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A Novel Hybrid Metaheuristic Based on Augmented Grey Wolf Optimizer and Cuckoo Search for Global Optimization

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***Abstract*— Grey Wolf Optimizer (GWO) is an intelligent metaheuristic approach which imitates the leadership hierarchy and cooperative hunting behavior of a group of Grey wolves(wolf-pack). An augmentation of GWO, named Augmented GWO (AGWO), was recently proposed which possesses greater exploration abilities. Nevertheless, in some cases, AGWO underperforms in the exploitation phase and stagnates at local optimum. The CS algorithm is a nature-inspired optimizing technique that mimics the unique nesting strategy of cuckoo birds and levy-flights. Both the algorithms possess powerful searching capabilities. In our research work, a novel hybrid metaheuristic, termed AGWOCS, is put forth, which combines the merits of both metaheuristics in order to attain global optimum effectively. The proposed algorithm amalgamates the exploring abilities of the AGWO with the exploiting abilities of the Cuckoo Search (CS). For the purpose of testing the proficiency of our proposed hybrid AGWOCS, twenty-three renowned benchmarking functions. It is compared with six other existing metaheuristics, including Standard GWO, Particle Swarm Optimization (PSO), Augmented-GWO (AGWO), Enhanced-GWO (EGWO), Hybrid GWO with CS (CS-GWO) and Hybrid PSO and GWO (GWOPSO). The simulation results indicate that AGWOCS surpasses other metaheuristics in terms of rapid convergence rates as well as avoiding local optimum stagnation.**

Keywords*— Augmented GWO; Cuckoo Search; AGWOCS; Metaheuristics.

I. Introduction

In recent years, a lot of research has been carried out world over within the domain of metaheuristic algorithms [1]. Every now and then, a new metaheuristic arises as a competitive metaheuristic to the its existing contemporaries.

Metaheuristics can generally be categorized into two categories: single-solution based metaheuristics and population-based metaheuristics[2]. In first group, single solutions are arbitrarily produced for specific problems and amended 'til the finest results are attained [3]. Furthermore, these metaheuristics stagnate within the local optimum. Meanwhile, for the second group, for a given state-space, a solution-set is arbitrarily produced and the solutions are improvised 'til the best results are attained [4]. These metaheuristics can even tackle several local optimums because of improved, manifold results throughout the iterations[5].

In addition, population-based metaheuristics inspect enormous quantity of state-space contrary to single-solution metaheuristics. Thus, the possibilities of finding global optimum are much higher for population-based metaheuristics. On account of these merits, the field of population-based metaheuristics has captured substantial interest of researchers. Their classification is done in accordance with, the biological mannerisms of these nature-inspired metaheuristics [6], group(swarm) intellect of candidates, concept of evolutionary metaheuristics and the logics of physics-based metaheuristics. The combined intellect serves as motivation for these group(swarm) intelligence metaheuristics [7]. These combined intellectual behaviours are discovered amongst foraging fish shoals, bird swarms, echolocating behaviour of bats, etc. Evolutionary metaheuristics rely upon the Darwinian principle of survival of the fittest, for the searching agents of a certain population [6]. The physics-based metaheuristics seek inspiration from principles of physics such as light refraction, electromagnetism, laws of gravitation, quantum theory, etc.

Up till now, ground-breaking studies have been carried out in the field of metaheuristics and numerous nature-inspired metaheuristics have been presented by several scholars. In spite of this substantial recent development, the everlasting question still remains that why do we require to design more and more metaheuristic approaches. It could be answered by the 'No Free Lunch' Theorem. The theorem specifies that, a single metaheuristic algorithm or an optimization technique can't deal with each and every optimization problem. It means that it might fare perfectly fine for a specific group of problems but not necessarily assure the same performance on other optimization problems. So, the NFL theorem enables scholars to design new metaheuristic approaches for solving optimization problems. Keeping that in mind, a noteworthy aspect is to design a metaheuristic by hybridising two or more metaheuristics for the purpose of upgrading their individual overall performances.

This encouraged us to propose us a novel hybrid metaheuristic for solving optimization problems. The objective of the present research work is to put forth an effective metaheuristic which amalgamates the utilities of existing algorithms AGWO and CS for global optimization. The remaining part of this paper is arranged as follows: Section II discusses the contextual details of AGWO and Cuckoo Search emphasising on their motivation as well as mathematical models. The proposed hybrid AGWOCS metaheuristic is detailed in Section III. Section IV focuses on experimentation results of AGWOCS on benchmark functions. Lastly, the conclusions are specified in Section V.

II. Preliminaries

A. Augmented Grey Wolf Optimizer(AGWO)

1) Overview of GWO and AGWO: GWO is a population-based metaheuristic technique developed by 'Seyedali Mirjalili' [8] in 2014, which imitates the leadership hierarchy and cooperative hunting behaviour of a pack of grey wolves. Generally, there are 5-12 wolves in a pack. The pack of wolves follows a stern four-level hierarchy which is led by their leader called Alpha wolf (α). The Alpha wolf oversees the decision-making tasks such as hunting, seeking shelter, distribution of food, etc. They are followed by Beta wolves (β) which assist the Alpha in different activities. They are the foremost contenders to become alpha in case the contemporary alphas die or their health declines. The third in the hierarchy are Delta wolves (δ), which ensure safety and wellbeing of the pack. They are supervised by both alpha as well as beta wolves. Finally, the remnants in the hierarchy are Omega wolves (ω), which act as scapegoats i.e. they are dominated by all the other wolves in the pack. The hierarchical structure of grey wolves is illustrated in Fig. 1.

