

# Trigonometry

**By: P.K Sir**

# Formulas

⇒ Trigonometric Ratios of some specific Angles :-

(विशिष्ट कोणों का त्रिकोणमितीय अनुपात) :-

	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
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$\uparrow \sin$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
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$\downarrow \cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
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$\uparrow \tan$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	अपरिभाषित
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## Formulas

### 3. Trigonometric Identities :-

त्रिकोणमितिय सर्वसमिकायें :-

1.  $\sin^2\theta + \cos^2\theta = 1$
2.  $\tan^2\theta + 1 = \sec^2\theta$
3.  $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
4.  $\sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y$
5.  $\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$
6.  $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \pm \tan x \cdot \tan y}$
7.  $2\sin A \cdot \cos B = \sin(A + B) + \sin(A - B)$
8.  $2\sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$
9.  $2\cos A \cdot \sin B = \sin(A + B) - \sin(A - B)$
10.  $2\cos A \cdot \cos B = \cos(A + B) + \cos(A - B)$

## Formulas

$$11. \sin C + \sin D = 2 \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$12. \sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

$$13. \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$14. \cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right)$$

$$15. \sin 2x = 2 \sin x \cdot \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

## Formulas

$$16. \cos 2x = \cos^2 x - \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$17. \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$18. \sin 3x = 3 \sin x - 4 \sin^3 x$$

$$19. \cos 3x = 4 \cos^3 x - 3 \cos x$$

$$20. \cos A \cdot \cos 2A \cdot \cos 4A = \frac{1}{4} \cos 3A$$

$$21. \sin A \cdot \sin 2A \cdot \sin 4A = \frac{1}{4} \sin 3A$$

$$22. \tan A \cdot \tan 2A \cdot \tan 4A = \tan^3 A$$

1. Convert  $15^\circ$  in Radian measure /  $15^\circ$  को रेडियन माप में बदलें।

(A)  $\frac{12}{\pi}$

(B)  $\frac{\pi}{12}$

(C)  $\frac{8\pi}{15}$

(D)  $\frac{8\pi}{21}$

**I**  $D \times \frac{\pi}{180} = R$

**II**  $R \times \frac{180}{\pi} = D$

$$15 \times \frac{\pi}{180} = \frac{\pi}{12}$$

२७<sup>c</sup> २७<sup>o</sup>

2. Convert  $\frac{3\pi}{4}$  in **degree** measure /  $\frac{3\pi}{4}$  की डिग्री में बदलें .

बदलें।

- (A)  $105^\circ$  (B)  $120^\circ$  (C)  $135^\circ$  (D)  $125^\circ$

$$\frac{3\pi}{4} \times \frac{45}{\pi} = 135^\circ$$

$$D \times \frac{\pi}{180} = R$$

$$R \times \frac{180}{\pi} = D$$

3. The two angles of a triangle are  $25^\circ$  and  $95^\circ$ . Find the third angle in Radian measure

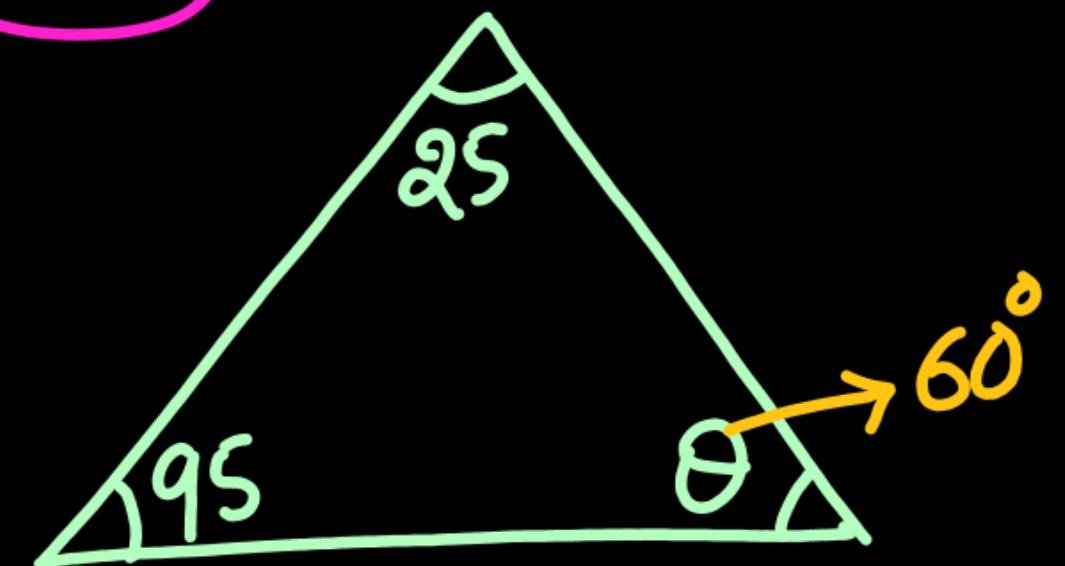
किसी त्रिभूज के दो कोण  $25^\circ$  एवं  $95^\circ$  है। तीसरा कोण रेडियन में क्या होगा ?

