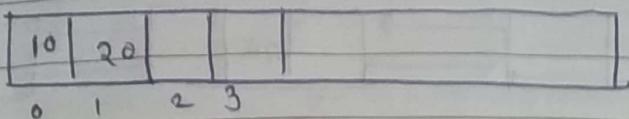


* LINKED LIST *

PAGE No.	/ / /
DATE	/ / /

In array time complexity is less as

A[10]



(1000 + (3 × 4)) → To find an element

↓ ↓ ↗
Base address index no. of bytes

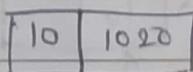
T.C. = O(1)

* Declaration for Linked List *

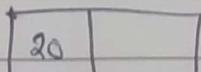
struct Node

{

int data;



struct Node *next;



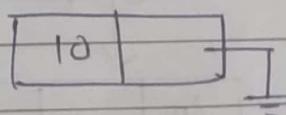
}

struct Node *newnode

newnode = (Node *) malloc(sizeof(struct Node))

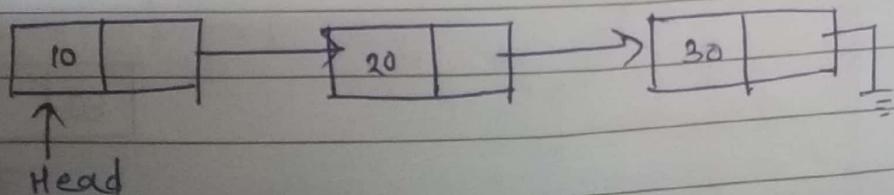
newnode → data = 10;

newnode → next = NULL;



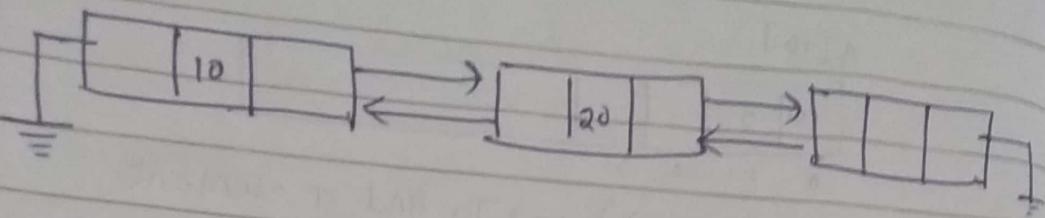
* Types of Linked List :-

① Singly Linked List :-



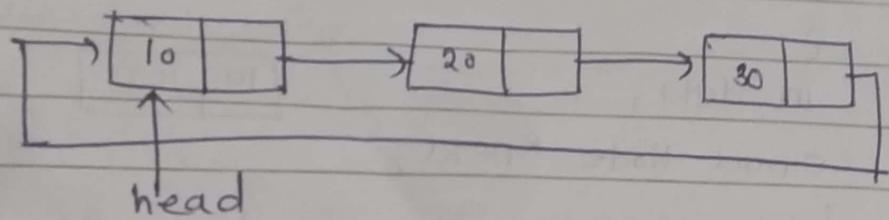
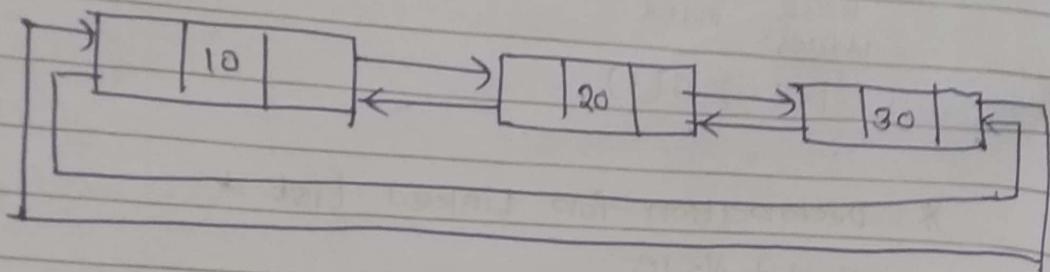
②

Doubly Linked List



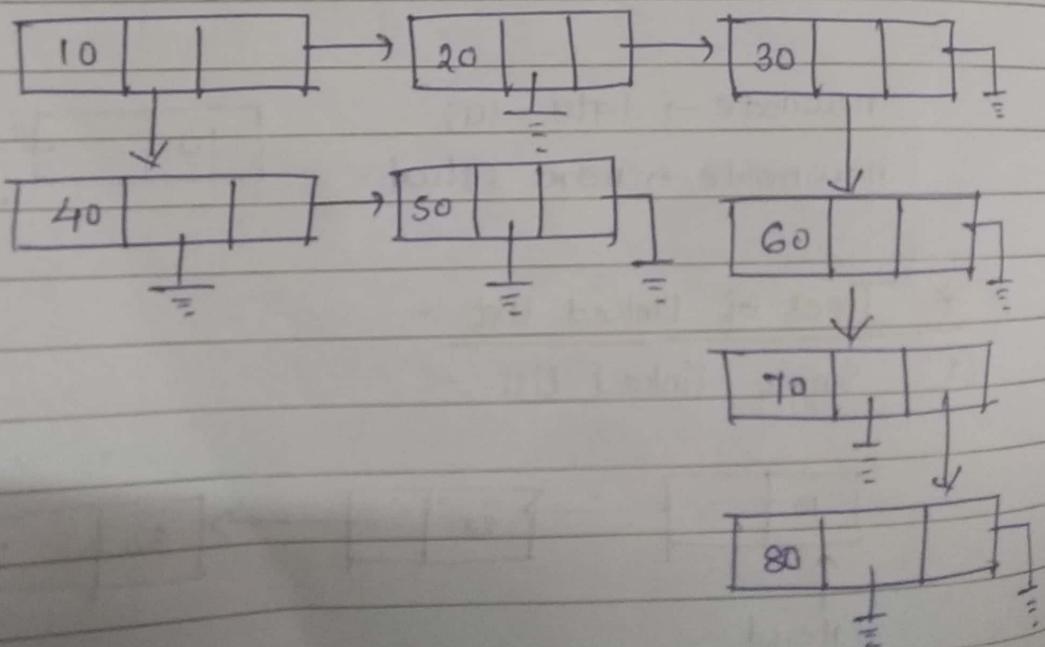
③

Circular Linked List



④

Generalized Linked List



* singly linked list :-

- creation & mode
- Insertion
- Deletion
- Display

main ()

{

struct Node *head;

insert (&head) // insert (head)

}

void insert (struct node **head)

{

struct node *new;

new = *head; // newhead

}

① Insertion

(a) begin

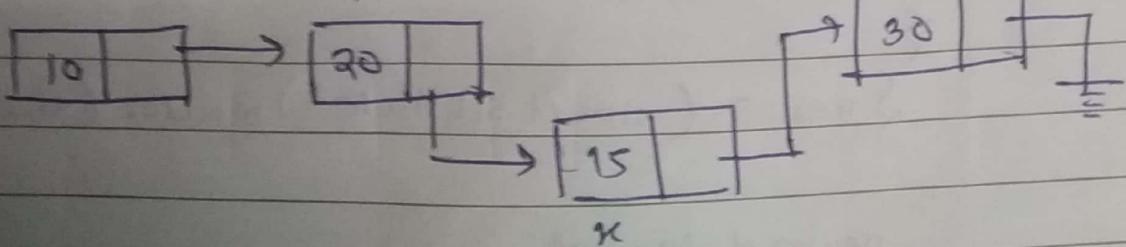
new → data = 5;

new → next = head;

head = new;

(b) between

p = head ?

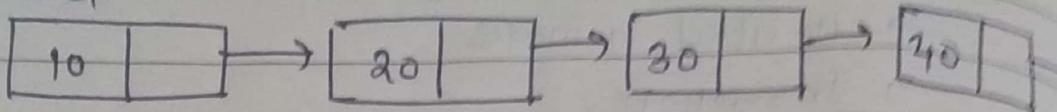


x → next = p → next

p → next = x;

- o Deleting the 1st node :-

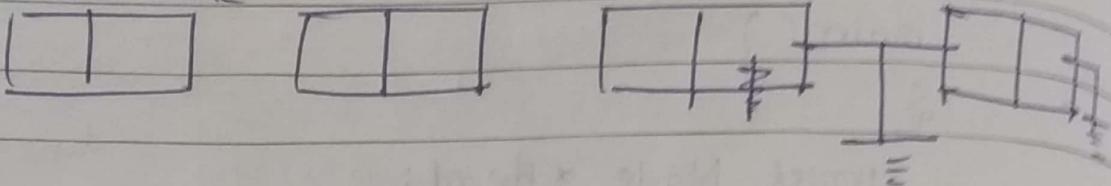
head



$q \rightarrow \text{next} = p \rightarrow \text{next}$

~~free (p);~~

last node



* Insertion Code :-

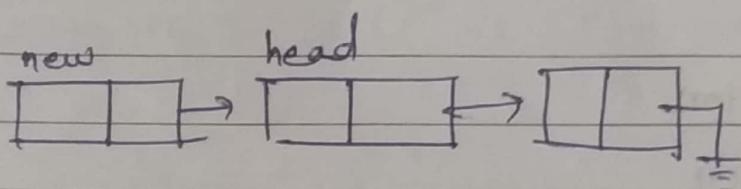
struct SLLNode

8

int data;

```
struct SLLnode *next;
```

3



```
void insert( struct SLLNode ** head , int val,  
           int pos )
```

$$\S \quad \text{int } k = 1;$$

```
struct SLLNode *new, *p;
```

1

```
snew = (struct SLLnode*) malloc(sizeof(struct SLLnode))
```

`new->data = val;`

`new->next = NULL;`

if (pos == 1)

{

 new->next = *head;

 *head = new;

 return;

}

 p = *head;

 while (p->next != NULL) && !(pos - 1)

{

 k++;

 p = p->next;

 if (p == NULL)

{

 printf ("Desired address not found");

 return;

}

 new->next = p->next;

 p->next = new;

}

main()

{ struct SLLNode * head;

 head = (struct SLLNode *) malloc (---);

 head->data = 1;

 head->next = NULL;

 insert (&head, 202);

}

* DLL :- (Doubly linked list)
struct DLLNode

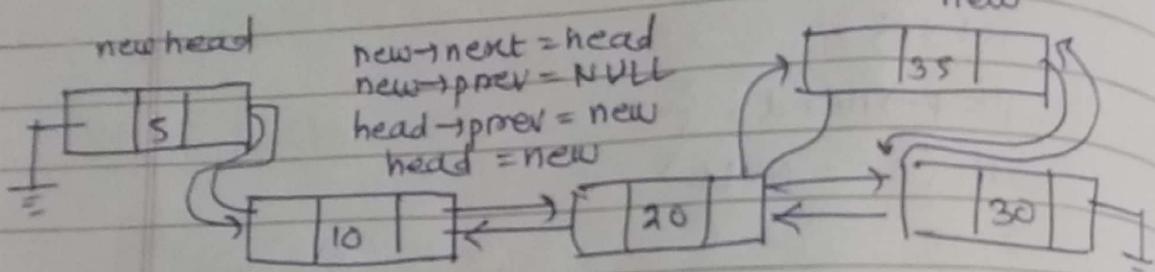
{

int data;

struct DLLNode *pprev, *next;

