3} \times 400 \times 9.24 \times \sin(36.87^\circ) = 3.84\text{ kVAR}$$

When capacitors are connected in delta to the same supply

$$pf = 0.95$$

$$\phi_2 = \cos^{-1}(0.95) = 18.19^\circ$$

$$\tan\phi_2 = \tan(18.19^\circ) = 0.33$$

Since capacitors do not absorb any power, power remains the same even when capacitors are connected. But reactive power changes.

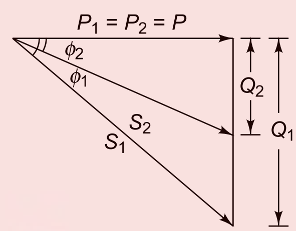


Fig. 5.15

$$P_2 = 5.12\text{ kW}$$

$$Q_2 = P_2 \tan\phi_2 = 5.12 \times 0.33 = 1.69\text{ kVAR}$$

Difference in reactive power is supplied by three capacitors.

$$Q = Q_1 - Q_2 = 3.84 - 1.69 = 2.15\text{ kVAR}$$

$$Q = \sqrt{3}\ V_L\ I_L \sin\phi$$

$$2.15 \times 10^3 = \sqrt{3} \times 400 \times I_L \times \sin(90^\circ)$$

$$I_L = 3.1\text{ A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = 1.79\text{ A}$$

$$I_{ph} = \frac{V_{ph}}{X_C} = V_{ph} \times 2\pi f C$$

$$C = \frac{I_{ph}}{V_{ph} \times 2\pi f} = \frac{1.79}{400 \times 2\pi \times 50} = 14.24\ \mu\text{F}$$

Useful Formulae

Star Connection

$$V_L = \sqrt{3}\, V_{ph}$$

$$I_L = I_{ph}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}}$$

$$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

$$Q = 3V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$$

$$S = 3V_{ph} I_{ph} = \sqrt{3}\, V_L I_L$$

Delta Connection

$$I_L = \sqrt{3}\, I_{ph}$$

$$V_L = V_{ph}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}}$$

$$P = 3V_{ph} I_{ph} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi$$

$$Q = 3V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi$$

$$S = 3V_{ph} I_{ph} = \sqrt{3}\, V_L I_L$$

Exercise 5.1

5.1 Three coils, each of 5 Ω resistance, and 6 Ω inductive reactance are connected in closed delta and supplied from a 440 V, three-phase system. Calculate the line and phase currents, the power factor of the system and the intake in watts.
[97.58 A, 56.33 A, 0.64 (lagging), 47.61 kW]

5.2 Three coils each having a resistance of 10 Ω and inductance of 0.02 H are connected (i) in star, (ii) in delta to a three-phase, 50 Hz supply, the line voltage being 500 volts. Calculate for each case the line current and the total power taken from the supply. *[(i) star : 24.46 A, 17.94 kW, (ii) delta : 73.39 A, 53.83 kW]*

5.3 A balanced delta-connected load of (8 + *j*6) ohms per phase is connected to a three-phase, 230 V supply with phase sequence *R-Y-B*. Find the line current, power factor, power, reactive volt-amperes and the total volt-amperes. Draw the phasor diagram. *[39.85 A, 0.8 (lagging), 12.74 kW, 9.52 kVAR, 15.86 kVA]*

5.4 A balanced star-connected load with (6 + *j*8)Ω per phase is connected to a three-phase, 440 V supply. Find the line current, power factor, power, reactive volt-amperes and volt-amperes total.
[25.404 A, 0.6(lagging), 11.616 kW, 15.488 kVAR, 19.360 kVA]

5.5 Calculate the active and reactive components of the current in each phase of a star-connected 5000 V, 3-phase, alternator supplying 3000 kW at a power factor of 0.8.
[250 A, 150A]

5.6 Three similar coils, connected in star, take a total power of 1.5 kW at a power factor of 0.2 lagging from a 3 phase, 440 V, 50 Hz supply. Calculate the resistance and inductance of each coil. *[5.16 Ω, 0.08 H]*

5.7 A balanced three-phase load connected in delta draws a power of 10 kW at 440 V at a power factor of 0.6 lead. Find the values of the circuit elements and the reactive volt-amperes drawn. *[20.91 Ω, 27.88 Ω, 13.33 kVAR]*

5.8 A balanced star-connected load, connected to a 400 V, 50 Hz, three-phase ac supply draws a phase current of 50 A at 0.6 power factor lagging. Calculate (i) phase voltage, (ii) total power, and (iii) parameters in the star-connected load. *[230.94 V, 20.84 kW, (2.77 + j3.7) Ω]*

5.9 Three equal star-connected inductors consume 8 kW power at 0.8 power factor when connected to 415 V, 3-phase, 3-wire, 50 Hz supply. Estimate the load parameters per phase and determine the line currents. *[6.2 Ω, 0.0386 H, 13.83A]*

5.10 A balanced three-phase, star-connected load of 150 kW takes a leading current of 100 A with a line voltage of 1100 V, 50 Hz. Find the circuit constant of the load per phase. *[5 Ω, 813 μF]*

5.11 Three pure elements connected in star draw *x* kVAR. What will be the value of elements that will draw the same kVAR, when connected in delta across the same supply? $[Z_\Delta = 3Z_Y]$

5.12 A balanced Wye-connected load with (10 + *j*20) ohms per phase is connected to a three-phase, 400 V supply. Determine the voltage across, current through and power dissipated in each resistor. Also, determine the total power. *[103.2 V, 10.32 V, 1067 W, 3201 W]*

5.13 A delta-connected three-phase load is supplied from a 3-phase, 400 volts balanced supply system. The line current is 20 A and power taken by the load is 10 kW. Find (i) impedance in each branch, (ii) line current, power factor and power consumed if the same load is connected in star. *[(24.95 + j24.05) Ω, 6.66 A, 0.72(lagging), 3323.21 W]*

5.14 A balanced star-connected load is supplied from a symmetrical 3-phase, 400 V system. The current in each phase is 30 A and lags 30° behind the phase voltage. Find (i) phase voltage, (ii) the circuit elements, and (iii) draw the vector diagram showing the currents and the voltages. *[230.94 V, 6.67 Ω, 3.849Ω]*

5.15 A 3-phase, delta-connected load having a (3 + *j*4) ohms per phase is connected across a 230 V, 3-phase source. Calculate the magnitude of the line current. *[76.21 A]*

5.16 A 220 V, 3-phase voltage is applied to a balanced delta-connected load. The rms value of the phase current is 20 ∠–30°A. Determine

(i) magnitude and phase of the line current
(ii) total power received by the three-phase load
(iii) value of the resistive portion of the phase impedance

Also, draw the phasor diagram showing clearly the line voltages, phase current and line currents. *[34.65 ∠–60°A, 11.43 kW, 9.53 Ω]*

5.17 A 3-phase, 37.3 kW, 440 V, 50 Hz induction motor operates on full load with an efficiency of 89% and at a power factor of 0.85 lagging. Calculate total kVA rating of capacitance required to raise power factor at 0.95 lagging. What will be the value of capacitance/phase if capacitors are (i) delta connected? (ii) star connected?

[12.19 kVA, 66.8 μF, 200.4 μF]

5.18 Three coils each having impedance $(4 + j3)$ ohms are connected in star to a 440 V, three-phase, 50 Hz balanced supply. Calculate the line current and active power. Now if three pure capacitors, each of C farads, connected in delta, are connected across the same supply, it is found that the total power factor of the circuit becomes 0.96 lag. Find the value of C. Also, find the total line current.

[50.8 A, 30.976 kW, 77.75 μF, 42.34 A]

5.12 MEASUREMENT OF THREE-PHASE POWER

In a three-phase system, total power is the sum of powers in three phases. The power is measured by wattmeter. It consists of two coils: (i) Current coil, and (ii) Voltage coil. Current coil is connected in series with the load and it senses current. Voltage coil is connected across supply terminals and it senses voltages.

There are three methods to measure three-phase power:

1. Three-wattmeter method
2. Two-wattmeter method
3. One-wattmeter method

5.12.1 Three-wattmeter Method

This method is used for balanced as well as unbalanced loads. Three wattmeters are inserted in each of the three phases of the load whether star connected or delta connected. Each wattmeter will measure the power consumed in each phase.

For balanced load, $W_1 = W_2 = W_3$

For unbalanced load, $W_1 \neq W_2 \neq W_3$

Total power $P = W_1 + W_2 + W_3$

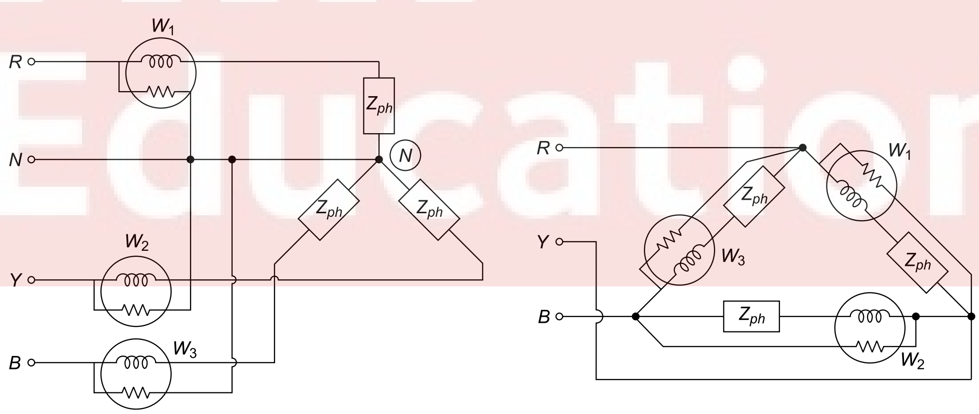


Fig. 5.16 *Three-wattmeter method*

5.12.2 Two-Wattmeter Method

This method is used for balanced as well as unbalanced load. The current coils of the two wattmeters are inserted in any two lines and the voltage coil of each wattmeter is joined to a third line. The load may be star or delta connected. The sum of the two wattmeter readings gives three-phase power.

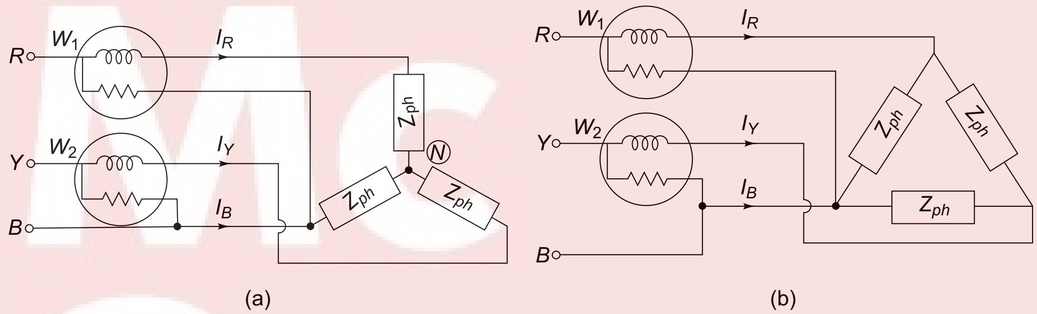


Fig. 5.17 *Two-wattmeter method*

Total power $\quad P = W_1 + W_2$

5.12.3 One-Wattmeter Method

This method is used for balanced load only. When load is balanced, total power is given by

$$P = 3\, V_{ph}\, I_{ph} \cos \phi$$

Hence, one wattmeter is used to measure power in one phase. The wattmeter reading is then multiplied by three to obtain three phase power.

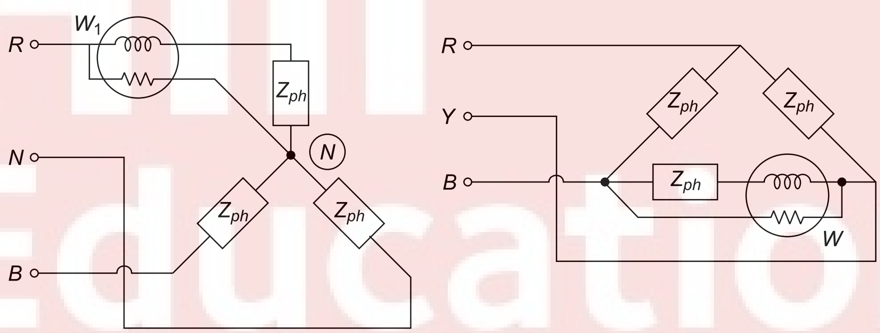


Fig. 5.18 *One-wattmeter method*

5.12.4 Measurement of Power by Two-wattmeter Method

Figure 5.17 shows a balanced star-connected load, the load may be assumed to be inductive. Let V_{RN}, V_{YN} and V_{BN} be the three phase voltages. I_R, I_Y and I_B be the phase currents. The phase currents will lag behind their respective phase voltages by angle ϕ.

Current through current coil of $W_1 = I_R$

Voltage across voltage coil of $W_1 = V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$

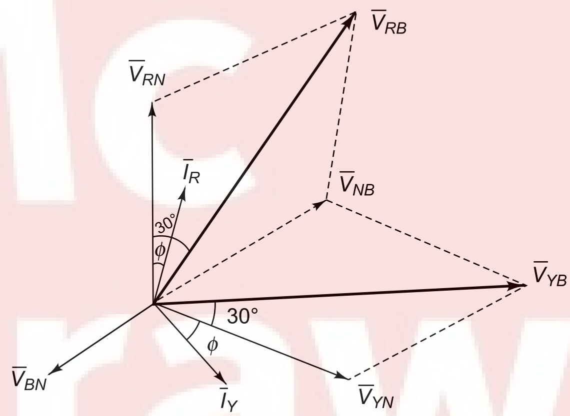


Fig. 5.19 *Phasor diagram*

From phasor diagram, it is clear that the phase angle between V_{RB} and I_R is $(30° - \phi)$

$$W_1 = V_{RB}\, I_R \cos (30° - \phi)$$

Current through current coil of $W_2 = I_Y$

Voltage across voltage coil of $W_2 = V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

From phasor diagram, it is clear that phase angle between V_{YB} and I_Y is $(30° + \phi)$

$$W_2 = V_{YB}\, I_Y \cos (30° + \phi)$$

But

$$I_R = I_Y = I_L$$

$$V_{RB} = V_{YB} = V_L$$

$$W_1 = V_L\, I_L \cos (30° - \phi)$$

$$W_2 = V_L\, I_L \cos (30° + \phi)$$

$$\begin{aligned} W_1 + W_2 &= V_L\, I_L\, [\cos (30° + \phi) + \cos (30° - \phi)] \\ &= V_L\, I_L\, (2 \cos 30° \cos \phi) = \sqrt{3}\, V_L\, I_L \cos \phi \end{aligned}$$

Thus, the sum of two wattmeter readings gives three-phase power.

5.12.5 Measurement of Power Factor by Two-wattmeter Method

(i) Lagging power factor

$$W_1 = V_L I_L \cos (30° - \phi)$$

$$W_2 = V_L I_L \cos (30° + \phi)$$

$$\therefore W_1 > W_2$$

$$W_1 + W_2 = \sqrt{3}\ V_L I_L \cos \phi$$

$$W_1 - W_2 = V_L I_L [\cos (30° - \phi) - \cos (30° + \phi) = V_L I_L \sin \phi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\tan \phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left(\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\text{pf} = \cos \phi = \cos \left\{\tan^{-1}\left(\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

(ii) Leading power factor

$$W_1 = V_L I_L \cos (30° + \phi)$$

$$W_2 = V_L I_L \cos (30° - \phi)$$

$$\therefore W_1 < W_2$$

$$W_1 + W_2 = \sqrt{3}\ V_L I_L \cos \phi$$

$$W_1 - W_2 = -V_L I_L \sin \phi$$

$$\tan \phi = -\sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}$$

$$\phi = \tan^{-1}\left\{-\sqrt{3}\frac{(W_1 - W_2)}{(W_1 + W_2)}\right\}$$

$$\text{pf} = \cos \phi = \cos \left\{\tan^{-1}\left(-\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

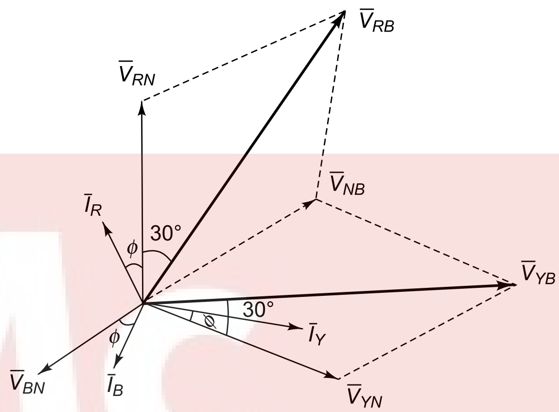


Fig. 5.20 *Phasor diagram*

Example 1

Two wattmeters are used to measure power in a three-phase balanced load. Find the power factor if (i) two readings are equal and positive, (ii) two readings are equal and opposite, and (iii) one wattmeter reads zero.

Solution (i) $W_1 = W_2$

(ii) $W_2 = 0$ $W_1 = -W_2$

(i) Power factor if two readings are equal and positive

$$W_1 = W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\ (0) = 0$$

$$\phi = 0°$$

$$\text{Power factor} = \cos\phi = \cos(0°) = 1$$

(ii) Power factor if two readings are equal and opposite

$$W_1 = -W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \infty$$

$$\phi = 90°$$

$$\text{Power factor} = \cos\phi = \cos(90°) = 0$$

(iii) Power factor if one wattmeter reads zero

$$W_2 = 0$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{W_1}{W_1}\right) = \sqrt{3}$$

$$\phi = 60°$$

$$\text{Power factor} = \cos\phi = \cos(60°) = 0.5$$

Example 2

What will be the relation between readings on the wattmeter connected to measure power in a three-phase balanced circuit with (i) unity power factor, (ii) zero power factor, and (iii) power factor = 0.5.

Solution (i) pf = 1

(ii) pf = 0

(iii) pf = 0.5

(i) Relation between wattmeter readings with power factor = 1

$$\cos\phi = 1$$

$$\phi = 0°$$

$$\tan\phi = \tan(0°) = 0$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$0 = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 = W_2$$

(ii) Relation between wattmeter readings with power factor = 0

$$\cos\phi = 0$$

$$\phi = 90°$$

$$\tan\phi = \tan(90°) = \infty$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 + W_2 = 0$$

$$W_1 = -W_2$$

(iii) Relation between wattmeter readings with power factor = 0.5

$$\cos\phi = 60°$$

$$\tan\phi = \tan(60°) = 1.732$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$1.732 = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$W_1 - W_2 = W_1 + W_2$$

$$W_2 = 0$$

Example 3

In a balanced three-phase system, the power is measured by two-wattmeter method and the ratio of two-wattmeter readings is 4:1. The load is inductive. Determine the load power factor.

Solution $$\frac{W_1}{W_2} = \frac{4}{1}$$

Load power factor

$$W_1 = 4W_2$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(3W_2)}{(5W_2)} = \sqrt{3}\left(\frac{3}{5}\right) = 1.039$$

$$\phi = 46.1°$$

$$\text{pf} = \cos\phi = \cos(46.1°) = 0.0693 \text{ (lagging)}$$

Example 4

Find the power and power factor of the balanced circuit in which the wattmeter readings are 5 kW and 0.5 kW, the latter being obtained after the reversal of the current coil terminals of the wattmeter.

Solution $$W_1 = 5 \text{ kW}$$

$$W_2 = 0.5 \text{ kW}$$

(i) Power

When the latter reading is obtained after the reversal of the current coil terminals of the wattmeter,

$$W_1 = 5 \text{ kW}$$

$$W_2 = -0.5 \text{ kW}$$

$$\text{Power} = W_1 + W_2 = 5 + (-0.5) = 4.5 \text{ kW}$$

(ii) Power factor

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(5 + 0.5)}{(5 - 0.5)} = 2.12$$

$$\phi = 64.72°$$

$$\text{Power factor} = \cos\phi = \cos(64.72°) = 0.43$$

Example 5

Two wattmeters connected to measure the input to a balanced, three-phase circuit indicate 2000 W and 500 W respectively. Find the power factor of the circuit (i) when both readings are positive and (ii) when the latter is obtained after reversing the connection to the current coil of one instrument.

Solution

$$W_1 = 2000 \text{ W}$$

$$W_2 = 500 \text{ W}$$

(i) Power factor of the circuit when both readings are positive

$$W_1 = 2000 \text{ W}$$

$$W_2 = 500 \text{ W}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(2000 - 500)}{(2000 + 500)} = 1.039$$

$$\phi = 46.102°$$

$$\text{Power factor} = \cos\phi = \cos(46.102°) = 0.693$$

(ii) Power factor of the circuit when the latter reading is obtained after reversing the connection to the current coil of one instrument

$$W_1 = 2000 \text{ W}$$

$$W_2 = -500 \text{ W}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{(2000 + 500)}{(2000 - 500)} = 2.887$$

$$\phi = 70.89°$$

$$\text{Power factor} = \cos\phi = \cos(70.89°) = 0.33$$

Example 6

A three-phase, 10 kVA load has a power factor of 0.342. The power is measured by the two-wattmeter method. Find the reading of each wattmeter when the (i) power factor is leading, and the (ii) power factor is lagging.

Solution

$$S = 10 \text{ kVA}$$

$$pf = 0.342$$

$$S = \sqrt{3}\,V_L I_L$$

$$10 \times 10^3 = \sqrt{3}\,V_L I_L$$

$$V_L I_L = 5.77 \text{ kVA}$$
$$\cos \phi = 0.342$$
$$\phi = 72°$$

(i) Reading of each wattmeter when the power factor is leading

$$W_1 = V_L I_L \cos (30° + \phi) = 5.77 \cos (30° + 70°) = -1 \text{ kW}$$
$$W_2 = V_L I_L \cos (30° - \phi) = 5.77 \cos (30° - 70°) = 4.42 \text{ kW}$$

(ii) Reading of each wattmeter when the power factor is lagging

$$W_1 = V_L I_L \cos (30° - \phi) = 4.42 \text{ kW}$$
$$W_2 = V_L I_L \cos (30° + \phi) = -1 \text{ kW}$$

Example 7

A three-phase, star-connected load draws a line curent of 20 A. The load kVA and kW are 20 and 11 respectively. Find the readings on each of the two wattmeters used to measure the three-phase power.

Solution

$$I_L = 20 \text{ A}$$
$$S = 20 \text{ kVA}$$
$$P = 11 \text{ kW}$$
$$S = \sqrt{3}\ V_L I_L$$
$$20 \times 10^3 = \sqrt{3}\ V_L I_L$$
$$V_L I_L = 11.55 \text{ kVA}$$
$$P = \sqrt{3}\ V_L I_L \cos \phi$$
$$11 \times 10^3 = 20 \times 10^3 \times \cos \phi$$
$$\cos \phi = 0.55$$
$$\phi = 56.63°$$
$$W_1 = V_L I_L \cos (30° - \phi) = 11.55 \times \cos (30° - 56.63°) = 10.32 \text{ kW}$$
$$W_2 = V_L I_L \cos (30° + \phi) = 11.55 \times \cos (30° + 56.63) = 0.68 \text{ kW}$$

Example 8

Calculate the total power and readings of the two wattmeters connected to measure power in three-phase balanced load, if the reactive power is 15 kVAR and load pf is 0.8 lagging.

Solution

$$Q = 15 \text{ kVAR}$$
$$\text{pf} = 0.8 \text{ (lagging)}$$

(i) Readings of the two wattmeters

$$\cos\phi = 0.8$$
$$\phi = 36.87°$$
$$Q = \sqrt{3}\ V_L I_L \sin\phi$$
$$15\times10^3 = \sqrt{3}\ V_L I_L \sin(36.87°)$$
$$V_L I_L = 14.43 \text{ kVA}$$
$$W_1 = V_L I_L \cos(30° - \phi) = 14.43\times10^3\times\cos(30° - 36.87°) = 14.03 \text{ kW}$$
$$W_2 = V_L I_L \cos(30° + \phi) = 14.43\times10^3\times\cos(30° + 36.87°) = 5.67 \text{ kW}$$

(ii) Total power

$$P = W_1 + W_2 = 14.03 + 5.67 = 19.7 \text{ kW}$$

Example 9

Two wattmeters are connected to measure power in a three-phase circuit. The reading of one of the wattmeters is 5 kW when the load power factor is unity. If the power factor of the load is changed to 0.707 lagging without changing the total input power, calculate the readings of the two wattmeters.

Solution (i) pf = 1, $W_1 = 5$ kW
(ii) pf = 0.707 (lagging)

(i) When the power factor is unity,

$$W_1 = W_2 = 5 \text{ kW}$$
$$\text{Power} = W_1 + W_2 = 5 + 5 = 10 \text{ kW} \qquad (1)$$

(ii) When the power factor is changed to 0.707 lagging,

$$\text{pf} = \cos\phi = 0.707 \text{ (lagging)}$$
$$\phi = 45°$$
$$\tan\phi = \tan(45°) = 1$$
$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$
$$1 = \sqrt{3}\,\frac{W_1 - W_2}{10}$$
$$W_1 - W_2 = \frac{10}{\sqrt{3}} = 5.77 \text{ kW} \qquad (2)$$

Solving Eq. (1) and (2),

$$W_1 = 7.89 \text{ kW}$$
$$W_2 = 2.11 \text{ kW}$$

Example 10

A three-phase RYB system has effective line voltage of 173.2 V. Wattmeters in line R and Y read 301 W and 1327 W respectively. Find the impedance of the balanced star-connected load.

Solution

$$V_L = 173.2 \text{ V}$$
$$W_1 = 301 \text{ W}$$
$$W_2 = 1327 \text{ W}$$

If the load is capacitive and pf is leading, then $W_1 < W_2$.

$$\tan\phi = -\sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = -\sqrt{3}\frac{(301-1327)}{(301+1327)} = -1.09$$
$$\phi = 47.47°$$
$$P = W_1 + W_2 = 301 + 1327 = 1628 \text{ W}$$
$$P = \sqrt{3}\ V_L I_L \cos\phi$$
$$1628 = \sqrt{3} \times 173.2 \times I_L \times \cos(47.47°)$$
$$I_L = 8.03 \text{ A}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{173.2}{\sqrt{3}} = 100 \text{ V}$$
$$I_{ph} = I_L = 8.03 \text{ A}$$
$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{100}{8.03} = 12.45\ \Omega$$

Example 11

Three coils each with a resistance of 10 Ω and reactance of 10 Ω are connected in star across a three-phase, 50 Hz, 400 V supply. Calculate (i) line current, and (ii) readings on the two wattmeters connected to measure the power.

Solution

$$R = 10\ \Omega$$
$$X_L = 10\ \Omega$$
$$V_L = 400 \text{ V}$$

For a star-connected load,

(i) Line current

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$\bar{Z}_{ph} = R + jX_L = 10 + j10 = 14.14\ \angle 45^\circ\ \Omega$$

$$Z_{ph} = 14.14\ \Omega$$

$$\phi = 45^\circ$$

$$\text{Power factor} = \cos\phi = \cos(45^\circ) = 0.707 \text{ (lagging)}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{14.14} = 16.33\text{ A}$$

$$I_L = I_{ph} = 16.33\text{ A}$$

(ii) Readings on the two wattmeters

$$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 16.33 \times 0.707 = 7998.83\text{ W}$$

$$W_1 + W_2 = 7998.83\text{ W} \tag{1}$$

Also,

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan 45^\circ = \sqrt{3}\,\frac{W_1 - W_2}{7998.83}$$

$$W_1 - W_2 = 4618.13\text{ W} \tag{2}$$

Solving Eqs (1) and (2),

$$W_1 = 6308.48\text{ W}$$

$$W_2 = 1690.35\text{ W}$$

Example 12

Three coils each having a resistance of 20 Ω and reactance of 15 Ω are connected in (i) star, and (ii) delta, across a three-phase, 400 V, 50 Hz supply. Calculate in each case, the readings on two wattmeters connected to measure the power input.

Solution

$$R = 20\ \Omega$$

$$X_L = 15\ \Omega$$

$$V_L = 400\text{ V}$$

(i) Readings on two wattmeters for a star-connected load

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\text{ V}$$

$$\bar{Z}_{ph} = 20 + j15 = 25\ \angle 36.87^\circ\ \Omega$$

$$Z_{ph} = 25\ \Omega$$

$$\phi = 36.87^\circ$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{25} = 9.24 \text{ A}$$

$$I_L = I_{ph} = 9.24 \text{ A}$$

$$W_1 = V_L I_L \cos(30° - \phi) = 400 \times 9.24 \times \cos(30° - 36.87°) = 3669.46 \text{ W}$$

$$W_2 = V_L I_L \cos(30° + \phi) = 400 \times 9.24 \times \cos(30° + 36.87°) = 1451.86 \text{ W}$$

(ii) Readings on two wattmeters for a delta-connected load

$$V_{ph} = V_L = 400 \text{ V}$$

$$Z_{ph} = 25\ \Omega$$

$$\phi = 36.87°$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{25} = 16 \text{ A}$$

$$I_L = \sqrt{3}\ I_{ph} = \sqrt{3} \times 16 = 27.72 \text{ A}$$

$$W_1 = V_L I_L \cos(30° - \phi) = 400 \times 27.72 \times \cos(30° - 36.87°) = 11008.39 \text{ W}$$

$$W_2 = V_L I_L \cos(30° + \phi) = 400 \times 27.72 \times \cos(30° + 36.87°) = 4355.57 \text{ W}$$

Example 13

Two wattmeters connected to measure three-phase power for star-connected load reads 3 kW and 1 kW. The line current is 10 A. Calculate (i) line and phase voltage (ii) resistance and reactance per phase.

Solution

$$W_1 = 3 \text{ kW}$$

$$W_2 = 1 \text{ kW}$$

$$I_L = 10 \text{ A}$$

(i) Line and phase voltage

$$\tan\phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\frac{(3-1)}{(3+1)} = 0.866$$

$$\phi = 40.89°$$

Power factor = $\cos\phi = \cos(40.89°) = 0.756$

$$P = W_1 + W_2 = 3 + 1 = 4 \text{ kW}$$

$$P = \sqrt{3}\ V_L I_L \cos\phi$$

$$4 \times 10^3 = \sqrt{3} \times V_L \times 10 \times 0.756$$

$$V_L = 305.48 \text{ V}$$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{305.48}{\sqrt{3}} = 176.37 \text{ V}$$

(ii) Resistance and reactance per phase

$$I_{ph} = I_L = 10 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{176.37}{10} = 17.637\ \Omega$$

$$R = Z_{ph} \cos \phi = 17.637 \times 0.756 = 13.33\ \Omega$$

$$X_L = Z_{ph} \sin \phi = 17.637 \times \sin (40.89°) = 11.55\ \Omega$$

Example 14

The power input to a 2000 V, 50 Hz, three-phase motor running on full load at an efficiency of 90% is measured by two wattmeters which indicate 300 kW and 100 kW respectively. Calculate the (i) input power, (ii) power factor, and (iii) line current.

Solution

$$V_L = 2000 \text{ V}$$

$$\eta = 0.9$$

$$W_1 = 300 \text{ kW}$$

$$W_2 = 100 \text{ kW}$$

(i) Input power

$$P = W_1 + W_2 = 300 + 100 = 400 \text{ kW}$$

(ii) Power factor

$$\tan \phi = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\frac{(300 - 100)}{(300 + 100)} = 0.866$$

$$\phi = 40.89°$$

$$\text{pf} = \cos \phi = \cos (40.89°) = 0.76 \text{ (lagging)}$$

(iii) Line current

$$P = \sqrt{3}\ V_L I_L \cos \phi$$

$$400 \times 10^3 = \sqrt{3} \times 2000 \times I_L \times 0.76$$

$$I_L = 151.93 \text{ A}$$

Example 15

A three-phase, 400 V, 50 Hz induction motor has a full load output of 14.9 kW at which the efficiency and power factor are 0.88 and 0.8 respectively. What is the line current? Find the readings on the two wattmeters connected to measure the power input to the motor.

Solution

$$V_L = 400 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$P_o = 14.9 \text{ kW}$$

$$\text{pf} = 0.8$$
$$\eta = 0.88$$

(i) Line current

$$\eta = \frac{P_o}{P_i}$$
$$0.88 = \frac{14.9 \times 10^3}{P_i}$$
$$P_i = 16.93 \text{ kW}$$
$$\text{pf} = \cos\phi = 0.8$$
$$\phi = 36.87°$$
$$P_i = \sqrt{3}\ V_L I_L \cos\phi$$
$$16.93 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.8$$
$$I_L = 30.55 \text{ A}$$

(ii) Readings on the two wattmeters

$$W_1 = V_L I_L \cos(30° - \phi) = 400 \times 30.55 \times \cos(30° - 36.87°) = 12.13 \text{ kW}$$
$$W_2 = V_L I_L \cos(30° + \phi) = 400 \times 30.55 \times \cos(30° + 36.87°) = 4.8 \text{ kW}$$

Example 16

A three-phase, 220 V, 50 Hz, 11.2 kW induction motor has a full load efficiency of 88 per cent and draws a line current of 38 A under full load, when connected to a three-phase, 220 V supply. Determine power factor at which the motor is operating. Find the reading on two wattmeters connected in the circuit to measure the input to the motor.

Solution

$$V_L = 220 \text{ V}$$
$$P_o = 11.2 \text{ kW}$$
$$\eta = 88\%$$
$$I_L = 38 \text{ A}$$

(i) Power factor at which the motor is operating

$$\eta = \frac{P_o}{P_i}$$
$$0.88 = \frac{11.2 \times 10^3}{P_i}$$
$$P_i = 12.73 \text{ kW}$$

But
$$P_i = \sqrt{3}\ V_L I_L \cos\phi$$
$$12.73 \times 10^3 = \sqrt{3} \times 220 \times 38 \times \cos\phi$$

$$\text{pf} = \cos\phi = 0.88 \text{ (lagging)}$$

(ii) Reading on two wattmeters

$$\phi = 28.36°$$

$$W_1 = V_L I_L \cos(30° - \phi) = 220 \times 38 \times \cos(30° - 28.36°) = 8356.58 \text{ W}$$

$$W_2 = V_L I_L \cos(30° + \phi) = 220 \times 38 \times \cos(30° + 28.36°) = 4385.49 \text{ W}$$

Useful Formulae

Two-wattmeter method

For lagging power factor	For leading power factor
$W_1 = V_L I_L \cos(30° - \phi)$	$W_1 = V_L I_L \cos(30° + \phi)$
$W_2 = V_L I_L \cos(30° + \phi)$	$W_2 = V_L I_L \cos(30° - \phi)$
$\phi = \tan^{-1}\left[\dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$	$\phi = \tan^{-1}\left[-\dfrac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$

Exercise 5.2

5.1 Three identical coils each having a resistance of 8 Ω and inductance of 0.02 H are connected in (i) star, and (ii) delta across a 3ϕ, 400 V, 50 Hz supply. Draw a neat phasor diagram and calculate the reading of two wattmeters connected to measure power. Also, calculate pf of the circuit.
[8.99 kW, 3.3 kW, 0.7866 (lagging), 26.98 kW, 10.14 kW, 0.786 (lagging)]

5.2 Three identical coils each having a reactance of 20 Ω and resistance of 10 Ω are connected in (i) star, (ii) delta across a 440 V, 3-phase line. Calculate for each method of connection the line current and readings of each of the two wattmeters connected. *[11.36 A, 4.17 kW, –299.07 W, 34.08 A, 12.52 kW, –897.14 W]*

5.3 A 3-phase motor load has a pf of 0.397 lagging. Two wattmeters connected to measure power show the input as 30 kW. Find the reading on each wattmeter.
[35 kW, –5 kW]

5.4 Each of the wattmeters connected to measure the input to a 3-phase induction motor reads 10 kW. If the power factor of the motor be changed to 0.866 lagging, determine the readings of the two wattmeters, the total input power remaining unchanged. *[6.67 kW, 13.33 kW]*

5.5 A 3-ϕ, star-connected load draws a line current of 25 A. The load kVA and kW are 20 and 16 respectively. Find the readings on each of the two wattmeters used to measure the 3ϕ power. *[11.46 kW, 4.54 kW]*

5.6 Three similar coils are star-connected to a 3ϕ 50 Hz supply. The line current taken is 25 A and the two wattmeters connected to measure the input indicate 5.185 kW and 10.37 kW respectively. Calculate (i) the line and phase voltages, and (ii) the resistance and reactance of each coil. *[415 V, 240 V, 5.31 Ω, 4.8 Ω]*

5.7 A three-phase, 500 V motor load has a power factor of 0.4. Two wattmeters connected to measure power show the input to be 30 kW. Find the reading on each instrument. *[35 kW, – 5kW]*

5.8 The power in a three-phase circuit is measured by two wattmeters. If the total power is 100 kW and power factor is 0.66 leading, what will be the reading of each wattmeter? *[17.26 kW, 82.74 kW]*

5.9 Two wattmeters are connected to measure the input to a 400 V, 3-phase connected motor outputting 24.4 kW at a power factor of 0.4 lag and 80% efficiency. Calculate (i) resistance and reactance of motor per phase, (ii) reading of each wattmeter.
[2.55 Ω, 5.58 Ω, 34915W, – 4850 W]

5.10 In a balanced 3-phase, 400 V circuit, the line current is 115.5 A. When power is measured by the two wattmeter method, one meter reads 40 kW and the other, zero. What is the power factor of the load? If the power factor were unity and the line current the same, what would be the reading of each wattmeter?
[0.5, 40 kW, 40 kW]

5.11 A 440 V, 3-phase, delta-connected induction motor has an output of 14.92 kW at pf of 0.82 and efficiency of 85%. Calculate the readings on each of the two wattmeters connected to measure the input. If another star-connected load of 10 kW at 0.85 pf lagging is added in parallel to the motor, what will be the current drawn from the line and the power taken from the line?
[12.35 kW, 5.26 kW, 43.56 A, 27.6 kW]

5.12 Balanced delta-connected impedances, each of $10 \angle 30° \ \Omega$ are connected across three-phase 400 V mains. Determine the two-wattmeter readings if the current coils of the two wattmeters are connected in lines *R* and *Y* and the pressure coils are connected between *R* and *B* and *Y* and *B* lines respectively.
[27.7 kW, 13.86 kW]

5.13 A balanced star-connected load, each phase having a resistance of 10 Ω and the inductive reactance of 30 Ω is connected to 400 V, 50 Hz supply. The phase rotation is *R*, *Y*, and *B*. Wattmeters connected to read total power have their current coils in the red and blue lines respectively. Calculate the reading on each wattmeter.
[2190 W, – 583W]

5.14 A balanced star-connected load is supplied from a symmetrical three-phase 400 V, 50 Hz supply system. The current in each phase is 20 A and lags behind its phase voltage by an angle of $2\pi/9$ radians. Calculate line voltage, phase voltage, current in each phase, load parameter, power in each phase, total power, readings of the wattmeters connected in the load circuit to measure the total power. Draw a neat circuit diagram and vector/phasor diagram.

