$$12 \longrightarrow 1, 2, 3, 4, 6, 12$$

गुणनखण्डों का औसत $= \dfrac{1+2+3+4+6+12}{6} = \dfrac{\text{कुल गुणनखण्डों का योग}}{\text{कुल गुणनखण्डों की सं०}}$

***

$$\boxed{\text{Average of factors} = \dfrac{\text{Sum of factors}}{T \cdot N \cdot F}}$$

**28.** Find the average of all factor of 720?

720 के सभी गुणखण्डों का औसत ज्ञात करो?

   (a)  12.5       (b) 70.6      (c) 80.6      (d) 55.6

$$720 \rightarrow 2^4 \times 3^2 \times 5^1$$

(i) T·N·F $\rightarrow 5 \times 3 \times 2 = 30$

(ii) योग $\rightarrow \left[\underset{2^0+2^1+2^2+2^3+2^4}{\overset{1+2+4+8+16}{}}\right] \times \left[\underset{3^0+3^1+3^2}{\overset{1+3+9}{}}\right] \times \left[\underset{5^0+5^1}{\overset{1+5}{}}\right]$

$$31 \times 13 \times 6$$

$$\dfrac{72 \times 10}{8 \times 9 \times 2 \times 5}$$

$$औसत = \dfrac{31 \times 13 \times 6}{\dfrac{30}{5}} = \dfrac{403}{5} = 80.6$$

Average of factors $= \dfrac{\text{sum of factors}}{T·N·F}$

## 60 के सभी गुणनखण्डों का औसत निकालें।

$60 \rightarrow 2^2 \times 3^1 \times 5^1$

(i) $T.N.F \rightarrow 3 \times 2 \times 2 = 12$

(ii) योग $\rightarrow \left[2^0 + 2^1 + 2^2\right] \times \left[3^0 + 3^1\right] \times \left[5^0 + 5^1\right]$

$$7 \times 4 \times 6$$

Average of factors $= \dfrac{\text{sum of factors}}{T.N.F}$

$$= \dfrac{7 \times \overset{2}{\cancel{4}} \times \cancel{6}}{\cancel{12}} = 14 \text{ Ans.}$$

$$
\begin{array}{r|l}
2 & 60 \\
\hline
2 & 30 \\
\hline
3 & 15 \\
\hline
 & 5
\end{array}
$$

**29.** If $N = 2^3 \times 3^5 \times 5^6 \times 7^2$, find number of factor that are perfect square? ☑️① ☑️② ☑️③ ☑️①

यदि $N = 2^{\frac{3}{2}} \times 3^{\frac{5}{2}} \times 5^{\frac{6}{2}} \times 7^{\frac{2}{2}}$ तब $N$ के वो गुणनखण्ड ज्ञात करों जो पूर्णत: वर्ग हो?

(a) 47     (b) 24     (c) 48 ✓     (d) 78

① $x^{2n} \longrightarrow$ perfect square

② $x^{3n} \longrightarrow$ perfect cube

Both $\rightarrow x^2, x^3 \Rightarrow x^6$

$2, 3 \xrightarrow{LCM} 6$

① perfect square

② perfect cube

③ Both $\rightarrow$ perfect cube and perfect square    yes/no.

# perfect square $\rightarrow 2 \times 3 \times 4 \times 2$
$= 48$

③ $x^{6n} \longrightarrow$ Both

**29.** If $N = 2^3 \times 3^5 \times 5^6 \times 7^2$, find number of factor that are perfect square?

यदि $N = 2^3 \times 3^5 \times 5^6 \times 7^2$ तब $N$ के वो गुणनखण्ड ज्ञात करों जो पूर्णतः वर्ग हो?