Besides maintaining a social hierarchy, collective hunting is also a fascinating aspect of the social behaviour of grey wolves. There are mainly three crucial phases in the collective hunting of grey wolves:

- Encirclement of prey
- Hunting the prey
- Attacking the prey

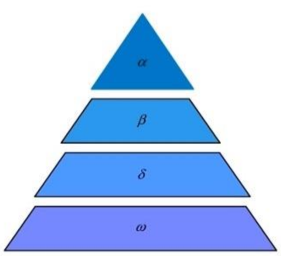


Fig 1. The hierarchical structure of grey wolves.

On the other hand, the Augmented GWO (or AGWO) metaheuristic is a newly introduced augmentation of GWO algorithm developed by 'Mohammed Qais' [9] in 2018. It's a population-based metaheuristic technique that possesses greater exploration capabilities in comparison to GWO. It's mathematical model is specified in the sub-sequent sections.

B. Cuckoo Search (CS)

The CS [10] is a nature-inspired optimization technique designed by 'Suash Deb' and 'Xin-She Yang' which mimics the exceptional reproducing strategy of cuckoo bird species as well as their Lévy-Flights. According to these reproducing strategies, cuckoos tend to lay their own eggs in other birds' nest instead of laying and hatching them in their own. Whenever the host is away from its nest, the cuckoo bird tends to lay its eggs in the host's nest in its absence. Occasionally, they may even eliminate the host's eggs. Sometimes hosts can get involved in a skirmish with the obtruder.

The Cuckoo Search is governed by the following assumptions:

- Firstly, cuckoo bird places single egg inside a nest at a time and that nest is chosen randomly.
- Secondly, the nests with the finest eggs (solution) are carried onward as successive generation.
- Thirdly, the existing host nests are fixed. The probability of detection of unfamiliar egg(s) is $P_D \in [0,1]$

In conformation with the aforementioned assumptions, the updation of nests is done using the following equations:

$$\vec{X_i}(t+1) = \vec{X_i}(t) + \alpha \oplus L\acute{e}vy(\lambda), i = 1, \dots, n \tag{1}$$

where $\vec{X_i}(t+1)$ signifies new solutions. $\vec{X_i}(t)$ represents the present solutions of the i^{th} cuckoo. $\oplus$ represents element-wise product. As, $\alpha > 0$ controls the size of steps, so generally its value is kept '1'.

The Lévy-Flights are illustrated using following equation:

$$L\acute{e}vy(\lambda) \sim u = t^{-\lambda}, \quad (1 < \lambda \le 3)$$

III. Proposed Approach

In our proposed approach, two population-based metaheuristics, CS and AGWO are hybridized. Population-based metaheuristics are used as they rapidly inspect enormous quantity of state-space. CS is embedded into the AGWO metaheuristic for strengthening and enhancing its capability to avoid entrapment within the local optima and converge towards global minimum. The exploration abilities of the CS are utilized to avoid these risks by guiding the wolves (or searching agents) towards the positions which are improved by the CS metaheuristic. The flowchart of the AGWOCS is illustrated in Figure 2.

A. Mathematical Model of Agwocs

$$\vec{D} = \left|\vec{C}.\vec{X_p}(t) - \vec{X}(t)\right| \tag{2}$$

$$\vec{X}(t+1) = \vec{X_p} - \vec{A}.\vec{D} \tag{3}$$

$$\vec{a} = 2 - cos(rand) \times t \,/Max\_iterations \tag{4}$$

$$\vec{A} = 2\vec{a}.\vec{r_1} - \vec{a} \tag{5}$$

$$\vec{C} = 2.\vec{r_2} \tag{6}$$

$$\vec{D_\alpha} = \left|\vec{C_1}.\vec{X_\alpha} - \vec{X}\right| , \vec{D_\beta} = \left|\vec{C_1}.\vec{X_\beta} - \vec{X}\right| \tag{7}$$

$$\vec{X_1} = \vec{X_\alpha} - \vec{A_1}.\vec{D_\alpha}, \vec{X_2} = \vec{X_\beta} - \vec{A_2}.\vec{D_\beta} \tag{8}$$

$$\vec{X}(t+1) = \frac{\vec{X_1} + \vec{X_2}}{2} \tag{9}$$

B. Improvement Over the Augmented-GWO Algorithm

The Cuckoo Search Algorithm is integrated with Augmented-GWO and it takes the control from here. The positions of the Alpha wolf (α) as well as Beta wolf (β) is updated using Cuckoo search (Eq. 10).

$$\vec{X_i}(t+1) = \vec{X_i}(t) + \alpha \oplus L\acute{e}vy(\lambda), i = 1, .., n \tag{10}$$

The Lévy-Flights are represented using the following Eqn:

$$Lévy(\lambda) \sim u = t^{-\lambda}, \qquad (1 < \lambda \leq 3) \tag{11}$$

The CS algorithm searches the state-space efficiently and performs updation of the position of nests using levy-flight and random walks.

