Assignment-10

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Download all python codes from

https://github.com/satyasm45/Summer−Internship/tree/main/Assignment−10/Codes

and latex-tikz codes from

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1 Question No. 2.35

A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week

2 Solution

- All the data can be tabularised as:

	Fabricating	Finishing	Profit
Model A	9	1	8000
Model B	12	3	12000
Max Hours	≤ 180	≤ 30	

TABLE 2.1: Labour Hours and Profit for each piece

- Let the number of pieces of model A manufactured be x and the number of pieces of model B manufactured be y such that :

$$x \geq 0 \tag{2.0.1}$$
$$y \geq 0 \tag{2.0.2}$$

- From the data given we have:

$$9x + 12y \leq 180 \tag{2.0.3}$$
$$\implies 3x + 4y \leq 60 \tag{2.0.4}$$

and,

$$x + 3y \leq 30 \tag{2.0.5}$$

$\therefore$ The maximizing function is:

$$\max Z = \begin{pmatrix} 8000 & 12000 \end{pmatrix} \mathbf{x} \tag{2.0.6}$$
$$s.t. \quad \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 60 \\ 30 \end{pmatrix} \tag{2.0.7}$$
$$-\mathbf{x} \leq \mathbf{0} \tag{2.0.8}$$

- The Lagrangian function can be given as:

$$\begin{aligned} &L(\mathbf{x}, \boldsymbol{\lambda}) \\ &= \begin{pmatrix} 8000 & 12000 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} - 60 \right] \right. \\ &+ \left[\begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} - 30 \right] \\ &+ \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.9}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} \tag{2.0.10}$$

- Now, we have

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 8000 + \begin{pmatrix} 3 & 1 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 12000 + \begin{pmatrix} 4 & 3 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} - 60 \\ \begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} - 30 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.11}$$

$\therefore$ The Lagrangian matrix is given by:-

$$\begin{pmatrix} 0 & 0 & 3 & 1 & -1 & 0 \\ 0 & 0 & 4 & 3 & 0 & -1 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -8000 \\ -12000 \\ 60 \\ 30 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.12}$$

- Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 4 & 3 \\ 3 & 4 & 0 & 0 \\ 1 & 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -8000 \\ -12000 \\ 60 \\ 30 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 4 & 3 \\ 3 & 4 & 0 & 0 \\ 1 & 3 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -8000 \\ -12000 \\ 60 \\ 30 \end{pmatrix} \tag{2.0.14}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{3}{5} & \frac{-4}{5} \\ 0 & 0 & \frac{-1}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{-1}{5} & 0 & 0 \\ \frac{-4}{5} & \frac{3}{5} & 0 & 0 \end{pmatrix} \begin{pmatrix} -8000 \\ -12000 \\ 60 \\ 30 \end{pmatrix} \tag{2.0.15}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \\ -2400 \\ -800 \end{pmatrix} \tag{2.0.16}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} -2400 \\ -800 \end{pmatrix} \prec \mathbf{0}$

- The Optimal solution is given by:

$$\mathbf{x} = \begin{pmatrix} 12 \\ 6 \end{pmatrix} \tag{2.0.17}$$

$$Z = \begin{pmatrix} 8000 & 12000 \end{pmatrix} \mathbf{x} \tag{2.0.18}$$

$$Z = \begin{pmatrix} 8000 & 12000 \end{pmatrix} \begin{pmatrix} 12 \\ 6 \end{pmatrix} \tag{2.0.19}$$

$$Z = \text{Rs}168000 \tag{2.0.20}$$

- So, to maximise profit
  Pieces of model **A** manufactured is $\boxed{x = 12}$
  and
  Pieces of model **B** manufactured is $\boxed{y = 6}$.
- The maximum profit per week is $\boxed{Z = \text{Rs}168000}$.

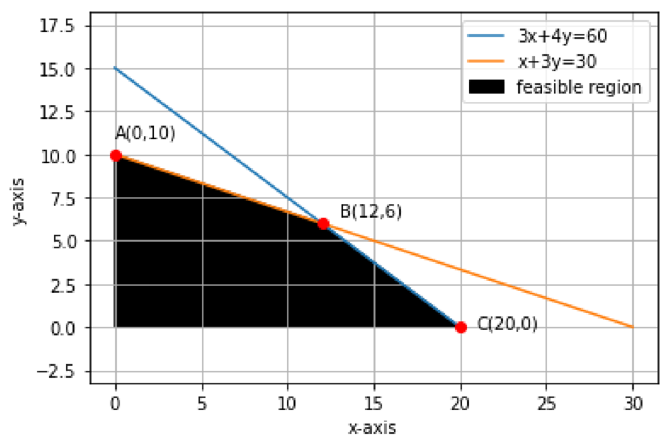

Fig. 2.1: Graphical Representataion