(a) 47      (b) 24      (c) 48      (d) 78

$$\boxed{1} \quad \boxed{2} \quad \boxed{3} \quad \boxed{1}$$

$$2^{\frac{3}{2}} \times 3^{\frac{5}{2}} \times 5^{\frac{6}{2}} \times 7^{\frac{2}{2}}$$

perfect square $\Rightarrow 2 \times 3 \times 4 \times 2$

$= 48$

# 

$$\overset{\checkmark}{\boxed{3}} \quad \overset{\checkmark}{\boxed{4}} \quad \overset{\checkmark}{\boxed{2}}$$

$$2^{\frac{7}{2}} \times 3^{\frac{8}{2}} \times 5^{\frac{5}{2}}$$

perfect square $\longrightarrow 4 \times 5 \times 3$

$$= 60$$

# $4^3 \times 3^5 \times 5^7 \implies 2^{\frac{6}{2}} \times 3^{\frac{5}{2}} \times 5^{\frac{7}{2}}$

$\boxed{3} \quad \boxed{2} \quad \boxed{3}$

$$(2^2)^3 = 2^6$$

perfect square $\longrightarrow 4 \times 3 \times 4$

$$= 48 \text{ Ans.}$$

**29.** If $N = 2^3 \times 3^5 \times 5^6 \times 7^2$, find number of factor that are perfect

cube ? $\boxed{1}$ ✓ $\boxed{1}$ ✓ $\boxed{2}$ ✓ $\boxed{0}$ ✓

यदि $N = 2^{\frac{3}{3}} \times 3^{\frac{5}{3}} \times 5^{\frac{6}{3}} \times 7^{\frac{2}{3}}$ तब $N$ के वो गुणनखण्ड ज्ञात करों जो पूर्णत: धन हो?

(a) 47      (b) 24      (c) 48      (d) 78    (e) 12

$$perfect\,cube \longrightarrow 2 \times 2 \times 3 \times 1$$
$$= 12 \underline{Ans.}$$

$$\boxed{2} \quad \boxed{1} \quad \boxed{2} \quad \boxed{2}$$

$$9\frac{7}{3} \times 2\frac{4}{3} \times 7\frac{6}{3} \times 11\frac{8}{3}$$

perfect cube $\Rightarrow 3 \times 2 \times 3 \times 3$

$$= 54 \underline{Ans}.$$

$$9^4 \rightarrow (3^{2 \times})^4$$
$$= 3^8$$

$$x^2, x^3 \rightarrow \boxed{x^6}$$

$$\# \quad 2^9 \times \underset{3^8}{9^4} \times 5^{11} \times 7^6 \Rightarrow 2^{\frac{9}{6}} \times 3^{\frac{8}{6}} \times 5^{\frac{11}{6}} \times 7^{\frac{6}{6}}$$

$$\boxed{1} \quad \boxed{1} \quad \boxed{1} \quad \boxed{1}$$

Both $\longrightarrow 2 \times 2 \times 2 \times 2 = 16 \underline{Ans}.$

$12 \rightarrow 1, 2, 3, 4, 6, 12$

$1 \times 12 = 12$

$2 \times 6 = 12$

$3 \times 4 = 12$

$12 \times 12 \times 12 = 12^3 = 1728$

# product of factors $= (Number)^{\frac{T.N.F}{2}}$

$= (12)^{\frac{6}{2}} = 12^3$
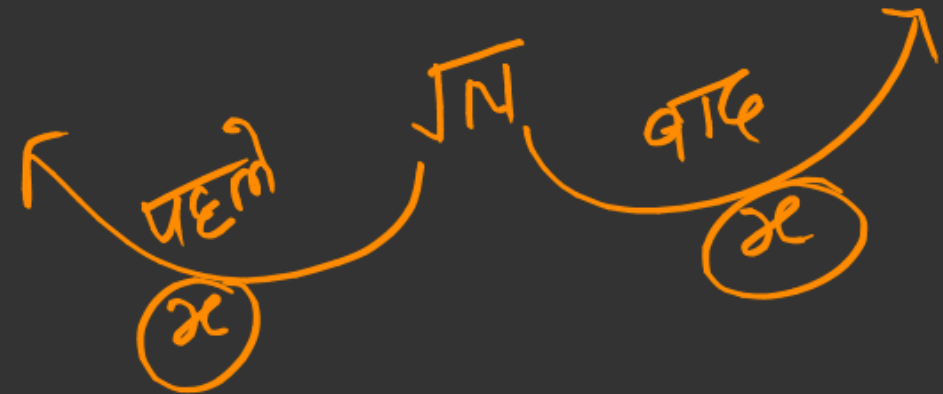
$= 1728 \ Ans.$

