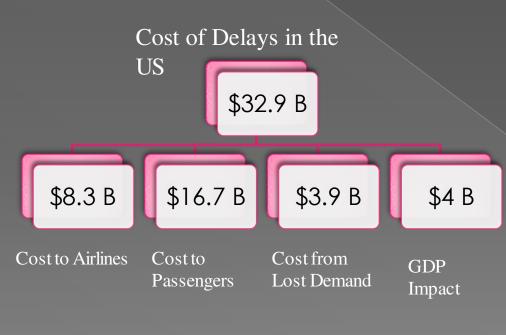


# USAARLINE DELAYS AND THEIR IMPACTS Explained using Multiple Regression and

Explained using Multiple Regression and Logistic Regression

-Satya M

### Why Airline Delays?





## How Different from Existing Models?

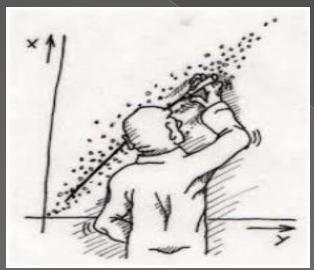
- I found 'N' number of projects in the internet trying to predict the Air Delay time based on various factors.
- However, I would focus on two important factors.
  - > AirTime
  - WeatherDelay
- I'm using the above factors for consideration as the same hasn't been used before in any model and they are quite difficult to predict.
- They play an significant factor while looking at Air Line information.

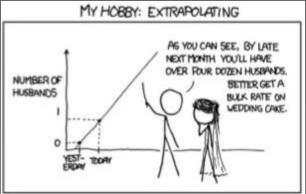
#### About the Dataset

- I used the dataset from Kaggle and it was populated using "The Flight Delay" project which is the most complete database of flight delays in the United States
- The dataset is separated into individual documents one record for every year and contains around 7 million records each.
- I'm considering the records only for the year 2008, as it holds the most recent data. Further, I'm looking at flight data with of Chicago and Washington DC.
- The source of data is given in the below link
- https://www.kaggle.com/giovamata/airlinedelaycauses



### What is Regression?

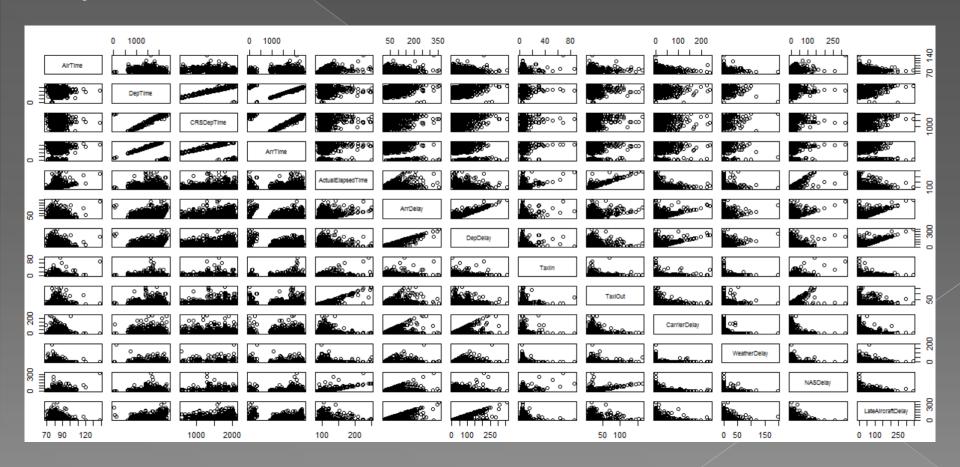




- \* Regression Analysis is the art and science of fitting straight lines to patterns of data.
- Regression Analysis is widely used for predicting and forecasting.
- ❖ In a linear regression model, the variable of interest which is the dependent variable is predicted from a single (simple linear regression) or multiple group of independent variables (multiple linear regression).
- ❖ Whereas, for Logistic regression, the dependent variable is categorical (i.e. qualitative data)
- ❖ In this case, I included the following as the dependent variable.
  - ❖ For Multiple Regression AirTime
  - For Logistic Regression WeatherDelay (For Delay 0, else 1)