[440 V, 254.034 V, 20 A, 9.73 Ω, 0.026 H, 3.892 kW, 11.676 kW, 8.666 kW, 3.009 kW]

Review Questions

5.1 Explain the following terms with reference to a polyphase system: (i) Balanced load (ii) Phase sequence (iii) Symmetrical system.

5.2 What are the advantages of a three-phase system over a single-phase system?

5.3 Prove that current in a neutral wire in a three-phase, four-wire balanced load system is zero.

5.4 Derive the relationship between phase and line quantities (voltage, current, power) for a balanced three-phase, delta-connected system. Also draw neat diagrams.

5.5 Deduce the relationship between phase and line quantities (voltage, current, power) for a balanced three-phase, star-connected system. Also draw neat diagrams.

5.6 Derive the relation between power in star and delta systems.

5.7 Explain merits of two-wattmeter method for power measurement, giving circuit diagram and phasor diagram.

5.8 Explain with phasor diagram of how two wattmeters can be used to measure power in a 3-phase system. Also explain the variations in the wattmeter readings with load power factors.

5.9 Derive the relation for total power and power factor in a 3-phase system with balanced load using two wattmeter method.

5.10 How do you measure power of a 3-phase balanced network by using a wattmeter with least number of wattmeters.

5.11 Explain the effect of power factor on wattmeter readings in three-phase power measurement by two-wattmeter method.

Objective-Type Questions

Choose the correct alternative in the following questions:

5.1 In a three-phase system, voltages differ in phase by

(a) 30° (b) 60° (c) 90° (d) 120°

5.2 The rated voltage of a 3-phase power system is given as

(a) rms phase voltage (b) peak phase voltage

(c) rms line to line voltage (d) peak line to line voltage

5.3 Total instantaneous power supplied by a 3-phase ac supply to a balanced *R–C* load is
(a) zero (b) constant
(c) pulsating with zero average (d) pulsating with non-zero average

5.4 Which of the following equations is valid for a 3-phase, balanced star-connected system?
(a) $I_R + I_Y + I_B = 0$ (b) $I_R + I_B = I_Y$
(c) $I_R + I_Y - I_B = 0$ (d) $\dfrac{V_R + V_B + V_Y}{Z} = I_N$

5.5 A three-phase load is balanced if all the three phases have the same
(a) impedance (b) power factor
(c) impedance and power factor (d) none of the above

5.6 The phase sequence of a three-phase system system shown in Fig. 5.21 is

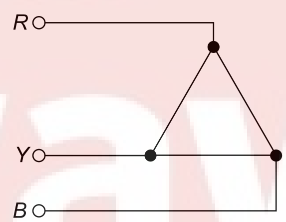


Fig. 5.21

(a) *RYB* (b) *RBY* (c) *BRY* (d) *YBR*

5.7 In a 3-phase system, $\overline{V}_{YN} = 100\ \angle{-120}°$V and $\overline{V}_{BN} = 100\ \angle 120°$ V. Then $\overline{V}_{YB}$ will be
(a) $170\ \angle\ 90°$ V (b) $173\ \angle -90°$ V
(c) $200\ \angle\ 60°$ V (d) none of the above

5.8 If a balanced delta load has an impedance of $(6 + j9)$ ohms per phase then the impedance of each phase in the equivalent star load is
(a) $(6 + j9)$ ohms (b) $(2 + j3)$ ohms
(c) $(2 + j8)$ ohms (d) $(3 + j4.5)$ ohms

5.9 Three equal impedances are first connected in delta across a 3-phase balanced supply. If the same impedances are connected in star across the same supply
(a) phase current will be one-third
(b) line current will be one-third
(c) power consumed will be one-third
(d) none of the above

5.10 Three identical resistors connected in star carry a line current of 12 A. If the same resistors are connected in delta across the same supply, the line current will be
(a) 12 A (b) 4 A (c) 8 A (d) 36 A

5.11 Three delta-connected resistors absorb 60 kW when connected to a 3-phase line. If the resistors are connected in star, the power absorbed is

(a) 60 kW (b) 20 kW (c) 40 kW (d) 180 kW

5.12 The power consumed in the star connected load shown in Fig. 5.22 is 690 W. The line current is

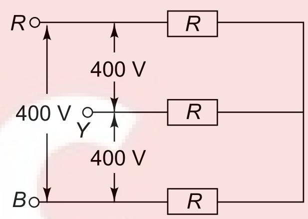


Fig. 5.22

(a) 2.5 A (b) 1 A (c) 1.725 A (d) none of the above

5.13 If the 3-phase balanced source in Fig. 5.23 delivers 1500 W at a leading power factor of 0.844 then the value of Z_L (in ohm) is approximately

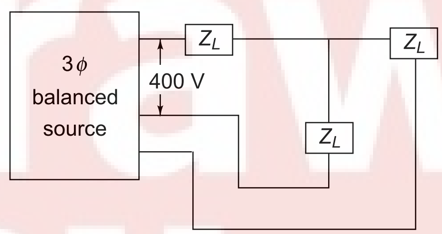


Fig. 5.23

(a) 90 $\angle 32.44°$ (b) 80 $\angle 32.44°$ (c) 80 $\angle -32.44°$ (d) 90 $\angle -32.44°$

5.14 If one of the resistors in Fig. 5.24 is open circuited, power consumed in the circuit is

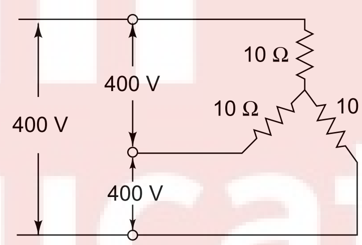


Fig. 5.24

(a) 8000 W (b) 4000 W (c) 16000 W (d) none of the above

5.15 For the three-phase circuit shown in Fig. 5.25, the ratio of the currents $I_R : I_Y : I_B$ is given by

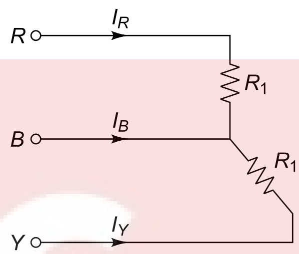


Fig. 5.25

(a) $1 : 1 : \sqrt{3}$ (b) $1 : 1 : 2$ (c) $1 : 1 : 0$ (d) $1 : 1 : \sqrt{\frac{3}{2}}$

5.16 The phase sequence *RYB* denotes that
(a) emf of phase *Y* lags behind that of phase *R* by 120°
(b) emf of phase *Y* leads that of phase *R* by 120°
(c) emf of phase *Y* and phase *R* are in phase
(d) none of the above

5.17 In the two wattmeter method of measurement, if one of the wattmeters reads zero, then power factor will be
(a) zero (b) unity (c) 0.5 (d) 0.866

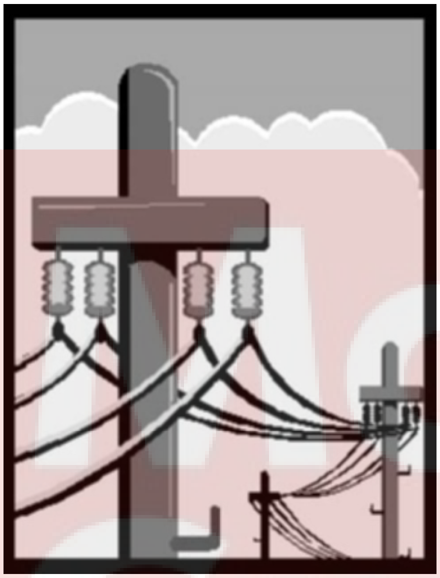
5.18 Two wattmeters, which are connected to measure the total power on a three-phase system, supplying a balanced load, read 10.5 kW and –2.5 kW, respectively. The total power and the power factor, respectively are
(a) 13 kW, 0.334 (b) 13 kW, 0.684
(c) 8 kW, 0.52 (d) 8 kW, 0.334

5.19 The minimum number of wattmeter(s) required to measure 3-phase, 3-wire balanced or unbalanced power is
(a) 1 (b) 2 (c) 3 (c) 4

5.20 One of the two wattmeters has read zero in the two-wattmeter method of power measurement. This indicated that the load phase angle is
(a) 0° (b) 30° (c) 60° (d) 90°

Answers to Objective-Type Questions

5.1 (d)	**5.2** (c)	**5.3** (b)	**5.4** (a)	**5.5** (c)	**5.6** (b)
5.7 (b)	**5.8** (b)	**5.9** (c)	**5.10** (d)	**5.11** (b)	**5.12** (b)
5.13 (d)	**5.14** (a)	**5.15** (a)	**5.16** (a)	**5.17** (c)	**5.18** (d)
5.19 (b)	**5.20** (c)				



Chapter 6

Single-Phase Transformers

Chapter Outline

6.1 SINGLE-PHASE TRANSFORMERS

A transformer is a static device which can transfer electrical energy from one circuit to another circuit without change of frequency. It can increase or decrease the voltage but with a corresponding decrease or increase in current. It works on the principle of mutual induction. It must be used with an input voltage that varies in amplitude, i.e., an ac voltage. A major application of transformers is to increase voltage before transmitting electrical energy over long distances through wires and to reduce voltage at places where it is to be used. Transformers are also used in electronic circuits to step down the supply voltage to a level suitable for the low-voltage circuits they contain. Signal and audio transformers are used to couple stages of amplifiers and to match devices such as microphones to the input of amplifiers.

6.2 CONSTRUCTION

A transformer mainly consists of two coils or windings placed on a common core. With the increase in size (capacity) and operating voltage, it also needs other parts such as a suitable tank, bushing, conservator, breather, etc. We will discuss two basic parts—core and windings.

6.2.1 Core

The composition of a transformer core depends on voltage, current and frequency. Commonly used core materials are soft iron and steel. Generally, air-core transformers are used when the voltage source has a high frequency (above 20 kHz). Iron-core transformers are usually used when the source frequency is low (below 20 kHz). In most transformers, the core is constructed of laminated steel to provide a continuous magnetic path. Silicon content in the steel increases its resistivity to eddy-current loss, thereby reducing eddy-current losses. To reduce eddy-current losses further, the core is laminated by a light coat of varnish or by an oxide layer on the surface. There are two main shapes of cores used in laminated steel-core transformers as shown in Fig 6.1 and Fig 6.2.

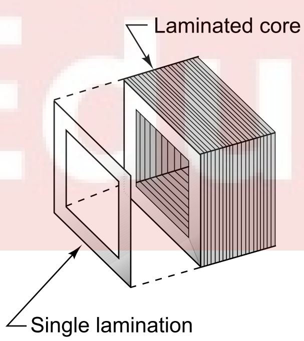


Fig. 6.1 *Hollow-core construction*

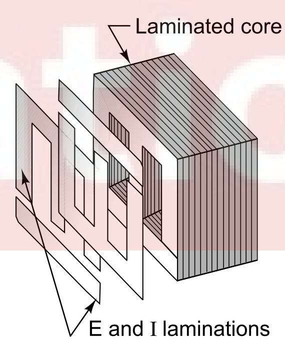


Fig. 6.2 *Shell-type core construction*

6.2.2 Transformer Windings

A transformer consists of two coils, called windings, which are wrapped around a core. The winding in which electrical energy is fed is called the *primary winding*. The winding which is connected to the load is called the *secondary winding*.

The primary and secondary windings are made up of an insulated copper conductor in the form of a round wire or strip. These windings are then placed around the limbs of the core. The windings are insulated from each other and the core, using cylinders of insulating material such as a press board or Bakelite.

For simplicity, the primary and secondary windings are shown on separate limbs of the core. If such an arrangement is used in actual practice, all the flux produced in the primary winding will not link with the secondary winding. Some of the flux will leak out through the air. Such flux is known as *leakage flux*. The more the value of leakage flux, poorer is the performance of the transformer. Hence, to reduce leakage flux, the windings are placed together on the same limb in actual transformers.

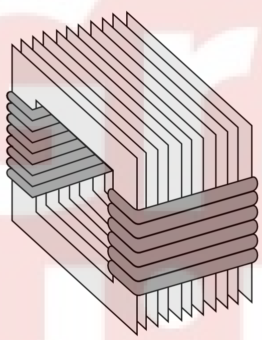


Fig. 6.3 *Core-type transformer*

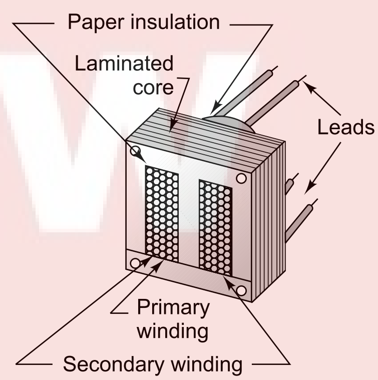


Fig. 6.4 *Shell-type transformer*

6.2.3 Comparison of Core-type and Shell-type Transformers

Core-type Transformer	*Shell-type Transformer*
1. It consists of a magnetic frame with two limbs.	It consists of a magnetic frame with three limbs.
2. It has a single magnetic circuit.	It has two magnetic circuits.
3. The winding encircles the core.	The core encircles most part of the winding.
4. It consists of cylindrical windings.	It consists of sandwich-type windings.
5. It is easy to repair.	It is not easy to repair.
6. It provides better cooling since windings are uniformly distributed on two limbs.	It does not provide effective cooling as the windings are surrounded by the core.
7. It is preferred for low-voltage transformers.	It is preferred for high-voltage transformers.

6.3 PRINCIPLE OF WORKING

When an alternating voltage V_1 is applied to a primary winding, an alternating current I_1 flows in it producing an alternating flux in the core. As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.

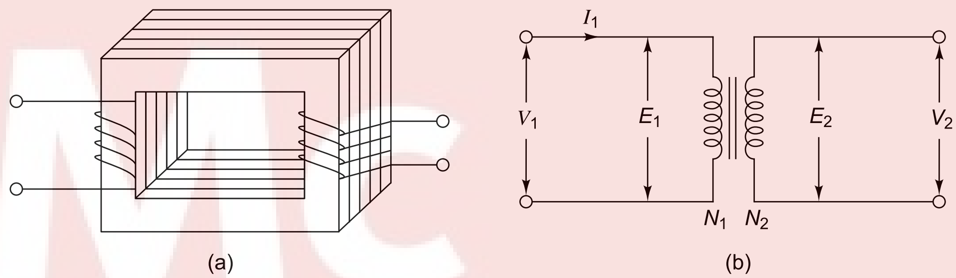


Fig. 6.5 *Working principle of a transformer*

$$e_1 = -N_1 \frac{d\phi}{dt}$$

where N_1 is the number of turns in the primary winding. The induced emf in the primary winding is nearly equal and opposite to the applied voltage V_1.

Assuming leakage flux to be negligible, almost the whole flux produced in primary winding links with the secondary winding. Hence, an emf e_2 is induced in the secondary winding.

$$e_2 = -N_2 \frac{d\phi}{dt}$$

where N_2 is the number of turns in the secondary winding. If the secondary circuit is closed through the load, a current I_2 flows in the secondary winding. Thus, energy is transferred from the primary winding to the secondary winding.

6.4 THE EMF EQUATION

As the primary winding is excited by a sinusoidal alternating voltage, an alternating current flows in the winding producing a sinusoidally varying flux ϕ in the core.

$$\phi = \phi_m \sin \omega t$$

As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.

$$\begin{aligned} e_1 &= -N_1 \frac{d\phi}{dt} \\ &= -N_1 \frac{d}{dt}(\phi_m \sin \omega t) \\ &= -N_1 \phi_m \omega \cos \omega t \\ &= N_1 \phi_m \omega \sin(\omega t - 90°) \\ &= 2\pi f \phi_m N_1 \sin(\omega t - 90°) \end{aligned}$$

Maximum value of induced emf = $2\pi f\,\phi_m\,N_1$

Hence, rms value of induced emf in primary winding is given by,

$$E_1 = \frac{E_{\max}}{\sqrt{2}} = \frac{2\pi f\,\phi_m\,N_1}{\sqrt{2}} = 4.44 f\,\phi_m\,N_1$$

Similarly, rms value of induced emf in the secondary winding is given by,

$$E_2 = 4.44\, f\,\phi_m\,N_2$$

Also, $$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f\,\phi_m$$

Thus, emf per turn is same in primary and secondary windings and an equal emf is induced in each turn of the primary and secondary windings.

6.5 TRANSFORMATION RATIO (K)

We know that,

$$E_1 = 4.44\, f\,\phi_m\,N_1$$

$$E_2 = 4.44\, f\,\phi_m\,N_2$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

where K is called the *transformation ratio*.

Neglecting small primary and secondary voltage drops,

$$V_1 \approx E_1$$

$$V_2 \approx E_2$$

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

In a transformer, losses are negligible. Hence, input and output can be approximately equated.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K$$

For step-up transformers,

$N_2 > N_1$ $\quad\quad$ $K > 1$

For step-down transformers,

$N_2 < N_1$ $\quad\quad$ $K < 1$

6.6 RATING OF A TRANSFORMER

Rating of a transformer indicates the output power from it. But for a transformer, load is not fixed and its power factor goes on changing. Hence, rating is not expressed in terms of power but in terms of product of voltage and current, called VA rating. This rating is generally expressed in kVA.

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

We can calculate full-load currents of primary and secondary windings from kVA rating of a transformer. *Full-load current* is the maximum current which can flow through the winding without damaging it.

$$\text{Full-load primary current} \qquad I_1 = \frac{\text{kVA rating} \times 1000}{V_1}$$

$$\text{Full-load secondary current} \qquad I_2 = \frac{\text{kVA rating} \times 1000}{V_2}$$

Example 1

What will be the secondary voltage at no load, if the primary of a 5 kVA, 220/110 V, 50 Hz transformer is fed at (i) 110 V, 50 Hz, and (ii) 220 V dc?

Solution

$$\text{kVA rating} = 5 \text{ kVA}$$

$$E_1 = 220 \text{ V}$$

$$E_2 = 110 \text{ V}$$

(i) Secondary voltage when $V_1 = 110$ V

For a transformer,

$$\frac{V_2}{V_1} = \frac{E_2}{E_1}$$

$$\frac{V_2}{110} = \frac{110}{220}$$

$$V_2 = 55 \text{ V}$$

(ii) Secondary voltage when $V_1 = 220$ V dc

When the transformer is fed 220 V dc, no emf is induced in the primary winding.

$$V_2 = 0$$

Example 2

It is desired to have 4.13 mWb maximum flux in the core of a transformer operating at 110 V and 50 Hz. Determine the required number of turns in the primary.

Solution

$$\phi_m = 4.13 \text{ mWb}$$

$$V_1 = 110 \text{ V}$$

$$f = 50 \text{ Hz}$$

For a transformer,

$$V_1 \approx E_1 = 110 \text{ V}$$
$$E_1 = 4.44\, f\, \phi_m\, N_1$$
$$110 = 4.44 \times 50 \times 4.13 \times 10^{-3} \times N_1$$
$$N_1 = 120$$

Example 3

A single-phase 50 Hz transformer has 80 turns on the primary winding and 280 turns in the secondary winding. The voltage applied across the primary winding is 240 V at 50 Hz. Calculate (i) maximum flux density in the core, and (ii) induced emf in the secondary. The net cross-sectional area of the core is 200 cm².

Solution

$$f = 50 \text{ Hz}$$
$$N_1 = 80$$
$$N_2 = 280$$
$$V_1 = 240 \text{ V}$$
$$A = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ m}^2$$

(i) Maximum flux density in the core
For a transformer,

$$V_1 \approx E_1 = 240 \text{ V}$$
$$E_1 = 4.44\, f\, \phi_m\, N_1 = 4.44\, f\, B_m\, A N_1$$
$$240 = 4.44 \times 50 \times B_m \times 200 \times 10^{-4} \times 80$$
$$B_m = 0.68 \text{ Wb/m}^2$$

(ii) Induced emf in the secondary

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{E_2}{240} = \frac{280}{80}$$
$$E_2 = 840 \text{ V}$$

Example 4

An 80 kVA, 3200/400 V, 50 Hz single-phase transformer has 111 turns on the secondary winding. Calculate (i) number of turns on primary winding, (ii) secondary current, and (iii) cross-sectional area of the core, if the maximum flux density is 1.2 teslas.

Solution

$$\text{kVA rating} = 80 \text{ kVA}$$
$$E_1 = 3200 \text{ V}$$

$$E_2 = 400 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$N_2 = 111$$
$$B_m = 1.2 \text{ T}$$

(i) Number of turns of primary winding

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{400}{3200} = \frac{111}{N_1}$$
$$N_1 = 888$$

(ii) Secondary current

For a transformer,

$$V_2 \approx E_2 = 400 \text{ V}$$
$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{80 \times 1000}{400} = 200 \text{ A}$$

(iii) Cross-sectional area of the core

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$
$$400 = 4.44 \times 50 \times 1.2 \times A \times 111$$
$$A = 0.0135 \text{ m}^2 = 135 \text{ cm}^2$$

Example 5

A 5 kVA, 240/2400 V, 50 Hz single-phase transformer has the maximum value of flux density as 1.2 teslas. If the emf per turn is 8, calculate (i) number of primary turns and secondary turns, (ii) cross-sectional area of the core, and (iii) primary and secondary current at full load.

Solution

$$\text{kVA rating} = 5 \text{ kVA}$$
$$E_1 = 240 \text{ V}$$
$$E_2 = 2400 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$B_m = 1.2 \text{ T}$$
$$\frac{E_1}{N_1} = 8$$

(i) Number of primary and secondary turns

$$\frac{E_1}{N_1} = 8 = \frac{240}{N_1}$$

$$N_1 = 30$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{2400}{240} = \frac{N_2}{30}$$

$$N_2 = 300$$

(ii) Cross-sectional area of the core

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$

$$2400 = 4.44 \times 50 \times 1.2 \times A \times 300$$

$$A = 0.03 \text{ m}^2$$

(iii) Primary and secondary currents at full load

For a transformer,

$$V_1 \approx E_1 = 240 \text{ V}$$

$$V_2 \approx E_2 = 2400 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{5 \times 1000}{240} = 20.83 \text{ A}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{5 \times 1000}{2400} = 2.08 \text{ A}$$

Example 6

A 250 kVA, 50 Hz single-phase transformer has ratio of secondary to primary turns as 0.1. The secondary voltage at no-load condition is 240 V. Calculate (i) primary voltage, and (ii) full-load primary and secondary currents.

Solution

$$\text{kVA rating} = 250 \text{ kVA}$$

$$\frac{N_2}{N_1} = 0.1$$

$$E_2 = 240 \text{ V}$$

(i) Primary voltage

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = 0.1$$

$$E_1 = 2400 \text{ V}$$

(ii) Full-load primary and secondary current

For a transformer,

$$V_1 \approx E_1 = 2400 \text{ V}$$

$$V_2 \approx E_2 = 240 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{250 \times 1000}{2400} = 104.17 \text{ A}$$

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{250 \times 1000}{240} = 1041.67 \text{ A}$$

Example 7

A 10 kVA, 3300/240 V, single-phase, 50 Hz transformer has a core area of 300 cm^2. The flux density is 1.3 T. Calculate (i) number of primary turns, (ii) number of secondary turns, and (iii) primary full-load current.

Solution

$$\text{kVA rating} = 10 \text{ kVA}$$
$$E_1 = 3300 \text{ V}$$
$$E_2 = 240 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$A = 300 \text{ cm}^2 = 300 \times 10^{-4} \text{ m}^2$$
$$B_m = 1.3 \text{ T}$$

(i) Number of primary turns

$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$
$$3300 = 4.44 \times 50 \times 1.3 \times 300 \times 10^{-4} \times N_1$$
$$N_1 = 381.15 \simeq 382$$

(ii) Number of secondary turns

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{240}{3300} = \frac{N_2}{382}$$
$$N_2 = 27.78 \simeq 28$$

(iii) Primary full-load current

For a transformer,

$$V_1 \approx E_1 = 3300 \text{ V}$$

$$I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{10 \times 1000}{3300} = 3.03 \text{ A}$$

Example 8

A 3300/250 V, 50 Hz, single-phase transformer has 125 cm^2 cross-sectional area of core and 70 turns on low-voltage side. Calculate (i) the value of maximum flux density, and (ii) number of turns on the high-voltage side.

Solution

$$E_1 = 3300 \text{ V}$$
$$E_2 = 250 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$A = 125 \text{ cm}^2 = 125 \times 10^{-4} \text{ m}^2$$
$$N_2 = 70$$

(i) Value of maximum flux density

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$
$$250 = 4.44 \times 50 \times B_m \times 125 \times 10^{-4} \times 70$$
$$B_m = 1.29 \text{ Wb/m}^2$$

(ii) Number of turns on high-voltage side

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$
$$\frac{250}{3300} = \frac{70}{N_1}$$
$$N_1 = 924$$

Useful Formulae

$$E_1 = 4.44\, f\, \phi_m\, N_1$$
$$E_2 = 4.44\, f\, \phi_m\, N_2$$
$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$
$$\text{kVA rating} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$
$$I_1 \text{ (full load)} = \frac{\text{kVA rating} \times 1000}{V_1}$$
$$I_2 \text{ (full load)} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Exercise 6.1

6.1 The required no-load ratio in a single-phase, 50 Hz, core-type transformer is 6000/250 V. Find the number of turns in each winding if the flux is to be about 0.06 Wb. *[450, 20]*

6.2 A 6600/600 V, 50 Hz, 1 ϕ transformer has a maximum flux density of 1.35 Wb/m^2 in its core. If the net cross-sectional area of iron in the core is 200 cm^2, calculate the number of turns in the primary and secondary windings of the transformer.

[1101, 100]

6.3 A 1 ϕ, 50 Hz transformer has 500 turns on the primary and 1000 turns on the secondary. The voltage per turn in the primary winding is 0.2 volt. Calculate (i) voltage induced in primary and secondary windings, (ii) maximum value of flux density if the cross-sectional area of the core is 200 cm^2, (iii) kVA rating of the transformer if the current in primary at full load is 10 A.

[100 V, 200 V, 9.09×10^{-4} Wb, 0.045 Wb/m^2, 1 kVA]

6.4 A 40 kVA, 3300/240 V, 50 Hz, 1-phase transformer has 660 turns on the primary. Determine (i) the number of turns on the secondary, (ii) the maximum value of flux in the core, and (iii) the approximate value of primary and secondary full-load current. Internal drops in the windings are to be ignored.

[48, 0.02 Wb, 12.12 A, 166.67 A]

6.5 A single-phase transformer has 350 primary and 1050 secondary turns. The net cross-sectional area of the core is 55 cm^2. If the primary winding be connected to a 400 V, 50 Hz single-phase supply, calculate (i) maximum value of flux density in the core, and (ii) the voltage induced in the secondary winding.

[0.93 Wb/m^2, 1200 V]

6.6 A 25 kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to a 3000 V, 50 Hz supply. Find the full-load primary and secondary currents, the secondary emf and the maximum flux in the core. *[8.33 A, 83.3 A, 300 V, 27 mWb]*

6.7 LOSSES IN A TRANSFORMER

There are two types of losses in a transformer:

(i) Iron or core loss
(ii) Copper loss

Iron Loss This loss is due to the reversal of flux in the core. The flux set-up in the core is nearly constant. Hence, iron loss is practically constant at all the loads, from no load to full load. The losses occuring under no-load condition are the iron losses because the copper losses in the primary winding due to no-load current are negligible. Iron losses can be subdivided into two losses:

(i) Hysteresis loss
(ii) Eddy-current loss

(1) *Hysteresis Loss* This loss occurs due to setting of an alternating flux in the core. It depends on the following factors:

(i) Area of the hysteresis loop of magnetic material which again depends upon the flux density

(ii) Volume of the core
(iii) Frequency of the magnetic flux reversal

(2) *Eddy–Current Loss* This loss occurs due to the flow of eddy currents in the core caused by induced emf in the core. It depends on the following factors:

(i) Thickness of lamination of core
(ii) Frequency of the magnetic flux reversal
(iii) Maximum value of flux density in the core
(iv) Volume of the core
(v) Quality of magnetic material used

Eddy-current losses are reduced by decreasing the thickness of lamination and by adding silicon to steel.

Copper Loss This loss is due to the resistances of primary and secondary windings.

$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2$$

where R_1 = primary winding resistance

R_2 = secondary winding resistance

Copper loss depends upon the load on the transformer and is proportional to square of load current of kVA rating of the transformer.

6.8 IDEAL AND PRACTICAL TRANSFORMERS

For an ideal transformer, (i) there will be no core loss and copper loss, and (ii) winding resistance and leakage flux are zero. But in a practical transformer, the windings have some resistance and there is always some leakage flux.

It has been assumed that all the flux linked with primary winding also links the secondary winding. However, all the flux linked with primary does not link the secondary. This is known as *primary leakage flux*. Similarly, when flux is set up in the secondary, all the flux linked with secondary does not link the primary, but part of it links with the secondary itself through air. This flux is known as *secondary leakage flux*. Leakage fluxes produce self-induced emf in their respective windings. It is, therefore, equivalent to an inductive coil in series with the respective winding.

A practical transformer consists of winding resistances R_1 and R_2 and leakage reactances X_1 and X_2 as shown in Fig. 6.6.

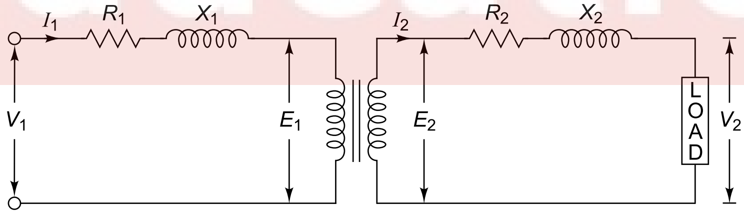


Fig. 6.6 *Resistances and leakage reactances of a practical transformer*

6.9 PHASOR DIAGRAM OF A TRANSFORMER ON NO LOAD

When the transformer is operating at no load, there is iron loss in the core and copper loss in the primary winding. Thus, primary input current I_0 has to supply iron loss in the core and a very small amount of copper loss in primary. Hence, the current I_0 has two components:

(i) a magnetising or reactive component I_μ, and

(ii) power or active component I_w.

The magnetising component I_μ is responsible for setting up flux in the core. It is in phase with the flux ϕ.

$$I_\mu = I_0 \sin \phi_0$$

The active component I_w is responsible for power loss in the transformer. It is in phase with V_1.

$$I_w = I_0 \cos \phi_0$$

Hence, no-load current I_0 is the phasor sum of I_μ and I_w.

$$\bar{I}_0 = \bar{I}_\mu + \bar{I}_w$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

The no-load current I_0 is very small as compared to full-load current I_1. Hence, copper loss is negligible and no-load input power is practically equal to iron loss or core loss in the transformer.

Iron loss $W_i = V_1 I_0 \cos \phi_0$ where $\cos \phi_0$ is power factor at no load.

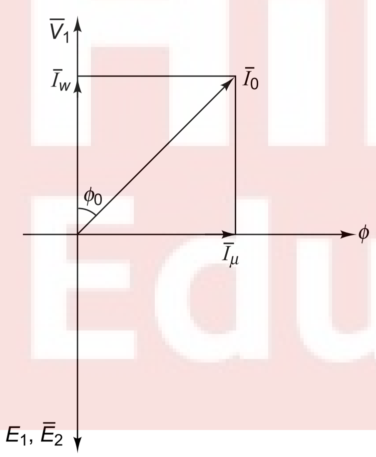


Fig. 6.7 *Phasor diagram*

Phasor Diagram Since the flux ϕ is common to both the windings, ϕ is chosen as a reference phasor. From emf equation of the transformer, it is clear that E_1 and E_2 lag the flux by 90°. Hence, emfs E_1 and E_2 are drawn such that these lag behind the flux ϕ by 90°. The magnetising component I_μ is drawn in phase with the flux ϕ. The applied voltage V_1 is drawn equal and opposite to E_1 as $V_1 \approx E_1$. The active component I_w is drawn in phase with voltage V_1. The phasor sum of I_μ and I_w gives the no-load current I_0.

Example 1

A 50 kVA, 2300/230 V, 50 Hz transformer takes 200 watts and 0.3 A at no load, when 2300 V are applied to the high-voltage side. The primary resistance is 3.5 Ω. Determine (i) core loss, and (ii) no-load pf.

Solution

$$W_i = 200 \text{ W}$$
$$I_0 = 0.3 \text{ A}$$
$$V_1 = 2300 \text{ V}$$
$$R_1 = 3.5\ \Omega$$

(i) Core loss

Copper loss in primary $= I_0^2 R_1 = (0.3)^2 \times 3.5 = 0.315$ W

Core loss $=$ Input power – Copper loss $= 200 - 0.315 = 199.685$ W

(ii) No load pf

$$W_i = V_1 I_0 \cos \phi_0$$
$$200 = 2300 \times 0.3 \times \cos \phi_0$$
$$\cos \phi_0 = 0.29 \text{ (lagging)}$$

Example 2

A single-phase transformer has a primary voltage of 230 V. No-load primary current is 5 A. No-load pf is 0.25. Number of primary turns are 200 and frequency is 50 Hz. Calculate (i) maximum value of flux in the core, (ii) core loss, and (iii) magnetising current.

Solution

$$V_1 = 230 \text{ V}$$
$$I_0 = 5 \text{ A}$$
$$\cos \phi_0 = 0.25$$
$$N_1 = 200$$
$$f = 50 \text{ Hz}$$

(i) Maximum value of flux in the core

For a transformer, $V_1 \approx E_1 = 230$ V

$$E_1 = 4.44 f \phi_m N_1$$
$$230 = 4.44 \times 50 \times \phi_m \times 200$$
$$\phi_m = 5.18 \text{ mWb}$$

(ii) Core loss

Neglecting primary copper loss,

$$W_i = V_1 I_0 \cos \phi_0 = 230 \times 5 \times 0.25 = 287.5 \text{ W}$$

(iii) Magnetising current

$$\cos \phi_0 = 0.25$$
$$\sin \phi_0 = 0.97$$

$$I_\mu = I_0 \sin \phi_0 = 5 \times 0.97 = 4.85 \text{ A}$$

6.10 PHASOR DIAGRAM OF A TRANSFORMER ON LOAD

When the transformer is loaded, a current I_2 will flow in the secondary winding. The secondary current I_2 sets up a secondary flux ϕ_2 that tends to reduce the flux ϕ produced by the primary current. Hence, induced emf E_1 in primary reduces. This causes more current to flow in the primary. Let the additional current in the primary be I_2'. This current I_2' is anti-phase with I_2 and sets up its own flux ϕ_2' which cancels the flux ϕ_2 produced by I_2.

Hence, the primary current I_1 is the phasor sum of the no-load current I_0 and the current I_2'.

$$\frac{N_2}{N_1} = \frac{I_1}{I_2} = \frac{I_0 + I_2'}{I_2} = \frac{I_2'}{I_2} = K$$

$$I_2' = KI_2$$

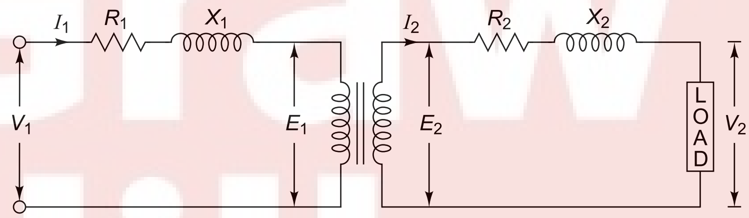


Fig. 6.8 *Practical transformer on load condition*

Figure 6.8 shows a practical transformer on load condition. When a transformer is loaded, the current I_2 flows in the secondary winding and the voltage V_2 appears across the load. Current I_2 is in phase with voltage V_2, if the load is resistive; it lags behind it, if load is inductive, and it leads, if load is capacitive.

Writing vector equations for primary and secondary sides,

$$\overline{V_1} = \overline{I_1 R_1} + \overline{I_1 X_1} + (-\overline{E_1})$$

$$\overline{E_2} = \overline{I_2 R_2} + \overline{I_2 X_2} + \overline{V_2}$$

where $\overline{I_1} = \overline{I_0} + \overline{I_2'}$

The phasor diagram of a transformer on load condition is drawn with the help of the above expressions.

