University Roll No. ____________________

END TERM EXAMINATIONS, 2016 – 2017

B. Tech. I Semester

Sub. Name: Engg. Mathematics - I **Sub. Code: AHM – 1101**

Time: 2 Hrs. 30 Minutes **Max. Marks: 40**

Note - Attempt ALL Sections.

SECTION A (1 x 16 = 16 marks)

Note: ALL questions of this section are **COMPULSORY.** Here $D = \frac{d}{dx}$

Q.1. $y^2(a-x) = x^2(a+x)$ is the equation of strophoid. What are its tangents at the origin?

Q.2. Assume that J_1 is the Jacobian of u, v with respect to x, y and J_2 is the Jacobian of x, y with respect to u, v. Can you tell, how are J_1 and J_2, related with each other?

Q.3. If $u = x^2 + y^2 + z^2$, show that $x\,u_x + y\,u_y + z\,u_z = 2$.

Q.4. If $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{1.44} \end{bmatrix}$, find the rank of A.

Q.5. Two eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find the eigen values of A^{-1}.

Q.6. If the characteristic equation of a matrix A is $\beta^2 + \beta + 1 = 0$, Form its matrix equation using Cayley – Hamilton theorem. Here β is a scalar.

Q.7. Define: (i) Hermitian matrix (ii) Singular matrix

Q.8. If $x = u(1 + v)$ and $y = v(1 + u)$ find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$

Q.9. If inverse of $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ is $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & k \\ 2 & 2 & 5 \end{bmatrix}$ then find the constant k.

Q.10. Examine the linear dependence of the following vectors:

$X = [1, 1, 1, 1]$, $Y = [0, 1, 1, 1]$

$Z = [0, 0, 1, 1]$ and $W = [0, 0, 0, 1]$

Q.11. Find the order and degree of the differential equation

$$(x + y)(dx - dy) = dx + dy$$

Q.12. An uncharged condenser of capacity C is charged by applying e.m.f. $E \sin \frac{t}{\sqrt{LC}}$ through leads of self – inductance L and negligible resistance. Determine the differential equation for the circuit, mentioning the initial conditions clearly.

Q.13. Find the value (s) of 'b' for which the differential equation

$(xy^2 + bx^2y)\,dx + (x^3 + x^2y)\,dy = 0$ remains exact.

Q.14. A differential equation whose auxiliary equation has the roots

$\pm 2i$ and $\pm 3i$, is written as $\frac{d^4y}{dx^4} + A\frac{d^2y}{dx^2} + By = Q$, where Q is any

function of x. Find the constants A and B.

Q.15. Solve the differential equation: $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$

Q.16. Find the periodic time of the motion described by the linear

differential equation $(D^2 + \mu^2)\,x = 0$, μ is a non – zero constant.

SECTION B **(3 x 4 = 12 marks)**

Note : Attempt any **FOUR** questions. Here $D = \frac{d}{dx}$

Q.1. Solve by the method of variation of parameters :

$$(D^2 + 1)\,y = \operatorname{cosec} x$$

Q.2. Solve the following system of simultaneous differential equations:

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

It is given that x = 2 and y = 0 when t = 0

Q.3. Solve the differential equation: $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log_e x}{x^2}$

Q.4. Solve the differential equation: $(D^2 + D + 1)y = (1 + e^x)^2$

Q.5. Test for exactness and hence solve the differential equation:

$$(2\,y^2 + x + x^8)\,dx + (y^8 - y + 4\,x\,y)\,dy = 0$$

SECTION C **(4 x 3 = 12 marks)**

Note : Attempt any **THREE** questions. Here $D = \frac{d}{dx}$

Q.1. Solve the following differential equation:

$$x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = x + \log_e x$$

Q.2. Solve the following system of simultaneous differential equations:

$$(D+3)x + Dy = \sin t$$

$$(D-1)x + y = \cos t$$

Q.3. Solve the following differential equations:

(a) $(D^2 - 2D + 1)\,y = x\,e^x \cos x$

(b) $(D^4 - n^4)\,y = \cos n\,x + e^{nx}$; n is a non – zero constant

Q.4. Test for exactness and hence solve the differential equation:

$$2\,y\,dx + x\,(2\,\log_e x - y)\,dy = 0$$

University Roll No. : ________________

END Term Examination, 2017 – 2018

Sub.:- Engineering Mathematics - I (AHM–1201)

Time: - 3 Hrs. **Course:- B. Tech. I Sem.** **Max. Marks:- 50**

Note - Attempt **BOTH** Sections.

