$$x + \frac{1}{x} = 4$$

$$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

$$52 \times 14 \quad -4$$

$$728 - 4 = 724 \underline{Ans}$$

① $x^2 + \frac{1}{x^2} = 4^2 - 2$
$$= 14$$

⑪ $x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$
$$= 64 - 12$$
$$= 52$$

#
① $x^5 - \frac{1}{x^5}$

⑪ $x^7 - \frac{1}{x^7}$

$$x + \frac{1}{x} = 5$$

$$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right)$$
$$- \left(x + \frac{1}{x}\right)$$

$$110 \times 23 - 5$$
$$= 2525$$

(i) $x^5 + \dfrac{1}{x^5} = \left(x^2 + \dfrac{1}{x^2}\right)\left(x^3 + \dfrac{1}{x^3}\right) - \left(x + \dfrac{1}{x}\right)$ ✓

(ii) $x^7 + \dfrac{1}{x^7} = \left(x^3 + \dfrac{1}{x^3}\right)\left(x^4 + \dfrac{1}{x^4}\right) - \left(x + \dfrac{1}{x}\right)$ ✓

(iii) $x^{11} + \dfrac{1}{x^{11}} = \left(x^5 + \dfrac{1}{x^5}\right)\left(x^6 + \dfrac{1}{x^6}\right) - \left(x + \dfrac{1}{x}\right)$

(iv) $x^{13} + \dfrac{1}{x^{13}} = \left(x^6 + \dfrac{1}{x^6}\right)\left(x^7 + \dfrac{1}{x^7}\right) - \left(x + \dfrac{1}{x}\right)$

$x^5 + \dfrac{1}{x} + x + \dfrac{1}{x^5} - x - \dfrac{1}{x}$

$\boxed{x^5 + \dfrac{1}{x^5}}$

① $(a+b)^2 = a^2 + 2ab + b^2 \Rightarrow (a+b)^2 = (a-b)^2 + 4ab$

② $(a-b)^2 = a^2 - 2ab + b^2 \Rightarrow (a-b)^2 = (a+b)^2 - 4ab$

③ $a^2 - b^2 = (a+b)(a-b)$

④ $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

⑤ $(a+b)^2 - (a-b)^2 = 4ab$

⑥ $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

⑦ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$\qquad = a^3 + b^3 + 3ab(a+b)$

⑧ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$\qquad = a^3 - b^3 - 3ab(a-b)$

$$\text{⑨} \quad a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$
$$= (a+b)\left[(a+b)^2 - 3ab\right]$$
$$= (a+b)\left[a^2 - ab + b^2\right]$$

$$\text{⑪} \quad a^3 + b^3 + c^3 - 3abc = (a+b+c)\left[a^2 + b^2 + c^2 - ab - bc - ca\right]$$

$$gf \rightarrow a + b + c = 0$$
$$\boxed{a^3 + b^3 + c^3 = 3abc}$$

$$\text{⑩} \quad a^3 - b^3 = (a-b)^3 + 3ab(a-b)$$
$$= (a-b)\left[(a-b)^2 + 3ab\right]$$
$$= (a-b)\left[a^2 + ab + b^2\right]$$

$$\text{⑫} \quad a^3 + b^3 + c^3 - 3abc = (a+b+c)\left[(a+b+c)^2 - 3(ab+bc+ca)\right]$$

$$\text{⑬} \quad a^3 + b^3 + c^3 - 3abc = \frac{1}{2}\left[a+b+c\right]\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

$$\text{⑭} \quad a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

$a = 99$ ⎤
$b = 98$ ⎬ 1
$c = 97$ ⎦ 2

$$a_1 b_1 c \rightarrow A.p \ series$$
$$3d^2$$
$$3 \times 1^2 = 3 \ Ans$$

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$$

$$= \frac{1}{2}\left[1 + 1 + 4\right]$$

$$= \frac{1}{2} \times 6 = 3 \ Ans.$$

$a = 999$
$b = 997$
$c = 995$
$a^2 + b^2 + c^2 - ab - bc - ca = 3 \times 2^2$
$= 3 \times 4$
$= 12$

1. Find the value of expression $x^4-17x^3+17x^2-17x+17$, when $x = 16$.

जब $\boxed{x = 16}$ हो, तो व्यंजक $x^4-17x^3+17x^2-17x+17$ का मान क्या होगा?