# Steps to Perform for Multiple Regression

 Draw a scatter plot and Observe whether you can fit a line to describe a pattern



- Build a multiple linear regression model , y = f(x) = β0 + β1X + e
- fit\_model <- Im(model\$AirTime ~ as.factor(model\$Month) +</p> as.factor(model\$DayofMonth) + as.factor(model\$DayOfWeek) + model\$DepTime + model\$CR\$DepTime + model\$ArrTime + model\$CRSArrTime + as.factor(model\$UniqueCarrier) + as.factor(model\$FlightNum) `+ model\$ActualElapsedTime + model\$CRSElapsedTime + model\$ArrDelay + model\$DepDelay + model\$TaxiIn + model\$TaxiOut + model\$CarrierDelay + model\$WeatherDelay + model\$NASDelay + model\$LateAircraftDélay)

#### Parameter Estimates

```
> # Dropping model$CarrierDelay
> fit_model <- lm(formula = model$AirTime ~
                             model$ArrDelay + model$DepDelay +
                             model$TaxiIn + model$TaxiOut)
> summary(fit_model)
call:
lm(formula = model$AirTime ~ +model$ArrDelay + model$DepDelay +
   +model$TaxiIn + model$TaxiOut)
Residuals:
           1Q Median 3Q
   Min
-7.1156 -1.6715 -0.3932 2.0813 8.4047
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
                          0.38413 268.13 <2e-16 ***
(Intercept)
             102.99754
model$ArrDelay 0.89767 0.01442 62.26 <2e-16 ***
model$DepDelay -0.90124 0.01424 -63.28 <2e-16 ***
model$TaxiIn -0.82366
                          0.02253 -36.56 <2e-16 ***
model$Taxiout -0.87367
                          0.01522 -57.40 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 2.866 on 1015 degrees of freedom
Multiple R-squared: 0.8025, Adjusted R-squared: 0.8018
F-statistic: 1031 on 4 and 1015 DF, p-value: < 2.2e-16
```

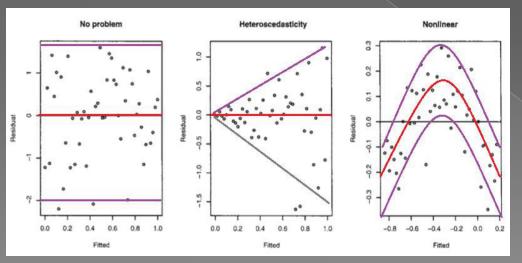
#### Goodness of Fit Test

- > A good model is determined by Goodness of fit.
- There are 3 ways to determine the same.
- These steps are to be performed sequentially.
  - 1. Overall Goodness using F-test
  - 2. Individual Parameter test
  - 3. Coefficient of determination (R2)

#### Residual Analysis

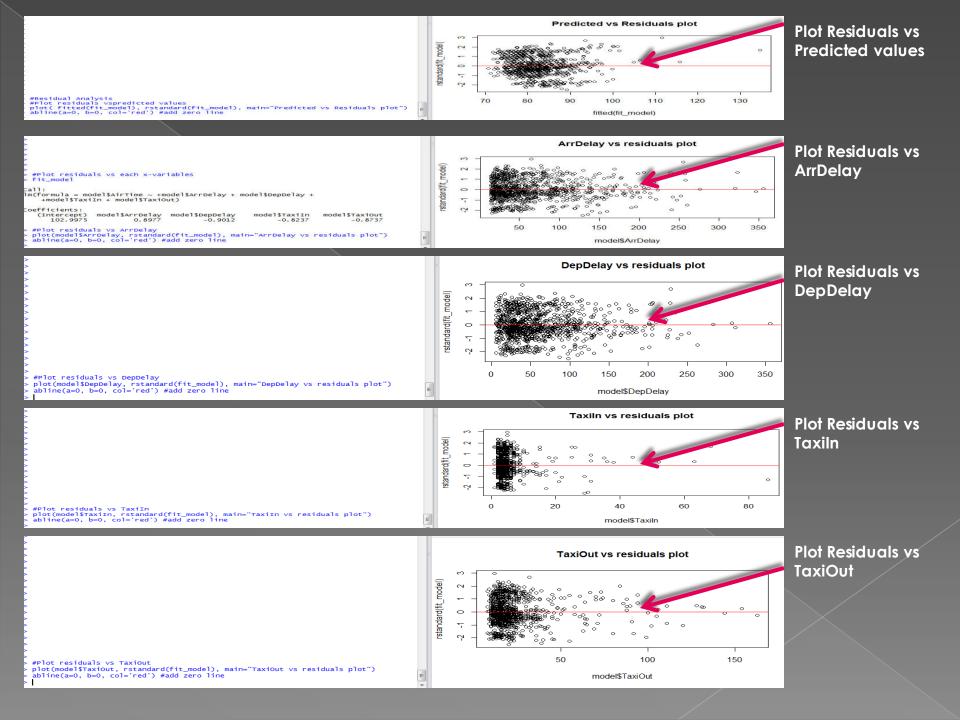
> A residual plot is a scatter plot of the residuals against the predicted value.

