

★

Data * Structure

①

25-4-2012

Data structure

1) Definition

a logical or mathematical model.

ex: stack

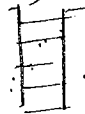


physical structure

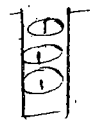
2) Definition

implementation of model in physical memory

Array stack



linked stack



Datatypes :- problem Solving

Q. Why data structures...

Sol. Solving a problem FAST by occupying optimum memory.

Time analysis

space analysis

ARRAYS

Q. Calculate the location of an element $A(0)$ in an array of a: array $[-5 \dots +5]$ elements and starting location is 1000 and each element occupies 2 memory locations.

Sol.

$$\text{upper boundary} - (\text{lower boundary}) + 1$$

$$\Rightarrow +5 - (-5) + 1$$

$$\Rightarrow 11$$

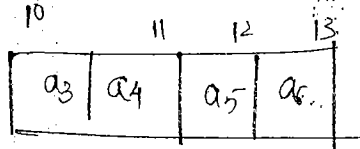
$$\text{loc } A(i) = \text{lb} + (i - \text{lb}) * C.$$

$$A(0) = 1000 + [0 - (-5)] * 2 \Rightarrow 1000 + 10$$

int 1
mt 2
float 4
double 8
long double 10

Scalar arithmetic

$$\text{loc } a_i = l_0 + (i - \text{lower boundary}) \times C$$



$$\text{loc } a_6 \rightarrow 10 + (6-3) \Rightarrow 13$$

$$\text{char} \rightarrow 10 + (1)_1 = 11$$

$$\text{int} \rightarrow 10 + (1)_{\times 2} = 12$$

$$\text{float} \rightarrow 10 + (1)_{\times 4} = 14$$

$$\text{double} \rightarrow 10 + (1)_{\times 8} = 18$$

$$\text{long double} \rightarrow 10 + (1)_{\times 16} = 26$$

Java Supports a Unicode. It is a Super Code of ASCII

- Unicode is universal Code

$$\text{loc } A(i) = l_0 + (i - l_b) \times C$$

$A = \text{array } [l_b \dots u_b] \text{ of elements}$

$l_0 = \text{Starting location}$

$l_b = \text{lower boundary}$

$C \rightarrow \text{Count}$

Row major order

Column major order

Q. Calculate the location of $A(0,5)$ element using RMO & CMO.
for $A = \text{array } [-2 \dots 2, 3 \dots 7]$ of elements and starting location is 1000 and each element occupies 2 memory cells.

Sol

2-D arrays

$A = \text{array } \left[\underbrace{l_{b_1} \dots u_{b_1}}_i, \underbrace{l_{b_2} \dots u_{b_2}}_j \right] \text{ of element}$

Row Column

$$u_{b_1} - l_{b_1} + 1$$

$$u_{b_2} - l_{b_2} + 1$$

$$r_1 \times r_2$$

(2)

* Scientific Notation

for RMO

$$\text{loc } A(i,j) = \text{lo} + \left[(i - lb_1) (ub_2 - lb_2 + 1) + (j - lb_2) \right] \times C.$$

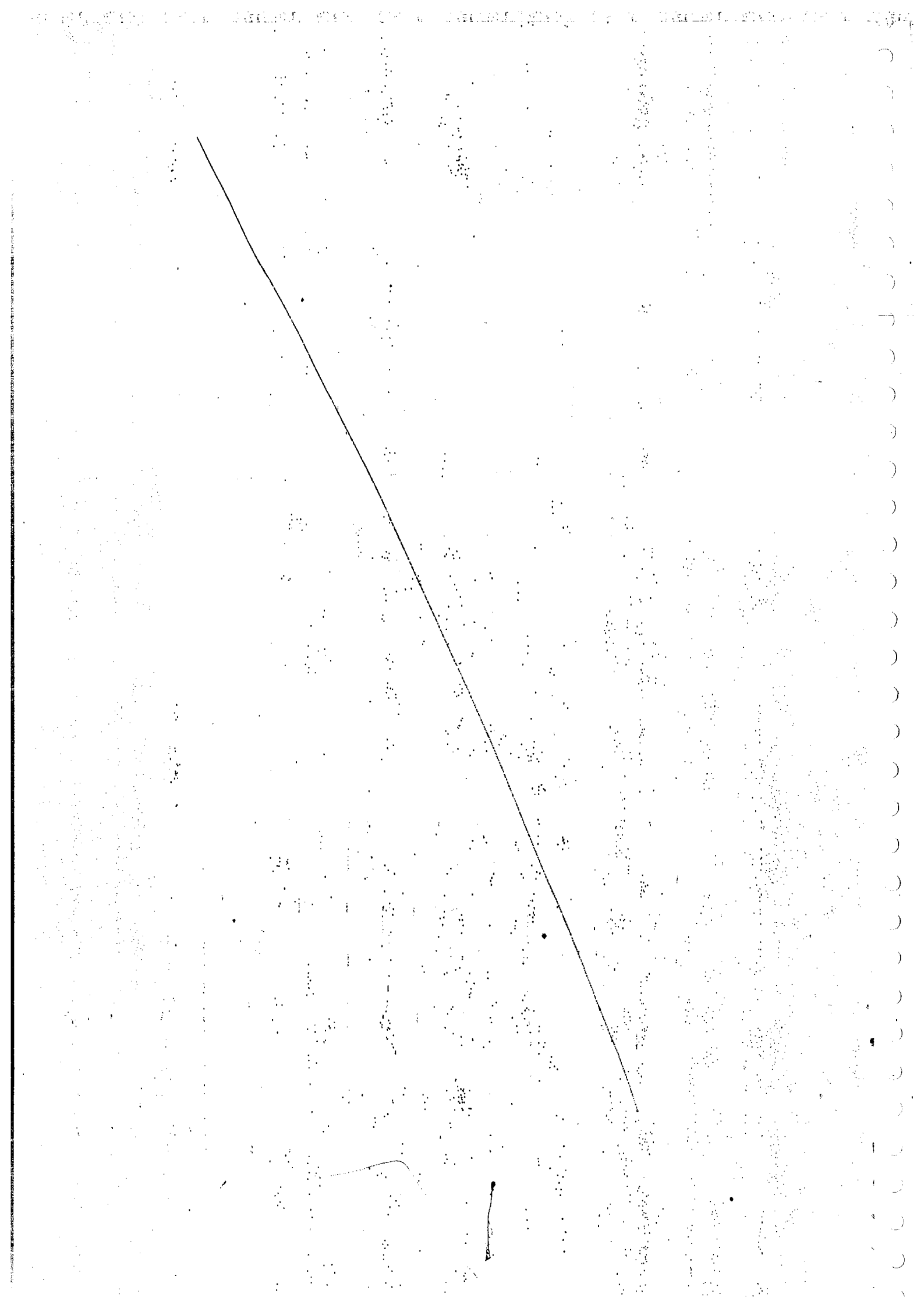
$$\text{loc } A(0,5) = 1000 + \left[(0 - (-2)) (2 - 3 + 1) + (5 - 3) \right] \times 2.$$

$$\text{loc } A(0,5) = 1024.$$

for CMO

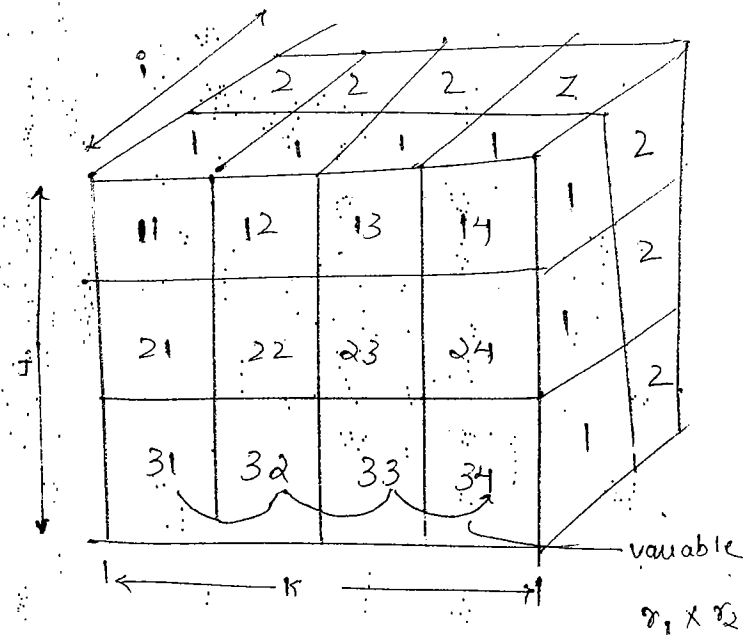
$$\text{loc } A(i,j) = \text{lo} + \left[(j - lb_2) (ub_1 - lb_1 + 1) + (i - lb_2) \right] \times C$$

$$\text{loc } A(0,5) = 1024$$



Date
27-4-2012

3-Dimensional array



3D RMO $\Rightarrow L_0 + (i-1) r_1 r_2$
 $+ (j-1) r_2$
 $+ (k-1)$

$2 \times 3 \times 4$
 planes \times rows \times columns
 $i \times j \times k$

• 3D Column measure order (CMO) \Rightarrow

$$L_0 + (k-1) r_1 r_2 + (j-1) r_1 + (i-1)$$

• 2D RMO $\Rightarrow L_0 + (i-1) r_2 + (j-1)$

• 2D CMO $\Rightarrow L_0 + (j-1) r_1 + (i-1)$

• STACK

ADT \rightarrow Abstract data type

ADT of Stack

• Definition :- One side open, the other side is closed.

\rightarrow LIFO or FILO model

\rightarrow Top pointer :- pointing to top most element among the available elements

• Operations :-

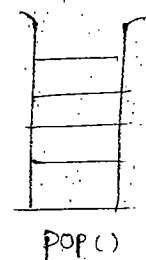
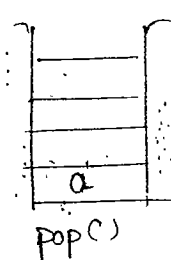
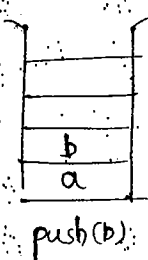
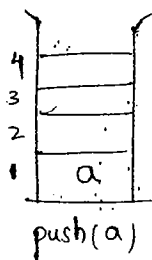
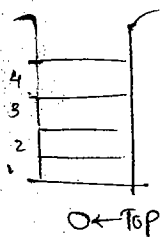
push(x) : insert an element 'x'

pop(x) : deletes the topmost element

• Application :-

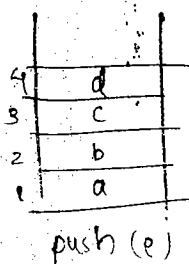
i) permutation ii) polish Notation iii) Recursion

Abstraction : hiding the internal implementation details



• b is deleted

0 \leftarrow initial position



push = insert

- ① of = size
- ② update
- ③ insert

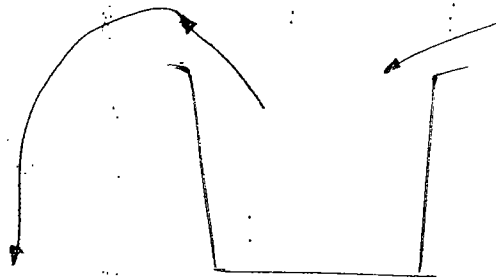
pop = delete

- ① of = initial
- ② delete
- ③ update

permutations :-

logic

Each and every element is pushed in



- Based on the desired Sequence, the elements are popped out

Q. identify invalid stack permutation for the input Sequence 1, 2, 3, 4.

- a) 3, 4, 2, 1 b) 4, 3, 1, 2 c) 2, 4, 3, 1 d) 1, 4, 3, 2
 e) 1, 4, 3, 5, 2

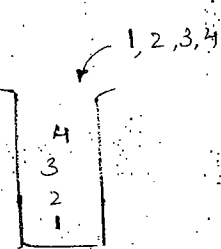
Sol

a) 3, 4, 2, 1

b) 4, 3, 1, 2

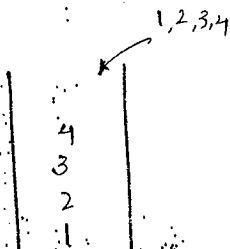
c) 2, 4, 3, 1

d) 1, 4, 3, 2



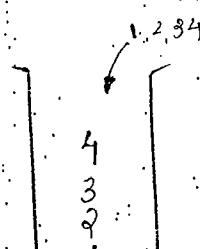
3, 4, 2, 1

Valid



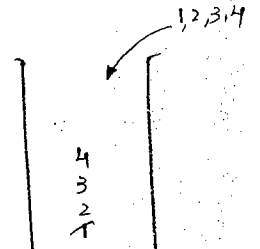
4, 3, 1, 2

Invalid



2, 4, 3, 1

Valid



1, 4, 3, 2

Valid

Q. identify valid/invalid stack operations

a) Is empty () → return true/false

b) Get top () → return top most element

i) Is empty (pop (push (push (s, x), y))) = True → invalid

ii) Get top (push (pop (push (pop (push (s, a), b), c))) = a → invalid

ii) polish notation :-

- Devised by polish logician, Lukasiewicz.
- it avoids the repeated scanning of infix expression.
- Compilers prefer $\left. \begin{array}{l} \text{prefix} \\ \text{postfix} \end{array} \right\}$ in one scan, we get result.

$\text{infix} \rightarrow a + b$
 $\text{prefix} \rightarrow +ab$
 $\text{postfix} \rightarrow ab+$

} Names w.r.t. operator.

$a + b \neq +ba$ This is called parsing error ; $tab = \text{valid}$.

Rule ① :- The relative position of Operands : Not disturbed.

Rule ② :- The relative position of operators : Disturbed according to the precedence & associativity rules.

Ex :- i) $a + b * c$ (precedence example)

prefix :-
 $a + b * c$
 $a + (*bc)$
 $+a * bc$
 $+a * bc$

post :-
 $a + b * c$
 $bc *$
 $a +$
 $a + bc *$
 $abc * +$

ii) $a * b / c$ (left associative)

prefix :-
 $a * b / c$
 $(a * b)$
 $*ab$
 $/c$
 $/ * abc$

post :-
 $a * b / c$
 $(a * b)$
 $ab *$
 $/c$
 $ab * c /$

iii) Right associative

as prefix

$a \uparrow b \uparrow c$
 $\uparrow bc$
 $a \uparrow \bigcirc$
 $\uparrow a \bigcirc$
 $\uparrow a \uparrow bc$

as postfix

$a \uparrow b \uparrow c$
 $\uparrow bc$
 $a \uparrow \bigcirc$
 $abc \uparrow \uparrow$

Q.2

i) $-b$

ii) $\log x$

iii) $x!$

iv) $\log x!$

$$a = -b + c * d / e - f \uparrow g \uparrow h + i * j$$

sol

infix	$-b$	$x!$	$\log x$
prefix	$-b$	$!x$	$\log x$
postfix	$b-$	$x!$	$x \log$

prefix

$\log x!$

$\log x!$

$!x$

$\log \bigcirc$

$\log \bigcirc$

$\log !x$

$a + b * c$

$b * c$

$x b c$

$a + \bigcirc$

$+ a \bigcirc$

$+ a x b c$

post

log x!

$a + b * c$

$b * c$

(x!)

x!

log

x! log

28.4.2012.

/ * : same level : left associativity

$$a = -b + c \times d / e - f \uparrow g \uparrow h + i * j$$

$a / b * c$

Sol

precedance table.

pref

H



- unary

↑ : Right

* / : Left associative

+ - : Left associative

=

$a / b * c$

ix) prefix

(-b)
-b

c * d
* c d
/ e
/ * c d e

g ↑ h

↑ g h

f ↑

↑ ↑ g h

i * j

* i j

+ (-b)

(/ * c d e)

$$\rightarrow = a + - + - b / \varnothing e d e \uparrow f \uparrow g h * i j$$

ii) postfix

-b	c*d	g+h	i*j
b-	cd*	gh+	ij*
	\bigcirc/e	f+ \bigcirc	
	$\bigcirc e/$	f \bigcirc +	
	cd*e/	fgh++	

b - cd*e/ + fgh++ ij* +

$$[a \ b - cd * e / + fgh ++ ij * + =]$$

$a = -b + c * d / e f + g + h + i * j \rightarrow \text{infix}$

~~a = b~~

i) prefix

$= a + -b / * c d e + f + g h * i j$

ii) postfix

$ab - cd * e / + fgh ++ - ij * + =$

8, 2, 3, ^, 1, 2, 3, +, 5, 1, *, +

Q.7 Give the stack top two elements after first * is evaluated

- a) 2, 3 b) 5, 1 c) 1, 2 d) ~~1, 5~~

Q.8 What is the final result if entire expression is evaluated.

- a) ~~12~~ b) 13 c) 14 d) 15

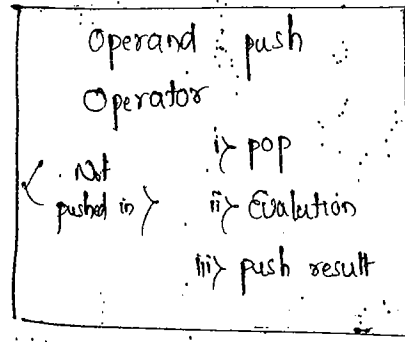
Q.9 Which of the following not possible stack content.

- a) 8 | 2 | 3 b) 7 | 5 | 1 c) ~~1 | 2 | 3 | *~~ d) 1 | 5

Evaluation

$$6 - 2 = 4$$

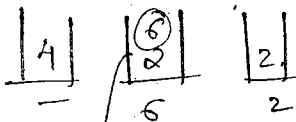
Concept



$$(6 - 2)$$

prefix

← 6, 2, -

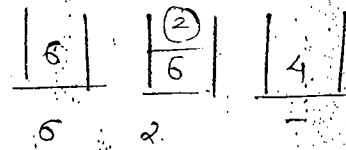


$$6 - 2 = 4$$

$$(6 - 2)$$

postfix

6, 2, - →

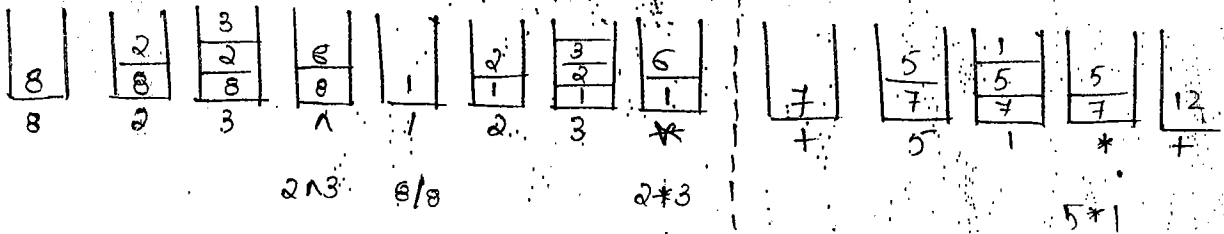


$$6 - 2 = 4$$

Sol

8, 2, 3, ^, /, 2, 3, V, +, 5, 1, *, +

i) post fix:



$$2 \wedge 3 = 8$$

$$2 * 3 = 6$$

$$5 * 1 = 5$$

Ex:- $1 + 2 * 3 - 4$

Sol

$$1 + 2 * 3 - 4$$

$$2 * 3$$

$$* 2 3$$

$$\text{○} + \text{○} - \text{○}$$

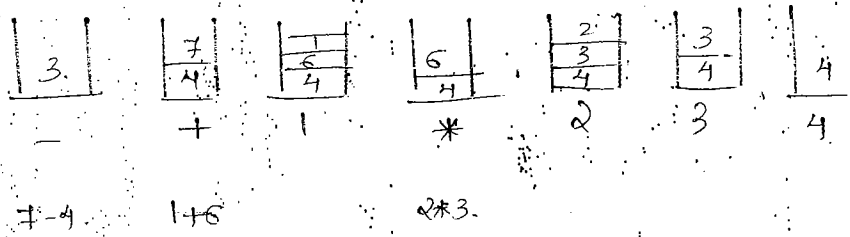
$$+ 1 * 2 3$$

$$\text{○} - \text{○}$$

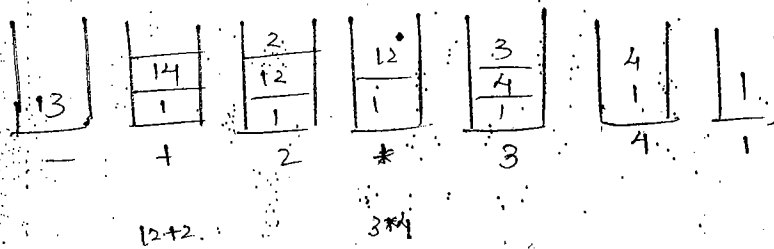
$$- + 1 * 2 3 4$$

9

Ex: $1 + 1 * 2 3 4$ (prefix evaluation)



ii) $1 + 2 * 3 4 1$ (prefix evaluation)



RECURSION

i) Direct Recursion

a) Tail Recursion

b) head Recursion

Ex: - factorial

Ex: Tower of Hanoi (ies)

ii) Indirect Recursion

Ex: - Trees

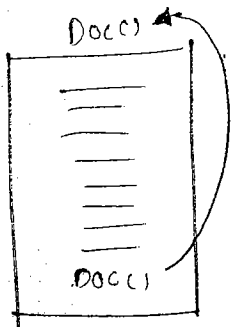
iii) Nested Recursion

Ex: Ackermann's function
(JTO) (ORDO)

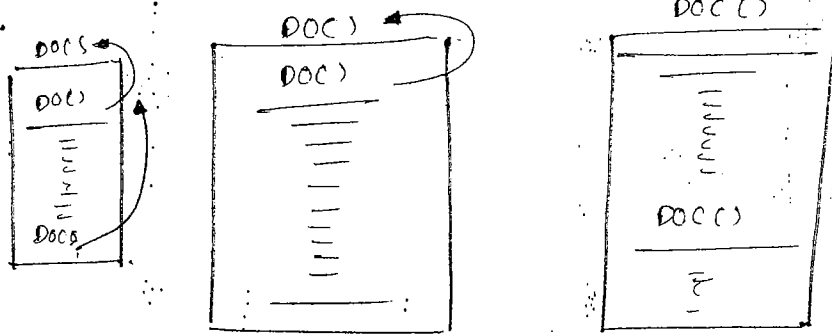
iv) Excessive Recursion

Ex: fibonacci

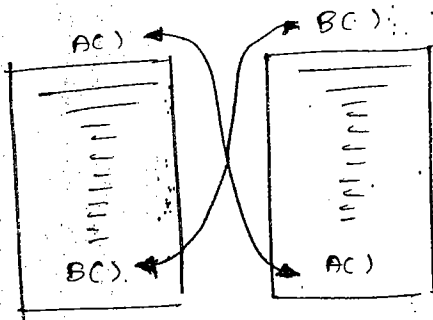
i) Tail Recursion



ii) Head Recursion (or other than last)
(non Tail)



iii) Indirect Recursion



Adv Excessive

- Does not remember previously evaluated values.

properties

- i) Base (or) Termination Condition (or) Anchor step.
- ii) Repeated step (or) Recursive step (or) inductive step.

$$\text{fact}(N) = \begin{cases} 1 & N=0 \text{ Anchor or Base} \\ N * \text{fact}(N-1) & \text{otherwise inductive step (Repeated)} \end{cases}$$

(8)

```
int fact (int N)
```

```
    if (N==0)
```

```
        Return 1;
```

```
    else Return N * fact (N-1);
```

```
}
```

$$f(4) = 4 * f(3)$$

$$3 * f(2)$$

$$2 * f(1)$$

$$1 * f(0)$$

$$4 * 3 * 2 * 1 * 1 = 24$$

Q7

```
Void Do (int N)
```

```
{
```

```
    if (N > 0)
```

```
        Do (N-1)
```

```
        PRINT : N
```

```
    } Do (N-1);
```

```
}
```

```
}
```

What is the o/p if Do(3)....?

Sol

N=3 > 0

```
{
```

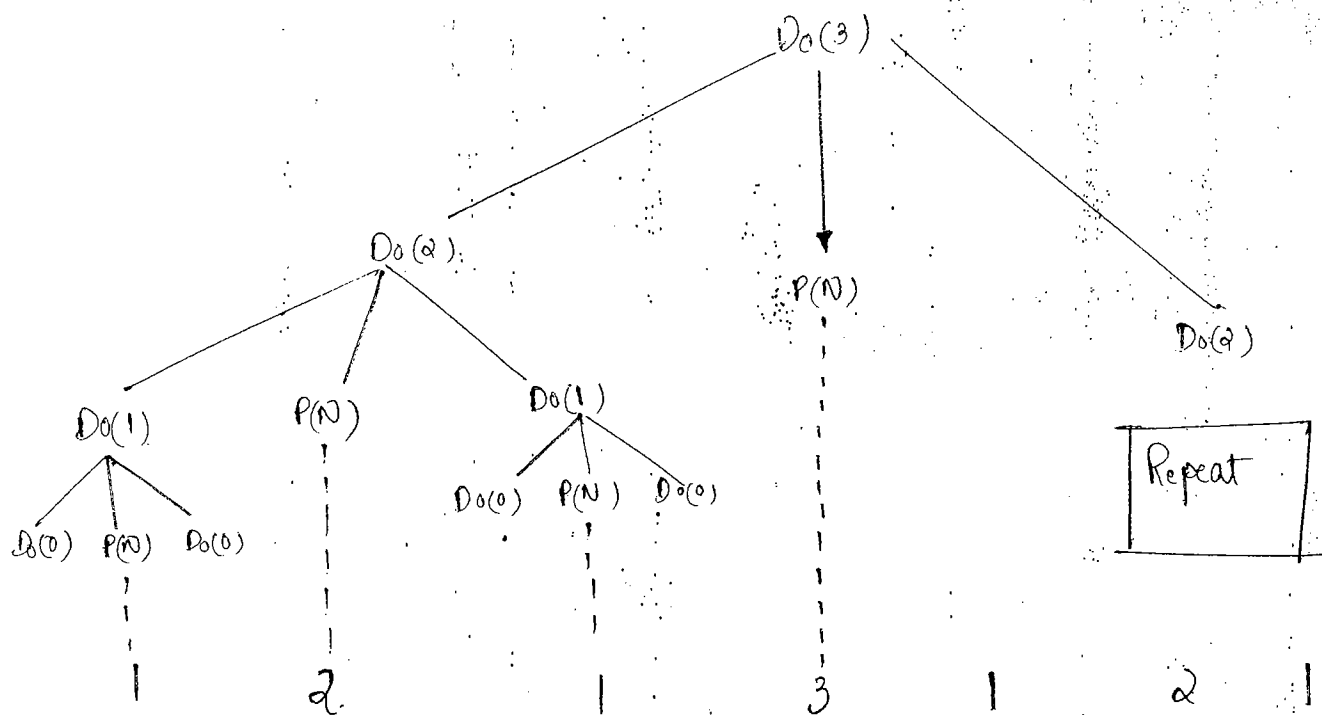
```
    2
```

```
    _____ 2
```

```
    1
```

```
    _____ 1
```

```
}
```



Ans is 1 2 1 3 1 2 1

$$fibonacci(N) = \begin{cases} 0, & N=0 \\ 1, & N=1 \\ fib(N-1) + fib(N-2), & N \geq 2 \end{cases}$$

Q. what is the o/p $X(X(5)) \dots$?

```

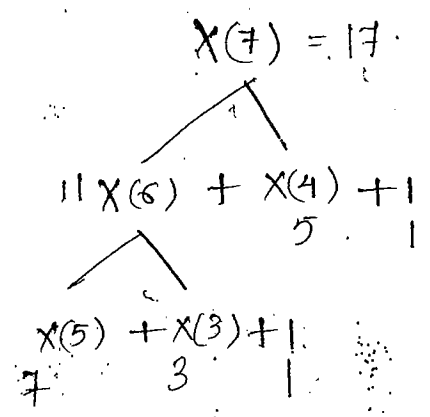
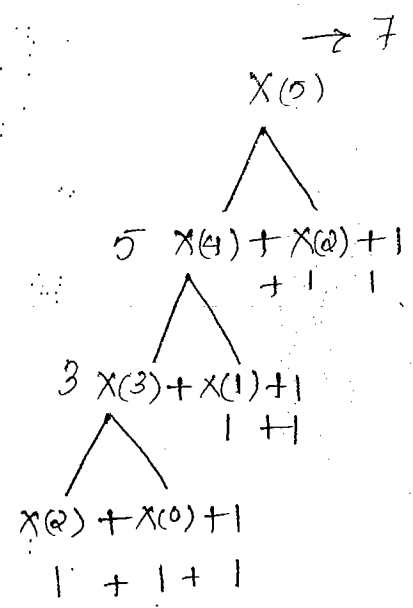
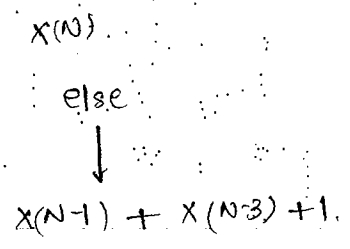
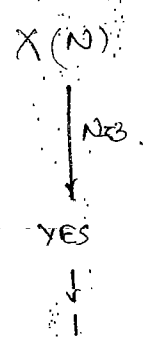
int X(int N)
{
    if N < 3
        Return 1;
    else
        Return X(N-1) + X(N-3) + 1;
}

```


9

30.4.2012

for



$X(X(5)) = X(7) = 17$

Return value = 17

No. of calls =

$X(5) = 7$

$X(7) = 17$

24

Q. what is the value of $F(6)$...

How many additions are there in evaluating $f(7)$...