$$\sqrt{16}$$

$16 \rightarrow 1, 2, 4, 8, 16$

$1 \times 16 = 16$
$2 \times 8 = 16$
$\sqrt{16} = 4$

$N^2 \rightarrow$ perfect square

पहले $x$    $\sqrt{N}$    बाद $x$

$T \cdot N \cdot F \rightarrow 2x+1$

$\rightarrow$ odd no.

Note:→ पूर्ण वर्ग संख्य के गुणनखण्डों की संख्या विषम संख्या में होती है

16 के सभी गुणनखण्डों का गुणनफल निकाले

$2^4$

$\boxed{T \cdot N \cdot F \to 5}$

$$\text{product of factors} = (\text{Number})^{\frac{T \cdot N \cdot F}{2}} = (16)^{\frac{5}{2}}$$

$$= (4^2)^{\frac{5}{2}}$$

$$= 4^{\frac{2 \times 5}{2}} = 4^5$$

$$= 2^{10}$$

$$= 1024$$

**30.** Find product of factor of 30?

30 के सभी गुणनखण्डों का गुणनफल ज्ञात करो?

(a) 27000     (b) 21256     (c) 78525     (d) 810000

$$30 \rightarrow 2^1 \times 3^1 \times 5^1$$

$$T.N.F \rightarrow 2 \times 2 \times 2 = 8$$

$$\text{product of factors} = (\text{Number})^{\frac{T.N.F}{2}}$$

$$= (30)^{\frac{8}{2}}$$

$$= 30^4$$

$$= 810000$$

**31.** Find sum of reciprocal of all factor of 100?

100 के गुणनखण्डों के व्युत्क्रमों का योगफल क्या होगा?

(a) $\dfrac{117}{100}$

(b) $\dfrac{157}{25}$

(c) $\dfrac{227}{100}$

(d) $\dfrac{217}{100}$

***

$$\text{Sum of Reciprocal of all factors} = \frac{\text{Sum of factors}}{\text{Number}} \Rightarrow \frac{7 \times 31}{100} = \frac{217}{100} = 2.17$$

$$100 \rightarrow 2^2 \times 5^2$$

$$\text{योग} \rightarrow \left[ \overset{1+2+4}{2^0 + 2^1 + 2^2} \right] \times \left[ \overset{1+5+25}{5^0 + 5^1 + 5^2} \right]$$

$$7 \times 31$$

\# 60 के सभी गुणनखण्डों के व्युत्क्रमों का योग

$$60 \rightarrow 2^2 \times 3^1 \times 5^1$$

$$\text{योग} \rightarrow \left[2^0 + 2^1 + 2^2\right] \times \left[3^0 + 3^1\right] \times \left[5^0 + 5^1\right]$$

$$7 \times 4 \times 6$$

$$\text{Sum of reciprocal of all factors} = \frac{\text{sum of factors}}{\text{Number}} = \frac{7 \times 4 \times 6}{\underset{\textstyle \boxed{10}}{60}} = \frac{28}{10} = 2 \cdot 8$$

