



# Knowledge Representation

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# Predicate Logic

- **Problem:** A, B and C belong to the Himalayan club. Every member in the club is either a mountain climber or a skier or both. A likes whatever B dislikes and dislikes whatever B likes. A likes rain and snow. No mountain climber likes rain. Every skier likes snow. *Is there a member who is a mountain climber and not a skier?*
- Given knowledge has:
  - **Facts**
  - **Rules**

# Predicate Logic: Example

- Let *mc* denote **mountain climber** and *sk* denotes **skier**. Knowledge representation in the given problem is as follows:
  1.  $member(A)$
  2.  $member(B)$
  3.  $member(C)$
  4.  $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
  5.  $\forall x[mc(x) \rightarrow \sim like(x, rain)]$
  6.  $\forall x[sk(x) \rightarrow like(x, snow)]$
  7.  $\forall x[like(B, x) \rightarrow \sim like(A, x)]$
  8.  $\forall x[\sim like(B, x) \rightarrow like(A, x)]$
  9.  $like(A, rain)$
  10.  $like(A, snow)$
  11. Question:  $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$
- We have to infer the 11<sup>th</sup> expression from given 10 expressions.
- Done through **Resolution Refutation** - empty clause.

# Representation

- AI agents deal with knowledge (data)
  - Facts (believe & observe knowledge)
  - Procedures (how to knowledge)
  - Meaning (relate & define knowledge)
- Right representation is crucial
  - Early realisation in AI
  - Wrong choice can lead to project failure
  - Active research area

# Predicate Logic Representation

- For certain problem solving techniques
  - ‘Best’ representation already known
  - Often a requirement of the technique
  - Or a requirement of the programming language (e.g. Prolog)
- Examples
  - First order theorem proving... first order logic
  - Inductive logic programming... logic programs
  - Neural networks learning... neural networks
- Some general representation schemes
  - Suitable for many different (and new) AI applications

# Some General Representations

1. Logical Representations
2. Production Rules
3. Semantic Networks
  - Conceptual graphs, frames
4. *Description Logics* (see textbook)

# What is a Logic?

- A language with concrete rules
  - No ambiguity in representation (may be other errors!)
  - Allows unambiguous communication and processing
  - Very unlike natural languages e.g. English
- Many ways to translate between languages
  - A statement can be represented in different logics
  - And perhaps differently in same logic
- **Expressiveness** of a logic
  - How much can we say in this language?
- Not to be confused with logical reasoning
  - Logics are languages, reasoning is a process (may **use** logic)

# Syntax and Semantics

- Syntax
  - Rules for constructing legal sentences in the logic
  - Which symbols we can use (English: letters, punctuation)
  - How we are allowed to combine symbols
- Semantics
  - How we interpret (read) sentences in the logic
  - Assigns a meaning to each sentence
- Example: “All lecturers are seven foot tall”
  - A valid sentence (syntax)
  - And we can understand the meaning (semantics)
  - This sentence happens to be false (there is a counterexample)



# Propositional Logic

- Syntax
  - Propositions, e.g. “it is wet”
  - Connectives: and, or, not, implies, iff (equivalent)  
 $\wedge \quad \vee \quad \neg \quad \rightarrow \quad \leftrightarrow$
  - Brackets, T (true) and F (false)
- Semantics (Classical AKA Boolean)
  - Define how connectives affect truth
    - “P and Q” is true if and only if P is true and Q is true
  - Use **truth tables** to work out the truth of statements

# Predicate Logic

- Propositional logic combines atoms
  - An atom contains no propositional connectives
  - Have no structure (today\_is\_wet, john\_likes\_apples)
- **Predicates** allow us to talk about objects
  - Properties: is\_wet(today)
  - Relations: likes(john, apples)
  - True or false
- In predicate logic each atom is a predicate
  - e.g. first order logic, higher-order logic