$f \rightarrow$ fibonacci function

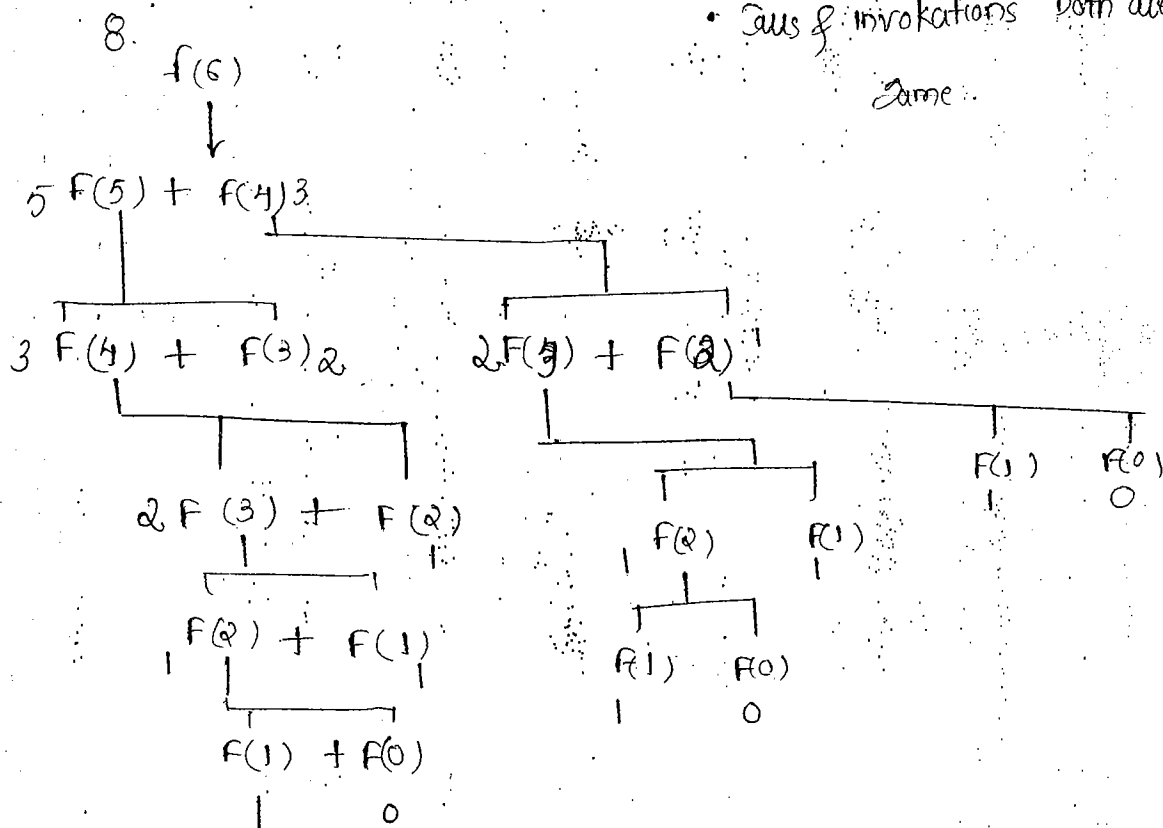
Given

$f(N) = \begin{cases} 0 & N=0 \\ 1 & N=1 \\ f(N-1) + f(N-2) & N > 1 \end{cases}$

Sol

$$f(N) = \begin{cases} 0 & N=0 \\ 1 & N=1 \\ f(N-1) + f(N-2) & N \geq 2 \end{cases}$$

• Caus & invokations both are same.



N	0	1	2	3	4	5	6	7	8
f(N)	0	1	1	2	3	5	8	13	21
Calls				5	9	15	25	41	
Additions				2	4	7	12	20	

Excessive :- Doesnot Remember the previously evaluates value.

✓ No. of Caus in $f(N) = 2 \cdot f(N+1) - 1$

✓ No of additions $f(N) = f(N+1) - 1$

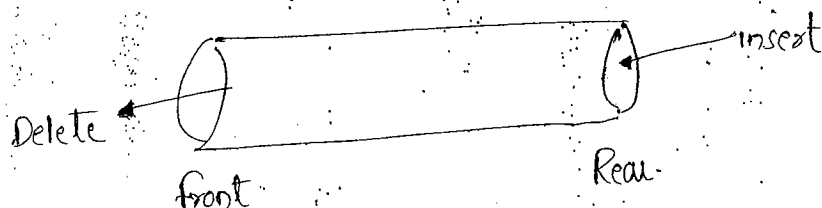
• QUEUE

→ FIFO or LIFO model.

→ Associated with Two pointers.

i) Rear pointer = insert an element 'x' → enqueue(x);

ii) front pointer = deletes the front element → dequeue();



Q. what is the status of 'Q' after the following sequence of steps. on a linear Queue.

i) addition of 'a' and then 'b'

ii) a deletion

iii) addition of 'y' and then 'z'

iv) a deletion

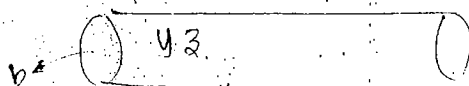
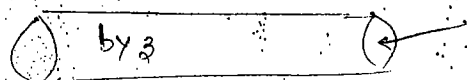
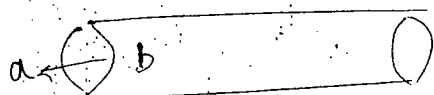
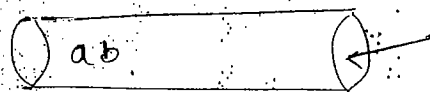
a) Error

b) yz

c) ☒ yz

d) ab

Sol



a deletion done not means that delete a → it means that delete the first element.

Ans :- yz

Q. what does $do()$ do on linear Queue.

Algorithm: Do (linear queue).

→ If Q is not Empty

Step(1) insert $temp = dequeue()$

Step(2) $do()$

Step(3) $enqueue(temp)$

Option:- a) Q contents remain intact

b) Q contents got reversed.

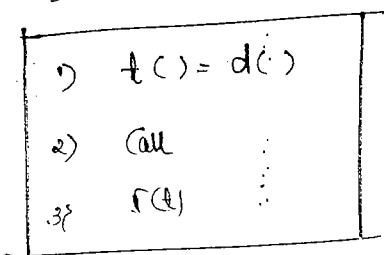
c) Q becomes Empty

d) Q contents doubled.

Sol

→ Recursion invokes stack implicitly

$do()$



Recursion (a)

$Queue()$

1) $t(a) = do()$

2) Call

3) $t(t)$

Recursion (b)

$Queue()$

1) $t(b) = do()$

2) Call

3) $t(t)$

Recursion (c)

$Queue()$

1) $t(c) = do()$

2) Call

3) $t(t)$

empty

$Queue()$

Stack is empty
now

$Queue()$

Q got reversed contents

Ans (B)

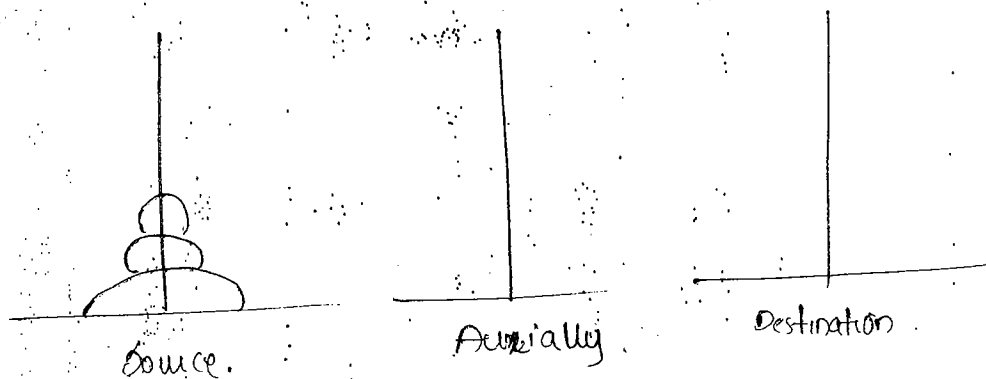
Observation

In program termination = ϕ is Empty

In Execution termination = Stack is Empty

Repeated

Towers of Hanoi



Rule \rightarrow Only one disc is allowed to lift & no restriction for direction

Observation \rightarrow



i) allowed

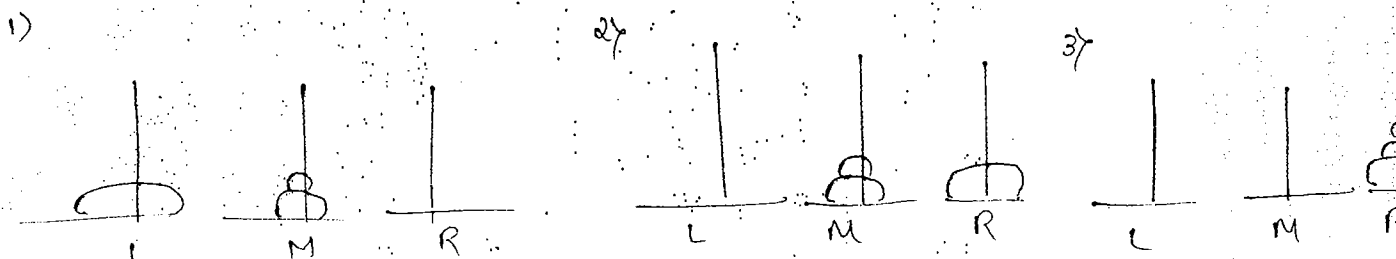
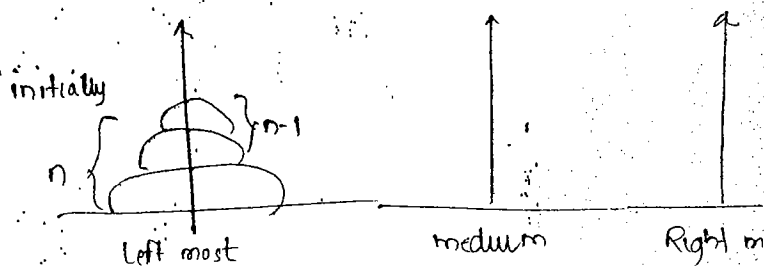


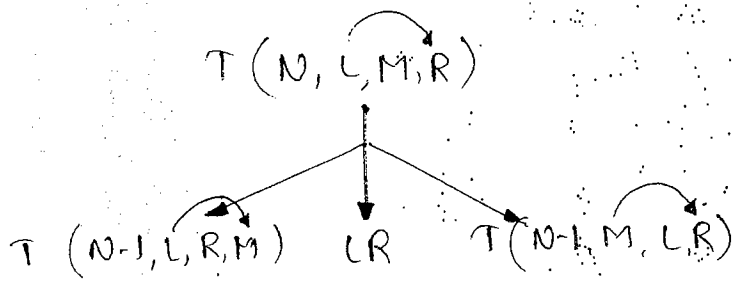
ii) Not allowed

Recursion (Stack application)

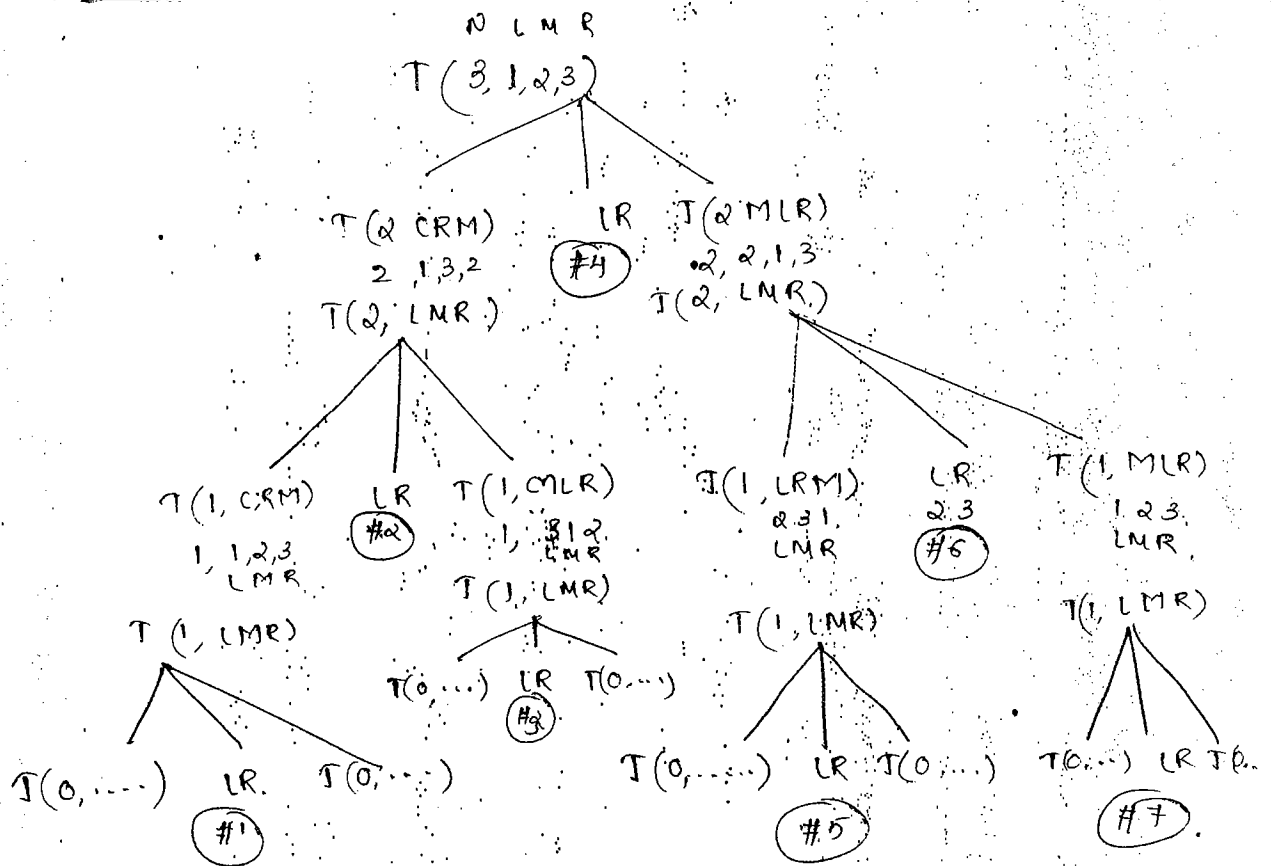
Algorithm (Recursive)

- 1) lift $(N-1)$ discs from L to M
- 2) Actual move from L to R
- 3) lift $(N-1)$ discs from M to R

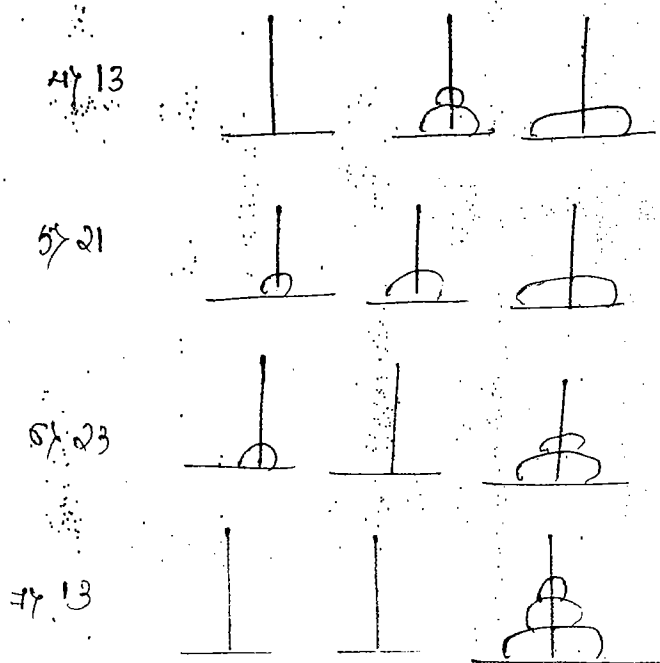
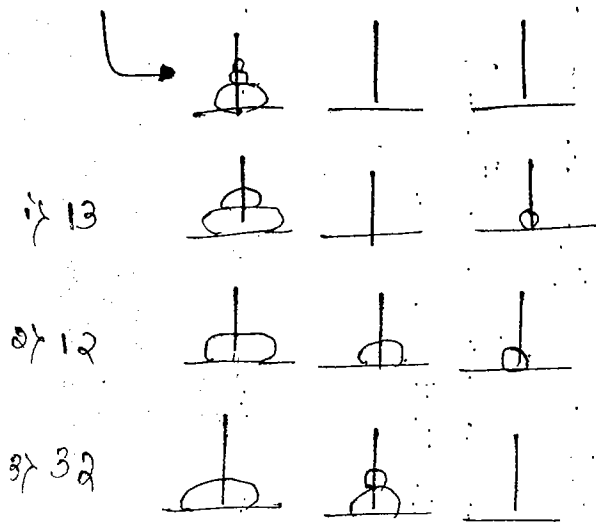




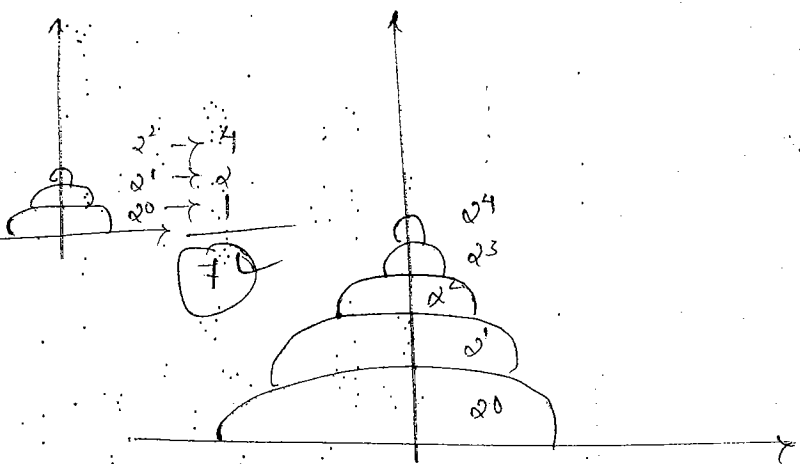
Example



Initially



moves	$2^n - 1$
3	$7 = 2^3 - 1$
4	$15 = 2^4 - 1$
5	$31 = 2^5 - 1$



Generally

Invocation or Ans.	$(N+1)$ $2^n - 1$
3	$15 = 2^4 - 1$
5	$31 = 2^5 - 1$

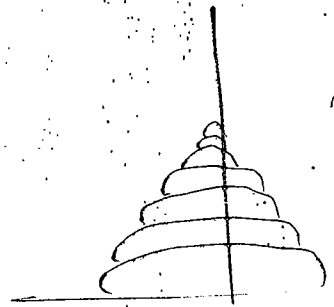
Observation

- Recursion tree traversal = pre order.
- Recursion is user friendly

Q1) No. of moves = $2^n - 1 \Rightarrow 7$ here $n=3$

Q2) No. of Ans = $2^{n+1} - 1 = 2^4 - 1 = 15$

Q3)



Each disc is moves. ... ?

2^i where $i = 0, 1, 2, 3, 4$

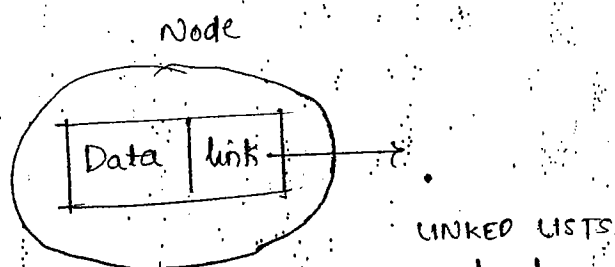
1.5.2012

• LINKED LISTS

array	pointer
static	Dynamic
Early binding	late binding
Compile time binding	Runtime binding

ex:- int a[1]; invalid
 ↑
 Size
int a[4]; ✓

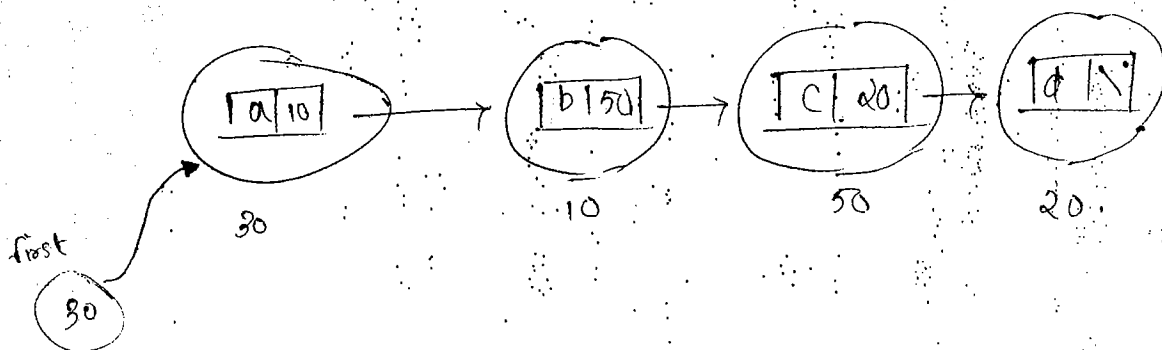
ex:- int *p;



Data structure :- Node in a self Referential structure.

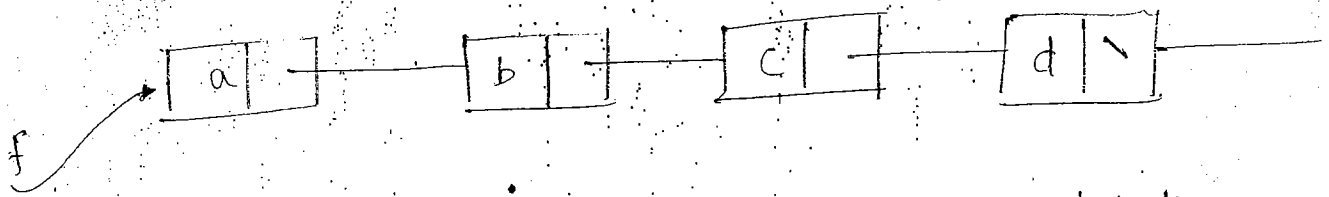
→ linked list :- Nodes are logically adjacent, physical Scattered.

• array is a physically adjacent & is contiguous i.e Continuous.

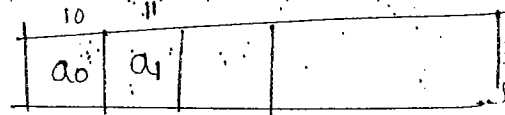


pictorial

• f is the pointer variable holding the address of very first node



Array = Contiguous = Continuous ; Array is physically adjacent



Single linked list (SLL) (disadvantage)

1) Stepping backwards is not possible (\rightarrow search(c)
 \rightarrow delete(c))

(we need to update downstream address in the upstream Node)

2) The link part of the last node : not utilized.

Hence Single linked list has become now Circular linked list.

Circular linked list (CLL)

• advantage of CLL

1) Stepping backwards is ~~not~~ possible by moving f

2) The last part of the node : utilized.

• disadvantage

1) Danger of falling into infinite loop.

Solution :- To head node is attached in the front, or Tail node is appended.

• yes 2009
 Inserting a new Node in a CLL requires the modification
 of _____ no. of link fields.

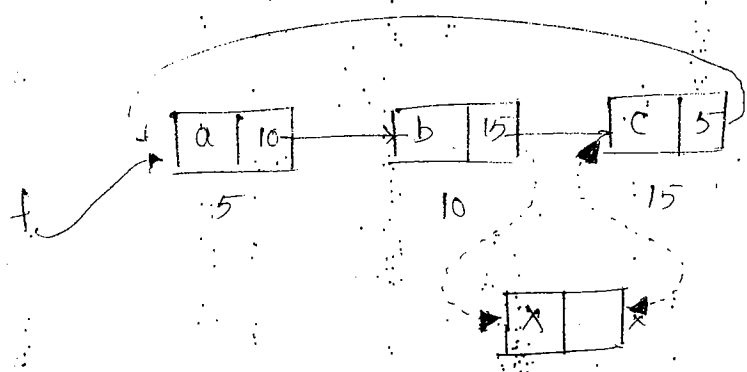
of 1

of 2

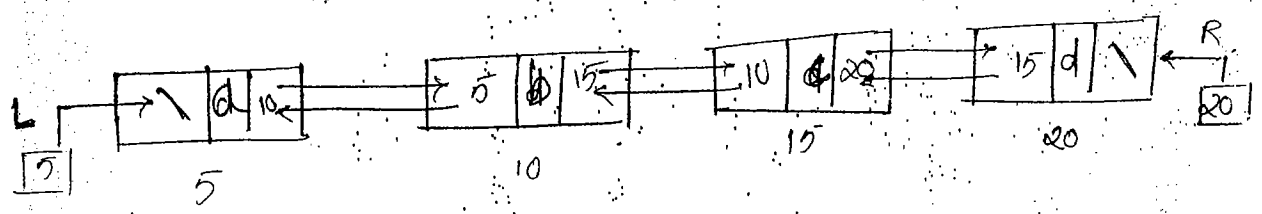
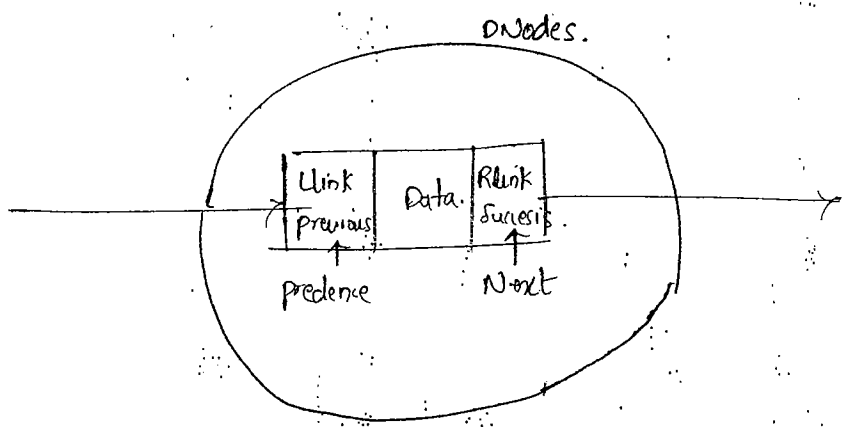
of 3

of 4

del



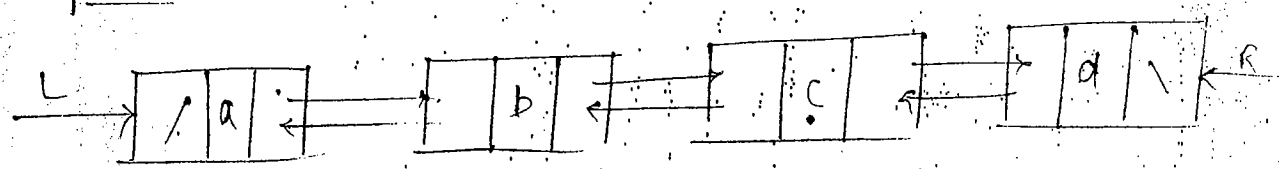
Doubly linked lists: DLL



$L \rightarrow$ address of left most Node.