#### Residual = Actual value - Predicted value



#### Goals in Residual Analysis

- Validate the constant variance
- Validate the linearity relationship
- Validate normal distribution of residuals (QQ-Plot)
- Identify potential outliers (Cook's Distance)



#### QQ Plot and Outliers

```
Normal Q-Q Plot

# QQ-Plot 
qq_plot <- (rstandard(fit_model))
qqnorm(qq_plot)
qqline(rstandard(fit_model), col='blue')

Theoretical Quantiles
```

```
> #Influential Points
> #Cook's Distance
> # n=1020, 4/n = 4/1020 = 0.003
> cook <- influence.measures(fit_model)</pre>
> summary(cook)
Potentially influential observations of
         lm(formula = model$AirTime ~ model$ArrDelay + model$DepDelay +
                                                                                 model$TaxiIn + model$TaxiOut) :
     dfb.1_ dfb.m$AD dfb.m$DD dfb.m$TI dfb.m$TO dffit
                                                                    cook.d hat
                                                           cov.r
53
      0.01
              0.00
                       0.00
                                 0.00
                                          0.00
                                                   -0.03
                                                            1.04 *
                                                                     0.00
                                                                            0.03 *
63
      0.04
              0.01
                      -0.01
                                 0.01
                                         -0.06
                                                   -0.15
                                                            1.03_*
                                                                     0.00
                                                                            0.03_*
      0.00
             0.00
                       0.00
                                 0.00
                                                   0.02
66
                                          0.00
                                                            1.02 *
                                                                     0.00
                                                                            0.01
                                                            1.01_*
                                                                    0.00
72
      0.00
             0.00
                       0.00
                                 0.00
                                          0.00
                                                   0.01
                                                                            0.01
             -0.03
                       0.02
                                          0.01
                                                  -0.36_*
103
      0.05
                                -0.22
                                                            1.00
                                                                     0.03
                                                                            0.02_*
115
      0.02
             -0.02
                       0.00
                                 0.04
                                          0.03
                                                  -0.21
                                                            1.01
                                                                     0.01
                                                                            0.02_*
                                                            1.04_* 0.01
121
                       0.03
                                 0.04
                                         -0.04
                                                   -0.20
      0.01
            -0.03
                                                                            0.03_*
```

### Regression Diagnostics

- R² / Adj R² Coefficient of Determination
  - > This is a measure of goodness of fit for a linear regression model.
  - Coefficient of determination is the calculation of the variation in the dependent variable to the variation in the independent variable. 0 means no linear relationship and 1 means perfect model. Usually R<sup>2</sup> value > 80% is considered a good model.
  - From the output, it is evident that 80.18% variation in Y is explained by X.

```
Residual standard error: 2.866 on 1015 degrees of freedom
Multiple R-squared: 0.8025, Adjusted R-squared: 0.8018
```

The only difference between R<sup>2</sup> and Adjusted R<sup>2</sup> is that adjusted R<sup>2</sup> increases only when the new term added improves the model. Hence, it is more reliable than R<sup>2</sup>

#### Taboo on R<sup>2</sup>

- High R² means a better model ?
- Low R<sup>2</sup> means a bad model ?
- I found the above taboos a myth, since post execution of my model, I'm able to find Adj- R² = 1 at a point. I had to recheck my X variables.
- Since they were highly collinear, I got 1 as the Adj- R² value, which is indeed not correct.