# First Order Logic

- More expressive logic than propositional
  - Used in this course (Lecture 6 on representation in FOL)
- **Constants** are objects: john, apples
- **Predicates** are properties and relations:
  - likes(john, apples)
- **Functions** transform objects:
  - likes(john, fruit\_of(apple\_tree))
- **Variables** represent any object: likes(X, apples)
- **Quantifiers** qualify values of variables
  - True for all objects (Universal):  $\forall X. \text{likes}(X, \text{apples})$
  - Exists at least one object (Existential):  $\exists X. \text{likes}(X, \text{apples})$

## Example: FOL Sentence

- “Every rose has a thorn”

$$\forall X.(rose(X) \rightarrow \exists Y.(has(X, Y) \wedge thorn(Y)))$$

- For all X
  - if (X is a rose)
  - then there exists Y
    - (X has Y) and (Y is a thorn)

## Example: FOL Sentence

- “On Mondays and Wednesdays I go to John’s house for dinner”

$$\forall X. ((is\_mon(X) \vee is\_wed(X)) \rightarrow eat\_meal(me, houseOf(john), X))$$

- Note the change from “and” to “or”
  - Translating is problematic

# Higher Order Logic

- More expressive than first order
- Functions and predicates are also objects
  - Described by predicates: `binary(addition)`
  - Transformed by functions: `differentiate(square)`
  - Can quantify over both
- E.g. define red functions as having zero at 17

$$\forall F. (red(F) \leftrightarrow F(0) = 17)$$

- Much harder to reason with

# Beyond True and False

- Multi-valued logics
  - More than two truth values
  - e.g., true, false & unknown
  - **Fuzzy logic** uses probabilities, truth value in  $[0,1]$
- Modal logics
  - Modal operators define mode for propositions
  - **Epistemic logics** (belief)
    - e.g.  $\Box p$  (necessarily p),  $\Diamond p$  (possibly p), ...
  - **Temporal logics** (time)
    - e.g.  $\Box p$  (always p),  $\Diamond p$  (eventually p), ...

# Logic is a Good Representation

- Fairly easy to do the translation when possible
- Branches of mathematics devoted to it
- It enables us to do logical reasoning
  - Tools and techniques come for free
- Basis for programming languages
  - Prolog uses logic programs (a subset of FOL)
  - $\lambda$ Prolog based on HOL



# Non-Logical Representations?

- Production rules
- Semantic networks
  - Conceptual graphs
  - Frames
- Logic representations have restrictions and can be hard to work with
  - Many AI researchers searched for better representations

# Production Rules

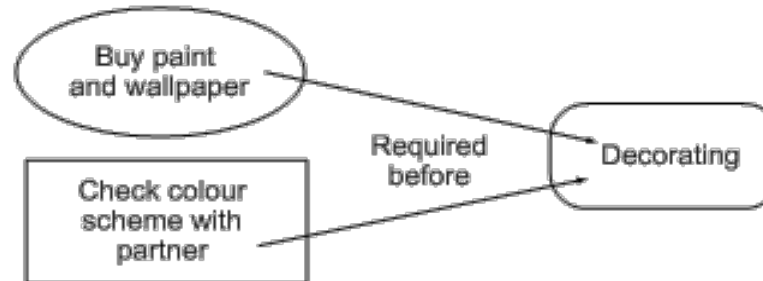
- Rule set of <condition,action> pairs
  - “if condition then action”
- Match-resolve-act cycle
  - **Match:** Agent checks if each rule’s condition holds
  - **Resolve:**
    - Multiple production rules may fire at once (**conflict set**)
    - Agent must choose rule from set (**conflict resolution**)
  - **Act:** If so, rule “fires” and the action is carried out
- Working memory:
  - rule can write knowledge to working memory
  - knowledge may match and fire other rules

# Production Rules Example

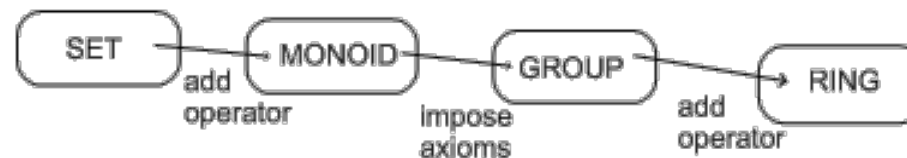
- IF (at bus stop AND bus arrives) THEN action(get on the bus)
- IF (on bus AND not paid AND have oyster card) THEN action(pay with oyster) AND add(paid)
- IF (on bus AND paid AND empty seat) THEN sit down
- conditions and actions must be clearly defined
  - can easily be expressed in first order logic!

