Assignment-4

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Download all python codes from

```
https://github.com/satyasm45/Summer−Internship/
    tree/main/Assignment−4/Codes
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and latex-tikz codes from

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https://github.com/satyasm45/Summer−Internship/
    tree/main/Assignment−4
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1 Question No. 2.30

Find the equation of the parabola with focus $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and directrix $\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -2$.

2 Explanation

Definition 1. *A parabola is a curve where any point is at an equal distance from: a fixed point (the focus* $\mathbf{F}$ *), and. a fixed straight line (the directrix* $\mathbf{n}^T \mathbf{x} = c$*).*

Lemma 2.1. *The distance of a point* $\mathbf{P}$ *from a line* $\mathbf{n}^T \mathbf{x} = c$ *is given by:*

$$\frac{|c - \mathbf{P}^T \mathbf{n}|}{||\mathbf{n}||} \tag{2.0.1}$$

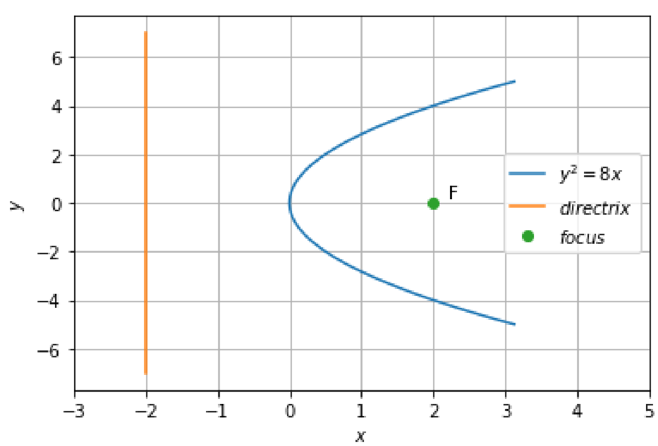

Fig. 2.1: Parabola $y^2 = 8x$

Theorem 2.1. *The equation of a parabola with focus* $\mathbf{F}$ *,directrix* $\mathbf{n}^T \mathbf{x} = c$ *and* $\lambda = ||\mathbf{n}||^2$ *is given by:*

$$\mathbf{x}^T (\lambda \mathbf{I} - \mathbf{n}\mathbf{n}^T)\mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} + \lambda ||\mathbf{F}||^2 - c^2 = 0 \tag{2.0.2}$$

Proof. Using Definition 1 and Lemma 2.1 for any point $\mathbf{x}$ on parabola we have:

$$||\mathbf{x} - \mathbf{F}||^2 = \frac{(c - \mathbf{x}^T \mathbf{n})^2}{||\mathbf{n}||^2} \tag{2.0.3}$$

$$\lambda (\mathbf{x} - \mathbf{F})^T (\mathbf{x} - \mathbf{F}) = (c - \mathbf{x}^T \mathbf{n})^2 \tag{2.0.4}$$

$$\lambda (\mathbf{x}^T \mathbf{x} - 2\mathbf{F}^T \mathbf{x} + ||\mathbf{F}||^2) = c^2 + (\mathbf{x}^T \mathbf{n})^2 - 2c\mathbf{x}^T \mathbf{n} \tag{2.0.5}$$

$$\lambda \mathbf{x}^T \mathbf{x} - (\mathbf{x}^T \mathbf{n})^2 - 2\lambda \mathbf{F}^T \mathbf{x} + 2c\mathbf{n}^T \mathbf{x} = c^2 - \lambda ||\mathbf{F}||^2 \tag{2.0.6}$$

$$\lambda \mathbf{x}^T \mathbf{I} \mathbf{x} - \mathbf{x}^T \mathbf{n}\mathbf{n}^T \mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} = c^2 - \lambda ||\mathbf{F}||^2 \tag{2.0.7}$$

$$\mathbf{x}^T (\lambda \mathbf{I} - \mathbf{n}\mathbf{n}^T)\mathbf{x} + 2(c\mathbf{n} - \lambda \mathbf{F})^T \mathbf{x} + \lambda ||\mathbf{F}||^2 - c^2 = 0 \tag{2.0.8}$$

□

Given information:

$$\mathbf{F} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = -2, \lambda = 1 \tag{2.0.9}$$

Substituting values of $\mathbf{F}, \mathbf{n}$,c,λ from(2.0.9):

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 0 = 0 \tag{2.0.10}$$

Replacing $\mathbf{x}$ by $\begin{pmatrix} x \\ y \end{pmatrix}$ in (2.0.10) gives:

$$y^2 = 8x \tag{2.0.11}$$