Steps for Drawing Phasor Diagrams

1. First draw $\overline{V_2}$ and then $\overline{I_2}$. The phase angle between $\overline{I_2}$ and $\overline{V_2}$ will depend on the type of load.

2. To $\overline{V_2}$, add the resistive drop $\overline{I_2 R_2}$, parallel to $\overline{I_2}$ and the inductive drop $\overline{I_2 X_2}$, leading $\overline{I_2}$ by 90° such that

$$\overline{E_2} = \overline{V_2} + \overline{I_2 R_2} + \overline{I_2 X_2}$$

3. Draw $\overline{E_1}$ on the same side such that $E_1 = \dfrac{E_2}{K}$
4. Draw $-\overline{E_1}$ equal and opposite to $\overline{E_1}$.
5. For drawing $\overline{I_1}$, first draw $\overline{I_0}$ and $\overline{I_2'}$ such that

$$I_2' = K\, I_2$$

6. Add $\overline{I_0}$ and $\overline{I_2'}$ using the parallelogram law of vector addition.

$$\overline{I_1} = \overline{I_0} + \overline{I_2'}$$

7. To $-\overline{E_1}$, add the resistive drop $\overline{I_1 R_1}$, parallel to $\overline{I_1}$ and the inductive drop $\overline{I_1 X_1}$, leading $\overline{I_1}$ by 90° such that

$$\overline{V_1} = -\overline{E_1} + \overline{I_1 R_1} + \overline{I_1 X_1}$$

8. Draw flux ϕ such that ϕ leads $\overline{E_1}$ and $\overline{E_2}$ by 90°.

Case (i) Resistive load (unity power factor) *Case (ii)* Inductive load (lagging power factor)

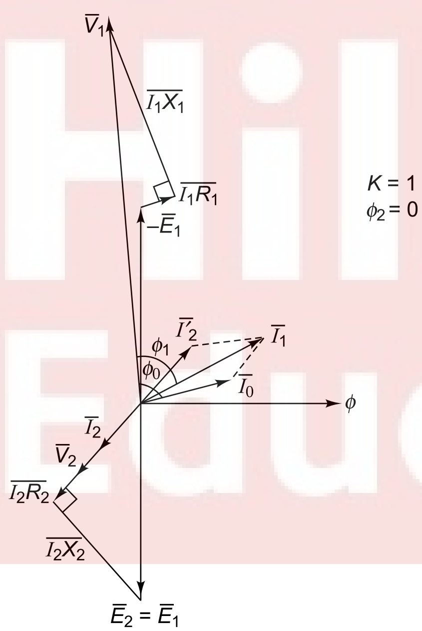


Fig. 6.9 *Phasor diagram for resistive load*

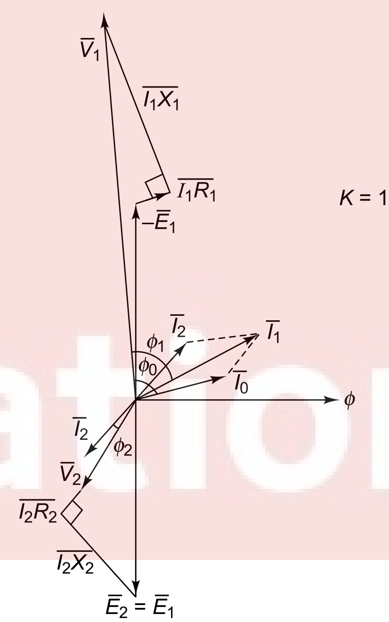


Fig. 6.10 *Phasor diagram for inductive load*

Case (iii) Capacitive load (leading power factor)

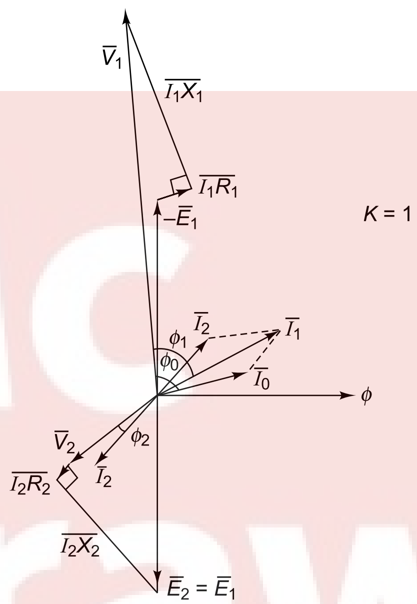


Fig. 6.11 *Phasor diagram for capacitive load*

6.11 EQUIVALENT CIRCUIT

Figure 6.12 shows a practical transformer. R_1 and R_2 represent the resistances of primary and secondary windings respectively. Similarly, X_1 and X_2 represent the leakage reactances of primary and secondary windings respectively.

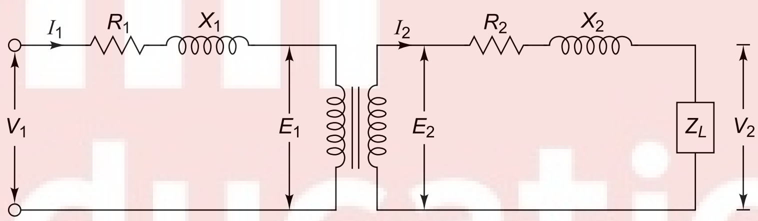


Fig. 6.12 *Practical transformer*

Figure 6.12 can be further modified to represent the no-load current I_0 and its component. The current I_0 is the phasor sum of currents I_w and I_μ. Hence, the current I_0 is simulated by the resistance R_0 taking working component I_w and inductance X_0, taking magnetising component I_μ connected in parallel across the primary circuit.

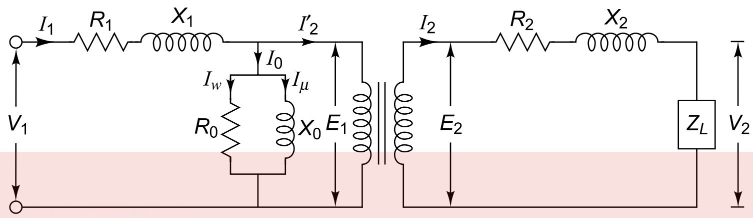


Fig. 6.13 *Practical transformer showing no-load current I_o and its component*

For convenience, all the quantities can be shown on only one side by transferring the quantities from one side to other without any power loss. The power loss in the secondary is $I_2^2\, R_2$. If R_2' is the resistance referred to primary which would have caused the same power loss as R_2 is secondary,

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2 = \frac{R_2}{K^2}$$

Similarly, $$X_2' = \frac{X_2}{K^2}$$

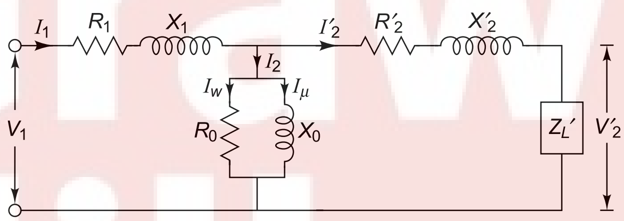


Fig. 6.14 *Modified circuit for primary winding*

Since all quantities are transferred to primary, the transformer need not be shown. The no-load current I_0 is very small compared to the full-load current I_1. Hence, drop across R_1 and X_1 due to I_0 can be neglected. Therefore, transferring R_0 and X_0 to the extreme left,

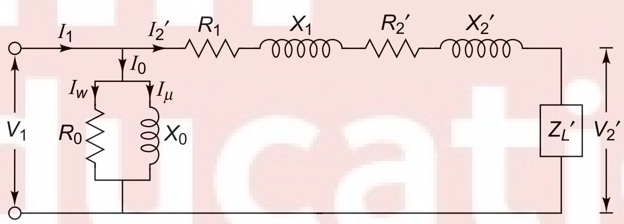


Fig. 6.15 *Modified circuit for primary winding*

The equivalent resistance referred to primary $R_{01} = R_1 + R_2' = R_1 + \dfrac{R_2}{K^2}$

The equivalent leakage reactance referred to primary $X_{01} = X_1 + X_2' = X_1 + \dfrac{X_2}{K^2}$

The equivalent impedance referred to primary $Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$

The equivalent circuit referred to primary is as shown in Fig. 6.16.

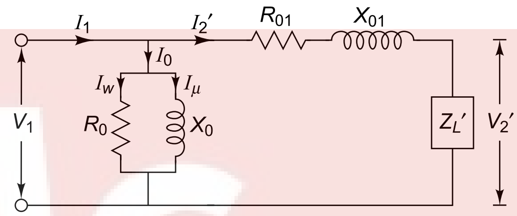


Fig. 6.16 *Equivalent circuit referred to primary winding*

Similarly, the equivalent circuit referred to secondary is as shown in Fig. 6.17.

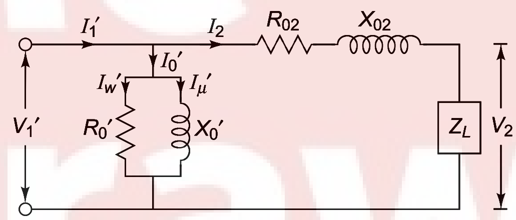


Fig. 6.17 *Equivalent circuit referred to secondary winding*

The equivalent resistance referred to secondary $R_{02} = R_2 + R_1' = R_2 + K^2 R_1 = K^2 R_{01}$

The equivalent leakage reactance referred to secondary $X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = K^2 X_{01}$

The equivalent impedance referred to secondary $Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = K^2 Z_{01}$

Note:

(i) While shifting any primary resistance or reactance to the secondary, multiply it by K^2.

(ii) While shifting any secondary resistance or reactance to the primary, divide it by K^2.

Example 1

A transformer has a turn ratio N_1:N_2 of 4. If a 50 Ω resistance is connected across the secondary, what is the resistance referred to primary?

Solution

$$\frac{N_1}{N_2} = 4$$

$$R = 50\ \Omega$$

$$K = \frac{N_2}{N_1} = \frac{1}{4}$$

Let the resistance R be connected across the secondary.

Equivalent resistance referred to primary = $R' = \frac{R}{K^2} = \frac{50}{(1/4)^2} = 800\ \Omega$

Example 2

A resistance connected across the secondary of an ideal transformer has a value of 800 Ω as referred to the primary. The same resistance when connected across the primary has a value of 3.125 Ω as referred to the secondary. Find the ratio of the transformer.

Solution Let R be the resistance connected to the secondary.

Then equivalent resistance referred to the primary = $\frac{R}{K^2}$

$$\frac{R}{K^2} = 800\ \Omega$$

If the resistance R is connected across the primary, the equivalent resistance referred to the secondary = K^2R

$$K^2R = 3.125$$

$$\frac{K^2R}{R/K^2} = \frac{3.125}{800}$$

$$K^4 = 3.90624 \times 10^{-3}$$

$$K = 0.25$$

Example 3

A 6600/400 V transformer has a primary resistance of 2.5 Ω and a reactance of 3.9 Ω. The secondary resistance is 0.01 Ω and the reactance is 0.025 Ω. Determine the equivalent circuit parameters referred to primary and secondary.

Solution

$$E_1 = 6600\ \text{V}$$

$$E_2 = 400\ \text{V}$$

$$R_1 = 2.5\ \Omega$$

$$X_1 = 3.9\ \Omega$$

$$R_2 = 0.01\ \Omega$$

$$X_2 = 0.025\ \Omega$$

$$K = \frac{E_2}{E_1} = \frac{400}{6600} = 0.06$$

(i) Equivalent resistance referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 2.5 + \frac{0.01}{(0.06)^2} = 5.28\ \Omega$$

(ii) Equivalent reactance referred to primary

$$X_{01} = X_1 + \frac{X_2}{K^2} = 3.9 + \frac{0.025}{(0.06)^2} = 10.84\ \Omega$$

(iii) Equivalent resistance referred to secondary

$$R_{02} = K^2 R_{01} = (0.06)^2 \times 5.28 = 0.02\ \Omega$$

(iv) Equivalent reactance referred to secondary

$$X_{02} = K^2 X_{01} = (0.06)^2 \times 10.84 = 0.04\ \Omega$$

Example 4

A 50 kVA, 4400/220 V transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$. The reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for the transformer, (i) full-load currents on primary and secondary side, (ii) equivalent resistances, reactances, impedances referred to primary side and secondary side, and (iii) total copper loss using individual resistances and equivalent resistances.

Solution kVA rating = 50 kVA

$$E_1 = 4400\ \text{V}$$
$$E_2 = 220\ \text{V}$$
$$R_1 = 3.45\ \Omega$$
$$R_2 = 0.009\ \Omega$$
$$X_1 = 5.2\ \Omega$$
$$X_2 = 0.015\ \Omega$$
$$K = \frac{E_2}{E_1} = \frac{220}{4400} = 0.05$$

(i) Full-load currents and primary and secondary side

For a transformer,

$$V_1 \simeq E_1 = 4400\ \text{V}$$
$$E_2 \simeq V_2 = 220\ \text{V}$$

$$\text{Full-load primary current } I_1 = \frac{kVA \text{ rating} \times 1000}{V_1} = \frac{50 \times 1000}{4400} = 11.36$$

$$\text{Full-load secondary current } I_2 = \frac{kVA \text{ rating} \times 1000}{V_2} = \frac{50 \times 1000}{220} = 227.27$$

(ii) Equivalent resistance, reactances, impedances referred to primary side and secondary side

$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(0.05)^2} = 7.05\ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 5.2 + \frac{0.015}{(0.05)^2} = 11.2\ \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(7.05)^2 + (11.2)^2} = 13.23\ \Omega$$

$$R_{02} = K^2 R_{01} = (0.05)^2 \times 7.05 = 0.02\ \Omega$$

$$X_{02} = K^2 X_{01} = (0.05)^2 \times 11.2 = 0.028\ \Omega$$

$$Z_{02} = K^2 Z_{01} = (0.05)^2 \times 13.23 = 0.03\ \Omega$$

(iii) Copper loss with individual resistances

$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (227.27)^2 \times 0.009$$
$$= 445.22 + 464.86 = 910.08\ \text{W}$$

(iv) Copper loss with equivalent resistances

$$W_{Cu} = I_1^2 R_{01} = I_2^2 R_{02} = (11.36)^2 \times 7.05 = 909.8\ \text{W}$$

6.12 VOLTAGE REGULATION

When a transformer is loaded, the secondary terminal voltage decreases due to a drop across secondary winding resistance and leakage reactance. This change in secondary terminal voltage from no load to full load conditions, expressed as a fraction of the no-load secondary voltage is called *regulation of the transformer*.

$$\text{Regulation} = \frac{\begin{pmatrix}\text{Secondary terminal}\\ \text{voltage on no load}\end{pmatrix} - \begin{pmatrix}\text{Secondary terminal voltage}\\ \text{on full-load condition}\end{pmatrix}}{\text{Secondary terminal voltage on no load}}$$

$$= \frac{E_2 - V_2}{E_2}$$

$$\text{Percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

6.12.1 Expression for Voltage Regulation

Consider a phasor diagram of transformer referred to secondary side on load condition (load is assumed to be inductive). With O as centre and radius OC, draw an arc cutting OA produced at M. From the point B, draw BD perpendicular on OA produced. Draw CN perpendiculor to OM and draw BL parallel to OM.

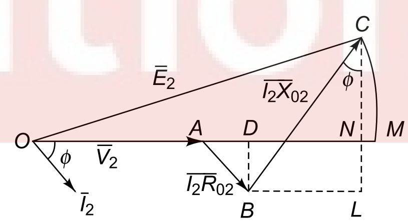


Fig. 6.18

$$\text{Total voltage drop} = E_2 - V_2 = OC - OA = OM - OA$$
$$= AM = AN + NM$$
$$\text{Approximate voltage drop} \approx AN \qquad (\because NM \text{ is very small})$$
$$= AD + DN = AD + BL$$
$$= I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$$
$$\% \text{ regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2} 100$$

For leading pf,

$$\text{Approximate voltage drop} = I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$$
$$\% \text{ regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{E_2} \times 100$$

Hence, in general,

$$\% \text{ regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} \times 100$$

'+' sign is used for lagging pf and '–' sign is used for leading pf.

On primary side, we can express regulation as,

$$\% \text{ regulation} = \frac{I_1 R_{01} \cos\phi \pm I_1 X_{01} \sin\phi}{V_1} \times 100$$

We can also express percentage regulation as,

$$\% \text{ regulation} = \frac{100 I_2 R_{02}}{E_2} \cos\phi \pm \frac{100 I_2 X_{02}}{E_2} \sin\phi$$
$$= v_r \cos\phi \pm v_x \sin\phi$$

where

$$v_r = \frac{100 I_2 R_{02}}{E_2} = \text{percentage resistive drop}$$
$$v_x = \frac{100 I_2 X_{02}}{E_2} = \text{percentage reactive drop}$$

Example 1

A 200 kVA, 2200/440 V, 50 Hz, single-phase transformer is operating at full load, 0.8 lagging pf. The voltage on secondary of the transformer at full load, 0.8 lagging pf is 400 V. Calculate voltage regulation of the transformer.

Solution

$$E_2 = 440 \text{ V}$$
$$V_2 = 400 \text{ V}$$
$$\text{Percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100 = \frac{440 - 400}{440} \times 100 = 9.09\%$$

Example 2

A single-phase, 440/220 V, 10 kVA, 50 Hz transformer has a resistance of 0.2 Ω and reactance of 0.6 Ω on h.v. side. The corresponding values of l.v. side are 0.04 Ω and 0.14 Ω. Calculate the percentage regulation on full load for (i) 0.8 lagging pf (ii) 0.8 leading pf, (iii) unity pf.

Solution kVA rating = 10 kVA

$$E_2 = 220 \text{ V}$$
$$E_1 = 440 \text{ V}$$
$$R_2 = 0.04\ \Omega$$
$$R_1 = 0.2\ \Omega$$
$$X_2 = 0.14\ \Omega$$
$$X_1 = 0.6\ \Omega$$

For a transformer $V_2 \approx E_2 = 220$ V

$$I_2 = \frac{\text{kVA rating} \times 1000}{V_2} = \frac{10 \times 1000}{220} = 45.45 \text{ A}$$

$$K = \frac{E_2}{E_1} = \frac{220}{440} = 0.5$$

$$R_{02} = R_2 + K^2 R_1 = 0.04 + (0.05)^2 \times 0.2 = 0.09\ \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 0.14 + (0.5)^2 \times 0.6 = 0.29\ \Omega$$

(i) Percentage regulation on full load for 0.8 lagging pf

$$\cos\phi = 0.8$$
$$\sin\phi = 0.6$$

$$\%\text{ regulation} = \frac{I_2 (R_{02}\cos\phi + X_{02}\sin\phi)}{E_2} \times 100$$
$$= \frac{45.45(0.09 \times 0.8 + 0.29 \times 0.6)}{220} \times 100$$
$$= 5.08\%$$

(ii) Percentage regulation on full load for 0.8 leading pf

$$\%\text{ regulation} = \frac{I_2 (R_{02}\cos\phi - X_{02}\sin\phi)}{E_2} \times 100$$
$$= \frac{45.45(0.09 \times 0.8 - 0.29 \times 0.6)}{220} \times 100$$
$$= -2.11\%$$

(iii) Percentage regulation on full load for unity pf,

$$\cos\phi = 1$$

$$\sin \phi = 0$$

$$\% \text{ regulation} = \frac{I_2 (R_{02} \cos \phi \pm X_{02} \sin \phi)}{E_2} \times 100$$

$$= \frac{45.45 (0.09 \times 1 - 0.29 \times 0)}{220} \times 100$$

$$= 1.86\%$$

Example 3

Calculate the regulation of a transformer in which resistive drop is 1% of the output and reactive drop is 5 % of the output, when the pf is (a) 0.8 lagging, (b) unity, and (c) 0.8 leading.

Solution

$$v_r = 1$$

$$v_x = 5$$

$$\% \text{ regulation} = v_r \cos \phi \pm v_x \sin \phi$$

(a) Percentage regulation for 0.8 lagging pf,

$$\cos \phi = 0.8$$

$$\sin \phi = 0.6$$

$$\% \text{ regulation} = 1 \times 0.8 + 5 \times 0.6 = 3.8\%$$

(b) Percentage regulation for unity pf,

$$\cos \phi = 1$$

$$\sin \phi = 0$$

$$\% \text{ regulation} = 1 \times 1 + 5 \times 0 = 1\%$$

(c) Percentage regulation for 0.8 leading pf,

$$\% \text{ regulation} = 1 \times 0.8 - 5 \times 0.6 = -2.2\%$$

Example 4

A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.

Solution

$$v_r = 5$$

$$v_x = 2.5$$

$$\% R = v_r \cos \phi + v_x \sin \phi \qquad (1)$$

Differentiating Eq. (1),

$$\frac{dR}{d\phi} = -v_r \sin \phi + v_r \cos \phi$$

For regulation to be maximum,

$$\frac{dR}{d\phi} = 0$$

$$-V_r \sin\phi + V_x \cos\phi = 0$$

$$\tan\phi = \frac{v_x}{v_r} = \frac{5}{2.5} = 2$$

$$\phi = 63.43°$$

$$\text{pf} = \cos\phi = \cos(63.43°) = 0.45$$

$$\sin\phi = 0.89$$

Maximum percentage regulation = $v_r \cos\phi + v_x \sin\phi = 2.5 \times 0.45 + 5 \times 0.89 = 5.58\%$

Example 5

A 230/460 V transformer has a primary resistance of 0.2 Ω and a reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when 10 A is supplied at 0.8 pf lagging.

Solution

$$E_1 = 230 \text{ V}$$

$$E_2 = 460 \text{ V}$$

$$R_1 = 0.2\ \Omega$$

$$R_2 = 0.75\ \Omega$$

$$X_1 = 0.5\ \Omega$$

$$X_2 = 1.8\ \Omega$$

$$I_2 = 10 \text{ A}$$

$$\cos\phi = 0.8$$

$$K = \frac{E_2}{E_1} = \frac{460}{230} = 2$$

$$R_{02} = R_2 + K^2 R_1 = 0.75 + (2)^2 \times 0.2 = 1.55\ \Omega$$

$$X_{02} = X_2 + K^2 X_1 = 1.8 + (2)^2 \times 0.5 = 3.8\ \Omega$$

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

For lagging pf,

$$E_2 - V_2 = I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$$

Secondary terminal voltage

$$\begin{aligned} V_2 &= E_2 - I_2 (R_{02} \cos\phi + X_{02} \sin\phi) \\ &= 460 - 10\,(1.55 \times 0.8 + 3.8 \times 0.6) \\ &= 424.8 \text{ V} \end{aligned}$$

Useful Formulae

$$W_i = V_1 I_0 \cos \phi_0$$

$$I_\mu = I_0 \sin \phi_0$$

$$I_w = I_0 \cos \phi_0$$

$$W_{Cu} = I_1^2 R_1 + I_2^2 R_2$$

$$W_{Cu} = I_1^2 R_{01} = I_2^2 R_{02}$$

$$R_{01} = R_1 + \frac{R_2}{K^2}$$

$$R_{02} = R_2 + K^2 R_1 = K^2 R_{01}$$

$$X_{01} = X_1 + \frac{X_2}{K^2}$$

$$X_{02} = X_2 + K^2 X_1 = K^2 X_{01}$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = K^2 Z_{01}$$

$$\% \text{ Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

$$= \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} \times 100$$

$$= \frac{I_1 R_{01} \cos\phi \pm I_1 R_{01} \sin\phi}{V_1} \times 100$$

Exercise 6.2

6.1 The no-load current of a transformer is 10 A at a pf of 0.25 lagging, when connected to a 400 V, 50 Hz supply. Calculate (a) magnetising component of no-load current, (b) iron loss, and (c) maximum value of flux in the core. Assume primary winding turns as 500. *[9.68 A, 1000 W, 3.6036 mWb]*

6.2 A 2200/250 V transformer takes 0.5 A at a pf of 0.3 on no load. Find magnetising and working components of no-load primary current. *[0.476 A, 0.15 A]*

6.3 The no-load current of a transformer is 4 A at 0.25 pf when supplied at 250 V, 50 Hz. The number of turns on the primary winding is 200. Calculate (i) rms value of flux in the core, (ii) core loss, and (iii) magnetising current. *[5.63 mWb, 250 W, 3.87 A]*

6.4 The values of the resistances of the primary and secondary windings of a 2200/ 200 V, 50 Hz single-phase transformer are 2.4 Ω and 0.02 Ω respectively. Find (i) equivalent resistance of primary referred to secondary, (ii) equivalent resistance of secondary referred to primary, (iii) total resistance referred to secondary, and (iv) total resistance referred to primary. *[0.0198 Ω, 2.42 Ω, 0.0398 Ω, 4.82 Ω]*

6.5 A 40 kVA transformer with a ratio of 2000/250 V has a primary resistance of 1.15 Ω and a secondary resistance of 0.0155 Ω. Calculate (i) the total resistance in terms of the secondary winding, (ii) total resistance drop on full load, and (iii) total copper loss on full load. *[0.0334 Ω, 5.35 V, 855.04 Ω]*

6.13 EFFICIENCY

Efficiency is defined as the ratio of output power to input power.

Efficiency $$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}}$$

$$= \frac{\text{Output}}{\text{Output + Copper loss + Iron loss}}$$

Also $$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{\text{Input} - \text{Copper loss} - \text{Iron loss}}{\text{Input}}$$

Condition for Maximum Efficiency We know that,

$$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}}$$

Considering secondary side of the transformer,

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}}$$

Differentiating both the sides w.r.t. I_2,

$$\frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}) V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2 I_2 R_{02})}{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02})^2}$$

For maximum efficiency, $\frac{d\eta}{dI_2} = 0$

$$(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}) V_2 \cos\phi_2 = V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2 I_2 R_{02})$$

$$V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02} = V_2 I_2 \cos\phi_2 + 2 I_2^2 R_{02}$$

$$W_i = I_2^2 R_{02}$$

Similarly on primary side,

$$W_i = I_1^2 R_{01}$$

Thus when copper loss = iron loss, the efficiency of the transformer is maximum.

Load Corresponding to Maximum Efficiency For maximum efficiency,

$$W_i = I_2^2 R_{02}$$

$$I_{2\,(\text{max. efficiency})} = \sqrt{\frac{W_i}{R_{02}}}$$

Multiplying both the sides by V_2,

$$V_2 I_{2\,(\text{max. efficiency})} = V_2 \sqrt{\frac{W_i}{R_{02}}}$$

$$\text{Load } VA_{\text{(max. efficiency)}} = V_2 I_2 \sqrt{\frac{W_i}{I_2^2 R_{02}}} = V_2 I_2 \sqrt{\frac{W_i}{W_{Cu}}}$$

$$\text{Load kVA}_{\text{(max. efficiency)}} = \text{Full-load kVA} \sqrt{\frac{W_i}{W_{Cu}}}$$

where W_i = iron loss

W_{Cu} = full-load copper loss

Note: The efficiency at any load is given by,

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

where x = ratio of actual to full load kVA

W_i = iron loss in kW

W_{Cu} = full-load copper loss in kW

Example 1

Iron loss of 80 kVA, 1000/250 V, single-phase, 50 Hz transformer is 500 W. The copper loss when the primary carries a current of 50 A is 400 W. Find (i) area of cross section of limb if working flux density is 1 T and there are 1000 turns on the primary, (ii) efficiency at full load and pf 0.8 lagging, and (iii) efficiency at 75% of full load and unity pf.

Solution

Full load kVA = 80 kVA

$E_1 = 1000$ V

$E_2 = 250$ V

$f = 50$ Hz

$W_i = 500 \text{ W} = 0.5 \text{ kW}$

$W_{Cu} = 400 \text{ W} = 0.4 \text{ kW}$

$I_1 = 50$ A

$B_m = 1$ T

$N_1 = 1000$

(i) Area of cross section of limb

$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$

$$1000 = 4.44 \times 50 \times 1 \times A \times 1000$$

$$A = 4.5 \times 10^{-3} \text{ m}^2$$

(ii) Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 80 \times 0.8}{1 \times 80 \times 0.8 + 0.5 + (1)^2 \times 0.4} \times 100$$

$$= 98.61\%$$

(iii) Efficiency at 75% of full load and unity pf

$$x = 0.75$$

$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.75 \times 80 \times 1}{0.75 \times 80 \times 1 + 0.5 + (0.75)^2 \times 0.4} \times 100$$

$$= 98.81\ \%$$

Example 2

A 100 kVA, single-phase transformer has iron loss of 600 W and a copper loss of 1.5 kW at full-load current. Calculate the efficiency at (i) full load and 0.8 lagging pf, and (ii) half load and unity pf.

Solution

$$\text{Full load kVA} = 100 \text{ kVA}$$

$$W_i = 600 \text{ W} = 0.6 \text{ kW}$$

$$W_{Cu} = 1.5 \text{ kW}$$

(i) Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 0.6 + (1)^2 \times 1.5} \times 100$$

$$= 97.44\ \%$$

(ii) Efficiency at half load and unity pf

$$x = 0.5$$

$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 100 \times 1}{0.5 \times 100 \times 1 + 0.6 + (0.5)^2 \times 1.5} \times 100$$

$$= 98.09\ \%.$$

Example 3

A 25 kVA, 2200/220 V, 50 Hz, single-phase transformer has a primary resistance of 1.8 Ω and a secondary resistance of 0.02 Ω. Calculate the efficiency of the transformer at (i) full load and unity pf, and (ii) half load and 0.8 lagging pf. Iron loss is 1000 watts.

Solution

$$\text{Full load kVA} = 25\text{ kVA}$$

$$E_1 = 2200\text{ V}$$

$$E_2 = 220\text{ V}$$

$$R_1 = 1.8\ \Omega$$

$$R_2 = 0.02\ \Omega$$

$$W_i = 1000\text{ W} = 1\text{ kW}$$

For a transformer,

$$E_2 \simeq V_2 = 220\text{ V}$$

$$I_2 = \frac{\text{Full load kVA} \times 1000}{V_2} = \frac{25 \times 1000}{220} = 113.64\text{ A}$$

$$K = \frac{E_2}{E_1} = \frac{220}{2200} = 0.1$$

$$R_{02} = R_2 + K^2 R_1 = 0.02 + (0.1)^2 \times 1.8 = 0.038\ \Omega$$

$$W_{Cu} = I_2^2 R_{02} = (113.64)^2 \times 0.038 = 0.49\text{ kW}$$

(i) Efficiency at full load and unity pf

$$x = 1$$

$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 25 \times 1}{1 \times 25 \times 1 + 1 + (1)^2 \times 0.49} \times 100$$

$$= 94.38\%$$

(ii) Efficiency at half load and 0.8 lagging pf

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 25 \times 0.8}{0.5 \times 25 \times 0.8 + 1 + (0.5)^2 \times 0.49} \times 100$$

$$= 89.91\%.$$

Example 4

A 250 kVA, single-phase transformer has 98.135% efficiency at full load and 0.8 lagging pf. The efficiency at half load and 0.8 lagging pf is 97.751%. Calculate the iron loss and full load copper loss.

Solution

$$\text{Full load kVA} = 250 \text{ kVA}$$

$$\eta_1 = 98.135\%$$

$$\eta_2 = 97.751\%$$

(i) Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta_1 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$98.135 = \frac{1 \times 250 \times 0.8}{1 \times 250 \times 0.8 + W_i + (1)^2 W_{Cu}} \times 100$$

$$W_i + W_{Cu} = 3.8 \text{ kW} \qquad (1)$$

(ii) Efficiency at half load and 0.8 lagging pf

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\eta_2 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$97.751 = \frac{0.5 \times 250 \times 0.8}{0.5 \times 250 \times 0.8 + W_i + (0.5)^2 W_{Cu}} \times 100$$

$$W_i + 0.25\ W_{Cu} = 2.3 \tag{2}$$

Solving Eqs (1) and (2),

$$W_{Cu} = 2 \text{ kW}$$

$$W_i = 1.8 \text{ kW}$$

Example 5

A 600 kVA, single-phase transformer has an efficiency of 92% at full load and also at half load, working at unity pf. Calculate the efficiency of the transformer at 60% full load and unity pf.

Solution

$$\text{Full load kVA} = 600 \text{ kVA}$$

$$\eta_1 = \eta_2 = 92\%$$

Efficiency at full load and unity pf

$$x = 1$$

$$\text{pf} = 1$$

$$\%\eta_1 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_i + (1)^2 W_{Cu}} \times 100$$

$$W_i + W_{Cu} = 52.2 \tag{1}$$

Efficiency at half load and unity pf

$$x = 0.5$$

$$\text{pf} = 1$$

$$\%\eta_2 = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$92 = \frac{0.5 \times 600 \times 1}{0.5 \times 600 \times 1 + W_i + (0.5)^2 W_{Cu}} \times 100$$

$$W_i + 0.25\ W_{Cu} = 26.1 \tag{2}$$

Solving Eqs (1) and (2),

$$W_{Cu} = 34.8 \text{ kW}$$
$$W_i = 17.4 \text{ kW}$$

Efficiency at 60% full load and unity pf

$$x = 0.6$$
$$\text{pf} = 1$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.6 \times 600 \times 1}{0.6 \times 600 \times 1 + 17.4 + (0.6)^2 \times 34.8} \times 100$$

$$= 92.32\ \%$$

Example 6

A 150 kVA, single-phase transformer has iron loss of 1.4 kW and full-load copper loss of 1.6 kW. Determine (i) the kVA load for maximum efficiency and the maximum efficiency at 0.8 lagging pf, and (ii) the efficiency at half full load and 0.8 lagging pf.

Solution

$$\text{Full load kVA} = 150 \text{ kVA}$$
$$W_i = 1.4 \text{ kW}$$
$$W_{Cu} = 1.6 \text{ kW}$$

(i) Load kVA for maximum efficiency and the maximum efficiency

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 150 \times \sqrt{\frac{1.4}{1.6}} = 140.31 \text{ kVA}$$

For maximum efficiency,

$$W_i = W_{Cu} = 1.4 \text{ kW}$$
$$\text{pf} = 0.8$$

$$\%\eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{140.31 \times 0.8}{140.31 \times 0.8 + 1.4 + 1.4} \times 100$$

$$= 97.57\%$$

(ii) Efficiency at half full load and 0.8 pf

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 150 \times 0.8}{0.5 \times 150 \times 0.8 + 1.4 + (0.5)^2 \times 1.6} \times 100$$

$$= 97.08\%.$$

Example 7

A 100 kVA, single-phase transformer has an efficiency of 97% at full load and 0.8 lagging pf. If the maximum efficiency occurs at 80% of full load at 0.8 lagging pf, calculate (i) iron loss and full load copper loss, (ii) maximum efficiency.

Solution

$$\text{Full load kVA} = 100 \text{ kVA}$$

$$\eta = 97\%$$

(i) Iron loss and full load copper loss

Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$97 = \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + W_i + (1)^2 \times W_{Cu}} \times 100$$

$$W_i + W_{Cu} = 2.474 \qquad (1)$$

(ii) Maximum efficiency occurs at 80 % of full load

$$W_i = (0.8)^2\, W_{Cu} = 0.64\, W_{Cu} \qquad (2)$$

Solving Eqs (1) and (2),

$$W_{Cu} = 1.508 \text{ kW}$$

$$W_i = 0.965 \text{ kW}$$

Maximum efficiency at 80 % full load and 0.8 lagging pf

$$\%\eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{0.8 \times 100 \times 0.8}{0.8 \times 100 \times 0.8 + 0.965 + 0.965} \times 100$$

$$= 97.07\%.$$

Example 8

The maximum efficiency of a 500 kVA, 3000/500 V, 50 Hz single-phase transformer is 98% and occurs at 3/4 full load, unity pf. If the impedance is 10%, calculate the regulation at full load, 0.8 lagging pf.

Solution

$$\% \eta_{max} = 98\%$$

$$\text{Full load kVA} = 500 \text{ kVA}$$

$$E_1 = 3000 \text{ V}$$

$$E_2 = 500 \text{ V}$$

Since, maximum efficiency occurs at 3/4 of full load and unity pf, iron loss is equal to copper loss at this load.

$$\% \eta_{max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$98 = \frac{3/4 \times 500 \times 1}{3/4 \times 500 \times 1 + 2W_i} \times 100$$

$$W_i = 3.826 \text{ kW}$$

Copper loss at 3/4 of full load = 3.826 kW

$$\text{Full-load copper loss} = \left(\frac{4}{3}\right)^2 \times 3.826 = 6.803 \text{ kW}$$

Percentage regulation at full load and 0.8 lagging pf

$$\% \text{ resistance} = v_r = \frac{I_2\ R_{02}}{E_2} \times 100 = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2\ R_{01}}{V_1 I_1} \times 100$$

$$= \% \text{ Cu loss at full load}$$

$$= \frac{6.803}{500} \times 100$$

$$= 1.36\%$$

$$\% \text{ reactance} = v_x = \sqrt{\% Z^2 - \% R^2}$$

$$= \sqrt{(10)^2 - (1.36)^2}$$

$$= 9.91\%$$

$$\cos \phi = 0.8 \qquad \sin \phi = 0.6$$

$$\% \text{ regulation} = v_r \cos \phi + v_x \sin \phi$$

$$= 1.36 \times 0.8 + 9.91 \times 0.6$$

$$= 7.034\%$$

Example 9

The maximum efficiency of a 100 kVA, 6600/250 V single-phase transformer occurs at half load and is 98 % at unity power factor. If the percentage impedance is 8 %, calculate the percentage regulation and efficiency on full load at 0.8 lagging pf.

