ASSIGNMENT-12

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1 QUESTION No-6.16

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent ?

2 Solution

Lemma 2.1. *Two events A and B are said to be independent if and only if* $\Pr(AB) = \Pr(A)\Pr(B)$.

Let X be a Random Variable the number on top face when die is tossed.

$$X\epsilon\{1,2,3,4,5,6\} \tag{2.0.1}$$

Also,

$$\Pr(X=i) = \begin{cases} \frac{1}{6} & 1 \le i \le 6 \\ 0 & otherwise \end{cases} \tag{2.0.2}$$

The probabilities of the various events are as follows.

$$\Pr(A) = \sum_{i=2,4,6} \Pr(X=i) = \frac{1}{2} \tag{2.0.3}$$

$$\Pr(B) = \sum_{i=1}^{3} \Pr(X=i) = \frac{1}{2} \tag{2.0.4}$$

$$\Pr(AB) = \Pr(X=2) = \frac{1}{6} \tag{2.0.5}$$

$$\Pr(A) \times \Pr(B) = \frac{1}{4} \tag{2.0.6}$$

Now to check whether the events are independent we use Lemma 2.1

$$\Pr(A) \times \Pr(B) \neq \Pr(AB) \tag{2.0.7}$$

Thus A and B are not independent.