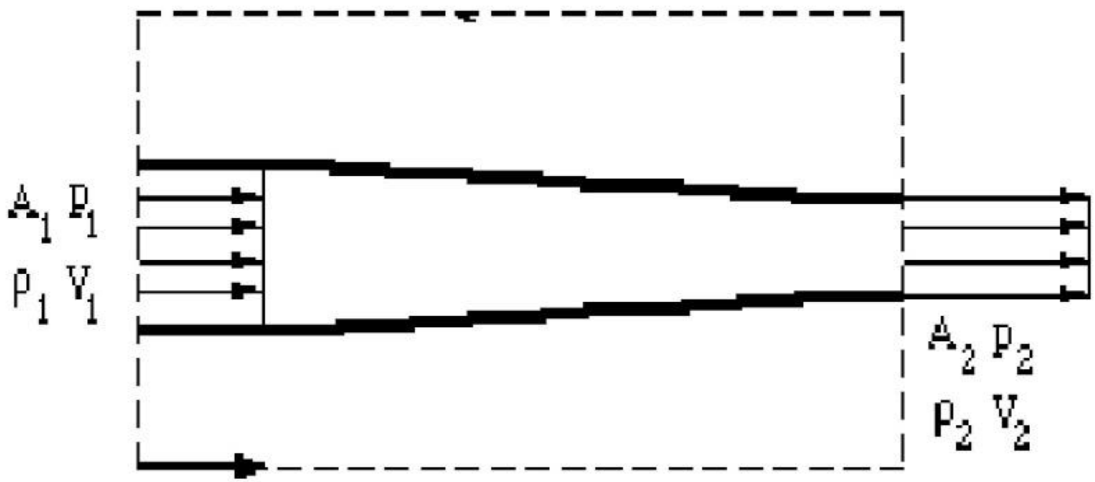
Theory of Wind Energy

The principles concerned with converting the potential energy of fluids into useful power relies on three basic fundamentals: conversation of mass, energy and momentum, so it is useful to discuss these before examining the operation of wind turbines.

Conservation of Mass:

The continuity equation applies the principle of conservation of mass to fluid flow. Consider a fluid flowing through a fixed conduit having one inlet and one outlet as shown in figure below



If the flow is steady i.e no accumulation of fluid within the control volume, then the rate of fluid flow at entry must be equal to the rate of fluid flow at exist for mass conservation. If the flow cross-sectional area A (m^2), and the fluid parcel travels a distance dL in time dt, then the volume flow rate (Vf m^3/s) is given by:

$$Vf = A.dL/dt$$

But since dL/dt is the fluid velocity (V, m/s) we can write: Vf = V x A

The mass flow rate (m, kg/s) is given by the product of density and volume flow rate. Between any two points within the control volume, the fluid mass flow rate can be shown to remain constant:

Or $\rho 1A1V1 = \rho 2A2V2$ ---------------(1)

Conservation of Energy:

Conservation of energy necessitates that the total energy of the fluid remains constant. However, there can be transformation from one form to another.

There are three forms of non-thermal energy for a fluid at any given point:-

The kinetic energy due to the motion of the fluid.
The potential energy due to the positional elevation above a datum.
The pressure energy due to the absolute pressure of the fluid at that point.

If all energy terms are written in the form of the head (potential energy), i.e., in meters of the fluid, then conservation of energy principle requires that:

$$\left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right) = \left(\frac{p}{\rho g}+\frac{V^2}{2g}+z\right) \qquad \text{------------(2a)}$$

This equation is known as the bernoulli equation and is valid if the two points of interest 1 & 2 are very close to each other and there is no loss of energy.

In a real situation, the flow will suffer a loss of energy due to friction (h_L) and obstruction between stations 1 & 2, hence

$$\left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right) = \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z\right) + h_L \qquad \text{-------------------(2b)}$$

Conservation of Momentum:

Consider a duct of length L , cross- sectional area A_c, surface area A_s, in which a fluid of density (ρ), is flowing at mean velocity V. the forces acting on a segment of the duct are that due to pressure difference and that due to friction at the walls in contact with the fluid.

