Homework 2 *Probability & Statistics for DS & AI, Michele Guindani, 2022*

Topics: Random Variables & Discrete Probability Distribution — Due on August 3, 2022

Note: Attach your code with your submission.

1. [2+2+2+2+2=10 points]

A representative from the National Football League's Marketing Division randomly selects people on a random street in Kansas City, Kansas until he finds a person who attended the last home football game. Let p, be the probability that he succeeds in finding such a person, equal 0.20. Now, let X denote the number of people he selects until he finds his first success.

(a) What is the probability mass function of X?

(b) What is the probability that the marketing representative must select 4 people before he finds one who attended the last home football game?

(c) What is the probability that the marketing representative must select more than 6 people before he finds one who attended the last home football game?

(d) How many people should we expect (that is, what is the average number) the marketing representative needs to select before he finds one who attended the last home football game?

(e) Now, let Y denote the number of people he selects until he finds $r = 3$ who attended the last home football game. What is the probability that $X = 10$?

2. [4 points]

Suppose your favorite brand of cereals is having a competition, where in each box of cereals there is a coupon with a positive integer number between 1 and N, and you win a prize if you collect coupons with all numbers between 1 and N. Moreover, when you buy a new box of cereals, the coupon inside the box independently from other boxes has the number i with probability $\frac{1}{N}$, for every i = 1,. . . , N. How many boxes do we need to buy in order to win a prize? [Hint: Define B as a random variable that denotes total number of boxes are needed to win a prize and express B as a sum of more simpler random variables.]

3. [4+3+3=10 points]

Suppose small aircraft arrive at an airport according to a Poisson process with rate $\lambda = 8$ per hour, so that the no of arrivals during of a time period of t hours is a Poisson random variable with parameter $\mu = 8t$.

(a) What is the probability that exactly 6 small aircraft arrive during a 1-h period? At least 6? At least 10?

(b) What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-min period?

(c) What is the probability that at least 20 small aircraft arrive during a 2.5-h period? That at most 10 arrive during this period?

4. [2+2+2+1+1=8 points]

An airport limousine can accommodate up to four passengers on any one trip. The company will accept a maximum of six reservations for a trip, and a passenger must have a reservation. From previous records, 20% of all those making reservations do not appear for the trip. In the following questions, assume independence:

(a) If six reservations are made, what is the probability that at least one individual with a reservation cannot be accommodated on the trip?

(b) If six reservations are made, what is the expected number of available places when the limousine departs?

(c) Suppose the probability distribution of the number of reservations made is given in the accompanying table.

Number of reservations	3	4	5	6
Probability	.1	.2	.3	.4

(d) Let X denote the number of passengers on a randomly selected trip. Obtain the probability mass function of X.

(e) We have assumed independence. Explain why there could be dependence.

5. **Simulation based problem:** [4+4=8 points]

A glass rod drops and breaks into 5 random pieces. Let's find the probability that the smallest piece has length below 0.03. [Hint: Try to define a function to calculate length of smallest piece.]

Modify the code in the above problem so that number of breaks is random, not just the break points. A reasonable model to try would be Poisson. However, the latter's support starts at 0, and we cannot have 0 pieces, so we need to model the number of pieces minus 1 (the number of break points) as Poisson.