$16+1$

(A) 1   (B) 2   (C) 3   (D) 0

$$x^4-17x^3+17x^2-17x+17$$

$$x^4-16x^3-x^3+16x^2+x^2-16x-x+16+1$$
$$\underset{x\times x^3}{} \quad \underset{x\times x^2}{} \quad \underset{-x\times x}{} \quad \underset{x}{}$$

$$x^4-x^4-x^3+x^3+x^2-x^2-x+x+1 = \boxed{1}$$

$\boxed{x=18}$

$x^4-19x^3+19x^2-19x+\boxed{22}$

$18+\boxed{4}$

4 Ans.

2. If $x = 997$, $y = 998$ and $z = 999$, then the value of $x^2 + y^2 + z^2 - xy - yz - zx$, is

यदि $x = 997$, $y = 998$ और $z = 999$ हो, तो $x^2 + y^2 + z^2 - xy - yz - zx$ का मान बतायें।

(A) 1   (B) 3   (C) 0   (D) -1

$$a^2 + b^2 + c^2 - ab - bc - ca$$
$$= \frac{1}{2}\left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right]$$

$$3a^2 = 3 \times 1^2$$
$$= 3$$

3. If $x = z = 225$ and $y = 226$ then the value of : $x^3 + y^3 + z^3 - 3xyz$.

यदि $x = z = 225$ और $y = 226$ हो, तो $x^3 + y^3 + z^3 - 3xyz$ का मान ज्ञात करें।

(A) 674   (B) 573   (C) 225   (D) 676

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2} \times (x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$$

$$= \frac{1}{2}(225 + 226 + 225)\left[1 + 1 + 0\right]$$

$$= \frac{1}{2} \times 676 \times 2$$

$$= 676$$

4. If, $a + b + c = 0$, then find $a^3 + b^3 + c^3$.

यदि $a + b + c = 0$ हो, तो $a^3 + b^3 + c^3$ निकालें।

(A) $2\,abc$ (B) abc (C) $3abc$ (D) $a+b+c$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)\left[a^2 + b^2 + c^2 - ab - bc - ca\right]$$

$$gf \rightarrow (a+b+c) = 0$$

$$\boxed{a^3 + b^3 + c^3 = 3abc}$$

5. If $a^2 + b^2 + c^2 = ab + bc + ca$, then find the value of $\dfrac{a+c}{b}$.

यदि $a^2 + b^2 + c^2 = ab + bc + ca$ हो, तो $\dfrac{a+c}{b}$ का मान ज्ञात करें।

$\dfrac{1+1}{1} = \dfrac{2}{1} = 2$

(A) −1   (B) 2   (C) 0   (D) 1

$$a^2 + b^2 + c^2 = ab + bc + ca$$

$a^2 = ab \quad\big|\quad b^2 = bc \quad\big|\quad c^2 = ca$

$a = b \quad\big|\quad b = c \quad\big|\quad c = a$

$\overline{OX}$

$a = b = c = 1$

$= 2 = 3$

$\dfrac{2+2}{2} = \dfrac{4}{2} = 2$

$\dfrac{3+3}{3} = \dfrac{6}{3} = 2$

6. If $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$, then $\dfrac{x^2}{9} + \dfrac{y^2}{25} + \dfrac{z^2}{16} = ?$

यदि $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$, तो $\dfrac{x^2}{9} + \dfrac{y^2}{25} + \dfrac{z^2}{16} = ?$

(A)   12   (B)   9   (C)   3   (D)   1

$(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$

$\underset{0}{\parallel} \qquad \underset{0}{\parallel} \qquad \underset{0}{\parallel}$

$(x-3)^2 = 0 \quad \bigg| \quad (y-5)^2 = 0 \quad \bigg| \quad (z-4)^2 = 0$

$x = 3 \qquad\qquad y = 5 \qquad\qquad z = 4$

$\dfrac{x^2}{9} + \dfrac{y^2}{25} + \dfrac{z^2}{16}$

$\dfrac{9}{9} + \dfrac{25}{25} + \dfrac{16}{16}$

$1 + 1 + 1 = 3$

7. If $x + y + z = 6$ and $x^2 + y^2 + z^2 = 20$, then find the value of $x^3 + y^3 + z^3 - 3xyz$.

यदि $x + y + z = 6$ और $x^2 + y^2 + z^2 = 20$ हो, तो $x^3 + y^3 + z^3 - 3xyz$ का मान क्या होगा ?

(A) 72   (B) 70   (C) 64   (D) 76

$$(a-b)^2 + (b-c)^2 + (c-a)^2 = 3(a^2+b^2+c^2) - (a+b+c)^2$$

$$(x+y+z)^2 = x^2+y^2+z^2 + 2(xy+yz+zx)$$

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$$

$$= \frac{1}{2}(x+y+z)\left[3(x^2+y^2+z^2) - (x+y+z)^2\right]$$

$$= \frac{1}{2} \times 6 \times [3 \times 20 - 36]$$

$$= 3 \times 24 = 72$$

$$a^2 + b^2 + c^2 + 4a - 6b + 2c + 14 = 0$$

$$(a^2 + 4a + 4) + (b^2 - 6b + 9) + (c^2 + 2c + 1) = 0$$

$$(a+2)^2 + (b-3)^2 + (c+1)^2 = 0$$

$(a+2)^2 = 0$	$(b-3)^2 = 0$	$(c+1)^2 = 0$
$a + 2 = 0$	$b - 3 = 0$	$c + 1 = 0$
$a = -2$	$b = 3$	$c = -1$