If the acceleration of the fluid is zero, the net forces acting on the element must be zero, hence

$$(p_1-p_2)A_c - (f\,\rho\,V^2/2\,)A_s = 0$$

$$\text{or} \qquad p1 - p2 = \rho\, g\, h_f$$

Where $hf = f\,(As/Ac)\,v2/2g$

For a pipe $As/\,Ac = \pi\,D\,L\,/\pi D2\,/4 = 4L/D$

Hence $hf = (4\,fL/D)\,V2/2g$ ------------(3)

This is known as Darcy formula.

The value of the friction factor (f) depends mainly on two parameters namely the value of the Reynolds number and the surface roughness.

The Reynolds number is defined in terms of the density, velocity of flow, diameter and the dynamic viscosity as follows:

$$Re = \frac{\rho V D}{\mu} \qquad \text{------------------}(4)$$

For *laminar* flow (i.e., Re < 2000),

$$f = \frac{16}{Re} \qquad \text{------------------}(5a)$$

While for a *smooth* pipe with ***turbulent*** (i.e., Re > 4000) flow,

$$f = \frac{0.079}{Re^{0.25}} \qquad --------- -(5b)$$

For Re > 2000 and Re< 4000, this region is known as the critical zone and the Value of the friction factor is certain.

In the turbulent zone, if the surface of the pipe is not perfectly smooth, then the value of the friction factor has to be determined from the **Moody diagram** (see overleaf). The **relative roughness** is the average height of the surface projections on the inside of the pipe (k) to the pipe diameter (D). In common with Reynolds number and friction factor this parameter is dimensionless.

Ideal Wind Power calculations:

In theory, Wind power (P) is calculated by the following general equation (the proof for which will be derived in the following section):

P = Cp * ½ ρ * A * V^3 ---------------(6)

Where

Cp	is the power coefficient
P	is the density of the oncoming air
A	swept area of the rotor
V	is the velocity of the wind

The actual power is further reduced by two more inefficiencies, due to the gear box losses and the generator efficiency.

The value of the ideal power is limited by what is know as Betz coefficient with a value of Cp = 0.59 as the highest possible conversion efficiency possible.

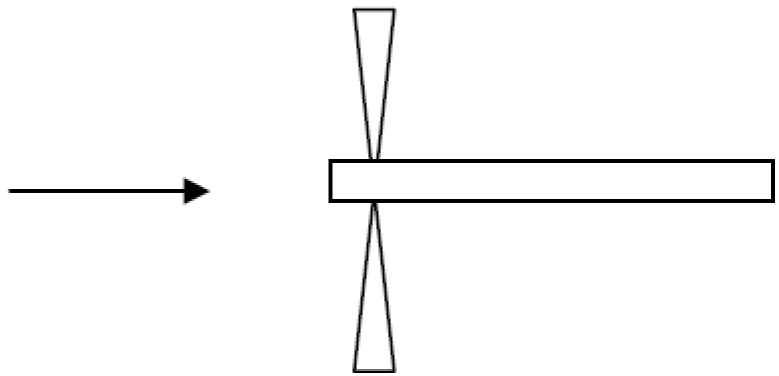
In practice, most wind turbines have efficiencies well below 0.5, depending on the type, design and operational conditions.

In the operational output range, wind power generated increases with wind speed of 5 m/s, the power output is proportionalwith5 cubed = 125, whereas at a wind speed of 10m/s, the power output is proportional to 1000. This shows that doubling the speed from 5 to 10 m/s resulted in a power increase of 8 folds. This highlights the importance of location when it comes to install wind turbines. The effect of the rotor diameter affect the power output in a square manner, i.e., doubling the rotor diameter results in increasing the power output by four times.

On the other hand, since power generated is related to wind speed by a cubic ratio. That means if your turbine is rated at producing 1KW at 12m/s then it will produce 125W at 6m/s and 15W at 3m/s.

Theory of Wind Turbines

A windmill extracts power from the wind by slowing down the wind. At stand still, the rotor obviously produces no power, and at very high rotational speeds the air is more or less blocked by the rotor, and again no power is produced.