$R \rightarrow$ address of Right most Node.

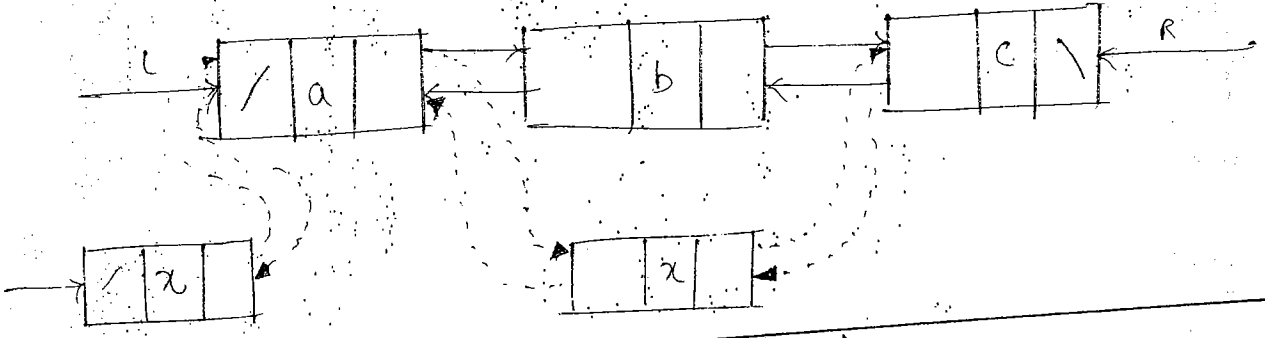
pictorial :-



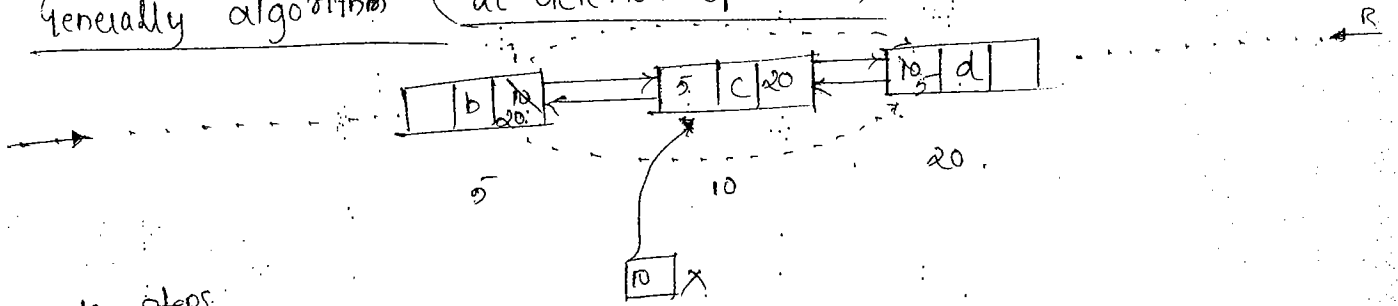
Qr

Operation	Left most	Right most	else middle
Insert	3	3	4
Delete	1	1	2.

Sol



Generally algorithm (at deletion operation)

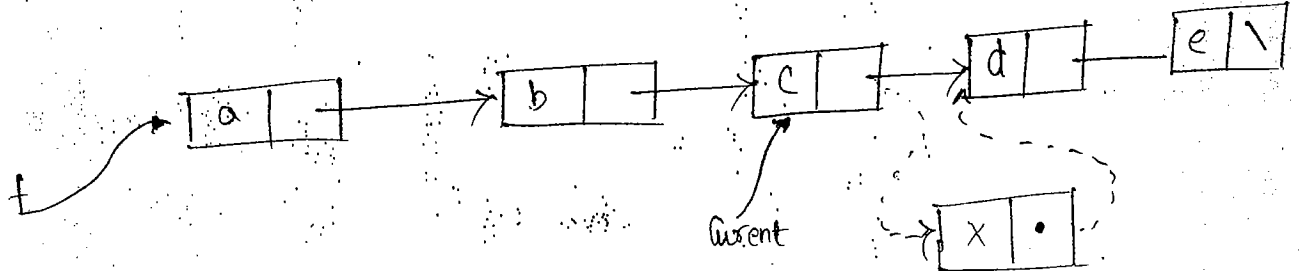


also steps

- i) $Rptr(Lptr(X)) \leftarrow Rptr(X)$
- ii) $Lptr(Rptr(X)) \leftarrow Lptr(X)$

X- address of Node being deleted in DLCL

Q) which of the following option inserts X after Current



- a) $Cur \leftarrow \text{new Node}(X, Cur)$
- b) $Cur \leftarrow \text{New Node}(X, Cur.Next)$
- c) $Cur.next \leftarrow \text{New Node}(X, Cur)$
- d) $Cur.next \leftarrow \text{NewNode}(X, Cur.next)$

get algorithm

- 1) Create a Node.
 - 2) place X
 - 3) update link part of New Node.
 - 4) update link part of Current Node.
-

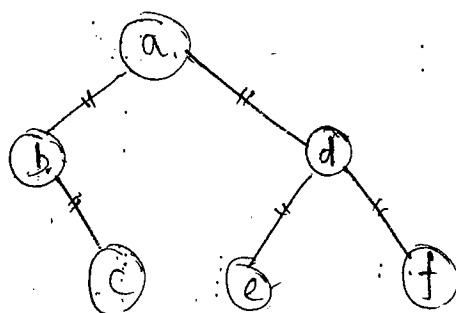
Obsevation :-

UNIX editor = vi (visual) early version was developed by using Circular doubly linked list.

Non linear linked lists :-

→ Binary tree.

atmost two children.



$$N = n_0 + n_1 + n_2$$

$$n_0 = 3$$

$$n_1 = 1$$

$$n_2 = 2$$

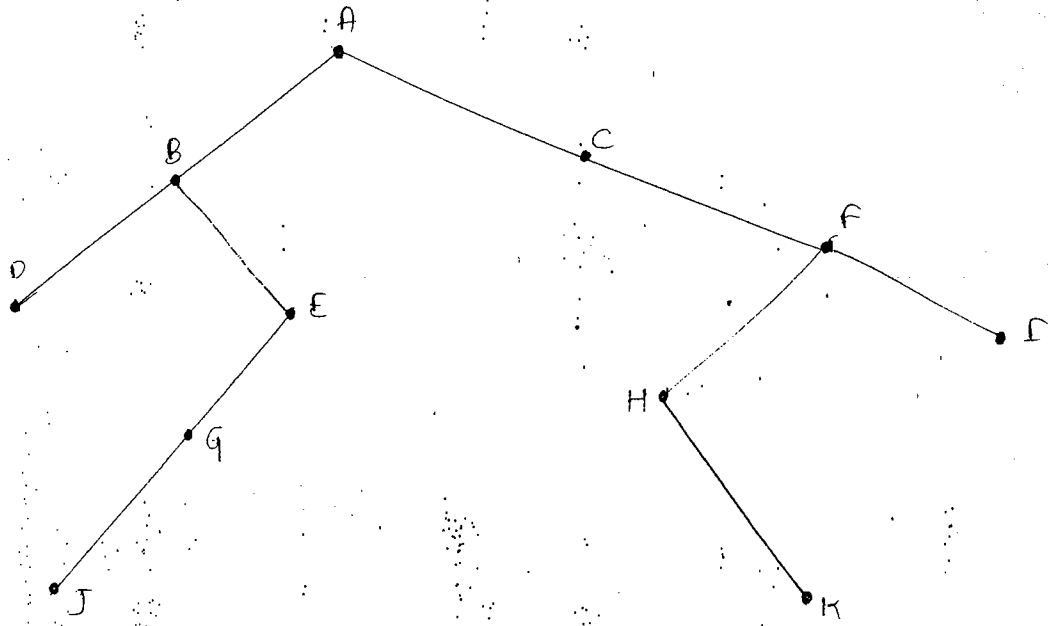
$$n = 6$$

No. of Branches

$$B = N - 1$$

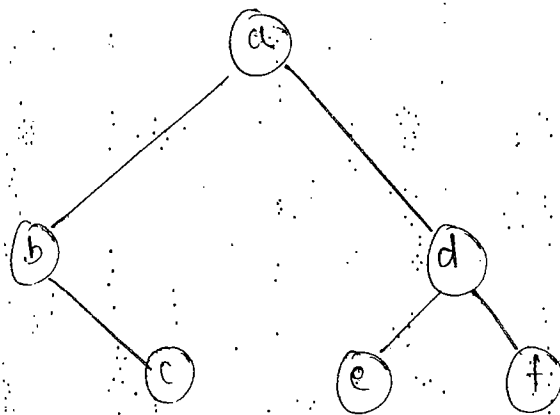
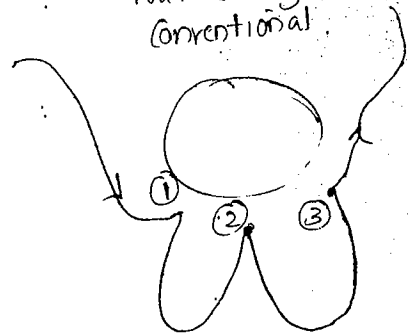
$$B = 5$$

Ques: post order. ?

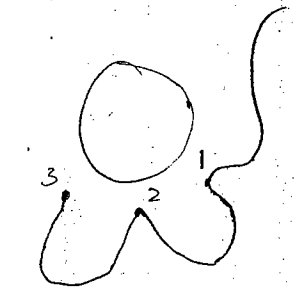


IN \rightarrow DBJGEACHKFI
 CIN \rightarrow IFKHCAEGJBD
 pre \rightarrow ABDEGJCFHKI
 post \rightarrow

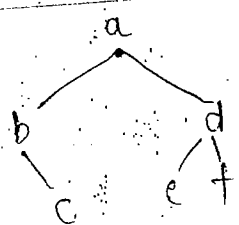
-walk through
Conventional



pre(1) \rightarrow a b c d e f
 in(2) \rightarrow b c a e d f
 post(3) \rightarrow e b e f d a



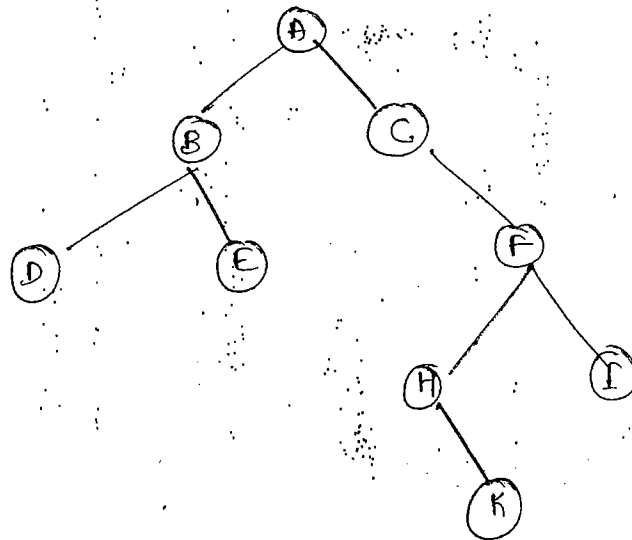
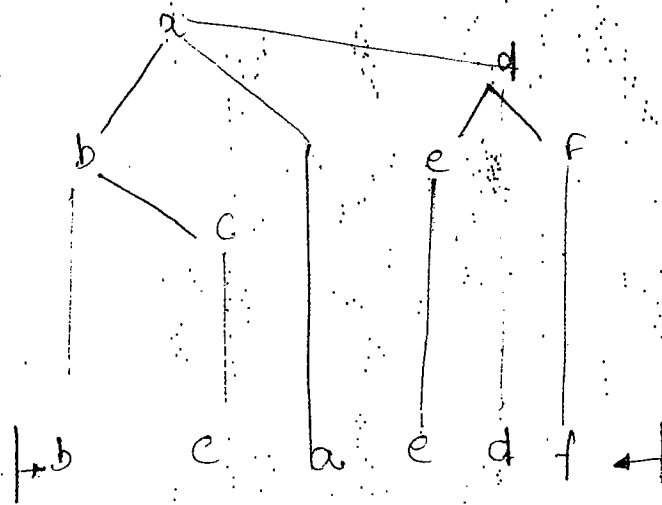
ii) Converse



Converse

Converse pre = a d f e b c
 in = f d e a c b
 post = f e d e b a

Valid only for in. (falling down) .

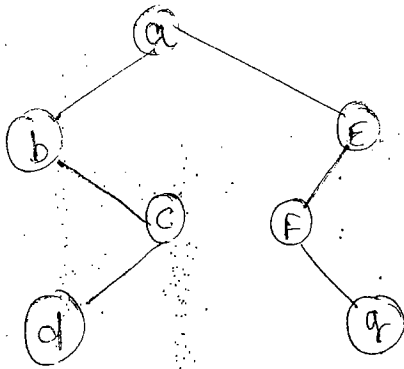


pre = ABDEGJCFHRI

post = DJGE BKHIFGA

Cpre = A C F I H K B E G J D. (previous question ans)

Cpost = I K H F C J G E D B A.



pre \rightarrow a b c d e f g

post \rightarrow a c b g f e a

cpre \rightarrow a e f g b c d

cpost \rightarrow g f e d c b a

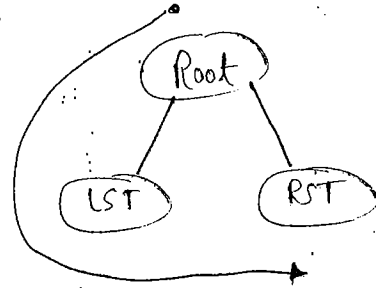
• Recursive algorithm

preorder :-

Steps 1) visit root

2) Traverse LST in preorder

3) " RST in preorder



• What kind of traversal does the following algorithm describes.

algorithm in pre traversal (root)

if (root is not null)

1) process (root)

2) pre traversal (LST)

3) pre traversal (RST)

end

Ans:- preorder



PF
LST
PF
RST
PF

pre

1) Root

2) LST

3) RST

cpre

1) Root

2) RST

3) LST

in

1) LST

2) Root

3) RST

cIn

1) RST

2) Root

3) LST

post

1) LST

2) RST

3) a

cpost

1) RST

2) LST

3) a

LST \rightarrow left subtree

RST \rightarrow Right subtree

2.5-2012

Q. Cal. the max. no. of nodes in a B.T. of given height H.

i) With Recursion

$$N(H) = \begin{cases} 1 & H=L=0 \\ N(H-1) + 2 & (H=L) > 0 \end{cases}$$

$$H=L=0$$

$$(H=L) > 0$$

Ans $\rightarrow 2^H$

ii) without Recursion

$$N_{max}(H) =$$

$$2^{H+1} - 1$$

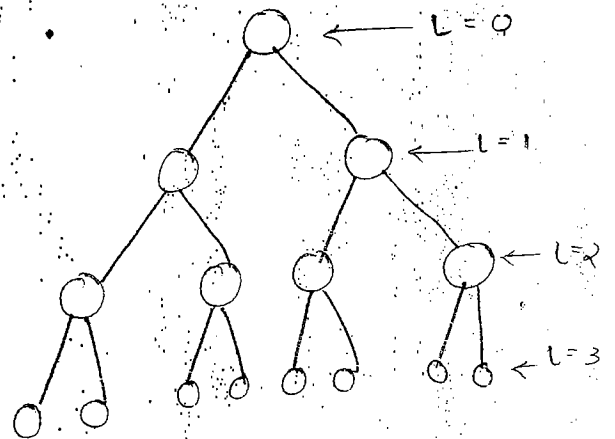
Observation:-

i) root is at level '0'

$$\text{levels} = \text{Height} = \text{Depth} = 2 \\ (0-2)$$

ii) root is at level '1'

$$\text{levels} = \text{Height} = \text{Depth} = 3 \\ (1, 2, 3)$$



$$N(3) = 7 + 2^3 = 15$$

$$N(H) = N(H-1) + 2^H$$

Ans: 2^H

ii)

L=H=0	Total
0	1
1	3
2	$7 = 2^3 - 1$
3	$15 = 2^4 - 1$

$$\rightarrow 2^{(H+1)} - 1$$

$$N(0) = 1$$

$$N(1) = N(0) + 2^1$$

$$\Rightarrow 3$$

$$N(2) = 3 + 2^2$$

$$\Rightarrow 7$$

$$L=0$$

$$L=1$$

$$L=1$$

$$L=0$$

$$L=1$$

$$L=2$$



Q. for a given 3- unlabelled nodes, how many distinct (separate) BTs can be formulated?

Sol

$n = \text{no. of nodes} = 5$

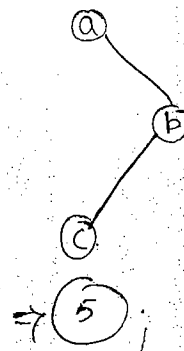
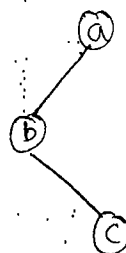
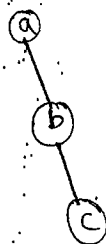
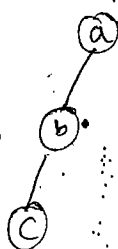
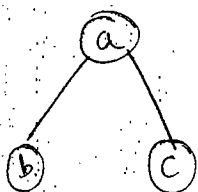
Try $n = 5$

$$\Rightarrow \frac{2nC_n}{n+1}$$

$$\Rightarrow \frac{10C_5}{6} = 7$$

$$\frac{10 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 6}$$

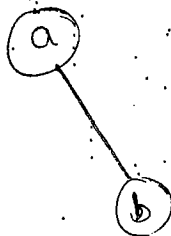
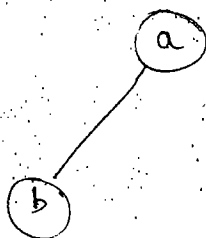
$\Rightarrow 12$



pre = abc

Q. for a given tree pre = ab, post = ba generate Binary Trees...

Sol



$$\begin{cases} \text{pre} = ab \\ \text{post} = ba \end{cases}$$

Observation :- To have unique pattern
 $\rightarrow \text{IN} + \text{pre}$
 or
 $\rightarrow \text{IN} + \text{post}$

pre = abcdef
 in = bcaedf

Q. Given : pre = abcdef ; IN = bcaedf } Generate BT and give post

Sol

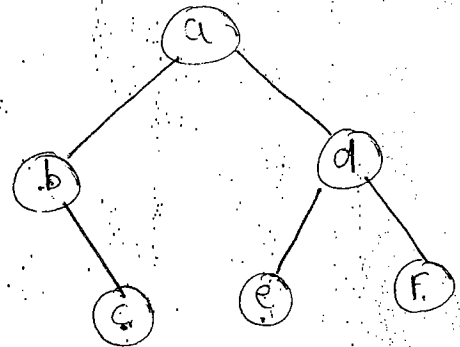
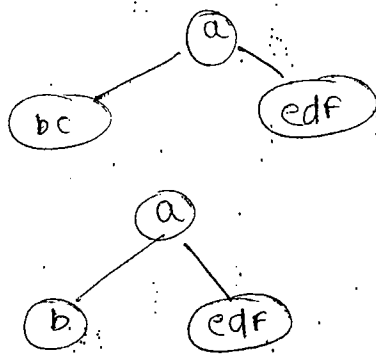
Concept :-

Step 1 :- Consider IN order.

pre :- abcdef
IN - ~~post~~ :- bcaedf

Step 2 :- look at pre to find Next root

IN :- bcaedf



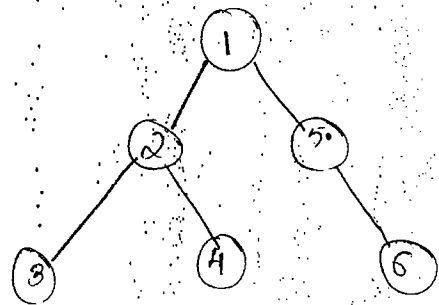
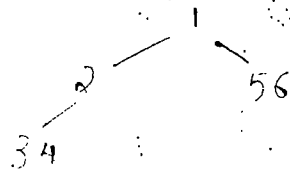
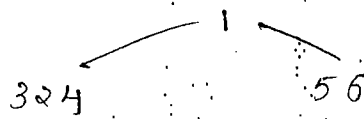
Check -
pre :- abcdef
IN :- bcaedf

post = cbefda

ii) pre = 1 2 3 4 5 6

IN ~~post~~ = 3 2 4 1 5 6

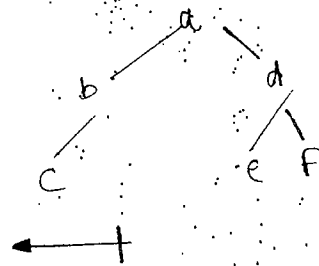
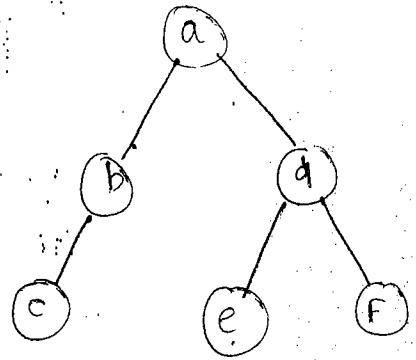
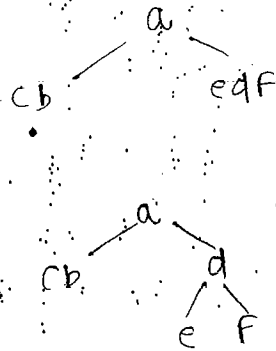
IN - 3 2 4 1 5 6



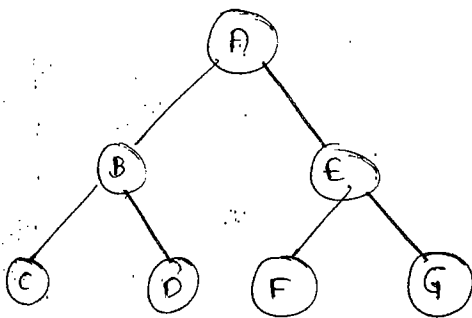
post = 3 4 2 6 5 1

iii) post = c b e f d a
IN = c b a e d f

IN \rightarrow c b a e d f



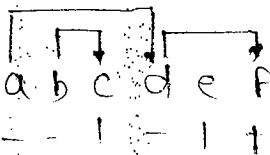
iv) post \Rightarrow C D B F G E A
Degree \Rightarrow 0 0 2 0 0 2 2



pre = A B C D E F G

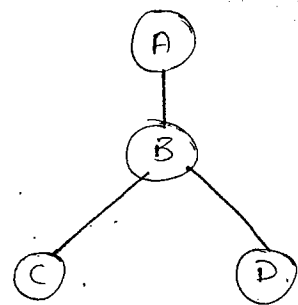
Ans

- RPT R
- pre \rightarrow a b c d e f
- IAC

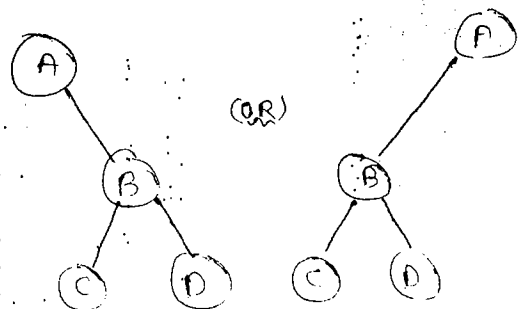


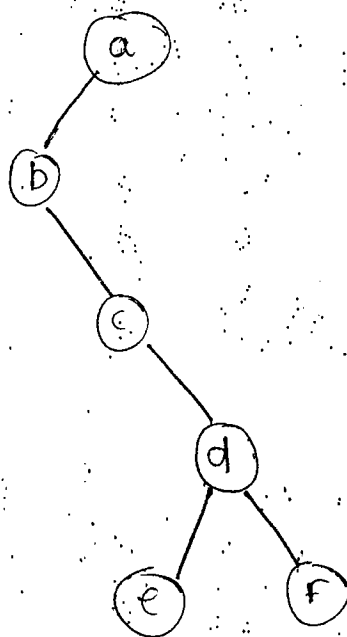
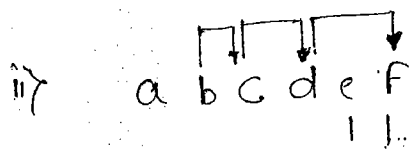
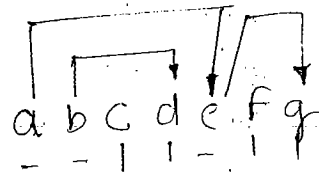
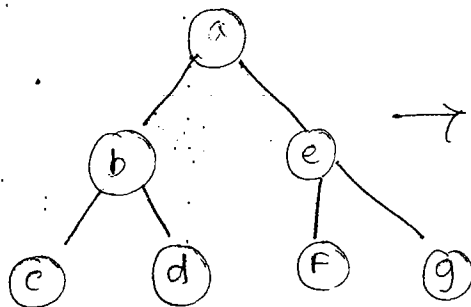
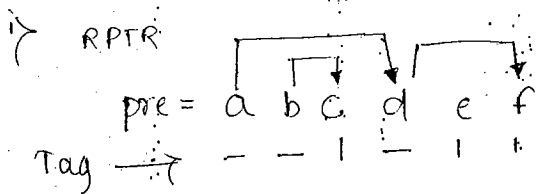
terminal nodes generate BT

v) post = C D B A
degree = 0 0 2 1



pre \rightarrow A B C D





→ 5 levels

at single order

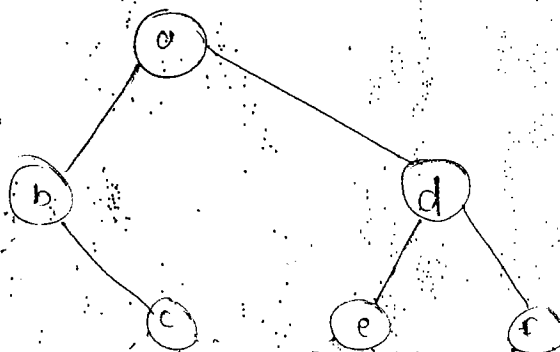
- 1 → LST
- 2 → PF
- 3 → RST

bcaedf

Q. Given

double order

- 1 → PF
- 2 → L
- 3 → PF
- 4 → R



sol.