};

① Insertion *begin*
 Between
 End



new->pprev = p;

new->next = p->next;

p->next->pprev = new;

p->next = new;

void DLL insert (struct Node DLL **head, int val,
 int pos);

Struct DLLNode *new, *temp;

new = malloc ();

new->next = new->pprev = NULL;

new->data = val;

if (pos == 1)

at

begin

{ new->next = * head;

(* head)->pprev = new;

* head = new;

return;

}

$p = \& \text{head};$
 $\text{while } (p \rightarrow \text{next} \neq \text{NULL}) \ \& \ k < (\text{pos}-1)$

{

$k++;$

$p = p \rightarrow \text{next};$

}

$\text{new} \rightarrow \text{prev} = p;$

$\text{new} \rightarrow \text{next} = p \rightarrow \text{next};$

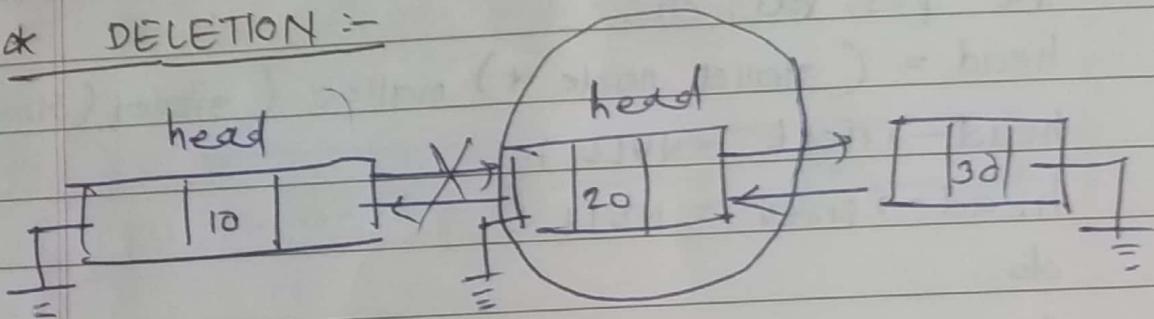
$\text{if } (p \rightarrow \text{next} \neq \text{NULL})$

$p \rightarrow \text{next} \rightarrow \text{prev} = \text{new};$

$p \rightarrow \text{next} = \text{new};$

}

* DELETION :-

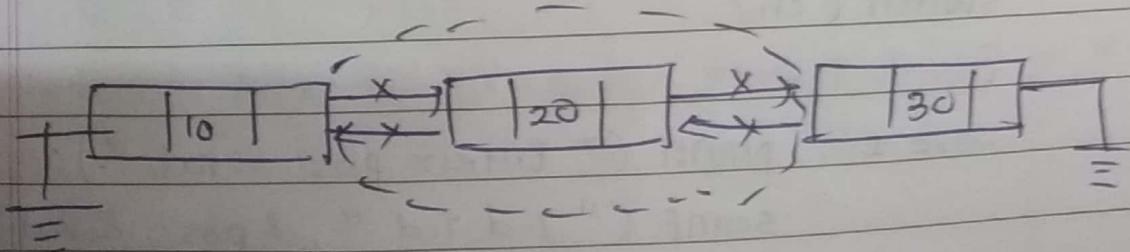


$p = \text{head};$

$\text{head} = \text{head} \rightarrow \text{next}$

$\text{head} \rightarrow \text{prev} = \text{NULL};$

$\text{free}(p);$



* Insertion of DLL using single pointers *

struct DLLnode

{

int data;

struct DLLnode *next, *prev;

}

struct DLLnode *insert(struct DLLnode *,
int, int);

main()

{

struct node *head;

int pos, val, ch;

head = (struct node *) malloc (sizeof (struct node));

head → next = NULL;

head → prev = NULL;

do

{

printf (" 1. Insert 2. Delete 3. Trans / display
4. Exit ");

scanf ("%d", &ch);

switch (ch)

{

case 1 : printf (" Enter pos & val ");

scanf ("%d %d ", &pos, &val);

head = insert (head, pos, val);

break;

case 2 : printf ("Enter node no. to be deleted");

scanf ("%d", &pos);

head = delete (head, pos);

break;

case 3 : display (head);

break;

}

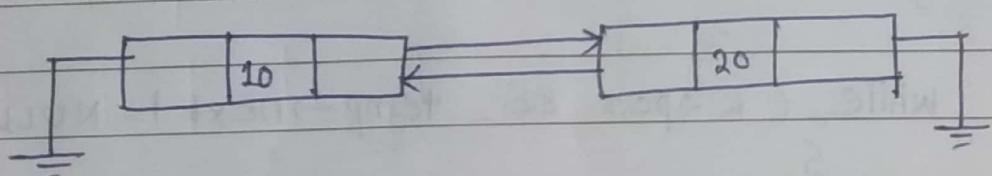
}

while (ch != 4);

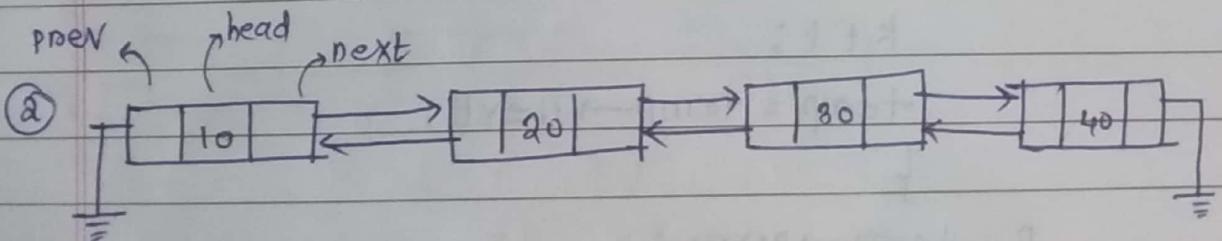
}

* Deletion from DLL *

①



②



struct DLLnode * delete (struct DLLnode *head,
int pos)

{

int k = 1;

struct node * p, * temp;

temp = head;

list empty {
 if (head == NULL)
 {
 printf ("\n list is empty ");
 return (head);
 }

1st node deletion {
 if (pos == 1)
 {
 head = head → next;
 if (head != NULL)
 head → prev = NULL;
 free (temp);
 return (head);
 }

while (k < pos && temp → next != NULL)

{

k++;

temp = temp → next;

{

p = temp → prev;

p → next = temp → next;

if (temp → next != NULL)

temp → next → prev = p;

free (temp);

return (head);

}

void display (struct DLLnode *head)

2

struct DLLnode *p ;

p = head ;

while (p != NULL)

3

printf (" %d " p->data);

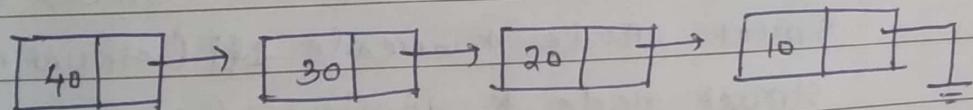
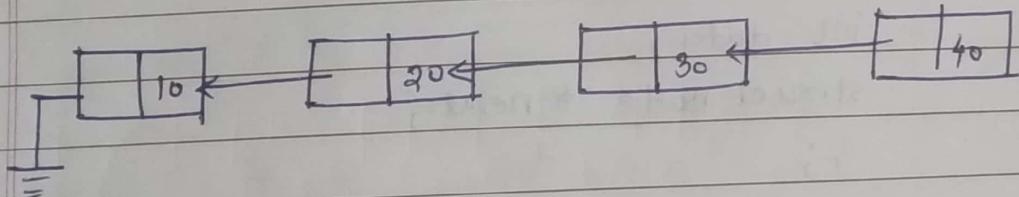
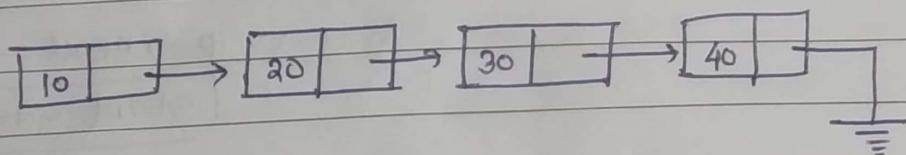
p = p->next ;

}

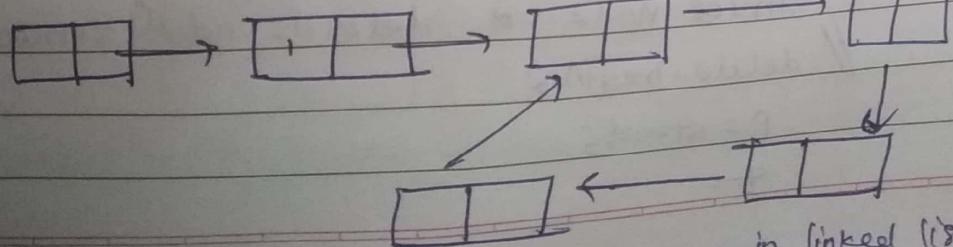
3

* Task

②

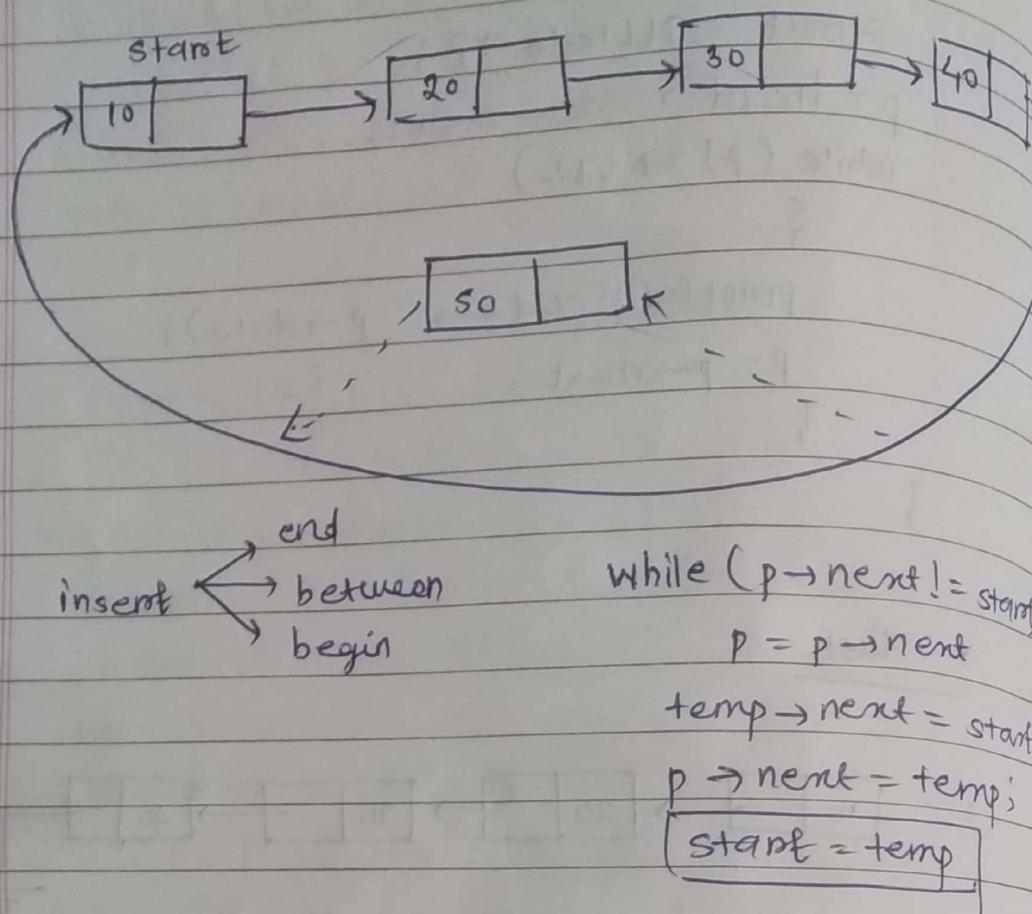


①



To detect whether there is a loop in linked list.