Ideal Wind Energy Theory

The power produced (P kin) by the wind turbine is the net kinetic energy change across the wind turbine (from initial air velocity of V1 to a turbine exit air velocity of V2) is given as:

$$\text{Pkin} = ½ * \text{m}[V1^2 - V2^2] \quad \text{------------------(7)}$$

The mass flow rate of wind energy is given by the continuity equation as the product of density, area swept by the turbine rotor and the approach air velocity as:

m = ρ * A * Va ------------------(8)

Hence the power becomes:

Pkin = ½ x ρ *A * Va * $[V1^2 - V2^2]$ -----------------------(9)

Since the rotor speed is the average speed (Va) between inlet and outlet:

Va = ½ [V1 + V2] --------------(10)

Hence the power is

Pkin = (1/2)* ρ *A*(V1 + V2)/2) * $[(V1)^2 - (V2)^2]$
= (1/4)* ρ *A*$[V1^3 - V2^3 - V1 * V2^2 + V1^2 * V2)]$
= (1/4)* ρ *A*$V1^3 * [1 - \left(\frac{V2}{V1}\right)^3 - \left(\frac{V2}{V1}\right)^2 + \left(\frac{V2}{V1}\right)]$ -----------------(11)

To find the maximum power extracted by the rotor, differentiate equation 11 with respect to V2 and equate it to zero

$$\frac{dP_{kin}}{dV_2} = \frac{1}{4} * \rho * A * (-3 * v_2^2 - 2 * V_1 * V_2 + V_1^2) = 0 \text{ --------------------(12)}$$

Since the area of the rotor (A) and the density of the air (r) cannot be zero, the expression in the bracket of equation 12 has to be zero. Hence, the quadratic equation becomes:

$$(3V_2 - V_1)(V_2 + V_1) = 0$$

Since $V_2 = -V_1$ is unrealistic in this situation, there is only one solution, equation 12 yields:

$$V_2 = \frac{1}{3} V_1 \text{ -----------------(13)}$$

Substitution of equation 13 into equation 11 results in:

$$P_{kin} = (0.5925) * 1/2[\mathrm{\rho} * \mathrm{A} * \mathrm{V}_1^3] \text{ ------------------------(14)}$$

The theoretical maximum fraction of the power in the wind which could be extracted by an ideal windmill is, therefore the fraction 0.5925 is called the *Betz Coefficient.* Because of aerodynamic imperfections in any practical machine and of mechanical losses, the power extracted is less than that calculated above. Figure 3.7 demonstrates the effect of wind turbine design implications on the resulting power that can be harnessed from the incoming wind. Efficient wind turbines depend on the production of that optimum speed ratio giving the maximum or near the maximum power possible.

Equation 14 clearly shows that:

- The power is proportional to the density (r) of the air which varies slightly with altitude and temperature.
- The power is proportional to the area (A) swept by the blades and thus to the square of the radius(R) of the rotor; and
- The power varies with the cube of the wind speed (V3). This means that the **power increases eightfold if the wind speed is doubled.** Hence, one has to pay particular attention in site selection.

Distinction between rated and actual power output of the turbine

The world's largest wind turbine generator has a rotor blade diameter of 126 meters and is located on offshore, at sea-level and so we know the air density is 1.2 kg/m^3. The turbine is rated at 5MW in 30mph (14m/s) winds,