Doc) → a b b c c a d e e d f f
(Double order)

Q.7

Given :- Triple order

Tc \rightarrow Triple order

To ()

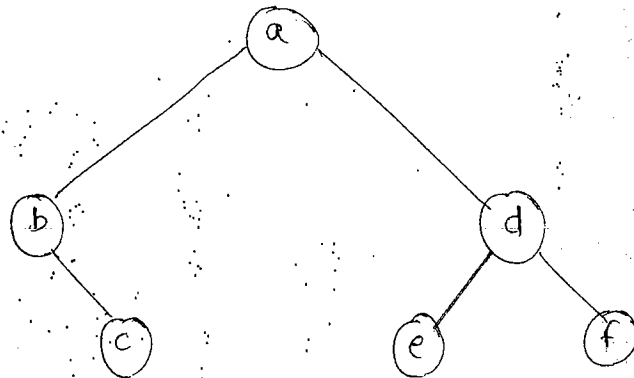
1 \rightarrow PF

2 \rightarrow L

3 \rightarrow RF

4 \rightarrow R

5 \rightarrow PF



\rightarrow abbcccbad eed fff d a

Q.8

Given :-

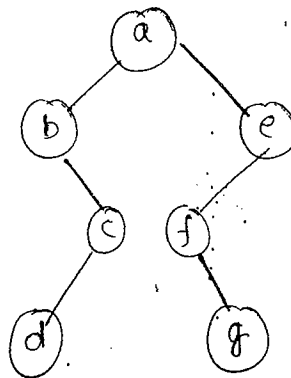
i) Do ()

1 \rightarrow PF

2 \rightarrow L

3 \rightarrow PF

4 \rightarrow R



Ans :- abbcd dca e f f g g e

ii) Triple order

To ()

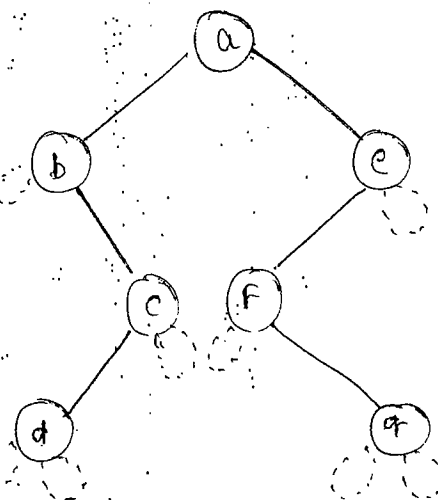
i) PF

ii) LST

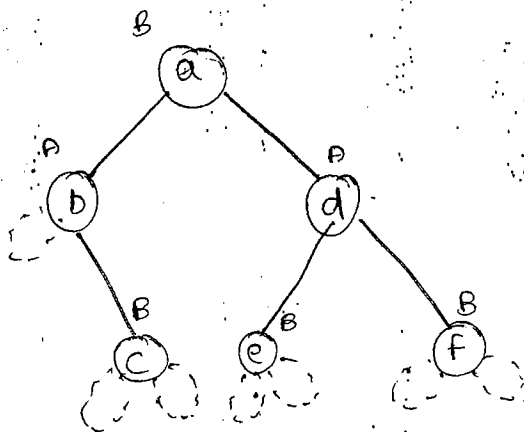
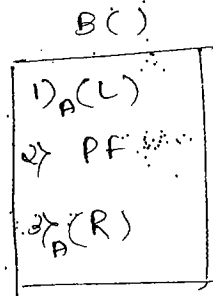
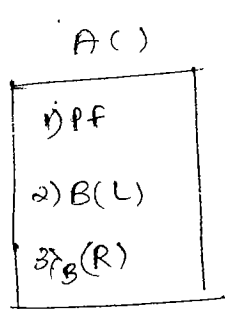
iii) PF

iv) RST

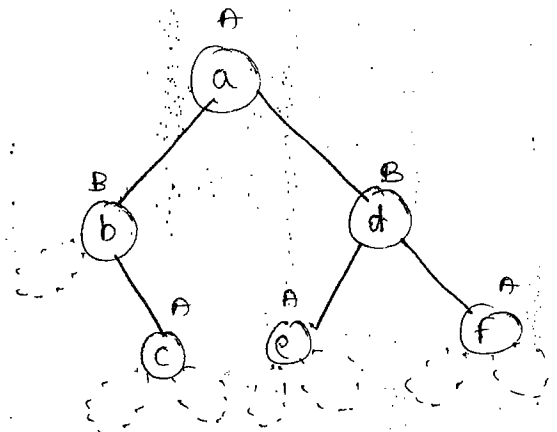
v) PF



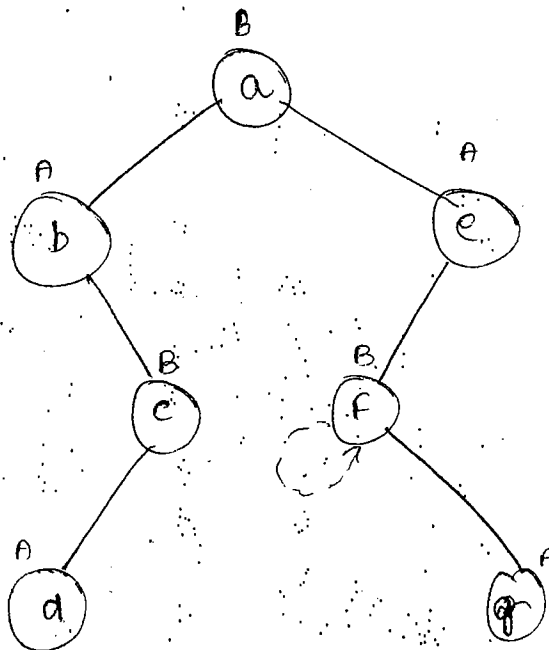
Indirect Recursion



$B(t) = bcadef$



$A(t) = abcdef$



$B(t) = bdc aefg$

3.5-2012

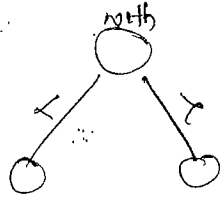
20

- BINARY SEARCH TREE (or) Lexically ordered (or) Dictionary Tree

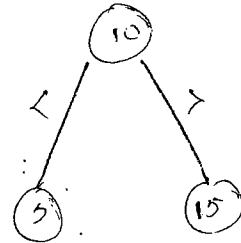
Definition :-

Lexical = word.

Binary Tree



Ex:-



IN:- 5, 10, 15

Significance :- Traverse inorder, get sorted.

Q. Create BST for the nodes: 40, 20, 30, 60, 70, 50, 10.

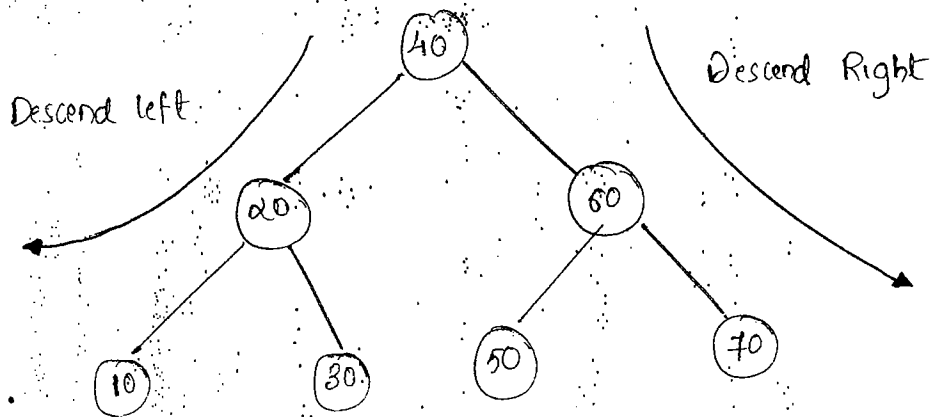
procedure: Create BST (or) insert a node in BST.

Step 1:- very first node = root node

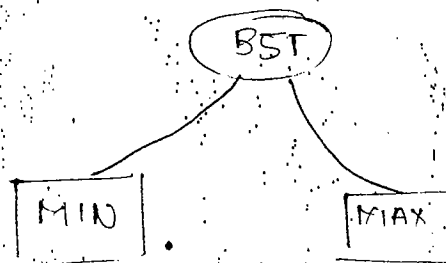
Step 2:- start comparing from the side root node, for inserting the

Subsequent Nodes.

Sol

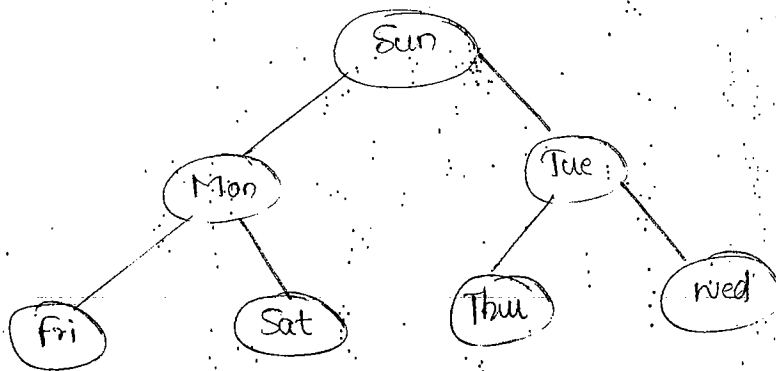


In-Sorted: 10 20 30 40 50 60 70



Q. Create LOBT or BST for Sun, Mon, Tue, Wed, Thu, Fri, Sat.

Sol



IN :- Fri MON Sat Sun Thur Tue wed.

ies
DEDO
JRO

Q. Identify Valid search Sequence on a BST for node 88.

a) 11, 11, 99, 22, 55, 95, 85, 88

b) 333, 33, 53, 93, 65, 95, 85

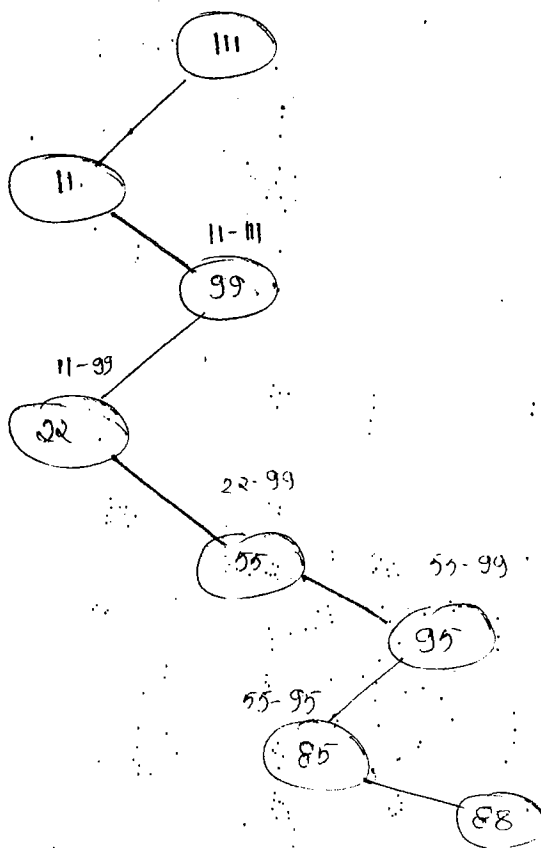
c) 1, 1111, 111, 91, 71, 101, 88

d) 40, 70, 100, 50, 80, 90, 85, 88

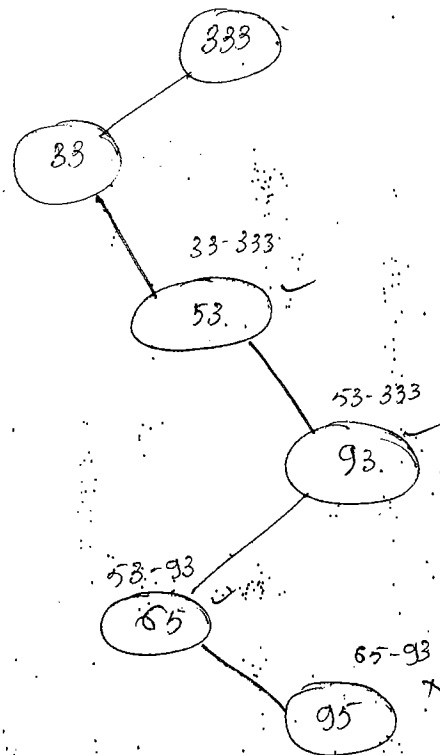
Sol

a) 11 11 99 22 55 95 85 88

for 88



333, 33, 53, 43, 65, 95, 88



Deletion procedure

Goal :- Even after deletion, the order should be intact. (remain unchanged)

Observation :- Tree can be disturbed but order remain unchanged.

A Node to be deleted: Action

i) No children : Simple delete it

ii) Only one child (or) subtree : Connect to its grand parent.

iii) Both children or subtrees : Replace with.

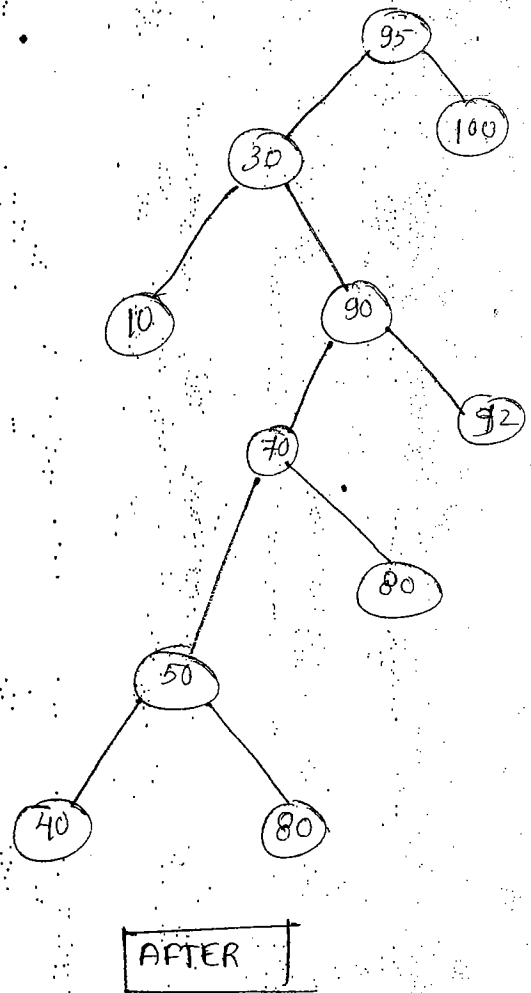
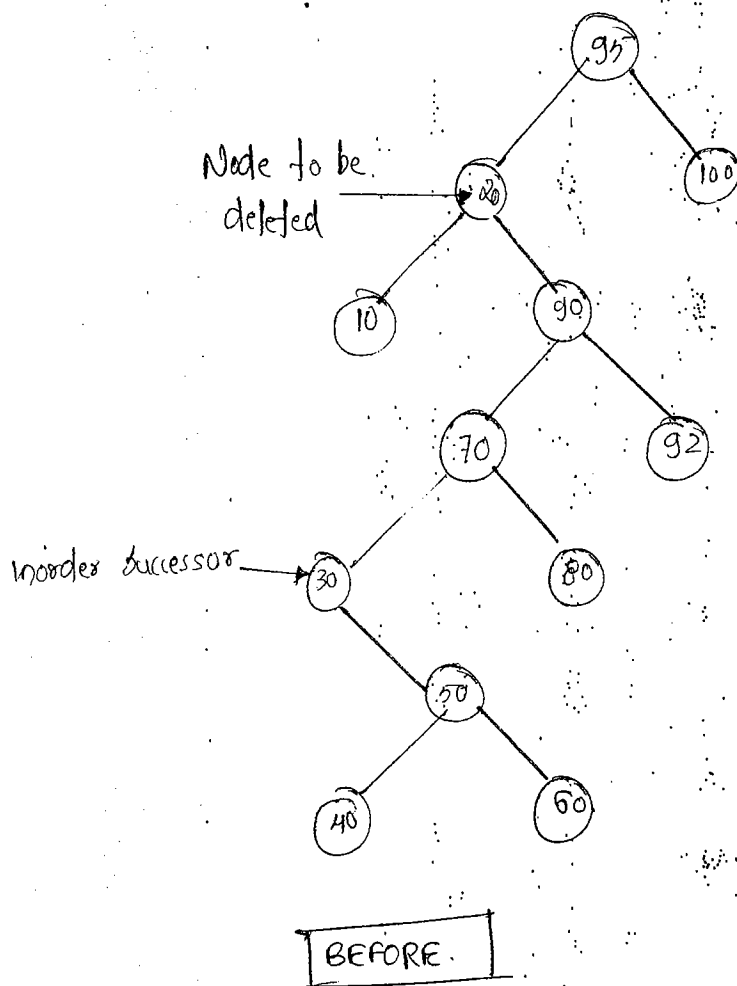
default action —→ in order successor (or) min. of RST

if insists, (asked) —→ in order predecessor (or) target of Ls

Logic for the inorder Successor

1) Jump Right.

2) Go on descend left.

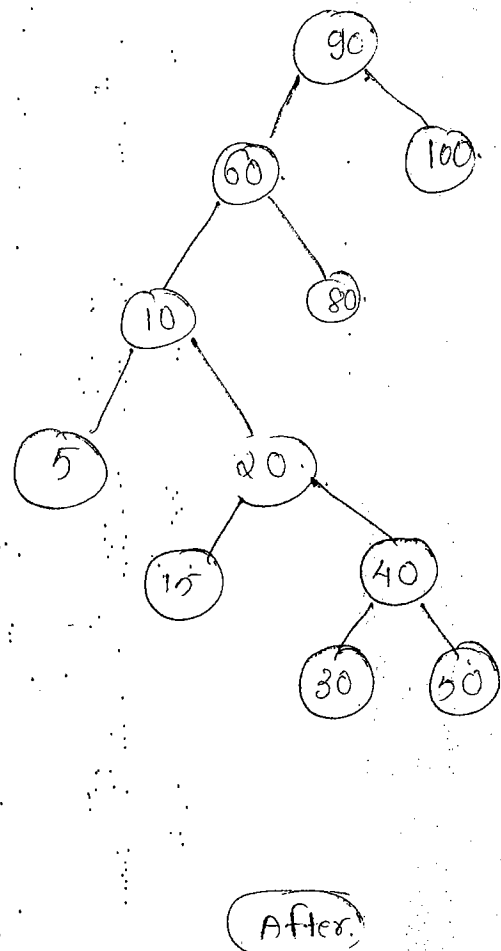
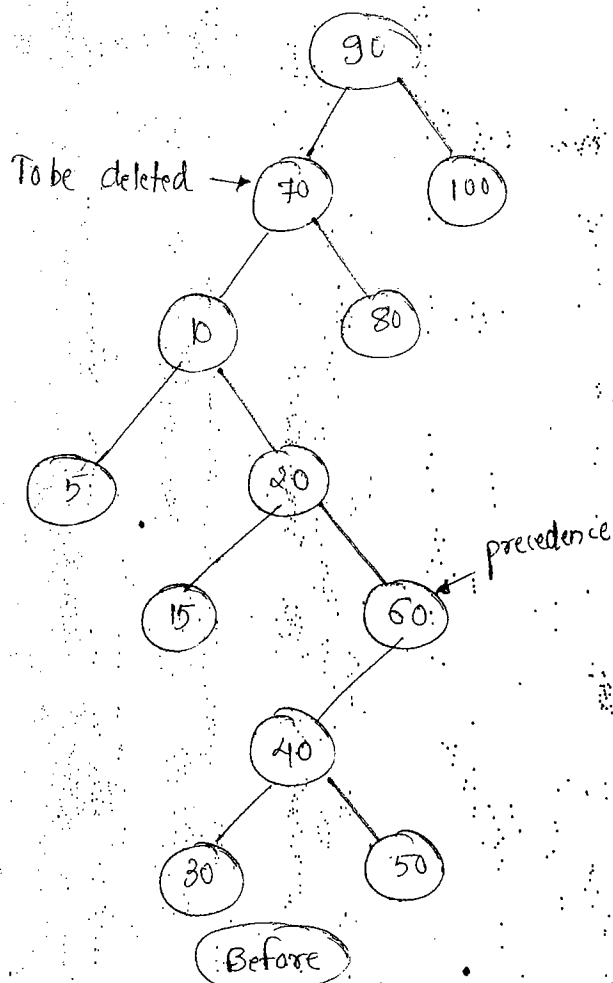


Q7 Inorder pred (or) largest of 7

1) Jump left

2) Go on descend Right

Sol if insisted to work over precedence.



$\times S_1 \rightarrow$ all CBTs are FBTs \rightarrow false
 $S_2 \rightarrow$ All FBTs are CBTs \rightarrow True

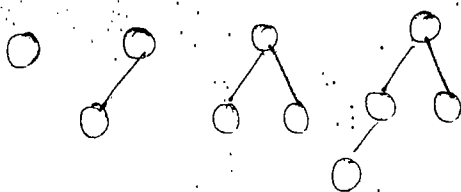
CBT
Complete BT

FBT
full BT

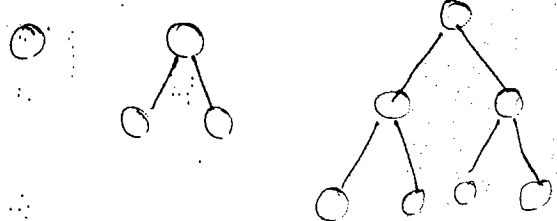
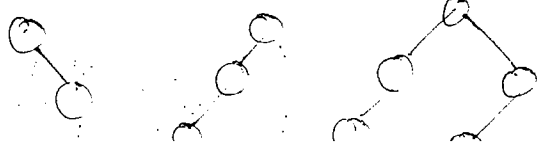
\rightarrow Seq (array) [Sequential]
 \rightarrow L to R in Sequence

Definition - of any level, the max. no. of nodes

Valid :-



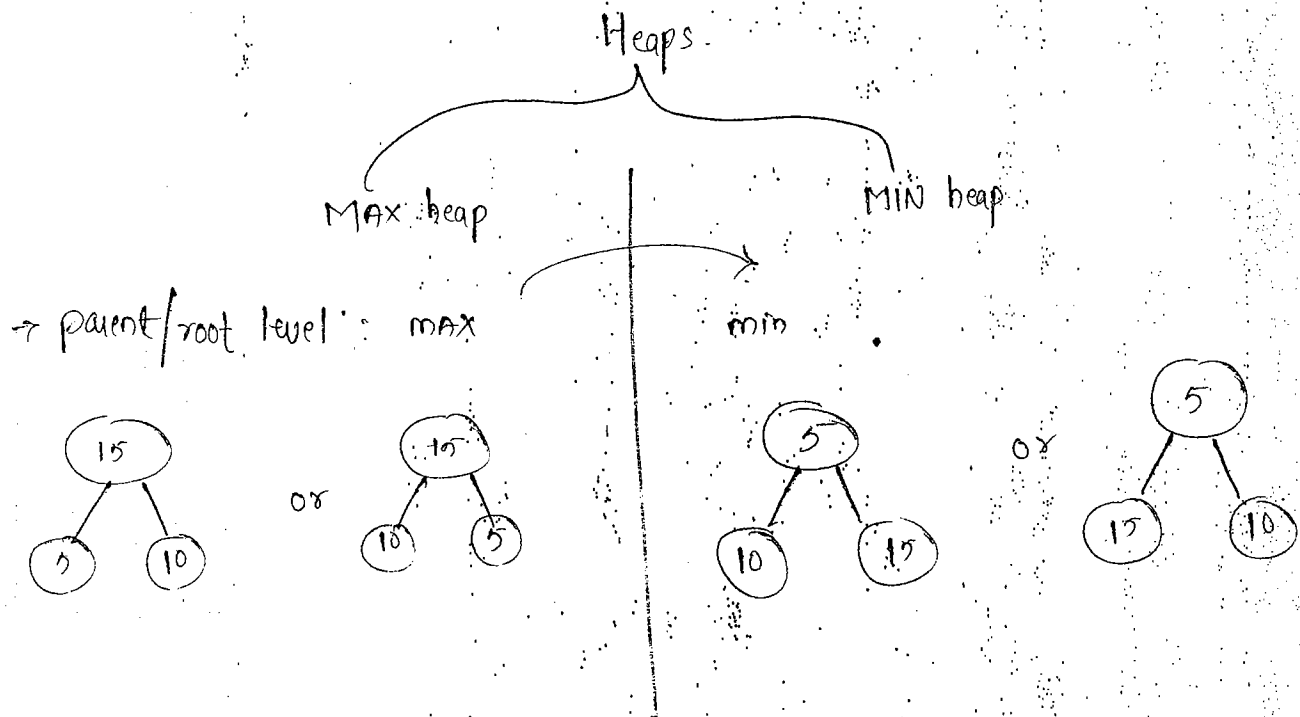
Invalid :-



• < HEAPS >

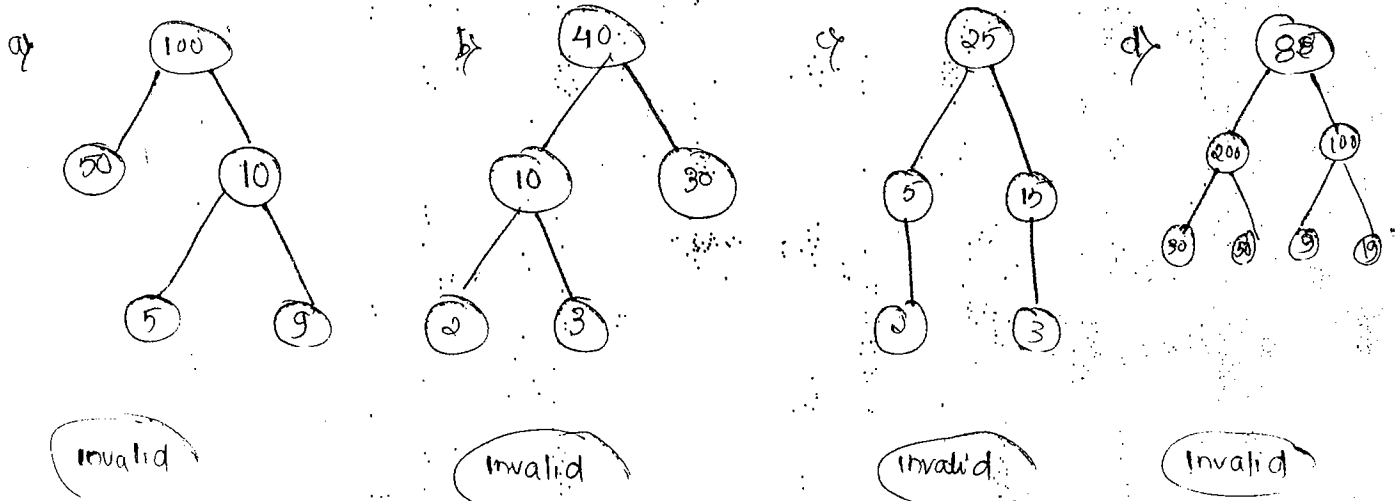
Definition :- Sequential Represent_{ation} (array) and Complete Binary tree.