* CIRCULAR LINKED LIST *



struct node

{

int data;

struct node * next;

}

struct node * insert_beg (struct node *);

struct node * insert_end (struct node *);

struct node * create_ll (struct node *);

struct node * display (struct node *);

struct node * delete_beg (struct node *);

struct node * delete_end (struct node *);

// delete-begin

p = start;

if = .

```
while (p->next != start)
    p = p->next;
    q = start;
    start = start->next;
    p->next = start;
    free(q);
// delete last
p = start;
while (p->next != start)

{
    q = p;
    p = p->next;
    q->next = p->next;
    free(p);
}
```

main()

```
{
```

switch(ch)

```
{
```

case 1: head = insert(head);
break;

case 2 :

```
head = insert-end(head);  
break;
```

```
struct node * insert-beg (struct node * head)
{

    struct node * new-node, * p;
```

```
int num;
printf ("Enter data");
scanf ("%d", &num);
new-node = malloc ( );
new-node → data = num;
if (head == NULL)
{
    new-node → next = new-node;
    head = new-node;
    return (head);
}
else
{
    p = head;
    while (p → next != head)
        p = p → next;
    new-node → next = head;
    p → next = new-node;
    head = new-node;
    return (head);
}
```

```
// create-cll
struct node * create-cll (struct node * head)
{
    struct node * new-node, * p;
    int num;
    printf ("Enter data");
```

```

scanf ("%d", &num);
while (num != -1)
{
    new_node = (struct node *) malloc (- - -);
    new_node->data = num;
    if (head == NULL)
    {
        new_node->next = new_node;
        head = new_node;
    }
    else
    {
        p = head;
        while (p->next != head)
            p = p->next;
        new_node->next = head;
        p->next = new_node;
    }
    printf ("Enter data");
    scanf ("%d", &num);
}
return (head);
}

```

struct node * delete_beg (struct node * head)

```

{
    struct node * new_node, * p;
    p = head;

```

```

if (head == NULL)
{
    printf (" linked is Empty ");
    return (head);
}

```

```

if (p->next == head)
{

```

```

    free(p);
    return ();
}
```

```

while (p->next != head)

```

```

    p = p->next;

```

```

    q = head;

```

```

    head = head->next;

```

```

    p->next = head;

```

```

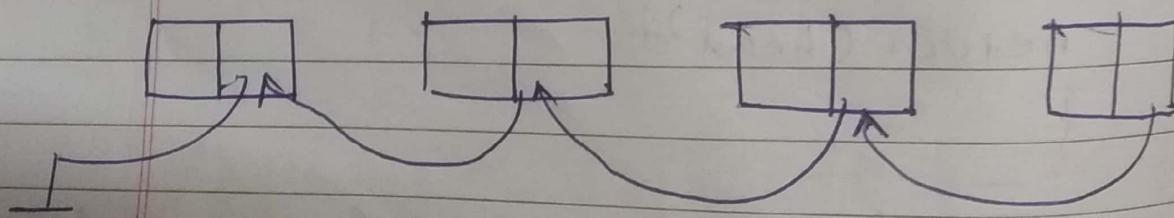
    free(q);

```

```

    return (head);
}
```

* Reverse



$p = \text{head};$

$q = p \rightarrow \text{next};$

$r = q \rightarrow \text{next};$

$p \rightarrow \text{next} = \text{NULL};$

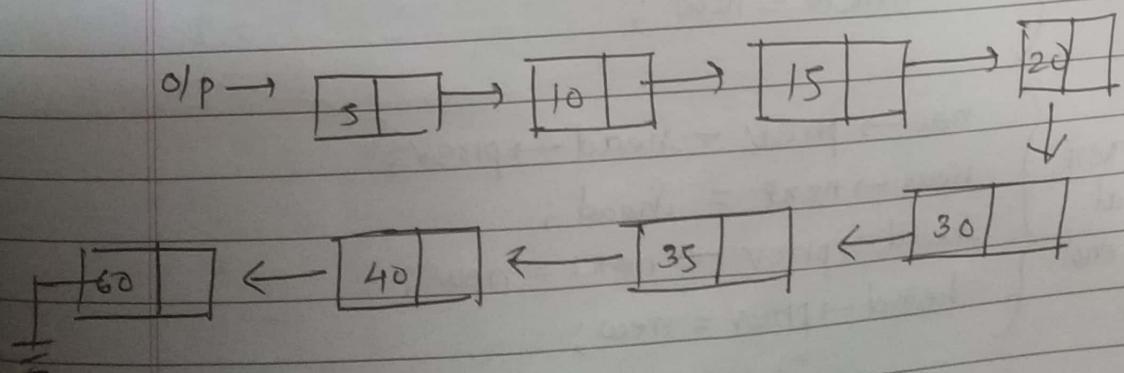
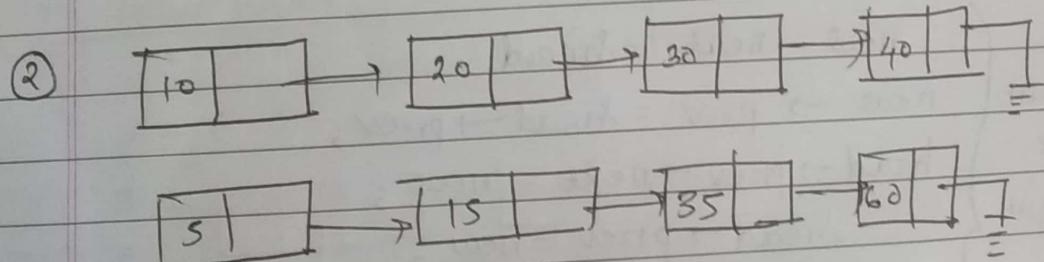
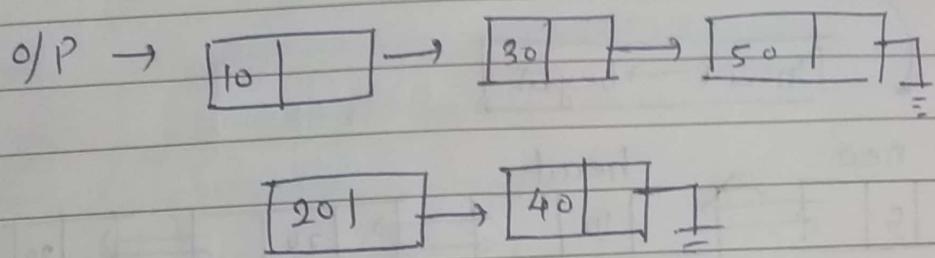
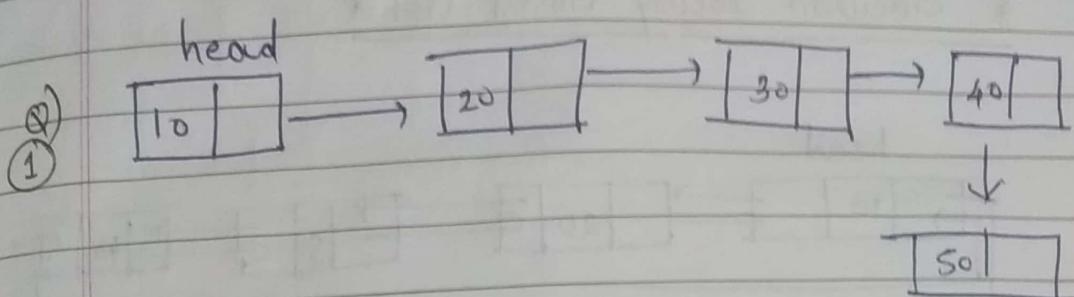
while ($q \neq \text{NULL}$)

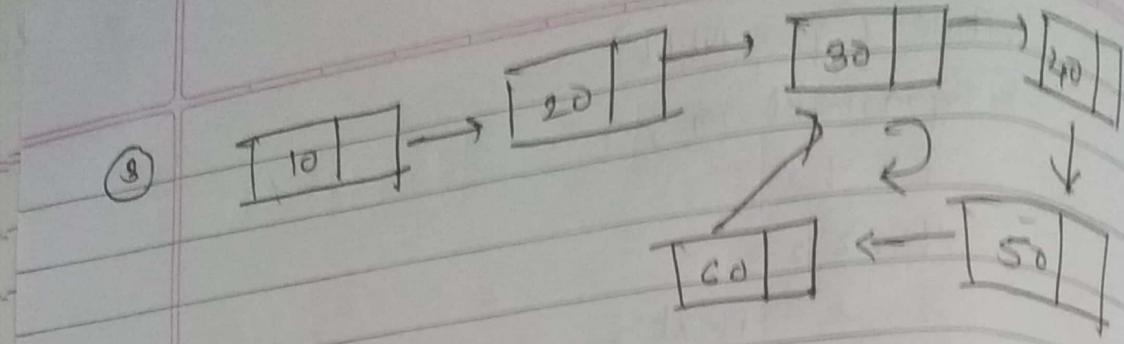
$\{ q = q \rightarrow \text{next} = p;$

$p = q;$

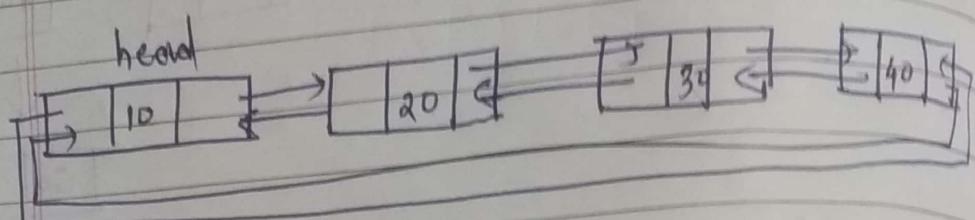
$q = q \rightarrow \text{next};$

}

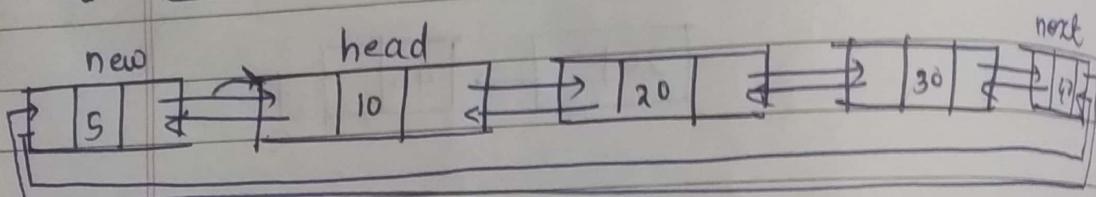




* Circular Doubly Linked List *



Insert → begin



insert at begin

$$\left. \begin{array}{l} \text{new} \rightarrow \text{next} = \text{head}; \\ \text{new} \rightarrow \text{prev} = \text{head} \rightarrow \text{prev}; \\ \text{head} \rightarrow \text{prev} \rightarrow \text{next} = \text{new}; \\ \text{head} \rightarrow \text{prev} = \text{new}; \\ \text{head} = \text{new}; \end{array} \right\}$$

insert at end.