(A)  $\frac{\pi}{3}$

(B)  $\frac{\pi}{2}$

(C)  $\frac{\pi}{4}$

(D)  $\frac{\pi}{6}$



$$95 + 25 + \theta = 180$$

$$120 + \theta = 180$$

$$\theta = 60^\circ$$

$$60 \times \frac{\pi}{180} = \frac{\pi}{3}$$

4. If  $\tan \theta = \frac{3}{4}$  and ' $\theta$ ' is acute, then cosec  $\theta$  is :-

यदि  $\tan \theta = \frac{3}{4}$  और ' $\theta$ ' न्यूनकोण है, तो cosec  $\theta$  है :-

(A)  $\frac{5}{4}$

(B)  $\frac{4}{3}$

(C)  $\frac{4}{5}$

~~(D)  $\frac{5}{3}$~~

3, 4, 5

9, 40, 41

5, 12, 13

10, 24, 26

6, 8, 10

11, 60, 61

7, 24, 25

12, 35, 37

8, 15, 17

$$\begin{aligned} \tan \theta &= \frac{3}{4} \rightarrow P \\ h &\rightarrow 5 \quad \sqrt{3^2 + 4^2} \\ h &= \sqrt{9+16} \\ &= \sqrt{25} \end{aligned}$$

$$P^2 + b^2 = h^2 \rightarrow \text{Triplet's}$$

$$\text{cosec } \theta = \frac{h}{P} = \frac{5}{3}$$

## C & d rule

$$\textcircled{I} \quad \frac{a}{b} = \frac{c}{d}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\textcircled{II} \quad \frac{a+b}{a-b} = \frac{c}{d}$$

$$\frac{a}{b} = \frac{c+d}{c-d}$$

$$\textcircled{III} \quad \frac{a}{b} = \frac{c+d}{c-d}$$

$$\frac{a+b}{a-b} = \frac{c}{d}$$

$$\textcircled{IV} \quad \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\frac{a}{b} = \frac{c}{d}$$

5. If  $\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta} = 8\frac{1}{3}$ , then  $\cos \theta = ?$

यदि  $\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta} = 8\frac{1}{3}$ , तो  $\cos \theta = ?$

- (A)  $\frac{5}{14}$     (B)  $\frac{9}{14}$  ~~(C)~~    (D)  $\frac{11}{14}$

$$\frac{\operatorname{cosec} \theta + \cot \theta}{\operatorname{cosec} \theta - \cot \theta} = \frac{25}{3}$$

$$\frac{\operatorname{cosec} \theta}{\cot \theta} = \frac{25+3}{25-3} = \frac{28}{22} = \frac{14}{11}$$

$$\frac{1}{\sin \theta \times \frac{\cos \theta}{\sin \theta}} = \frac{1}{\cos \theta} = \frac{14}{11} \Rightarrow \cos \theta = \frac{11}{14}$$

6 . If  $\tan \theta = \frac{a}{b}$ , then find  $\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta}$  ?

यदि  $\tan \theta = \frac{a}{b}$  हो, तो  $\frac{\cancel{a} \sin \theta + \cancel{b} \cos \theta}{\cancel{a} \sin \theta - \cancel{b} \cos \theta}$  कितना होगा ?

