

Ans 1

Multinomial $(20, 0.3, 0.4, 0.1)$

$$0.3 + 0.4 + 0.1 = 0.8 \neq 1$$

So there are 4 components in

Multinomial $(20, 0.3, 0.4, 0.1)$

With $p_1 = 0.3, p_2 = 0.4, p_3 = 0.1, p_4 = 0.2$

Ques 2

Let $f_i(x)$ $i = 1, 2, 3, 4$ be four

bivariate gaussian distributions with mean $\mu_1, \mu_2, \mu_3, \mu_4$ & covariance matrix corresponding to them is $\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4$

Define hidden variable

$$z_i = \begin{cases} i & \text{if } x_i \sim f_i \\ 0 & \text{o.w} \end{cases}$$

$$P(x_i < X_i < x_i + \Delta x_i, z_i | \theta)$$

$$P(x_i < X_i < x_i + \Delta x_i | z_i, \theta) P(z_i = z_i)$$

$$\begin{cases} p_1 f_1(x_i) & x_i \sim f_1 \\ p_2 f_2(x_i) & x_i \sim f_2 \\ p_3 f_3(x_i) & x_i \sim f_3 \\ p_4 f_4(x_i) & x_i \sim f_4 \end{cases} \quad p_1 + p_2 + p_3 + p_4 = 1$$

Now

$$L(\theta) = p_1 f_1(x_i) I(z_i=1) + p_2 f_2(x_i) I(z_i=2) + p_3 f_3(x_i) I(z_i=3) + p_4 f_4(x_i) I(z_i=4)$$

$$L(\theta) = [p_1 f_1(x_i)]^{I(z_i=1)} [p_2 f_2(x_i)]^{I(z_i=2)} [p_3 f_3(x_i)]^{I(z_i=3)} [p_4 f_4(x_i)]^{I(z_i=4)}$$