$$\left. \begin{array}{l} \text{new} \rightarrow \text{prev} = \text{head} \rightarrow \text{prev}; \\ \text{new} \rightarrow \text{next} = \text{head}; \\ \text{head} \rightarrow \text{prev} \rightarrow \text{next} = \text{new}; \\ \text{head} \rightarrow \text{prev} = \text{new}; \end{array} \right\}$$

Deleting at front

first node deletion {

- head = head \rightarrow next ;
- head \rightarrow prev = p \rightarrow prev ;
- p \rightarrow prev \rightarrow next = head ;
- free(p) ;

Deleting last end

last node deletion {

- p = head \rightarrow prev ;
- p \rightarrow prev \rightarrow next = head ;
- h \rightarrow prev = p \rightarrow prev ;
- free(p) ;

* Generalized Linked List (GLL) *

A = ()

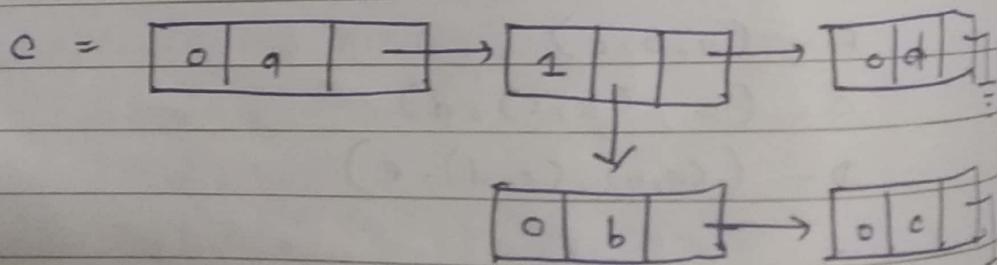
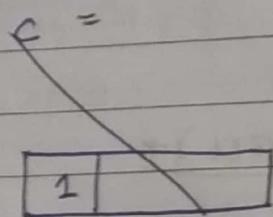
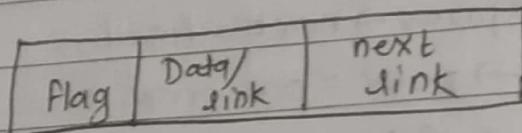
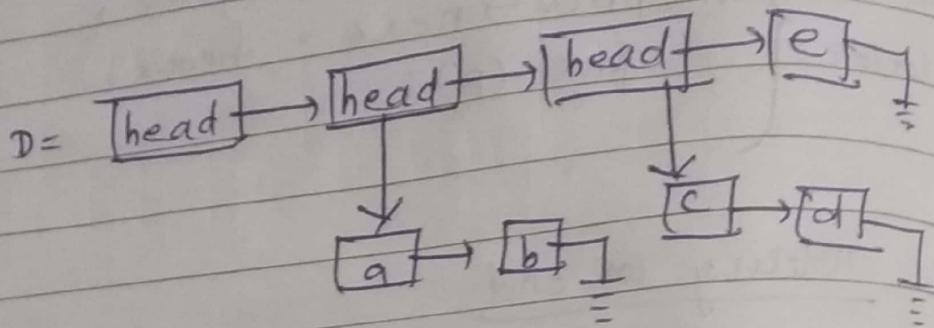
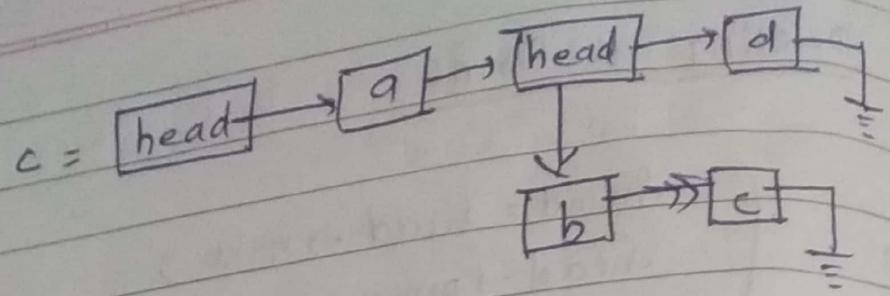
B = (a, b, c)

C = (a, (b, c), d)

D = ((a, b), (c, d), e)

A = head

B = head \rightarrow a \rightarrow b \rightarrow c



struct node

{ int flag;

union {

int data;

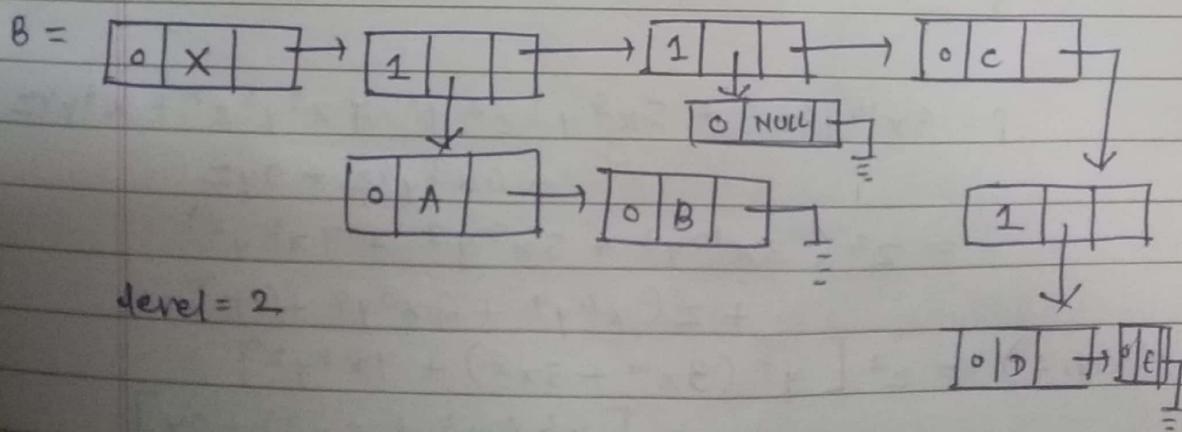
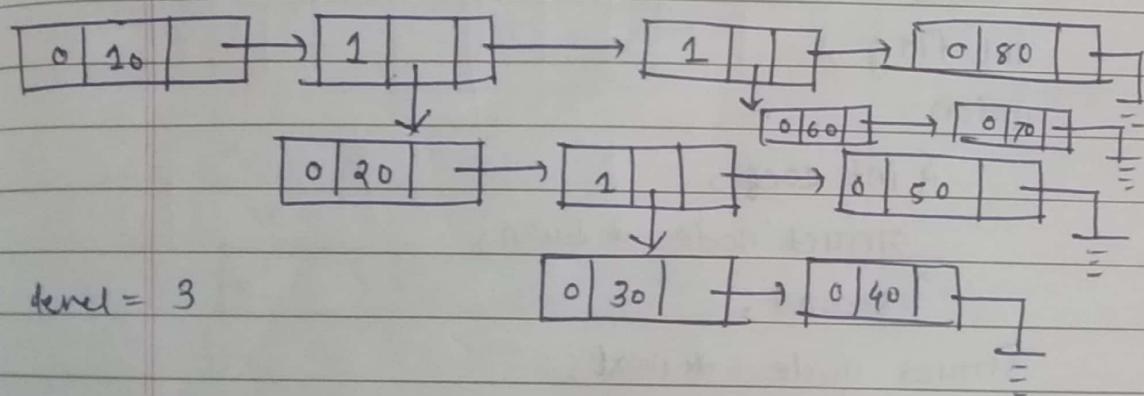
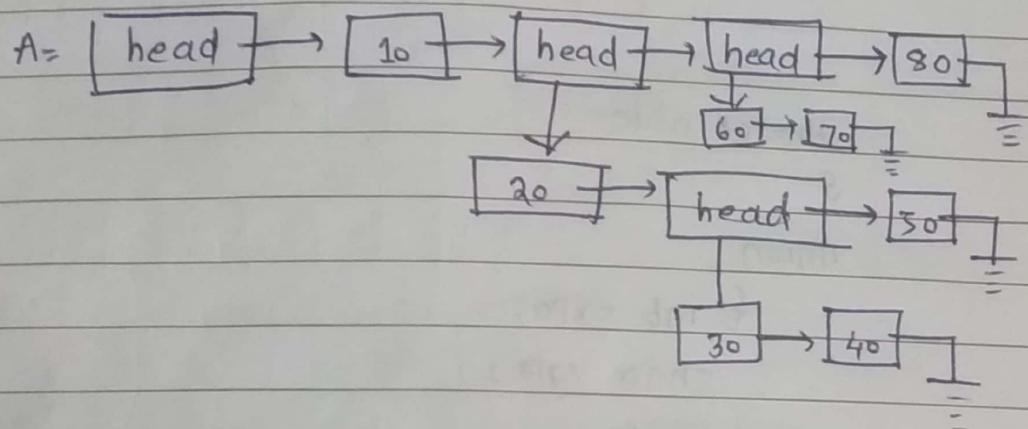
struct node * deun;

} un;

struct node *next;
};

* A = { 10, (20, (30, 40), 50), (60, 70), 80 }

B = { x, (A, B), (), C, (D, E) }



level = no. of down links + 1 .

$$* \quad 3x^{10}y^3z^2 + 5x^8y^3z^2 + 7x^8y^2z^2 + x^4y^4z \\ + 6x^3y^4z + 9yz$$

expo/ variable	flag	<u>coef</u> down	next link
-------------------	------	---------------------	--------------

struct node

{

union

{ int expo;
char var; }

} first;

int flag;

union

{ int coef;

struct node *down;

} second;

struct node *next;