### Multicollinearity



- Multicollinearity is an undesirable situation where the correlations among the independent variables are strong.
- When two X variables are highly collinear, be it negative or positive, they essentially convey the same information and when this happens, I found the regression results to be paradoxical.
- Situations that indicate Multicollinearity:
  - High F-test, but none of the X variables are significant.
  - Appearance of NA in the parameter estimates.

# Problems due to Multicollinearity

- Multicollinearity misleadingly inflates the standard errors of coefficients.
- Thus, it makes some variables statistically insignificant while they should be otherwise significant.
- It is like two or more people are singing loudly at the same time. One cannot discern which is which. They offset each other.





## How to detect Multicollinearity?

- ❖ Variation Inflation Factors (VIF) >= 5 indicates Multicollinearity.
- If there are two or more variables that has VIF greater than or around 5, one of the variables must be removed first.
- To determine the best one to remove, remove each one individually.
- Select the regression equation with highest R<sup>2</sup>.



- Evaluations and Predictions
  - Hold Out Evaluation (Large Data)
  - N-Fold Evaluation (Small Data)
- I performed Hold Out Evaluation.



#### Model Selection

- Search Algorithms
  - Best Subset Regression
  - Backward Elimination
  - Stepwise Regression/Forward Selection
- Model Selection Methods
  - > Adj-R<sup>2</sup>
  - Mallow's Cp Statistics
  - > AIC and BIC criterion
  - > PRESS Statistics

```
> #Selecting the best model based on Adj-R2
> summary(fit model)
Call:
lm(formula = model$AirTime ~ ++model$ArrDelav + model$DepDelav +
   ++model$TaxiIn + model$TaxiOut)
Residuals:
  Min 10 Median 30 Max
-7.1156 -1.6715 -0.3932 2.0813 8.4047
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 102.99754 0.38413 268.13 <2e-16 ***
model$ArrDelay 0.89767 0.01442 62.26 <2e-16 ***
model$DepDelay -0.90124 0.01424 -63.28 <2e-16 ***
model$TaxiIn -0.82366 0.02253 -36.56 <2e-16 ***
model$TaxiOut -0.87367 0.01522 -57.40 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '' 1
Residual standard error: 2.866 on 1015 degrees of freedom.
Multiple R-squared: 0.8025, Adjusted R-squared: 0.8018
F-statistic: 1031 on 4 and 1015 DF, p-value: < 2.2e-16
> #As Backward elimination has high Adj-R2(80.18%), so we take this model into consideration
> summary(fit model2) #final reduced model by forward selection
Call:
 lm(formula = model$AirTime ~ model$ActualElapsedTime + model$DepDelay +
    model$TaxiOut + model$ArrDelay)
 Residuals:
   Min 10 Median 30 Max
-42.783 -1.536 -0.052 1.799 17.442
 Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
               21.66587 4.35185 4.979 7.52e-07 ***
 (Intercept)
model$ActualElapsedTime 0.67723 0.04124 16.422 < 2e-16 ***
model$DepDelay 0.12173 0.04268 2.852 0.00443 **
                     -0.54940 0.01590 -34.550 < 2e-16 ***
modelSTaxiOut
model$ArrDelay -0.13474 0.04261 -3.162 0.00161 **
 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' 1
Residual standard error: 3.878 on 1015 degrees of freedom
Multiple R-squared: 0.6386, Adjusted R-squared: 0.6372
 F-statistic: 448.3 on 4 and 1015 DF, p-value: < 2.2e-16
```

```
sten/fit model2 direction="backward", trace=TRUE)
Start: AIC=2769.58
model$AirTime ~ model$ActualElapsedTime + model$DepDelay + model$TaxiOut +
   model$ArrDelay
                     Df Sum of Sq RSS AIC
<none>
                                15261 2769.6
                   1 122.3 15383 2775.7
- model$DepDelay
- model$ArrDelay
                         150.3 15411 2777.6
- model$ActualElapsedTime 1 4054.5 19315 3007.9
- model$TaxiOut 1 17947.0 33208 3560.6
Call:
lm(formula = model$AirTime ~ model$ActualElapsedTime + model$DepDelay +
   model$TaxiOut + model$ArrDelay)
Coefficients:
          (Intercept) model$ActualElapsedTime model$DepDelay
                                                              model$TaxiOut model$ArrDelay
             21.6659
                                   0.6772
                                                       0.1217
                                                                           -0.5494
                                                                                                -0.1347
> > step(fit model1,scope=list(upper=fit model2, lower=~1), direction= "forward", trace=TRUE)
Start: AIC=3619.02
model$AirTime ~ model$ActualElapsedTime
                Df Sum of Sq RSS AIC
+ model$TaxiOut 1 19493.0 15809 2801.6
+ model$ArrDelay 1 982.0 34320 3592.2
+ model$DepDelay 1 857.7 34444 3595.9
<none>
                               35302 3619.0
Step: AIC=2801.6
model$AirTime ~ model$ActualElapsedTime + model$TaxiOut
                Df Sum of Sq RSS AIC
+ model$ArrDelay 1 426.14 15383 2775.7
+ model$DepDelay 1 398.13 15411 2777.6
                               15809 2801.6
<none>
Step: AIC=2775.73
model$AirTime ~ model$ActualElapsedTime + model$TaxiOut + model$ArrDelay
                Df Sum of Sq RSS AIC
+ model$DepDelay 1 122.31 15261 2769.6
<none>
                               15383 2775.7
Step: AIC=2769.58
model$AirTime ~ model$ActualElapsedTime + model$TaxiOut + model$ArrDelay +
    model$DepDelay
Call:
lm(formula = model$AirTime ~ model$ActualElapsedTime + model$TaxiOut +
    model$ArrDelay + model$DepDelay)
Coefficients:
```