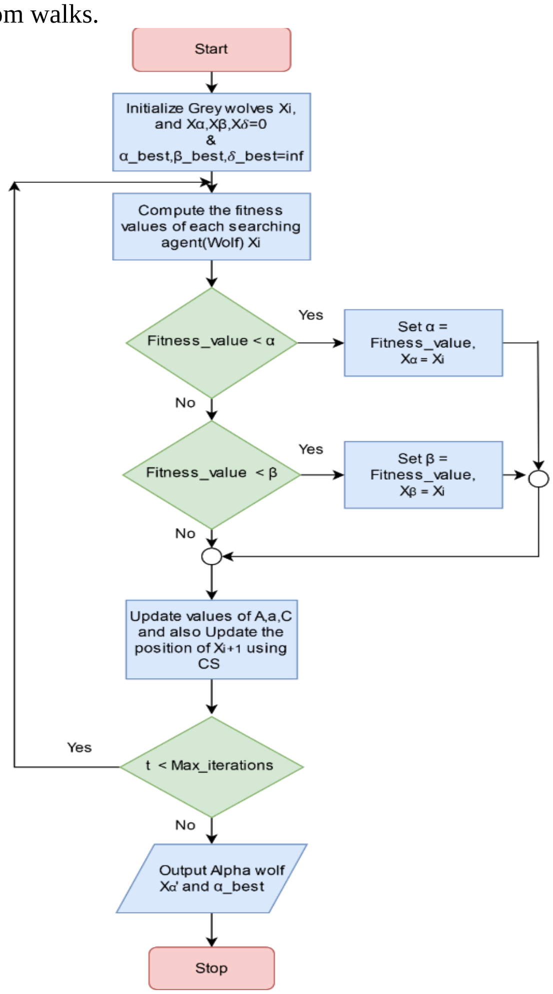


Fig 2. The Flowchart of Hybrid AGWOCS algorithm.

IV. RESULT & DISCUSSIONS

In order to test the efficacy of our proposed approach, a set of '23' extensively used benchmark functions have been utilized and applied upon AGWOCS and also compared with six other existing metaheuristics. The Benchmark functions are classified into the following three classes, namely unimodal (Fig. 3), multi-modal (Fig. 4) and fixed dimensional multi-modal functions (Fig. 5).

Functions	Ranges	Dimensions	f_{min}
$f_1(z) = \Sigma_{i=1}^{D} z_i^2$	[-100,100]	30	0
$f_2(z) = \Sigma_{i=1}^{D} \lvert z_i \rvert - \Pi_{i=1}^{D} \lvert z_i \rvert$	[-10,10]	30	0
$f_3(z) = \Sigma_{i=1}^{D} (\Sigma_{j-1}^{i} z_j)^2$	[-100,100]	30	0
$f_4(z) = \max_i(\lvert z_i \rvert, 1 \leq i \leq D)$	[-100,100]	30	0
$f_5(z) = \Sigma_{i=1}^{D-1}[100 z_{i+1} - z_i^2]^2 + (z_i - 1)^2$	[-30,30]	30	0
$f_6(z) = \Sigma_{i=1}^{D}([z_i + 0.5])^2$	[-100,100]	30	0
$f_7(z) = \Sigma_{i=1}^{D} i z_i^4 + \mathrm{rand}(0,1)$	[-1.28,1.28]	30	0

Fig 3. Unimodal functions.

Functions	Ranges	Dimensions	f_{min}
$f_8(z) = \Sigma_{i=1}^{D} - z_i \sin\sqrt{(\lvert z_i \rvert)}$	[-500,500]	30	-418.982
$f_9(z) = \Sigma_{i=1}^{D}(z_i^2 - 10\cos(2\pi z_i) + 10)$	[-5.12,5.12]	30	0
$f_{10}(z) = -20 exp\left(-0.2 \times \sqrt{\frac{1}{n}\Sigma_{i=1}^{D} z_i^2}\right) - exp\left(\frac{1}{n}\Sigma_{i=1}^{D}\cos(2\pi z_i)\right) + 20 + e$	[-32,32]	30	0
$f_{11}(z) = \frac{1}{4000}\Sigma_{i=1}^{D} z_i^2 - \Pi_{i=1}^{D}\cos\left(\frac{z_i}{\sqrt{i}}\right)+1$	[-600,600]	30	0
$f_{12}(z) = \frac{\pi}{n}\{10\sin(\pi x_1) + \Sigma_{i=1}^{D-1}(x_1 - 1)^2[1 + 10\sin^2(\pi x_{i+1})] + (x_n - 1)^2\} + \Sigma_{i=1}^{D} u(z_i, 10,100,4)$ $x_i = 1.0 + \frac{z_i + 1}{4.0}$, $u(z_i, a, k, m) = \begin{cases} k(z_i - a)^m, & z_i > a \\ 0 - a, & z_i < a \\ k(-z_i - a)^m, & z_i < -a \end{cases}$	[-50,50]	30	0
$f_{13}(z) = 0.1\{sin^2(3\pi z_i)\Sigma_{i=1}^{D}(z_i - 1.0)^2[1.0 + sin^2(3\pi z_i + 1.0)] + (z_n - 1)^2[1.0 + sin^2(2\pi z_n)]\} + \Sigma_{i=1}^{D} u(z_i, 5,100,4)$	[-50,50]	30	0

Fig 4. Multi-modal functions.