- i) i th Node children : $2i$, $2i+1$
- ii) j th Node parent : $\lfloor j/2 \rfloor$



Q Identify MAX heap

sol

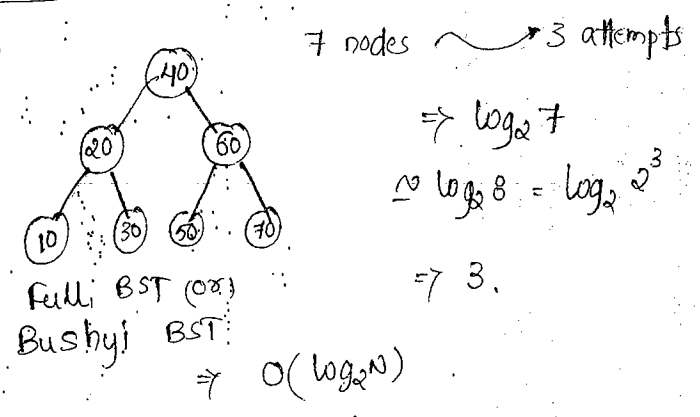
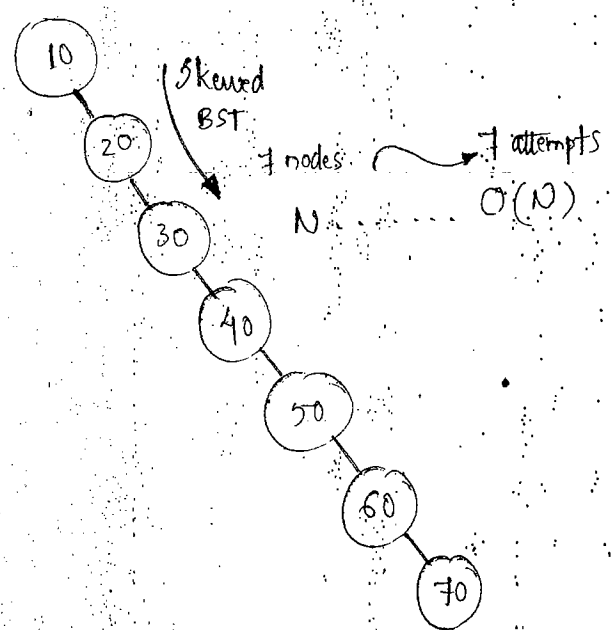


Q. Create BST

10, 20, 30, 40, 50, 60, 70

40, 20, 30, 60, 70, 50, 10

Search for 70



Goal - Apply rotations, Skewed BST ~ Bushy BST : search time $O(\log_2 N)$

• Height Balanced BST - AVL Tree

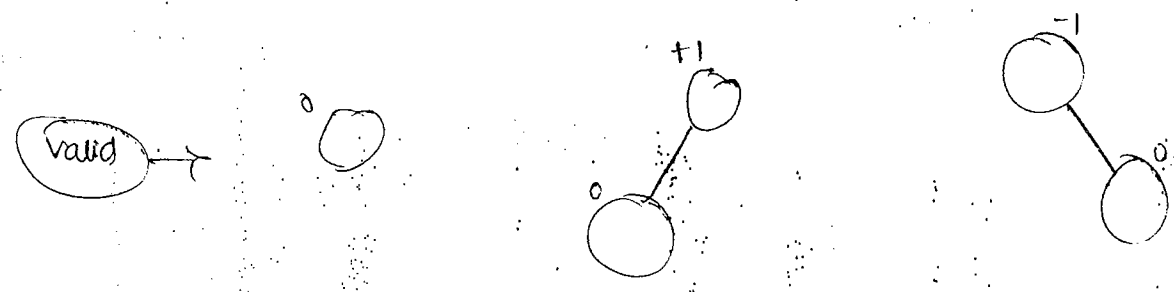
→ Devised by Russian mathematicians adelson
velski
landi

Definition :-

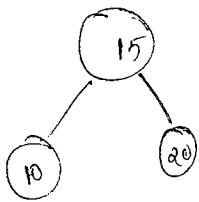
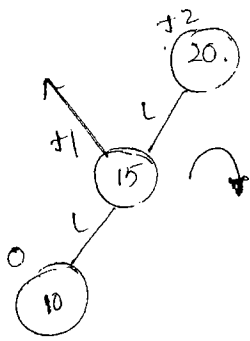
- BST
- with Balance factor

where $b_L = \text{height of LST (or) Level of the left Side Tree}$
 $b_R = \text{height of RST (or) Level of the right Side Tree}$

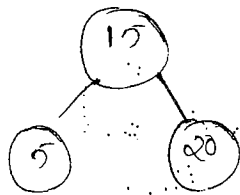
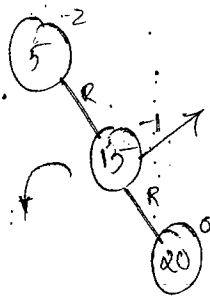
- ± 2 node is violated Node



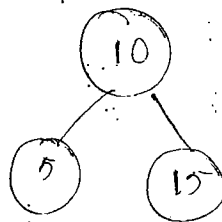
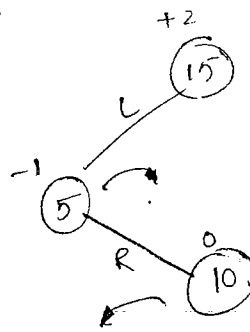
I) LL imbalance



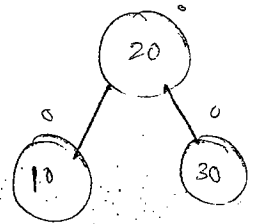
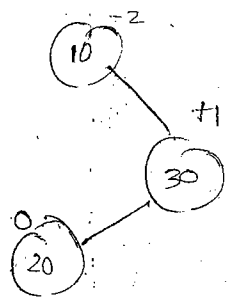
II) RR imbalance



III) LR imbalance

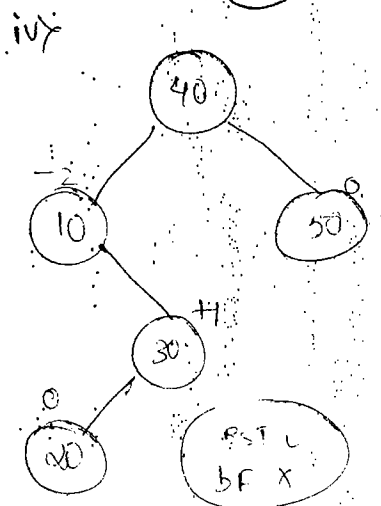
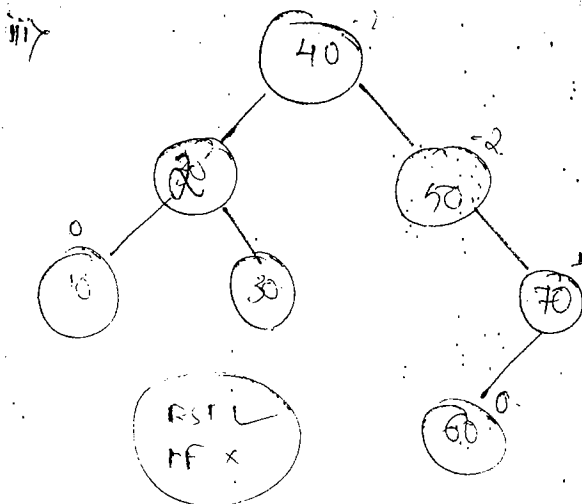
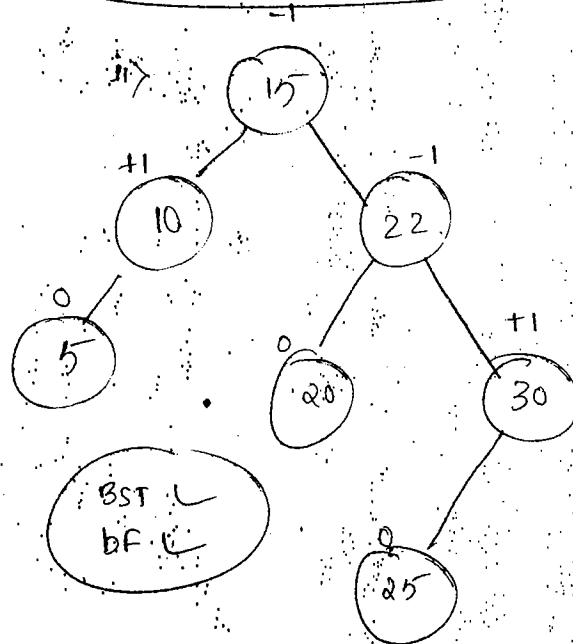
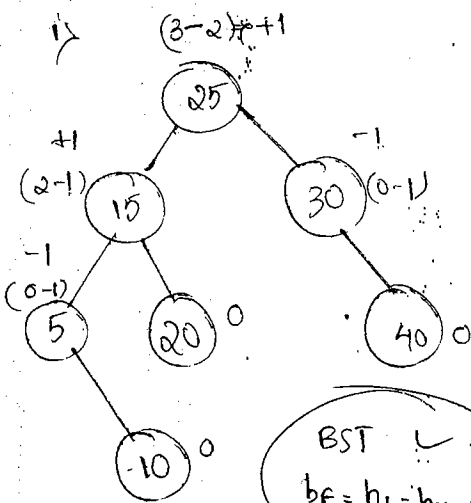


IV) RL imbalance



Q. identify valid AVL Trees.

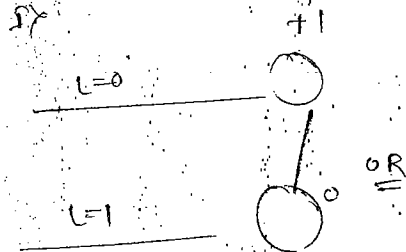
Imp
+2 \Rightarrow Violation/stop



Q. 1/2007

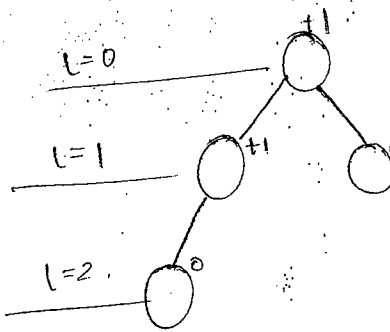
Q. Calculate minimum no. of nodes for a given height (5) root is at level 0.

Sol



$$N_{min}(1) = 2.$$

ii)



$$N_{min}(2) = 4.$$

$$N_{min}(2) = 1 + N(1) + N(0)$$

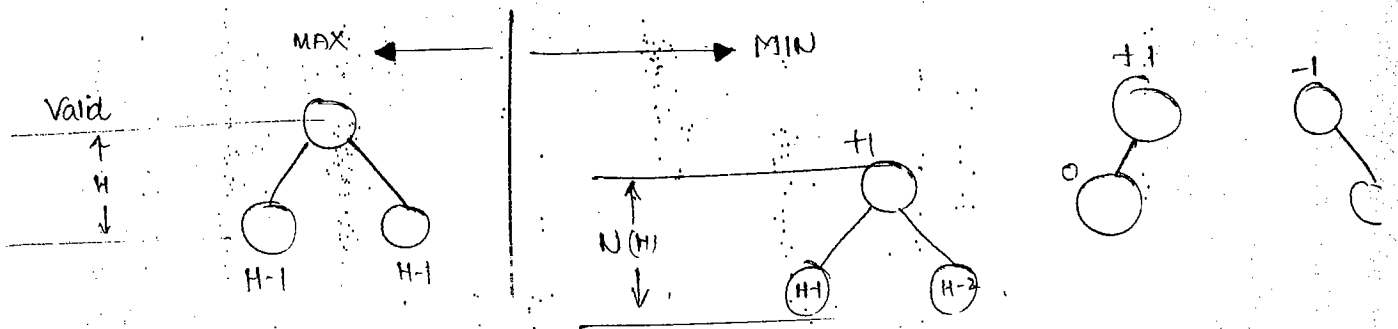
$$\Rightarrow 1 + 2 + 1$$

$$\Rightarrow 4.$$

iii)

$$N_{min}(3) = 7$$

$$N_{min}(H) = 1 + N(H-1) + N(H-2)$$



$$1 + N(H-1) + N(H-2)$$

Fibonacci $f(0) = f(0-1) + f(0-2)$

$$N_{min}(H) = 1 + N(H-1) + N(H-2)$$

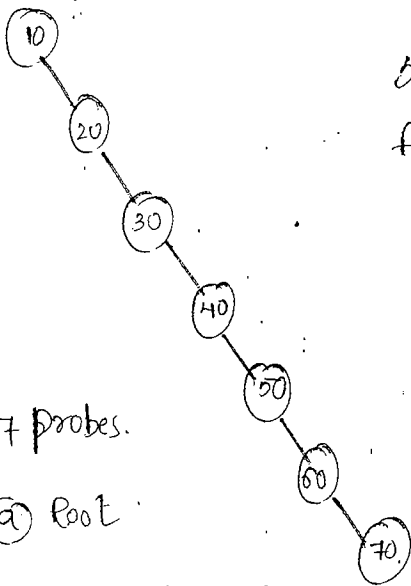
H	0	1	2	3	4	5
$N_{min}(H)$	1	2	4	7	12	20

< SPLAY TREE >

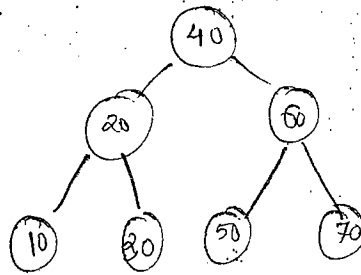
→ In the world of information technology it has been observed that the same record is going to be accessed for several times. Then splaying that record towards root requires the following rotations

Rotations

1) zig 2) zig-zig 3) zig-zag



Search for 70.
for 1000 times



1000 times \times 3 attempt = 3000 probes

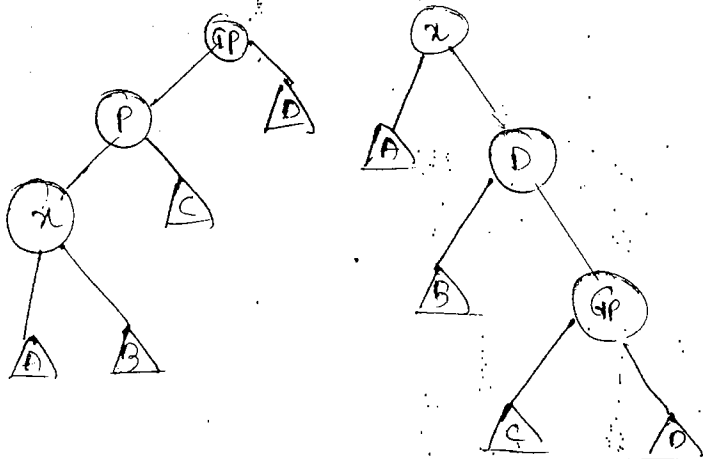
1 time = 7 probes.

Splay 70 to root

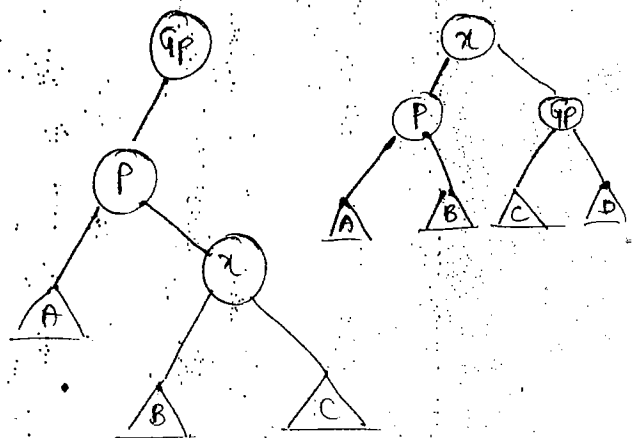
999 times \times 1 attempt \times 999

1000 times = 1006 attempts / probe

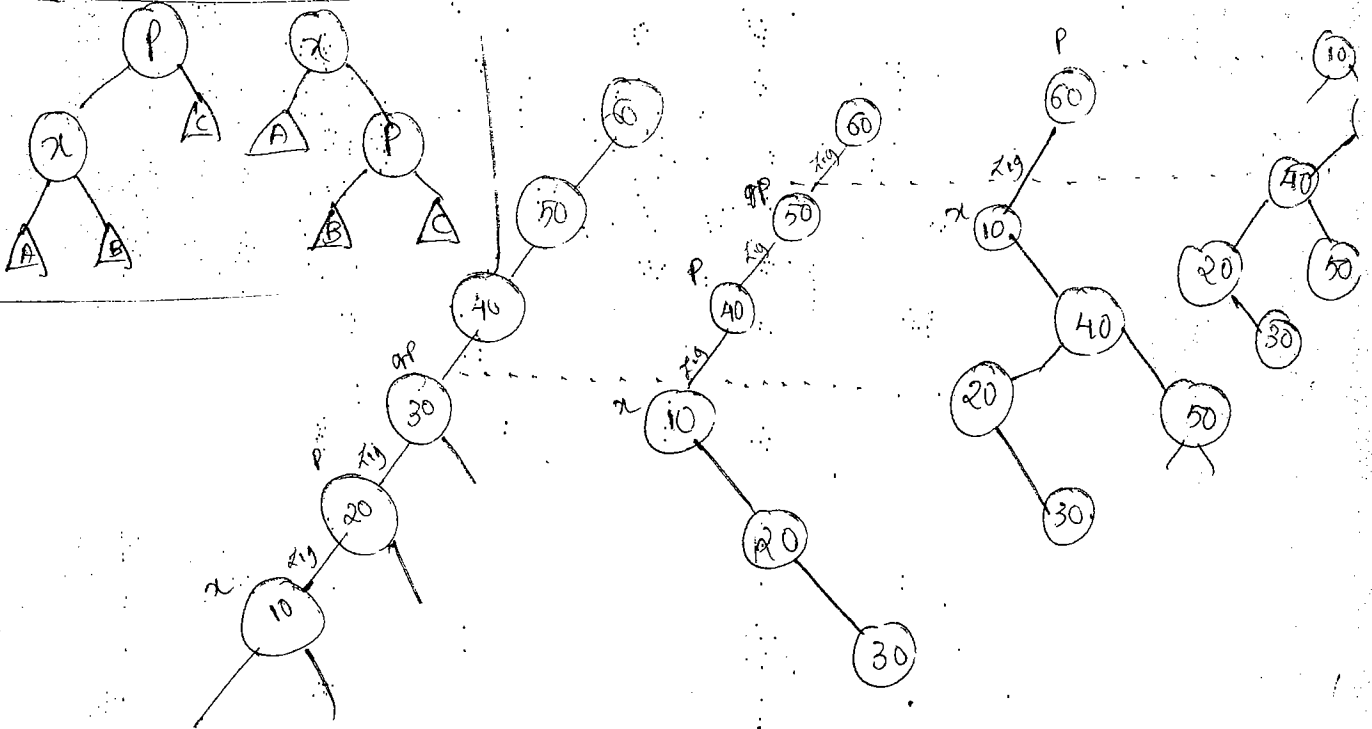
I - Zig-Zig



II - Zig-Zag



Using this method.



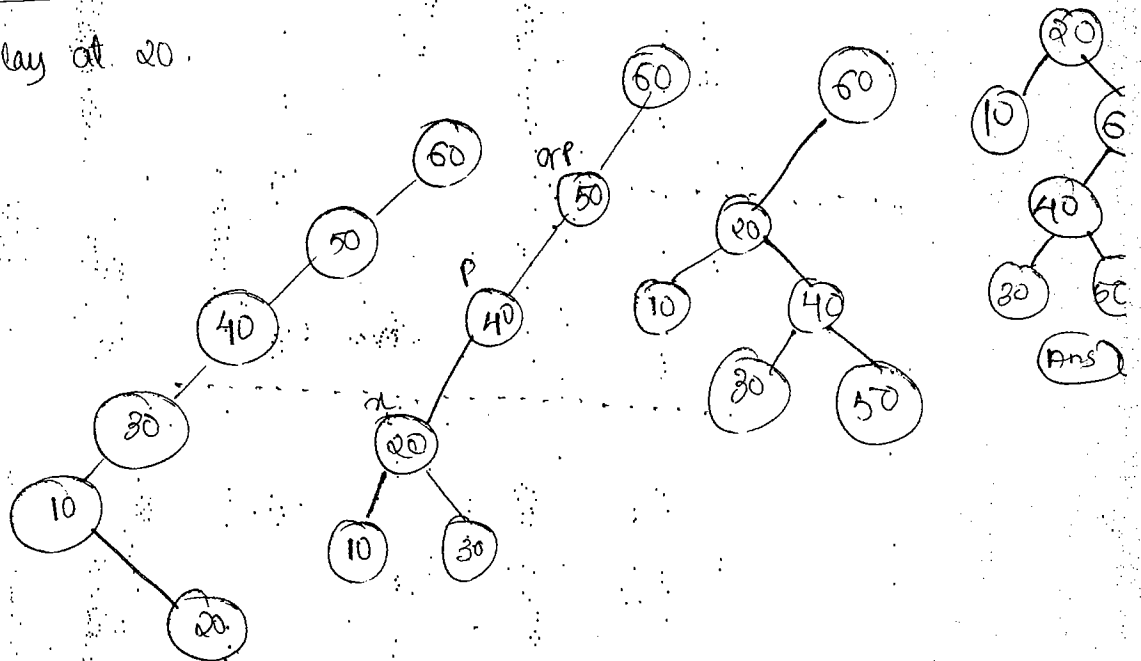
Splay tree =

BST =

IN = 10, 20, 30, 40, 50, 60.

Q. Find Splay at 20.

- i) Zig-Zag
- ii) Zig-Zig
- iii) Zig



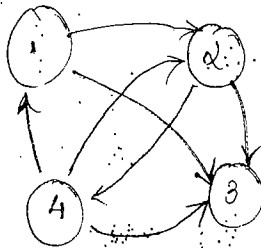
• < GRAPHS >

Representation

i) Sets

Vertices	adjacent
1	{2, 3}
2	{3, 4}
3	\emptyset
4	{1, 2, 3}

Given: Directed graph



ii) Adjacent matrix

	1	2	3	4
1		T	T	F
2	F		T	T
3	F	F		F
4	T	T	T	

Graph Traversals

DFS

i) Depth first Search

- ii) Similar to preorder
- iii) Recursive stack

Termination :- Stack empty
in algorithm: Each

BFS

ii) Breadth first Search

- ii) level order
- iii) iterative Queue

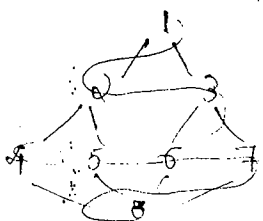
\emptyset is empty

in algorithm: all

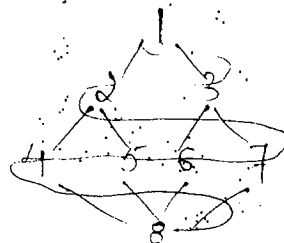
BFS

L.R

R.L



op \rightarrow 1, 2, 3, 4, 5, 6, 7, 8



op \rightarrow 1, 3, 2, 7, 6, 5, 4, 8

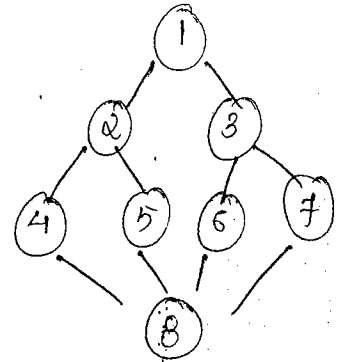
Q. Identify valid BFS search Sequences. if starting vertex is v_1

a) 1 3 2 7 6 4 5 8 \rightarrow valid

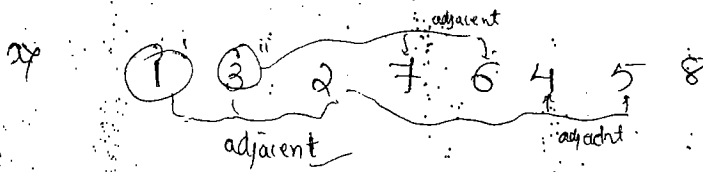
b) 1 2 3 5 4 7 6 8 \rightarrow valid

c) 1 2 3 7 6 5 4 8 \rightarrow invalid

d) 1 3 2 4 5 7 6 8 \rightarrow invalid



Sol



(1 3 2 7 6 4 5 8)

- adjacent of 1 is 2,3, next is 3,2
- adjacent of 3 is 6,7,
- adjacent of 2 is 4,6,

b) 5 2 8 4 1 6 7 3 \rightarrow valid

adjacent of 5 is 2,8, adjacent of 2 is 4,1, adjacent of 8 is 6,7, hence valid.

c) 5 2 8 4 1 3 6 7 \rightarrow wrong

d) 5 8 2 4 7 6 1 3 \rightarrow valid

e) 5 8 2 4 6 1 3 7 \rightarrow wrong.

4.5.2012

• Depth first search

→ Recursive Routine (stack)

Graph	Tree
vertex explore	node visit

procedure DFS(v)

visited(v) = True

for each adjacent w for v

if not visited(w)

Call DFS(w)

end.

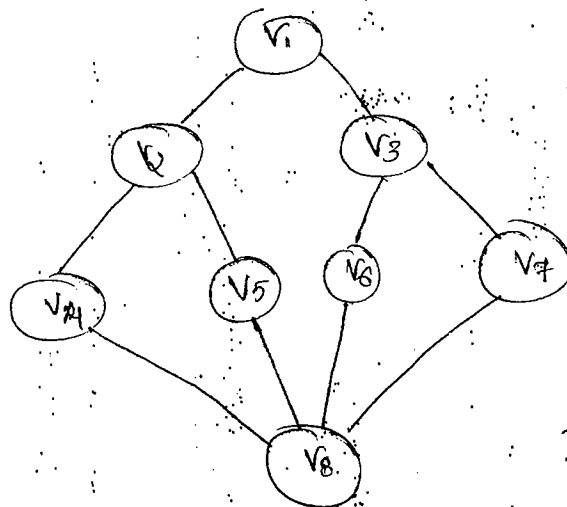


fig. 4.5

Observation

→ step back only when the adjacent is already visited

→ move forward (Depth) ; atleast on unexplored adjacent is there.

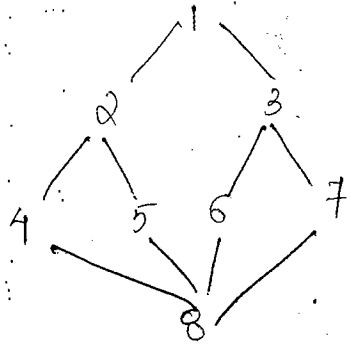
St. vertex ($v1$)

if Valid : 1 2 5 8 4 6 3 7

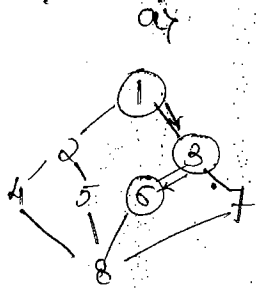
if Invalid : 1 2 5 8 4 6 7 3

Q. identify valid/invalid DFS Search Sequences, if
 starting vertex (v₁)

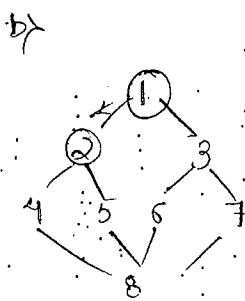
- a) 1 3 6 7 8 5 4 2
 b) 1 2 3 6 7 5 4 8
 c) 1 2 5 4 8 6 3 7
 d) 1 3 6 8 7 5 2 4



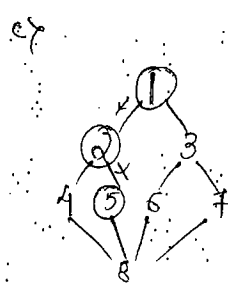
Sol



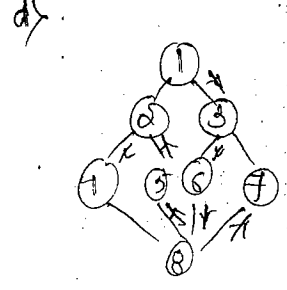
• 6 to 7 can't move
 hence invalid



• hence invalid



• hence invalid



• hence valid

• in case d, after 7 we have to go to 5, already both the nodes are visited hence step back to 8 then move to 5 then 2 & 4

~~e) 3, 1~~

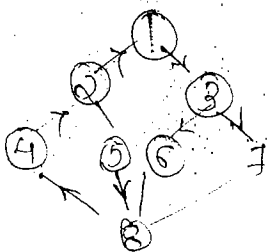
Q. starting vertex is (v₅)

- a) 5 2 4 8 6 3 7 1
 c) 5 8 4 2 1 3 6 7

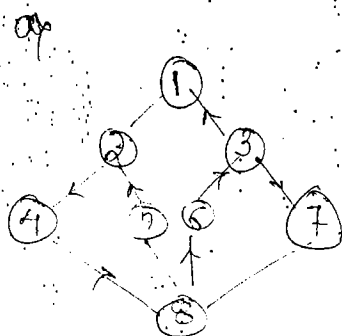
b) 5 8 6 3 7 1 2 4

d) 5 2 1 3 6 7 8 4

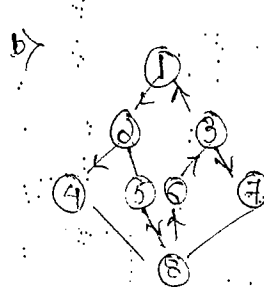
Sol



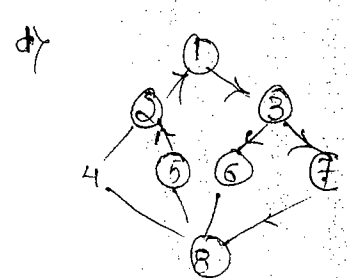
Valid



Valid



Valid



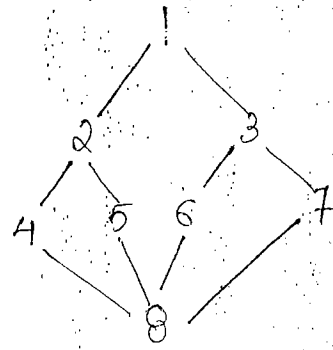
invalid

Q Identify valid/invalid DFS/BFS Search Sequence.

Q 7 3 8 1 2 4 5 6

6 is missing hence not BFS.
 () 7 3 8 1 2 4 5 6

Ans: neither BFS nor DFS.



Q 2 4 5 1 8 6 7 3

• neither DFS nor BFS

Q 4 2 5 8 6 3 7 1

• valid for DFS

• HUFFMAN (TREE) ALGORITHM (prefix tree : Binary tree)

Q Encode H.T for the given data.

Symbols	a	b	c	d	e	f	g
frequency	10	15	12	3	4	13	1

Definition: An algorithm constructs an optimal prefix code by repeatedly merging the minimum height trees.

