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Assignment 9

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Download all python codes from

https://github.com/satyasm45/Summer-Internship/ tree/main/Assignment-9/Codes

and latex-tikz codes from

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1 Question No. 2.56

Solve: $x-2y \le 3$, $3x+4y \ge 12$, $x \ge 0$, $y \ge 1$.

2 Explanation

1) Solving first pair of inequality:

$$-x + 2y \ge -3 3x + 4y \ge 12$$
 (2.0.1)

Solution: Let $u_1 \ge 0, u_2 \ge 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \ge \mathbf{0} \tag{2.0.2}$$

(2.0.8) can then be expressed as

$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} \ge \begin{pmatrix} -3 \\ 12 \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} -1 & 2\\ 3 & 4 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} -3\\ 12 \end{pmatrix} \tag{2.0.4}$$

or,
$$\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \mathbf{u}$$
 (2.0.5)

resulting in

$$\mathbf{x} = \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -3 \\ 12 \end{pmatrix} + \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \mathbf{u} \quad (2.0.6)$$

or,
$$\mathbf{x} = \begin{pmatrix} 3.6 \\ 0.3 \end{pmatrix} + \frac{-1}{10} \begin{pmatrix} 4 & -2 \\ -3 & -1 \end{pmatrix} \mathbf{u}$$
 (2.0.7)

2) Similarly, Solving second pair of inequality:

$$x \ge 0$$

$$y \ge 1$$
 (2.0.8)

Solution: Let $u_1 \ge 0, u_2 \ge 0$. This may be expressed as

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \succeq \mathbf{0} \tag{2.0.9}$$

(2.0.8) can then be expressed as

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} \ge \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.11}$$

or,
$$\mathbf{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mathbf{u}$$
 (2.0.12)

From (2.0.7) and (2.0.12), solution of the given system of inequalities can be found out graphically by intersection as shown by the below figures generated by Python:

As seen from 2.3 the solution region is bounded by line segments AB and BC and the line x-2y=3.Beyond A the region expands infinitely along the Y axis,Beyond C the region includes all the portion above the line x-2y=3.

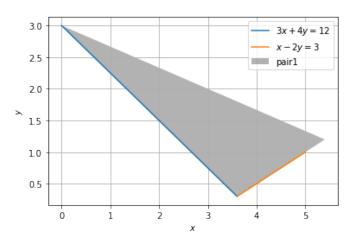


Fig. 2.1: Inequality pair 1

The common region shown by 2.3 is the solution of set of inequalities.

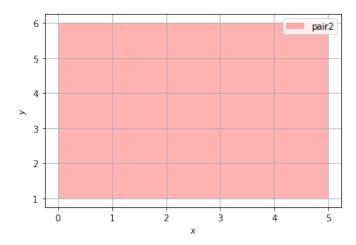


Fig. 2.2: Inequality pair 2

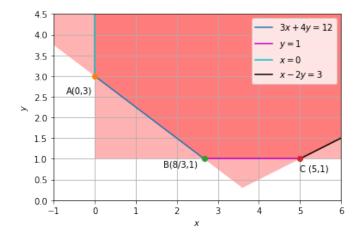


Fig. 2.3: Intersection of 2.1 and 2.2