SECTION A (7 x 5 = 35 marks)

Note: ALL questions of this section are **COMPULSORY.**

Q.1. Consider the curve:

$$(x-y)^2(x^2+y^2)-10(x-y)x^2+12y^2+2x+y=0$$

(a) Find asymptotes of this curve parallel to coordinate axes. (1)

(b) Find oblique asymptotes of the given curve. (2)

(c) Check if the given curve passes through the origin? If yes, then find the tangent (s) at the origin. (2)

Q.2. If $u = x\phi\left(\frac{y}{x}\right)+\psi\left(\frac{y}{x}\right)$, find:

(a) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ (2)

(b) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ (3)

Q.3. Solve the following system of equations by matrix method:

$$\begin{bmatrix} 2 & 1 & 2 & 1 \\ 6 & -6 & 6 & 12 \\ 4 & 3 & 3 & -3 \\ 2 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6 \\ 36 \\ -1 \\ 10 \end{bmatrix} \qquad (5)$$

Q.4. (a) Solve the differential equation: (3)

$$(xy^2 \sin xy + y\cos xy)\, dx + (x^2 y\sin xy - x\cos xy)\, dy = 0$$

(b) Show that matrix A has one repeated eigen value.
Find the Eigen vector corresponding to that eigen value:

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \qquad (2)$$

OR,

Q.4. Solve the differential equations:

(a) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 4x + 2^x$ (3)

(b) $\frac{d^6y}{dx^6} - \frac{d^4y}{dx^4} - x^2 = 0$ (2)

Q.5. Apply the method of variation of parameters to solve:

(i) $\frac{d^2y}{dx^2} + y = x$ (2)

(ii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ (3)

Q.6. Solve the Euler – Cauchy differential equation: (5)

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$$

Q.7. An uncharged condenser of capacity C is charged by applying an e. m. f. $E\sin\frac{t}{\sqrt{LC}}$ through leads of self–inductance L and negligible resistance. Prove that, at time t, the charge on one of the plates is

$$\frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}}-\frac{t}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}\right] \quad (5)$$

SECTION B (15 marks)

Note : Attempt ALL questions. Marks are shown against them.

Q.1. Solve the differential equation: $\frac{d^2x}{dt^2}+x=0$ given that

x = 2 at t = 0 and also, $x=-2$ at t = $\frac{\pi}{2}$. (2)

Q.2. A particle begins to move from a distance 'a' towards a fixed point which repels it with retardation $\mu^2 x$. If its initial velocity is aμ, show that it will continually approach the fixed centre but will never reach it. (2)

Q.3. Solve the differential equation: (2)

$$\frac{d^2y}{dx^2}+4y=e^x+\sin 2x$$

Q.4. Solve the differential equation: (3)

$$\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}=n^2y$$

Q.5. Solve by changing the independent variable: (3)

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2$$

Q.6. Solve the following system of simultaneous differential

equations:

$$\frac{dx}{dt}=-4(x+y)$$
$$\frac{dx}{dt}+4\frac{dy}{dt}=-4y$$

with conditions

$x(0)=1$ and $y(0)=0$ (3)

Printed Pages: 03 University Roll No: ....

End Term Examination, Odd Semester 2018-19

B.Tech. I Year I Semester,

Engg. Mathematics I (BMAS-0101)

Time: 3 Hour Maximum Marks: 50

Section-A **7 X 5=35 Marks**

Note: Attempt All questions

1. Trace the curve $y^2 = x^3$.

OR

If $v = \log_e \sin\left[\dfrac{\pi\left(2x^2 + y^2 + xz\right)^{\frac{1}{2}}}{2\left(x^2 + xy + 2yz + z^2\right)^{\frac{1}{3}}}\right]$, prove that when x= 0,

y=1, z=2; $x\dfrac{\partial v}{\partial x} + y\dfrac{\partial v}{\partial y} + z\dfrac{\partial v}{\partial z} = \dfrac{\pi}{12}$.

2. If $u = x^2 + y^2 + z^2, v = x + y + z, w = xy + yz + zx$, then show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = 0$. Is u, v, w functionally related ? If so find the relation between them.

3. Find the values of a and b if the equations: x + y + 2 z =2, 2x-y+3z=2, 5x-y+az=b have (i) no solution (ii) unique solution and (iii) infinite number of solutions.