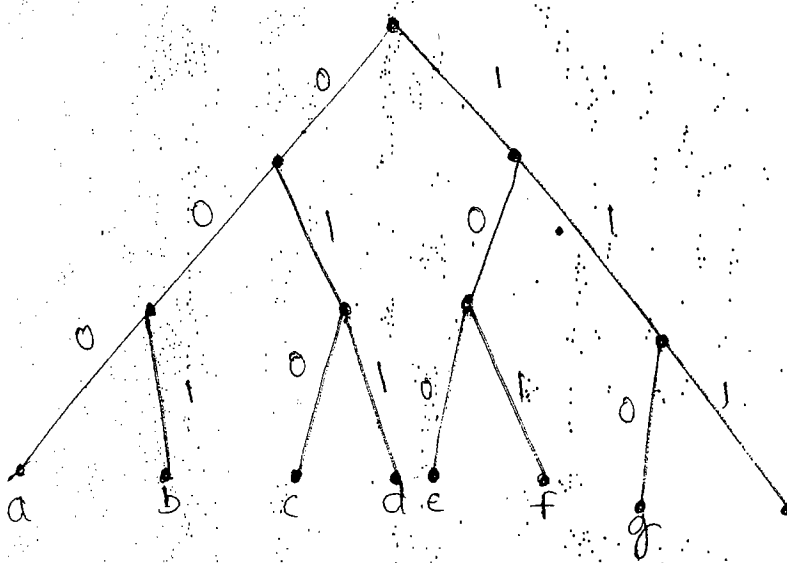
A binary tree (ty):- A data structure in which a left branch represents '0' and a right branch represents '1'. The path to ~~be~~ a node ~~indication~~ indicate its representation.

Goal: To reduce the no. of bits required (Zip or Compression technique)
logic: More ^{repeated, or} frequency character should be represented in less no. of b and vice versa.

i) Without huffman

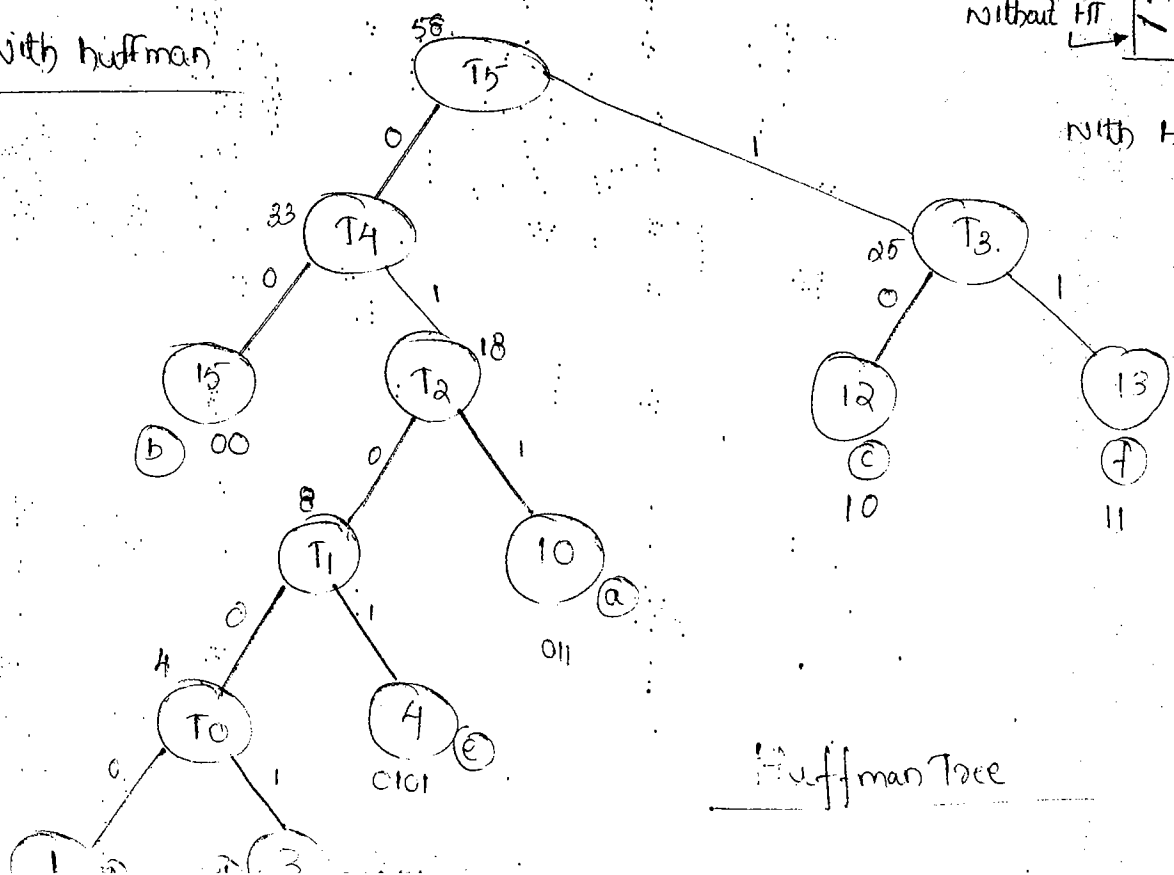
→ Refer above table.

Normal tree



	Symbols	frequency	no. of bits	Total
011	a	10	000	30
00	b	15	001	45
10	c	12	010	36
01001	d	3	011	9
0101	e	4	100	12
11	f	13	101	39
01000	g	1	010	3

ii) With huffman



Without HT

With HT

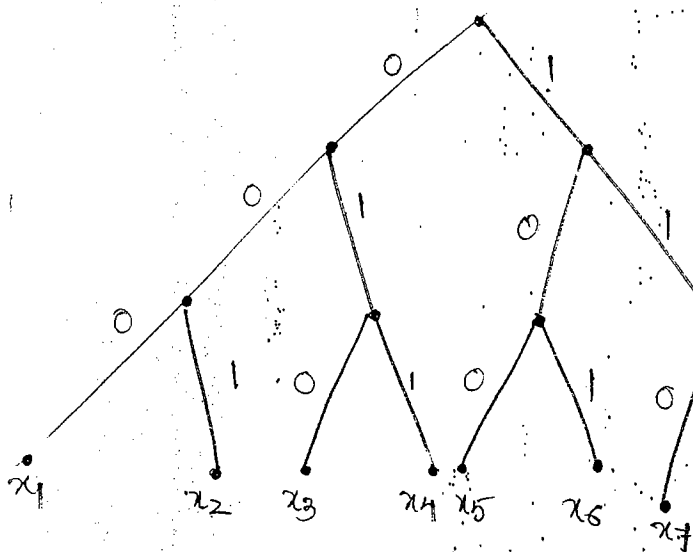
Huffman Tree

Q.7 Encode H.T for the given data

Symbols	x_1	x_2	x_3	x_4	x_5	x_6	x_7
probabilities	0.35	0.3	0.2	0.1	0.04	0.005	0.005

602

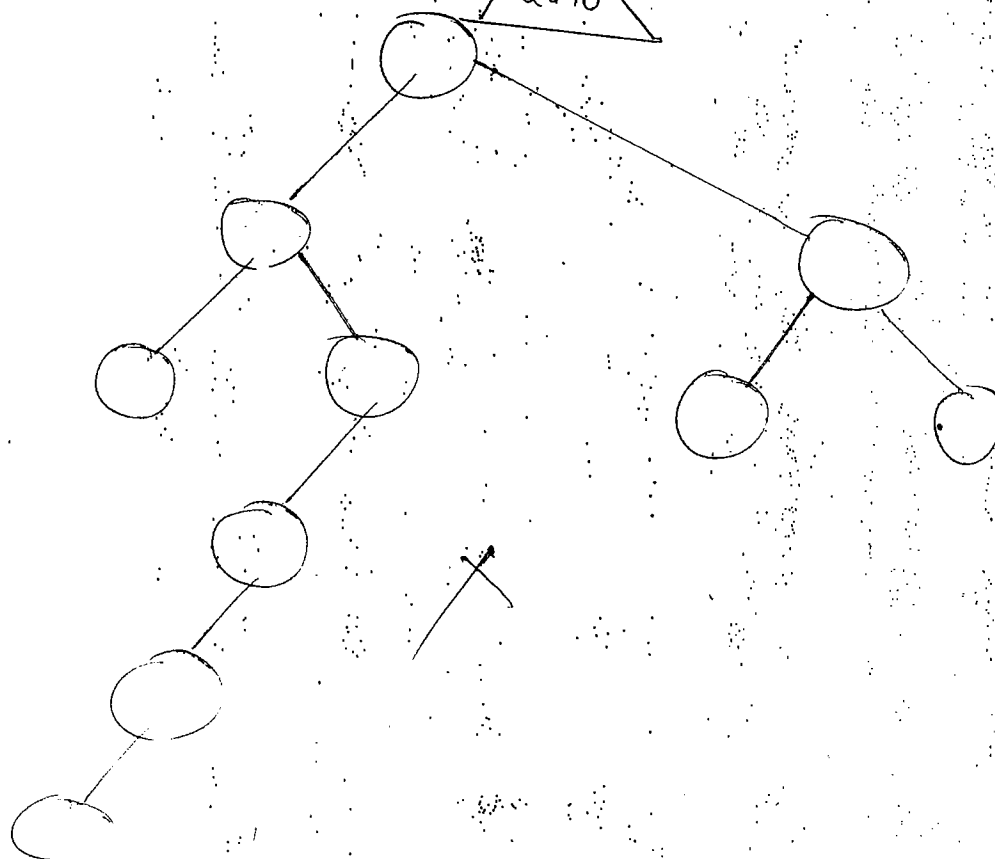
ix) without Huffman



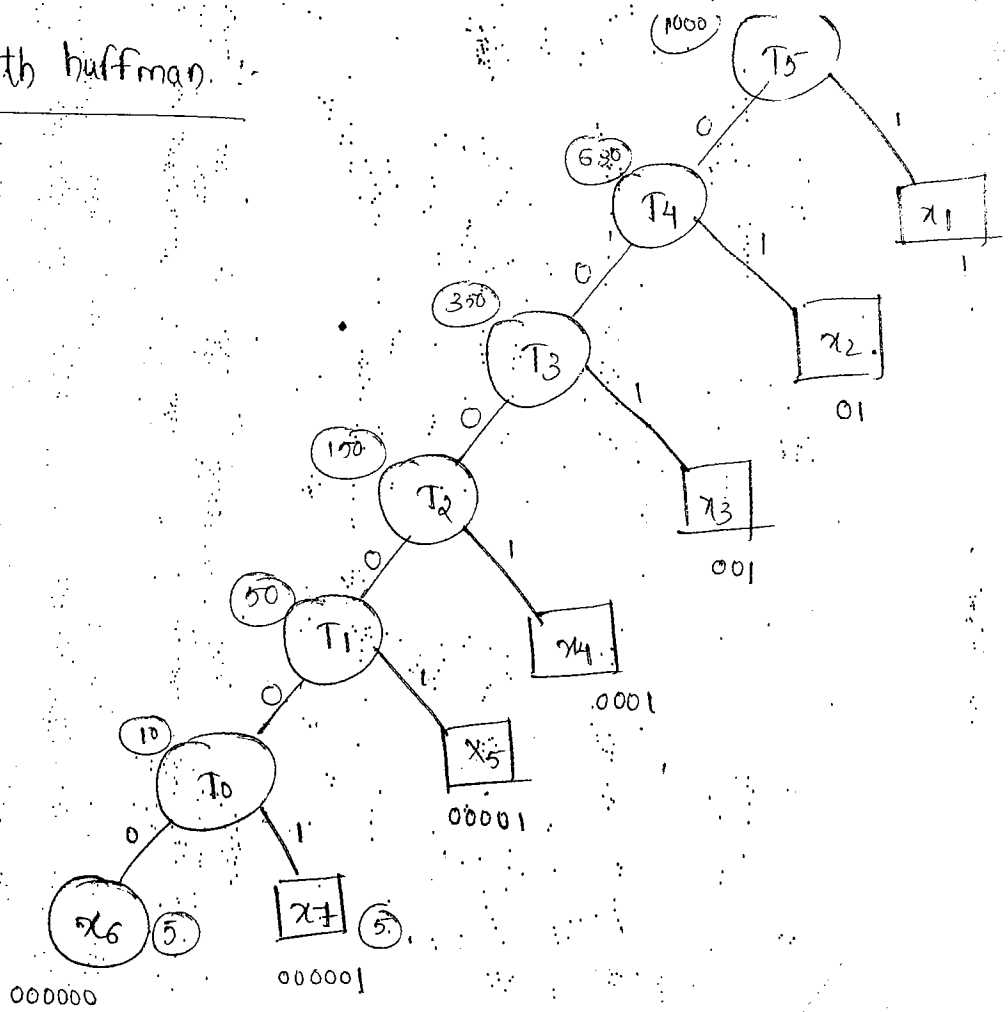
		Symbols	frequency	No. of bits	Total
350	1	x_1	0.35	000	1.05
600	01	x_2	0.3	001	0.9
500	001	x_3	0.2	010	0.6
400	0001	x_4	0.1	011	0.3
200	00001	x_5	0.04	100	0.12
30	000001	x_6	0.005	101	0.015
30	000000	x_7	0.005	110	0.015

2210

3000

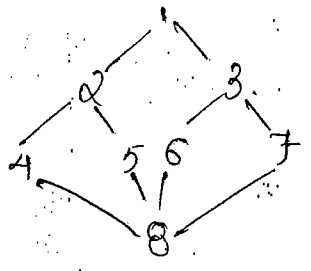


ing with huffman.



huffman Tree.

Pr

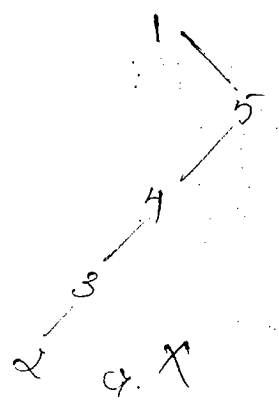
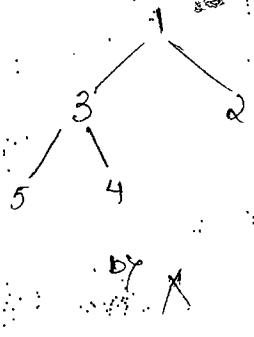
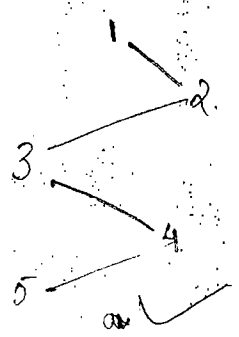


~~1~~ 5 2 4 8 6 3 7 1
~~x2~~ 5 8 6 7 3 1 2 4
~~3~~ 5 8 4 2 1 3 6 7
~~x4~~ 5 8 7 6 3 2 5 4

Pr

PAST 1 5 4 3 2 1
 IN 1 3 5 4 2
 what is BT...?

50



• < HASHING >

→ Searching Technique

→ Goal Search time = $O(1)$ but not fulfilled

HF → Hash function = key to address transformation

$$H(x) = X \bmod m$$

Where $x = \text{Key}$

$m = \text{hash table size}$

Searching

1) Linear Search

2) Sequential Search

key	address
69	4
47	2
50	0
18	3
71	1

0	50
1	71
2	47
3	18
4	69

here $m = 5 (0-4)$

Search for 69

$$69 \div 5$$

$$\begin{array}{r} 5 \overline{) 69} \quad (13 \\ 65 \\ \hline 4 \end{array}$$

4 is the address location where 69 is stored.

Q. Find the suitable HF for storing the elements in the address

range from 1 to 1000

a) $H(x) = X \bmod 1000$ $0-999$

b) $H(x) = (X+1) \bmod 1000$ $1-1000$

c) $H(x) = X \bmod (1000+1)$ $0-1000$

d) $H(x) = (X \bmod 1000) + 1$ $1-1000$

Sol

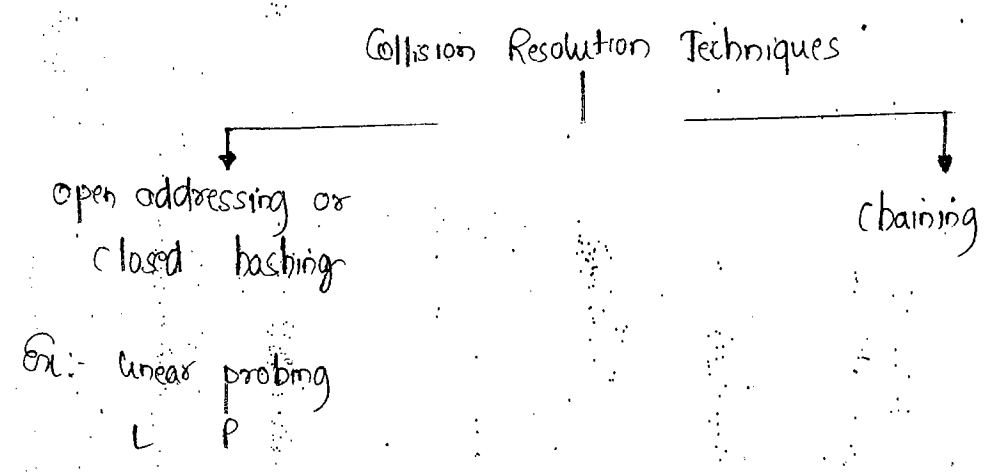
Qr find the no. of collisions for storing the following keys with a hash function $H(K) = K \text{ mod } 13$.

K = Key	130	36	98	12	9	32	45	71	200	300
Location	0	10	5	12	9	6	6	6	5	1
							①	②	③	

Sol

No. of collisions = 3.

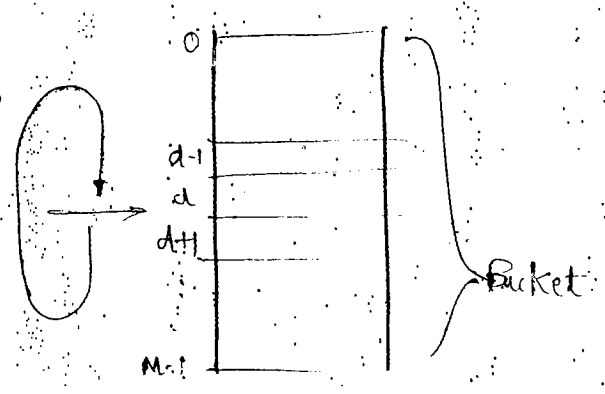
• < COLLISION RESOLUTION TECHNIQUES >



Defination :-

LP :- Once the bucket is full, then linearly come down to find the next empty bucket.

Sequence :- $d, d+1, d+2, \dots, m-1, 0, 1, \dots, d-1$



Q. The Key 17, 18, 13, 2, 3, 23, 15 are inserted into an empty hash table of length 10 with $H(K) = K \% 10$ using linear probing.

What is the Resultant Table.

Sol.

Key	17	18	13	2	3	23	15
$H(K) = K \% 10$	7	8	3	2	3	3	5

advantage

- Once the empty bucket is encountered then the element is not found in the entire table without

Comparing all the buckets.

probes	
0	
1	
2	2
3	13
4	3
5	23
6	15
7	17
8	18
9	

Q. Which of the following choices given a possible order in which the Key valid could have been inserted into the hash table given below.

a) 46 42 34 52 23 33

b) 34 42 23 52 33 46

c) 46 34 42 23 52 33

d) 42 46 33 23 34 52

Sol.

a)

Key	46	42	34	52	23	33
Location	6	2	4	2	3	3

• Invalid.

b)

Key	34	42	23	52	33	46
Location	4	2	3	2	3	6

• Invalid

c)

Key	46	34	42	23	52	33
Location	6	4	2	3	2	3

• Invalid → Come down and store in next address.

d)

Key	42	46	33	23	34	52
Location	2	6	3	3	4	2

• Invalid.

0	
1	
2	12
3	23
4	34
5	52
6	46
7	33
8	
9	

Observation :-

To have the above same effect, how many such different insertion sequences could have been: then $= 30$

Case (i) To insert variable 52

Variable	fixed
42, 23, 34	52, 46, 33

$3! = 6$

Case (ii) To insert 33.

it should definitely come after 5.

Variable	fixed
46, 42, 23, 34	52, 33.

$4! = 24$

Case (i) and Case (ii) are mutually exclusive.

$$So = 6 + 24 = 30$$

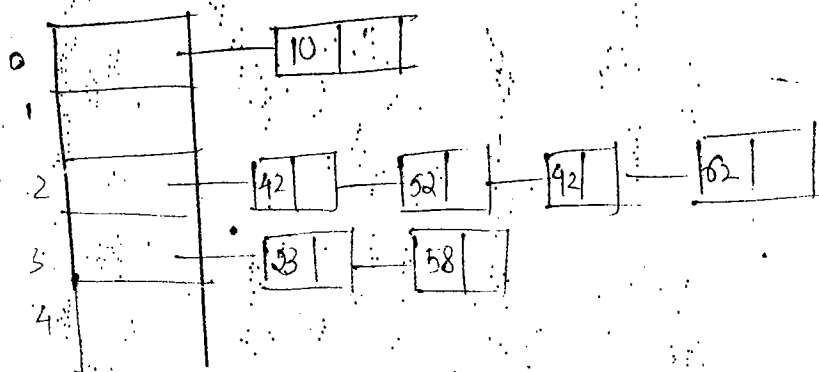
Disadvantage of open addressing or closed hashing

- 1) Overflow.
- 2) Deletion not possible
- 3) Collided records requires more probes.

Advantage of chaining

- 1) No overflow
- 2) deletion is possible
- 3) Collided records requires less probes

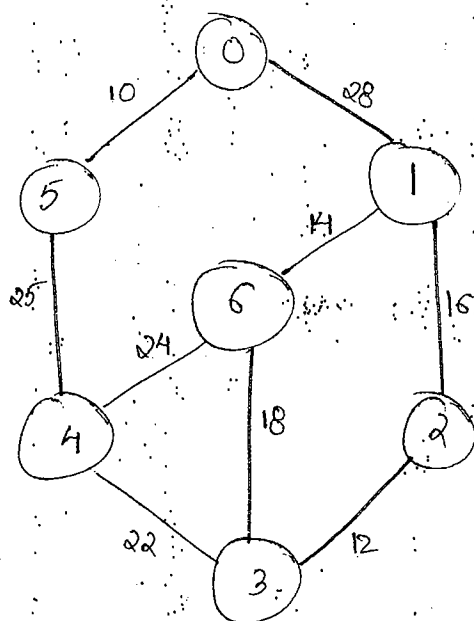
K	23	42	52	92	58	62
K-1.5	3	2	2	2	3	2



5-4-2012

• < MINIMUM COST SPANNING TREES >

- 1) we must use only edges with in the edges.
- 2) we must use exactly $(n-1)$ edges.
- 3) " " " not use edges that would cause cycles.



Kruskal algorithm

- A set of selected edges form a Forest.

Prim's algorithm

- A set of selected edges forms a Tree.

edges	Cost ascending order (min)	edges	Corresponding Cost order
(0,5)	10 → Add	(3,4)	22 → Add
(2,3)	12 → Add	(4,6)	24 → Discarded
(1,6)	14 → Add	(4,5)	25 → A Now $N-1$ edges, rest of
(1,2)	16 → Add	(0,1)	28 → Not considered
(3,6)	18 → Discarded (because cycle)		

• always start with minimum edge only.

ii) PRIM algorithm

eligible edges

0 = {10, 28}

5 = {25}

4 = {22, 24}

3 = {12, 18}

2 = {15}

1 = {14}

stop

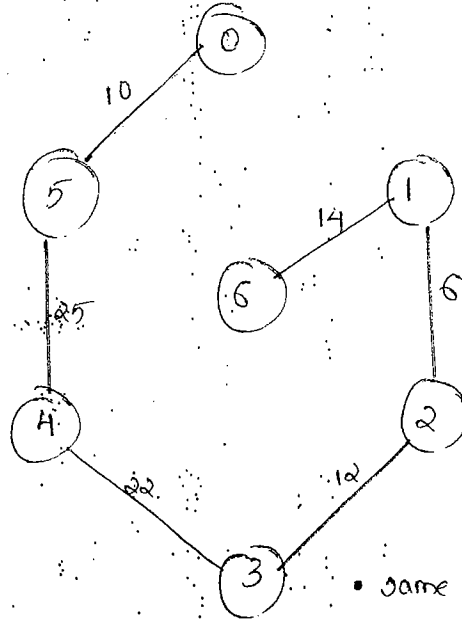
Observation

it can be started from any vertex.

$$\eta = x(0.66) = 7$$

$$e = x - 1 = 6$$

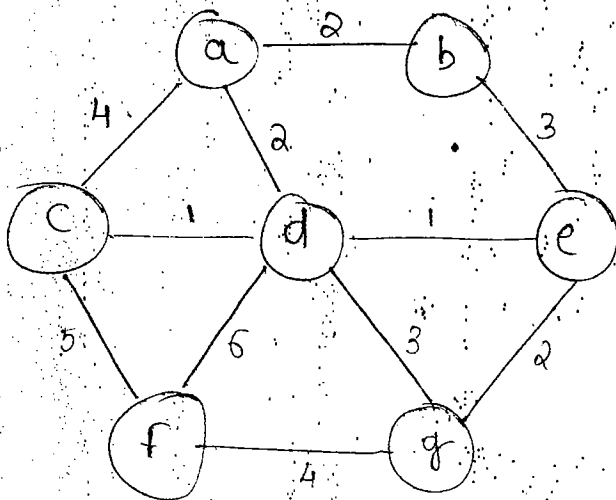
e → edge.
n → node



• same for both algorithms

Q.1

Sol



• SEARCH

Search

Linear or Sequential search

Binary search

• Input = unsorted

• Time = $O(N)$

$O \rightarrow$ order

• Input = Sorted

• Time = $O(\log n)$

Termination

while ($L \leq H$)

{

$$M = \left\lfloor \frac{L+H}{2} \right\rfloor$$

if $X < A[M]$

$H = M - 1$

else if $X > A[M]$

$L = M + 1$

else found : M position

}

Q.

	1	2	3	4	5	6	7	8	9	10
A	75	151	203	275	318	489	524	591	667	727

Sol

ex: Search for 275

low \rightarrow 1

high \rightarrow 10

L	H	M
1	10	5
1	4	2
3	4	3
4	4	(4)

ex 2: Search for 555

L \leq H	M
1 \leq 10	5
6 \leq 10	8
6 \leq 7	6
7 \leq 7	7
8 \leq 7	

stop: Not found

• <SORTING>

- 1) Selection Sort
- 2) Bubble Sort
- 3) Insertion
- 4) Radix
- 5) Merge
- 6) Heap sort

Selection Sort

Select the minimum and place in its appropriate position by swapping.

	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
1	47	<u>11</u>								
2	23	23	<u>23</u>							
3	11	74	<u>74</u>	<u>36</u>						
4	65	42	42	<u>42</u>	<u>38</u>					
5	38	65	65	65	<u>65</u>	<u>42</u>				
6	44	38	38	<u>38</u>	<u>42</u>	<u>65</u>	<u>44</u>			
7	36	44	44	44	44	<u>44</u>	<u>65</u>	<u>65</u>		
8	74	36	<u>36</u>	74	74	74	74	74	<u>74</u>	
9	99	99	99	99	99	99	99	99	<u>99</u>	87
10	87	87	87	87	87	87	87	87	<u>87</u>	99

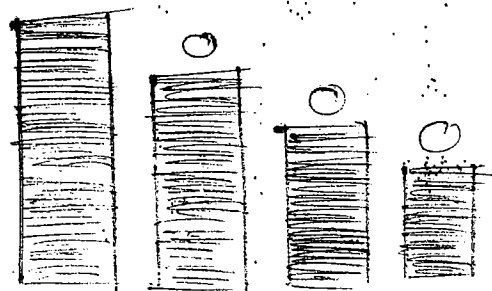
Bubble Sort

from unsorted to sorted. the movement of the elements are bubbling up as if a bubble moves from down to top.

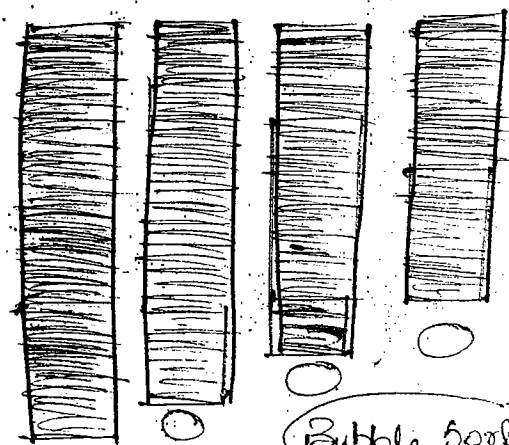
$a(i) > a(i+1)$ if no exchange, then stop.
 exchange;

• No. exchange stop.

