

Assignment-6

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Download all python codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-6/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-6>

The equation of tangents are given by:

$$\mathbf{n}^T(\mathbf{x} - \mathbf{q}) = 0 \quad (2.0.11)$$

$$\begin{pmatrix} -2 & 1 \end{pmatrix}(\mathbf{x} - \mathbf{q}_1) = 0 \quad (2.0.12)$$

$$\begin{pmatrix} -2 & 1 \end{pmatrix}(\mathbf{x} - \mathbf{q}_2) = 0 \quad (2.0.13)$$

where \mathbf{q}_1 and \mathbf{q}_2 are given by (2.0.9) and (2.0.10)

1 QUESTION No. 2.41

Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0$

2 EXPLANATION

The equation of curve:

$$y + \frac{2}{x-3} = 0 \quad (2.0.1)$$

$$xy - 3y + 2 = 0 \quad (2.0.2)$$

Comparing with the standard equation :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.3)$$

$$\mathbf{V} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}, f = 2 \quad (2.0.4)$$

$$\mathbf{V}^{-1} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad (2.0.5)$$

The direction and normal vectors of tangent with slope 2:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad (2.0.6)$$

From conics table in manual:

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2} \quad (2.0.7)$$

and the desired points of contact are

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa \mathbf{n} - \mathbf{u}) \quad (2.0.8)$$

$$\mathbf{q}_1 = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{q}_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (2.0.10)$$

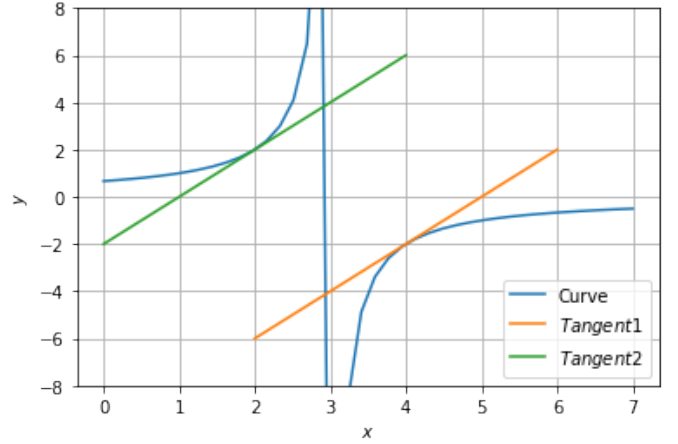


Fig. 2.1: The tangents to the curve with slope 2