4. Use Cayley - Hamilton theorem to find the matrix

$A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 - 2A + I$, if the matrix

$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

5. Solve the following by the method of variation of parameters:

$\frac{d^2y}{dx^2} + 1 = x$.

6. Solve the following simultaneous differential equations:

$\frac{d^2y}{dt^2} + x = \cos t$ and $\frac{d^2x}{dt^2} + y = \sin t$

7. Solve the following homogeneous differential equation

$x^3\frac{d^3y}{dx^3} - 3x\frac{dy}{dx} + 3y = 16x + 9x^2 \log_e x, x > 0$.

Section-B

Note: Attempt All questions. **3 x 2 = 6 Marks**

1a. Solve the differential equation $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{-2x} + e^{3x}$.

1b. Find the solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = \sin 3x \cos 2x$.

1c. Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$.

Note: Attempt All questions. **3 x 3 = 9 Marks**

2a. Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2e^{2x} + \sin^2 x.$

2b. $\frac{d^2y}{dx^2} + 4y = \tan 2x$

2c. A spring for which the spring constant k is 700 Newton per meter hangs in a vertical position with its upper end fixed to a support. A mass of 20 kg is attached to the lower end and system brought to rest. Find the position of the mass at time t, if a force 70 sin 2t Newton is applied to the support.

Printed Pages: **University Roll No: ……………**

End Term Examination, Odd Semester 2019-20

B.Tech. I Year (I Semester)

BMAS 0101: Engineering Mathematics I

Time: 3 Hour **Maximum Marks: 50**

Section- A

Note: Attempt ANY FIVE Questions. (5 x 4 = 20 Marks)

1) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree.

2) If $u = x + 2y + z$, $v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$;
Show that u, v, w are not independent. Find the relation between them.

3) Determine the values of λ and μ such that the system
$2x - 5y + 2z = 8, 2x + 4y + 6z = 5, x + 2y + \lambda z = \mu$;
Has (i) no solution (ii) a unique solution (iii) infinite number of solutions.

4) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

5) Solve the differential equation
$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$

6) Solve the simultaneous differential equations
$\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$; given that $x = 2, y = 0$ when $t = 0$.

Section- B

Note: Attempt ALL Questions. (5 x 3 = 15 Marks)

1) Solve the differential equation $\frac{d^2y}{dx^2} + a^2 y = \sec ax$.

2) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, then using Cayley-Hamilton theorem find the value of

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

3) Solve the differential equation

$$(y \sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$$

4) Solve the differential equation $(D^3 - 3D^2 + 3D - 1)y = \sinh x + 5\log 2$

5) Solve the differential equation $(D^2 - 4D + 4)y = 8(e^{2x} + \sin 2x)$

Section- C

Note: Attempt ANY THREE Questions. (3 x 5 = 15 Marks)

1) Solve the differential equation by changing the independent variable

$$\frac{d^2y}{dx^2} + (3 \sin x - \cot x)\frac{dy}{dx} + 2y \sin^2 x = e^{-\cos x} \sin^2 x$$

2) An uncharged condenser of capacity C is charged by applying an e.m.f. $E \sin\frac{t}{\sqrt{LC}}$, through leads of self-inductance L and negligible resistance. Prove that at time t, the charge on plates is $\frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos\frac{t}{\sqrt{LC}}\right]$.

3) Solve the differential equation $(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0$.

4) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

Hence find A^{-1}.

Course Name: MATRICES, DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Course Outcome

CO1- Know the rank of a matrix and its applications in solving systems of linear equations
CO2- Find the Eigen values and Eigen vectors of a square matrix
CO3- Solve linear ordinary and partial differential equations of higher orders
CO4- Classify the linear partial differential equations as elliptic, parabolic and hyperbolic
CO5- Apply Laplace transform to Engineering problems using properties
CO6- Apply Inverse Laplace transform to Engineering problems

Printed Pages: 4 **University Roll No. ....**

End Term Examination, Even Semester 2022-23
B.Tech. Year-I, Semester-II
Subject Code - BMAS 1105
Subject Name- MATRICES, DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Time: 3 Hours **Maximum Marks: 50**

Instruction for students:

Attempt All Sections.