> #Backward - Stepwise

#### Model Evaluation

#### Hold Out Evaluation

```
> y1=predict.glm(fit model,test.data)
Warning message:
'newdata' had 204 rows but variables found have 1020 rows
> y2=predict.glm(fit model2,test.data)
Warning message:
'newdata' had 204 rows but variables found have 1020 rows
> y=test.data[,8]
> rmse1 <- sqrt((y-y1)%*%(y-y1))/nrow(test.data)</p>
> rmse2 <- sqrt((y-y2)%*%(y-y2))/nrow(test.data)
> rmse1
        [,1]
[1.1 1.34746
> rmse2
        [,1]
[1.] 1.28515
```

# Measuring Predictive Performance

#### Root Mean Square Error:

Best model minimizes RMSE

$$RMSE = \sqrt{\frac{\sum_{i=1}^{m} (y_i - \hat{y}_i)^2}{m}}$$

Mean Absolute Error

Best model minimizes MAE

$$MAE = \frac{\sum_{i=1}^{m} |y_i - \hat{y}_i|}{m}$$

#### Logistic Regression

- Predictor Categorical or Numeric
- Response Categorical
- Relationship between response (binary) and predictor(s)

Model for probability p=Pr(Y=1):

$$\log(\frac{p}{1-p}) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + e$$

- Based on AIC or BIC value
- In this case, AIC value is used

```
> #Try with backward selection model
> step(full,direction = "backward",trace = F)
Call: glm(formula = train_data$Wdelay ~ train_data$Month + train_data$DayofMonth +
    train_data$UniqueCarrier + train_data$FlightNum + train_data$ActualElapsedTime +
    train_data$CRSElapsedTime + train_data$ArrDelay + train_data$CarrierDelay +
    train_data$NASDelay, family = "binomial", data = train_data,
    control = list(maxit = 50)
Coefficients:
                 (Intercept)
                                          train_data$Month
                                                                    train_data$DayofMonth
                   3.5873941
                                                 -0.4126614
                                                                               -0.0202636
  train_data$UniqueCarrierMQ
                                train_data$UniqueCarrierUA
                                                                     train_data$FlightNum
                  -2.5555662
                                                 -0.6128026
                                                                                0.0005114
train_data$ActualElapsedTime
                                 train_data$CRSElapsedTime
                                                                      train_data$ArrDelay
                   0.0797740
                                                 -0.1159837
                                                                                0.0100691
     train_data$CarrierDelay
                                       train_data$NASDelay
                  -0.0361235
                                                 -0.0844027
Degrees of Freedom: 815 Total (i.e. Null); 805 Residual
Null Deviance:
                                AIC: 520.8
Residual Deviance: 498.8
```

```
> full1 <- glm(train_data$Wdelay ~ train_data$Month+train_data$ArrDelay+train_data$DepDelay+train_data$Carrier
Delay+train_data$NASDelay,data = train_data, family = "binomial", control = list(maxit = 50))
> summary(full1)
Call:
glm(formula = train_data$Wdelay ~ train_data$Month + train_data$ArrDelay +
   train_data$DepDelay + train_data$CarrierDelay + train_data$NASDelay,
   family = "binomial", data = train_data, control = list(maxit = 50))
Deviance Residuals:
   Min
            10 Median
                                    Max
-2.0153 -0.5361 -0.3468 -0.1928
                                3.2395
Coefficients:
                      Estimate Std. Error z value Pr(>|z|)
(Intercept)
                     -0.430183 0.291422 -1.476 0.139904
                     -0.433393 0.071232 -6.084 1.17e-09 ***
train data$Month
train_data$ArrDelay
                     0.081100 0.021615 3.752 0.000175 ***
                     train_data$DepDelay
train_data$NASDelav
                   -0.073820 0.022929 -3.219 0.001284 **
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
   Null deviance: 637.84 on 815 degrees of freedom
Residual deviance: 521.19 on 810 degrees of freedom
AIC: 533.19
Number of Fisher Scoring iterations: 6
```

#### McFadden's R-square

```
> mcFmodel <- glm(formula = train_data$Wdelay ~ train_data$Month + train_data$DayofMonth + train_data$UniqueCa
rrier + train_data$FlightNum + train_data$ActualElapsedTime + train_data$CRSElapsedTime + train_data$ArrDelay
+ train_data$CarrierDelay + train_data$NASDelay, family = "binomial", data = train_data, control = list(maxit
= 50))
> mcFnmodel <- glm(train_data$Wdelay ~ 1, family = "binomial")
> 1-logLik(mcFmodel)/logLik(mcFnmodel)
'log Lik.' 0.2179834 (df=11)
> |
```

#### 95% CI for the coefficients

```
> #Confidence Intervals For Model Coefficients
> confint(mcFmodel)
Waiting for profiling to be done...
                                      2.5 %
                                                  97.5 %
                              -7.0824671151 14.272594321
(Intercept)
train_data$Month
                              -0.5691316353 -0.263144833
train_data$DayofMonth
                              -0.0452549713
                                             0.004323568
                              -5.0361771094 -0.315940003
train_data$UniqueCarrierMQ
                              -1.2386408595
                                             0.023931861
train_data$UniqueCarrierUA
train_data$FlightNum
                              -0.0001104443
                                             0.001136609
train_data$ActualElapsedTime
                               0.0366785209
                                             0.126898499
train_data$CRSElapsedTime
                              -0.2250645625 -0.008808043
                               0.0057002817
                                             0.014555780
train_data$ArrDelay
train_data$CarrierDelay
                              -0.0539527266 -0.021690803
train_data$NASDelay
                              -0.1349492785 -0.038537181
```

0.97994035

1.00051151

1.01011998

 Computing exp(coefficients) to analyze change in odds for changes in X

```
> exp(coef(mcFmodel))
                                         train_data$Month
                                                                  train_data$DayofMonth
                 (Intercept)
                 36.13977707
                                               0.66188636
                                                                   train_data$FlightNum
  train_data$UniqueCarrierMQ
                               train_data$UniqueCarrierUA
                  0.07764825
                                               0.54183018
train_data$ActualElapsedTime
                                train_data$CRSElapsedTime
                                                                    train_data$ArrDelay
                  1.08304225
                                               0.89048976
     train_data$CarrierDelay
                                      train_data$NASDelay
                  0.96452116
                                               0.91906105
```

