First Term Examination, 2016 – 2017

B. Tech. II Semester

Sub. Name: Engg. Mathematics - II **Sub. Code: AHM – 2101**

Time: 1 Hr. 30 Min. **Max. Marks: 20**

Note - Attempt ALL Sections.

SECTION A (1 x 5 = 5 marks)

Note: ALL questions of this section are **COMPULSORY.**

Q.1. Consider the following series:

(i) $1 - 2 + 3 - 4 + \ldots..$ (ii) $1 - (1/2) + (1/3) - (1/4) + \ldots\ldots.$

(iii) $1 - 1 + 1 - 1 + 1 - 1 + \ldots\ldots..$

In which of the above series, is the Leibnitz test for testing the convergence of series applicable? Justify your answer.

Q.2. Find the nth term of the series:

$$\frac{\alpha.\beta}{1.\gamma}x + \frac{\alpha(\alpha+1).\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2 + \frac{\alpha(\alpha+1)(\alpha+2).\beta(\beta+1)(\beta+2)}{1.2.3.\gamma(\gamma+1)(\gamma+2)}x^3 + \ldots..$$

Q.3. Test the convergence of the series: $1 + 2 + 3 + 4 + \ldots..$

Q.4. State Logarithmic test for finding the nature of a positive term infinite series.

Q.5. Test whether we can apply Cauchy condensation test in determining the convergence of the following series?

$$\frac{(\log 2)^2}{2^2} + \frac{(\log 3)^2}{3^2} + \frac{(\log 4)^2}{4^2} + \ldots\ldots + \frac{(\log n)^2}{n^2} + \ldots.$$

If yes, write your observations.

SECTION B (2 x 3 = 6 marks)

Note : Attempt any **THREE** questions.

Q.1. Test the convergence of the series: $\frac{|\underline{2}}{3}+\frac{|\underline{3}}{3^2}+\frac{|\underline{4}}{3^3}+......$

Q.2. Prove that the series $\sum_{1}^{\infty}(1+\frac{1}{n})^{n^2}$ is divergent.

Q.3. Test the convergence of the series: $\frac{14}{1^3}+\frac{24}{2^3}+\frac{34}{3^3}+.........+\frac{10n+4}{n^3}+.....$

Q.4. Prove that the series $\frac{2^p}{1^q}+\frac{3^p}{2^q}+\frac{4^p}{3^q}+.......$ is convergent if q > p +1 and divergent otherwise. Here p and q are positive numbers.

SECTION C (3 x 3 = 9 marks)

Note : Attempt any **THREE** questions.

Q.1. Test the convergence of the series: $\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+........$

Q.2. Prove that the series $1+\frac{2^2}{3^2}+\frac{2^2.4^2}{3^2.5^2}+\frac{2^2.4^2.6^2}{3^2.5^2.7^2}+......$ is divergent.

Q.3. Prove that the series

$$1+\frac{\alpha+1}{\beta+1}+\frac{(\alpha+1)(2\alpha+1)}{(\beta+1)(2\beta+1)}+\frac{(\alpha+1)(2\alpha+1)(3\alpha+1)}{(\beta+1)(2\beta+1)(3\beta+1)}+......,\alpha>0,\beta>0$$

converges if $\beta>\alpha>0$ and diverges if $\alpha\geq\beta>0$

Q.4. Test the convergence of the series: $\frac{x}{1}+\frac{1}{2}.\frac{x^3}{3}+\frac{1.3}{2.4}.\frac{x^5}{5}+\frac{1.3.5}{2.4.6}.\frac{x^7}{7}+.....$

Printed Pages: 02 University Roll No....................