Solution

$$\eta_{max} = 98\%$$
$$\text{Full load kVA} = 100 \text{ kVA}$$
$$E_1 = 6600 \text{ V}$$
$$E_2 = 250 \text{ V}$$

Since, maximum efficiency occurs at half load and unity pf, iron loss is equal to copper loss at this load.

$$\%\ \eta_{max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$98 = \frac{1/2 \times 100 \times 1}{1/2 \times 100 \times 1 + 2W_i} \times 100$$

$$W_i = 0.51 \text{ kW}$$

Copper loss at half load = 0.51 kW

Full-load copper loss $= (2)^2 \times 0.51 = 2.04$ kW

Efficiency at full load and 0.8 lagging pf

$$x = 1$$
$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 0.51 + (1)^2 \times 2.04} \times 100$$

$$= 96.91\%$$

Percentage regulation at full load and 0.8 lagging pf

$$\%\ R = v_r = \%\ \text{Cu loss} = \frac{I_1^2\, R_{01}}{V_1\, I_1} \times 100 = \frac{2.04}{100} \times 100 = 2.04\%$$

$$\%\ X = v_x = \sqrt{\%Z^2 - \%R^2} = \sqrt{(8)^2 - (2.04)^2} = 7.74\ \%$$

$$\cos\phi = 0.8 \qquad \sin\phi = 0.6$$

$$\%\ \text{regulation} = v_r \cos\phi + v_x \sin\phi$$

$= 2.04 \times 0.8 + 7.74 \times 0.6$

$= 6.28\%$

Example 10

A 300 kVA, single-phase transformer has a percentage resistance of 1.5% and maximum efficiency occurs at a load of 173.2 kVA. Find the efficiency at full load and 0.8 lagging pf.

Solution Full load kVA = 300 kVA

Load kVA = 173.2 kVA

$\% R = 1.5$

% resistance = % Cu loss

$$= \frac{\text{Full-load copper loss}}{\text{Full-load kVA}} \times 100$$

$$1.5 = \frac{\text{Full-load copper loss}}{300} \times 100$$

Full-load copper loss $W_{Cu} = 4.5$ kW

Also, for maximum efficiency,

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$173.2 = 300 \times \sqrt{\frac{W_i}{4.5}}$$

$$W_i = 1.5 \text{ kW}$$

Efficiency at full load and 0.8 lagging pf

$$x = 1$$

$$\text{pf} = 0.8$$

$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 300 \times 0.8}{1 \times 300 \times 0.8 + 1.5 + (1)^2 \times 4.5} \times 100$$

$$= 97.6\%$$

Example 11

The parameters of the equivalent circuit of 150 kVA, 2400/240 V transformer are as shown in the Fig. 6.19.

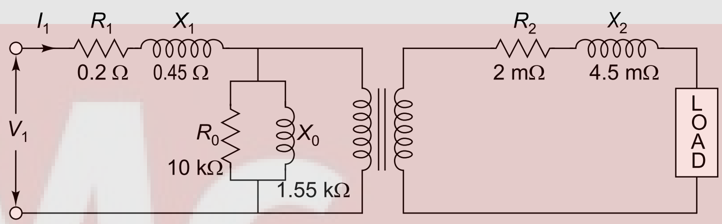


Fig. 6.19

Using the circuit referred to primary, determine voltage regulation and efficiency of the transformer operating at rated load with 0.8 lagging pf.

Solution

(i) Percentage regulation at rated load and 0.8 lagging pf

$$K = \frac{240}{2400} = 0.1$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.2 + \frac{2 \times 10^{-3}}{(0.1)^2} = 0.4\ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.45 + \frac{4.5 \times 10^{-3}}{(0.1)^2} = 0.9\ \Omega$$

$$I_1 = \frac{150 \times 1000}{2400} = 62.5\text{ A}$$

$$\cos\phi = 0.8 \qquad \sin\phi = 0.6$$

$$\%\text{ regulation} = \frac{I_1(R_{01}\cos\phi + X_{01}\sin\phi)}{V_1} \times 100$$

$$= \frac{62.5(0.4 \times 0.8 + 0.9 \times 0.6)}{2400} \times 100$$

$$= 2.24\%$$

(ii) Efficiency at rated load and 0.8 lagging pf

$$I_w = \frac{2400}{10 \times 10^3} = 0.24\text{ A}$$

$$I_w = I_0 \cos\phi_0 = 0.24\text{ A}$$

$$W_i = V_1 I_0 \cos\phi_0 = 2400 \times 0.24 = 576\text{ W} = 0.576\text{ kW}$$

$$W_{Cu} = I_1^2 R_{01} = (62.5)^2 \times 0.4 = 1562.5\text{ W} = 1.5625\text{ kW}$$

$$x = 1$$
$$\text{pf} = 0.8$$
$$\%\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$
$$= \frac{1 \times 150 \times 0.8}{1 \times 150 \times 0.8 + 0.576 + (1)^2 \times 1.5625} \times 100$$
$$= 98.25\ \%$$

Useful Formulae

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100$$
$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + W_i + W_{Cu}} \times 100$$
$$= \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

where W_i and W_{Cu} are in kW.

and $$x = \frac{\text{Actual kVA}}{\text{Full-load kVA}}$$

Condition for maximum efficiency,

$$W_i = W_{Cu}$$

Load kVA (maximum efficiency)

$$= \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$\eta_{\max} = \frac{\text{Load kVA (max.efficiency)} \times \text{pf}}{\text{Load kVA(max.efficiency)} \times \text{pf} + 2W_i} \times 100$$

Exercise 6.3

6.1 A 100 kVA transformer has iron loss of 2 kW and full-load copper loss of 1 kW. Calculate the efficiency of the transformer at (i) full-load unity pf, and (ii) half-load unity pf. *[97.087%, 95.69%]*

6.2 Calculate the efficiency of a transformer at half load and quarter load for 0.71 lagging pf for a 800 kVA, 1100/250 V, 50 Hz single-phase transformer, whose losses are as follows: Iron loss = 800 W and copper loss at full load = 800 W. *[99.64%, 99.40%]*

6.3 A 50 kVA, 2300/230 V, 50 Hz, single-phase transformer has a primary resistance of 2 Ω and a secondary resistance of 0.02 Ω. Calculate the efficiency of the transformer at (i) full load, and (ii) half load when the pf of the load is 0.8. Given that the iron loss is 412 watts. *[94.56%, 95.76%]*

6.4 A 40 kVA transformer has iron loss of 450 W and full-load copper loss of 850 W. If power factor of the load is 0.8 lagging, calculate (i) full-load efficiency, (ii) the load at which maximum efficiency occurs, and (iii) the maximum efficiency.

[96.09%, 29.104 kVA, 96.278%]

6.5 A 50 kVA transformer has an efficiency of 98% at full load 0.8 pf. and 97% at half load 0.8 pf. Determine full-load copper loss and iron loss. Find the load at which maximum efficiency occurs and also find the maximum efficiency.

[0.264 kW, 0.552 kW, 72.29 kVA, 98.12%]

6.6 The efficiency of a 220 kVA, 1100/220 V transformer is maximum of 98% at 50% of rated load. Calculate (i) core loss, and (ii) efficiency at rated load.

[1.12 kW, 97.51%]

6.7 Calculate the efficiencies at half, full and $1\frac{1}{4}$ load of a 100 kVA transformer for power factors of unity. The copper loss is 1000 W at full load and the iron loss is 1000 W. *[97.56%, 98.04%, 97.98%]*

6.8 In a 25 kVA, 2000/200 V transformer, the iron and copper losses are 350 W and 400 W respectively. Calculate the efficiency on unity power factor at (i) full load, (ii) half load, and (iii) determine the load for maximum efficiency and the iron and copper loss in this case. *[97.1%, 96.5%, 23.4 kVA, 350 W, 350 W]*

6.9 The efficiency of 400 kVA, 50 Hz, 1 phase transformer is 98.77% delivering FL at 0.8 pf and 99.13% at half load at UPF. Determine maximum efficiency at 0.8 pf.

[1.012 kW, 2.973 kW, 98.93%]

6.10 Calculate the efficiency at full load and one-fourth load at (i) unity pf, and (ii) 0.71 lagging pf, for a 80 kVA, 1100/250 V, 50 Hz single-phase transformer, whose losses are as follows:

Iron losses = 800 W

Total copper losses with 160 A in the low-voltage winding = 200 W

[(i) 98.04%, 97.57%, 95.92%, (ii) 97.25%, 96.61%, 94.36%]

6.11 A 200 kVA transformer has an efficiency of 98% at full load. If the maximum efficiency occurs at three quarters of full load, calculate the efficiency at half load. Assume negligible magnetising current and pf at all loads. *[97.9%]*

6.14 OPEN CIRCUIT (OC) TEST

The purpose of this test is to determine (i) iron loss or core loss (W_i) (ii) magnetising resistance R_0, and (iii) magnetising reactance X_0.

Figure 6.20 shows the circuit diagram for conducting OC test on the transformer. In this test, one winding (usually high-voltage winding) is left open and the other winding is connected to a supply of normal voltage and frequency. An ammeter, voltmeter and wattmeter are connected on this side. The ammeter indicates no-load current drawn by the transformer. As the no-load current is usually 3 to 5% of the full-load current, copper losses are negligible and the wattmeter indicates iron loss.

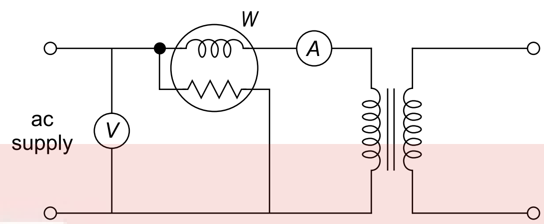


Fig. 6.20 *O.C. test*

Calculations

(i) When meters are connected on the primary side

Wattmeter reading = W_i

Voltmeter reading = V_1

Ammeter reading = I_0

$$W_i = V_1 I_0 \cos \phi_0$$

$$\cos \phi_0 = \frac{W_i}{V_1 I_0}$$

$$I_w = I_0 \cos \phi_0 \qquad I_\mu = I_0 \sin \phi_0$$

$$R_0 = \frac{V_1}{Iw}$$

$$X_0 = \frac{V_1}{I_\mu}$$

(ii) When meters are connected on the secondary side

Wattmeter reading = W_i

Voltmeter reading = V_2

Ammeter reading = I'_0

$$W_i = V_2 I'_0 \cos \phi'_0$$

$$\cos \phi'_0 = \frac{W_i}{V_2 I'_0}$$

$$I'_w = I'_0 \cos \phi'_0 \qquad I'_\mu = I'_0 \sin \phi'_0$$

$$R'_0 = \frac{V_2}{I'_w} \qquad X'_0 = \frac{V_2}{I'_\mu}$$

$$R_0 = \frac{R'_0}{K^2} \qquad X_0 = \frac{X'_0}{K^2}$$

6.15 SHORT-CIRCUIT (SC) TEST

The purpose of this test is to determine (i) full-load copper loss, (ii) equivalent resistance R_{01} or R_{02}, and (iii) equivalent reactance X_{01} or X_{02}.

Figure. 6.21 shows the circuit diagram for conducting an SC test on the transformer. In this test, one winding (usually low-voltage winding) is short circuited, while a low voltage is applied to the other winding. The applied voltage is slowly increased until full-load current flows in this winding and hence, through the other winding. Normally, the applied voltage is 5 to 10% of the rated voltage of this winding. Hence, fluxes produced in the core are small and the iron losses are very small. Thus, wattmeter indicates full-load copper loss.

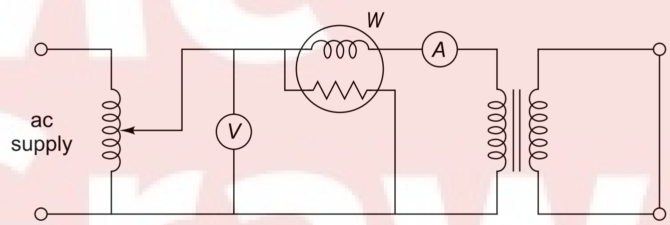


Fig. 6.21 *SC test*

Calculations

Wattmeter reading = W_{sc}

Voltmeter reading = V_{sc}

Ammeter reading = I_{sc}

(i) When meters are connected on the primary side

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2}$$

(ii) When meters are connected on the secondary side

$$Z_{02} = \frac{V_{sc}}{I_{sc}}$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2}$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2}$$

Example 1

A 5 kVA, 1000/200 V, 50 Hz, single-phase transformer gives the following test results:

OC test (LV side)	*200 V,*	*1.2 A,*	*90 W*
SC test (HV side)	*50 V,*	*5 A,*	*110 W*

Determine efficiency at half load at 0.8 pf lagging.

Solution From OC test (meters are connected on LV side, i.e., secondary),

$$W_i = 90 \text{ W} = 0.09 \text{ kW}$$

From SC test (meters are connected on HV side, i.e., primary),

$$W_{\text{sc}} = 110 \text{ W}$$

$$\text{Full-load current} = \frac{\text{kVA rating} \times 1000}{V_1}$$

$$I_1 = \frac{5 \times 1000}{1000} = 5 \text{ A}$$

$$W_{\text{Cu}} = W_{\text{sc}} = 110 \text{ W} = 0.11 \text{ kW}$$

Efficiency at half load and 0.8 pf lagging

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\% \ \eta = \frac{x \times \text{Full-load kVA} \times \text{pf}}{x \times \text{Full load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 5 \times 0.8}{0.5 \times 5 \times 0.8 \times 0.09 + (0.5)^2 + 0.11} \times 100$$

$$= 94.45 \ \%$$

Example 2

A 5 kVA, 200/400 V, 50 Hz, single-phase transformer gives the following test results:

OC test (L V side)	*200 V,*	*0.7 A,*	*60 W*
SC test (HV side)	*22 V,*	*16 A,*	*120 W*

(i) Draw the equivalent circuit of the transformer and insert all parameter values.
(ii) Find efficiency and regulation at 0.9 pf (lead) if operating at rated load.
(iii) Find current at which efficiency is maximum.

Solution

(i) Equivalent circuit of the transformer

From OC test (meters are connected on LV side, i.e., primary),

$$W_i = 60 \text{ W} \qquad V_1 = 200 \text{ V} \qquad I_0 = 0.7 \text{ A}$$

$$\cos \phi_0 = \frac{W_i}{V_1 I_0} = \frac{60}{200 \times 0.7} = 0.43$$

$$\sin \phi_0 = 0.9$$

$$I_w = I_0 \cos \phi_0 = 0.7 \times 0.43 = 0.3 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.3} = 666.67\ \Omega$$

$$I_\mu = I_0 \sin \phi_0 = 0.7 \times 0.9 = 0.63 \text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.63} = 317.46\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary),

$$W_{sc} = 120 \text{ W} \qquad V_{sc} = 22 \text{ V} \qquad I_{sc} = 16 \text{ A}$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{22}{16} = 1.375\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{120}{(16)^2} = 0.47\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.375)^2 - (0.47)^2} = 1.29\ \Omega$$

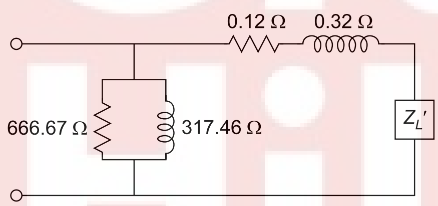


Fig. 6.22

$$K = \frac{400}{200} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.47}{(2)^2} = 0.12\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.29}{(2)^2} = 0.32\ \Omega$$

(ii) Efficiency at rated load and 0.9 pf leading

$$W_i = 60 \text{ W} = 0.06 \text{ kW}$$

Since meters are connected on secondary in SC test,

$$I_2 = \frac{5 \times 1000}{400} = 12.5 \text{ A}$$

$$W_{Cu} = I_2^2 R_{02} = (12.5)^2 \times 0.47 = 73.43 \text{ W} = 0.073 \text{ kW}$$

$$x = 1$$

$$\text{pf} = 0.9$$

$$\%\ \eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 5 \times 0.9}{1 \times 5 \times 0.9 + 0.06 + (1)^2 \times 0.073} \times 100$$

$$= 97.13\%$$

Regulation at rated load and 0.9 pf lead

$$\cos \phi = 0.9$$

$$\sin \phi = 0.44$$

$$\% \text{ regulation} = \frac{I_2 (R_{02} \cos \phi - X_{02} \sin \phi)}{E_2} \times 100$$

$$= \frac{12.5(0.47 \times 0.9 - 1.29 \times 0.44)}{400} \times 100$$

$$= -0.45\ \%$$

(iii) Current at maximum efficiency

$$W_i = I_2^2 R_{02}$$

$$I_2 = \sqrt{\frac{W_i}{R_{02}}} = \sqrt{\frac{60}{0.47}} = 11.3 \text{ A}$$

Example 3

A 5 kVA, 250/500 V, 50 Hz, single-phase transformer gives the following test results:

No-load test (LV side)	*250 V,*	*0.75 A,*	*60 W*
Short Circuit test (HV side)	*9 V,*	*6 A,*	*21.6 W*

Calculate (i) The equivalent circuit constants and insert these on the equivalent circuit diagram

(ii) Efficiency at 60% of full-load unity pf

(iii) Maximum efficiency and the load at which it occurs

(iv) The secondary terminal voltage on full load at pf of 0.8 lagging, unity and 0.8 leading

Solution

(i) Equivalent circuit constants

From no-load test (meters are connected on LV side, i.e., primary),

$W_i = 60$ W $\qquad V_1 = 250$ V $\qquad I_0 = 0.75$ A

$$\cos \phi_0 = \frac{W_i}{V_1 I_0} = \frac{60}{250 \times 0.75} = 0.32$$

$$\sin \phi_0 = 0.95$$

$$I_w = I_0 \cos \phi_0 = 0.75 \times 0.32 = 0.24 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{250}{0.24} = 1041.66\ \Omega$$

$$I_\mu = I_0 \sin\phi_0 = 0.75 \times 0.95 = 0.71 \text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{250}{0.71} = 351.84\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary),

$W_{sc} = 21.6$ W $\quad V_{sc} = 9$ V $\quad I_{sc} = 6$ A

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{9}{6} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{21.6}{(6)^2} = 0.6\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.6)^2} = 1.37\ \Omega$$

$$K = \frac{500}{250} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.6}{(2)^2} = 0.15\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.37}{(2)^2} = 0.34\ \Omega$$

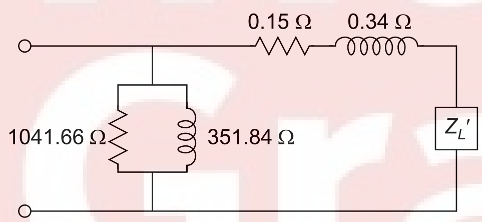


Fig. 6.23

(ii) Efficiency at 60 % of full load and unity factor

$$x = 0.6$$

$$W_i = 60 \text{ W} = 0.06 \text{ kW}$$

Since meters are connected on HV side in SC test,

$$I_2 = \frac{5 \times 1000}{500} = 10 \text{ A}$$

$$W_{Cu} = I_2^2 R_{02} = (10)^2 \times 0.6 = 60 \text{ W} = 0.06 \text{ kW}$$

$$\text{pf} = 1$$

$$\%\ \eta = \frac{x \times \text{full load kVA} \times \text{pf}}{x \times \text{full load kVA} \times \text{pf} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.6 \times 5 \times 1}{0.6 \times 5 \times 1 + 0.06 + (0.6)^2 \times 0.06} \times 100$$

$$= 97.35\ \%$$

(iii) The load corresponding to maximum efficiency

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 5 \times \sqrt{\frac{60}{60}} = 5 \text{ kVA}$$

For maximum efficiency,

$$W_i = W_{Cu} = 60 \text{ W} = 0.06 \text{ kW}$$

$$\text{pf} = 1$$

$$\% \, \eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{5 \times 1}{5 \times 1 + 0.06 + 0.06} \times 100$$

$$= 97.65 \, \%$$

(iv) Secondary terminal voltage

$$E_2 - V_2 = I_2 \, (R_{02} \cos \phi \pm X_{02} \sin \phi)$$

$$V_2 = E_2 - I_2 \, (R_{02} \cos \phi \pm X_{02} \sin \phi)$$

For $\quad \text{pf} = 0.8$ lagging

$$\cos \phi = 0.8$$

$$\sin \phi = 0.6$$

$$E_2 = 500 \text{ V}$$

$$V_2 = 500 - 10(0.6 \times 0.8 + 1.37 \times 0.6) = 486.98 \text{ V}$$

For $\quad \text{pf} = 0.8$ leading,

$$V_2 = 500 - 10(0.6 \times 0.8 - 1.37 \times 0.6) = 503.42 \text{ V}$$

For unity pf,

$$V_2 = 500 - 10(0.6 \times 0.8 + 0) = 494 \text{ V}$$

Example 4

A single-phase, 50 kVA, 2400/120 V, 50 Hz transformer gives the following results:

OC test with instruments on LV side	*120 V,*	*9.65 A,*	*396 W*
SC test with instruments on HV side	*92 V,*	*20.8 A,*	*810 W*

Calculate (i) Equivalent circuit constants
(ii) Draw equivalent circuit
(iii) The efficiency when rated kVA is delivered to a load having a pf of 0.8 lagging
(iv) The voltage regulation
(v) kVA at maximum efficiency

Solution (i) Equivalent circuit constants

From OC test (meters are connected on LV side, i.e., secondary),

$$W_i = 396 \text{ W} \qquad V_2 = 120 \text{ V} \qquad I'_0 = 9.65 \text{ A}$$

$$\cos \phi'_0 = \frac{396}{120 \times 9.65} = 0.34$$

$$\sin \phi'_0 = 0.94$$

$$I'_w = I'_0 \cos \phi'_0 = 9.65 \times 0.34 = 3.28 \text{ A}$$

$$R'_0 = \frac{V_2}{I'_w} = \frac{120}{3.3} = 36.36\ \Omega$$

$$I'_\mu = I'_0 \sin \phi'_0 = 9.65 \times 0.94 = 9.07\ \text{A}$$

$$X'_0 = \frac{V_2}{I'_\mu} = \frac{120}{9.07} = 13.23\ \Omega$$

$$K = \frac{120}{2400} = 0.05$$

$$R_0 = \frac{R'_0}{K^2} = \frac{36.36}{(0.05)^2} = 14.54\ \text{k}\Omega$$

$$X_0 = \frac{X'_0}{K^2} = \frac{13.23}{(0.05)^2} = 5.29\ \text{k}\Omega$$

From SC test (meters are connected on primary),

$$W_{sc} = 810\ \text{W} \qquad V_{sc} = 92\ \text{V} \qquad I_{sc} = 20.8\ \text{A}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{92}{20.8} = 4.42\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{810}{(20.8)^2} = 1.87\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(4.42)^2 - (1.87)^2} = 4\ \Omega$$

(ii) Equivalent circuit

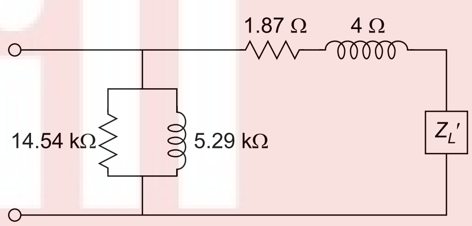


Fig. 6.24

(iii) Efficiency at full load and 0.8 pf lagging

$$x = 1$$

$$W_i = 396\ \text{W} = 0.396\ \text{kW}$$

Since meters are connected on the primary side in SC test,

$$I_1 = \frac{50 \times 1000}{2400} = 20.8\ \text{A}$$

$$W_{Cu} = I_1^2 R_{01} = (20.8)^2 \times 1.87 = 809\ \text{W} = 0.81\ \text{kW}$$

$$\text{pf} = 0.8$$

$$\% \ \eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 50 \times 0.8}{1 \times 50 \times 0.8 + 0.396 + (1)^2 \times 0.81} \times 100$$

$$= 97.07 \ \%$$

(iv) Voltage regulation

$$\cos \phi = 0.8$$

$$\sin \phi = 0.6$$

$$\% \text{ regulation} = \frac{I_1 (R_{01} \cos \phi + X_{01} \sin \phi)}{V_1} \times 100$$

$$= \frac{20.8(1.87 \times 0.8 + 4 \times 0.6)}{2400} \times 100$$

$$= 3.38 \ \%$$

(v) Load kVA at maximum efficiency,

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 50 \times \sqrt{\frac{0.396}{0.81}} = 34.96 \text{ kVA}$$

Example 5

The instrument readings obtained from open-circuit test and short-circuit test on a 10 kVA, 450/120 V, 50 Hz single-phase ac transformer are as follows:

OC test (LV side)	$V_0 = 120$ V	$I'_0 = 4.2$ A	$W_0 = 80$ W
SC test (HV side)	$V_{sc} = 9.65$ V	$I_{sc} = 22.2$ A	$W_{sc} = 120$ W

Compute the following: *(i) Equivalent circuit constants*
(ii) Draw the equivalent circuit
(iii) Efficiency and voltage regulation at 80 % lagging pf load
(iv) Efficiency at half full load at 80 % lagging pf load
(v) The maximum efficiency at 0.8 pf lagging

Solution (i) Equivalent circuit constants

From OC test (meters are connected on LV side, i.e., secondary),

$$W_0 = 80 \text{ W} \qquad V_0 = 120 \text{ V} \qquad I'_0 = 4.2 \text{ A}$$

$$\cos \phi'_0 = \frac{W_0}{V_0 I'_0} = \frac{80}{120 \times 4.2} = 0.16$$

$$\sin \phi'_0 = 0.99$$

$$I'_w = I'_0 \cos \phi'_0 = 4.2 \times 0.16 = 0.67 \text{ A}$$

$$R_0' = \frac{V_0}{I_w'} = \frac{120}{0.67} = 180.03\ \Omega$$

$$I_\mu' = I_0' \sin \phi_0' = 4.2 \times 0.99 = 4.16\ \text{A}$$

$$X_0' = \frac{V_0}{I_\mu'} = \frac{120}{4.16} = 28.85\ \Omega$$

$$K = \frac{120}{450} = 0.27$$

$$R_0 = \frac{R_0'}{K^2} = \frac{180.03}{(0.27)^2} = 2469.55\ \Omega$$

$$X_0 = \frac{X_0'}{K^2} = \frac{28.85}{(0.27)^2} = 395.75\ \Omega$$

From SC test (meters are connected on HV side, i.e., primary),

$$Z_{01} = \frac{V_{\text{sc}}}{I_{\text{sc}}} = \frac{9.65}{22.2} = 0.43\ \Omega$$

$$R_{01} = \frac{W_{\text{sc}}}{I_{\text{sc}}^2} = \frac{120}{(22.2)^2} = 0.24\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(0.43)^2 - (0.24)^2} = 0.36\ \Omega$$

(ii) Equivalent circuit

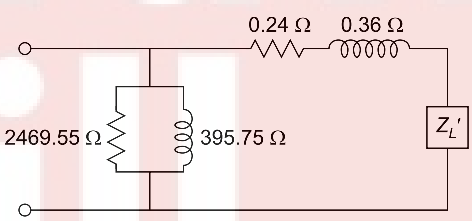


Fig. 6.25

(iii) Efficiency and voltage regulation at 80% lagging pf load

$$W_0 = 80\ \text{W} = 0.08\ \text{kW}$$

Since meters are connected on the primary side in SC test,

$$I_1 = \frac{10 \times 1000}{450} = 22.22\ \text{A}$$

$$W_{Cu} = I_1^2 R_{01} = (22.22)^2 \times 0.24 = 118.49 = 0.12\ \text{kW}$$

Efficiency at 80% lagging pf load

$$x = 1$$

$$\text{pf} = 0.8$$

$$\eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} \times \text{pf} + W_0 + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 10 \times 0.8}{1 \times 10 \times 0.8 + 0.08 + (1)^2 \times 0.12} \times 100$$

$$= 97.56\ \%$$

Voltage regulation

$$\cos\phi = 0.8$$

$$\sin\phi = 0.6$$

$$\%\ \text{regulation} = \frac{I_1 (R_{01}\cos\phi + X_{01}\sin\phi)}{V_1} \times 100$$

$$= \frac{22.22(0.24 \times 0.8 + 0.36 \times 0.6)}{450} \times 100$$

$$= 2.01\ \%$$

(iv) Efficiency at half full load for 80 % lagging pf load

$$x = 0.5$$

$$\text{pf} = 0.8$$

$$\%\ \eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} + W_0 + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 10 \times 0.8}{0.5 \times 10 \times 0.8 + 0.08 + (0.5)^2 \times 0.12} \times 100$$

$$= 97.34\ \%$$

(v) Maximum efficiency at 0.8 pf lagging

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}} = 10 \times \sqrt{\frac{80}{120}} = 8.16\ \text{kVA}$$

$$\%\ \eta_{max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{8.16 \times 0.8}{8.16 \times 0.8 + 0.08 + 0.08} \times 100$$

$$= 97.61\ \%$$

Example 6

Obtain the equivalent circuit of a 200/400 V, 50 Hz, single-phase transformer from the following test:

OC test	*200 V,*	*0.7 A,*	*70 W*	*on LV side*
SC test	*15 V,*	*10 A,*	*85 W*	*on HV side*

Calculate the secondary voltage when delivering 5 kW, 0.8 pf lagging, the primary voltage being 200 V.

Solution From OC test (meters are connected on LV side, i.e., primary),

$$W_i = 70 \text{ W}, \quad V_1 = 200 \text{ V}, \quad I_0 = 0.7 \text{ A}$$

$$\cos\phi = \frac{W_i}{V_i I_0} = \frac{70}{200 \times 0.7} = 0.5$$

$$\sin\phi = 0.87$$

$$I_w = I_0 \cos\phi_0 = 0.7 \times 0.5 = 0.35 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.35} = 571.43\ \Omega$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.87 = 0.61 \text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.61} = 327.87\ \Omega$$

From SC test (meters are connected on HV side, i.e., secondary),

$$W_{sc} = 85 \text{ W} \quad V_{sc} = 15 \text{ V} \quad I_{sc} = 10 \text{ A}$$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{15}{10} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{85}{(10)^2} = 0.85\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.85)^2} = 1.24\ \Omega$$

$$K = \frac{400}{200} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.85}{(2)^2} = 0.21\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.24}{(2)^2} = 0.31\ \Omega$$

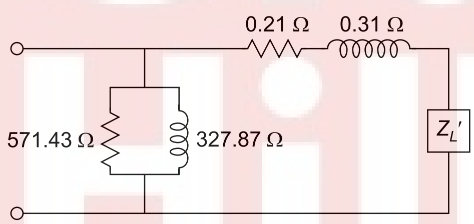


Fig. 6.26

$$I_2 = \frac{5000}{400 \times 0.8} = 15.63 \text{ A}$$

$$V_2 = E_2 - I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$$
$$= 400 - 15.63 (0.85 \times 0.8 + 1.24 \times 0.6)$$
$$= 377.74 \text{ V}$$

Example 7

A 40 kVA single-phase transformer with voltages of 11 kV/440 V has the following test results:

OC test (Instruments on LV side)	*440 V,*	*1.1 A,*	*145 W*
SC test (Instruments on HV side)	*100 V,*	*3 A,*	*90 W*

Deduce an approximate circuit referred to the LV side. For this transformer, calculate maximum efficiency.

Solution (i) Approximate equivalent circuit

From OC test (meters are connected on LV side, i.e., secondary).

$W_0 = 145$ W $\qquad V_0 = 440$ V $\qquad I'_0 = 1.1$ A

$$\cos \phi'_0 = \frac{W_0}{V_0 I'_0} = \frac{145}{440 \times 1.1} = 0.3$$

$$\sin \phi'_0 = 0.95$$

$$I'_w = I'_0 \cos \phi'_0 = 1.1 \times 0.3 = 0.33 \text{ A}$$

$$R'_0 = \frac{V_0}{I'_w} = \frac{440}{0.33} = 1333.33\ \Omega$$

$$I'_\mu = I'_0 \sin \phi'_0 = 1.1 \times 0.95 = 1.05 \text{ A}$$

$$X'_0 = \frac{V_0}{I'_\mu} = \frac{440}{1.05} = 419.05\ \Omega$$

From SC test (meters are connected on the HV side, i.e., primary)

$W_{sc} = 90$ W $\qquad V_{sc} = 100$ V $\qquad I_{sc} = 3$ A

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{100}{3} = 33.33\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{90}{(3)^2} = 10\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(33.33)^2 - (10)^2} = 31.79\ \Omega$$

$$K = \frac{440}{11 \times 10^3} = 0.04$$

$$R_{02} = K^2 R_{01} = (0.04)^2 \times 10 = 0.016\ \Omega$$

$$X_{02} = K^2 X_{01} = (0.04)^2 \times 31.79 = 0.05\ \Omega$$

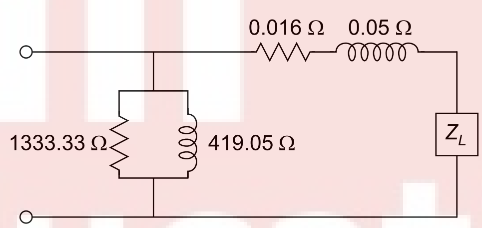


Fig. 6.27

(ii) Maximum efficiency

$$W_i = 145 \text{ W}$$

Since meters are connected on the primary in SC test,

$$I_1 = \frac{40 \times 1000}{11 \times 1000} = 3.64 \text{ A}$$

$$W_{Cu} = I_1^2 R_{01} = (3.64)^2 \times 10 = 132.5 \text{ W}$$

$$\text{Load kVA} = \text{Full-load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$= 40 \times \sqrt{\frac{145}{132.5}} = 41.84 \text{ kVA}$$

$$\% \, \eta_{\max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{41.84 \times 1}{41.84 \times 1 + 0.145 + 0.145} \times 100$$

$$= 99.31 \, \%$$

Example 8

The results of OC and SC test on a 25 kVA, 440/220V, 50 Hz transformer are on follows:

OC test	*220 V,*	*9.6 A,*	*710 W,*	*LV side*
SC test	*42 V,*	*57 A,*	*1030 W,*	*HV side*

Obtain the parameters of the exact equivalent circuit referred to the high-voltage side.

Solution From OC test (meters are connected on the LV side, i.e., secondary),

$$W_i = 710 \text{ W} \qquad V_2 = 220 \text{ V} \qquad I_0' = 9.6 \text{ A}$$

$$\cos \phi_0' = \frac{W_i}{V_2 I_0'} = \frac{710}{220 \times 9.6} = 0.34$$

$$\sin \phi_0' = 0.94$$

$$I_w' = I_0' \cos \phi_0' = 9.6 \times 0.34 = 3.26 \text{ A}$$

$$R_0' = \frac{V_2}{I_w'} = \frac{220}{3.26} = 67.48 \, \Omega$$

$$I_\mu' = I_0' \sin \phi_0' = 9.6 \times 0.94 = 9.02 \text{ A}$$

$$X_0' = \frac{V_2}{I_\mu'} = \frac{220}{9.02} = 24.39 \, \Omega$$

$$K = \frac{220}{440} = 0.5$$

$$R_0 = \frac{R_0'}{K^2} = \frac{67.48}{(0.5)^2} = 269.92 \, \Omega$$

$$X_0 = \frac{X_0'}{K^2} = \frac{24.39}{(0.5)^2} = 97.56 \, \Omega$$

From SC test (meters are connected on HV side, i.e., primary).

$$W_{sc} = 1030 \text{ W} \qquad V_{sc} = 42 \text{ V} \qquad I_{sc} = 57 \text{ A}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{42}{57} = 0.74\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{1030}{(57)^2} = 0.32\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(0.74)^2 - (0.32)^2} = 0.67\ \Omega$$

Example 9

The two windings of a 2400/240 V, 48 kVa, 50 Hz transformer have resistances of 0.6 and 0.025 Ω for the high and low-voltage winding respectively. The transformer requires that 238 V be impressed on the high-voltage coil in order that the rated current be circulated in the short-circuit low-voltage winding.

(i) Calculate the equivalent leakage reactance referred to high-voltage side.

(ii) How much power is needed to circulate valid current on short circuit?

(iii) Compute the efficiency at full load when the pf is 0.8 lagging. Assume that core loss equals the copper loss.

