(Q). Find θ and P if ($\sqrt{3}$ 1)X = -2 is equivalent to (cosθ sinθ)X = P ?

Given, ($\sqrt{3}$ 1)X = -2 is equivalent to (cosθ sinθ)X = P

Let,

$$X = \begin{bmatrix} a \\ b \end{bmatrix} \tag{1}$$

From, ($\sqrt{3}$ 1)X = -2

$$A = \begin{bmatrix} \sqrt{3} & 1 \end{bmatrix} \tag{2}$$

$$B = \begin{bmatrix} -2 \end{bmatrix} \tag{3}$$

We know that,

$$AX = B \ (for\ linear\ equations) \tag{4}$$

$$\begin{bmatrix} \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -2 \end{bmatrix} \tag{5}$$

$$\sqrt{3}\ a + b = -2 \tag{6}$$

From , (cosθ sinθ)X = P

$$\begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} P \end{bmatrix} \tag{7}$$

$$a\cos\theta + b\sin\theta = P \tag{8}$$

From , eqns. (6, 8)

$$\cos\theta = \sqrt{3} \tag{9}$$

(Eqn.(9) is not possible as range of cosθ is [-1,1].)

Now, Divide eqn.(6) with 2 on both sides, resulting the cosine value that falls in it's range.

$$\frac{\sqrt{3}}{2}\ a + \frac{1}{2}\ b = -1 \tag{10}$$

By comparing eqns. (8, 10). We get,

$$\cos\theta = \frac{\sqrt{3}}{2};\ \sin\theta = \frac{1}{2} \tag{11}$$

$$\theta = cos^{-1}(\frac{\sqrt{3}}{2}) = sin^{-1}(\frac{1}{2}) = 30^{\circ} \tag{12}$$

$$P = -1 \tag{13}$$