Functions	Ranges	Dimensions	f_{min}
$f_{14}(z) = \left(\frac{1}{500} + \Sigma_{j=1}^{25}\frac{1}{j + \Sigma_{i=1}^{2}(z_i - a_{ij})^6}\right)^{-1}$	[-65,65]	2	1
$f_{15}(z) = \Sigma_{i=1}^{11}\left[a_i - \frac{z_i(b_i^2 + b_i z_2)}{b_i^2 + b_i z_3 + z_4}\right]^2$	[-5,5]	4	0.00030
$f_{16}(z) = 4z_1^2 - 2.1 * z_1^4 + \frac{1}{3}z_1^6 + z_1 * z_2 - 4 * z_2^2 + 4 * z_2^4$	[-5,5]	2	-1.0316
$f_{17}(z) = \left(z_2 - \frac{5.10}{4\pi^2}z_1^2 + \frac{5}{\pi}z_1 - 6.0\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cos z_1 - 10.0$	[-5,5]	2	0.398
$f_{18}(z) = [1.0 + (z_1 + z_2 + 1.0)^2(19.0 - 14z_1 + 3z_1^2 - 14z_2 + 6z_1z_2 + 3z_2^2)] \times [30.0 + (2z_1 - 3z_2)^2 \times (18.0 - 32z_1 + 12z_1^2 + 48z_2 - 36z_1z_2 + 27z_2^2)]$	[-2,2]	2	3
$f_{19}(z) = -\Sigma_{i=1}^{4} c_i\, exp\left(-\Sigma_{j=1}^{3} a_{ij}(z_j - p_{ij})^2\right)$	[1,3]	3	-3.86
$f_{20}(z) = -\Sigma_{i=1}^{4} c_i\, exp\left(-\Sigma_{j=1}^{6} a_{ij}(z_j - p_{ij})^2\right)$	[0,1]	6	-3.32
$f_{21}(z) = -\Sigma_{i=1}^{5}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	[0,10]	4	-10.1532
$f_{22}(z) = -\Sigma_{i=1}^{7}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	[0,10]	4	-10.4028
$f_{23}(z) = -\Sigma_{i=1}^{10}[(X - a_i)(X - a_i)^T + c_i]^{-1}$	[0,10]	4	-10.5363

Fig 5. Fixed Dimension Multi-modal functions.

A. *Experimental results of Metaheuristics upon Unimodal functions*

The experimentational results of the metaheuristics on uni-modal functions have been revealed in the Table I. The best results achieved are indicated by the bold values. It can be determined from the Table I that:

1) The AGWOCS metaheuristic technique surpasses GWO[8], PSO[11], AGWO[9], EGWO[12], CS-GWO[13] and finally, GWOPSO[14] on almost all the testing functions.

2) The mean, standard deviation as well as the best values over the course of 20 runs for the benchmarking functions, from F1 to F7, are specified in the Table below.

3) Furthermore, the comparison of convergence characteristics of the AGWOCS metaheuristic with GWO, PSO, AGWO, EGWO, CS-GWO, GWOPSO are precisely shown in Figure 6.

B. *Experimental results of Metaheuristics upon Multi modal functions*

Multi-modal testing functions are generally applied in order to benchmark the exploration as well as avoiding local optimum entrapment. The experimentational results of the metaheuristics upon multi-modal testing functions have been revealed in the Table I. It can be deduced from the Table I that the AGWOCS technique exhibits superior performance in comparison to GWO, PSO, EGWO, AGWO, CS-GWO and finally, GWOPSO in majority of the testing scenarios. Consequently, the results are indicative of the fact that AGWOCS metaheuristic possesses outstanding exploring abilities that enables it to search within the favorable regions in the search spaces.