Solution (i) The equivalent leakage reactance referred to high-voltage side.

$$R_1 = 0.6\ \Omega$$

$$R_2 = 0.025\ \Omega$$

$$K = \frac{240}{2400} = 0.1$$

Since rated current flows in short-circuited low voltage winding, i.e., primary winding,

$$I_1 = \frac{48000}{2400} = 20 \text{ A}$$

$$I_{sc} = I_1 = 20 \text{ A}$$

$$V_{sc} = 238 \text{ V}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{238}{20} = 11.9\ \Omega$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.6 + \frac{0.025}{(0.1)^2} = 3.1\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(11.9)^2 - (3.1)^2} = 11.49\ \Omega$$

(ii) Power needed to circulate valid current on short circuit

$$W_{sc} = I^2_{sc} R_{01} = (20)^2 \times 3.1 = 1240 \text{ W}$$

(iii) Efficiency at full load

$$x = 1$$

$$\text{pf} = 0.8$$

$$\% \, \eta = \frac{x \times \text{full-load kVA} \times \text{pf}}{x \times \text{full-load kVA} + W_i + x^2 \, W_{Cu}} \times 100$$

$$= \frac{1 \times 48 \times 0.8}{1 \times 48 \times 0.8 + 1.24 + (1)^2 \times 1.24} \times 100$$

$$= 93.93\%$$

Exercise 6.4

6.1 A 4 kVA, 200/400 V, 50 Hz, single-phase transformer gave the following test results:

OC test (LV side)	200 V,	0.7 A,	60 W
SC test (HV side)	9 V,	6 A,	21.6 W

Calculate

(i) magnetising current and the component corresponding to iron loss at normal voltage and frequency

(ii) efficiency on full load at unity pf

(iii) secondary terminal voltage on full load at power factors of unity, 0.8 lagging and 0.8 leading

[(i) 0.63 A, 0.3 A, (ii) 97.08%, (iii) 394 V, 386.95 V, 403.44 V]

6.2 A 10 kVA, 500/2000 V, 50 Hz single-phase transformer gave the following results:

OC test : 500 V, 120 W	on primary side
SC test : 15 V, 20 A, 100 W	on primary side

Determine

(i) Efficiency on full-load unity power factor

(ii) Secondary terminal voltage on full load at unity pf, 0.8 lagging and 0.8 leading pf *[97.9%, 1980 V, 1950 V, 2018 V]*

6.3 A 100 kVA, 6600/330 V, 50 Hz, single-phase transformer took 10 A and 436 W at 100 V in a short-circuit test, the figures referring to the high-voltage side. Calculate the voltage to be applied to the high-voltage side on full load at power factor 0.8 lagging when the secondary terminal voltage is 330 V. *[6734 V]*

6.4 A 50 Hz, single-phase transformer has a turn ratio of 6. The resistances are 0.9 Ω and 0.03 Ω and the reactances are 5 Ω and 0.13 Ω for high-voltage and low-voltage winding respectively. Find (i) the voltage to be applied to the high-voltage side to obtain full-load current of 200 A in the low-voltage winding on short circuit, (ii) power factor in the short circuit. *[330 V, 0.2]*

6.5 A 5 kVA, 1000/200 V, 50 Hz, single-phase transformer gave the following test results:

OC test (LV side)	200 V,	1.2 A,	90 W
SC test (HV side)	50 V,	5 A,	110 W

Determine efficiency at half-load at 0.8 pf lagging. *[94.45%]*

6.6 Draw the equivalent circuit of transformer referred to primary side of 4 kVA, 200/400 V, 50 Hz transformer which gave the following test results:

OC test (on LV side)	200 V,	0.7 A,	70 W
SC test (on HV side)	15 V,	10 A,	80 W

[571 Ω, 330 Ω, 0.2 Ω, 0.317 Ω]

6.7 The following figures were obtained from tests on a 30 kVA, 3000/110 V transformer:

OC test	3000 V,	0.5 A,	350 W
SC test	150 V,	10 A,	500 W

Calculate the efficiency of the transformer at (a) full load 0.8 pf, (b) half-load unity pf. Also, calculate the kVA output at which the efficiency is maximum.

[96.56%, 97%, 25.1 kVA]

Review Questions

6.1 Explain the working principle of a transformer.

6.2 Explain what happens if a dc voltage is applied to a transformer.

6.3 Differentiate between shell-type and core-type transformers.

6.4 Derive an emf equation for a single-phase transformer and explain voltage and current ratio of an ideal transformer.

6.5 Show that the emf per turn in a transformer is $4.44 f \phi_m$, where f is the frequency of supply and ϕ_m is maximum flux associated with transformer winding.

6.6 What do you understand by an ideal transformer?

6.7 Draw and explain phasor diagram of a transformer for

(a) Unity power factor or resistive load
(b) Lagging power factor or inductive load
(c) Leading power factor or capacitive load

6.8 Develop the approximate equivalent circuit of a transformer. How does it help in deciding the regulation of a transformer?

6.9 Define voltage regulation and derive its expression.

6.10 Explain various losses in a transformer.

6.11 What do you understand by efficiency of a transformer? Derive the condition for maximum efficiency.

Objective-Type Questions

Choose the correct alternative in the following questions:

6.1 When the primary of a transformer is connected to a dc supply, the
(a) primary draws small current
(b) primary leakage reactance is increased
(c) core losses are increased
(d) primary may burn out

6.2 The function of oil in a transformer is
(a) to provide insulation and cooling
(b) to provide protection against lightning
(c) to provide protection against short circuit
(d) to provide lubrication

6.3 In a transformer, the primary and the secondary voltages are
(a) 60° out of phase (b) 90° out of phase
(c) 180° out of phase (d) always in phase

6.4 The core flux of a practical transformer with a resistive load
(a) is strictly constant with load changes
(b) increases linearly with load
(c) increases the square root of the load
(d) decreases with increase of load

6.5 The inductive reactance of a transformer depends on
(a) electromotive force (b) magnetomotive force
(c) magnetic flux (d) leakage flux

6.6 For an ideal transformer the windings should have
(a) maximum resistance on primary side and least resistance on secondary side
(b) least resistance on primary side and maximum resistance on secondary side
(c) equal resistance on primary and secondary side
(d) no ohmic resistance on either side

6.7 If the applied voltage to a primary transformer is increased by keeping the V/f ratio fixed, then the magnetising current and the core loss will respectively:
(a) decrease and remain the same
(b) increase and decrease
(c) both remain the same
(d) remain the same and increase

6.8 If the applied voltage to a certain transformer is increased by 50% and the frequency is reduced to 50% (assuming that the magnetic circuit remains unsaturated), the maximum core flux density will

(a) change to three times the original value
(b) change to 1.5 times the original value
(c) change to 0.5 times the original value
(d) remain the same as the original value

6.9 The low-voltage winding of a 400 V/230 V, 1 phase, 50 Hz transformer is to be connected to a 25 Hz supply. In order to keep the magnetisation current at the same level in both the cases, the voltage at 25 Hz should be

(a) 230 V (b) 460 V (c) 115 V (d) 65 V

6.10 A voltage $v = 400 \sin 314.16t$ is applied to a 1-phase transformer on no-load. If the no-load current of the transformer is $2 \sin (314.16\, t - 85°)$, the magnetisation branch impedance will be approximately equal to

(a) $141 \angle 90°\ \Omega$ (b) $200 \angle -85°\ \Omega$ (c) $200 \angle 85°\ \Omega$ (d) $282 \angle -80°\ \Omega$

6.11 The eddy current losses in a transformer is reduced

(a) if laminations are thick
(b) if the number of turns in primary winding is reduced
(c) if the number of turns in secondary winding is reduced
(d) if laminations are thin

6.12 The efficiency of a transformer will be maximum when

(a) leakage reactances of the two windings are equal
(b) resistances of the two windings are equal
(c) copper loss is equal to constant loss
(d) none of the above

6.13 The efficiency of a transformer is usually in the range of

(a) 50 to 60% (b) 60 to 80% (c) 80 to 90% (d) 90 to 98%

6.14 The full-load copper loss of a transformer is twice its core loss. The efficiency will be maximum at

(a) 25% of full load (b) 50% of full load
(c) 70.7% of full load (d) 141% of full load

6.15 The full-load copper loss and iron loss of a transformer are 6400 W and 5000 W respectively. The copper loss and iron loss at half load will be respectively

(a) 3200 W and 2500 W (b) 3200 W and 5200 W
(c) 1600 W and 1250 W (d) 1600 W and 5000 W

6.16 The load at which maximum efficiency occurs in case of a 100 kVA transformer with iron loss of 1 kW and full load copper loss of 2 kW is

(a) 100 kVA (b) 70.7 kVA (c) 50.5 kVA (d) 25.2 kVA

6.17 A 300 kVA transformer has 95% efficiency at full load, 0.8 pf lagging and 96% efficiency at half load, unity pf. The iron loss W_i and copper loss W_{cu} in kW, under full-load operation are

(a) 4.12, 8.15 (b) 6.59, 9.21 (c) 8.51, 4.12 (d) 12.72, 3.07

6.18 A single-phase transformer has a maximum efficiency of 90% at full load and unity power factor. Efficiency at half load at the same power factor is

(a) 86.7 % (b) 88.26 % (c) 88.9 % (d) 87.9 %

6.19 The efficiency of a 100 kVA transformer is 0.98 at full as well as at half load. For this transformer at full load, the copper loss

(a) is less than core loss (b) is equal to core loss

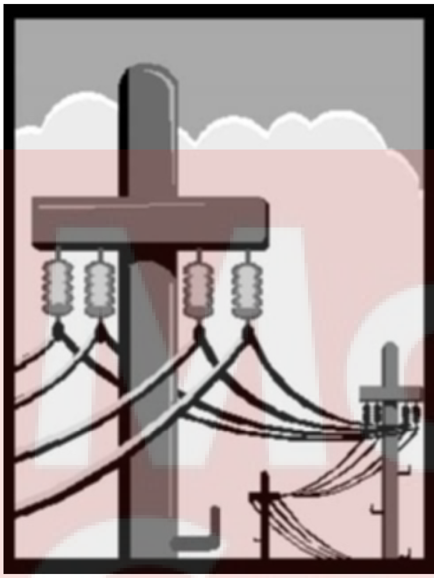
(c) is more than core loss (d) none of the above

6.20 If P_1 and P_2 be the iron and copper losses of a transformer at full-load and the maximum efficiency is at 75% of the full load, ratio of P_1 and P_2 will be

(a) $\frac{9}{16}$ (b) $\frac{10}{16}$ (c) $\frac{3}{4}$ (d) $\frac{3}{16}$

Answers to Objective-Type Questions

6.1 (d)	**6.2** (a)	**6.3** (c)	**6.4** (a)	**6.5** (d)	**6.6** (d)
6.7 (d)	**6.8** (a)	**6.9** (c)	**6.10** (c)	**6.11** (d)	**6.12** (c)
6.13 (d)	**6.14** (c)	**6.15** (d)	**6.16** (b)	**6.17** (c)	**6.18** (d)
6.19 (c)	**6.20** (a)				



Chapter 7

Electronics

Chapter Outline

7.1 SEMICONDUCTORS

The conductivity of a material is proportional to the concentration of free charge carriers. For a good conductor, concentration of free charge carriers is very large, whereas for an insulator, it is very small. For semiconductors, conductivity lies in between the conductivities of conductors and insulators. Silicon and germanium are two examples of semiconductors.

Silicon and germanium have four valence electrons. They combine with the valence electrons of other atoms to form covalent bonds. Due to these bonds, the valence electrons are not free to move. Hence, some amount of energy is needed to break the covalent bonds. The energy required to break a covalent bond is about 1.1 eV for silicon and 0.7 eV for germanium. When the covalent bonds are broken, free electrons and holes are generated. The concentration of electrons is same as the concentration of holes in the pure semiconductor. Such a type of semiconductor is called an *intrinsic semiconductor*.

The conductivity of a semiconductor is increased by adding a small amount of pentavalent (antimony, phosphorus or arsenic) or trivalent (boron, gallium or indium) impurities. This process is known as doping. Such a type of semiconductor is called an *extrinsic semiconductor*.

In the case of a pentavalent impurity, four valence electrons will form covalent bonds with the neighbouring atoms and the fifth valence electron will serve as a free electron. Such a type of semiconductor is referred to as an *n-type semiconductor*.

In the case of a trivalent impurity, three valence electrons will form covalent bonds with the neighbouring atoms, causing a vacancy (hole) in the fourth covalent bond. Such type of semiconductor is referred to as a *p-type semiconductor*.

7.2 SEMICONDUCTOR DIODES

A semiconductor diode is formed by introducing an *n*-type impurity into one side and a *p*-type impurity into the other side of a single crystal of a semiconductor, as shown in Fig. 7.1.

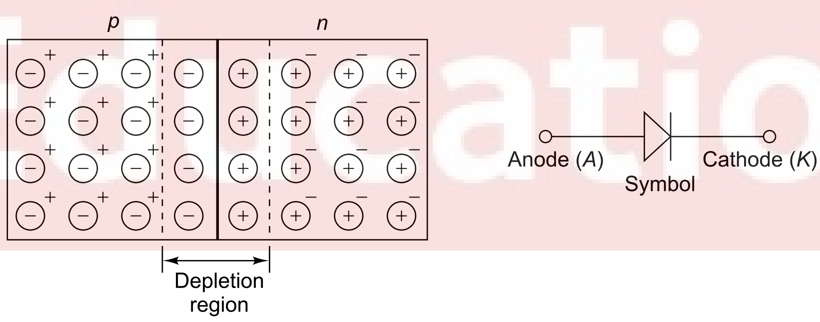


Fig. 7.1 *Semiconductor diode*

We know that *n*-type material has a high concentration of free electrons while *p*-type material has a high concentration of holes. Therefore, at the *p*-*n* junction, there is a tendency for the free electrons to diffuse over to the *p*-side and holes to *n*-side. This process is called *diffusion*. As the free electrons move across the junction from *n*-type to *p*-type, a positive charge is built on the *n*-side of the junction. At the same time, the free holes cross the junction and a negative charge is established on the *p*-side of the junction. These recombinations near the junction do not continue for a long time because positive charge on *n*-side repels holes to cross from *p*-type to *n*-type and negative charge on *p*-side repels free electrons to enter from *n*-type to *p*-type. Thus, the region at the junction contains only negative and positive charge. This region is called *depletion region* or *space charge region* as the mobile charge carriers (free electrons and holes) have been depleted in this region.

Due to positive and negative charge in the depletion region, an electric field is formed. Therefore, an electric potential is established across the junction which is called *barrier potential*. The potential is about 0.2–0.3 V for germanium and 0.6–0.8 V for silicon.

7.2.1 Working and Characteristics

Forward Biasing A *p*-*n* junction is said to be forward biased, when the *p*-side is positive with respect to the *n*-side. This applied voltage establishes an electric field which acts against the field due to the potential barrier. Therefore, the resulting field is weakened. As the potential barrier voltage is very small (0.3 V for Ge and 0.7 V for Si), a small forward voltage is sufficient to completely eliminate the barrier. Once the potential barrier is eliminated, current starts flowing in the circuit.

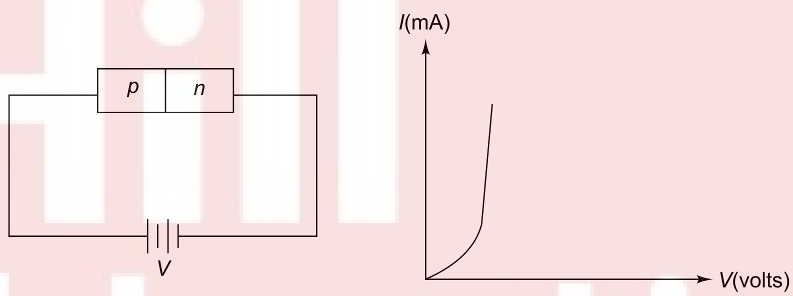


Fig. 7.2 *Forward characteristics*

Reverse Biasing A *p*-*n* junction is said to be reverse biased when the *p*-side is negative with respect to the *n*-side. The applied reverse voltage establishes an electric field which acts in the same direction as the field due to the potential barrier.

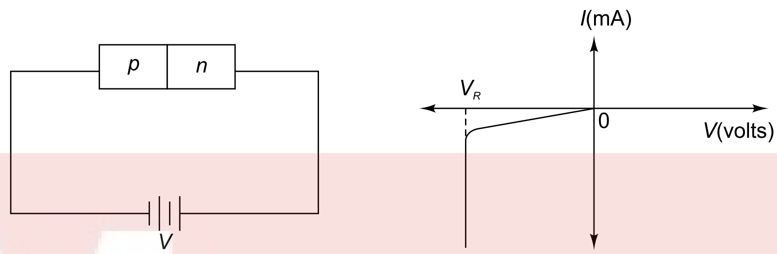


Fig. 7.3 *Reverse characteristics*

Therefore, the resulting field is strengthened. The increased potential barrier prevents the flow of charge carriers across the junction. Hence, no current flows in the circuit. However, in practice, a very small current flows due to minority carriers. If reverse voltage is increased continuously, at one stage the junction breaks down and heavy current starts flowing.

7.2.2 Important Terms in *p-n* Junction Diode

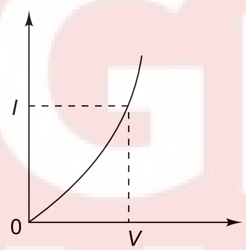


Fig. 7.4 *DC or static resistance*

DC or Static Resistance The resistance offered by the diode to dc supply, when forward biased, is known as dc or static resistance. It is the ratio of dc voltage across the diode to the dc current flowing through it.

$$R_f = \frac{V}{I}$$

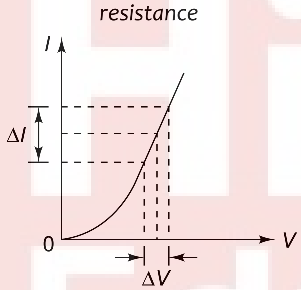


Fig. 7.5 *AC or dynamic resistance*

AC or Dynamic Resistance The resistance offered by the diode to a changing signal, when forward biased, is known as ac or dynamic resistance. If ΔV is the change in voltage and ΔI is the change in current, the dynamic resistance r_f is given by,

$$r_f = \frac{\Delta V}{\Delta I}$$

Transition Capacitance When a *p-n* junction is reversed biased, the majority carriers move away from the junction. Hence, depletion region increases in size. As the reverse voltage increases, the depletion region consisting of positive and negative charge also increases. Thus, capacitance exists across the junction due to the presence of immobile charges. This capacitance is called transition or space charge capacitance (C_T).

Diffusion Capacitance When a *p-n* junction is forward biased, electrons from *n*-side will move into *p*-side and holes from *p*-side will move into *n*-side. As the forward voltage increases, the number of majority carriers crossing the junction also increases. The change in the amount of charge Q due to change in applied voltage is called diffusion capacitance (C_D).

Reverse Saturation Current When a *p-n* junction is reverse biased, there is no current due to majority carriers, yet there is small amount of current due to flow of minority carriers across the junction. This current is called *reverse saturation current*. Since minority carriers are thermally generated, reverse saturation current is extremely temperature dependent.

7.3 RECTIFIERS

A rectifier is a device that converts ac voltage into dc voltage. The element used for rectification is a diode. The diode can be used as rectifier as it conducts in one direction only. When the diode is forward biased, it allows the current to flow through it. When it is reverse biased, no current flows through it. There are two types of rectifiers:

(i) Half-wave rectifier
(ii) Full-wave rectifier

7.4 HALF-WAVE RECTIFIERS

Figure 7.6 shows a half-wave rectifier. It uses only one diode.

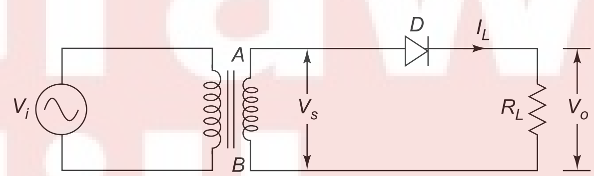


Fig. 7.6 *Half-wave rectifier*

Working During the positive half-cycle of ac supply, terminal *A* is positive w.r.t *B*. The diode *D* is forward biased and current flows through the load.

During the negative half-cycle, the terminal *B* is positive w.r.t *A*. The diode *D* is reverse biased and no current flows through the load.

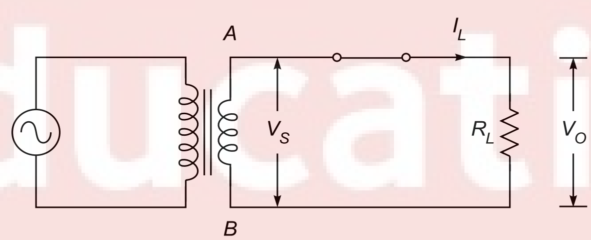


Fig. 7.7 *Half-wave rectifier in positive half cycle*

The output voltage appears across the load during the positive half cycle of ac supply only. Hence, the circuit is called half-wave rectifier. The output of the rectifier is pulsating in nature, i.e., it contains ac as well as dc components.

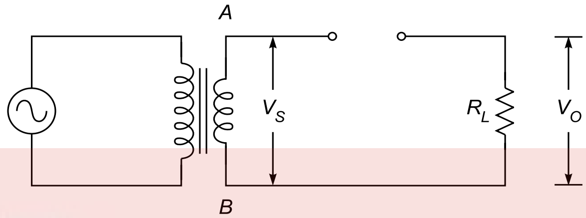


Fig. 7.8 *Half-wave rectifier in negative half cycle*

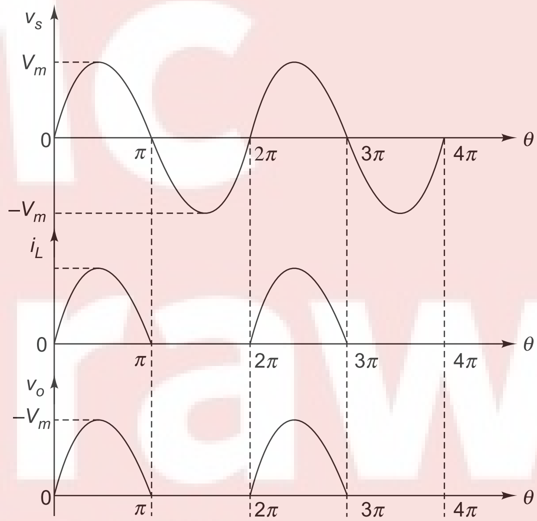


Fig. 7.9 *Half-wave rectifier waveforms*

7.4.1 Analysis of a Half-Wave Rectifier

The load current flowing through the half wave rectifier in one complete cycle is

$$I = I_m \sin\theta \qquad 0 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

DC or Average Value of Load Current (I_{dc}) It is the average value of the load current flowing through the circuit in one cycle.

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i_L\, d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{2\pi}[-\cos\pi + \cos 0]$$

$$= \frac{I_m}{\pi}$$

RMS Value of Load Current (I_{rms}) It is the rms value of the load current flowing through the circuit in one cycle.

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_L^2 \, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta + \int_{\pi}^{2\pi} 0 \, d\theta\right]}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\pi - \frac{\sin 2\pi}{2} + \frac{\sin 0}{2}\right]}$$

$$= \sqrt{\frac{I_m^2}{4}}$$

$$= \frac{I_m}{2}$$

Output Voltage (V_{dc}) It is the voltage available across the load.

$$V_{dc} = I_{dc} R_L$$

$$= \frac{I_m}{\pi} R_L$$

$$= \frac{1}{\pi}\frac{V_m}{(R_f + R_s + R_L)} R_L \qquad \left(\because I_m = \frac{V_m}{R_f + R_s + R_L}\right)$$

where R_f = Diode forward resistance

R_s = Secondary winding resistance

R_L = Load resistance

$$V_{dc} = \frac{V_m}{\pi}\left(\frac{R_f + R_s + R_L - R_f - R_s}{R_f + R_s + R_L}\right)$$

$$= \frac{V_m}{\pi}\left(1 - \frac{R_f + R_s}{R_f + R_s + R_L}\right)$$

$$= \frac{V_m}{\pi} - \frac{V_m}{\pi(R_f + R_s + R_L)}(R_f + R_s)$$

$$= \frac{V_m}{\pi} - I_{dc}(R_f + R_s)$$

At no load, $I_{dc} = 0$

$$V_{dc} = \frac{V_m}{\pi}$$

Rectifier Efficiency (η) It is defined as the ratio of dc output power to ac input power.

$$\eta = \frac{\text{dc output power}}{\text{ac input power}}$$

$$= \frac{P_{dc}}{P_{ac}}$$

$$P_{dc} = I^2_{dc} R_L$$

$$= \frac{I_m^2}{\pi^2} R_L$$

$$P_{ac} = I^2_{rms}(R_f + R_s + R_L)$$

$$= \left(\frac{I_m}{2}\right)^2 (R_f + R_s + R_L)$$

$$= \frac{I_m^2}{4}(R_f + R_s + R_L)$$

$$\eta = \frac{\frac{I_m^2}{\pi^2} R_L}{\frac{I_m^2}{4}(R_f + R_s + R_L)}$$

$$= \frac{4}{\pi^2} \frac{R_L}{R_f + R_s + R_L}$$

$$= \frac{4}{\pi^2} \frac{1}{\frac{R_f + R_s}{R_L} + 1}$$

$$= \frac{4}{\pi^2}$$

$$= 0.406 \qquad (\because R_f + R_s << R_L)$$

$$\%\ \eta = 40.6\%$$

Ripple Factor (r) The purpose of a rectifier is to convert ac into dc. The output of a rectifier is pulsating in nature, i.e., it contains dc as well as ac components. The ac component is called ripple which is removed with the help of a filter circuit. The ratio of rms value of ac component present in the dc waveform to the dc component of the waveform is known as ripple factor.

$$\text{Ripple factor } r = \frac{\text{rms value of ac component of wave}}{\text{dc component of wave}}$$

$$= \frac{I_{\text{ac, rms}}}{I_{\text{dc}}}$$

The output current in a half-wave rectifier is given by,

$$i_L = I_{\text{dc}} + i_{\text{ac}}$$

rms value of the output current is given by,

$$I_{\text{rms}} = \sqrt{I_{\text{dc}}^2 + I_{\text{ac, rms}}^2}$$

$$I_{\text{ac, rms}} = \sqrt{I_{\text{rms}}^2 - I_{\text{dc}}^2}$$

$$\frac{I_{\text{ac, rms}}}{I_{\text{dc}}} = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1}$$

$$r = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1}$$

For a half-wave rectifier,

$$r = \sqrt{\frac{\left(\frac{I_m}{2}\right)^2}{\left(\frac{I_m}{\pi}\right)^2} - 1}$$

$$= 1.21$$

Peak Inverse Voltage (PIV) It is defined as the maximum reverse voltage that can be applied across the diode without damaging it. For a half-wave rectifier.

$$\text{PIV} = V_m$$

Transformer Utilisation Factor (TUF) It indicates the amount of utilization of transformer in the circuit.

$$\text{TUF} = \frac{\text{dc power delivered to the load}}{\text{ac power rating of transformer secondary}}$$

$$= \frac{P_{\text{dc}}}{P_{\text{ac}}(\text{rated})}$$

The rated voltage of the secondary is $\frac{V_m}{\sqrt{2}}$, but actual rms current flowing through the winding is only $\frac{I_m}{2}$, not $\frac{I_m}{\sqrt{2}}$.

$$\text{TUF} = \frac{\left(\frac{I_m}{\pi}\right)^2 R_L}{\frac{V_m}{\sqrt{2}}\frac{I_m}{2}}$$

$$= \frac{\frac{I_m^2}{\pi^2} R_L}{\frac{I_m(R_f + R_s + R_L)}{\sqrt{2}}\frac{I_m}{2}} \qquad [\because V_m = I_m(R_f + R_s + R_L)]$$

$$= \frac{2\sqrt{2}}{\pi^2}\frac{R_L}{R_f + R_s + R_L}$$

$$= \frac{2\sqrt{2}}{\pi^2}\frac{1}{\frac{R_f + R_s}{R_L} + 1}$$

$$= \frac{2\sqrt{2}}{\pi^2}$$

$$= 0.287 \qquad (\because R_f + R_s << R_L)$$

Voltage Regulation The variation of dc output voltage as a function of the dc load current is called voltage regulation.

$$\%\text{ regulation} = \frac{V_{\text{dc}_{\text{no load}}} - V_{\text{dc}_{\text{full load}}}}{V_{\text{dc}_{\text{full load}}}} \times 100$$

$$= \frac{\frac{V_m}{\pi} - \left[\frac{V_m}{\pi} - I_{\text{dc}}(R_f + R_s)\right]}{I_{\text{dc}} R_L} \times 100$$

$$= \frac{R_f + R_s}{R_L} \times 100$$

7.5 CENTRE-TAPPED FULL-WAVE RECTIFIERS

Figure 7.10 shows a centre-tapped full-wave rectifier. It uses two diodes.

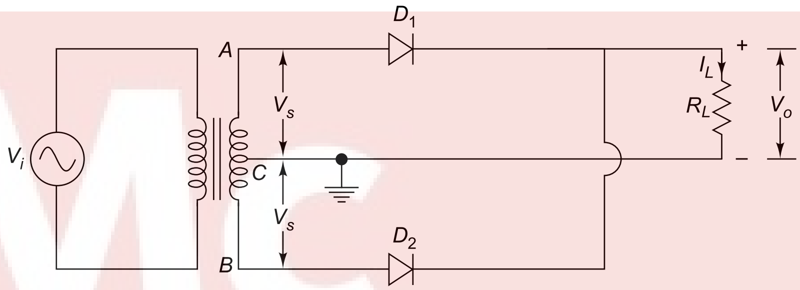


Fig. 7.10 *Centre-tapped full-wave rectifier*

Working During the positive half-cycle, the terminal *A* is positive w.r.t *C* and the terminal *B* is negative w.r.t *C*. The diode D_1 is forward biased and D_2 is reverse biased. The current flows through diode D_1 and load.

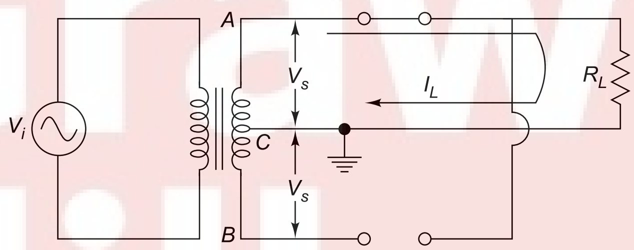


Fig. 7.11 *Centre-tapped full-wave rectifier in positive half cycle*

During the negative half-cycle, the terminal *B* is positive w.r.t *C*, and the terminal *A* is negative w.r.t *C*. The diode D_2 is forward biased and the diode D_1 is reverse biased. The current flows through the diode D_2 and load.

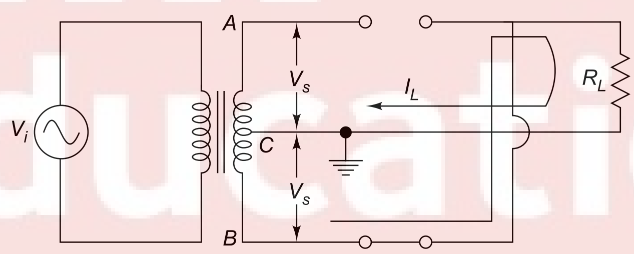


Fig. 7.12 *Centre-tapped full-wave rectifier in negative half cycle*

The current flows through R_L in both half cycles. Thus, ac waveform is converted into dc and the circuit is called a full-wave rectifier.

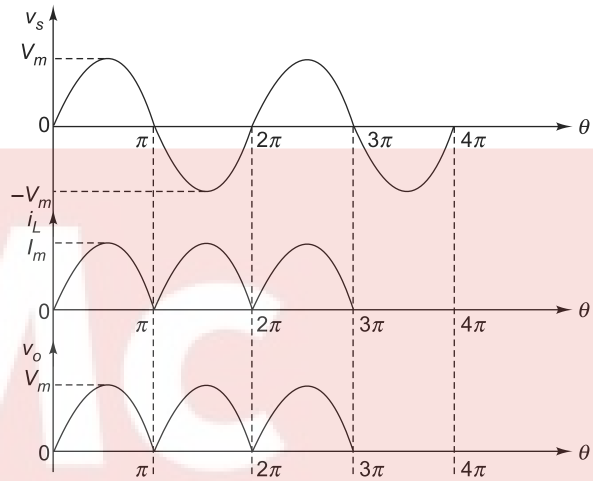


Fig. 7.13 *Centre-tapped full-wave rectifier waveforms*

7.5.1 Analysis of a Full-Wave Rectifier

The load current flowing through the full wave rectifier in one complete cycle is

$$I = I_m \sin \theta \qquad 0 < \theta < \pi$$

DC Value of Load Current (I_{dc}) It is the average value of load current flowing through the circuit in one cycle.

$$I_{dc} = \frac{1}{\pi}\int_0^{\pi} i_L \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\pi + \cos 0]$$

$$= \frac{2I_m}{\pi}$$

RMS Value of Load Current (I_{rms}) It is the rms value of the load current flowing through the circuit in one cycle.

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_L^2 \, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \frac{I_m}{\sqrt{2}}$$

Output Voltage (V_{dc}) It is the voltage available across the load.

$$V_{\text{dc}} = I_{\text{dc}} R_L$$

$$= \frac{2I_m}{\pi} R_L$$

$$= \frac{2}{\pi}\frac{V_m}{(R_f + R_s + R_L)} R_L$$

where R_f = Diode forward resistance
R_s = Secondary winding resistance
R_L = Load resistance

$$V_{\text{dc}} = \frac{2V_m}{\pi}\left(1 - \frac{R_f + R_s}{R_f + R_s + R_L}\right)$$

$$= \frac{2V_m}{\pi} - I_{\text{dc}}(R_f + R_s)$$

At no load $I_{\text{dc}} = 0$

$$V_{\text{dc}} = \frac{2V_m}{\pi}$$

Rectifier Efficiency (η) It is defined as the ratio of dc output power to ac input power.

$$\eta = \frac{\text{dc output power}}{\text{ac input power}}$$

$$= \frac{P_{\text{dc}}}{P_{\text{ac}}}$$

$$P_{\text{dc}} = I_{\text{dc}}^2 R_L$$

$$= \left(\frac{2I_m}{\pi}\right)^2 R_L$$

$$= \frac{4I_m^2}{\pi^2} R_L$$

$$P_{\text{ac}} = I_{\text{rms}}^2 (R_f + R_s + R_L)$$

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 (R_f + R_s + R_L)$$

$$= \frac{I_m^2}{2} (R_f + R_s + R_L)$$

$$\eta = \frac{\frac{4I_m^2}{\pi^2} R_L}{\frac{I_m^2}{2} (R_f + R_s + R_L)}$$

$$= \frac{8}{\pi^2} \frac{R_L}{R_f + R_s + R_L}$$

$$= \frac{8}{\pi^2}$$

$$= 0.812 \qquad (\because R_f + R_s << R_L)$$

$$\% \eta = 81.2\%$$

Ripple Factor (r) The ratio of rms value of ac component present in the dc waveform to the dc component of the waveform is known as ripple factor.

$$r = \sqrt{\left(\frac{I_{\text{rms}}}{I_{\text{dc}}}\right)^2 - 1}$$

$$= \sqrt{\frac{\left(\frac{I_m}{\sqrt{2}}\right)^2}{\left(\frac{2I_m}{\pi}\right)^2} - 1}$$

$$= 0.48$$

Peak Inverse Voltage (PIV) It is defined as the maximum reverse voltage that can be applied across the diode without damaging it. For a centre-tapped full-wave rectifier,

$$\text{PIV} = 2V_m$$

Transformer Utilisation Factor (TUF) It indicates the amount of utilization of transformer in the circuit.

$$\begin{aligned}
\text{TUF} &= \frac{\text{dc power delivered to the load}}{\text{ac power rating of transformer secondary}} \\
&= \frac{I_{\text{dc}}^2 R_L}{V_{\text{rms}} I_{\text{rms}}} \\
&= \frac{\left(\frac{2I_m}{\pi}\right)^2 R_L}{\frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}} \\
&= \frac{\frac{4I_m^2}{\pi^2} R_L}{\frac{I_m (R_f + R_s + R_L)}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}} \\
&= \frac{8}{\pi^2} \frac{R_L}{R_f + R_s + R_L} \\
&= \frac{8}{\pi^2} \\
&= 0.812 \qquad (\because R_f + R_s << R_L)
\end{aligned}$$

The primary of the transformer is feeding two half-wave rectifiers separately.

$$\begin{aligned}
\text{TUF} &= 2 \times \text{TUF of half-wave rectifier} \\
&= 2 \times 0.287 = 0.574
\end{aligned}$$

$$\begin{aligned}
\text{Average TUF for full-wave rectifier} &= \frac{0.574 + 0.812}{2} \\
&= 0.693
\end{aligned}$$

Voltage Regulation The variation of dc output voltage as a function of dc load current is called voltage regulation.

$$\%\ \text{regulation} = \frac{V_{\text{dc}_{\text{no load}}} - V_{\text{dc}_{\text{full load}}}}{V_{\text{dc}_{\text{full load}}}} \times 100$$

$$= \frac{\frac{2V_m}{\pi} - \left[\frac{2V_m}{\pi} - I_{\text{dc}}(R_f + R_s)\right]}{I_{dc} R_L} \times 100$$

$$= \frac{R_f + R_s}{R_L} \times 100$$

7.6 BRIDGE RECTIFIERS

The bridge rectifier is essentially a full wave rectifier. Figure 7.14 shows a bridge type full-wave rectifier. It uses four diodes.

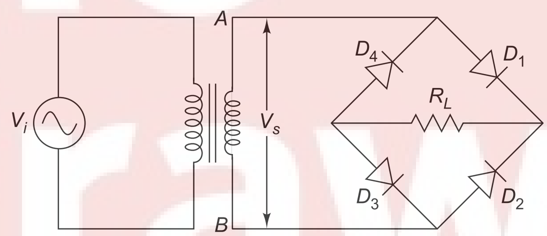


Fig. 7.14 *Bridge rectifier*

Working During the positive half-cycle, terminal A is positive w.r.t B. Diodes D_1 and D_3 are forward biased and diodes D_2 and D_4 are reverse biased. Current flows through diode D_1, load R_L and diode D_3.

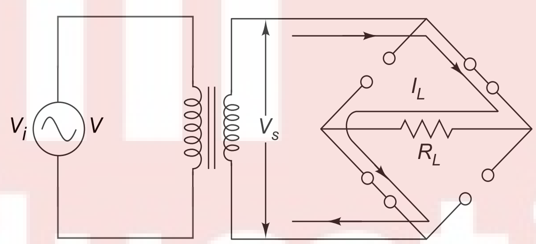


Fig. 7.15 *Bridge rectifier in positive half cycle*

During the negative half cycle, terminal A is negative w.r.t B. Diodes D_2 and D_4 are forward biased and diodes D_1 and D_3 are reverse biased. Current flows through D_2, load R_{L} and D_4.