Selection Sort



• definitely $(N-1)$ Rounds.



Bubble Sort

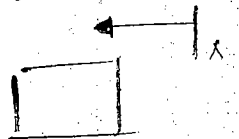
	①	②	③	④	⑤	⑥
42	23	23	11	23	23	23
23	42	11	23	38	36	36
74	11	42	38	36	38	38
11	65	38	42	42	42	42
65	38	44	36	44	44	44
38	44	36	44	65	65	65
44	36	65	65	74	74	74
36	74	74	74	87	87	87
99	87	87	87	99	99	99
87	99	99	99			

no
exchange then stop.

iii) insertion Sort

- insert a node by comparing with its predecessor.

logic



Given -

42	23	74	11	65
----	----	----	----	----

42

23	42
----	----

23	42	74
----	----	----

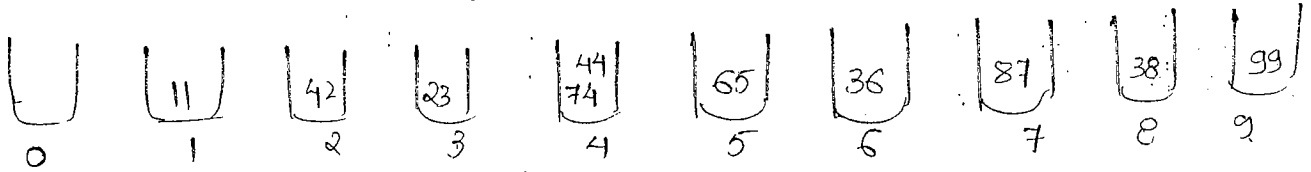
11	23	42	74
----	----	----	----

11	23	42	65	74
----	----	----	----	----

4. Radix Sort

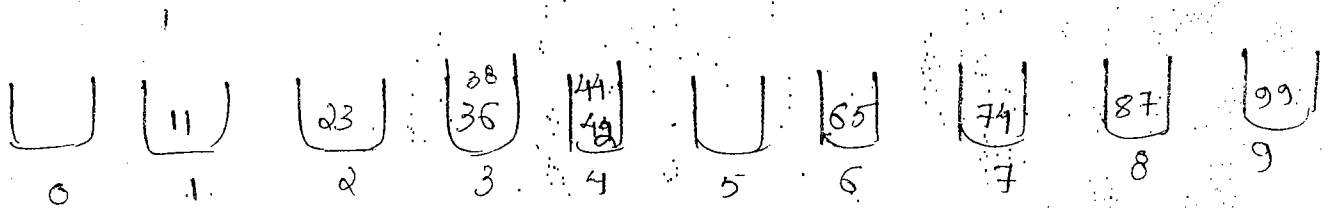
I Round :- unit place

merging bottom to top.



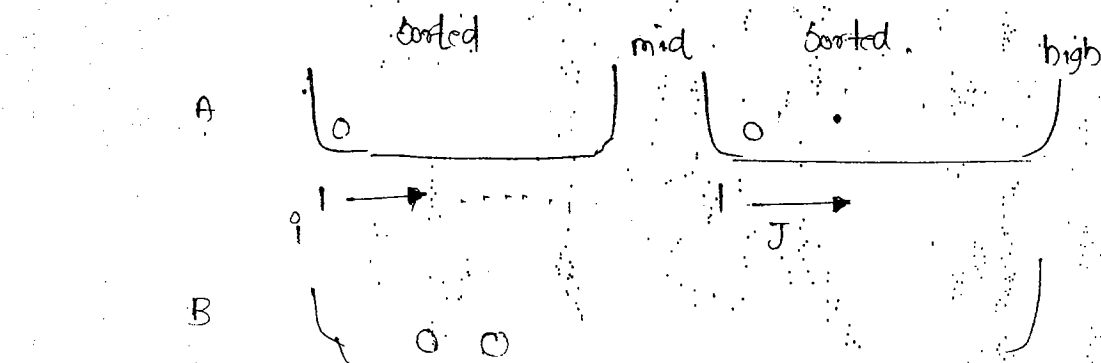
→ 11, 42, 23, 74, 44, 65, 36, 87, 38, 99.

II Round :- Tenth place



→ 11, 23, 36, 38, 42, 44, 65, 74, 87, 99

5. Merging Sort.



while ($i \leq \text{mid}$ & $j \leq \text{high}$)

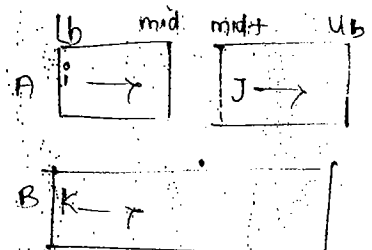
if $A[i] < A[j]$

$B[k] \leftarrow A[i]; i \leftarrow i + 1$

else

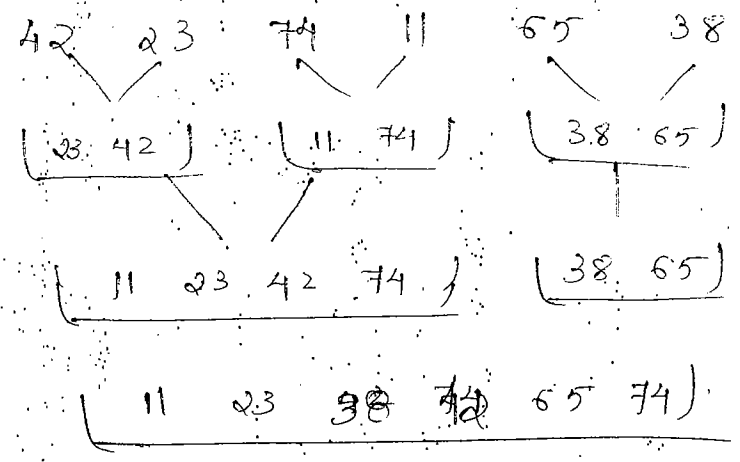
$B[k] \leftarrow A[j]; j \leftarrow j + 1$

$k \leftarrow k + 1$



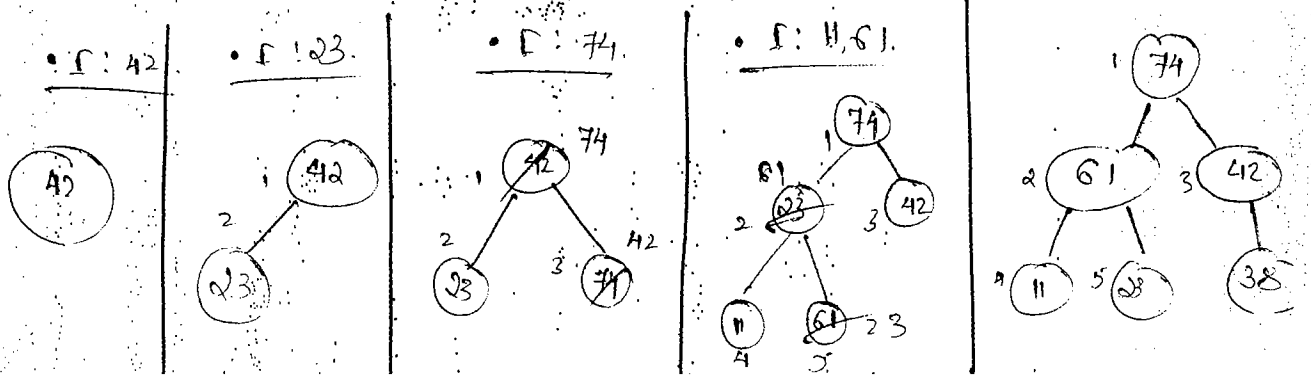
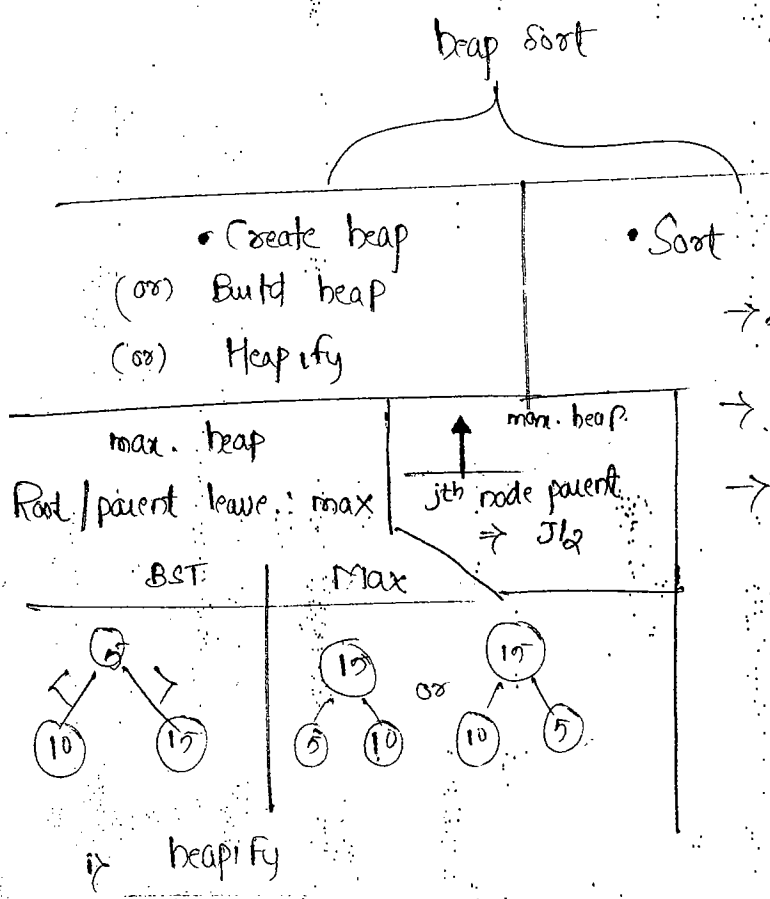
Heap Sort

Two way merge

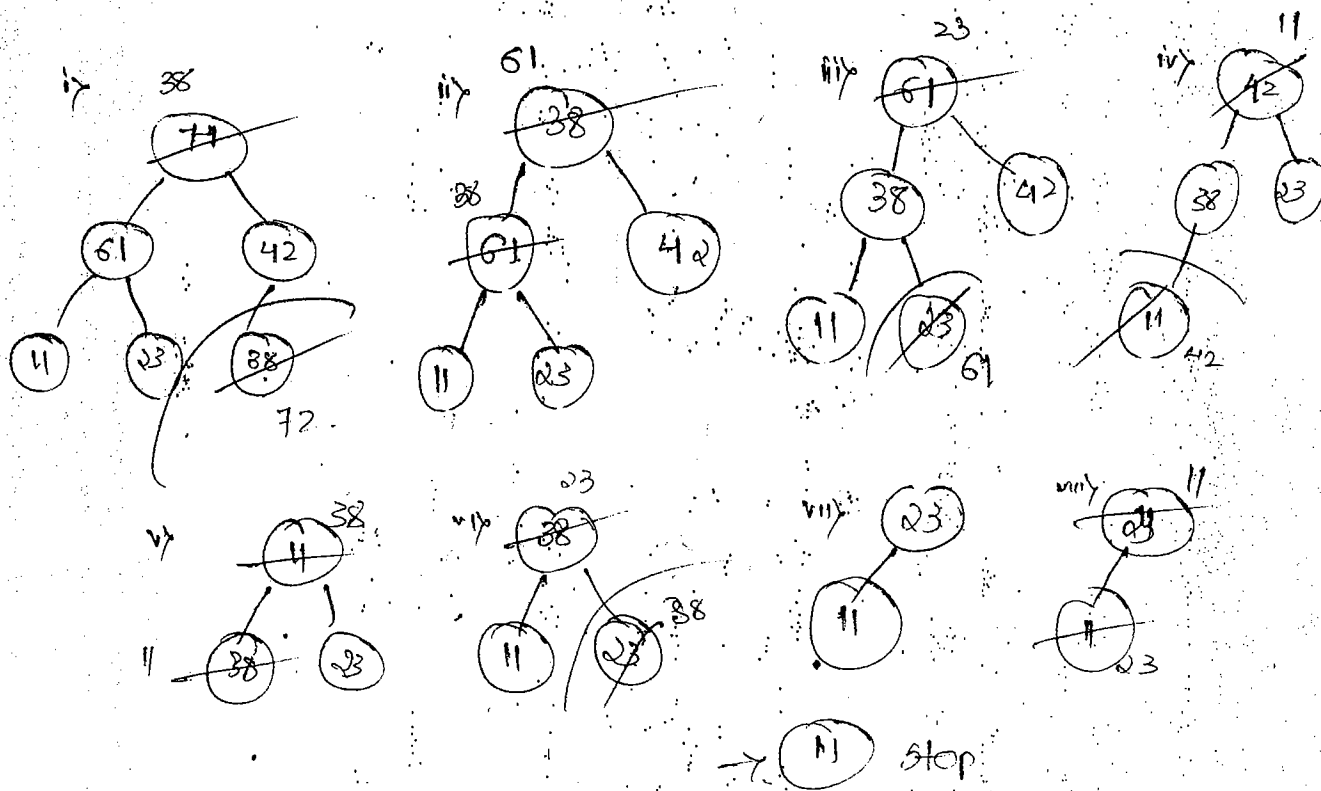
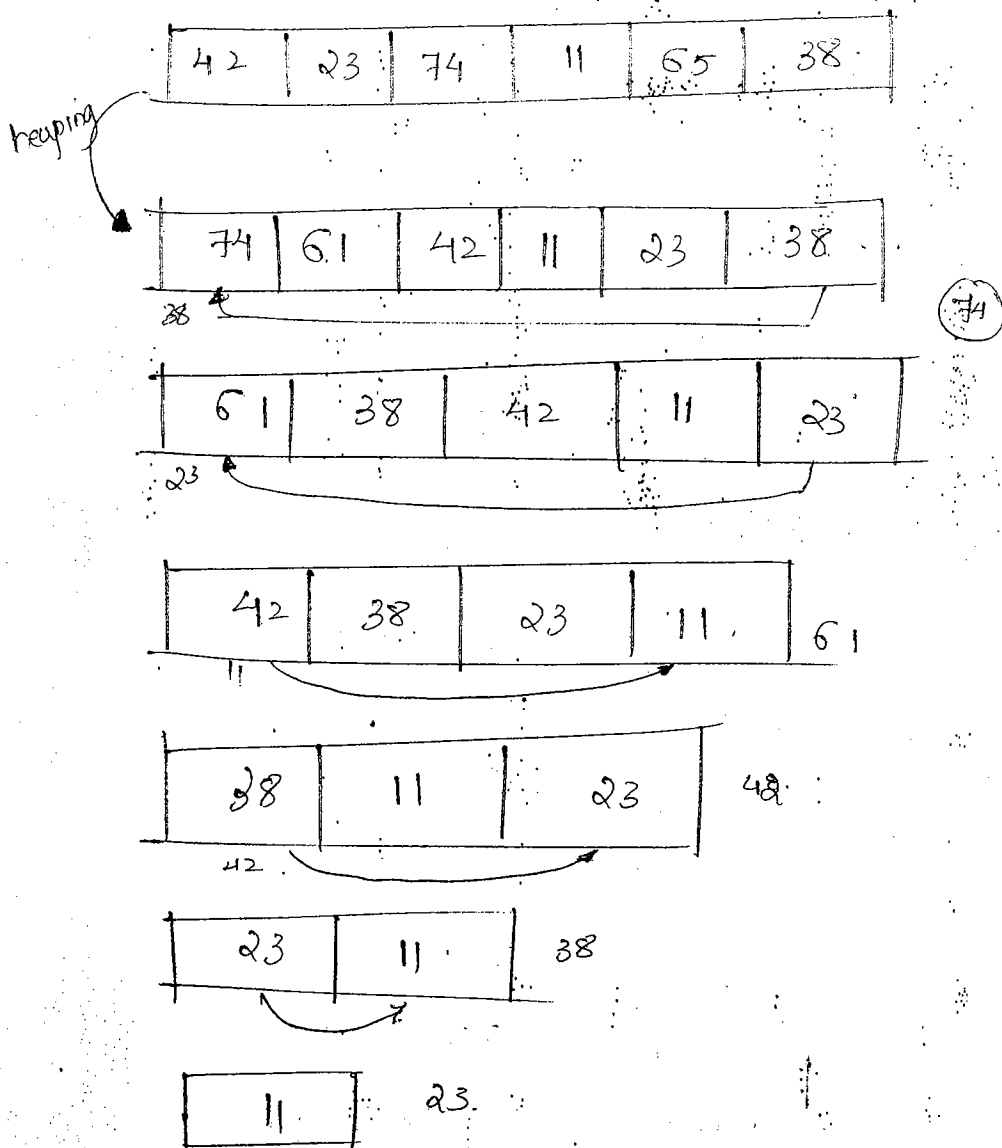


Heap Sort

Sequential Representation of Binary
Seq. Rep: CBF



11/7 sort :-



1.5.2012

< Elements of high level programming : Pascal and 'c' >

Machine	year	inventor	Country
1) Abacus	5000 years ago	-	china
2) Napier bones	1619	John Napier	Scotland
3) pascal line	1647	Blaise pascal	France
4) Differential Engine	1821	Charles Babbage father of Computer	England
5) Analytical Engine	1834		

→ 1st programmer : lady Ada Lovelace (1825-1852 : 36 yrs)

Generation of Computers

<u>Generation</u>	<u>year</u>	<u>characteristics</u>	<u>Model</u>						
I	1940-55	large and heavy electronic Vacuum tubes	UNIVAC						
II	after 55	Transistor	IBM 7070						
		<ul style="list-style-type: none"> IBM :- international Business machines 1924 : started : American Scientist : Herman Hollerith universal automatic Computer (ENIAC :- Electronic Numerical Integrator and Computer) 							
III	after 1960	IC	IBM 370						
IV	after 1970	LSI (µP)	classification of Computer <table border="1"> <tr> <td> based on <table> <tr> <td>speed</td> <td rowspan="3">{</td> <td rowspan="3">mini mini mini</td> </tr> <tr> <td>accuracy</td> </tr> <tr> <td>memory</td> </tr> </table> </td> </tr> </table>	based on <table> <tr> <td>speed</td> <td rowspan="3">{</td> <td rowspan="3">mini mini mini</td> </tr> <tr> <td>accuracy</td> </tr> <tr> <td>memory</td> </tr> </table>	speed	{	mini mini mini	accuracy	memory
based on <table> <tr> <td>speed</td> <td rowspan="3">{</td> <td rowspan="3">mini mini mini</td> </tr> <tr> <td>accuracy</td> </tr> <tr> <td>memory</td> </tr> </table>	speed	{	mini mini mini	accuracy	memory				
speed	{			mini mini mini					
accuracy									
memory									
V	at present	A.I : artificial intelligence	super Computer.						

ALGOL 60 : 1960

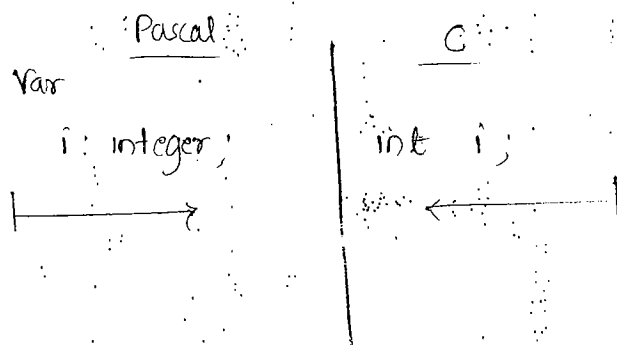
BCPL 1967 : Martin Richards : Basic Combined programming language

B: 1970 : Ken Thomson

C: 1972 : Dennis Ritchie

→ Pascal 1971 : Niklaus Wirth
(to develop OS : UNIX)

Java: 1994 : James Gosling of Sun micro system.



• C language is the mirror image of pascal language

C Operators and Expressions

precedence

Associativity

high

16> postfix

→ left

15> prefix, unary, !, &

→ Right

14> () Typecast

→ Right

13> * / %

12> + -

11> << >>

10> Relative <, ≤, >, ≥ =

9> == !=

8> bit level { & AND
^ EXOR
| OR

5> logical { &&
||

3> assignment

LEFT

RIGHT

low

Compound Assignment

i) $x += y \Rightarrow x = x + y$

iv) $x *= y \Rightarrow x = x * y$

ii) $x /= y \Rightarrow x = x / y$

v) $x -= y \Rightarrow x = x - y$

iii) $x *= y \Rightarrow x = x * y$

Q. If $x=2$ then $x *= 3+2$

Sol

$x *= 3+2$	
$x *= 5$ $x = x * 5$ $\Rightarrow 2 * 5$ $x = 10$	$x = x * 3 + 2$ $\Rightarrow 2 * 3 + 2$ $\Rightarrow 6 + 2$ $x = 8$

• Valid

• Invalid because '+' is higher precedence over '*' Compound assignment

Q. $x=40, y=4, z=4$ then $PRINT\ x=y=z$

Sol

$x=40, y=4, z=4$

i) $x=y=z$

ii) $x=1, y=4, z=4$

$x = (4=4)$

$PRINT\ x=y=z$

$x=1$

$1 == 4$ (false)

• output = 0

Observation

Computer	True	False
PRINT	1	0
READ	Non zero	0 or Null

Q. $x=y=z=1$ then $++x \ \&\& \ ++y \ || \ ++z$

Sol

$x=y=z=1$

$\therefore ++x \ \&\& \ ++y \ || \ ++z$

- false ANDING with whatever is FALSE
- True ORing with whatever is True

$\therefore (2, 2, 1)$

$\Rightarrow 2 \ \&\& \ 2 \ || \ \text{whatever}$

$\Rightarrow T \ \&\& \ T \ || \ \text{whatever}$

unary $(++)$ increment operator

	post fix	prefix	Remark
stand alone	$x++$ $x = x + 1$	$++x$ $x = x + 1$	} No difference
Association	$y = x++$ • present value of x is assigned and later x is incremented	$y = ++x$ here first incremented and assigned	

Ex :- $\text{if } x = 5$

PRINT $++x$

PRINT x

O/P $\rightarrow 6, 6$

ii) $x = 5$

PRINT $x++$

PRINT x

O/P $\rightarrow 5, 6$

iii) $x = 5$

$x++$

PRINT x

O/P $\rightarrow 6$

iv) $x = 5$

$y = x--$

PRINT x, y

O/P $\rightarrow 4, 5$

v) $x = 5$

$y = --x$

PRINT x, y

O/P $\rightarrow 4, 4$

$x = 5$

$x--$

PRINT x

$x \rightarrow 4$

standalone

Q. $x=y=z=1$ then $++x \parallel ++y \&\& ++z$

Sol

$$x=y=z=1$$

$$++x \parallel ++y \&\& ++z$$

\Rightarrow

$$2 \parallel \&\&$$

True coming whatever.

$$2, 1, 1$$

ii) $++x \&\& ++y \&\& ++z$

Sol

$$++x \&\& ++y \&\& ++z$$

$$2, 2, 2$$

iii) $++x \parallel ++y \parallel ++z$

Sol

$$2, 1, 1$$

Q. $x=y=z=-1$ then

i) $++x \&\& ++y \parallel ++z \rightarrow 0, -1, 0$

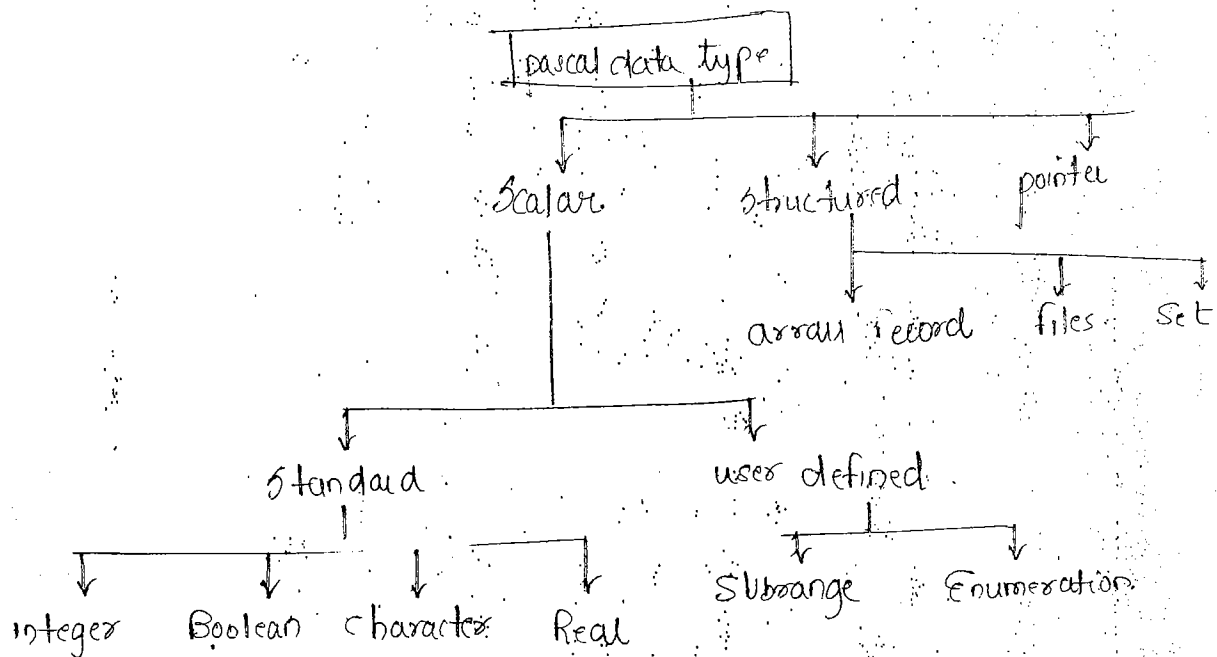
ii) $++x \parallel ++y \&\& ++z \rightarrow 0, 0, -1$

iii) $++x \&\& ++y \&\& ++z \rightarrow 0, -1, -1$

iv) $++x \parallel ++y \parallel ++z \rightarrow 0, 0, 0$

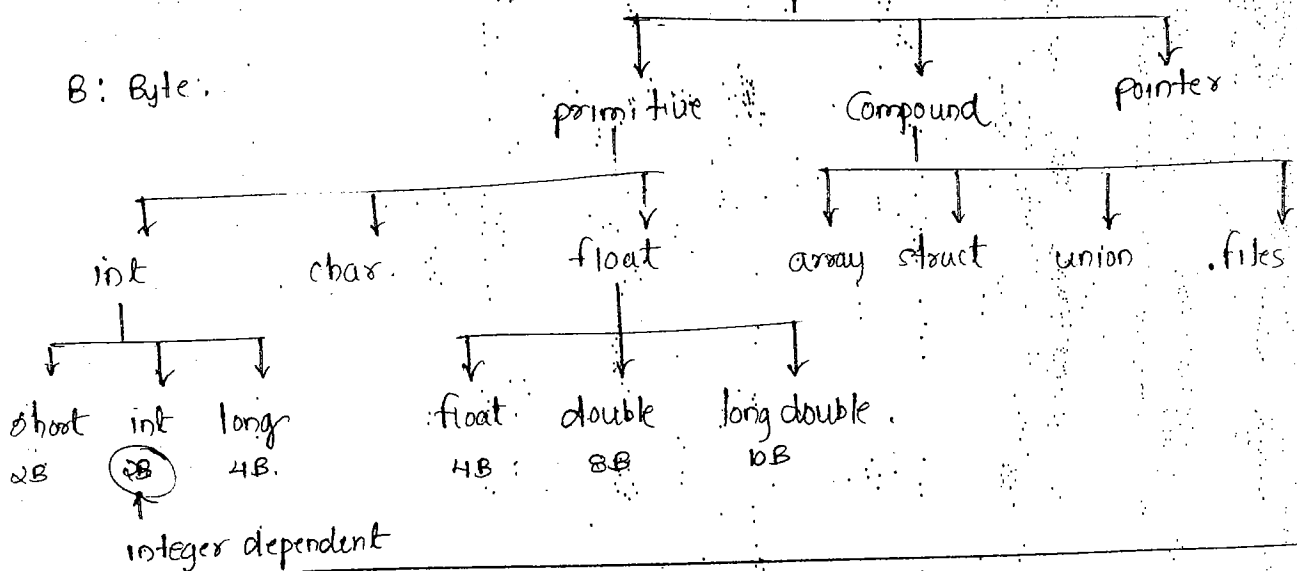
pascal data types

C: Data types



C: Data type

B: Byte.