# Graphical Representation

- Humans draw diagrams all the time, e.g.
  - Causal relationships

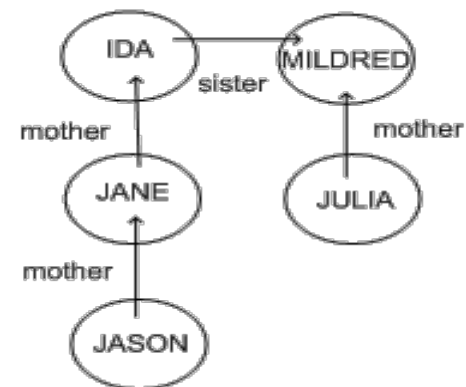
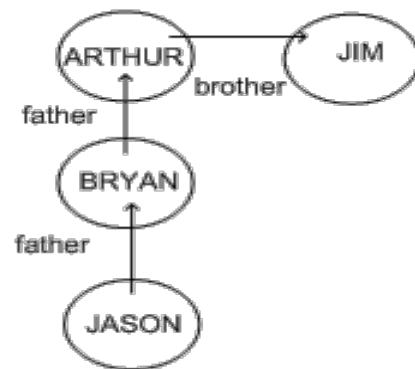


- And relationships between ideas



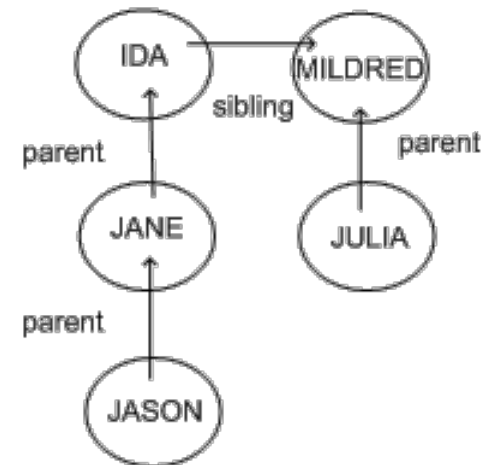
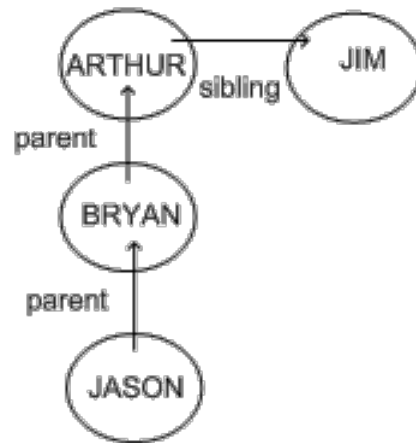
# Graphical Representation

- Graphs easy to store in a computer
- To be of any use must impose a formalism



- Jason is 15, Bryan is 40, Arthur is 70, Jim is 74
- How old is Julia?

# Semantic Networks



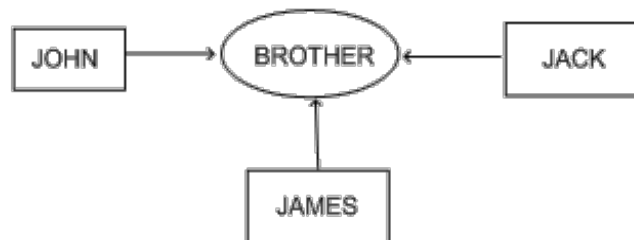
- Because the syntax is the same
  - We can guess that Julia's age is similar to Bryan's
- Formalism imposes restricted syntax

# Semantic Networks

- Graphical representation (a graph)
  - Links indicate subset, member, relation, ...
- Equivalent to logical statements (usually FOL)
  - Easier to understand than FOL?
  - Specialised SN reasoning algorithms can be faster
- Example: natural language understanding
  - Sentences with same meaning have same graphs
  - e.g. Conceptual Dependency Theory (Schank)