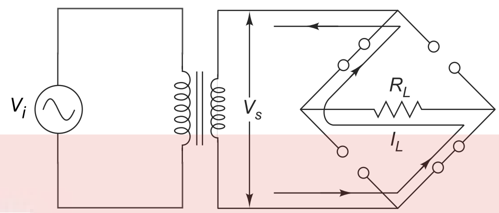


Fig. 7.16 *Bridge rectifier in negative half cycle*

Thus, current flows in both half cycles of ac supply. Output voltage is available in both half cycles and hence, the circuit is called a full-wave rectifier.

Various Parameters of a Bridge Rectifier

$$I_{\text{dc}} = \frac{2I_m}{\pi}$$

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$$

$$V_{\text{dc}} = I_{\text{dc}} R_L = \frac{2V_m}{\pi} - I_{\text{dc}}(2R_f + R_s)$$

$$V_{\text{dc (NL)}} = \frac{2V_m}{\pi}$$

$$\%\ \eta = 81.2\ \%$$

$$r = 0.48$$

$$\text{PIV} = V_m$$

$$\text{TUF} = 0.812$$

$$\%\ \text{regulation} = \frac{2R_f + R_s}{R_L} \times 100$$

7.7 FILTERS

The output of a rectifier is pulsating in nature, i.e., it contains ac as well as dc components. At the output, we want only dc components. Therefore, we have to remove (filter) ac components. The circuit used for this purpose is known as filter.

Generally, inductors and capacitors are used as filter components. The inductor acts as a short circuit for dc but has a large impedance for ac. Similarly, the capacitor acts as an open circuit for dc and almost short circuit for ac. Hence, inductance cannot be placed in a shunt arm across the load, otherwise dc will be shorted. Therefore, it is always connected in series with the load. Similarly, capacitance is open for dc, i.e., it blocks dc. Hence, it cannot be connected in series with the load. It is always connected in shunt arm, parallel to the load.

There are four types of filters:

1. Inductor filter
2. Capacitor filter
3. Choke input or LC filter
4. C-L-C or π filter

7.7.1 Inductor Filter

Figure 7.17 shows an inductor filter. An inductor filter consists of a choke in series with the load. The rectified output contains ac as well as dc components. The inductor acts as short circuit for dc components. All the dc components can reach the output. But inductor opposes ac components and part of ac components is blocked by it. Only small amount of ac components reach the output. Thus, ac components are reduced or filtered by the inductor.

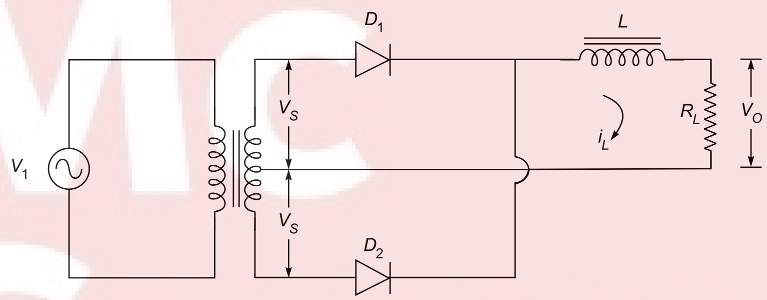


Fig. 7.17 *Inductor filter*

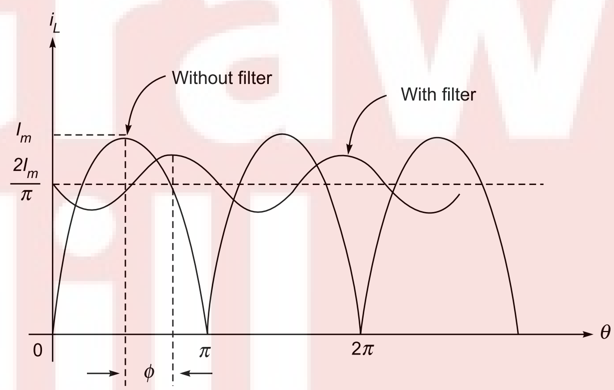


Fig. 7.18 *Inductor-filter waveform*

7.7.2 Capacitor Filter

In a capacitor filter, the capacitor is connected in parallel with the load R_L. During the positive half-cycle, the diode D_1 is forward biased and the diode D_2 is reverse biased. Therefore, current, flows through D_1, charging the capacitor C to the maximum value V_m. Since capacitor and load R_L are in parallel, the voltage across the capacitor will appear as output voltage. Output voltage increases up to point A. After point A, input voltage starts decreasing but the capacitor is already charged to the value V_m. Hence, the diode D_1 is reverse biased in the positive half-cycle itself and capacitor discharges slowly through R_L.

At the same time, positive half-cycle is over and the negative half-cycle begins. Due to this, D_1 continues to be reverse biased. Now, the diode D_2 is also reverse biased due to voltage across the capacitor. Therefore, the capacitor continues to discharge and reaches the point B. At this time, voltage at the point B is more than the capacitor voltage and so the diode D_2 is forward biased and again, the capacitor charges to the value V_m. The process repeats. Thus, amount of fluctuation is less at the output, i.e., ac component is reduced after connecting the capacitor.

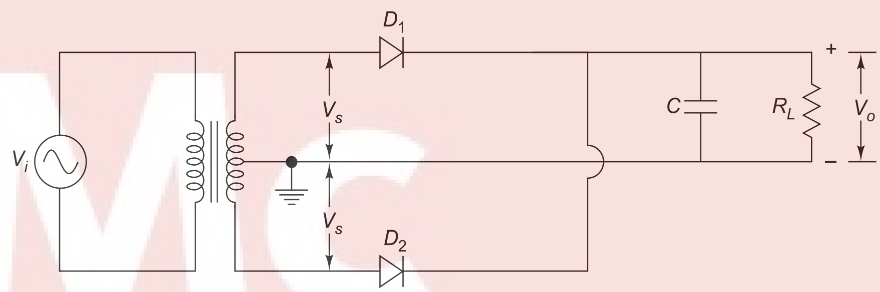


Fig. 7.19 *Capacitor filter*

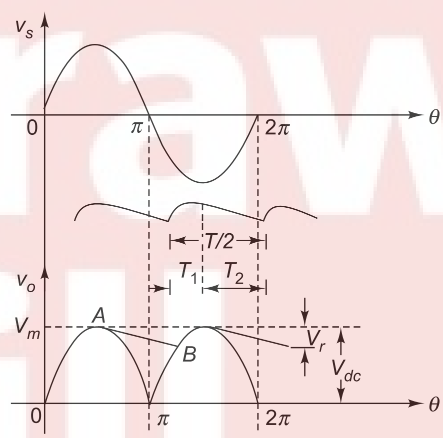


Fig. 7.20 *Capacitor-filter waveforms*

7.8 BIPOLAR JUNCTION TRANSISTORS (BJT)

A bipolar junction transistor is a three-terminal semiconductor device containing two *p-n* junctions. When a *p*-type layer is placed between two *n*-type layers, an *npn* transistor is formed. Similarly, when an *n*-type layer is placed between two *p*-type layers, a *pnp* transistor is formed.

In each type of transistor, the middle region is called the *base* of the transistor and other two regions are called *emitter* and *collector*. The physical size of the collector is greater than both the emitter and base. The emitter is heavily doped, while the base is lightly doped. The doping of the collector is in between that of the emitter and base.

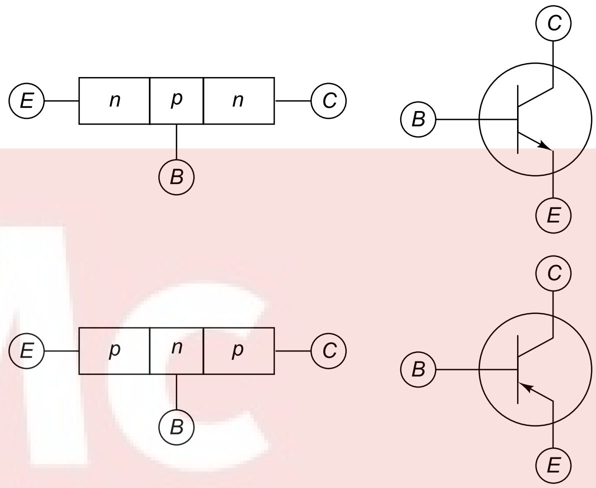


Fig. 7.21 *Bipolar junction transistors*

In the symbol shown, the arrowhead in the emitter shows the direction of conventional current which is opposite to the flow of electrons. In an *npn* transistor, conventional current flows out of the emitter while in a *pnp* transistor, the conventional current flows into the emitter.

7.8.1 Working

For proper working of a BJT, the emitter-base junction is forward biased and the collector-base junction is reverse biased.

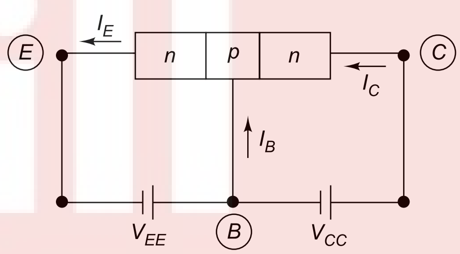


Fig. 7.22 *Working of BJT*

When the emitter-base junction is forward biased, electrons from an *n*-type emitter flow towards the base and holes from the *p*-type base flow towards the emitter. Since the base region is very thin and lightly doped, very few of the electrons recombine with holes and most of the electrons diffuse to the reverse-biased collector base junction.

The base current is equivalent to the number of recombinations in the base. Hence, applying KCL to the transistor,

$$I_E = I_B + I_C$$

The collector current I_C is composed of two parts: (i) the fraction of emitter current which reaches the collector, and (ii) leakage current which flows due to minority carriers.

$$I_C = \alpha I_E + I_{\text{CBO}}$$

Usually, I_{CBO} is very small.

$$I_C \approx \alpha I_E$$

$$\alpha = \frac{I_C}{I_E}$$

where α is the dc current gain of CB configuration. The value of α lies between 0.9 to 0.995.

7.8.2 Regions of Operation of BJT

Any transistor can be operated in three regions:

Cut-off In this region, both emitter-base and collector-base junctions are reverse biased.

Active In this region, the emitter-base junction is forward biased and the collector-base junction is reverse biased.

Saturation In this region, both emitter-base and collector-base junctions are forward biased.

7.8.3 BJT Configurations

Depending upon the common terminal, there are three possible BJT configurations:

(i) Common Base (CB) configuration
(ii) Common Emitter (CE) configuration
(iii) Common Collector (CC) configuration

Common Base Configuration In this configuration, the base is common to both input and output.

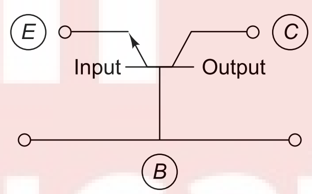


Fig. 7.23 *Common base configuration*

Current amplification factor (α) It is defined as the ratio of change in collector current to the change in emitter current at constant collector base voltage V_{CB}.

$$\alpha_{\text{ac}} = \left.\frac{\Delta I_C}{\Delta I_E}\right|_{V_{\text{CB}} = \text{constant}}$$

If only dc values are considered,

$$\alpha_{\text{dc}} = \frac{I_C}{I_E}$$

Common Emitter Configuration In this configuration, the emitter is common to both input and output.

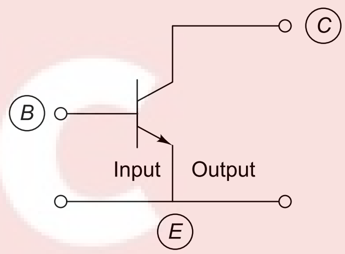


Fig. 7.24 *Common emitter configuration*

Current amplification factor (β) It is defined as the change in collector current to the change in base current at constant collector emitter voltage V_{CE}.

$$\beta_{\text{ac}} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{\text{CE}} = \text{constant}}$$

If only dc values are considered,

$$\beta_{\text{dc}} = \frac{I_C}{I_B}$$

Common Collector Configuration In this configuration, the collector is common to both input and output.

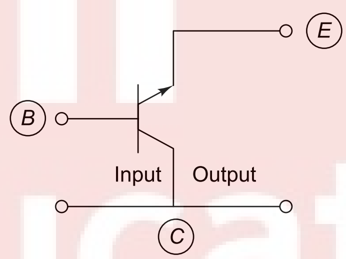


Fig. 7.25 *Common collector configuration*

Current amplification factor (γ) It is defined as the ratio of change in emitter current to the change in base current at constant collector emitter voltage V_{CE}.

$$\gamma_{\text{ac}} = \left.\frac{\Delta I_E}{\Delta I_B}\right|_{V_{\text{CE}} = \text{constant}}$$

If only dc values are considered,

$$\gamma_{dc} = \frac{I_E}{I_B}$$

7.8.4 Relation Between α and β

We know that,

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{I_C/I_E}{1 - I_C/I_E}$$

$$\beta = \frac{\alpha}{1-\alpha} \qquad \left(\because \alpha = \frac{I_C}{I_E}\right)$$

7.8.5 Expression for Collector Current (I_C)

We know that,

$$I_E = I_B + I_C \tag{7.1}$$

For CB configuration,

$$I_C = \alpha I_E + I_{CBO} \tag{7.2}$$

Substituting I_E in Eq. (7.2),

$$I_C = \alpha(I_B + I_C) + I_{CBO}$$

$$I_C = \alpha I_B + \alpha I_C + I_{CBO}$$

$$I_C(1-\alpha) = \alpha I_B + I_{CBO}$$

$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO}$$

$$= \beta I_B + (\beta + 1) I_{CBO}$$

7.9 COMMON-EMITTER CONFIGURATION CHARACTERISTICS

Figure 7.26 shows the experimental set-up to draw input and output characteristics in a CE configuration.

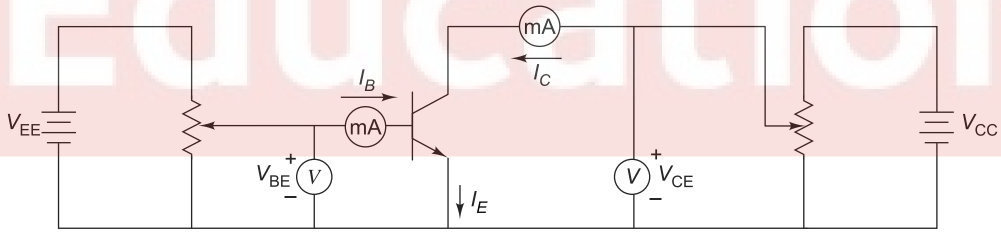


Fig. 7.26 *CE configuration*

Input Characteristics It is the graph of input current I_B v/s input voltage V_{BE} at a constant output voltage V_{CE}. It resembles the characteristics of a forward-biased diode. The input current I_B increases as the input voltage V_{BE} increases for a fixed value of V_{CE}.

As the reverse-bias voltage V_{CE} increases, depletion region in the collector-base increases. Hence, the width of the base available for conduction decreases. Hence, I_B decreases due to Early effect and the graph shifts towards the X-axis.

Dynamic input resistance

$$r_i = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE} = \text{constant}}$$

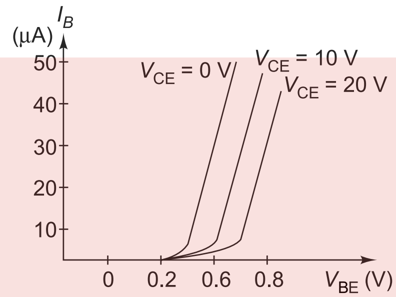


Fig. 7.27 *Input characteristics*

Output Characteristics It is the graph of output current I_C v/s output voltage V_{CE} for given values of I_B. The output characteristics has three different regions.

Cut-off region In this region, both the junctions are reverse biased. When the emitter-base junction is reverse biased, the current due to majority carrier, i.e., I_B is zero. Since collector-base junction is reverse biased, the current due to minority carriers flows from collector to emitter which is represented as I_{CEO}.

Active region In this region, the emitter-base junction is forward biased and the collector-base junction is reverse biased. As I_B is maintained constant, the current I_C increases as the reverse bias voltage V_{CE} increases.

Saturation region In this region, both the junctions are forward biased. When V_{CE} is reduced to a small value such as 0.2 V, the collector-base junction is actually forward biased ($\because V_{CB} = V_{CE} - V_{BE} = 0.2 - 0.7 = -0.5$ V). In this region, there is a large change in the collector current I_C with a small change in V_{CE}.

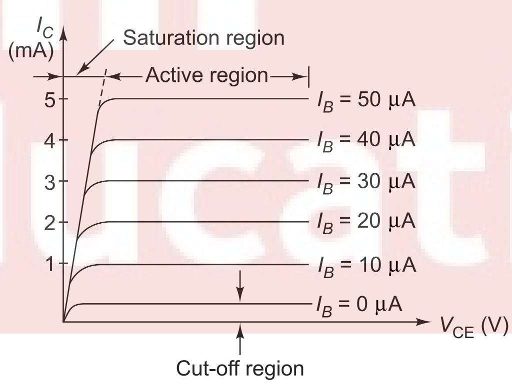


Fig. 7.28 *Output characteristics*

Output resistance $$r_0 = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_B = \text{constant}}$$

Current gain $$\beta_{dc} = \left.\frac{I_C}{I_B}\right|_{V_{CE} = \text{constant}}$$

$$\beta_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE} = \text{constant}}$$

Review Questions

7.1 Explain the working of a *p-n* junction diode with characteristics.
7.2 Explain the following terms with reference to *p-n* junction diode:
(a) Static resistance
(b) Dynamic resistance
(c) Transition capacitance
(d) Diffusion capacitance
(e) Reverse saturation current.
7.3 Draw the circuit diagram of a half-wave rectifier with diode. Draw the waveforms for input voltage, output current and output voltage. Explain its working.
7.4 Derive the following for a half-wave rectifier:
(a) Average value of load current
(b) RMS value of load current
(c) Output voltage
(d) Rectifier efficiency
(e) Ripple factor
7.5 Explain the working of a full-wave rectifier with diagram. Draw the waveforms for input voltage and output voltage.
7.6 Explain operation of diode bridge rectifier with neat circuit diagram and waveform.
7.7 Derive the following for a full-wave rectifier:
(a) Average value of load current
(b) RMS value of load current
(c) Output voltage
(d) Rectifier efficiency
(e) Ripple factor
7.8 Compare half-wave rectifier, centre-tapped full-wave rectifier and a bridge rectifier.
7.9 Explain the working of a full-wave rectifier with capacitor filter.

7.10 Explain the construction and working of a bipolar junction transistor.

7.11 Define and derive the relationship between α and β of a transistor.

7.12 Explain the input and output characteristics of an *npn* transistor in a common emitter configuration. Clearly mark various regions on the characteristics. Show how different parameters can be determined from the above characteristics.

Objective-Type Questions

Choose the correct alternative in the following questions:

7.1 In a junction diode
(a) the depletion capacitance increases with increase in the reverse bias
(b) the depletion capacitance decreases with increase in the reverse bias
(c) the diffusion capacitance increases with increase in the forward bias
(d) the diffusion capacitance is much higher than the depletion capacitance when it is forward biased

7.2 A BJT is said to be operating in the saturation region when
(a) both the junctions are reverse biased
(b) base-emitter junction is reverse biased and base-collection junction is forward biased
(c) base-emitter junction is forward biased and base-collector junction is reverse biased
(d) both the junctions are forward biased

7.3 If the BJT is in saturation then
(a) I_C is always equal to $\beta_{dc} I_B$ (b) I_C is always equal to $-\beta_{dc} I_B$
(c) I_C is greater than or equal to $\beta_{dc} I_B$ (d) I_C is less than or equal to $\beta_{dc} I_B$

7.4 An HWR uses a diode with a forward resistance R_f.
The voltage is $V_m \sin \omega t$ and the load resistance is R_L.
The dc current is given by
(a) $\dfrac{V_m}{\sqrt{2}\, R_L}$ (b) $\dfrac{V_m}{\pi(R_f + R_L)}$ (c) $\dfrac{2V_m}{\pi}$ (d) $\dfrac{V_m}{R_L}$

7.5 Choose the correct match for input resistance of various transistor configuration shown below.

Configuration	*Input resistance*
(P) Common base	(1) Low
(Q) Common collector	(2) Moderate
(R) Common emitter	(3) High

(a) $P-1, Q-2, R-3,$ (b) $P-1, Q-3, R-2$
(c) $P-2, Q-3, R-1$ (d) $P-3, Q-1, R-2$

7.6 Which of the following statements are correct for basic transistor configurations?
(a) CB amplifier has low input resistance and a low current gain
(b) CC amplifier has low output resistance and a low current gain
(c) CE amplifier has very poor voltage gain but very high input resistance
(d) the current gain of CB amplifier is higher than the current gain of CC amplifier

7.7 The early effect in a bipolar function transistor is caused by
(a) fast turn ON (b) fast turn OFF
(c) collector base reverse bias (d) large emitter base forward bias

7.8 The centre-tap full-wave single-phase rectifier circuit uses 2 diodes as shown in Fig. 7.29. The RMS voltage across each diode is

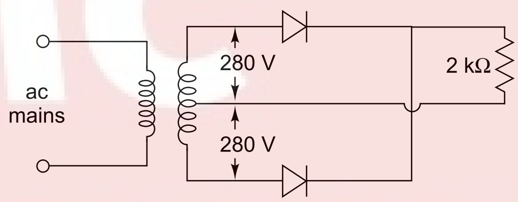


Fig. 7.29

(a) 790.7 V (b) 395.3 V (c) 280 V (d) 201.3 V

7.9 If $\alpha = 0.98$, $I_{CO} = 6\ \mu$A and $I_B = 100\ \mu$A for a transistor then the value of I_C will be
(a) 2.3 mA (b) 3.1 mA (c) 4.6 mA (d) 5.2 mA

7.10 For the BJT, which of the following relation is true?

(a) $\alpha = \dfrac{\beta}{1-\beta}$ (b) $\alpha = \beta(1+\beta)$ (c) $\beta = \dfrac{\alpha}{1-\alpha}$ (d) $\beta = \dfrac{\alpha}{1+\alpha}$

7.11 As compared to a full-wave rectifier using two diodes, the four-diode bridge rectifier has the dominant advantage of
(a) higher current carrying (b) lower PIV
(c) lower ripple factor (d) higher efficiency

7.12 The depletion region, or space charge region, or transition region, in a semiconductor *p-n* junction diode has
(a) electrons and holes (b) positive ions and electrons
(c) positive and negative ions (d) negative ions and holes

7.13 In the CE configuration of a transistor amplifier, there is
(a) no phase change between input and output voltages
(b) 180° phase change between input and output currents
(c) no phase change between input and output current and no phase change between input and output voltages
(d) no phase change between input and output currents and 180° phase change between input and output voltages

7.14 The peak input voltage to a full-wave bridge rectifier is 1000 V at 50 Hz. The dc output voltage and ripple frequency respectively are
(a) 318 V and 50 Hz (b) 318 V and 100 Hz
(c) 636 V and 50 Hz (d) 636 V and 100 Hz

7.15 Early effect in BJT refers to
(a) avalanche breakdown (b) thermal runaway
(c) base narrowing (d) Zener breakdown

7.16 Consider the following statements:
An applied bias voltage in a *p-n* junction diode (*n* region positive w. r. t *p* region) results in
(i) increase in potential barrier
(ii) reduction in space charge layer width
(iii) increase in space charge layer width
(iv) increase in magnitude of electric field
Which of the statements given above are correct?
(a) (i) and (ii) (b) (i) and (iii) (c) (i) and (iv) (d) (i), (iii) and (iv)

Answers to Objective-Type Questions

7.1 (b)	**7.2** (d)	**7.3** (d)	**7.4** (b)
7.5 (b)	**7.6** (a)	**7.7** (c)	**7.8** (b)
7.9 (d)	**7.10** (c)	**7.11** (b)	**7.12** (c)
7.13 (d)	**7.14** (b)	**7.15** (c)	**7.16** (b)

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Solved Question Paper
(Mumbai University–December, 2012)

1. (a) Find the current flowing through the 5 Ω resistance using source transformation. **(03)**

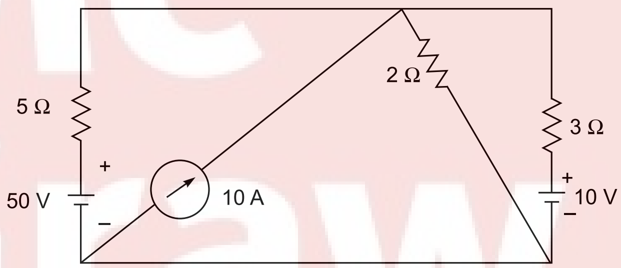


Fig. 1

Ans. Refer Example 4 on page 2.54.

(b) State and explain maximum power transfer theorem. **(03)**

Ans. Refer Section 2.9 on page 2.151.

(c) An alternating voltage is given by $v = 141.4 \sin 314t$. Find: **(03)**

(i) Frequency

(ii) rms value

(iii) Average value

(iv) Instantaneous value of voltage, when t is 3 ms.

Ans. $v = 141.4 \sin 314t$

(i) Comparing with the equation, $v = V_m \sin 2\pi f t$,

$$2\pi f = 314$$

$$f = \frac{314}{2\pi} = 49.97 \text{ Hz}$$

(ii) $V_m = 141.4$ V

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} = 99.98 \text{ V}$$

(iii) $V_{\text{avg}} = \dfrac{2V_m}{\pi} = \dfrac{2 \times 141.4}{\pi} = 90.02$ V

(iv) When $t = 3$ ms,

$$v = 141.4 \sin 314 \times 3 \times 10^{-3} = 114.36 \text{ V}$$

(d) Define the equation for resonance frequency (f_r) in a parallel resonance circuit. **(02)**

Ans. Refer Section 4.9 on page 4.93.

(e) Write voltage, current and power relation in a balanced delta-connected load. **(03)**

Ans. Refer Section 5.8 on page 5.7.

(f) Derive emf equation for single-phase transformer. **(04)**

Ans. Refer Section 6.4 on page 6.4.

(g) Draw complete *V–I* characteristics of a diode. **(02)**

Ans. Refer Section 7.2.1 on page 7.3.

2. (a) Determine the value of resistance *R* as shown in Fig. 2 using KVL and KCL. **(06)**

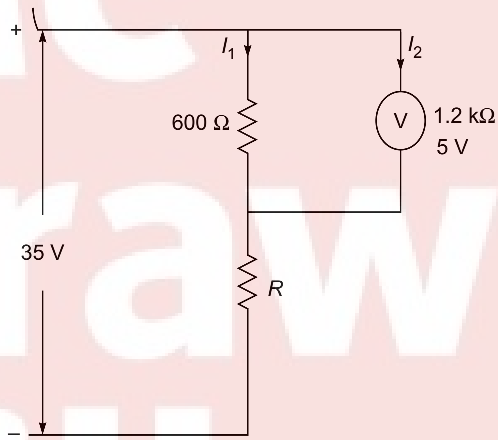


Fig. 2

Ans.

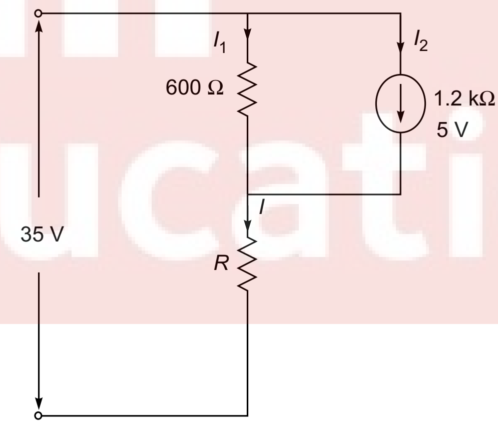


Fig. 3

$$I_2 = \frac{5}{1.2 \times 10^{-3}} = 4.17 \text{ mA}$$

$$I_1 = \frac{5}{600} = 8.33 \text{ mA}$$

$$I = I_1 + I_2 = 4.17 + 8.33 = 12.5 \text{ mA}$$

Applying KVL to the circuit,

$$35 - 600 I_1 - RI = 0$$

$$35 - 600(8.33 \times 10^{-3}) - R(12.5 \times 10^{-3}) = 0$$

$$R = 2.4 \text{ k}\Omega$$

(b) A 100 Ω resistor is connected in series with a choke coil. When a 400 V, 50 Hz supply is applied to this combination, the voltage across the resistance and the choke coil are 200 V and 300 V, respectively. Find the power consumed by the choke coil. Also calculate the power factor of the choke coil and power factor of the circuit. **(08)**

Ans. Refer Example 13 on page 4.19.

(c) Draw the phasor diagram of a single-phase transformer on resistive load (unity power factor) and inductive load (lagging power factor). **(06)**

Ans. Refer Section 6.10 on page 6.16.

3. (a) Three similar coils, connected in star, take a total power of 1.5 kW at p.f. of 0.2 lagging from a three-phase, 440 V, 50 Hz supply. Calculate the resistance and inductance of each coil. **(08)**

Ans.

$$P = 1.5 \text{ kW}$$

$$\text{pf} = 0.2 \text{ (lagging)}$$

$$V_L = 440 \text{ V}$$

$$f = 50 \text{ Hz}$$

For a star-connected load.

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$1.5 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.2$$

$$I_L = 9.84 \text{ A}$$

$$I_{ph} = I_L = 9.84 \text{ A}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{254.03}{9.84} = 25.82 \ \Omega$$

$$\phi = \cos^{-1}(0.2) = 78.46°$$

$$\bar{Z}_{ph} = Z_{ph} \angle\phi = 25.82 \angle 78.46° = (5.17 + j25.3) \ \Omega$$

$$R_{ph} = 5.17 \ \Omega$$

$$
\begin{aligned}
X_{L_{\text{ph}}} &= 25.3\ \Omega \\
X_{L_{\text{ph}}} &= 2\pi f L_{\text{ph}} \\
25.3 &= 2\pi \times 50 \times L_{\text{ph}} \\
L_{\text{ph}} &= 0.08\ \text{H}
\end{aligned}
$$

(b) A 230/110 V, single-phase transformer takes an input of 350 VA at no load and at rated voltage. The core loss is 110 W. Find (i) the iron loss component of no-load current, (ii) magnetizing component of no-load current, and (iii) no-load power factor. **(06)**

Ans.

$$
\begin{aligned}
S &= 350\ \text{VA} \\
W_i &= 110\ \text{W} \\
E_1 &= 230\ \text{V} \\
S &= V_1 I_o = E_1 I_o \\
350 &= 230 \times I_o \\
I_o &= 1.52\ \text{A} \\
W_i &= V_1 I_o \cos \phi_o \\
110 &= 350 \cos \phi_o \\
\cos \phi_o &= 0.314 \\
\text{No-load pf} &= 0.314 \\
I_\omega &= I_o \cos \phi_o = 1.52 \times 0.314 = 0.48\ \text{A} \\
I_\mu &= I_o \sin \phi_o = 1.52 \times \sin(\cos^{-1} 0.314) = 1.44\ \text{A}
\end{aligned}
$$

(c) Define filter and write down types of filters. **(02)**

Ans. Refer Section 7.7 on page 7.17.

(d) Explain input characteristics of common-emitter configuration. **(04)**

Ans. Refer Section 7.9 on page 7.23.

4. (a) Calculate R_{xy} for the circuit shown in **Fig**. 4. **(07)**

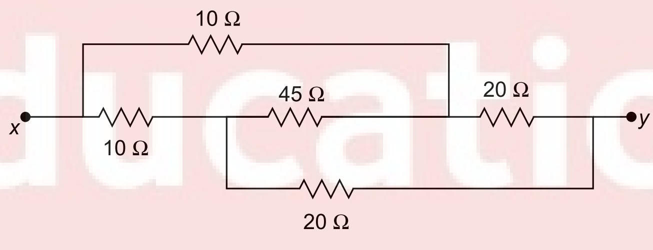


Fig. 4

Ans. Redrawing the network.

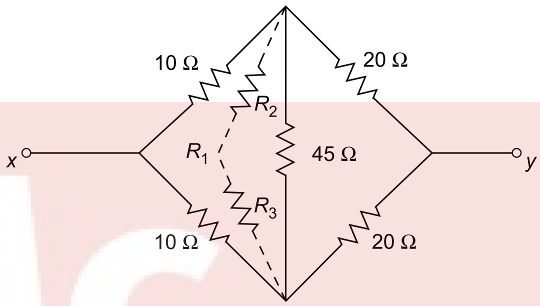


Fig. 5

Converting the delta network formed by resistors of 10 Ω, 10 Ω and 45 Ω into an equivalent star network.

$$R_1 = \frac{10 \times 10}{10 + 10 + 45} = 1.54\ \Omega$$

$$R_2 = R_3 = \frac{10 \times 45}{10 + 10 + 45} = 6.92\ \Omega$$

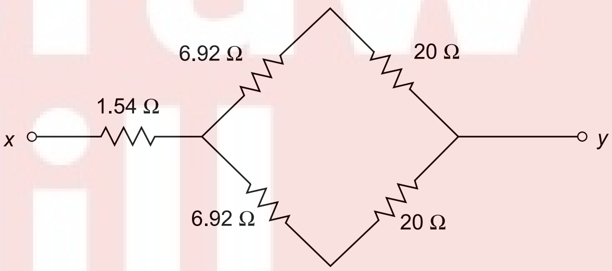


Fig. 6

Simplifying the network.

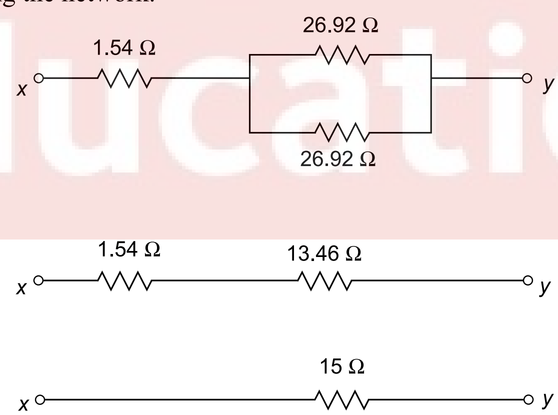


Fig. 7

$$R_{xy} = 15\ \Omega$$

(b) A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current. **(05)**

Ans.

$$V_{\text{rm}} = 150\text{ V}$$
$$f = 50\text{ Hz}$$
$$L = 0.2\text{ H}$$
$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$$
$$V_m = V_{\text{rms}}\sqrt{2} = 150\sqrt{2} = 212.13\text{ V}$$
$$I_m = \frac{V_m}{X_L} = \frac{212.13}{62.83} = 3.38\text{ A}$$
$$v = V_m \sin 2\pi f t = 212.13 \sin 2\pi \times 50 \times t = 212.13 \sin 100\pi t$$
$$i = I_m \sin(2\pi f t - 90°) = 3.38 \sin(100\pi t - 90°)$$

(c) In a balanced three-phase circuit, power is measured by two wattmeters, the ratio of two wattmeter readings is 2 : 1. Determine the power factor of the system. **(04)**

Ans.

$$\frac{W_1}{W_2} = \frac{2}{1}$$
$$W_1 = 2W_2$$
$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{W_2}{3W_2} = \sqrt{3}\left(\frac{1}{3}\right) = 0.577$$
$$\phi = 30°$$
$$\text{pf} = \cos\phi = \cos(30°) = 0.866\ (\text{lagging})$$

(d) Explain with circuit diagram and waveform, the working of a centre tap full-wave rectifier. **(04)**

Ans. Refer Section 7.5 on page 7.11.