(A)  $\frac{a^2 + b^2}{a^2 - b^2}$

(C)  $\frac{b}{a^2 + b^2}$

(B)  $\frac{a^2 - b^2}{a^2 + b^2}$

(D)  $\frac{a}{a^2 + b^2}$

$$\tan \theta = \frac{a}{b}$$

$$\frac{a \tan \theta + b}{a \tan \theta - b} = \frac{a \times \frac{a}{b} + b}{a \times \frac{a}{b} - b} = \frac{\frac{a^2}{b} + b}{\frac{a^2}{b} - b} = \frac{a^2 + b^2}{a^2 - b^2}$$

6 . If  $\tan \theta = \frac{a}{b}$ , then find  $\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta}$  ?

यदि  $\tan \theta = \frac{a}{b}$  हो, तो  $\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta}$  कितना होगा ?

(A)  $\frac{a^2 + b^2}{a^2 - b^2}$

(B)  $\frac{a^2 - b^2}{a^2 + b^2}$

(C)  $\frac{b}{a^2 + b^2}$

(D)  $\frac{a}{a^2 + b^2}$

$$\frac{\sin \theta}{\cos \theta} \Rightarrow \frac{a}{b}$$

$$\frac{a \sin \theta + b \cos \theta}{a \sin \theta - b \cos \theta}$$

$$\frac{a \times a + b \times b}{a \times a - b \times b} = \frac{a^2 + b^2}{a^2 - b^2}$$

7. If  $\cos \theta = \frac{x^2 - y^2}{x^2 + y^2}$ , then,  $\cot \theta = ?$

यदि  $\cos \theta = \frac{x^2 - y^2}{x^2 + y^2}$ , तो  $\cot \theta = ?$

(A)  $\frac{2xy}{x^2 - y^2}$

(B)  $\frac{2xy}{x^2 - y^2}$

(C)  $\frac{x^2 - y^2}{x^2 + y^2}$

(D)

$$\cos \theta = \frac{b}{h} = \frac{x^2 - y^2}{x^2 + y^2}$$

$P \rightarrow 2xy$

$$\begin{aligned} & x^2 - y^2 \quad 2xy \\ & (x^2 - y^2), 2ab, (a^2 + b^2) \\ & \text{Triplet is} \end{aligned}$$

$$\cot \theta = \frac{b}{P} = \frac{x^2 - y^2}{2xy}$$

$$\sin \theta = \frac{x^2 - 1}{x^2 + 1}$$

$$\cos \theta = \frac{x^2}{x^2 + 1}$$

8. If  $\tan \alpha + \cot \alpha = \sqrt{3}$ , then  $\tan^3 \alpha + \cot^3 \alpha = ?$

यदि  $\tan \alpha + \cot \alpha = \sqrt{3}$ , हो, तो  $\tan^3 \alpha + \cot^3 \alpha = ?$

~~(A)~~ 0

(B)  $\sqrt{3}$

(C)  $3\sqrt{3}$

(D) 1

$$\tan \alpha = x$$

$$x^3 + \frac{1}{x^3}$$

$$\tan \alpha + \cot \alpha = \sqrt{3}$$

$$x + \frac{1}{x} = \sqrt{3}$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= (\sqrt{3})^3 - 3x\sqrt{3} \\ &= 3\sqrt{3} - 3\sqrt{3} \\ &= 0 \end{aligned}$$

