Homework 2

1. Determine whether or not u and v is linearly dependent, where

$$u = (3, 2), v = (3, 6)$$

2. Determine where or not the vectors $u = (2, 4, 1), v = (1, 3, 2), w = (1, 1, -1)$ in $\mathbb{R}^3$ are linearly dependent.

3. Draw the following four points in the Cartesian coordinates (X-axis and Y-axis):

$$a = [1, 0], \quad b = [0, 1], \quad c = [1, 1], \quad d = [0, 0]$$

Multiply each point by 0.5 and draw the results on the same plot.

4. Draw the following two vectors in Cartesian coordinates (X-axis and Y-axis):

a) $\boldsymbol{u} = [4\sqrt{2},\ 2\sqrt{2}]$

b) $\boldsymbol{v} = [0,\ 6\sqrt{2}]$

Rotate $\boldsymbol{u}$ and $\boldsymbol{v}$ 45 degree in the anticlockwise direction around the origin.

(Hint: $Rotated_u = Ru'$, where $= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$, $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$)

Draw the rotated vectors on the same plot you have drawn for $\boldsymbol{u}$ and $\boldsymbol{v}$.

5. Recall:

- Consider the vector equation

$$x_1\boldsymbol{u}_1 + x_2\boldsymbol{u}_2 + \cdots + x_d\boldsymbol{u}_d = 0,$$

  where $\boldsymbol{u}_1, \boldsymbol{u}_{2,\ldots,}\ \boldsymbol{u}_d$ are vectors, and $x_1, x_2, \cdots, x_d$ are scalars.

  The vectors $\boldsymbol{u}_1, \boldsymbol{u}_{2,\ldots,}\ \boldsymbol{u}_d$ are called linear independent if $x_1 = 0, x_2 = 0, \cdots, x_d = 0$.

- The maximal number of linearly independent columns of a matrix is called rank.

Find the rank for the following matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 4 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

(Advanced: Note you may use Gaussian elimination to find rank and linear independence.)

6. Investigate properties of SVD.

Let $M = \begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}$. The results of SVD, [U,S,V] = svd(M), are shown as follows:

U:

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0.5390    0.8423
0.8423   -0.5390
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S:

```
6.1088         0
     0    2.9466
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V:

```
0.7777   -0.6287
0.6287    0.7777
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Compute:

(a) UU^T

(b) VV^T

(c) M^T

(d) VSU^T

(e) Compute S(1,1)*U(:,1) and M*V(:,1). Are the two values equal?

(f) $S1 = \begin{bmatrix} 6.1088 & 0 \\ 0 & 0 \end{bmatrix}$, what's the difference between $S1$ and S?

(g) $US1V^T$ and set the values to M1

(h) M-M1

(i) Compute the squared Frobenius of M, denoted as $||M||_F^2$, that is the sum of squared each element in M

(j) Compute the squared Frobenius of M-M1

(k) Compute $||M||_F^2 - S(1,1)^2$