Algebra

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$$x + \frac{1}{x} = 5$$

$$x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5$$

$$125 - 15$$

$$110$$

$$x + \frac{1}{x} = a$$

$$x^2 + \frac{1}{x^2} = a^2 - 2$$

$$x^3 + \frac{1}{x^3} = a^3 - 3a$$

$$x - \frac{1}{x} = a$$

$$x^2 + \frac{1}{x^2} = a^2 + 2$$

$$x^3 - \frac{1}{x^3} = a^3 + 3a$$

Low power to High power

$x + \frac{1}{x} = 3$

$x^2 + \frac{1}{x^2} = 3^2 - 2$
$= 7$

$x^4 + \frac{1}{x^4} = 7^2 - 2$
$= 47$

$x + \frac{1}{x} = 5$

$x^2 + \frac{1}{x^2} = 5^2 - 2$
$= 23$

$x^4 + \frac{1}{x^4} = 23^2 - 2$
$= 529 - 2$
$= 527$

$x + \frac{1}{x} = 3$

$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$

$x^4 + \frac{1}{x^4} = 7^2 - 2 = 47$

$x^8 + \frac{1}{x^8} = 47^2 - 2$
$= 2209 - 2$
$= 2207$

$x - \frac{1}{x} = 1$

$x^2 + \frac{1}{x^2} = 1^2 + 2 = 3$

$x^4 + \frac{1}{x^4} = 3^2 - 2 = 7$

$x^8 + \frac{1}{x^8} = 7^2 - 2 = 47$

$x^{16} + \frac{1}{x^{16}} = 47^2 - 2$
$= 2207$

$x - \frac{1}{x} = 2$

$x^2 + \frac{1}{x^2} = 2^2 + 2 = 6$

$x^4 + \frac{1}{x^4} = 6^2 - 2 = 34$

$x^8 + \frac{1}{x^8} = 34^2 - 2$
$= 1156 - 2$
$= 1154$

$x^5 + \frac{1}{x^5} = 9$

$x^{7.5} - \frac{1}{x^{7.5}} = 5$

$x^{10} + \frac{1}{x^{10}} = 9^2 - 2$

$= 81 - 2$

$= 79$

$x^{15} + \frac{1}{x^{15}} = 5^2 + 2 = 27$

$x^{1.25} + \frac{1}{x^{1.25}} = 15$

$x^{2.5} + \frac{1}{x^{2.5}} = 15^2 - 2$

$225 - 2$

223

Highest power to Lowest power

$$x^4 + \frac{1}{x^4} = 47$$

$x^4 + \frac{1}{x^4}$

47

$x^2 + \frac{1}{x^2}$

$\sqrt{47+2}$

$= 7$

$x + \frac{1}{x}$

$\sqrt{7+2}$

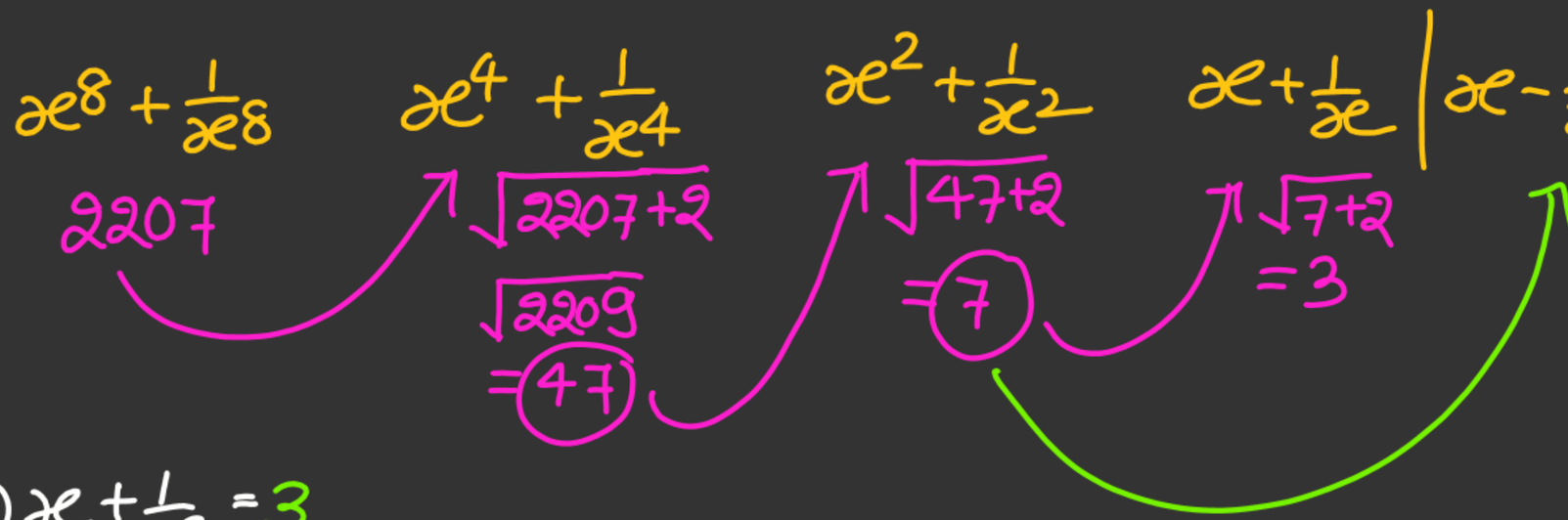
$= 3$ Ans

$x - \frac{1}{x}$

$\sqrt{7-2}$

$\sqrt{5}$

$x^8 + \frac{1}{x^8} = 2207$



(i) $x + \frac{1}{x} = 3$

(ii) $x - \frac{1}{x} = \sqrt{5}$

$x^{16} + \frac{1}{x^{16}} = 23$

$x^8 - \frac{1}{x^8} = \sqrt{23-2}$

$= \sqrt{21}$

$$x^{27} + \frac{1}{x^{27}} = 47$$

$$x^{13.5} - \frac{1}{x^{13.5}} = \sqrt{47-2}$$

$$= \sqrt{45}$$

$$= 3\sqrt{5}$$

Same power

(i) $x^n + \frac{1}{x^n} = A$

$x^n - \frac{1}{x^n} = \sqrt{A^2-4}$

