

Assignment 9

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Download all python codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-9/Codes>

and latex-tikz codes from

<https://github.com/satyasm45/Summer-Internship/tree/main/Assignment-9/Codes>

1 QUESTION No. 2.56

Solve: $x-2y \leq 3$, $3x+4y \geq 12$, $x \geq 0$, $y \geq 1$.

2 EXPLANATION

1) Solving first pair of inequality:

$$\begin{aligned} -x + 2y &\geq -3 \\ 3x + 4y &\geq 12 \end{aligned} \quad (2.0.1)$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \quad (2.0.2)$$

(2.0.8) can then be expressed as

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} -3 \\ 12 \end{pmatrix} \quad (2.0.3)$$

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} \quad (2.0.4)$$

$$\text{or, } \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \mathbf{u} \quad (2.0.5)$$

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.6)$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 3.6 \\ 0.3 \end{pmatrix} + \frac{-1}{10} \begin{pmatrix} 4 & -2 \\ -3 & -1 \end{pmatrix} \mathbf{u} \quad (2.0.7)$$

2) Similarly, Solving second pair of inequality:

$$\begin{aligned} x &\geq 0 \\ y &\geq 1 \end{aligned} \quad (2.0.8)$$

Solution: Let $u_1 \geq 0, u_2 \geq 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \geq \mathbf{0} \quad (2.0.9)$$

(2.0.8) can then be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \geq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.11)$$

$$\text{or, } \mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mathbf{u} \quad (2.0.12)$$

From (2.0.7) and (2.0.12), solution of the given system of inequalities can be found out graphically by intersection as shown by the below figures generated by Python:

As seen from 2.3 the solution region is bounded by line segments AB and BC and the line $x-2y=3$. Beyond A the region expands infinitely along the Y axis, Beyond C the region includes all the portion above the line $x-2y=3$.

Also Point D shows the importance of $\begin{pmatrix} 3.6 \\ 0.3 \end{pmatrix}$ derived in (2.0.7)

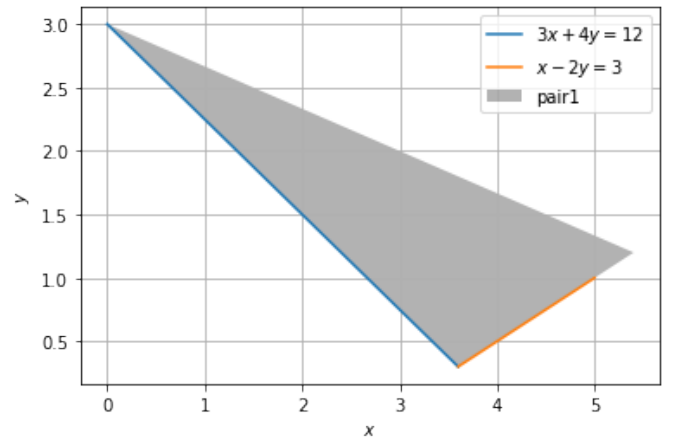


Fig. 2.1: Inequality pair 1

The common region shown by 2.3 is the solution of set of inequalities.

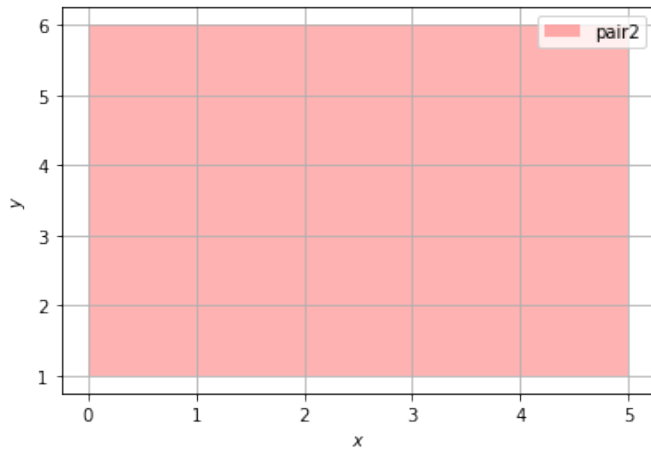


Fig. 2.2: Inequality pair 2

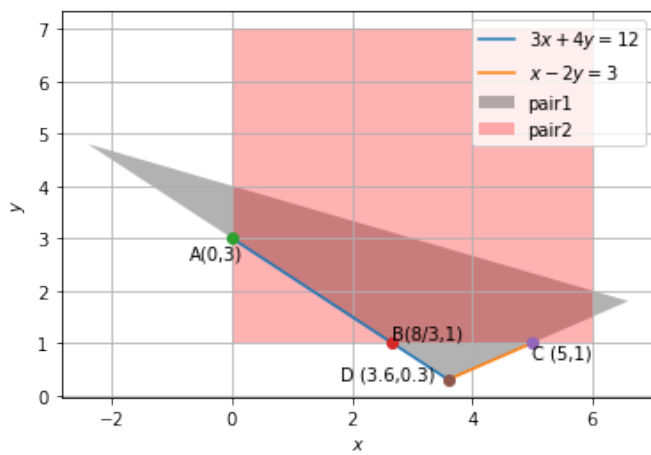


Fig. 2.3: Intersection of 2.1 and 2.2