}

$$P = 3x^{10}y^3z^2 + 5x^8y^3z^2 + 7x^8y^2z^2 + x^4y^4z$$

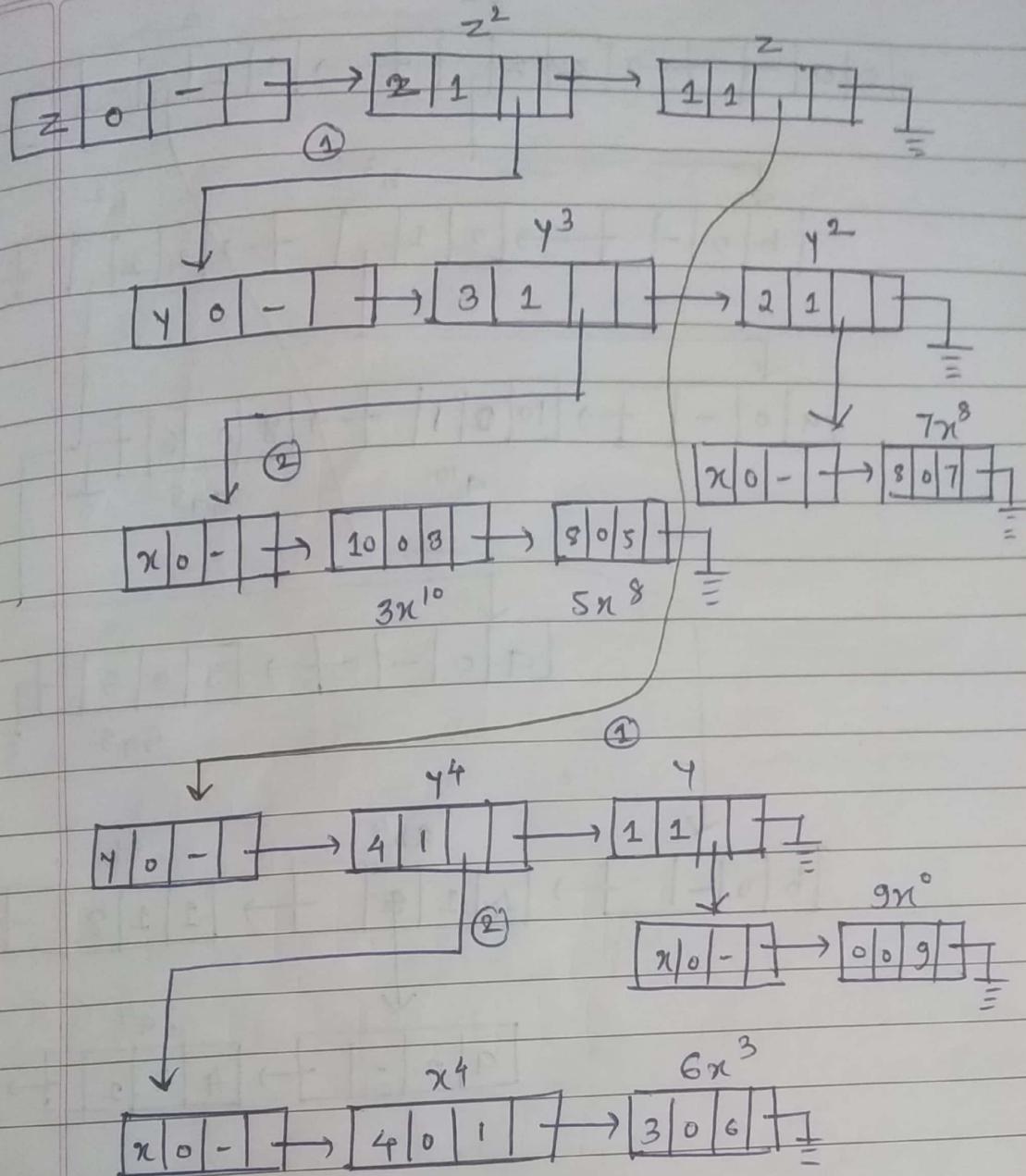
$$+ 6x^3y^4z + 9yz$$

$$= z^2 (3x^{10}y^3 + 5x^8y^3 + 7x^8y^2)$$

$$+ z (x^4y^4 + 6x^3y^4 + 9y)$$

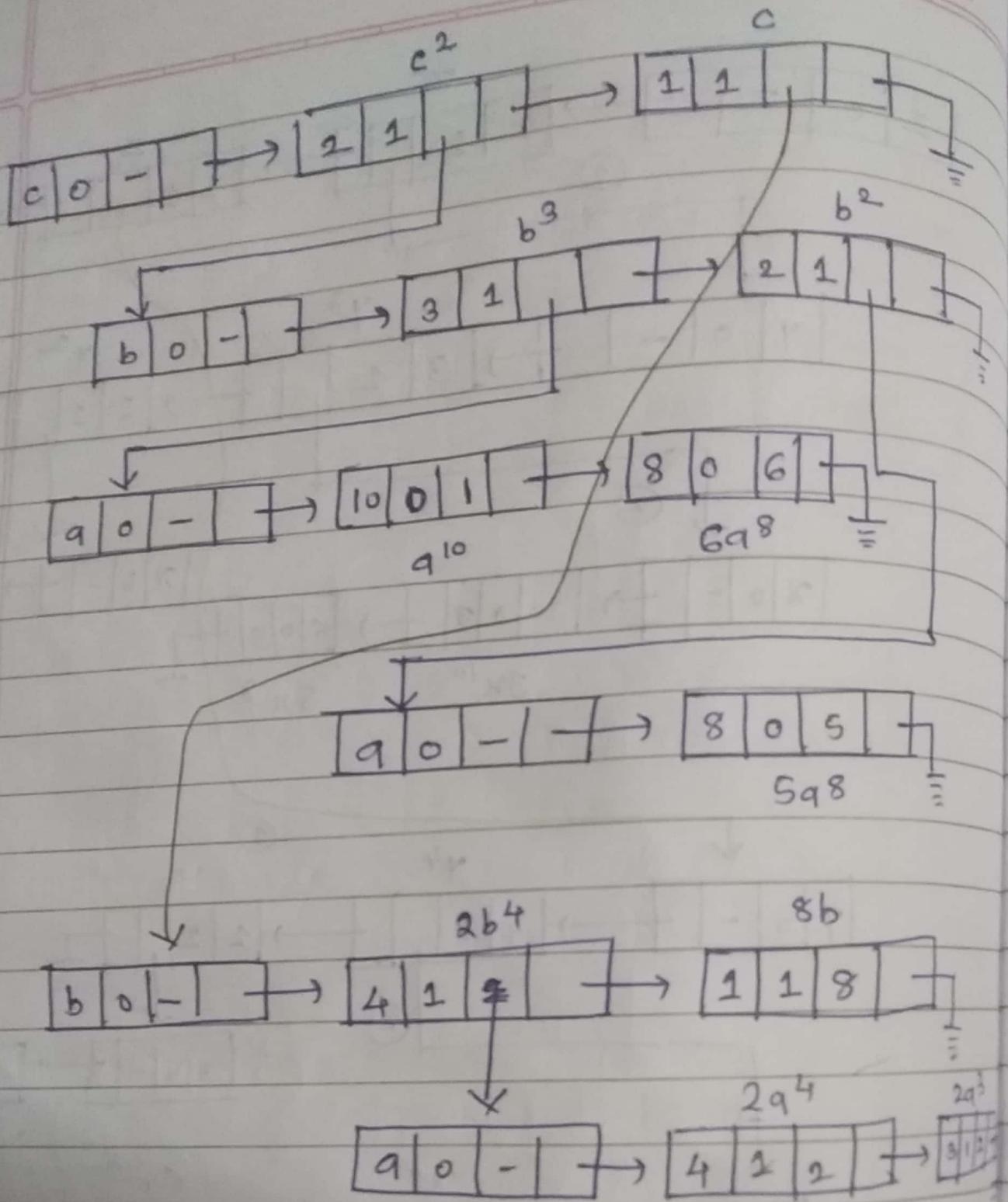
$$P(x, y, z) = z^2 [y^3 (3x^{10} + 5x^8) + 7x^8y^2]$$

$$+ z [y^4 (x^4 + 6x^3) + 9y]$$



$$\text{depth} = \text{level} = 2 + 1 = \underline{\underline{3}}$$

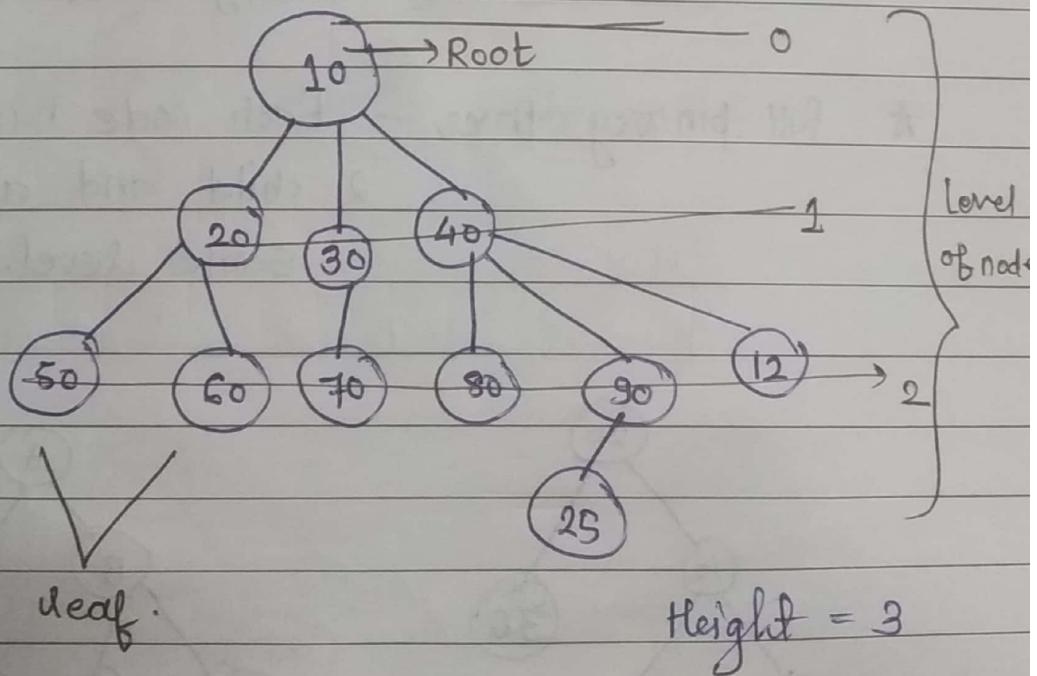
$$\begin{aligned}
 * P(a, b, c) &= a^{10}b^3c^2 + 6a^8b^3c^2 + 5a^8b^2c^2 \\
 &\quad + 2a^4b^4c + 2a^3b^4c + 8bc \\
 &= c^2 (a^{10}b^3 + 6a^8b^3 + 5a^8b^2) \\
 &\quad + c (2a^4b^4 + 2a^3b^4 + 8b) \\
 &= c^2 [b^3 (a^{10} + 6a^8) + 5a^8b^2] \\
 &\quad + c [2b^4 (2a^4 + 2a^3) + 8b] \\
 &\quad + c [b^4 (2a^4 + 2a^3) + 8b]
 \end{aligned}$$