TABLE I. EXPERIMENTATION RESULTS

Function		**GWO[8]**	**EGWO[12]**	**AGWO[9]**	**PSO[11]**	**GWOCS[13]**	**GWOPSO[14]**	**AGWOCS**
f 1	Mean	1.0956e-27	8.126e-30	6.869e-09	5.668e-07	3.634e-28	7.959e3	1.513e-44
	S.D.	2.3235e-27	3.342e-29	2.329e-08	1.499e-06	6.737e-28	1.414e4	3.219e-44
	Best	1.486e-29	6.970e-34	1.993e-23	5.81e-10	7.348e-30	1.579e-18	**1.493e-46**
f 2	Mean	6.414e-17	2.580e-18	4.472e11	1.9141e-4	3.4422e-17	6.837e4	3.7613e-27
	S.D.	2.9732e-17	9.465e-18	1.999e12	4.3677e-4	1.8031e-17	2.408e5	3.4918e-27
	Best	2.0271e-17	1.792e-20	9.903e-12	4.3947e-07	1.3935e-17	2.0222e-09	**2.4547e-28**
f 3	Mean	1.5908e-05	2.6447e-3	5.4545e4	1.2466e1	4.9573e-07	7.2214e3	1.0119e-09
	S.D.	3.3157e-05	6.3269e-3	4.8523e4	2.2911e1	8.0808e-07	1.7350e4	2.9121e-09
	Best	1.1272e-08	6.757e-10	1.896e1	1.9526e-3	2.6375e-09	1.9384e-06	**9.1524e-16**
f 4	Mean	5.2611e-07	5.8834e-3	8.5414e0	3.4672e-1	2.2647e-07	1.4079e1	1.6027e-11
	S.D.	3.71e-07	8.1364e-3	2.0640e1	3.7854e-1	1.9677e-07	2.5526e1	3.6286e-11
	Best	8.5261e-08	1.399e-06	9.887e-05	1.4163e-2	2.1173e-08	1.6549e-4	**1.8258e-13**
f 5	Mean	2.70545e1	2.77302e1	2.76864e1	1.07723e1	2.69566e1	3.2698e4	2.70427e1
	S.D.	8.5037e-1	9.8688e-1	6.42e-1	1.29546e1	9.0299e-1	8.7386e4	6.3709e-1
	Best	2.54395e1	2.60944e1	2.69498e1	**7.6121e-3**	2.58403e1	2.5361e1	2.58667e1
f 6	Mean	7.2019e-1	3.1828e0	1.7649e0	3.0392e-07	8.4894e-1	3.97863e2	1.3828e0
	S.D.	3.6332e-1	6.6516e-1	4.2622e-1	8.6301e-07	3.4552e-1	1.2871e3	3.6622e-1
	Best	9.5885e-05	2.2725e0	1.0822e0	**2.7937e-11**	2.4908e-1	4.9926e-05	6.119e-1
f 7	Mean	1.8551e-3	7.1653e-3	7.5852e-3	9.6957e-3	1.7647e-3	2.8168e-1	1.3895e-3
	S.D.	8.7175e-4	4.2499e-3	3.3765e-3	7.3955e-3	7.7358e-4	5.9849e-1	8.3274e-4
	Best	6.245e-4	1.6222e-3	2.5187e-3	1.795e-3	7.8808e-4	3.5425e-3	**2.8497e-4**
f 8	Mean	-6.2096e3	-5.9965e3	-5.8650e3	-1.2569e4	-1.14842e4	-7.0538e3	-7.1600e3
	S.D.	4.53379e2	1.3461e2	5.5558e1	2.9616e-3	7.173073e2	1.01588e3	6.70012e2
	Best	-7.22253e3	-6.2523e3	-5.9840e3	**-1.2569e4**	-1.23466e4	-8.6493e3	-8.4836e3
f 9	Mean	1.9332e0	1.7027e2	1.8455e2	8.868e-4	1.8958e0	1.0843e2	0.00
	S.D.	2.6467e0	3.7633e1	1.1845e2	3.577e-3	4.242e0	1.1476e2	0.00
	Best	5.6843e-14	1.1167e2	2.0673e1	5.9914e-08	5.6843e-14	1.6262e1	**0.00**
f 10	Mean	1.0445e-13	1.80918e1	1.99596e1	1.4734e-1	8.793e-14	4.8556e0	9.0594e-15
	S.D.	2.3742e-14	5.7656e0	6.1902e-4	4.8594e-1	1.6199e-14	6.8614e0	2.6031e-15
	Best	6.4837e-14	2.220e-14	1.99588e1	3.4932e-06	5.4179e-14	9.7082e-10	**7.9936e-15**
f 11	Mean	3.708e-3	1.3477e-2	1.4113e-2	1.464e-2	4.6324e-3	3.10514e1	0.00
	S.D.	9.4582e-3	1.6177e-2	1.9559e-2	1.8456e-2	7.9451e-3	9.12179e1	0.00
	Best	**0.00**	**0.00**	**0.00**	7.9234e-10	**0.00**	3.1086e-15	**0.00**
