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End Term Examination, Even Semester 2016-17

B. Tech. I-Year, II-Semester

AHM 2101: Engineering Mathematics – II

Time: 2½ Hours **Max. Marks: 40**

Section – A

Note: Attempt ALL Questions. **(1x16=16)**

I. Test the convergence of infinite series

$$\log\frac{1}{2}-\log\frac{2}{3}+\log\frac{3}{4}-\log\frac{4}{5}+\ldots$$

II. Test the convergence of the infinite series

$$2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\ldots\ldots$$

III. State the logarithmic test for infinite positive term series.

IV. What is comparison test for determining the convergence or divergence of an infinite series? How is this test related to p - series test (or hyper harmonic test)?

V. Test the convergence of geometric series

$$1+r+r^2+r^3+\ldots \quad \text{if } -1<r<1.$$

Section – B

Note: Attempt Any FOUR Questions. (3x4=12)

I. Find the work done by the force $\vec{F} = \cos x\,\hat{i} - y\,\hat{j} + xz\,\hat{k}$ in moving a particle along curve $x = t,\ y = -t^2,\ z = t^3$ from $t = 0$ to $t = 1$.

II. Find directional derivative of $\phi(x, y, z) = x^2 y - z + \sin z$ at point P $(1,1,0)$ in the direction of point Q $(2,0,1)$.

III. Discuss the physical meaning of curl of vector function.

IV. Find Fourier series for $f(x) = x^2, -\pi \le x \le \pi; f(x + 2\pi) = f(x)$

Hence deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{12}$

V. Obtain half range Fourier cosine series of $f(x) = x, 0 < x < \pi,$

$f(x + 2\pi) = f(x).$ Hence evaluate $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$

Section – C

Note: Attempt Any THREE Questions. (4x3=12)

I. A fluid motion is given by $\vec{V} = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$.

(a) Is this motion irrotational? If so, find the velocity potential.

(b) Is the motion possible for an incompressible fluid?

VI. Test the convergence of $\sum (-1)^{n+1} \frac{1}{n}$

VII. Evaluate the integral $\int_0^1 \int_0^2 \int_0^1 x^2 y^3 \sqrt{z}\, dx\, dy\, dz$

VIII. Find: $\{\text{grad } f(r)\} \times \vec{r}$ where f (r) is any function of r and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

IX. Evaluate : $\int_0^\infty e^{-x} x^{2n+1} dx$

X. Convert the integral $\int_0^2 \int_{y=0}^{\sqrt{2x-x^2}} \int_0^{a-x^2-y^2} (x^2 + y^2)\, dz\, dy\, dx$ to cylindrical coordinates.

XI. Evaluate $\int_0^{\pi/2} \sin^4\theta . \cos^3\theta \; d\theta$ using Beta and/or Gamma function.

XII. Prove that $\Gamma(n+1) = n\Gamma(n)$

XIII. Find the value of constant a if vector $\vec{F} = x\hat{i} + ay\hat{j} + z\hat{k}$ is solenoidal.

XIV. Find unit normal vector to the surface $x^2 + y^2 + \sin z = 14$ at point $(1, 1, \pi)$.

XV. Find Fourier coefficient a_0 in the expansion of the function $f(x) = x^2, -\pi \le x \le \pi$, $f(x + 2\pi) = f(x)$.

XVI. Show that $\nabla|\vec{r}| = \frac{\vec{r}}{|\vec{r}|}$, $\vec{r}$ being position vector.

II. State the Green's theorem and verify it for $\bar{F}=e^{x}\left(\sin y\,\hat{i}+\cos y\,\hat{j}\right)$ around the curve C, where C is the boundary of the rectangle with vertices $(0,0),(2,0),\left(2,\frac{\pi}{2}\right),\left(0,\frac{\pi}{2}\right)$.

III. Verify Gauss' Divergence Theorem for $\bar{F}=4xz\hat{i}-y^{2}\hat{j}+yz\hat{k}$ taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1,

z = 0 and z = 1.

IV. Find Fourier series for the function

$$f(x)=\begin{Bmatrix} 0, -\pi \le x < 0 \\ \sin x, 0 < x \le \pi \end{Bmatrix}; f(x+2\pi)=f(x)$$

Hence, deduce that

$$\frac{\pi-2}{4}=\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\dots$$

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END Term, Even Semester Examination, 2018–2019

Sub.:- Engineering Mathematics II (BMAS–0102)

Time: - 3 Hrs. **Course:- B. Tech. II Sem.** **Max. Marks:- 50**

Note - Attempt **BOTH** Sections. The terms have their usual meanings.

SECTION A (7 x 5 = 35 marks)

Note: ALL questions of this section are **COMPULSORY.** Each question of this section is of **Five marks.**

Q.1. Change the variables x, y, z to r, θ, φ by the equations,

$$x = r\sin\theta\cos\varphi,$$
$$y = r\sin\theta\sin\varphi,$$
$$z = r\cos\theta,$$

and evaluate the following integral

$$\iiint \frac{dx\,dy\,dz}{(x^2+y^2+z^2)}$$

taken throughout the volume of the sphere

$$x^2+y^2+z^2=4$$

Q.2. Evaluate using Beta and Gamma functions:

(a) $\int_0^2 x\,(8-x^3)^{\frac{1}{3}}\,dx$ (b) $\int_0^\infty \frac{dx}{1+x^4}$

Q.3. Examine the convergence of the following infinite series:

$$\sum_{n=1}^{\infty} \frac{1^2.3^2.5^2 \ldots\ldots (2n-1)^2}{2^2.4^2.6^2 \ldots\ldots (2n)^2}\, x^{n-1}$$

Q.4. Show that the vector

$$\vec{A} = (6\,xy + z^3)\,\hat{\imath} + (3\,x^2 - z)\,\hat{\jmath} + (3xz^2 - y)\hat{k}$$

is irrotational. Also, find the scalar potential φ such that

$$\vec{A} = \nabla\varphi$$

Q.5. Prove that: div (grad r^n) = $\nabla^2 r^n$ = n (n + 1) r^{n-2}

where, $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ and $r = |\vec{r}|$. Also show that,

$$\nabla^2\left(\frac{1}{r}\right) = 0$$

Q.6. Verify Gauss' Divergence theorem for

$$\vec{F} = 4xz\,\hat{\imath} - y^2\hat{\jmath} + yz\,\hat{k}$$

taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1.