#### Change in Odds

```
> exp(confint(mcFmodel))
Waiting for profiling to be done...
                                               97.5 %
                                   2.5 %
(Intercept)
                             0.000839699 1.579461e+06
train data$Month
                             0.566016734 7.686306e-01
train_data$DayofMonth
                             0.955753761 1.004333e+00
                             0.006498544 7.291032e-01
train_data$UniqueCarrierMQ
train_data$UniqueCarrierUA
                             0.289777799 1.024221e+00
                             0.999889562 1.001137e+00
train_data$FlightNum
train_data$ActualElapsedTime 1.037359478 1.135302e+00
train_data$CRSElapsedTime
                             0.798464666 9.912306e-01
train_data$ArrDelay
                             1.005716559 1.014662e+00
train_data$CarrierDelay
                             0.947476896 9.785428e-01
train_data$NASDelay
                             0.873760229 9.621959e-01
```

#### VIF

```
> vif(mcFmodel)
           GVIF Df GVIF^{(1/(2*Df))}
       1.179723 1
                           1.086150
mon
dom
       1.020857
                           1.010375
                           1.388165
       3.713334
uq
fn
       3.336038
                           1.826482
      17.387084
                           4.169782
aet
crset 1.330932
                           1.153660
ad
       1.172485
                           1.082813
cd
       1.057806
                           1.028497
      16.854155
                           4.105381
nasd
```

Sensing how strong the predictor is

```
> n <- 1020
> set.seed(1021)
                                              > x <- 1*(runif(n)<0.5)
> n <- 1020
> x <- 1*(runif(n)<0.5)
                                              > pr <- (x==1)*0.99+(x==0)*0.01
> pr <- (x==1)*0.9+(x==0)*0.1
                                              > y <- 1*(runif(n) < pr)
> v <- 1*(runif(n) < pr)
                                              > mod <- glm(y~x, family="binomial")
> mod <- glm(y~x, family="binomial")
                                              > nullmod <- glm(y~1, family="binomial")</pre>
> nullmod <- glm(y~1, family="binomial")</pre>
> 1-logLik(mod)/logLik(nullmod)
                                              > 1-logLik(mod)/logLik(nullmod)
'log Lik.' 0.4928337 (df=2)
                                              'log Lik.' 0.8893839 (df=2)
```

```
> set.seed(1021)
> n <- 1020
> x <- 1*(runif(n)<0.5)
> x <- 1*(runif(n)>0.5)
> pr <- (x==1)*0.9+(x==0)*0.1
> y <- 1*(runif(n) < pr)
> mod <- glm(y~x, family="binomial")
> nullmod <- glm(y~1, family="binomial")
> 1-logLik(mod)/logLik(nullmod)
'log Lik.' 0.4765672 (df=2)
> |
```

Predicting values based on equation

```
> pred_dataglm <- data.frame(mon=6,dom=30,uq="AA",fn=1448,aet=101,crset=105,ad=39,cd=10,nasd=0)</pre>
> predict(mcFmodel, pred_dataglm,se.fit = TRUE,interval=c("none","confidence","prediction"), level=0.95,type="
response")
$fit
0.05490634
$se.fit
0.02068382
$residual.scale
[1] 1
> predict(mcFmodel, pred_dataglm, type="response")
0.05490634
> pred_dataglm <- data.frame(mon=6,dom=5,uq="AA",fn=580,aet=98,crset=105,ad=120,cd=0,nasd=0)
> predict(mcFmodel, pred_dataglm, type="response")
0.1364101
> pred_dataglm <- data.frame(mon=1,dom=3,uq="UA",fn=602,aet=102,crset=103,ad=24,cd=0,nasd=0)
> predict(mcFmodel, pred_dataglm, type="response")
0.3189236
> pred_dataglm <- data.frame(mon=5,dom=2,uq="MQ",fn=3908,aet=129,crset=100,ad=160,cd=0,nasd=29)</pre>
> predict(mcFmodel, pred_dataglm, type="response")
0.2283462
> pred_datag1m <- data.frame(mon=3,dom=28,uq="UA",fn=622,aet=95,crset=106,ad=42,cd=0,nasd=0)</pre>
> predict(mcFmodel, pred_dataglm, type="response")
0.05702596
```