f 12	Mean	4.7471e-2	2.8971e0	1.7179e-1	5.0961e-08	4.2658e-2	4.2729e6.	9.8737e-2
	S.D.	2.9785e-2	3.3638e0	5.7325e-2	1.6891e-07	2.2187e-2	1.7089e7	2.9176e-2
	Best	1.089e-2	2.6064e-1	9.5185e-2	**1.7369e-11**	1.3604e-2	6.2117e-06	6.0001e-2
f 13	Mean	6.1534e-1	2.5675e0	1.448e0	2.7479e-3	6.1613e-1	1.4739e7	1.0405e0
	S.D.	2.6083e-1	6.2019e-1	2.8139e-1	4.8808e-3	2.3914e-1	6.1622e7	1.2548e-1
	Best	1.9713e-1	1.7823e0	8.9315e-1	**3.231e-11**	1.7762e-1	1.9255e-4	7.7047e-1
f 14	Mean	4.3744e0	6.5077e0	1.2956e0	9.98e-1	6.4644e0	2.2763e0	2.9627e0
	S.D.	4.3356e0	5.3384e0	7.2686e-1	8.82e-17	5.0714e0	2.6214e0	3.1609e0
	Best	**9.98e-1**	**9.98e-1**	**9.98e-1**	**9.98e-1**	**9.98e-1**	**9.98e-1**	**9.98e-1**
f 15	Mean	7.362e-3	2.3698e-3	5.8995e-4	3.100e-4	4.4372e-4	2.5949e-3	6.0105e-4
	S.D.	9.788e-3	6.1666e-3	1.8736e-4	6.601e-6	3.5928e-4	6.0937e-3	4.3373e-4
	Best	3.075e-4	3.0755e-4	3.1212e-4	**3.074e-4**	3.075e-4	3.075e-4	3.1225e-4
f 16	Mean	-1.031e0	-1.031e0	-1.031e0	-1.031e0	-1.031e0	-1.031e0	-1.031e0
	S.D.	2.406e-8	9.8257e-9	7.0999e-6	2.03e-16	3.8823e-8	2.0886e-5	4.7349e-6
	Best	**-1.031e0**	**-1.031e0**	**-1.031e0**	**-1.031e0**	**-1.031e0**	**-1.031e0**	**-1.031e0**
f 17	Mean	3.979e-1	3.978e-1	3.9833e-1	3.978e-1	3.978e-1	3.9819e-1	3.9829e-1
	S.D.	9.467e-5	1.2679e-6	7.3283e-4	0.00	1.8184e-6	9.0878e-4	3.857e-4
	Best	3.978e-1	3.978e-1	3.979e-1	3.978e-1	3.978e-1	3.978e-1	**3.9791e-1**
f 18	Mean	3.0001e0	9.75e0	3.000e0	3.000e0	3.000e0	3.000e0	3.000e0
	S.D.	6.394e-5	1.93415e1	2.0671e-5	1.04e-15	4.0527e-5	6.8267e-5	1.897e-5
	Best	**3.000e0**	**3.000e0**	**3.000e0**	**3.000e0**	**3.000e0**	**3.000e0**	**3.000e0**
f 19	Mean	-3.862e0	-3.862e0	-3.8601e0	-3.862e0	-3.8624e0	-3.8586e0	-3.8615e0
	S.D.	2.002e-3	5.7917e-6	2.4309e-3	2.23e-15	1.1375e-3	3.8951e-3	2.1792e-3
	Best	-3.8628	-3.8628	-3.8628	-3.8628	-3.8628	-3.8628	**-3.8629**
f 20	Mean	-3.260e0	-3.2721e0	-3.1825e0	-3.302e0	-3.322e0	-3.1614e0	-3.272e0
	S.D.	8.264e-2	6.2966e-2	1.9923e-1	4.771e-2	8.445e-06	1.8386e-1	4.2858e-2
	Best	**-3.322e0**	**-3.322e0**	-3.3007e0	**-3.322e0**	**-3.322e0**	**-3.322e0**	-3.3104e0
f 21	Mean	-9.6435	-5.6472	-6.8854	-10.1532	-7.8865	-6.2932	-5.9185
	S.D.	1.5617	3.4937	1.8463	2.79e-15	2.9255	3.3299	1.9085
	Best	-1.016e1	**-1.0153e1**	-8.7833e0	-1.016e1	-1.0152e1	**-1.0153e1**	-9.188e0
f 22	Mean	-10.1349	-8.6087	-7.4574	-10.4029	-8.2835	-8.4476	-6.3999
	S.D.	1.188	3.2232	1.5738	3.36e-15	2.6605	2.8648	2.0608
	Best	-10.4025	**-10.4029**	-9.7506	**-10.4029**	-10.4028	**-10.4029**	-9.3993
f 23	Mean	-10.5345	-6.7493	-7.5661	-10.5364	-8.8052	-7.4843	-6.5742
	S.D.	1.305e-3	3.5927	1.8002	3.13e-15	2.7519	3.7842	1.8562
	Best	-10.5362	**-10.5364**	-10.0284	**-10.5364**	-10.5359	**-10.5364**	-9.752