I Term Examination, Even - Semester, 2018-19

Program:- B. Tech Year:- I Sem:- II

Subject:-Engineering Mathematics-II Code:- BMAS-0102

Time: - 60 Min. Max. Marks: 15

Section- A

Note: Attempt ALL Questions. **(2x3=6 Marks)**

Q.1. Using Beta and Gamma functions, evaluate

(a) $\int_0^\infty x^{\frac{1}{4}} e^{-\sqrt{x}} dx$

(b) $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{\frac{1}{2}} dx$

Q.2. Test the convergence of the series:

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\dots$$

Q.3. Test the series whether it is convergent or divergent.

$$\sum_{n=1}^{\infty}\left(\frac{1+nx}{n}\right)^n$$

Section- B

Note: Attempt ALL Questions. (3x3=9 Marks)

Q.1. Test the series for convergence and divergence:

$$\frac{1^2}{4^2}+\frac{1^2.5^2}{4^2.8^2}+\frac{1^2.5^2.9^2}{4^2.8^2.12^2}+\frac{1^2.5^2.9^2.13^2}{4^2.8^2.12^2.16^2}+...$$

Q.2. Test the series for convergence and divergence:

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+...$$

Q.3. (a) Using Gamma function, prove that:

$$\int_0^{\frac{\pi}{2}}\sqrt{\tan\theta}\ d\theta=\frac{\pi}{\sqrt{2}}$$ (2 Marks)

(b) State De Morgan and Bertrand's test. (1 Mark)

Printed Pages: 03 University Roll No. :- ____________________

Mid Term Examination, 2019 - 20
B. Tech. I Year II Semester

Subject Name and Code: Engineering Mathematics II (BMAS 0102)

Time: 2 Hrs. Note: Attempt **ALL** sections. **Total Marks: 30**

SECTION A

(2 x 3 = 6 marks)

Instructions: This section has **3** questions. Attempt **ALL** questions. The symbols have their usual meanings.

Q.1. What is Leibnitz test? Test the following infinite series for convergence: (2)

$$\frac{2}{1^3} - \frac{3}{2^3} + \frac{4}{3^3} - \frac{5}{4^3} + \cdots$$

Q.2. Use Beta and Gamma functions to evaluate the integral:

$$\int_0^1 x^5 \left(1 - x^3\right)^{10} dx$$ (2)

Q.3. Show that: (2)

$$\iint_R r^2 \sin\theta \, dr \, d\theta = \frac{2a^3}{3}$$

where R is the region bounded by the semi-circle $r = 2a\cos\theta$ above the initial line.

SECTION B (3 x 3 = 9 marks)

Instructions: This section has **3** questions. Attempt **ALL** questions.

Marks are indicated against each question.

Q.1. Test the following infinite series for convergence and divergence:

$$\frac{(1+a).(1+b)}{1.2.3}+\frac{(2+a).(2+b)}{2.3.4}+\frac{(3+a).(3+b)}{3.4.5}+\cdots$$

where a and b are non-zero, fixed and finite numbers. (3)

Q.2. Prove that: (1.5 + 1.5)

(a) $\beta(m+1,n)+\beta(m,n+1)=\beta(m,n)$

(b) $\int_0^\infty \frac{e^{-st}}{\sqrt{t}}\,dt=\sqrt{\frac{\pi}{s}}\ \ , s>0$

Q.3. Evaluate the triple integral: (3)

$$\int_0^1\int_{y^2}^1\int_0^{1-x} x\ dz\,dx\,dy$$

SECTION C (3 x 5 = 15 marks)

Instructions: This section has **4** questions. Attempt **ANY THREE** questions. Marks are indicated against each question.

Q.1. (i) Test the convergence of the infinite series: (4)

$$\frac{1^2}{4^2}+\frac{1^2.5^2}{4^2.8^2}+\frac{1^2.5^2.9^2}{4^2.8^2.12^2}+\frac{1^2.5^2.9^2.13^2}{4^2.8^2.12^2.16^2}+\cdots$$

(ii) State Cauchy's root test for determining the convergence and divergence of the infinite positive term series. (1)

Q.2. (i) Use Beta and Gamma functions to evaluate: (3)

$$\int_0^\infty \frac{x^2}{(1+x^4)^3}\,dx$$

(ii) Prove that: $\Gamma\frac{1}{2}=\sqrt{\pi}$ (2)

Q.3. Change the order of integration of the following double integral and hence evaluate the same:

$$\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy \qquad (5)$$

Q.4. (i) Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle

$x^2+y^2=a^2$. (3)

(ii) Calculate the volume of the solid bounded by (2)

$$x=0,\ y=0, z=0 \text{ and } x+y+z=1.$$