Ob servation

pascal	C
Boolean	No (redefine using Enumeration)
real	float
Record	struct
{Not (Record variant) Sides}	union
	No

pascal

Var

i: integer;
r: real;
c: character;
b: boolean

Ex:-

i = 10;
r = 2.8;
c := 'a';
b := false

C-language

int

char

float

i = 10;

c = 'a';

f = 2.8;

i) pascal

Var

d: digits;
f: family;

Type

digits = 0 9;
 family = (father, mother, brother, sister);

Sub range

enumeration list down

ii) C-language

enum family { father, mother, brother, sister };

enum months { Jan, feb, march, ... };

lower boundary = 0

enum months { Jan = 1, feb, march };

Type

one dimensional = array [1...4] of integers;

Two dimensional = array [1...2] of
array [1...3] of integers;

Var

a: one dimen;

b: Two dimen;

int a[4];

int b[2][3];

a ₀	a ₁	a ₂	a ₃
----------------	----------------	----------------	----------------

00	01	02
10	11	12

OR

Var

a: array [1...4] of integers;

- array is an homogeneous elements (Similar data type)
- structure or record = heterogeneous elements (Dissimilar data type)

ix in C language

struct sample

{

char c;

int i;

};

prototype

memory is not allocated

struct sample s;

Declaration

memory is allocation

s.c = 'a';

s.i = 10;

└ Direct Selector

Definition

ix in pascal

Type sample = record

c: character;

i: integer;

end;

Prototype

var s: sample;

Declaration

with s do

begin

c := 'a';

i := 10;

end

Definition

Q. Write a pascal program for Student's record.

Sol

```

Type Student
Sample = record
    name = array [1..3] of characters;
    Marks = array [1..2] of integers;
End;

Var x: Student;

With x do
begin
    name := "KARAN";
    marks := 90;
End
    
```

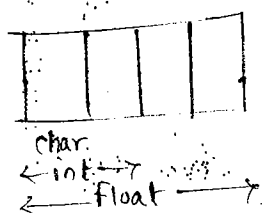
C: UNION

- heterogeneous
- memory is allocated for the larger component and all components share the memory
- better utilization of available memory
- always holds the latest value.

Ex:- union Sample

```

char c;
int i;
float f;
    
```



union Sample s;

s.c = a;
s.i = 10;

C: STRUCTURE

- heterogeneous
- memory is allocated for the components or field or members

Ex:- struct Sample

```

char c;
int i;
float f;
    
```

c() // 1
i() // 2
f(100) // 1
7

struct Sample s;

s.c = a;
s.i = 10;

Pascal : RECORD VARIANT

allow us to set up a record whose precise structure may slightly be different for different variables

→ Achieved by using Case.

Rules :-

- 1) variant is allowed only in one field.
- 2) must be last record.

Or type status = (married, divorced, widowed);
marital = record.

name : array [1..20] of characters;

age : integer;

Sex : (male, female);

Case 'status' of

married : (children : integer;

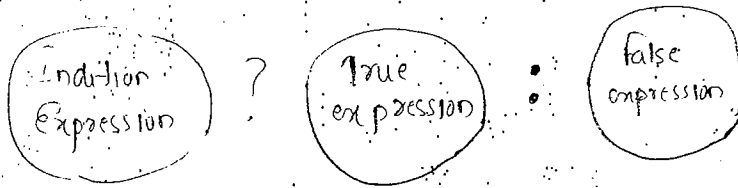
Spouse name : array [1..20] of characters;);

divorced : (divorced date : record; month : integer; year : integer;
end;

);

widow : (year of death : integer; insurance : boolean;);
end;

Ternary Operator (or) Conditional Operator

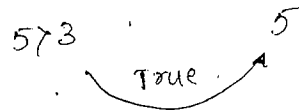


Ex:-

$$\text{Max} = (a > b) ? a : b$$

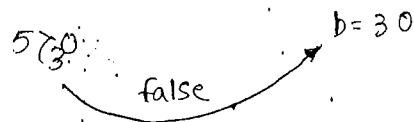
Max = 5

Say: $a = 5$ $b = 3$



Max = 30

$a = 5$ $b = 30$



Example :-

i) $a = 2$ $b = 3$ $c = 4$

$$c += a * b ? a-- : ++b$$



$a = 2$

$b = 4$

$c += 4$

$c = c + 4$

$\rightarrow 8$

2, 4, 8

ii) $c * = a < b ? a-- : ++b$



$c * = 2$

$c = c * 2$

$c = 2 * 2$

$a = 2$

$c = 8$

1, 3, 8

9.5.2012

< STORAGE CLASSES IN C >

- i) Automatic
- ii) Static
- iii) Register
- iv) Extern.

Local variable :- Variables are declared in a block and visibility is confined to that block.

Global variable :- visibility is applicable to the entire program and can be accessed from any block/function.

Automatic

- local variable
- At the time declaration, automatically created and when it goes out of scope, automatically destroyed / vanished.
- Auto keyword : optional.
- When not initialized, it holds Junk values
- Everytime reinitialized

Static

- local variable
- As the name indicates static it persists even after out of scope.
- Mandatory
- When not initialized it holds '0'
- Only on the first invocation, it is initialized

automatic

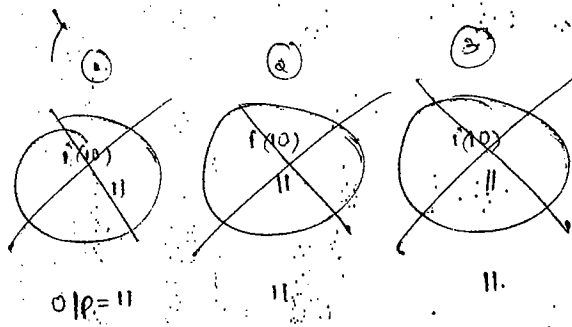
```

Void Do()
{
    int i = 10;
    printf("%d", ++i);
}
    
```

Void main()

```

{
    Do();
    Do();
    Do();
}
    
```



Static

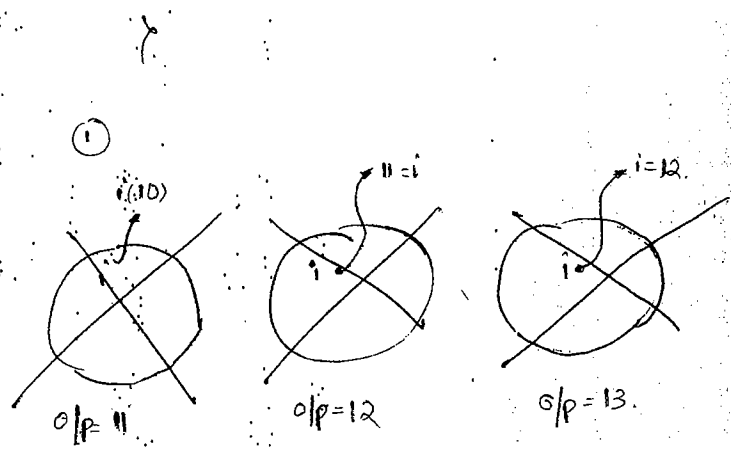
```

Void Do()
{
    static int i = 10;
    printf("%d", ++i);
}
    
```

Void main()

```

{
    Do();
    Do();
    Do();
}
    
```



< ARRAYS AND POINTER >

pointer variable : variable holds address of another variable (data type)

$P * q \rightarrow$ multiplication

$\text{int } *i \rightarrow$ reference address of another variable

$*P = 20 \rightarrow$ Dereference operator. (Referencing by the address)
value stored in the reference of address.

• One byte is mapped to another byte is called loosely typed

C: is a loosely typed but pointer is a strongly typed.

Observation :- on 16 bit machine

$\text{char } *C;$

$\text{int } *i;$

$\text{float } *f;$

$\text{char } C; \quad // 1B$

$\text{int } i; \quad // 2B$

$\text{float } f; \quad // 4B$

• all pointers are Capable of holding addresses and address size is machine dependent

C: loosely typed but with pointers strongly typed but pascal & C++ is strongly type

name = inside

$\&$ = outside

$*$ = Content (Jump)

$\text{int } i;$

$\text{int } *P;$

$P = 100$ // absolute addressing is invalid

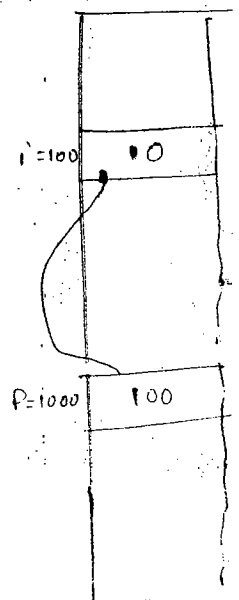
$P = \&i$

$*P = 10$

int

$P \rightarrow \text{int}$

$P(100) \rightarrow 100$



$\text{printf}("%d", i); \text{PF}("%u", \&i); \text{PF}("%u", P);$
 $\text{PF}("%d", *P); \text{PF}("%u", \&P);$
 $\text{op}; 10, 100, 100, 100, 1000$

[]	*
Early binding	late binding
Static binding	Dynamic
Compile time binding	Run time
a[i] user friendly operator	*[a+i] machine friendly operator
Read Only; Can't write	pointer variable
Constant pointer a+i : valid a = a+i : invalid	p = p+i : valid

a = Name of array \Rightarrow starting address of array

a = &a[0];

int a[] = {10, 11, 12, 13};

int i = 3;

printf("%d", i[a]);

$$\text{loc } a(i) = \text{lo} + (i - \text{lb}) * C$$

$$\text{loc } a(i) = \&a(i) + (i - 0)$$

\rightarrow scalar arithmetic

$$\text{loc } a(i) = a + i$$

$$a[i] = *(a+i)$$

i[a]

$$\Rightarrow *(i+a)$$

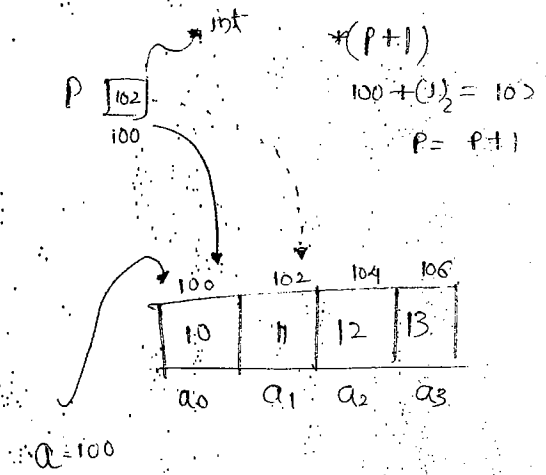
$$\Rightarrow *(a+i)$$

$$\Rightarrow a[i]$$

Example 2

```
int *p;
p = &a[0];  (OR)
p = a
```

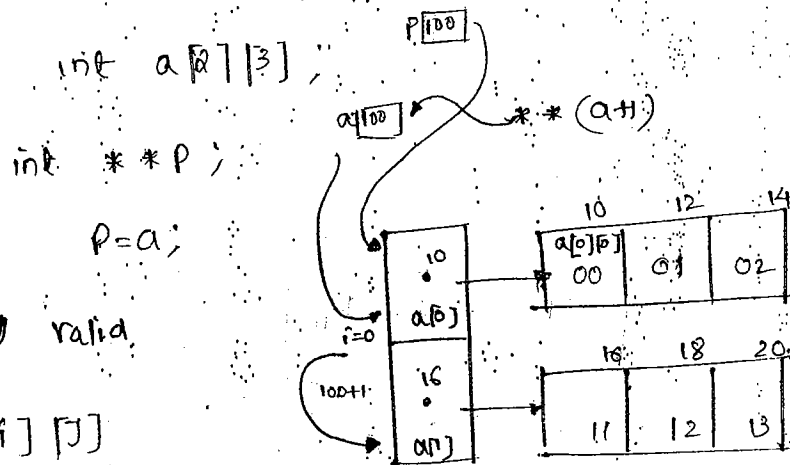
```
printf("%.d", *(p+1));
printf("%.d", p[1]);
above : same
```



Valid
`a+1`
 $100 + (1) * 2$
 $\Rightarrow 102$

2-D dimensional array and pointer

`p[i][j] \Rightarrow *p[j] \Rightarrow **p` all are same.



```
a[i][j]
*(i+j)
*(a[i]+j)
*(* (a+i) + j)
```

`a[0] = *(a+0) = *a`

```
*a
**a
*(*a+1)
** (a+1)
**a + 1
```

```
a[0]
* [* (a+0)+0]
* (a[0]+1) = a[0][1]
* (a[1]+0)  $\Rightarrow$  a[1][0]
a[0][0] + 1
```

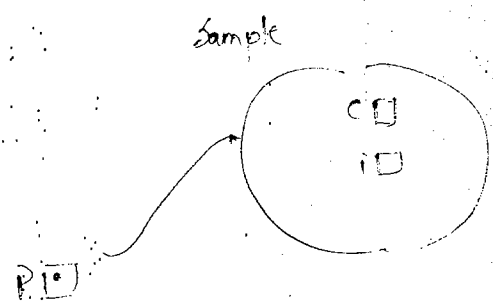
pointer and structure

```

struct Sample
{
    char c;
    int i;
};
    
```

```

struct Sample s;
struct Sample * p;
p = &s;
    
```



$*p.c = 'a'$
 $*p.i = 10$

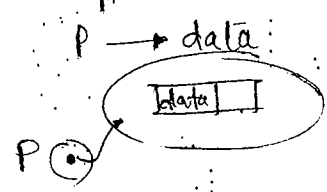
} invalid

above = Segmentation fault
core dumped

• is higher precedence over *

$(*p).c = 'a'$ (or) $p \rightarrow c = 'a'$
 $(*p).i = 10$ (or) $p \rightarrow i = 10$

→ indirect selection
 → user friendly
 operator



→ Direct Selection
 → machine friendly
 $(*p).data$

Subroutines or Subprocedure or routines

i) pascal

pascal (it doesn't support without keywords)

procedure

→ Does not return a value but
Carry out certain process

→ ^{procedure}
Swap (a, b)
process

function

→ Return a value

function

Ex:- 1 fact (4)

return = $4 \times 3 \times 2 \times 1 = 24$

ii) C

• Supports procedure and function without keywords

Void Swap (a, b)
↑
procedure

Ex:-

int Max()

↑
return type integer function

P: function

function MAX (a: integer; b: integer):
integer;

begin

if (a > b)

then MAX := a;

else

MAX := b;

end;

C: function

int MAX (int a, int b)

{

if (a > b)

Return a;

else

Return b;

}

P: procedure

```
procedure Do(a: integer, b: integer)
begin
  _____
  _____
  _____
end;
```

C- procedure

```
void (Do int a, int b)
{
  process
}
```

< RECURSION > (or) Non-iterative statements

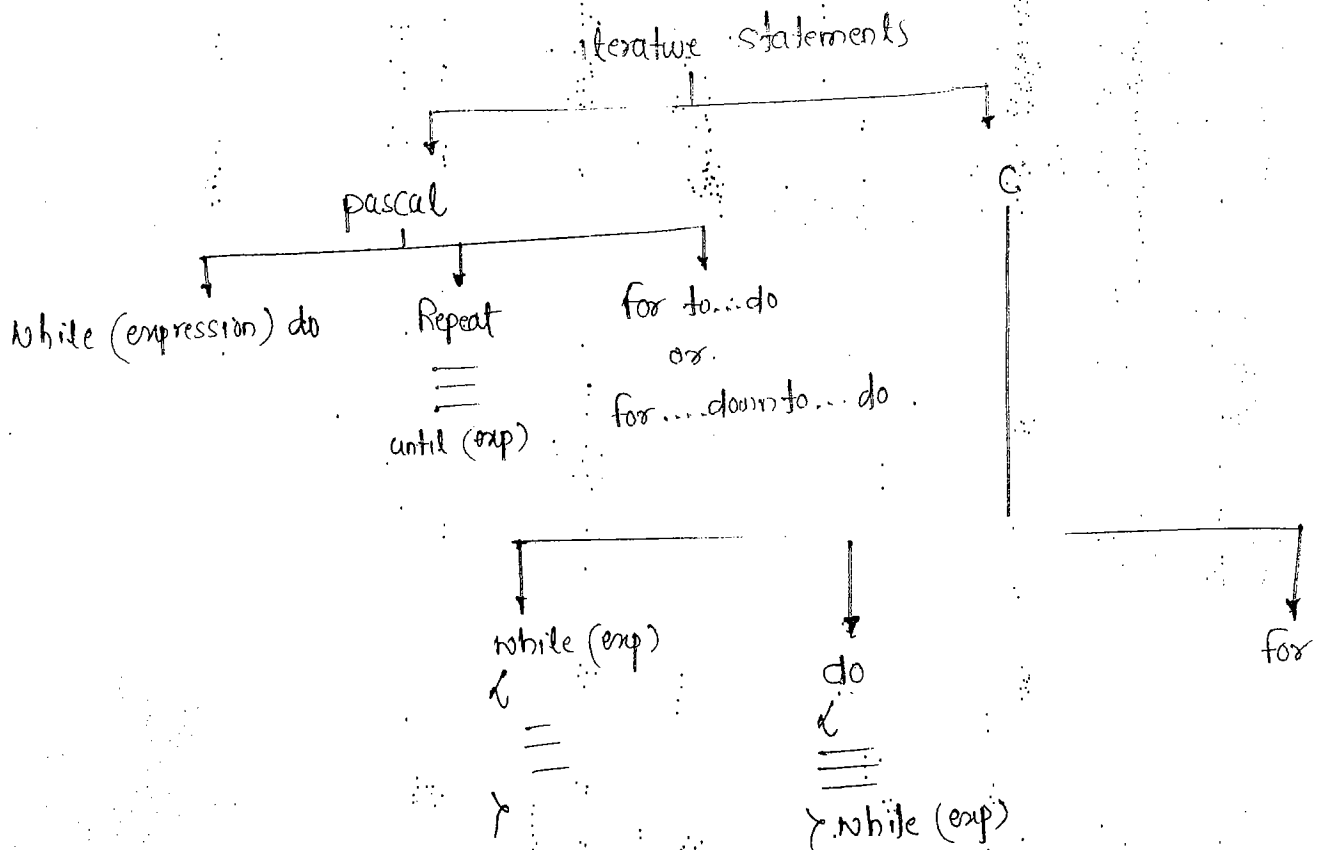
⇒ pascal

```
function power(x: integer; y: integer): integer;
begin
  if (N=0)
  then power:=1;
  else
    if (x=0)
    then power:=0;
    else if (N>0)
    then power:=x*power(x, N-1);
    elseif (N<0)
    then power:= 1/x * power(x, N+1);
end;
```

⇒ C

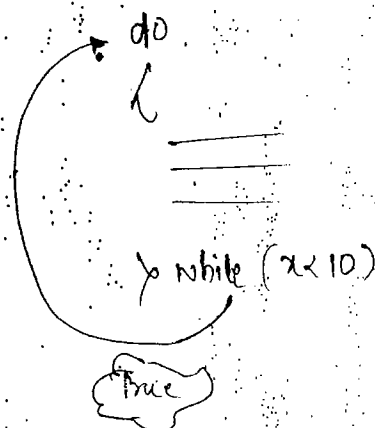
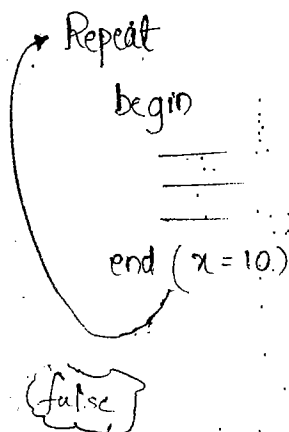
```
int pow(int x, int N)
{
  int (N=0)
  Return 1;
  else if (x==0)
  return 0;
  else if (N>0)
  return x*pow(x, N-1);
  else if (N<0)
  return 1/x * pow(x, N+1);
}
```

< iterative statements (or) NON RECURSION >



while	do while
1) Top testing	1) Bottom testing
2) Once condition is true then only the body of the loop is executed.	2) Min. of once, the body of the loop is executed. Then condition is evaluated.

Observation:



Q. write a program to print sum of first 100 integers both in pascal and C language.

Sol.

pascal language

```
17 program sum (input, output)
   var
     i, sum := integer
   begin
     i := 1
     sum := 0;
     while (i ≤ 5) do
       begin
         sum := sum + i;
         i := i + 1;
       end;
     writeln (sum);
   end

   for i := 1 to 5 do
     begin
       sum := sum + i;
     end
```

C language

```
17 #include <stdio.h>
   void main()
   {
     int i, sum;
     i = 1;
     sum = 0;
     while (i ≤ 5)
     {
       sum = sum + i;
       ++i;
     }
     printf ("%d", sum);
   }
```

for (initialization; condition; updation)

for (i = 1; i ≤ 5; ++i)

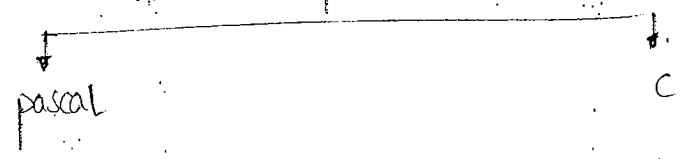
Q. when do you refer for

Sol. when we know well in advance for how many time the body of the loop is executed.

Ex: Print primes below first 100 integer.

10.5.2012

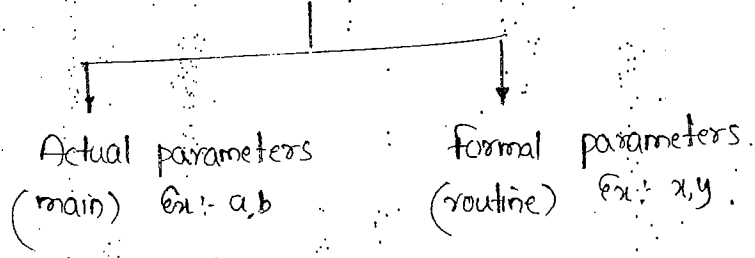
parameter passing Technique.



- Call by (pass by) value (cr)
- Call by variable (cr) reference

→ Only Call by value
but in c++, we can call by value and reference.

arguments = parameters



pascal

program ~~for~~ (input, output);

Sol

```
var a, b: integer;  
procedure change (var
```

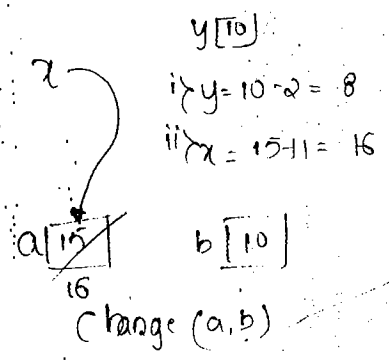
```
begin  
  y = y - x;  
  x = x + 1;  
end;
```

```
begin  
  a := 15;  
  b := 10;  
  write (a, b);  
  change (a, b);  
  write (a, b);  
end
```

(Variable)
pass by reference pass by value.
x: integer; y: integer)

Logic :-

change (a, 10)
change (x, y)



Call by value

i> Whenever there is a invocation from the main, the actual parameters values are passed hence the name Call by value.

ii> memory is allocated for formal parameters and the passed values are dumped into them.

iii> Whatever updations are ^{done} ~~there~~ for the formal parameters will not effect the actual parameter.

Call by reference (Variable)

i> whenever there is a invocation from the main, the formal parameters will reference to actual parameter. hence the name Call by reference.

ii> memory is not allotted for the formal parameters.

iii> whatever updations are done for formal parameters will effect the actual parameter.

Call by value

```
program pai(input, output)
```

```
var a, b: integer;
```

```
procedure change(a: integer)
```

```
begin
```

```
  a = a + 5;
```

```
end;
```

```
begin
```

```
  x := 3;
```

```
  write(x);
```

```
  change(x);
```

```
  write(x);
```

```
end.
```

change for
Call by ref.

change(3)

change(a)

a(3) =

3 + 5 = 8

x = 3

o/p = 3

change(x)

o/p = 3

Call by reference

```
(var a: integer)
```

```
change(x)
```

```
change(var a)
```

```
  a = a + 5
```

```
  a = 8
```

x 3 = 8

o/p = 3

change(x);

o/p = 8

Q1 ES. 2003

Sol

var i, j : integer

procedure y (p, q : integer) \Rightarrow y (var p : integer; var q : integer)

begin

p := p - q;

q := p + q;

p := q - p;

end

begin

i := 2;

j := 3;

y (i, j)

end

if both parameters to y are passed by reference, then what are the values of i, j.

a) 0, 2

b) ~~1, 5~~

c) 2, 3

d) ~~3, 2~~

Q2 ES. 1999

Sol

function Calc (var A : real; B : real) : real
var x, y : integer

begin

x := 3.0;

y := 3.0;

Calc := 5.0 * A + (B - A)

end

if this function was called

X := 7.0;

Y := 10;

R := Calc(X, Y)

The value of R would be

Calc block.

Cal (x, 1.0)

Cal (A, B)

local {
x = 3
y = 3
A = 7; B = 1
5 * 7 + (1 - 7)
= 35 - 6
= 29
x = 7.0; y = 1.0; may

a) 15

b) ~~25~~

c) 13

d) 31

Q. write a pascal program to swap the two given number.

Sol

```
program pas(input, output)
```

```
var a, b : integer;
```

```
procedure SWAP (var x: integer; var y: integer)
```

```
var t : integer
```

```
begin
```

```
  t := y;
```

```
  y := x;
```

```
  x := t;
```

```
end;
```

```
begin
```

```
  a := 2; b := 3;
```

```
  write('Before Swap');
```

```
  write(a, b);
```

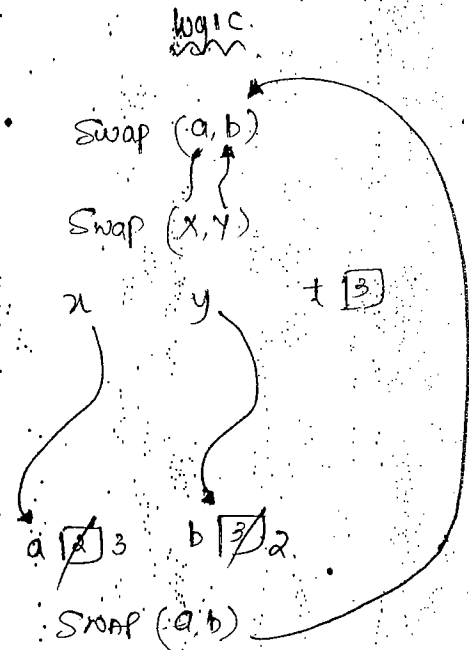
```
  SWAP(a, b);
```

```
  writeln('After Swap');
```

```
  write(a, b);
```

```
end
```

if var is not there o/p will be (2,3)



O/P = 3, 2

C++

stand alone

```
int &x = a;
```

x is an alias to 'a'

by passing

```
swap(int &x, int &y)
```

```
Swap(a, b)
```


Q. Write the above same program for Swapping the integer in C & C++.

Sol.

in C++

```
#include <iostream.h>
```

```
void Swap (int &x, int &y)
```

```
{
    int t;
```

```
    t = y;
```

```
    y = x;
```

```
    x = t;
```

```
}
```

```
void main()
```

```
{
```

```
    int a, b;
```

```
    a = 2; b = 3;
```

```
    cout << "before Swapping"
```

```
    cout << a; cout << b; swap(a, b);
```

```
    cout << "after Swapping";
```

```
    cout << a << b;
```

```
}
```

in C language

```
#include <stdio.h>
```

```
void Swap (int *x, int *y)
```

```
{
    int temp;
```

```
    temp = *y;
```

```
    *y = *x;
```

```
    *x = temp;
```

```
}
```

```
void main()
```

```
{
```

```
    int a = 2; b = 3;
```

```
    pf(" before Swapping
```

```
    a = %d, b = %d", a, b);
```

```
    swap(&a, &b);
```

```
    pf(" after Swapping
```

```
    a = %d, b = %d", a, b);
```

```
}
```

• C doesnot supports call by reference.