Distinct prime Factor $\Rightarrow$ विशिष्ट अभाज्य गुणनखण्ड

or

Unique prime factors

$12 \rightarrow 2^2 \times 3^1$

Base में prime

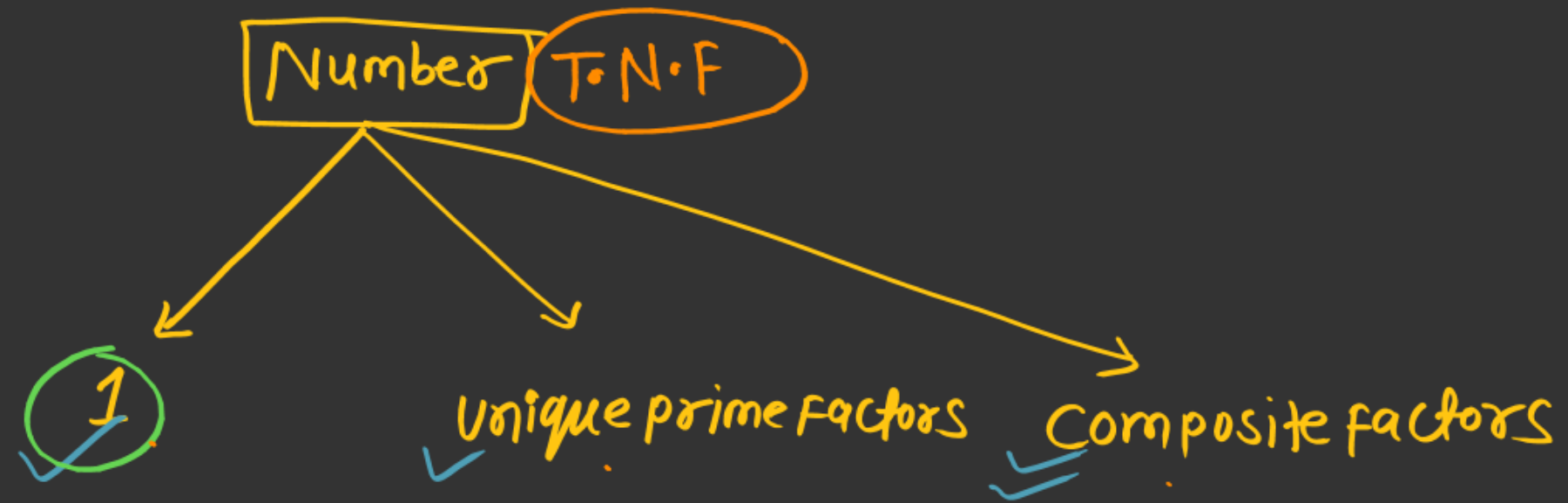Unique prime factors $\longrightarrow 2$

120 में unique prime factors

$120 \rightarrow 2^3 \times 3^1 \times 5^1$

Unique prime factors $= 3$

$12 \rightarrow 1, 2, 3, 4, 6, 12$

Distinct prime factors $= 2$

Number T·N·F

1

Unique prime factors    Composite factors

1 is  Neither prime,
Nor composite

Total no. of composite factors = T·N·F – Unique prime factors – 1
कुल भाज्य गुणनखण्डो की सं०

\# 60 के कुल भाज्य गुणनखण्डों की सं० निकालें।

$$60 \rightarrow 2^2 \times 3^1 \times 5^1$$

$$T.N.F \rightarrow 3 \times 2 \times 2 = 12$$

$$T.N.C.F \Rightarrow T.N.F - \text{Unique prime} - 1 = 12 - 3 - 1 = 8 \text{ Ans.}$$

Total no. of composite factors

\# $720 \rightarrow 2^4 \times 3^2 \times 5^1$

T.N.F $\rightarrow 5 \times 3 \times 2$

$\qquad = 30$

Composite factors $\rightarrow$ T.N.F $-$ Unique prime $-1$

$\qquad = 30 - 3 - 1$

$\qquad = 26$ Ans.

\# $210$ के composite no. of factors

$$
\begin{array}{r|r}
2 & 210 \\
\hline
3 & 105 \\
\hline
5 & 35 \\
\hline
 & 7
\end{array}
$$

$210 \rightarrow 2^1 \times 3^1 \times 5^1 \times 7^1$

T.N.F $\rightarrow 2 \times 2 \times 2 \times 2 = 16$

T.C.N.F $\rightarrow$ T.N.F $-$ Unique prime $-1$

$\qquad = 16 - 4 - 1$

$\qquad = 11$ Ans

**32.** Find no of zero when multiplay first 50 prime number.

प्रथम 50 अभाज्य संख्याओं के गुणनफल के अंत में कितने शून्य होगे।

(a) 1      (b) 10      (c) 15      (d) 20

No. of zero

अंत से शून्यों की सं०

22 Aug.

1 pm

19 Aug.