Section – A

Attempt All Questions. 4 X 5 = 20 Marks

No.	Detail of Question	Marks	CO	BL	KL
1	A matrix of an image with positive and negative values is presented as: $$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$ In this matrix, the positive values will be given in nuances from black to white, and negative – from black to yellow. Scaling may be done according to the biggest matrix elements magnitude. Find Eigen values and Eigen vectors for this matrix of an image.	4	2	An	F
2	Consider the following boundary value problem. $(\cos x - x\sin x + y^2)dx + 2xy\,dy = 0;\ y(\pi) = a$ Determine its particular solution, if possible.	4	3	E	C

3	Assume that an object weighing 2 lb. stretches a spring 6 inches. If the spring is released from the equilibrium position with an upward velocity of 16 ft. /sec and the motion of the spring is governed by the differential equation, $$\frac{1}{16}\frac{d^2x}{dt^2}+4x=0$$ Can you determine the period of the motion?	4	3	U	M
4	Show that the following differential equation $$\frac{\partial^2 z}{\partial v^2}+2v\frac{\partial^2 z}{\partial u \partial v}+(1-u^2)\frac{\partial^2 z}{\partial u^2}=0$$ is, (i) elliptic for the values of u and v in the region $u^2+v^2<1$, (ii) parabolic on the boundary of the circular region $u^2+v^2=1$, and (iii) hyperbolic in the region $u^2+v^2>1$.	4	4	An	C
5	The following function f (p) is the image of an object function F (t) in the frequency domain. $$f(p)=\frac{6}{2p-3}-\frac{3+4p}{p^2-16}+\frac{8-6p}{16p^2+9}$$ What will be the object function of this image in the time domain? Use Inverse Laplace transform.	4	6	A	P

Section – B

Attempt All Questions.

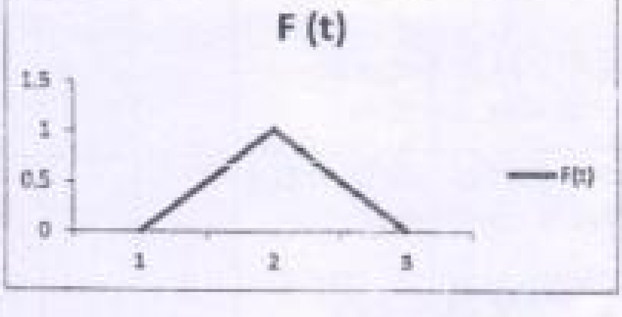
3 X 5 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
6	Combining complementary function and particular integral while finding the solution of a linear partial differential equation with constant coefficients and then equating the sum to the dependent variable, we get its complete solution. Hence solve the following: $$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=e^{3x+2y}.$$	3	3	E	C

7	Classify the following partial differential equation as hyperbolic, parabolic or elliptic. $$u_{xx} - 2u_{xy} + u_{yy} = 0$$ The terms have their usual meanings.	3	4	An	F
8	State second shifting property of Laplace transform and apply it to find the Laplace transform of $F(t) = \begin{cases} (t-1)^2, & t > 1 \\ 0, & 0 < t < 1 \end{cases}$	3	5	R	P
9	Evaluate the real integral using the technique of Laplace transform: $$\int_0^\infty \frac{\sin t}{t} dt$$ Hence or otherwise, find: $$\int_0^\infty e^{-t} \frac{\sin t}{t} dt$$ **OR,** A periodic function of period 2 is given as follows: $$F(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$ Obtain its image $f(p) = L[F(t)]$ in frequency domain. L is Laplace transformation operator.	3	5	A	F
10	A non-homogeneous linear partial differential equation is given below. Solve it completely. $$(D^2 - DD' + D' - 1)\, z = \cos(x + 2y)$$ The terms have their usual meanings.	3	3	A	P

Section – C

Attempt All Questions. 5 X 3 = 15 Marks

No.	Detail of Question	Marks	CO	BL	KL
11	Discuss the nature and solve the linear partial differential equation: $$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x\ \cos 2y + (2x + y)^2$$ **OR,** Apply convolution theorem to show that: $$\int_0^t u^2 \cos(t-u)\ du = 2(t - \sin t)$$	5	4	E	C
12	What will be the inverse Laplace transform of the following function? $$f(p) = log\left(1 + \frac{1}{p^2}\right)$$ Can you regain this function f (p) from the obtained result? If yes, then obtain it also using Laplace transform. **OR.** If $\frac{\partial^2 z}{\partial x^2} = r$, $\frac{\partial^2 z}{\partial y^2} = t$, and $\frac{\partial^2 z}{\partial x \partial y} = s$, then find the complete solution of the following linear differential equation of II order: $$r + s - 6t = y \cos x$$	5	6	A	P
13	Express the function F (t) , shown in the diagram below, in terms of unit step function and obtain its Laplace transform: 	5	5	C	M