5. (a) Determine the value of R for maximum power transfer. Also find the magnitude of maximum power transferred. **(08)**

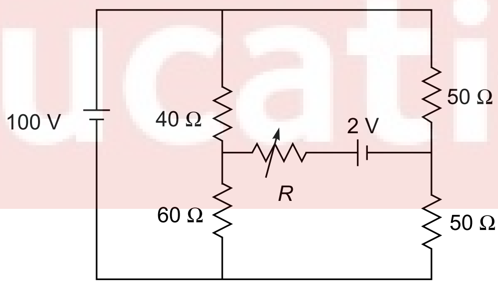


Fig. 8

Ans. **Step I:** Calculation of V_{Th}

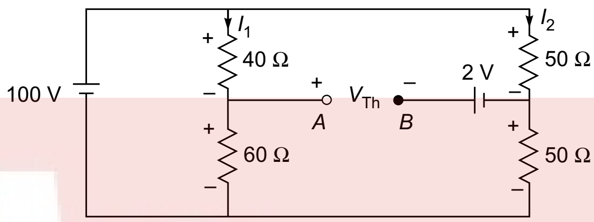


Fig. 9

$$I_1 = \frac{100}{40+60} = 1 \text{ A}$$

$$I_2 = \frac{100}{50+50} = 1 \text{ A}$$

Writing the V_{Th} equation,

$$-40I_1 + V_{Th} - 2 + 50I_2 = 0$$

$$-40(1) + V_{Th} - 2 + 50(1) = 0$$

$$V_{Th} = -8 \text{ V}$$

$$= 8 \text{ V (terminal } B \text{ is positive w.r.t. } A)$$

Step II: Calculation of R_{Th}

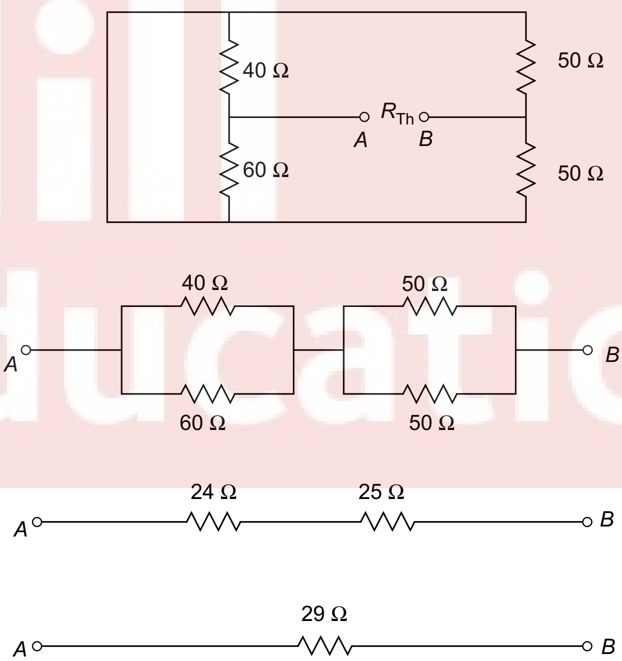


Fig. 10

$$R_{Th} = 49\ \Omega$$

Step III: Value of R

For maximum power transfer,

$$R = R_{Th} = 49\ \Omega$$

Step IV: Calculation of P_{max}

Fig. 11

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(8)^2}{4 \times 49} = 0.33\ \text{W}$$

(b) Calculate the branch current I_1 and I_2 for the circuit shown in Fig. 12. **(04)**

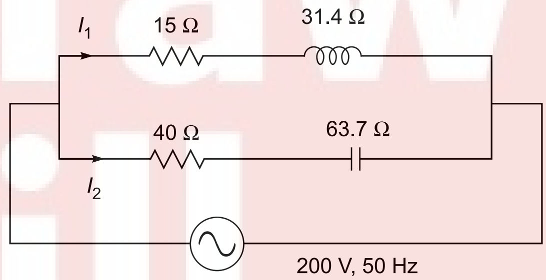


Fig. 12

Ans.

$$\bar{Z}_1 = 15 + j31.4 = 34.8\angle 64.47^\circ\ \Omega$$

$$\bar{Z}_2 = 40 - j63.7 = 75.22\angle -57.87^\circ\ \Omega$$

$$\bar{V} = 200\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{200}{34.8\angle 64.47^\circ} = 5.75\angle -64.47^\circ\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{200}{75.22\angle -57.87^\circ} = 2.66\angle 57.87^\circ\ \text{A}$$

(c) A 30 kVA, 2400/120 V, 50 Hz, transformer has a high-voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low-voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Calculate for the transformer: **(08)**

(i) Equivalent resistance as referred to both primary and secondary

(ii) Equivalent reactance as referred to both primary and secondary

(iii) Equivalent impedance as referred to both primary and secondary

(iv) Copper loss at full load

Ans. kVA rating = 30 kVA

$$E_1 = 2400 \text{ V}$$

$$E_2 = 120 \text{ V}$$

$$R_1 = 0.1 \ \Omega$$

$$X_1 = 0.22 \ \Omega$$

$$R_2 = 0.035 \ \Omega$$

$$X_2 = 0.012 \ \Omega$$

$$K = \frac{E_2}{E_1} = \frac{120}{2400} = 0.05$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.1 + \frac{0.035}{(0.05)^2} = 14.1 \ \Omega$$

$$R_{02} = K^2 R_{01} = (0.05)^2 \times 14.1 = 0.035 \ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.22 + \frac{0.012}{(0.05)^2} = 5.02 \ \Omega$$

$$X_{02} = K^2 X_{01} = (0.05)^2 \times 5.02 = 0.013 \ \Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(14.1)^2 + (5.02)^2} = 14.97 \ \Omega$$

$$Z_{02} = K^2 Z_{01} = (0.05)^2 \times 14.97 = 0.037 \ \Omega$$

$$V_1 \simeq E_1 = 2400 \text{ V}$$

$$\text{Full-load current } I_1 = \frac{\text{kVA rating} \times 1000}{V_1} = \frac{30 \times 1000}{2400} = 12.5 \text{ A}$$

$$W_{cu} = I_1^2 R_{01} = (12.5)^2 \times 14.1 = 2.2 \text{ kW}$$

6. (a) Find the current through the 3 Ω resistor using the superposition theorem. **(07)**

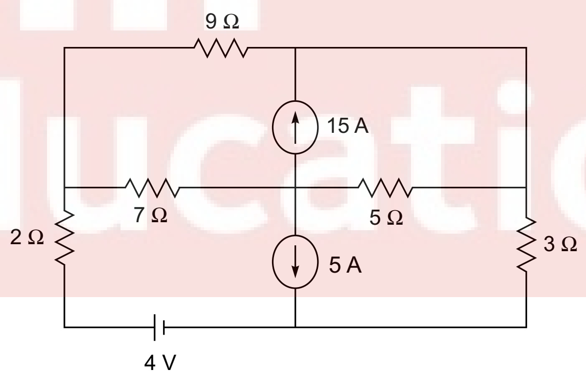


Fig. 13

Ans. When the 4 V source is acting alone,

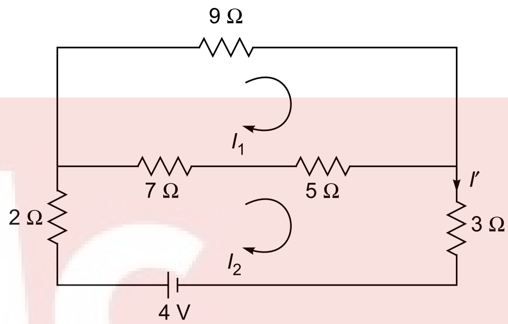


Fig. 14

Writing the KVL equation in matrix form,

$$\begin{bmatrix} 21 & -12 \\ -12 & 17 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

$$I' = I_2 = 0.39 \text{ A} \quad (\downarrow)$$

Step II: When the 15 A source is acting alone

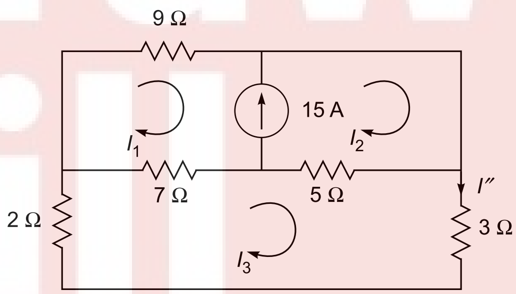


Fig. 15

Writing the current equation for the upper loop,

$$I_2 - I_1 = 15 \qquad \ldots(1)$$

Writing the voltage equation for the upper loop,

$$-9I_1 - 5(I_2 - I_3) - 7(I_1 - I_3) = 0$$

$$-16I_1 - 5I_2 + 12I_3 = 0 \qquad \ldots(\text{ii})$$

Applying KVL to Mesh 3,

$$-2I_3 - 7(I_3 - I_1) - 5(I_3 - I_2) - 3I_3 = 0$$

$$-7I_1 - 5I_2 + 17I_3 = 0 \qquad \ldots(\text{iii})$$

Solving Eqs (i), (ii) and (iii),

$$I'' = I_3 = 3.17 \text{ A} \quad (\downarrow)$$

Step III: When the 5 A source is acting alone

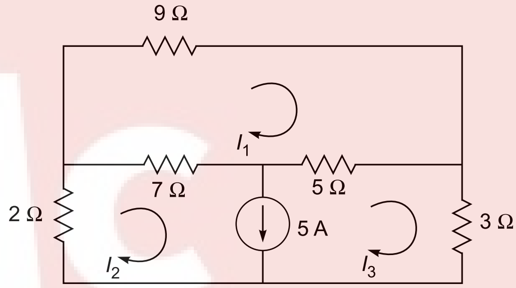


Fig. 16

Applying KVL to Mesh 1,

$$-9I_1 - 5(I_1 - I_3) - 7(I_1 - I_2) = 0$$

$$21I_1 - 7I_2 - 5I_3 = 0 \qquad \ldots\text{(i)}$$

Writing the current equation for the lower loop,

$$I_2 - I_3 = 5 \qquad \ldots\text{(ii)}$$

Writing the voltage equation for the lower loop,

$$-2I_2 - 7(I_2 - I_1) - 5(I_3 - I_1) - 3I_3 = 0$$

$$12I_1 - 9I_2 - 8I_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I''' = I_3 = -2.46 \text{ A} \quad (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I''' = 0.39 + 3.17 - 2.46 = 1.1 \text{ A}$$

$$V_{3\,\Omega} = 3I = 3(1.1) = 3.3 \text{ V}$$

(b) A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 230 V, 50 Hz supply, the maximum current obtained by varying the inductance is 2 A. The voltage across the capacitor is 500 V. Calculate the resistance, inductance and capacitance of the circuit. **(07)**

Ans. Refer Example 4 on page 4.88.

(c) Explain measurement of three-phase power using two-wattmeter method. **(06)**

Ans. Refer Section 5.12.4 on page 5.38.

Solved Question Paper
(Mumbai University–December, 2013)

1. **(a)** Using source transformation, find *I*. **(03)**

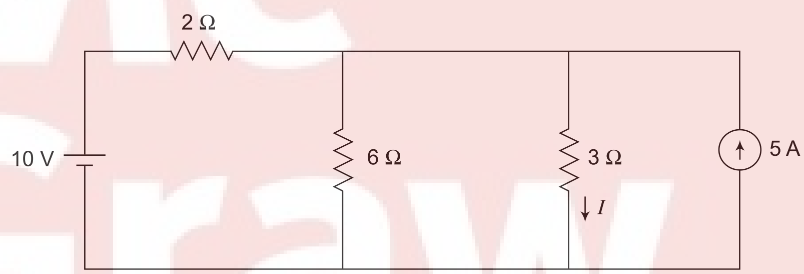


Fig. 1

Ans.

Converting the series combination of the voltage source of 10 V and the resistor of 2 Ω into equivalent current source and resistor,

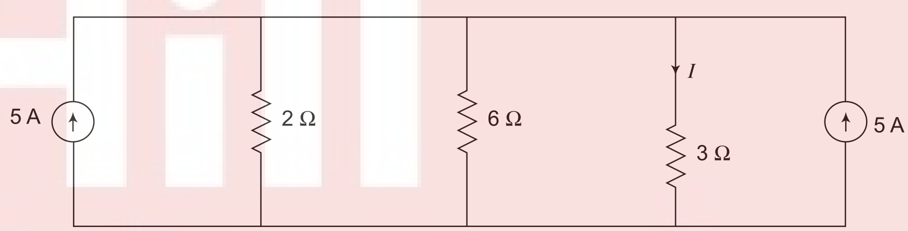


Fig. 2

Adding the two current sources,

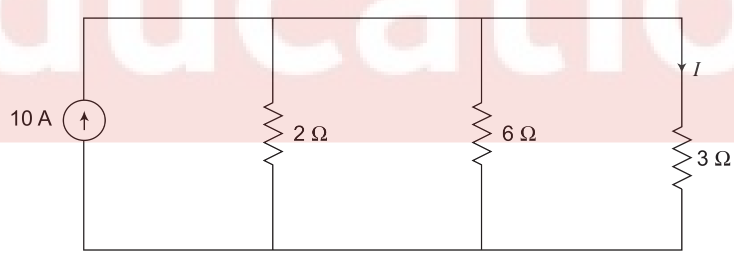


Fig. 3

Simplifying the circuit,

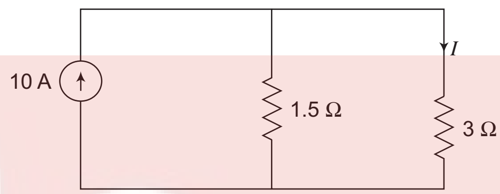


Fig. 4

By current-division rule,

$$I = 10 \times \frac{1.5}{1.5+3} = 3.33 \text{ A}$$

(b) State and explain Norton's theorem. **(03)**

Ans. Refer Section 2.8 on page 2.136.

(c) Derive an expression for the average value of a sinusoidally varying current in terms of peak value. **(03)**

Ans. Refer Section 3.4.1 on page 3.6.

(d) Derive the condition for resonance in a series circuit. **(03)**

Ans. Refer Section 4.8 on page 4.81.

(e) Give the relation between line current and phase current, line voltage and phase voltage in a balanced star- and delta-connected load. **(02)**

Ans. Refer Section 5.7 on page 5.5, and Section 5.8 on page 5.7.

(f) What are assumptions for an ideal transformer? **(04)**

Ans. Refer Section 6.8 on page 6.13.

(g) Draw and explain the circuit diagram for a half-wave rectifier. **(02)**

Ans. Refer Section 7.4 on page 7.5.

2. (a) Find the currents I_1, I_2, I_3 in the given circuit by node-voltage method. **(06)**

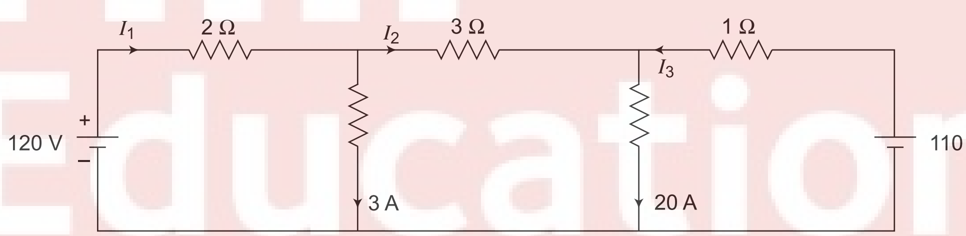


Fig. 5

Ans. Refer similar Example 6 on page 2.42.

[Ans. 7A, 4A, 16A]

(b) For the circuit shown, determine the **(08)**

(i) Supply frequency (f)

(ii) Coil resistance (r)
(iii) Supply voltage (v)

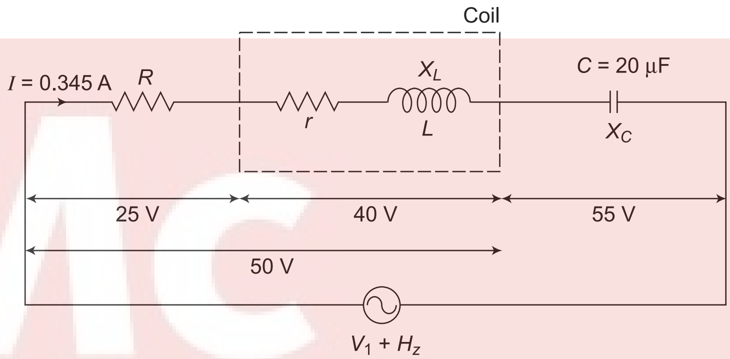


Fig. 6

Ans. Refer similar Example 13 on page 4.53.
[Ans. 50 Hz, 15.95 Ω, 35.05 V]

(c) Draw and explain phasor diagram of a single-phase practical transformer when **(06)**
(i) on no load
(ii) leading power factor load

Ans. (i) Refer Section 6.9 on page 6.14.
(ii) Refer Section 6.10 on page 6.18.

3. (a) Find the values of circuit elements and reactive volt-ampere drawn for a balanced 3-phase load connected in delta and drawing a power of 12 kW at 440 V. The power factor is 0.7 leading. **(08)**

Ans.
$$P = 12 \text{ kW}$$
$$V_L = 440 \text{ V}$$
$$\text{pf} = 0.7 \text{ (leading)}$$

For a delta-connected load,

(i) Values of circuit elements

$$V_L = V_{\text{ph}} = 440 \text{ V}$$
$$P = \sqrt{3} V_L I_L \cos\phi$$
$$12 \times 10^3 = \sqrt{3} \times 440 \times I_L \times 0.7$$
$$I_L = 22.49 \text{ A}$$
$$I_{\text{ph}} = \frac{I_L}{\sqrt{3}} = \frac{22.49}{\sqrt{3}} = 12.98 \text{ A}$$
$$Z_{\text{ph}} = \frac{V_{\text{ph}}}{I_{\text{ph}}} = \frac{440}{12.98} = 33.9\ \Omega$$
$$R_{\text{ph}} = Z_{\text{ph}} \cos\phi = 33.9 \times 0.7 = 23.73\ \Omega$$

$$X_{ph} = Z_{ph} \sin\phi = 33.9 \times \sin(\cos^{-1}0.7) = 33.9 \times 0.71 = 24.07\ \Omega$$

(ii) Reactive voltampere drawn

$$Q = \sqrt{3}V_L I_L \sin\phi = \sqrt{3} \times 440 \times 22.49 \times 0.71 = 12.17 \text{ kVAR}$$

(b) The following results were obtained on a 40 kVA, 2400/120 V transformer,
OC test: 120 V, 9.65 A and 396 W (on LV side)
SC test: 92 V, 20.8 A and 810 W (on HV side)
Calculate the parameters of approximate equivalent circuit referred to HV side. **(06)**

Ans. Refer similar Example 4 on page 6.49.

$$\begin{bmatrix} R_0 = 14.54\ \text{k}\Omega, & X_0 = 5.29\ \text{k}\Omega, \\ R_{01} = 1.87\ \Omega, & X_{01} = 4.007\ \Omega \end{bmatrix}$$

(c) Explain series inductor filter. **(02)**

Ans. Refer Section 7.7.1 on page 7.18.

(d) Explain circuit diagram and working of CE configuration of BJT. **(04)**

Ans. Refer Section 7.8.3 and 7.9 on page 7.22 and 7.23 respectively.

4. (a) Determine current through the 20 Ω resistor in the following circuit. **(07)**

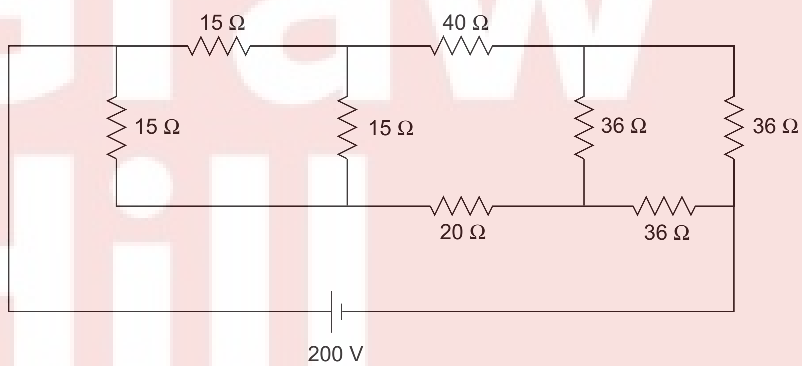


Fig. 7

Ans. Converting the two outer delta networks into equivalent star networks,

$$R_{Y1} = \frac{15 \times 15}{15 + 15 + 15} = 5\ \Omega$$

$$R_{Y2} = \frac{36 \times 36}{36 + 36 + 36} = 12\ \Omega$$

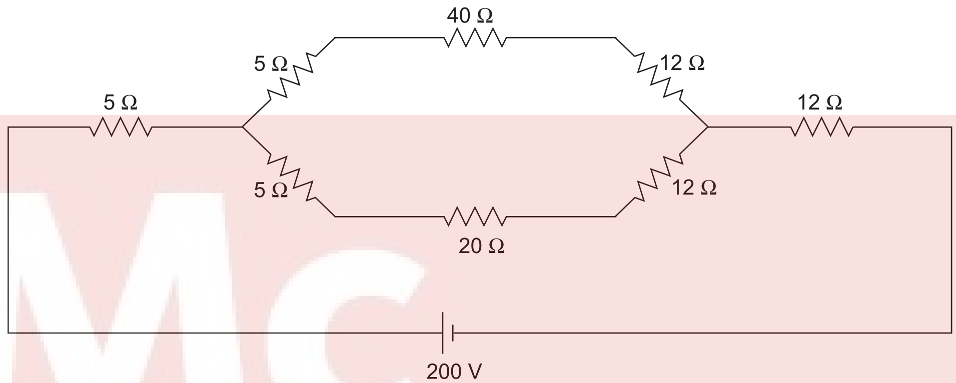


Fig. 8

Simplifying the network,

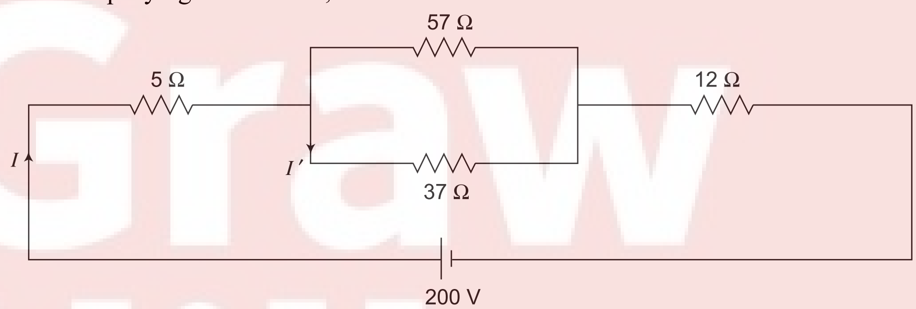


Fig. 9

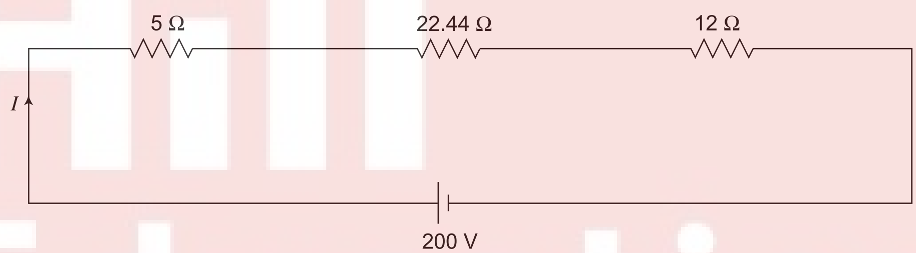


Fig. 10

$$I = \frac{200}{5+22.44+12} = 5.07 \text{ A}$$

By current-division rule,

$$I_{20\,\Omega} = I_{37\,\Omega} = 5.07 \times \frac{57}{57+37} = 3.07 \text{ A}$$

(b) Two currents are represented by $I_1 = 15\sin\left(\omega t + \frac{\pi}{3}\right)$ and $I_2 = 25\sin\left(\omega t + \frac{\pi}{4}\right)$. These currents are fed into a common conductor. Find the total current. If the conductor has a resistance of 50 Ω, what will be energy loss in 10 hours? **(05)**

Ans. Refer similar Example 7 on page 3.44.

[$i = 39.68 \sin(\omega t + 50.62°)$, 1.41×10^9 J].

(c) In a three-phase power measurement by two-wattmeter method, both the wattmeters read the same value. What is the power factor of the load? Justify your answer. **(04)**

Ans. Refer Example 1 on page 5.40.

(d) Explain the circuit diagram and waveforms of the Bridge rectifier. **(04)**

Ans. Refer Section 7.6 on page 7.16.

5. (a) For the given circuit, find the value of 'R_L' so that maximum power is dissipated in it. Also, find P_{max}. **(08)**

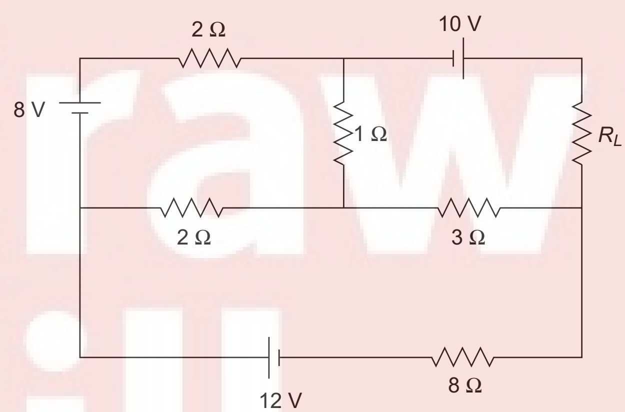


Fig. 11

Ans. **Step I:** Calculation of V_{Th}

Removing the resistor R_L from the network,

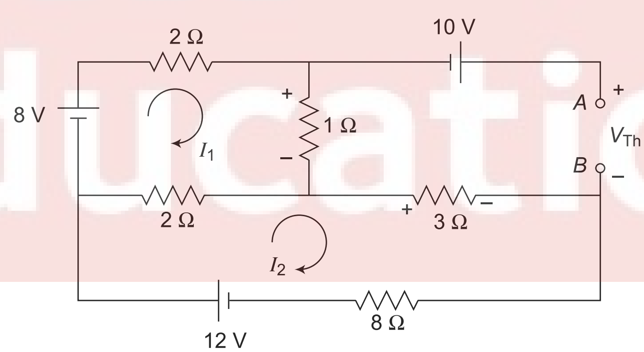


Fig. 12

Applying KVL to Mesh 1,

$$8 - 2I_1 - 1I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 8 \quad (1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 3I_2 - 8I_2 + 12 = 0$$

$$-2I_1 + 13I_2 = 12 \quad (2)$$

Solving Eqs. (1) and (2)

$$I_1 = 2.1 \text{ A}$$

$$I_2 = 1.25 \text{ A}$$

Writing V_{Th} equation,

$$1I_1 + 10 - V_{Th} + 3I_2 = 0$$

$$V_{Th} = 1I_1 + 10 + 3I_2 = 1(2.1) + 10 + 3(1.25) = 15.85 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the voltage sources by short circuits,

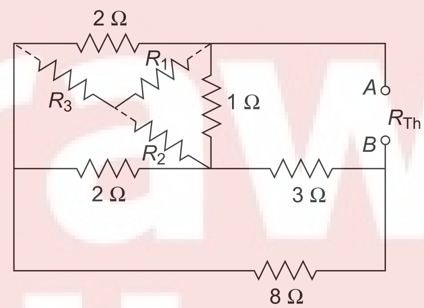


Fig. 13

Converting the delta network formed by resistors of 2 Ω, 1 Ω and 2 Ω into equivalent star network,

$$R_1 = \frac{2 \times 1}{2+1+2} = 0.4\ \Omega$$

$$R_2 = \frac{2 \times 1}{2+1+2} = 0.4\ \Omega$$

$$R_3 = \frac{2 \times 2}{2+1+2} = 0.8\ \Omega$$

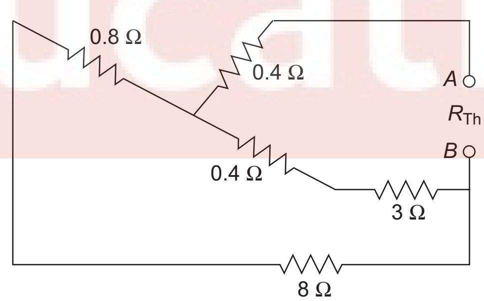


Fig. 14

Simplifying the network,

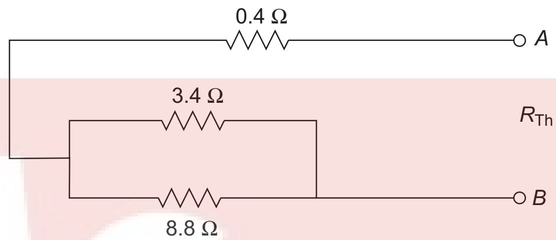


Fig. 15

Fig. 16

$$R_{Th} = 2.85\ \Omega$$

Step III: Calculation of R_L

For maximum power transfer,

$$R_L = R_{Th} = 2.85\ \Omega$$

Step IV: Calculation of P_{max}

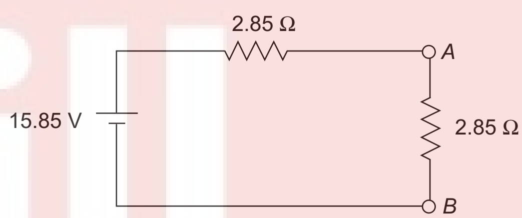


Fig. 17

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(15.85)^2}{4 \times 2.85} = 22.04\ \text{W}$$

(b) With proper phase diagrams, explain the behaviour of a pure capacitor in an ac circuit. **(04)**

Ans. Refer Section 4.3 on page 4.5.

(c) Derive the condition for maximum efficiency of a transformer. Also derive equation for load at maximum efficiency. **(08)**

Ans. Refer Section 6.13 on page 6.29.

6. (a) Determine current in the 1 Ω resistor using the superposition theorem. **(07)**

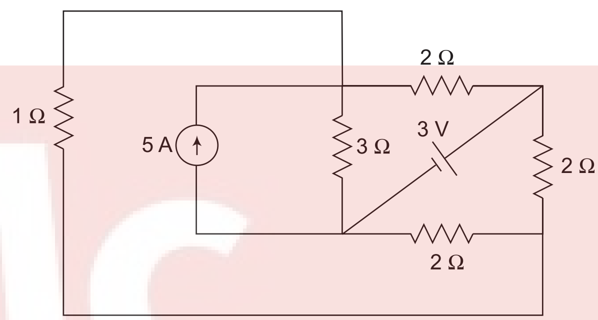


Fig. 18

Ans. Step 1: When the 5 A source is acting alone

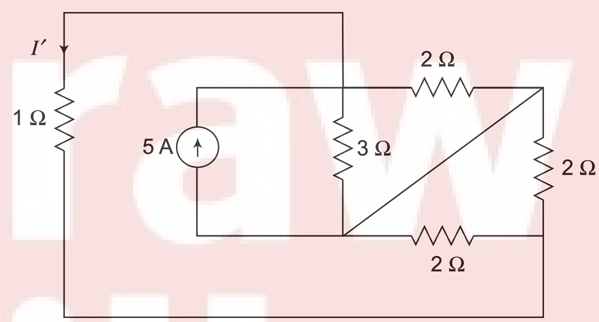


Fig. 19

Simplifying the network,

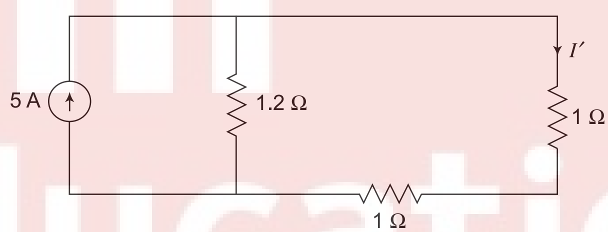


Fig. 20

By current-division rule,

$$I' = 5 \times \frac{1.2}{1.2+1+1} = 1.875 \text{ A } (\downarrow)$$

Step II: When the 3 V source is acting alone

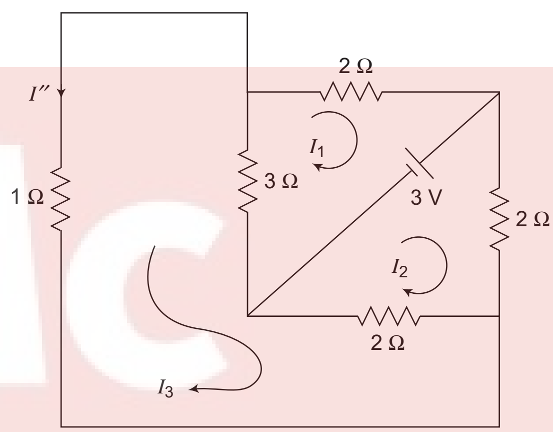


Fig. 21

Applying KVL to Mesh 1,

$$-2I_1 - 3 - 3(I_1 - I_3) = 0$$

$$5I_1 - 3I_3 = -3 \tag{1}$$

Applying KVL to Mesh 2,

$$3 - 2I_2 - 2(I_2 - I_3) = 0$$

$$4I_2 - 2I_3 = 3 \tag{2}$$

Applying KVL to Mesh 3,

$$-3(I_3 - I_1) - 2(I_3 - I_2) - I_3 = 0$$

$$-3I_1 - 2I_2 + 6I_3 = 0 \tag{3}$$

Solving Eqs (1), (2) and (3),

$$I_1 = -0.66\ \text{A}$$

$$I_2 = 0.7\ \text{A}$$

$$I_3 = -0.09\ \text{A}$$

$$I'' = -I_3 = 0.09\ \text{A}$$

Step III: By superposition theorem,

$$I = I' + I'' = 1.875 + 0.09 = 1.965\ \text{A}\ (\downarrow)$$

(b) An inductive coil of 10 Ω resistance and 0.1 H inductance is connected in parallel with a 150 μF capacitor to a variable frequency, and 200 V supply. Find the resonance frequency at which the total current taken from the supply is in phase with the supply voltage. Also find value of this current. Draw the phasor diagram. **(07)**

Ans. $R = 10\ \Omega$

$L = 0.1\ \text{H}$

$C = 150\ \mu F$
$V = 200\ V$

(i) Resonance frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1 \times 150 \times 10^{-6}} - \left(\frac{10}{0.1}\right)^2}$$

$$= 37.89\ \text{Hz}$$

(ii) Value of current

$$Z_D = \frac{L}{CR} = \frac{0.1}{150 \times 10^{-6} \times 10} = 66.67\ \text{A}$$

$$I = \frac{V}{Z_D} = \frac{200}{66.67} = 3\text{A}$$

(iii) Phasor diagram

$$\bar{Z}_{\text{coil}} = 10 + j2\pi \times 37.89 \times 0.1 = 25.82\ \angle 67.22°\ \Omega$$

$$\bar{Z}_C = -j\frac{1}{2\pi \times 37.89 \times 150 \times 10^{-6}} = -j28 = 28\angle -90°\ \Omega$$

$$\bar{I}_{\text{coil}} = \frac{200}{25.82\angle 67.22°} = 7.75\angle -67.22°\ \Omega$$

$$\bar{I}_C = \frac{200}{28\angle -90°} = 7.14\angle 90°\ \Omega$$

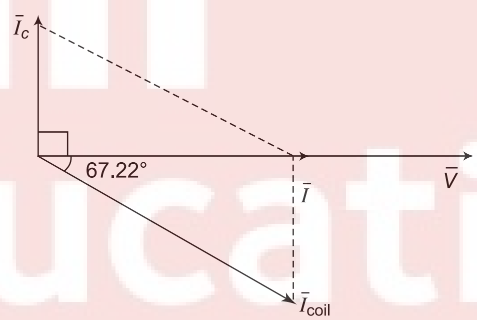


Fig. 22

(c) Two wattmeters are connected to measure power in a three-phase circuit. The reading of one of the wattmeters is 7 kW when load power factor is unity. If the power factor of the load is changed to 0.707 lagging without changing the total input power, calculate the readings of the two wattmeters. **(06)**

Ans. $W_1 = 7$ kW

When power factor is unity,

$$W_1 = W_2 = 7 \text{ kW}$$

$$P = W_1 + W_2 = 7 + 7 = 14 \text{ kW.}$$

Now, pf = 0.707 (lagging)

$$\phi = \cos^{-1}(0.707) = 45°$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

$$\tan 45° = \sqrt{3}\,\frac{W_1 - W_2}{14}$$

$$W_1 - W_2 = 8.08 \text{ kW}$$

$$W_1 + W_2 = 14 \text{ kW}$$

$$\therefore \quad W_1 = 11.04 \text{ kW}$$

$$W_2 = 2.96 \text{ kW}$$

Solved Question Paper
(Mumbai University–May, 2013)

1. (a) Using source transformation, convert the circuit given below to a single voltage source in series with a resistor. **(03)**

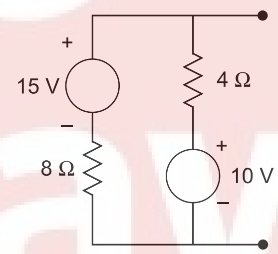


Fig. 1

Ans. By source transformation,

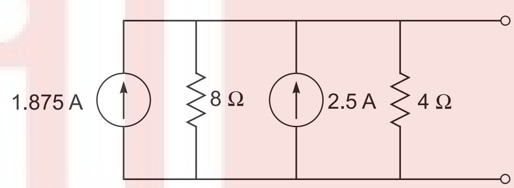


Fig. 2

Adding two current sources and simplifying,

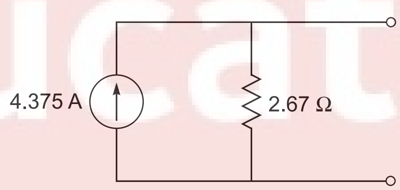


Fig. 3

Again by source transformation,

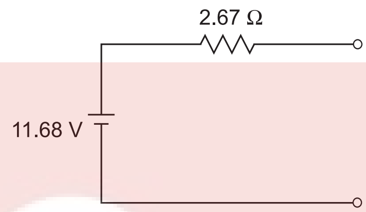


Fig. 4

(b) Derive the condition for maximum power transfer through the network. **(03)**

Ans. Refer Section 2.9 on page 2.151.

(c) Determine the rms value of the voltage waveform shown below: **(03)**

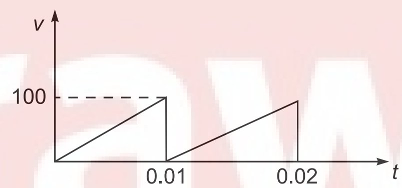


Fig. 5

Ans. $v(t) = \dfrac{100}{0.01} t = 10000\, t \qquad 0 < t < 0.01$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(f)dt}$$

$$= \sqrt{\frac{1}{0.01}\int_0^{0.01} (10000t)^2 dt}$$

$$= 57.74 \text{ V}$$

(d) Give the comparison between series and parallel resonance circuits. **(03)**

Ans. Refer Section 4.9 on page 4.96.

(e) Draw the phasor diagram of a three-phase star-connected load with lagging power factor. **(02)**

Ans. Refer Section 5.7 on page 5.6.

(f) State the working principle of a transformer and derive the expression for the emf induced. **(04)**

Ans. Refer Sections 6.3 and 6.4 on page 6.4.