Moreover, avoiding local optimum stagnation in case of the enhanced hybrid AGWOCS is also brilliant owing to its capability of jumping out of local optimum and swiftly discovering the global optimum in most of the multi-modal testing functions. The mean, standard deviation as well as the best values over the course of 20 runs for the benchmarking functions, $f8$, $f9$, $f10$, $f11$, $f12$ and $f13$ are specified above.

C. *Experimental results of the Metaheuristics on Fixed dimension Multi-modal functions*

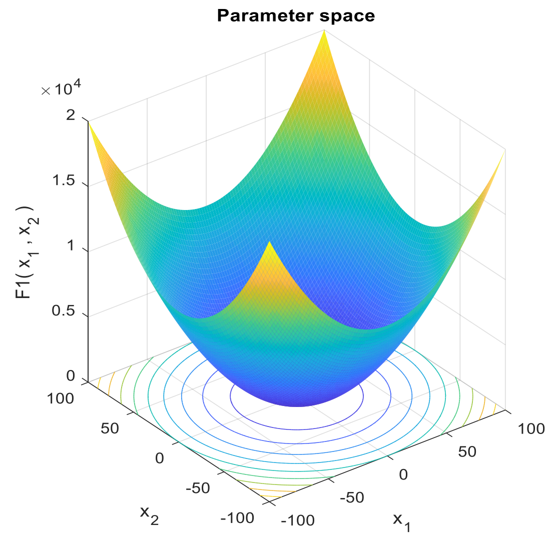
The fixed dimensional multi-modal functions benchmark the collective working of both exploitation and exploration. The investigational outcomes of the metaheuristics on fixed-dimensional multi-modal testing (composite) functions have been described in the Table I. From the Table I, it can be inferred that the AGWOCS method surpasses the other metaheuristics in the testing scenarios. From the consideration of the results on fixed-dimensional testing functions, it can be concluded that AGWOCS metaheuristic effectively balances between exploitation as well as exploration in order to emphasize on highly advantageous regions with in the searching spaces.

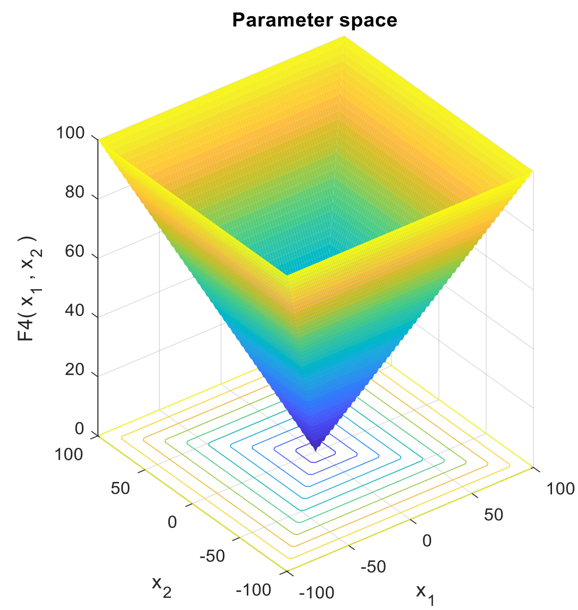
It is evident that AGWOCS has rapid convergence and enhanced global searching ability on the fixed-dimensional testing functions. Moreover, the novel metaheuristic is specifically exceptional for solving multi-modal optimization problems. Thus, it offers a unique methodology for solving high complexity optimizing problems.

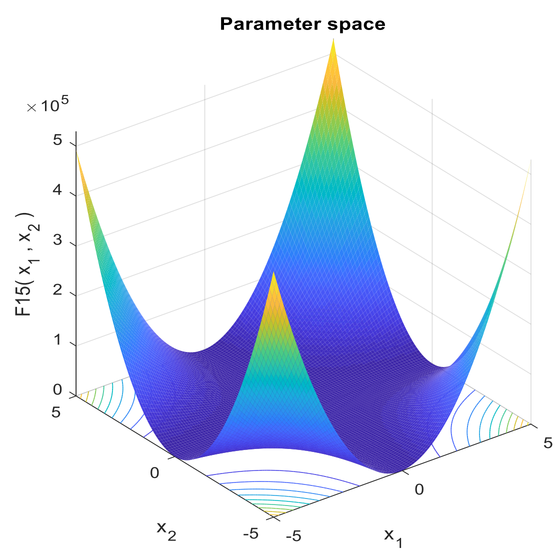
D. *The Merits of the AGWOCS metaheuristic*

The foremost reason behind the exceptional functioning of the AGWOCS, while dealing with global optimizing problems are specified below:

- The AGWO, which imitates the cooperative hunting behaviour of a group of grey wolves, has the benefit of a rapid converging rates.
- The CS metaheuristic technique possesses the ability to successfully avoid getting trapped within the local optimum and expand its global searching ability.
- The AGWOCS utilizes the position obtained by the alpha wolf and updated by the CS metaheuristic that in turn accelerates, the individual converging rates of AGWO and CS.