### Algebra

$$\textcircled{i} x + \frac{1}{x} = 1$$

$$x^3 = -1$$

$$\textcircled{iii} x + \frac{1}{x} = \pm \sqrt{3}$$

$$x^6 = -1$$

$$\textcircled{ii} x + \frac{1}{x} = -1$$

$$x^3 = 1$$

$$\cos \theta + \sec \theta = 1$$

$$\cos^3 \theta + \sec^3 \theta =$$

$$\cos\theta + \sec\theta = 1$$

$$\cos^3\theta + \sec^3\theta$$

$\boxed{\cos\theta = x}$

$$x + \frac{1}{x} = 1$$

$$x^3 + \frac{1}{x^3} = 1^3 - 3 \times 1$$

$$= 1 - 3$$

$$= -2$$

#

$$\sin\theta - \csc\theta = 1$$

$$x - \frac{1}{x} = 1$$

$$x^2 + \frac{1}{x^2} = 1^2 + 2 = 3$$

$$\sin^4\theta + \csc^4\theta =$$

$$x^4 + \frac{1}{x^4} = 3^2 - 2$$

$$= 7 \text{ Ans.}$$

$\sin\theta = x$

9. If  $\sin \theta + \sin^2 \theta = 1$ , then  $\cos^2 \theta + \cos^4 \theta = ?$

यदि  $\sin \theta + \sin^2 \theta = 1$ , तो  $\cos^2 \theta + \cos^4 \theta$  किसके तुल्य है?

- (A)  $\frac{\cos^2 \theta}{\sin \theta}$
- (B)  $\frac{\sin \theta}{\cos^2 \theta}$
- (C) 1
- (D) None of these

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$$\textcircled{I} \quad \sin \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta + \cos^4 \theta = 1$$

$$\textcircled{II} \quad \cos \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta + \sin \theta = 1$$

$$\begin{aligned} \sin^2 \theta + \sin^2 \theta &= 1 \\ \cos^2 \theta &= 1 - \sin^2 \theta \end{aligned}$$

$$\sin \theta + \sin^2 \theta = 1$$

$$\sin \theta = 1 - \sin^2 \theta = \cos^2 \theta$$

$$\sin \theta = \cos^2 \theta$$

$$\cos^2 \theta + \cos^4 \theta$$

$$\cos^2 \theta + (\cos^2 \theta)^2$$

$$\sin \theta + \sin^2 \theta = 1 \text{ Ans.}$$

$$\csc^2 \theta - \cot^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

$$\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

$$*\ast\ast \quad \sec \theta + \tan \theta = \alpha$$

$$\sec \theta - \tan \theta = \frac{1}{\alpha}$$

$$(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$$

$$\csc \theta + \cot \theta = \frac{1}{\csc \theta - \cot \theta}$$

$$*\ast\ast \quad \csc \theta + \cot \theta = \alpha$$
$$\csc \theta - \cot \theta = \frac{1}{\alpha}$$

$$\frac{\sec \theta + \tan \theta = 5}{\sec \theta - \tan \theta = \frac{1}{5}} \quad \sec \theta = \frac{13}{5}$$

$$\cancel{\sec \theta} = 5 + \frac{1}{5} = \frac{26}{5} \cancel{\sec \theta} = \frac{13}{5}$$

10. If  $\sec \theta + \tan \theta = 3$ , then find  $\sec \theta$ .

यदि  $\sec \theta + \tan \theta = 3$  हो, तो  $\sec \theta$  का मान क्या होगा?

(A) 3

(B)  $\frac{3}{5}$

(C)  $\frac{5}{3}$

(D) 1

$\sec \theta + \tan \theta = 3$

$\sec \theta - \tan \theta = \frac{1}{3}$

$$\cancel{\sec \theta} = 3 + \frac{1}{3} = \frac{10}{3}$$

$\sec \theta = \frac{5}{3}$

$$\begin{aligned}
 \sec \theta &= ? \\
 \sin \theta + \cos \theta &= \frac{3}{5} \\
 \cosec \theta + \cot \theta &= 4 \\
 \cosec \theta - \cot \theta &= \frac{1}{4} \\
 \hline
 2\cosec \theta &= 4 + \frac{1}{4} = \frac{17}{4} \\
 \cosec \theta &= \frac{17}{8} = \frac{h}{P}
 \end{aligned}$$

$b = 15$

$$\sec \theta = \frac{h}{b} = \frac{17}{15}$$

#  $\sin\theta \pm \cos\theta = \alpha$

$$\sin\theta \pm \cos\theta = \sqrt{2 - \alpha^2}$$

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$$\sin\theta + \cos\theta = \frac{3}{5}$$

$$\begin{aligned}\sin\theta - \cos\theta &= \sqrt{2 - \frac{9}{25}} = \sqrt{\frac{50-9}{25}} \\ &= \sqrt{\frac{41}{25}} \\ &= \frac{\sqrt{41}}{5}\end{aligned}$$