$$a + b + c =$$
$$-2 + 3 - 1 = 0 \text{ Ans.}$$

$$a^2 + b^2 + c^2 \boxed{+4}a \boxed{-6}b \boxed{+2}c + 14 = 0$$

$$\# a^2 + b^2 + c^2 \boxed{-6}a \boxed{-8}b \boxed{+4}c + 29 = 0$$

① $a = \dfrac{-\cancel{4}}{\cancel{2}} = -2$

$a = \dfrac{+\cancel{6}}{\cancel{2}} = 3$

② $b = \dfrac{+\cancel{6}}{2} = 3$

$b = \dfrac{+\cancel{8}}{\cancel{2}} = 4$

③ $c = \dfrac{-\cancel{2}}{\cancel{2}} = -1$

$c = \dfrac{-4}{2} = -2$

$$(a+b-c)^2 =$$
$$(3+4+2)^2 = 9^2 = 81$$

$$4a^2 + 1b^2 + 1c^2 - 12a - 6b + 8c + 34 = 0$$

$$a = \frac{+12}{8} = \frac{3}{2}$$

$$b = \frac{+6}{2} = 3$$

$$c = \frac{-8}{2} = -4$$

$$a + b - c = \frac{3}{2} + 3 + 4$$

$$1.5 + 3 + 4$$

$$= 8.5$$

8. If $x^2 + y^2 + 1 = 2x$, then $x^5 + y^{15} = ?$

यदि $x^2 + y^2 + 1 = 2x$, तो $x^5 + y^{15} = ?$

(A) 0   (B) 2   (C) -1 (D) 1

$$x^2 + y^2 - 2x + 1 = 0$$

$$x \rightarrow \frac{+2}{2} = 1$$

$$y \rightarrow \frac{0}{2} = 0$$

$$(1)^5 + 0^{15}$$

$$1 + 0 = 1 \ Ans.$$

9. If $x = 3 + 2\sqrt{2}$, then $\dfrac{x^6 + x^4 + x^2 + 1}{x^3} = ?$

यदि $x = 3 + 2\sqrt{2}$, तो $\dfrac{x^6 + x^4 + x^2 + 1}{x^3} = ?$ $x^3 + x + \dfrac{1}{x} + \dfrac{1}{x^3} = x^3 + \dfrac{1}{x^3} + x + \dfrac{1}{x}$

$$198 + 6$$

(A) 192   (B) 204   (C) 198   (D) 216 $= 204$

$$x = 3 + 2\sqrt{2}$$

$$\frac{1}{x} = 3 - 2\sqrt{2}$$

$$x + \frac{1}{x} = 6$$

$$x^3 + \frac{1}{x^3} = 6^3 - 3 \times 6 = 198$$

10. If $x - \dfrac{1}{x} = 1$, then $\dfrac{x^4 - \dfrac{1}{x^2}}{3x^2 + 5x - 3} = ?$

यदि $x - \dfrac{1}{x} = 1$, तो $\dfrac{x^4 - \dfrac{1}{x^2}}{3x^2 + 5x - 3} = \dfrac{x\left[x^3 - \dfrac{1}{x^3}\right]}{x\left[3x + 5 - \dfrac{3}{x}\right]} = \dfrac{x^3 - \dfrac{1}{x^3}}{3\left(x - \dfrac{1}{x}\right) + 5} = \dfrac{4}{3 \times 1 + 5}$

(A) 0　　(B) $\dfrac{1}{2}$　　(C) $\dfrac{1}{4}$　　(D) $\dfrac{3}{4}$

$= \dfrac{4}{8}$

$= \dfrac{1}{2}$

$x - \dfrac{1}{x} = 1$

$x^3 - \dfrac{1}{x^3} = 1^3 + 3 \times 1$

$= 1 + 3$

$= 4$

11. If $a + \dfrac{1}{a} = \sqrt{3}$, then find the value of $(a^{18} + a^{12} + a^6 + 1)$.

यदि $a + \dfrac{1}{a} = \sqrt{3}$ हो, तो $(a^{18} + a^{12} + a^6 + 1)$ का मान बतायें।

$-x + x$

(A) 0   (B) 1   (C) -1 (D) 4

$$a + \frac{1}{a} = \sqrt{3}$$

$$a^3 + \frac{1}{a^3} = (\sqrt{3})^3 - 3\sqrt{3}$$
$$= 3\sqrt{3} - 3\sqrt{3} = 0$$

$$\boxed{a^3 + \frac{1}{a^3} = 0}$$

$$a^3 + \frac{1}{a^3} = 0$$

$$\frac{a^6 + 1}{a^3} = 0$$

$$a^6 + 1 = 0$$

$$\boxed{a^6 = -1}$$

18

$12 - 18$

19