Homework 3 *Probability & Statistics for DS & AI, Michele Guindani, 2022*

Topics: Continuous Probability Distribution & Central Limit Theorem Due on August 12, 2022

Note: Attach your code with your submission.

1. [2+2+2+2+2=10 points]

   The time spent waiting between events is often modeled using the exponential distribution. For example, suppose that an average of 30 customers per hour arrive at a store and the time between arrivals is exponentially distributed.

   (a) On average, how many minutes elapse between two successive arrivals?
   (b) When the store first opens, how long on average does it take for three customers to arrive?
   (c) After a customer arrives, find the probability that it takes less than one minute for the next customer to arrive.
   (d) After a customer arrives, find the probability that it takes more than five minutes for the next customer to arrive.
   (e) Is an exponential distribution reasonable for this situation?

2. [4+4=8 points]

   A certain public parking garage charges parking fees of \$1.50 for the first hour, and \$1 per hour after that. Suppose parking times T are exponentially distributed with mean 1.5 hours. Let W denote the total fee paid. Let's Find $E(W)$ and $Var(W)$ using Montecarlo approximation. Compare these values with the exact values.

3. [2+3+3=8 points]

   (a) Suppose $X \sim Exp(\lambda)$, then show that $P(X > s + t | X > s) = P(X > t)$
   (b) Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of $10,000$ miles. If a person desires to take a 5000-mile trip, what is the probability that he or she will be able to complete the trip without having to replace the car battery? What can be said when the distribution is not exponential?

4. [6 points]

   The lifetime of a special type of battery is a random variable with mean 40 hours and standard deviation 20 hours. A battery is used until it fails, at which point it is replaced by a new one. Assuming a stockpile of 25 such batteries, the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained from using all of them.

5. [10 points]

   Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Use Central Limit Theorem (CLT) to approximate the probability and then compare it to the exact solution. [Hint: We can think Binomial as a sum of independent Bernouli]

6. **Simulation based problem:** [4+4=8 points]

   The variation in a certain electrical current source X (in milliamps) can be modeled by the pdf

$$f(x) = \begin{cases} 1.25 - 0.25x & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose a simulation of X is required as part of some larger system analysis. Find the distribution of average current and compare it with the theoretical result.