Challenge Problem 1

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1 CHALLENGE QUESTION 1

Show that the matrix $(t\mathbf{I} - \mathbf{n}\mathbf{n}^T)$ in the given document is a rank 1 matrix for a parabola where t=$\|\mathbf{n}\|^2$.

2 EXPLANATION

Given :

$$\mathbf{V} = (t\mathbf{I} - \mathbf{n}\mathbf{n}^T) \qquad (2.0.1)$$

where t=$\|\mathbf{n}\|^2$. Now $\mathbf{V}$ is a 2×2 matrix.Using Rank-Nullity theorem: Rank($\mathbf{V}$)+Nullity($\mathbf{V}$)=2

$$\mathbf{V} \neq \mathbf{0} \qquad (2.0.2)$$
$$\left|\mathbf{V}\right| = 0 \qquad (2.0.3)$$

(2.0.2) indicates Rank($\mathbf{V}$)$\neq$ 0 and (2.0.3) indicates Nullity($\mathbf{V}$)$\neq$ 0 .So, Rank($\mathbf{V}$)=Nullity($\mathbf{V}$)=1 for parabola.For other conics:

$$t = \frac{\|\mathbf{n}\|^2}{e^2} \qquad (2.0.4)$$
$$\left|\mathbf{V}\right| \neq 0 \qquad (2.0.5)$$

(2.0.5) indicates Nullity($\mathbf{V}$)=0 as $\mathbf{V}$ is invertible.So,for conics other than parabola Rank($\mathbf{V}$)=2.