$x^2 + \frac{1}{x^2} = 7$

$x^2 - \frac{1}{x^2} = \sqrt{7^2-4}$

$= \sqrt{45}$

$= 3\sqrt{5}$

(ii) $x^n - \frac{1}{x^n} = A$

$x^n + \frac{1}{x^n} = \sqrt{A^2+4}$

$x^{19} - \frac{1}{x^{19}} = 5$

$x^{19} + \frac{1}{x^{19}} = \sqrt{5^2+4}$

$= \sqrt{29}$

$x^{729} - \frac{1}{x^{729}} = 8$

$x^{729} + \frac{1}{x^{729}} = \sqrt{8^2+4}$

$= \sqrt{68}$

$= 2\sqrt{17}$

$x + \frac{1}{x} = 3$

$x - \frac{1}{x} = \sqrt{3^2 - 4}$
$= \sqrt{5}$

$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$

$x^2 - \frac{1}{x^2} = \sqrt{7^2 - 4}$
$= \sqrt{45}$
$= 3\sqrt{5}$

$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$
$= 3 \times \sqrt{5}$
$= 3 \times \sqrt{5}$

$x + \frac{1}{x} = 3$

(i) $x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$

$18 \times 7 \quad -3$

$126 - 3 = 123$ Ans

$x^2 + \frac{1}{x^2} = 3^2 - 2$

$= 7$

$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3$

$= 27 - 9$

$= 18$

$x^4 + \frac{1}{x^4} = 7^2 - 2$

$= 47$

(ii) $x^7 + \frac{1}{x^7} = \left(x^4 + \frac{1}{x^4}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$

$47 \times 18 - 3$

$846 - 3$

843 Ans

$x + \frac{1}{x} = 4$

$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$

$52 \times 14 \quad -4$

$728 - 4 = 724$ Ans

① $x^2 + \frac{1}{x^2} = 4^2 - 2$

$= 14$

② $x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$

$= 64 - 12$

$= 52$

#

① $x^5 - \frac{1}{x^5}$

② $x^7 - \frac{1}{x^7}$

$x + \frac{1}{x} = 5$

$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$

$110 \times 23 - 5$

$= 2525$

(i) $x^5+\frac{1}{x^5} = \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$ ✓

(ii) $x^7+\frac{1}{x^7} = \left(x^3+\frac{1}{x^3}\right)\left(x^4+\frac{1}{x^4}\right)-\left(x+\frac{1}{x}\right)$ ✓

$x^5+\frac{1}{x}+x+\frac{1}{x^5}-x-\frac{1}{x}$

$x^5+\frac{1}{x^5}$

(iii) $x^{11}+\frac{1}{x^{11}} = \left(x^5+\frac{1}{x^5}\right)\left(x^6+\frac{1}{x^6}\right)-\left(x+\frac{1}{x}\right)$

(iv) $x^{13}+\frac{1}{x^{13}} = \left(x^6+\frac{1}{x^6}\right)\left(x^7+\frac{1}{x^7}\right)-\left(x+\frac{1}{x}\right)$