**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans.- mean= *μ* = 45

But actual mean is 45+10 minutes as servicing time required is 45 min after 10 min a car is dropped.

SD= *σ* = 8

Probability that the service manager cannot meet his commitment = P(X>60)= 1- P(X<60)

=1- stats.norm.cdf(60,loc=55,scale=8)

**Required prob =0.26599**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans.- A.

Probability that employees at the processing center are older than 44 = P(X>44)

= 1- P(X<44) = 0.158655

Probability that employees at the processing center are between 38 and 44= P(38<X<44)=

0.34134

Hence, statement A is **False.**

Ans- B.

Probability that employees under the age of 30 at the center= P(X<30)

= stats.norm.cdf(30, loc=38, scale=6) = 0.09121

The proportion of 36 employees out of 400 is= 36/400= 0.09

Hence, statement B is **True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans- The distribution of a *X*1 is N(aμ , a2 σ2 )

The distribution of 2 *X*1 is N(2μ , 4σ2 )

The distribution of  *X*1 + *X*2 is N(2μ , 2σ2 )

The difference between 2 *X*1 and *X*1 + *X*2 is that is *X*1 + *X*2  sum of two iid normal random variables.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans.- probability of the random variable taking a value between the interval is 0.99

Reuired interval can be obtained as

stats.norm.interval(0.99, loc = 100, scale = 20)

= 48.5, 151.5

**Option D.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans.-A.

Profit1 ~ N(5, 32)

Profit2 ~ N(7, 42)

Distribution of profit of company can be given as sum of distributions of the two divisions

Annual profit ~ Profit1 + Profit2

Annual profit ~ N(5+7, 9+16) = N(8,25)

Required range can be obtained as

stats.norm.interval(0.95, loc = 12, scale = 5)

Required range = [ 2.2, 21.8]

**Required range for rupees =[ 99 , 981]**

B.

Z value for 5th percentile is = -1.645

X=mean + (-1.645)\*SD

X= 3.775

X for rupees= 3.775\*45 = 169.875

**5th percentile = 170**

C.

Making loss= P(X<0)

stats.norm.cdf(0,5,3)= 0.0477

stats.norm.cdf(0,7,4) =0.0400

**Division 2 will face more loss.**