* TREES *

* Basic Terminologies

- Root
- Parent
- Siblings
- Path
- Degree of Node
- Height
- Ancestors
- Descendents



siblings → common parent

leaf → Node having no child

degree = deg. of ancestors + deg. of descendants

* Types of Trees

(1) Binary Tree — 2 child or less

(2) m-way tree

— node m childs

Binary Search
tree

strict binary
tree

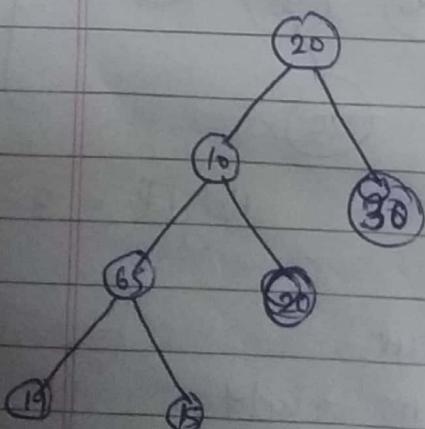
complete bin

Full binary tree

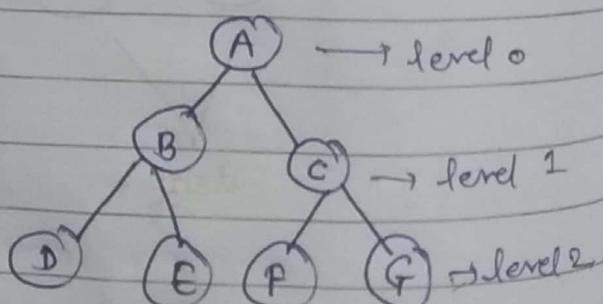
* strict binary tree — each node has exactly two or no child.

* complete binary tree — leaf nodes are at height $h / h - 1$.

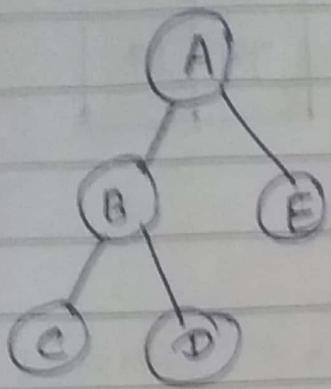
* full binary tree — Each node has exactly 2 child and all are at same level.



Strict binary
tree

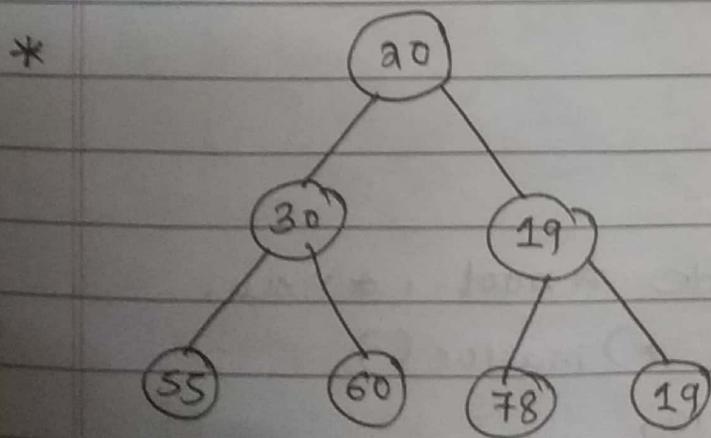
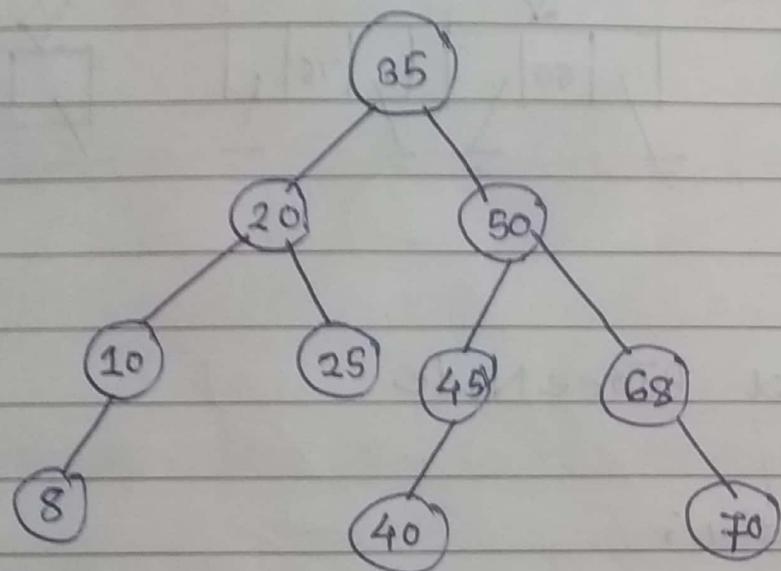


Full binary
tree



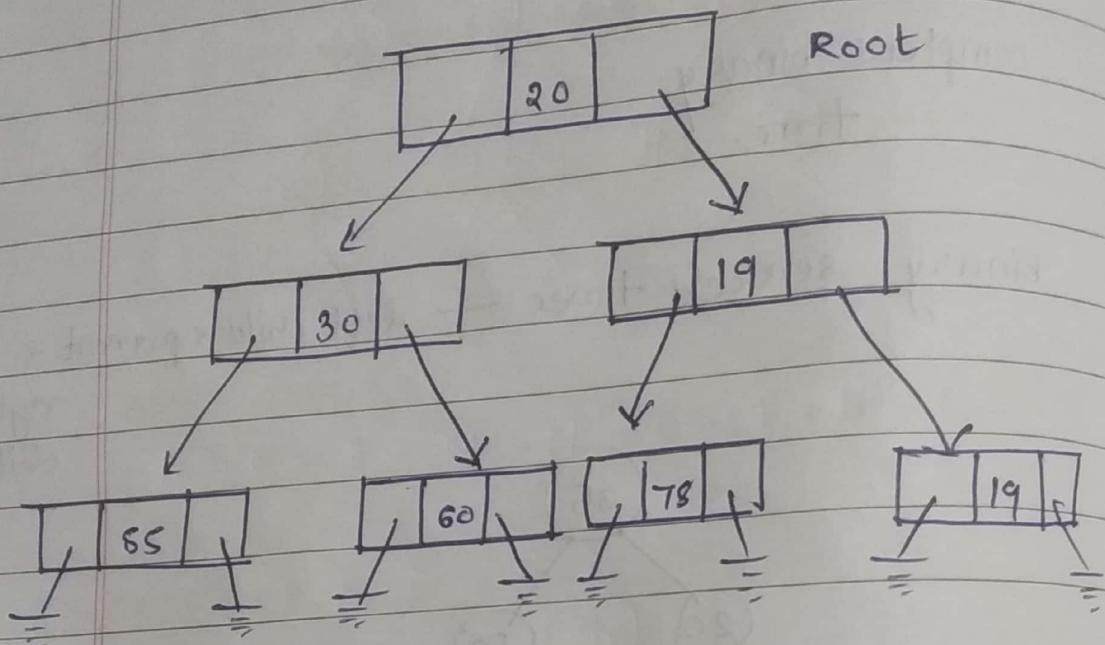
complete binary tree

* binary search tree — left child < parent < right child.



20	80	19	55	60	78	19	

Array



struct BTeeNode

{

int data ;

struct BTeeNode * left, * right ;

} ;

main()

{

struct BTeeNode *root , *new;

root = (struct *) malloc()

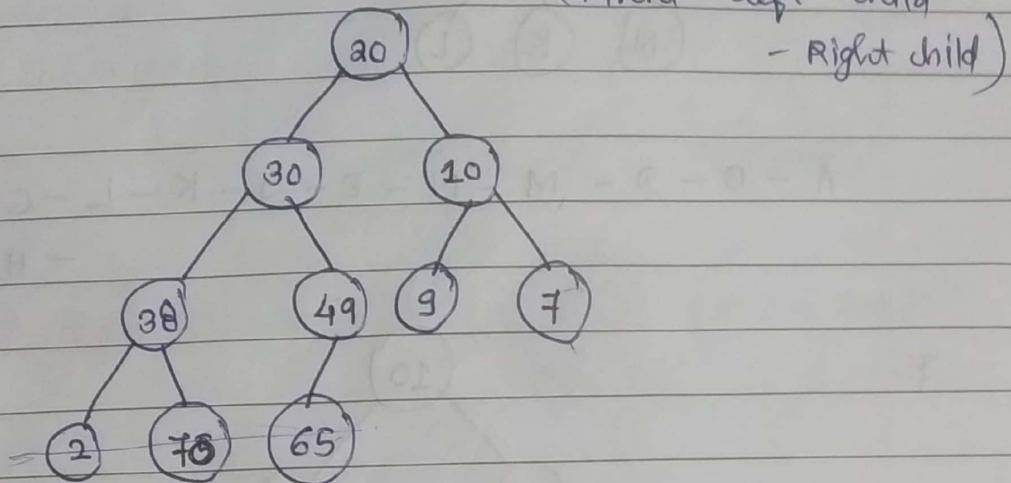
root → data = 10;

root → left

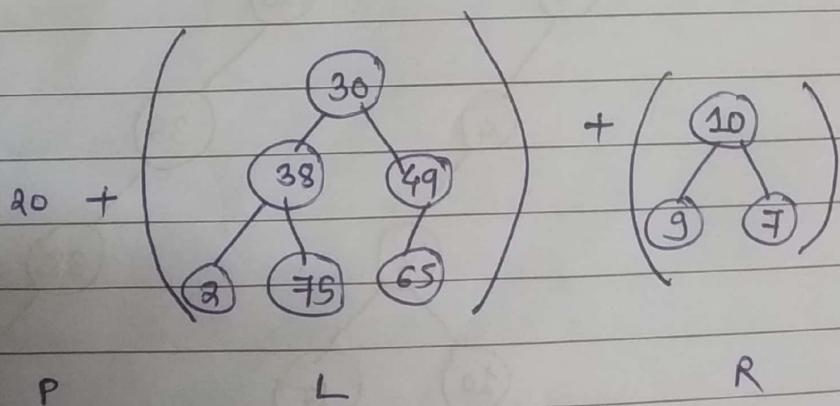
* Binary Tree traversals

① Preorders traversals (P - L - R)

(Parent - left child
- Right child)



