

Assignment-5

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Download all python codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-5/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-5>

1 QUESTION No. 2.37

Find the equation of the ellipse, with major axis along the x-axis and passing through the points $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

2 EXPLANATION

Let

$$\mathbf{p} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{q} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \quad (2.0.1)$$

This is a standard ellipse given by :

$$\mathbf{x}^T \mathbf{D} \mathbf{x} = 1, \quad \mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \lambda_1, \lambda_2 > 0 \quad (2.0.2)$$

$\therefore \mathbf{p}, \mathbf{q}$ satisfy (2.0.2),

$$\mathbf{p}^T \mathbf{D} \mathbf{p} = 1, \quad (2.0.3)$$

$$\mathbf{q}^T \mathbf{D} \mathbf{q} = 1 \quad (2.0.4)$$

which then can be expressed as:

$$\begin{aligned} \mathbf{p}^T \mathbf{P} \mathbf{d} &= 1, \\ \mathbf{q}^T \mathbf{Q} \mathbf{d} &= 1 \end{aligned} \quad (2.0.5)$$

where:

$$\mathbf{d} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -1 & 0 \\ 0 & 4 \end{pmatrix}. \quad (2.0.6)$$

(2.0.5) can then be expressed as:

$$\begin{pmatrix} \mathbf{p}^T \mathbf{P} \\ \mathbf{q}^T \mathbf{Q} \end{pmatrix} \mathbf{d} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.7)$$

$$\begin{pmatrix} 16 & 9 \\ 1 & 16 \end{pmatrix} \mathbf{d} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (2.0.8)$$

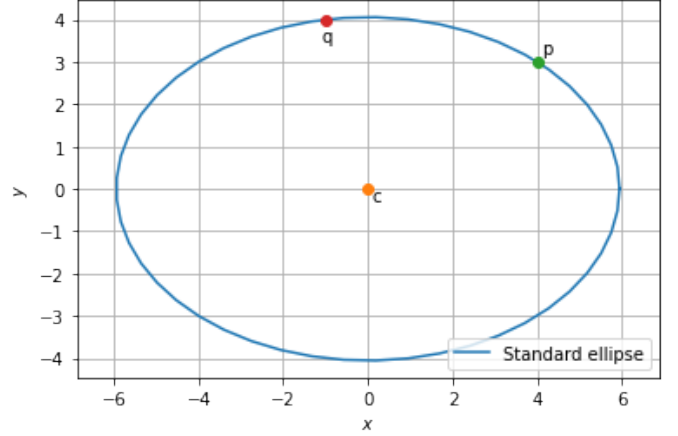


Fig. 2.1: Standard Ellipse

Forming the augmented matrix and performing row reduction,

$$\begin{pmatrix} 16 & 9 & 1 \\ 1 & 16 & 1 \end{pmatrix} \xrightarrow[R_2 \leftarrow R_2 - 16R_1, R_2 \leftarrow -R_2]{R_2 \leftrightarrow R_1} \begin{pmatrix} 1 & 16 & 1 \\ 0 & 247 & 15 \end{pmatrix} \quad (2.0.9)$$

$$\xrightarrow{R_1 \leftarrow 247R_1 - 16R_2} \begin{pmatrix} 247 & 0 & 7 \\ 0 & 247 & 15 \end{pmatrix} \quad (2.0.10)$$

$$\Rightarrow \mathbf{d} = \frac{1}{247} \begin{pmatrix} 7 \\ 15 \end{pmatrix}, \text{ or, } \mathbf{D} = \frac{1}{247} \begin{pmatrix} 7 & 0 \\ 0 & 15 \end{pmatrix} \quad (2.0.11)$$

So, equation of ellipse is given by:

$$\mathbf{x}^T \begin{pmatrix} 7 & 0 \\ 0 & 15 \end{pmatrix} \mathbf{x} = 247 \quad (2.0.12)$$

Replacing \mathbf{x} by $\begin{pmatrix} x \\ y \end{pmatrix}$ in (2.0.12) gives:

$$7x^2 + 15y^2 = 247 \quad (2.0.13)$$

The ellipse parameters center and axes are obtained from conics table in manual as

$$\mathbf{c} = \mathbf{0}; \frac{1}{\sqrt{\lambda_1}} = \sqrt{\frac{247}{7}}, \frac{1}{\sqrt{\lambda_2}} = \sqrt{\frac{247}{15}}. \quad (2.0.14)$$