# Conceptual Graphs

- Semantic network where each graph represents a single proposition
- Concept nodes can be
  - Concrete (visualisable) such as restaurant, my dog Spot
  - Abstract (not easily visualisable) such as anger
- Edges do not have labels
  - Instead, conceptual relation nodes
  - Easy to represent relations between multiple objects

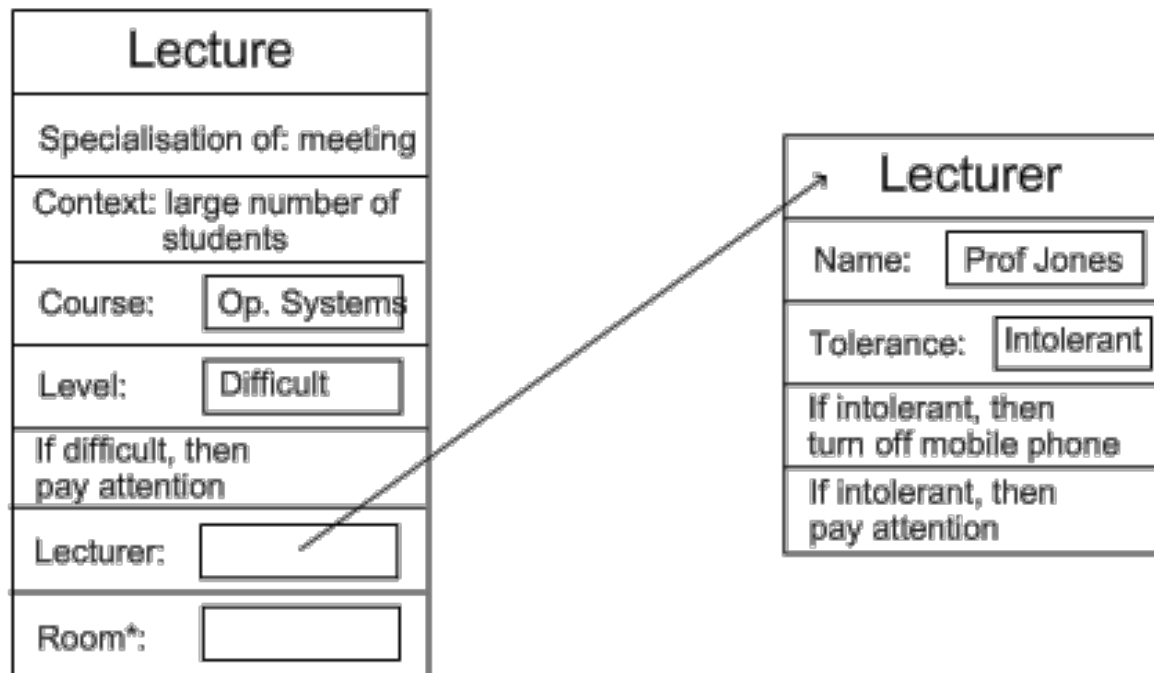




# Frame Representations

- Semantic networks where nodes have structure
  - Frame with a number of slots (age, height, ...)
  - Each slot stores specific item of information
- When agent faces a new situation
  - Slots can be filled in (value may be another frame)
  - Filling in may trigger actions
  - May trigger retrieval of other frames
- Inheritance of properties between frames
  - Very similar to objects in OOP

# Example: Frame Representation



# Flexibility in Frames

- Slots in a frame can contain
  - Information for choosing a frame in a situation
  - Relationships between this and other frames
  - Procedures to carry out after various slots filled
  - Default information to use where input is missing
  - Blank slots: left blank unless required for a task
  - Other frames, which gives a hierarchy
- Can also be expressed in first order logic

# Representation & Logic

- AI wanted “non-logical representations”
  - Production rules
  - Semantic networks
    - Conceptual graphs, frames
- But all can be expressed in first order logic!
- Best of both worlds
  - Logical reading ensures representation well-defined
  - Representations specialised for applications
  - Can make reasoning easier, more intuitive