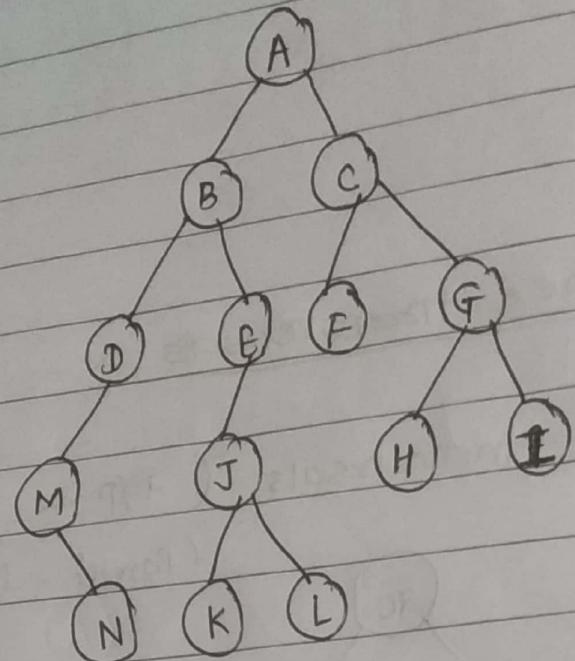
$\Rightarrow 20 - 30 - 38 - 2 - 75 - 49 - 65 - 10 - 9 - 7$



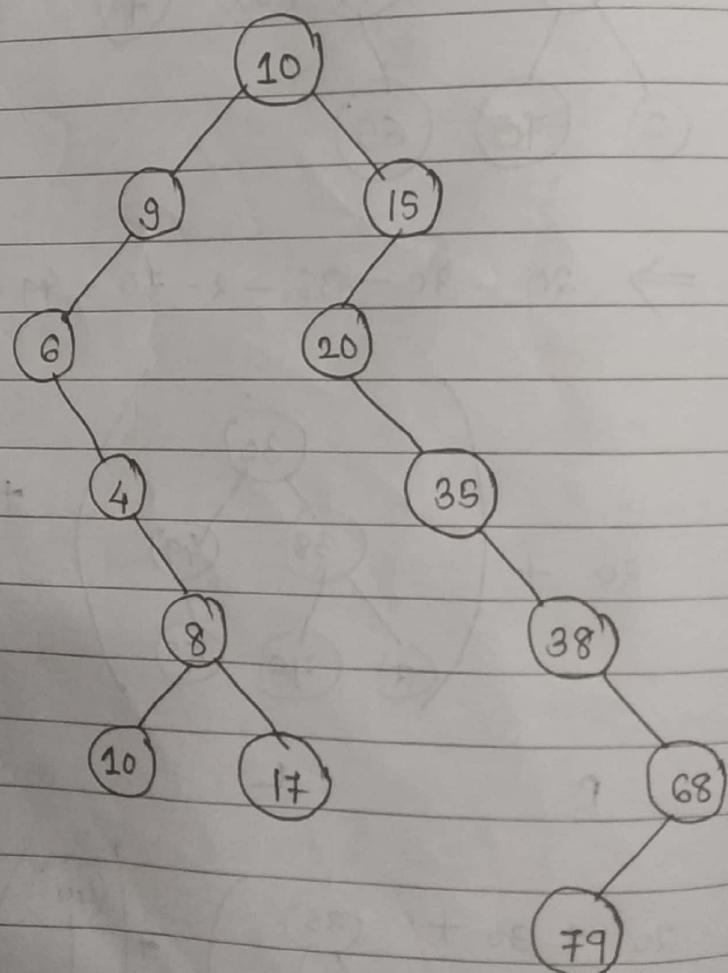
$20 + 30 + \left(\begin{array}{c} 38 \\ 2 \\ 75 \end{array} \right) + \left(\begin{array}{c} 49 \\ 65 \end{array} \right) + 10 + 9 + 7$

$20 + 30 + 38 + 2 + 75 + 49 + 65 + 10 + 9 + 7$

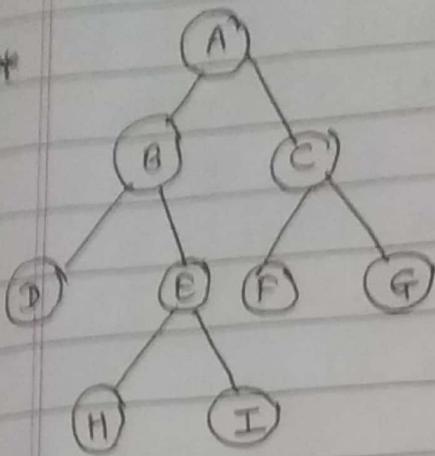
$\Rightarrow 20 - 30 - 38 - 2 - 75 - 49 - 65 - 10 - 9 - 7$



A - B - D - M - N - E - J - K - L - C - F - G
- H - I



10 - 9 - 6 - 4 - 8 - 10 - 17 - 15 - 20 - 35 - 38 - 68 - 79



* Display $\Rightarrow A + \text{pre}(\text{root} \rightarrow \text{left}) + \text{pre}(\text{root} \rightarrow \text{right})$

$\text{pre}(\text{root} \rightarrow \text{left})$

$\text{pre}(\text{root} \rightarrow \text{right})$

$\Rightarrow A + B + \text{pre}(D) + \text{pre}(E) + \text{pre}(F) + \text{pre}(G)$

$\Rightarrow A - B - D - E - H - I - C - F - G$

* Recursive Approach *

struct node

{

int data;

struct node * left, * right }

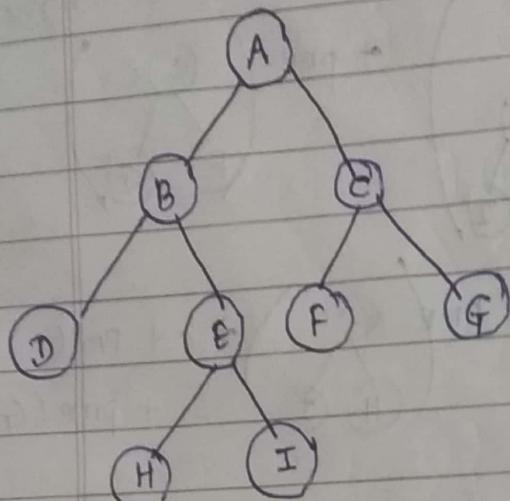
void preorder (struct node * root)

{

if (root)

```
{printf ("% .d ", root->data);  
preorder (root->left);  
preorder (root->right);  
}
```

* Non-Recursive



while (1)

{

 while (root)

```
{ printf (root);
```

```
    push (root);
```

```
    root = root->left;
```

}

 if (stack empty)

```
        break;
```

```
    root = pop (s);
```

```
    root = root->right;
```

}

void preorders (struct node * root)

{

 while (1)

{

 while (root)

 printf ("%d", root->data);

 push (s, root);

 root = root->left;

}

 if (EmptyStack)

 break;

 root = pop (s);

 root = root->right;

}

 deleteStack (s);

}

struct stack

{ int top ;

 struct node * data [30] ;

};

struct node

{ int data ;

 struct node * left , * right ;

};

struct stack * s = NULL ;

struct node * pop (struct stack * s)

{

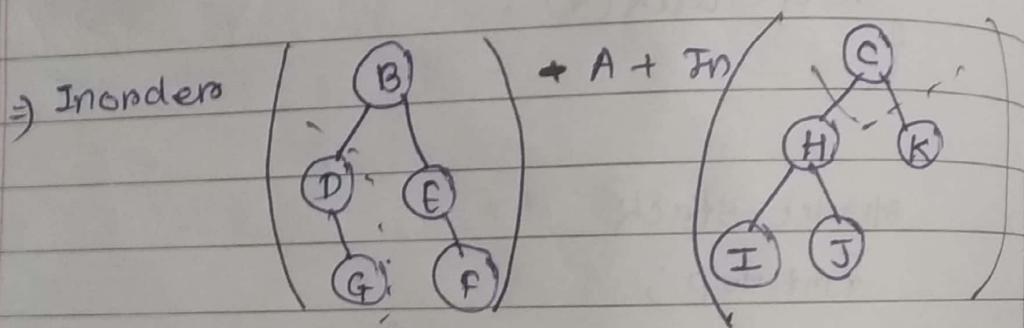
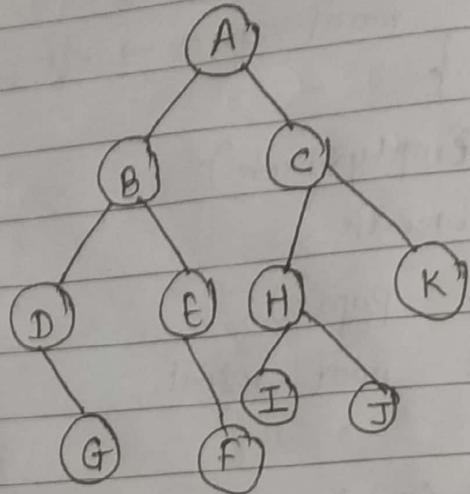
 struct node * temp ;

```

temp = s->data [s->top];
s->top = (s->top) - 1;
return (temp);
}

```

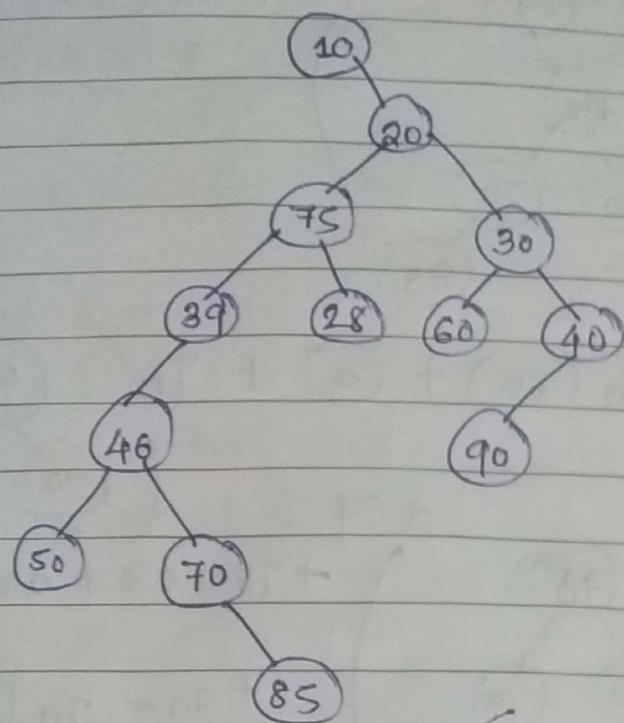
② Inorder Traversal (L-P-R)