Q.7. Solve the partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x\,\partial y} = y\cos x$$

OR,

$$\left(D^2 + 2DD' + D'^2 - 2D - 2D'\right) z = \sin(x + 2y)$$

SECTION B (15 marks)

Note: Attempt **ALL** questions. Marks are shown against them.

Q.1. Solve the partial differential equation: (2)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 2\cos x \cos y$$

Q.2. If $\overrightarrow{V_1}$ and $\overrightarrow{V_2}$ are the vectors joining the fixed points

(x_1, y_1, z_1) and (x_2, y_2, z_2) respectively to a variable point

(x, y, z); prove that,

$$div\left(\overrightarrow{V_1} \times \overrightarrow{V_2}\right) = 0. \qquad (2)$$

Q.3. Solve the partial differential equation:

$$(D - D' - 2)(D - D' - 1)\, z = e^{x+2y} \qquad (2)$$

where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.

Q.4. Solve the Lagrange's partial differential equation:

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. (3)

Q.5. Verify Stoke's theorem for

$$\vec{F} = x^2\,\hat{\imath} + xy\,\hat{\jmath}$$

integrated round the square whose sides are x = 0, y = 0,

x = a, y = a in the plane z = 0. (3)

Q.6. Find the work done in moving a particle in the force field

$$\vec{F} = 3x^2\,\hat{\imath} + (2xz - y)\,\hat{\jmath} + z\,\hat{k}$$

along the curve defined by

$$x^2 = 4y, 3x^3 = 8z$$

from x = 0 to x = 2. (3)

B.Tech I year.

Course Name:

Course Outcome

CO1- Know the rank of a matrix and its applications in solving systems of linear equations.
CO2-Understand complex matrices.
CO3-Find the Eigen values and Eigen vectors of a square matrix.
CO4-Solve ordinary and partial differential equations of higher orders.
CO5-Classify the linear partial differential equations as elliptic, parabolic, and hyperbolic.
CO6- Solve the linear differential equations of second order in a series.

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End Term Examination, Even Semester 2021-22

B.Tech., I Year, II Semester

Subject Code & Subject Name- BMAS 1102,Engg. Mathematics II

Time: 3 Hours **Maximum Marks: 50**

Instruction for students:

(1) Attempt all the sections.

(2) Marks of the questions and internal choice are indicated in each section.

Section – A

Attempt All Questions $4 \times 5 = 20$ Marks

No.	Detail of Question	Marks	CO	BL	KL
1	Solve the following system of equations (if possible) using the Gauss elimination method. $$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$	4	1	A	C
2	Evaluate the Eigen values and corresponding Eigen vectors of the following matrix. $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$	4	3	E	P
3	The forced oscillations of a mechanical system with periodic input are governed by the non-homogeneous equation $2\frac{d^2y}{dx^2} + c\frac{dy}{dx} + 8y = 8\cos\omega t$, where $c > 0$. Obtain its general solution when $c = 0$ (force un-damped solution).	4	4	C	F
4	Solve $(D^2 - 2DD' + D'^2)z = 12xy$, where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$.	4	4	A	C
5	Classify the following partial differential equation: $t\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + x\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial x} = 0$ Or Classify the singular points of the following equation. $x^2\frac{d^2y}{dx^2} + ax\frac{dy}{dx} + by = 0, a, b\ constants.$	4	5 OR 6	U	C

Section – B

Attempt All Questions 3 × 5 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
1	Find the solution of the following bi-harmonic partial differential equation. $(D^4 - 2D^2D'^2 + D'^4)z = 0, where D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}.$	3	4	An	C
2	Determine the Particular integral of $(D^3 - 10D^2D' + D'^3)z = \cos(2x + 3y), where D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}.$	3	4	A	P
3	Define ordinary and singular points of the differential equation $P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0.$	3	6	R	F
4	Write a short note on the application of partial differential equations in real life.	3	5	M	C
5	Find the power series solution about $x = 0$, of the differential equation $y'' - 4y = 0$.	3	6	An	C

Section – C

Attempt All Questions 5 × 3 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
1	Solve $\frac{d^2y}{dx^2} - 2x^2\frac{dy}{dx} + 4xy = x^2 + 2x + 2$, in powers of x.	5	6	A	P
2	Solve $(2D^2 - DD' - 3D'^2)z = 5\frac{e^x}{e^y}, where D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}.$	5	4	A	C
3	Use separation of variable technique to evaluate $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ with $u(x, 0) = 4e^{-x}$.	5	4	E	P

CO – Course Outcome, BL – Abbreviation for Bloom's Taxonomy Level (R-Remember, U-Understand, A-Apply, An-Analyze, E-Evaluate, C-Create), KL – Abbreviation for Knowledge Level (F-Factual, C-Conceptual, P-Procedural, M-Metacognitive). However, For Engg. Courses in addition to F, C, P & M include D-Fundamental Design Principles, S-Criteria and Specifications, PC-Practical Constraints, DI-Design Instrumentalities