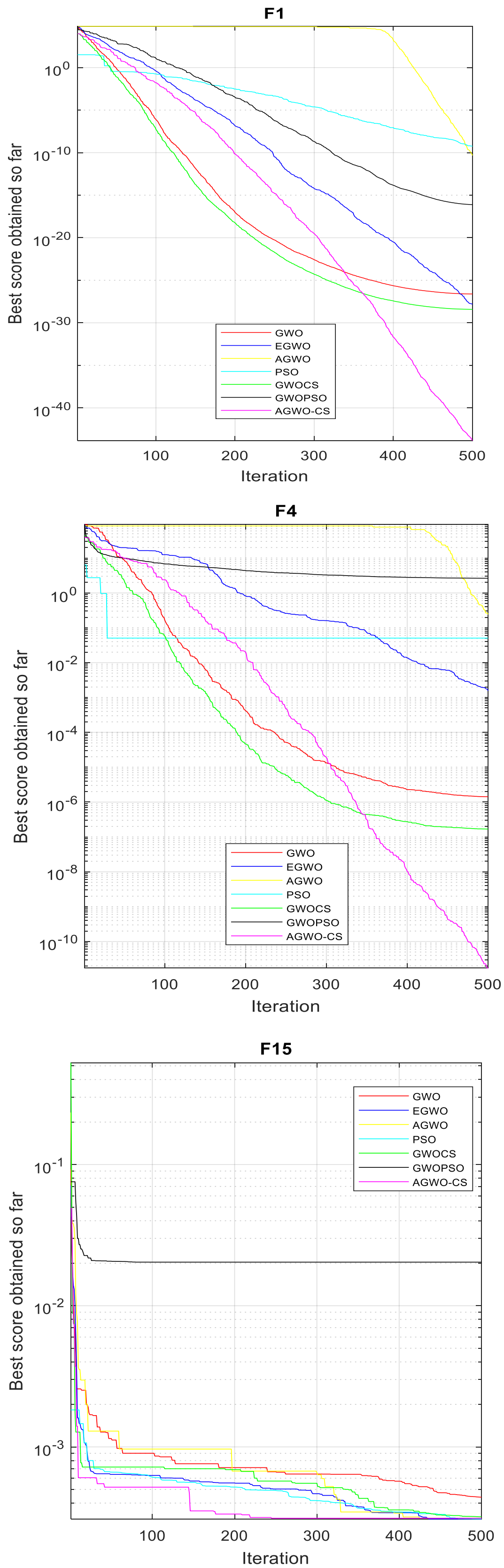


Fig 6. 3-D images and Convergence Curves of some Testing functions.

V. Conclusions and Future Scope

This paper puts forth a novel hybrid metaheuristic technique combining Augmented Grey Wolf Optimizer (AGWO) along with the Cuckoo Search (CS). The AGWOCS metaheuristic amalgamates the exploring abilities of AGWO along with the exploiting abilities of the Cuckoo Search (CS). For the purpose of testing its competence, AGWOCS is employed upon twenty-three typical benchmark functions. The experimental outcomes of the hybrid metaheuristic are compared with six metaheuristics, namely, GWO, AGWO, EGWO, PSO, GWOPSO and GWOCS. The experimental results disclose that AGWOCS is capable of providing extremely viable results in comparison with renowned metaheuristics such as Standard GWO, Particle Swarm Optimization (PSO), Augmented-GWO (AGWO), Enhanced-GWO (EGWO), Hybrid CS-GWO and Hybrid GWO and PSO (GWOPSO). Firstly, the outcomes of unimodal functions indicate exceptional exploiting abilities of AGWOCS algorithm. Secondly, the exploring capabilities of AGWOCS are affirmed by the outcomes of multimodal functions. Thirdly, the outcomes of the fixed dimensional multi-modal functions specify enhanced local optima dodging. The analysis of the convergence curves validates the convergence of AGWOCS. In future works, AGWOCS would be employed upon various intricate real-world problems.

References

[1] Yang XS (2010) Engineering Optimization: An Introduction with Metaheuristic Applications

[2] Dhiman G (2019) ESA : a hybrid bio - inspired metaheuristic optimization approach for engineering problems. Springer London

[3] Gao Z, Zhao J (2019) An Improved Grey Wolf Optimization Algorithm with Variable Weights. 2019:

[4] Saremi S, Zahra S (2014) Evolutionary population dynamics and grey wolf optimizer.

[5] S. Sharma, "QRP: QPSO Based Routing Protocol for Energy Efficiency in Wireless Body Area Networks," in International Conference on Computing Science, Communication and Security, 2021, pp. 205–221.

[6] Yang XS (2014) Nature-Inspired Optimization Algorithms

[7] Seyyedabbasi, Amir, and Farzad Kiani. "I-GWO and Ex-GWO: improved algorithms of the grey wolf optimizer to solve global optimization problems." *Engineering with Computers* (2019): 1-24.

[8] Mirjalili S, Mirjalili SM, Lewis A. Grey wolf optimizer. Advances in engineering software. 2014 Mar 1;69:46-61.

[9] M. H. Qais, H. M. Hasanien, and S. Alghuwainem, "Augmented grey wolf optimizer for grid-connected PMSG-based wind energy conversion systems." *Appl. Soft Comput. J.*, 2018.

[10] Yang X, Deb S (2009) Engineering Optimisation by Cuckoo Search. 1–17

[11] J. Kennedy and R. Eberhart, "Particle Swarm Optimization, Proceedings of IEEE International Conference on Neural Networks Vol. IV: 1942–1948.," *Neural Networks*, 1995.

[12] Q. Li *et al.*, "An Enhanced Grey Wolf Optimization Based Feature Selection Wrapped Kernel Extreme Learning Machine for Medical Diagnosis," *Comput. Math. Methods Med.*, 2017.

[13] H. Xu, X. Liu, J. Su, and A. Overview, "An Improved Grey Wolf Optimizer Algorithm Integrated with Cuckoo Search," no. 61602162, pp. 490–493, 2017.

[14] F. Ahmet, Ş. Fatih, G. Asım, S. Yüksel, and T. Yiğit, "A novel hybrid PSO – GWO algorithm for optimization problems," *Eng. Comput.*, vol. 0, no. 0, p. 0, 2018.