Assignment 9

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Download all python codes from

https://github.com/satyasm45/Summer−Internship/tree/main/Assignment−9/Codes

and latex-tikz codes from

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1 Question No. 2.56

Solve: x-2y ≤ 3, 3x+4y ≥ 12, x ≥ 0, y ≥ 1.

2 Explanation

1) Solving first pair of inequality:

$$\begin{aligned} -x + 2y &\geq -3 \\ 3x + 4y &\geq 12 \end{aligned} \tag{2.0.1}$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \tag{2.0.2}$$

(2.0.1) can then be expressed as

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -3 \\ 12 \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} \tag{2.0.4}$$

$$\text{or, } \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \mathbf{u} \tag{2.0.5}$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \mathbf{u} \tag{2.0.6}$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 3.6 \\ 0.3 \end{pmatrix} + \frac{-1}{10} \begin{pmatrix} 4 & -2 \\ -3 & -1 \end{pmatrix} \mathbf{u} \tag{2.0.7}$$

2) Similarly,Solving second pair of inequality:

$$\begin{aligned} x &\geq 0 \\ y &\geq 1 \end{aligned} \tag{2.0.8}$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \tag{2.0.9}$$

(2.0.8) can then be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.11}$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mathbf{u} \tag{2.0.12}$$

From (2.0.7) and (2.0.12),solution of the given system of inequalities can be found out graphically by intersection as shown by the below figures generated by Python:

As seen from figure 2.3 the solution region is bounded by line segments AB and BC and the line $\begin{pmatrix} 1 & -2 \end{pmatrix} \mathbf{x} = 3$.Beyond A the region expands infinitely along the Y axis,Beyond C the region includes all the portion above the line $\begin{pmatrix} 1 & -2 \end{pmatrix} \mathbf{x} = 3$.

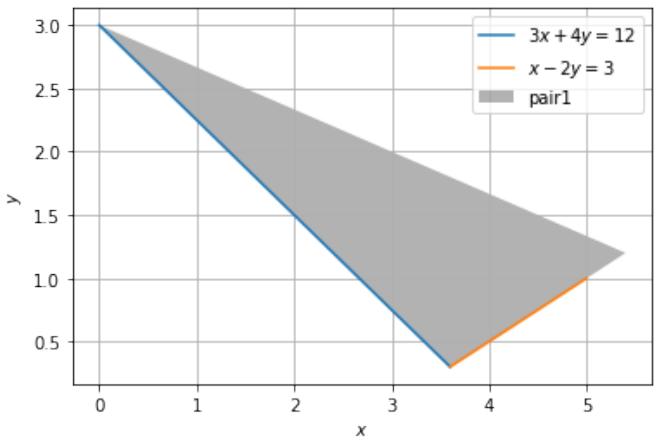

Fig. 2.1: Inequality pair 1

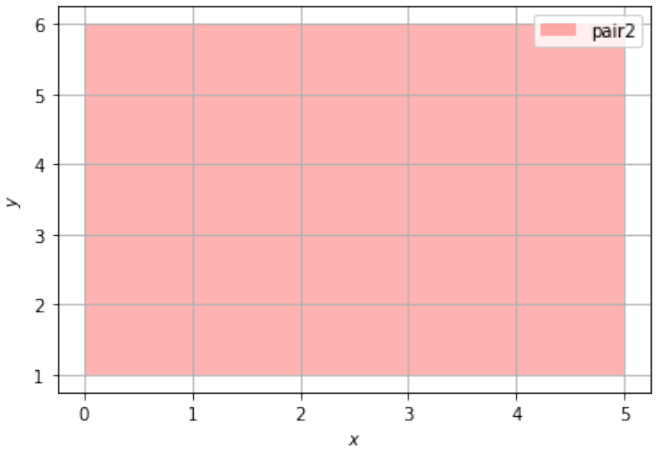

Fig. 2.2: Inequality pair 2

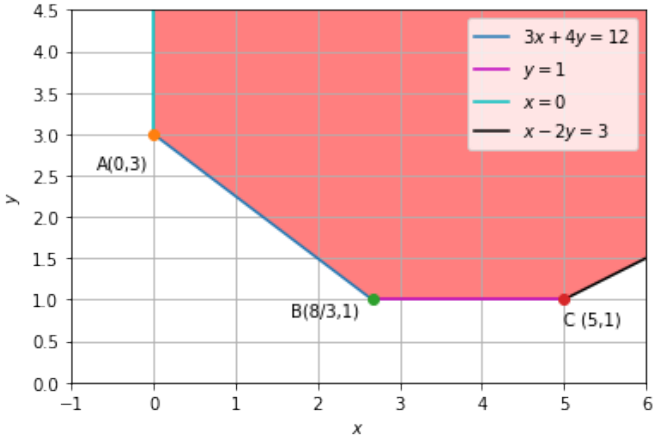

Fig. 2.3: Intersection of 2.1 and 2.2