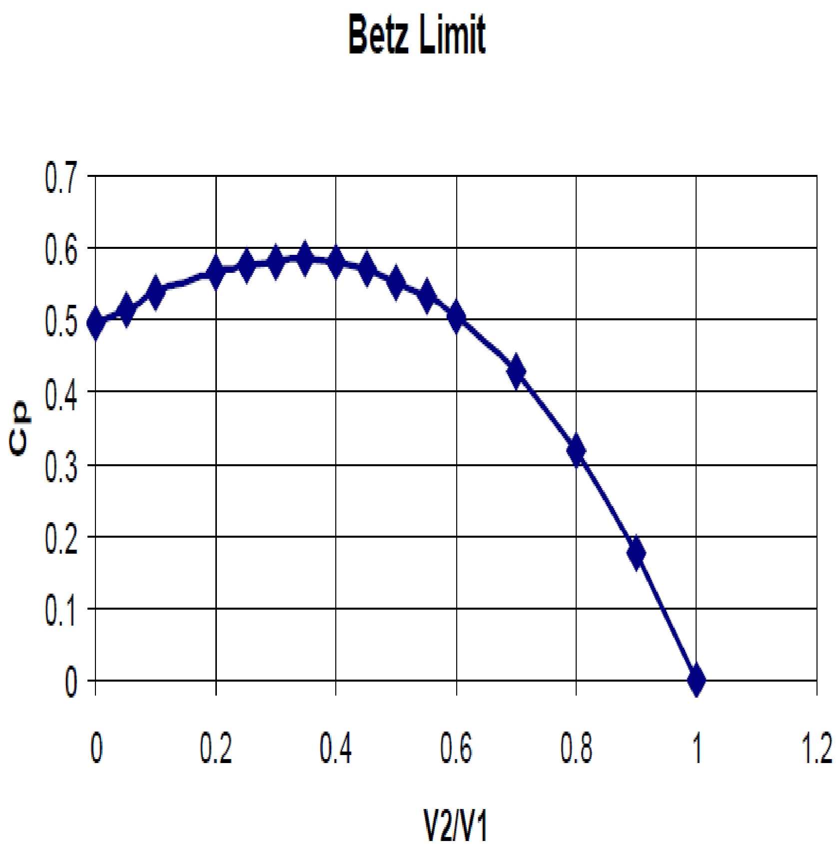
Rotor Swept area (A) = (π * 126^2) /4 = 1246 m^3
Wind power = 0.5 x A X ρ x V^3
= 0.5 x 12469 x 1.2 x $(14)^3$ = 20.5MW

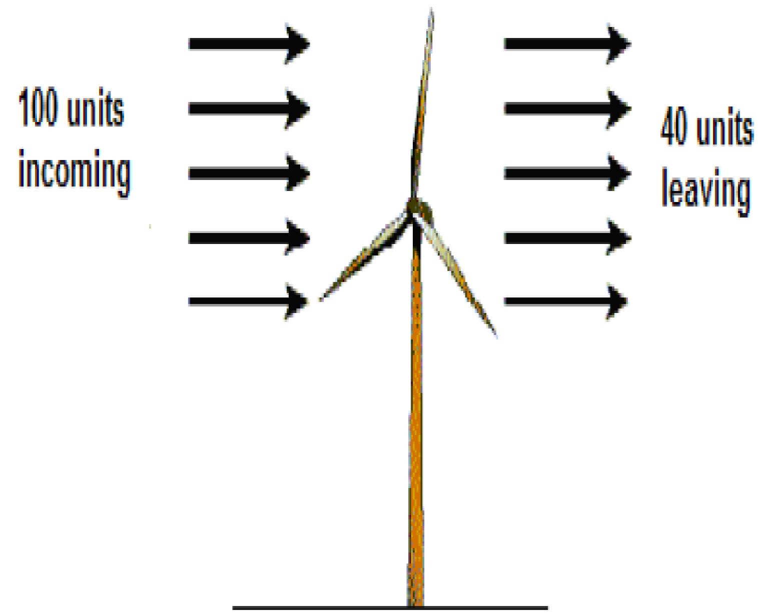
Why is the power of the wind (20MW) so much larger than the rated power of the turbine generator (5MW)?

The answer lies in the fact that the Betz limit and inefficiencies in the system seriously absorbs over 60% of the apparent power.

There are two further factors to be considered when estimating the power output from a turbine, thee first is the mechanical transmission and the second is the generator's efficiency, both of which are less than unity, hence the real power is proportionately less than the ideal value.

The capacity factor, Cf. assuming a 5kW wind turbine generates annually 10MWh, if that same installation had run – theoretically – 24 hours a day and 365 days a year at full load, it would have generated 43.8MWh. The capacity factor (Cf) is 10/43.8 = 0.23. Typical values for Cf between 0.2 and 0.4 in the united kingdom, depending on the exact location.





Betz Limit on wind energy efficiency and its Implications