(g) Define ripple factor and voltage regulation for rectifier circuits. **(02)**

Ans. Refer Section 7.4 on pages 7.9 and 7.10.

2. (a) For the network given below, find current through the 3 Ω resistor using nodal analysis. **(06)**

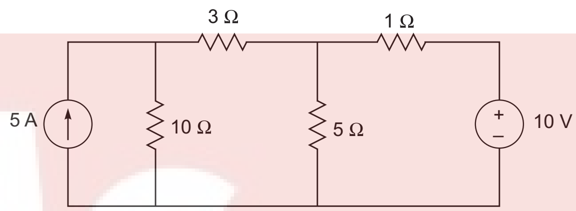


Fig. 6

Ans. Applying KCL at Node 1,

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$\left(\frac{1}{10} + \frac{1}{3}\right)V_1 - \frac{1}{3}V_2 = 5$$

$$0.43V_1 - 0.33V_2 = 5 \qquad \ldots\text{(i)}$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$-\frac{1}{3}V_1 + \left(\frac{1}{3} + \frac{1}{5} + 1\right)V_2 = 10$$

$$-0.33V_1 + 0.65V_2 = 10 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

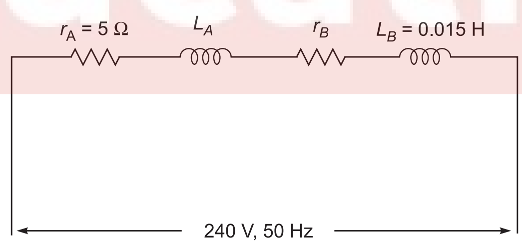
$$V_1 = 38.39 \text{ V}$$

$$V_2 = 34.88 \text{ V}$$

$$I_{3\,\Omega} = \frac{V_1 - V_2}{3} = \frac{38.39 - 34.88}{3} = 1.17 \text{ A}$$

(b) Two coils *A* and *B* are connected in series across a 240 V, 50 Hz supply. The resistance of *A* is 5 Ω and inductance of *B* is 0.015 H. If the input from supply is 3 kW and 2 kVAR, find inductance of *A* and resistance of *B*. Calculate the voltage across each coil. **(08)**

Ans.



$P = 3$ kW

$Q = 2$ kVAR

Fig. 7

$$\tan\phi = \frac{Q}{p} = \frac{2}{3} = 0.67$$

$$\phi = 33.69°$$

$$P = VI\cos\phi$$

$$3000 = 240 \times I \times \cos(33.69°)$$

$$I = 15.02 \text{ A}$$

$$Z_T = \frac{V}{I} = \frac{240}{15.02} = 15.98\ \Omega$$

$$\bar{Z}_T = Z_T\angle\phi = 15.98\angle 33.69 = (13.3 + j8.86)\ \Omega$$

$$r_T = r_A + r_B = 13.3\ \Omega$$

$$r_B = 8.3\ \Omega$$

$$X_B = 2\pi f L_B = 2\pi \times 50 \times 0.015 = 4.71\ \Omega$$

$$X_T = X_A + X_B = 8.86$$

$$X_A = 4.15\ \Omega$$

$$X_A = 2\pi f L_A$$

$$4.15 = 2\pi \times 50 \times L_A$$

$$L_A = 0.013 \text{ H}$$

$$Z_A = \sqrt{r_A^2 + X_A^2} = \sqrt{(5)^2 + (4.15)^2} = 6.5\ \Omega$$

$$Z_B = \sqrt{r_B^2 + X_B^2} = \sqrt{(8.3)^2 + (4.71)^2} = 9.54\ \Omega$$

$$V_A = Z_A I = 6.5 \times 15.02 = 97.63 \text{ V}$$

$$V_B = Z_B I = 9.54 \times 15.02 = 143.29 \text{ V}$$

(c) A 3000/200 V, 50 Hz, single-phase transformer has a cross-sectional area of 150 cm^2 for the core. If the number of turns on the low-voltage winding is 80, determine the number of turns on the high-voltage winding and maximum value of flux density in the core. **(06)**

Ans.

$$E_1 = 3000 \text{ V}$$

$$E_2 = 200 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$A = 150 \text{ cm}^2 = 150 \times 10^{-4} \text{ m}^2$$

$$N_2 = 80$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{200}{3000} = \frac{80}{N_1}$$

$$N_1 = 1200$$

$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$
$$3000 = 4.44 \times 50 \times B_m \times 150 \times 10^{-4} \times 1200$$
$$B_m = 0.75 \text{ Wb/m}^2$$

3. (a) Each phase of a delta-connected load consists of a 50 mH inductor in series with a parallel combination of a 5 Ω resistor and a 5 μF capacitor. The load is connected to a three-phase, 550 V, 50 Hz ac supply.

Find (i) phase current, (ii) line current, (iii) power drawn, (iv) power factor, (v) reactive power and kVA rating of the load. **(08)**

Ans. Refer Example 8 on page 5.18.

(b) A 5 kVA, 1000/200 V, 50 Hz, single-phase transformer gives the following test results:

OC test (LV side)	200 V	1.2 A	90 W
SC test (HV side)	50 V	5 A	110 W

Determine efficiency as half load at 0.8 p.f. lagging. **(06)**

Ans. Refer Example 1 on page 6.45.

(c) What is the function of a filter in rectifier circuits? Explain with appropriate waveforms. **(02)**

Ans. Refer Section 7.7 on page 7.17.

(d) Draw and explain output characteristics of a transistor in CE configuration. **(04)**

Ans. Refer Section 7.9 on page 7.24.

4. (a) For the circuit shown below, find the resistance between terminals A and B. **(07)**

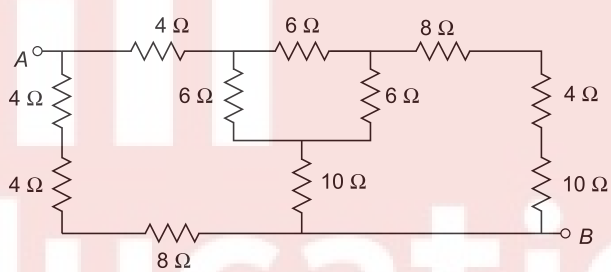


Fig. 8

Ans. Refer Example 7 on page 2.69.

(b) The voltage drops across four series-connected impedances are given: **(05)**

$$v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right),\ v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right)$$

$$v_3 = 100\cos\left(\omega t + \frac{\pi}{4}\right),\ v_4 = V_{4m}\sin(\omega t + \phi_4)$$

Calculate the values of V_{4m} and ϕ_4 if the voltage applied across the series circuit is $140\sin\left(\omega t + \frac{3\pi}{5}\right)$

Ans. Refer Example 8 on page 3.44.

(c) Draw the circuit for measurement of three-phase power using two wattmeters and state its advantages over other methods of three-phase power measurement. **(04)**

Ans. Refer Section 5.12 on page 5.36.

(d) Draw and explain half-wave rectifier with appropriate waveforms. **(04)**

Ans. Refer Section 7.4 on page 7.5.

5. (a) Using Norton's theorem, calculate the current flowing through the 15 Ω load resistor in the given circuit. **(08)**

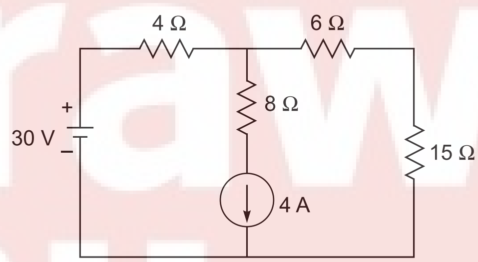


Fig. 9

Ans. **Step I:** Calculation at I_N

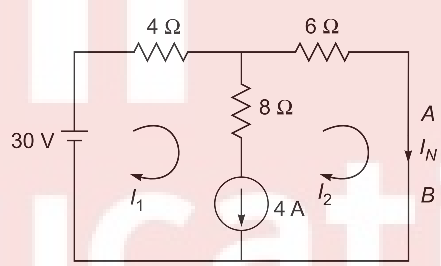


Fig. 10

Writing the current equation for the loop,

$$I_1 - I_2 = 4 \qquad \ldots\text{(i)}$$

Writing the voltage equation for the loop,

$$30 - 4I_1 - 6I_2 = 0$$

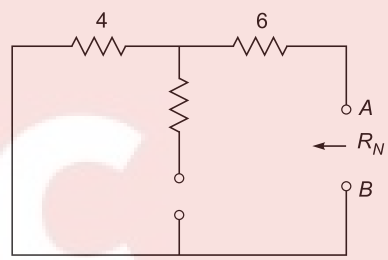
$$4I_1 + 6I_2 = 30 \qquad \ldots\text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_1 = 5.4\text{ A}$$

$$I_2 = 1.4 \text{ A}$$
$$I_N = I_2 = 1.4 \text{ A}$$

Step II: Calculation of R_N



$R_N = 10\ \Omega$

Fig. 11

Step III: Calculation of I_L

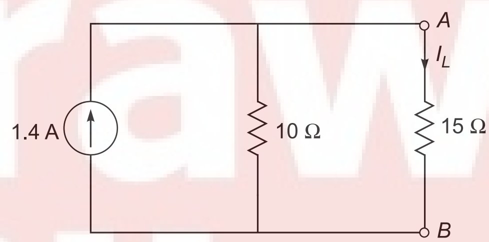


Fig. 12

$$I_L = 1.4 \times \frac{10}{10+15} = 0.56 \text{ A}$$

(b) A 46 mH inductive coil has a resistance of 10 Ω. (i) How much current will it draw if connected across a 100 V, 60 Hz supply? (ii) What is the power factor of the coil? (iii) Determine the value of capacitance that must be connected across the coil to make the power factor of overall circuit units. **(04)**

Ans.

$$L = 46 \text{ mH}$$
$$r = 10\ \Omega$$
$$V = 100 \text{ V}$$
$$f = 60 \text{ Hz}$$
$$X_L = 2\pi f L = 2\pi \times 60 \times 46 \times 10^{-3} = 17.34\ \Omega$$

$$Z = \sqrt{r^2 + X_L^2} = \sqrt{(10)^2 + (17.34)^2} = 20.02\ \Omega$$

$$\text{pf} = \cos\phi = \frac{r}{Z} = \frac{10}{20.02} = 0.5 \text{ (lagging)}$$

When a capacitor is connected across a coil to make power factor of overall circuit unity, the circuit is in resonance.

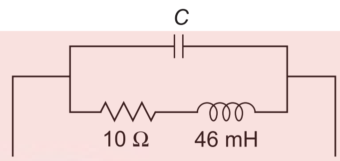


Fig. 13

$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$$

$$60 = \frac{1}{2\pi}\sqrt{\frac{1}{46 \times 10^{-3} \times C} - \frac{(10)^2}{(46 \times 10^{-3})^2}}$$

$$C = 114.79\ \mu\text{F}$$

(c) A 30 kVA, 2400/120 V, 50 Hz transformer has a high-voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low-voltage winding resistance is 0.035 Ω and the leakage reactance is 0.042 Ω. Calculate the equivalent winding resistance, reactance and impedance referred to (i) high-voltage side, (ii) low-voltage side, and (iii) total copper loss of the transformer. **(08)**

Ans. Refer Solved Question Paper of 2012, Q .5(c)

6. (a) Determine current through $R_L = 2\ \Omega$ in the circuit shown below using superposition theorem. **(07)**

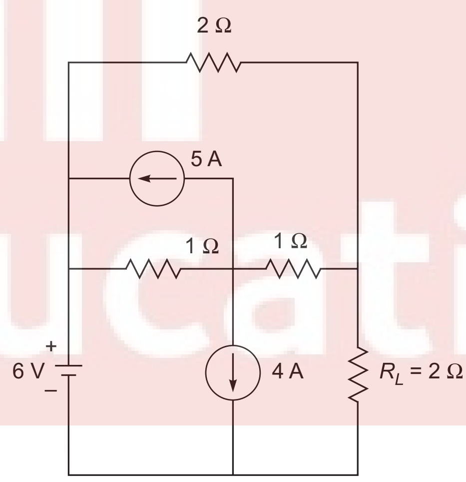


Fig. 14

Ans. **Step I:** When the 6 V source is acting alone

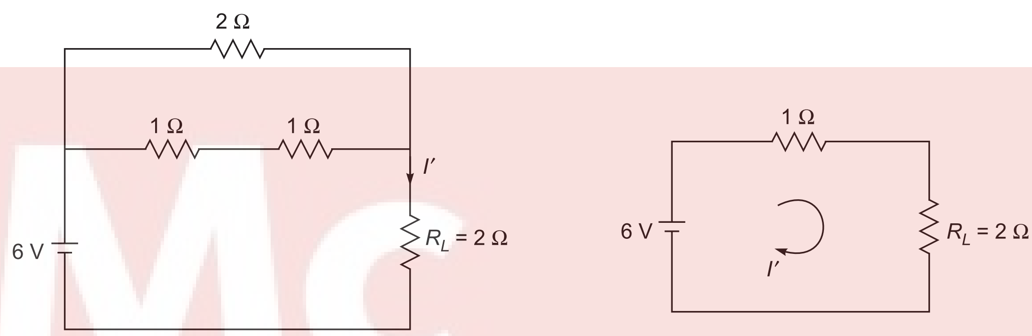


Fig. 15

$$I' = \frac{6}{1+2} = 2\text{ A} \quad (\downarrow)$$

Step II: When the 4 A source is acting alone

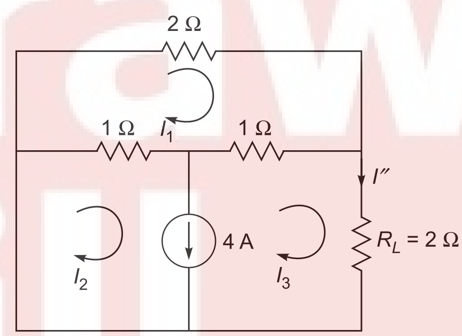


Fig. 16

Applying KVL to Mesh 1,

$$4I_1 - I_2 - I_3 = 0 \qquad \ldots\text{(i)}$$

Writing the current equation for the lower loop,

$$I_2 - I_3 = 4 \qquad \ldots\text{(ii)}$$

Writing the voltage equation for the lower loop,

$$-1(I_2 - I_1) - 1(I_3 - I_1) - 2I_3 = 0$$

$$2I_1 - I_2 - 3I_3 = 0 \qquad \ldots\text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I'' = I_3 = -0.67\text{ A} \quad (\downarrow)$$

Step III: When the 5 A source is acting alone

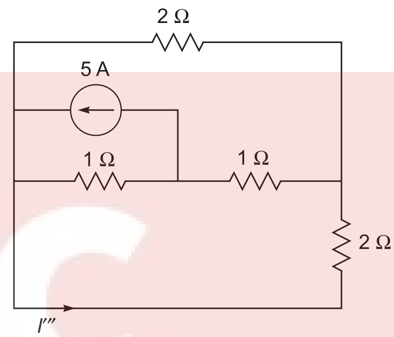


Fig. 17

Simplifying the circuit,

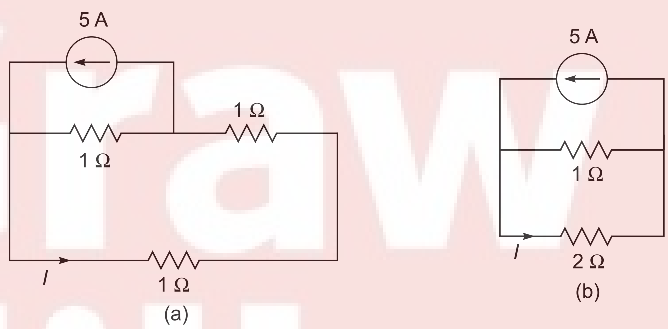


Fig. 18

$$I = 5 \times \frac{1}{1+2} = 1.67 \text{ A} \quad (\uparrow)$$

$$I''' = -1.67 \times \frac{1}{2} = -0.84 \text{ A} \quad (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I''' = 2 - 0.67 - 0.84 = -0.49 \text{ A} \quad (\downarrow)$$

(b) An inductor having a resistance of 25 Ω and Q_0 of 10 at a resonant frequency of 10 kHz is fed from a 100 V supply. Calculate (i) value of series capacitance required to produce resonance with the coil, (ii) the inductance of the coil, (iii) Q_0 using L/C ratio, (iv) voltage across capacitor, and (v) voltage across the coil. **(07)**

Ans.

$$R = 25\ \Omega$$

$$Q_o = 10$$

$$f_o = 10 \text{ kHz}$$

$$V = 100$$

$$Q_o = \frac{V_{Co}}{V}$$

$$10 = \frac{V_{Co}}{100}$$

$$V_{Co} = 1000 \text{ V}$$

$$I_o = \frac{V}{R} = \frac{100}{25} = 4 \text{ A}$$

$$V_{Co} = I_o X_{Co}$$

$$1000 = 4X_{Co}$$

$$X_{Co} = 250\ \Omega$$

$$X_{Co} = \frac{1}{2\pi f_o C}$$

$$250 = \frac{1}{2\pi \times 10 \times 10^3 \times C}$$

$$C = 6.37 \times 10^{-8} \text{ F}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$10 \times 10^3 = \frac{1}{2\pi\sqrt{L \times 6.37 \times 10^{-8}}}$$

$$L = 3.98 \text{ mH}$$

$$Q_o = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{25}\sqrt{\frac{3.98 \times 10^{-3}}{6.37 \times 10^{-8}}} = 10$$

$$V_{Lo} = V_{Co} = 1000 \text{ V}$$

(c) The input power of a three-phase motor was measured by the two-wattmeter method. The readings of two wattmeters are 5.2 kW and –1.7 kW and the line voltage is 415 V. Calculate the total active power, power factor and line current. **(06)**

Ans.

$$W_1 = 5.2 \text{ kW}$$

$$W_2 = -1.7 \text{ kW}$$

$$V_L = 415 \text{ V}$$

$$P = W_1 + W_2 = 5.2 - 1.7 = 3.5 \text{ kW}$$

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\left(\frac{5.2 + 1.7}{5.2 - 1.7}\right) = 3.41$$

$$\phi = 73.68°$$

$$\text{pf} = \cos\phi = \cos(73.68°) = 0.28 \text{ (lagging)}$$

$$P = \sqrt{3}\,V_L I_L \cos\phi$$

$$3.5 \times 10^3 = \sqrt{3} \times 415 \times I_L \times 0.28$$

$$I_L = 17.39 \text{ A}$$

Solved Question Paper
(Mumbai University– May, 2014)

1. (a) Using source conversion, reduce the circuit shown in the figure into a single current source in parallel with single resistance. **(03)**

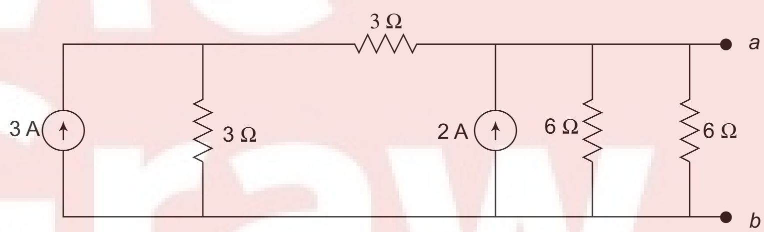


Fig. 1

Ans. Converting the parallel combination of the current source of 3 A and the resistor of 3 Ω into an equivalent series combination of voltage source and resistor,

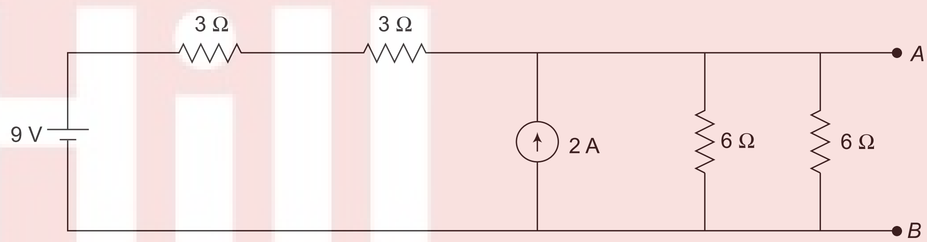


Fig. 2

Again by source transformation,

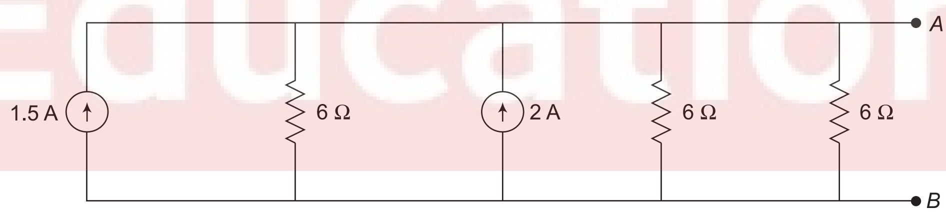


Fig. 3

Adding the two current sources and by series-parallel reduction technique,

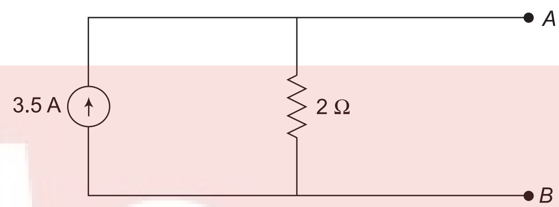


Fig. 4

(b) Derive the condition for maximum power transfer through the network. **(03)**

Ans. Refer Section 2.9 on page 2.151.

(c) An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of the current is 40 Hz. Find the maximum value of the alternating current. **(03)**

Ans. Refer Example 1 on page 3.6.

(d) Derive the equation for resonance frequency [f_r] in a parallel resonance circuit. **(03)**

Ans. Refer Section 4.9 on page 4.93.

(e) Three identical coils each of [4.2 + j5.6] ohms are connected in star across a 415 V, 3-phase, 50 Hz supply. Determine (i) V_{ph}, (ii) I_{ph}, and (iii) power factor. **(02)**

Ans. $\bar{Z}_{\text{ph}} = 4.2 + j5.6 = 7\ \angle 53.13°\ \Omega$

$V_L = 415$ V

$f = 50$ Hz

For a star-connected load,

$$\text{(i) } V_{\text{ph}} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6 \text{ V}$$

$$\text{(ii) } I_{\text{ph}} = \frac{V_{ph}}{Z_{ph}} = \frac{239.6}{7} = 34.23 \text{ A}$$

(iii) pf = cos ϕ = cos (53.13°) = 0.6 (lagging)

(f) What are the losses in a transformer? Explain why the rating of transformer is expressed in kVA not in kW. **(04)**

Ans. Refer Section 6.6 on page 6.5 and Section 6.7 on page 6.12.

(g) Draw complete VI characteristics of a diode. **(02)**

Ans. Refer Section 7.2.1 on page 7.3.

2. **(a)** Determine the potential different V_{AB} for the given network. **(06)**

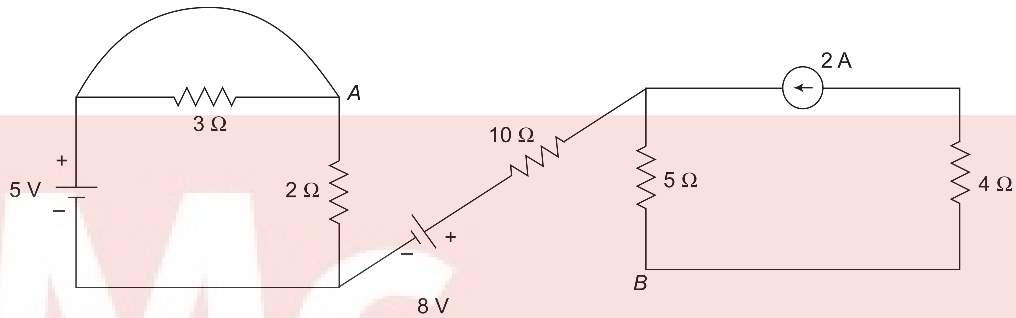


Fig. 5

Ans. Refer Example 7 on page 2.8.

(b) When a resistor and an inductor in series are connected to a 240 V supply, a current of 3 A flows lagging 3.7° behind the supply voltage, while voltage across the inductor is 171 volts. Find the resistance of resistor, and resistance and reactance of the inductor. **(08)**

Ans. Refer Example 15 on page 4.21.

(c) Draw the phasor diagram of a single-phase transformer on resistive load [unity power factor] and inductive load [lagging power factor]. **(06)**

Ans. Refer Section 6.10 on page 6.16.

3. (a) Three similar coils, connected in star, take a total power of 18 kW at a power factor of 0.866 lagging from a three-phase, 400-volt, 50 Hz system. Calculate the resistance and inductance of each coil. Also draw the phasor diagram showing the currents and voltages. **(08)**

Ans.

$$P = 18 \text{ kW}$$
$$\text{pf} = 0.866 \text{ (lagging)}$$
$$V_L = 440 \text{ V}$$
$$f = 50 \text{ Hz}$$

For a star-connected load,

(i) Resistance and inductance of each coil

$$V_{\text{ph}} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94 \text{ V}$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi$$

$$18 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.866$$

$$I_L = 30 \text{ A}$$

$$I_{\text{ph}} = I_L = 30 \text{ A}$$

$$Z_{\text{ph}} = \frac{V_{\text{ph}}}{I_{\text{ph}}} = \frac{230.94}{30} = 7.7\ \Omega$$

$$\begin{aligned}
\phi &= \cos^{-1}(0.866) = 30^\circ \\
\bar{Z}_{ph} &= Z_{ph} < \phi = 7.7 \angle 30^\circ = 6.67 + j3.85\ \Omega \\
R_{ph} &= 6.67\ \Omega \\
X_{ph} &= 3.85\ \Omega \\
X_{ph} &= 2\pi f L \\
3.85 &= 2\pi \times 50 \times L \\
L &= 12.25\ \text{mH}
\end{aligned}$$

(ii) Phasor diagram

Refer Fig 5.10 on page 5.6.

$$\phi = 30^\circ$$

(b) A 5 kVA 200/400 volt, 50 Hz, single-phase transformer gave the following test results.

OC test [LV Side]	200 V	0.7 A	60 W
SC test [HV side]	22 V	16 A	120 W

(i) Draw the equivalent circuit of the transformer referred to LV side and insert all parameters values.

(ii) Efficiency at 0.9 power factor leading if operating at rated load. **(06)**

Ans. Refer Example 2 on page 6.45.

(c) What is the function of a filter in a rectifier circuit? Draw the circuit of a rectifier with inductor filter. **(02)**

Ans. Refer Sections 7.7 and 7.7.1 on page 7.17 and 7.18 respectively.

(d) Explain with a circuit diagram the working of CE configuration of BJT. **(04)**

Ans. Refer Sections 7.8.3 and 7.9 on page 7.21 and 7.23 respectively.

4. (a) Find an equivalent resistance between A and B. **(07)**

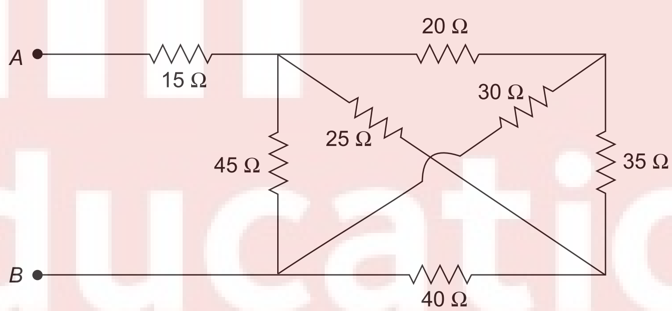


Fig. 6

Ans. Refer Example 6 on page 2.67.

(b) A circuit consists of three parallel branches. The branch currents are given as $i_1 = 10 \sin \omega t$, $i_2 = 20 \sin(\omega t + 60^\circ)$, and $i_3 = 75 \sin(\omega t - 30^\circ)$. Find the resultant current and express it in the form $i = I_m \sin(\omega t + \phi)$. If the supply frequency is 50 Hz, calculate the resultant current when (i) $t = 0$, and (ii) $t = 0.001$ second. **(05)**

Ans. Refer similar Example 9 on page 3.45.

(c) A 3 phase, 10 kVA load has a power factor of 0.342. The power is measured by two-wattmeter method. Find the reading of each wattmeter when, **(04)**

(i) power factor is leading

(ii) power factor is lagging

Ans. Refer Example 6 on page 5.43.

(d) Explain the working of a centre-tap full-wave rectifier with waveforms. **(04)**

Ans. Refer Section 7.5 on page 7.11.

5. (a) Find the current through the 60 Ω resistance by using Thevenin's theorem. **(08)**

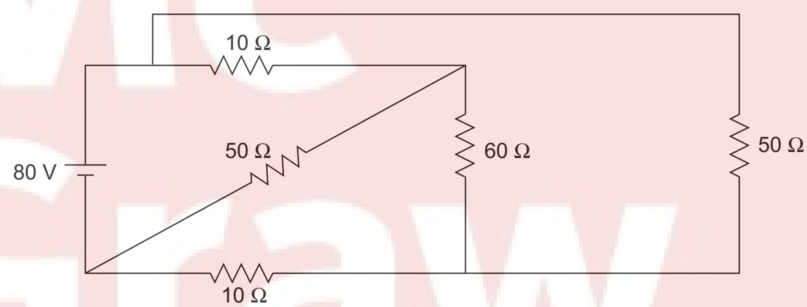


Fig. 7

Ans. Step I: Calculation of V_{Th}

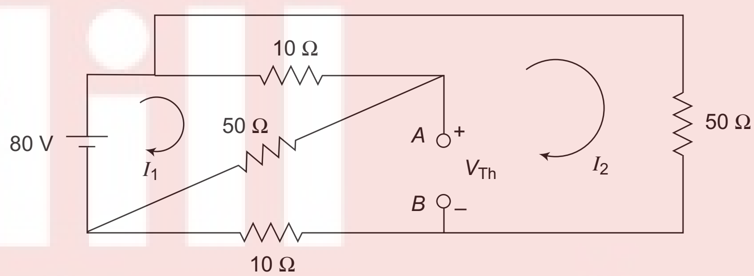


Fig. 8

Writing KVL equation in matrix form,

$$\begin{bmatrix} 60 & -0 \\ -0 & 120 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 0 \end{bmatrix}$$

$I_1 = 2.67$ A

$I_2 = 1.33$ A

Writing V_{Th} equation,

$80 - 10\,(I_1 - I_2) - V_{Th} - 10I_2 = 0$

$V_{Th} = 80 - 10(I_1 - I_2) - 10I_2$

$= 80 - 10(2.67 - 1.33) - 10\,(1.33) = 53.3$ V

Step II: Calculation of R_{Th}

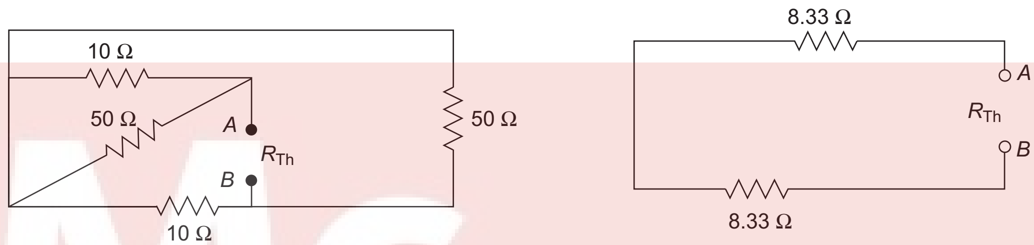


Fig. 9

$R_{Th} = 16.66\ \Omega$

Step III: Calculation of I_L

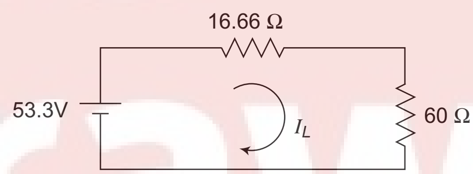


Fig. 10

$$I_L = \frac{53.3}{16.66 + 60} = 0.7\ \text{A}$$

(b) Find currents I_1 and I_2 shown in the figure. **(04)**

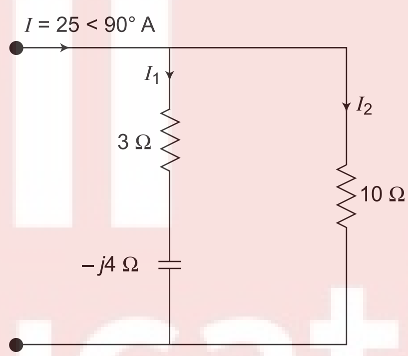


Fig. 11

Ans. Refer Example 12 on page 4.66.

(c) A 50 kVA, 4400/220 volt transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$. The reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for the transformer,

(i) full load currents on primary and secondary side,

(ii) equivalent resistance, reactances, impedances referred to primary side and secondary side, and

(iii) total copper loss using individual resistance and equivalent resistances.

Ans. Refer Example 4 on page 6.22.

6. (a) Find the current through the 6 Ω resistor using superposition theorem. **(07)**

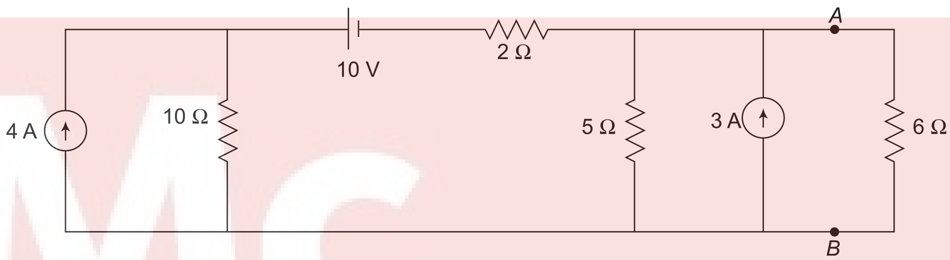


Fig. 12

Ans. **Step 1**: When the 4 A source is acting alone

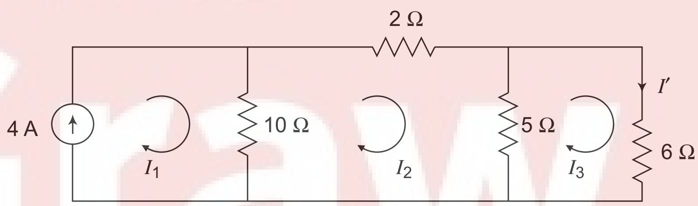


Fig. 13

Writing equations in matrix form,

$$\begin{bmatrix} 1 & 0 & 0 \\ -10 & 17 & -5 \\ 0 & -5 & 11 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$I' = I_3 = 1.23 \text{ A} \quad (\downarrow)$$

Step II: When the 10 V source is acting alone

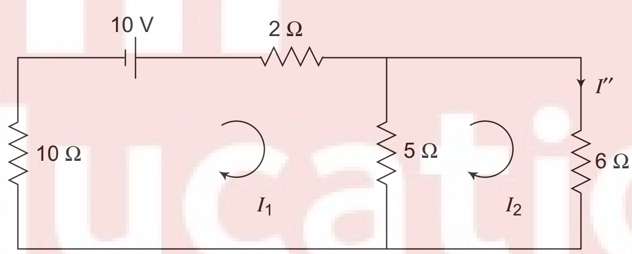


Fig. 14

Writing KVL equation in matrix form,

$$\begin{bmatrix} 17 & -5 \\ -5 & 11 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

$$I'' = I_2 = -0.31 \text{ A} \, (\downarrow)$$

Step III: When the 3 A source is acting alone

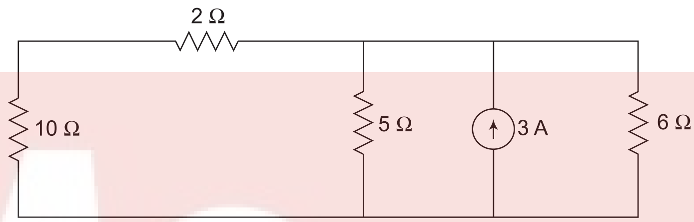


Fig. 15

By series-parallel reduction technique,

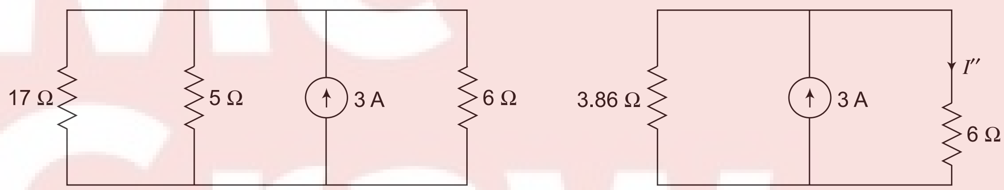


Fig. 16

By current-division rule,

$$I''' = 3\times\frac{3.86}{3.86+6} = 1.17 \text{ A } (\downarrow)$$

Step IV: By superposition theorem,

$$I = I' + I'' + I''' = 1.23 - 0.31 + 1.17 = 2.09 \text{ A } (\downarrow)$$

(b) A coil of 31.8 mH inductances with a resistance of 12 Ω is connected in parallel with a capacitor across a 250-volt, 50 Hz supply. Determine the value of capacitance, if no reactive current is taken from the supply. **(07)**

Ans. $L = 31.8$ mH

$R = 12\ \Omega$

$V = 250$ V

$f = 50$ Hz

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$50 = \frac{1}{2\pi}\sqrt{\frac{1}{31.8\times 10^{-3}\times C} - \left(\frac{12}{31.8\times 10^{-3}}\right)^2}$$

$C = 130.43\ \mu$F

(c) Explain measurement of three-phase power using the two-wattmeter method. **(06)**

Ans. Refer Section 5.12.4 on page 5.38.