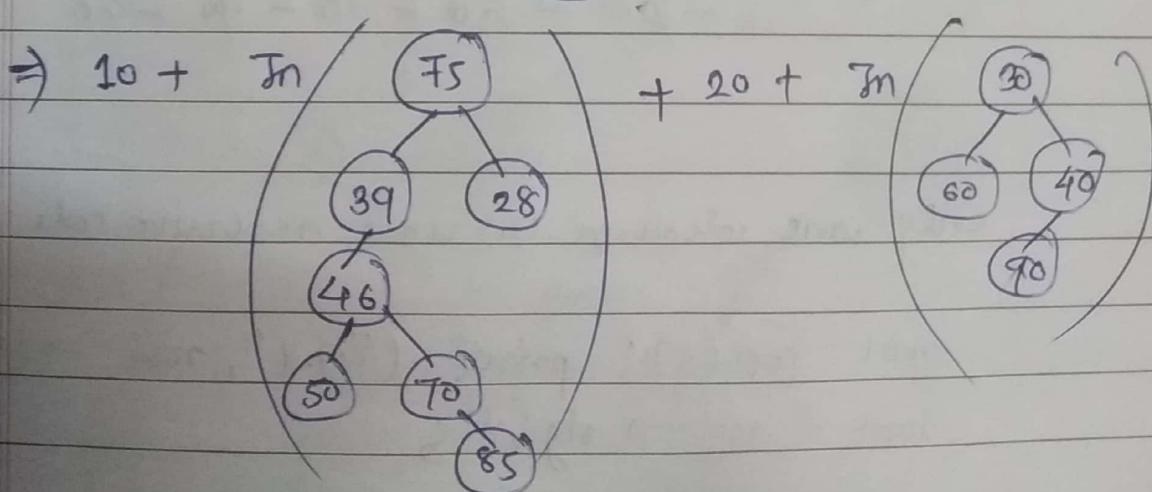
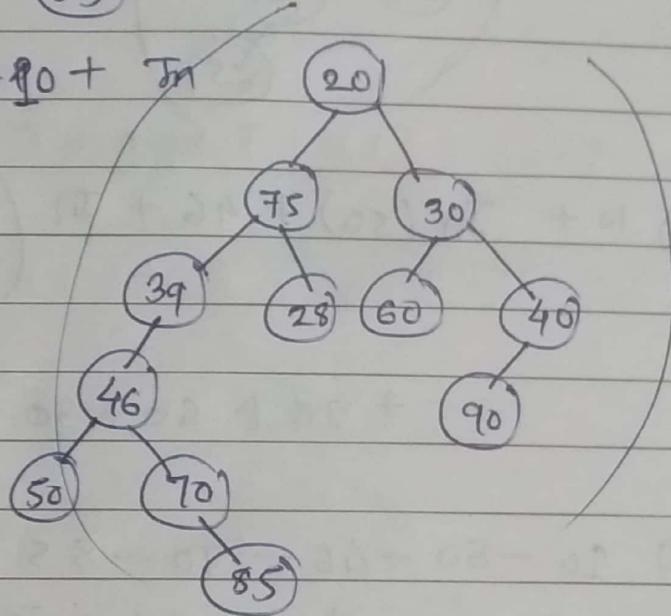
$$\Rightarrow \text{In}((D, G)) + B + \text{In}((E, F)) + A + \text{In}((H, I, J)) + C + K$$

$$\Rightarrow D + G + B + E + F + A + I + H + J + C + K$$

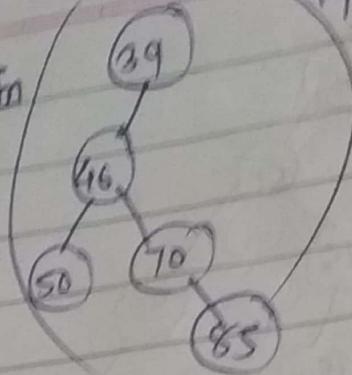
$$\Rightarrow D - G - B - E - F - A - I - H - J - C - K$$



$$\Rightarrow \text{In}(\text{null}) + 10 + \text{In}$$



$$\Rightarrow 10 + T_n \quad + 75 + T_n(28) + 20$$



$$+ T_n(60) + 30 + T_n \quad + 40$$

$$\Rightarrow 10 + T_n \quad + 39 \quad + 75 + 28 + 20 + 60 \\ + 30 + T_n(90) + 40$$

$$\Rightarrow 10 + T_n(50) + 46 + T_n \quad + 39 \quad + 75 + 28 \\ (70) \quad (85)$$

$$+ 20 + 60 + 30 + 90 + 40$$

$$\Rightarrow 10 - 50 - 46 - 70 - 85 - 39 - 75 - 28 \\ - 20 - 60 - 30 - 90 - 40$$

only one change in non recursive code:

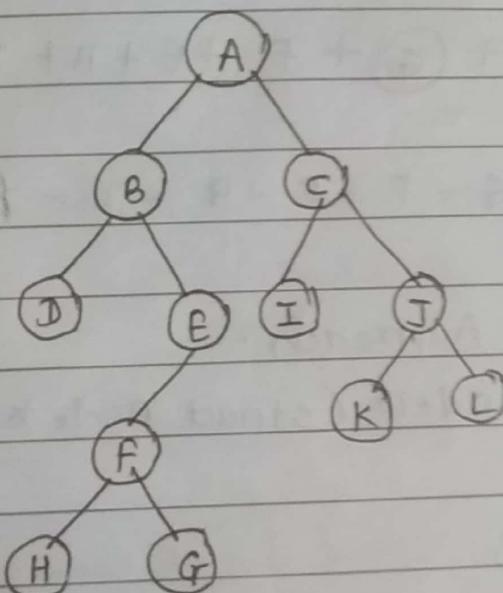
```
root = pop(s); printf ("l.d", root->left);
```

```
root = root->right;
```

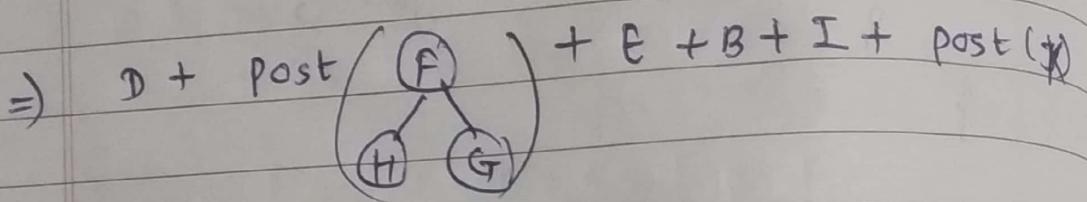
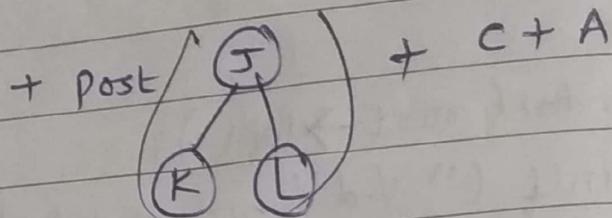
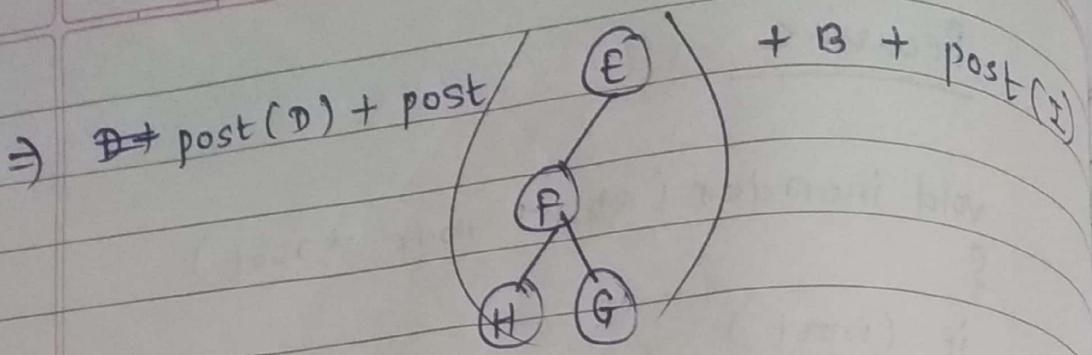
Recursive

```
void inorder (struct node *root)
{
    if (root)
    {
        inorder (root->left);
        printf ("% .d", root->data);
        inorder (root->right);
    }
}
```

* Postorder Traversal * (L-R-P)



=) post + post + A



$+ \text{post}(L) + J + C + A$

$\Rightarrow D + H + G + F + E + B + I + K + L + C + A$

$\Rightarrow D - H - G - F - E - B - I - K - L - C - A$

\Rightarrow Recursive Approach.

void postorder (struct node *root)

{

 if (root)

{

 postorder (root->left);

 postorder (root->right);

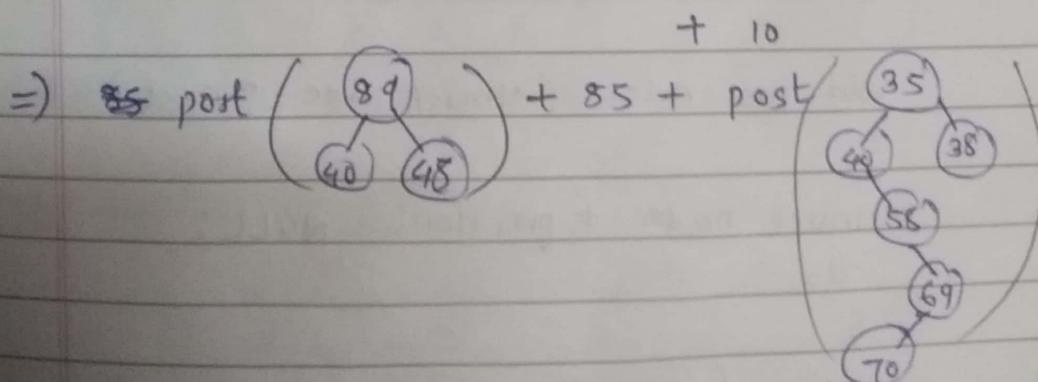
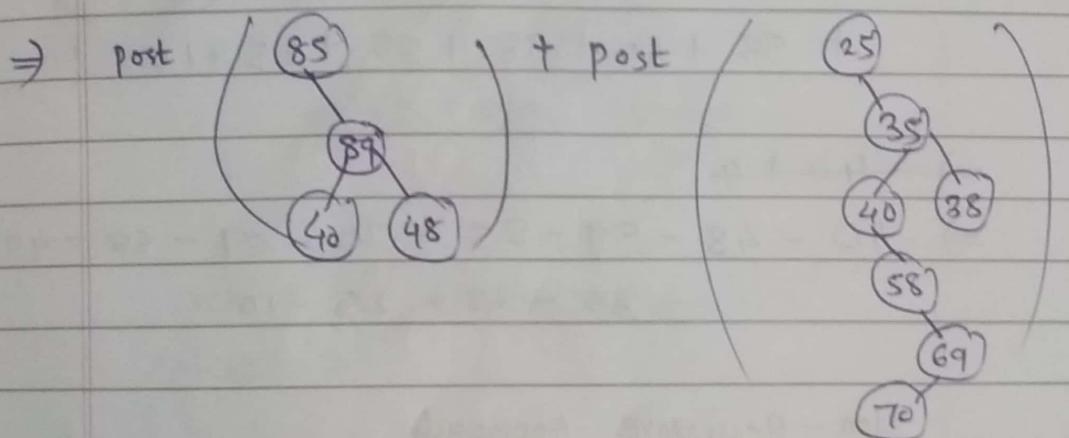
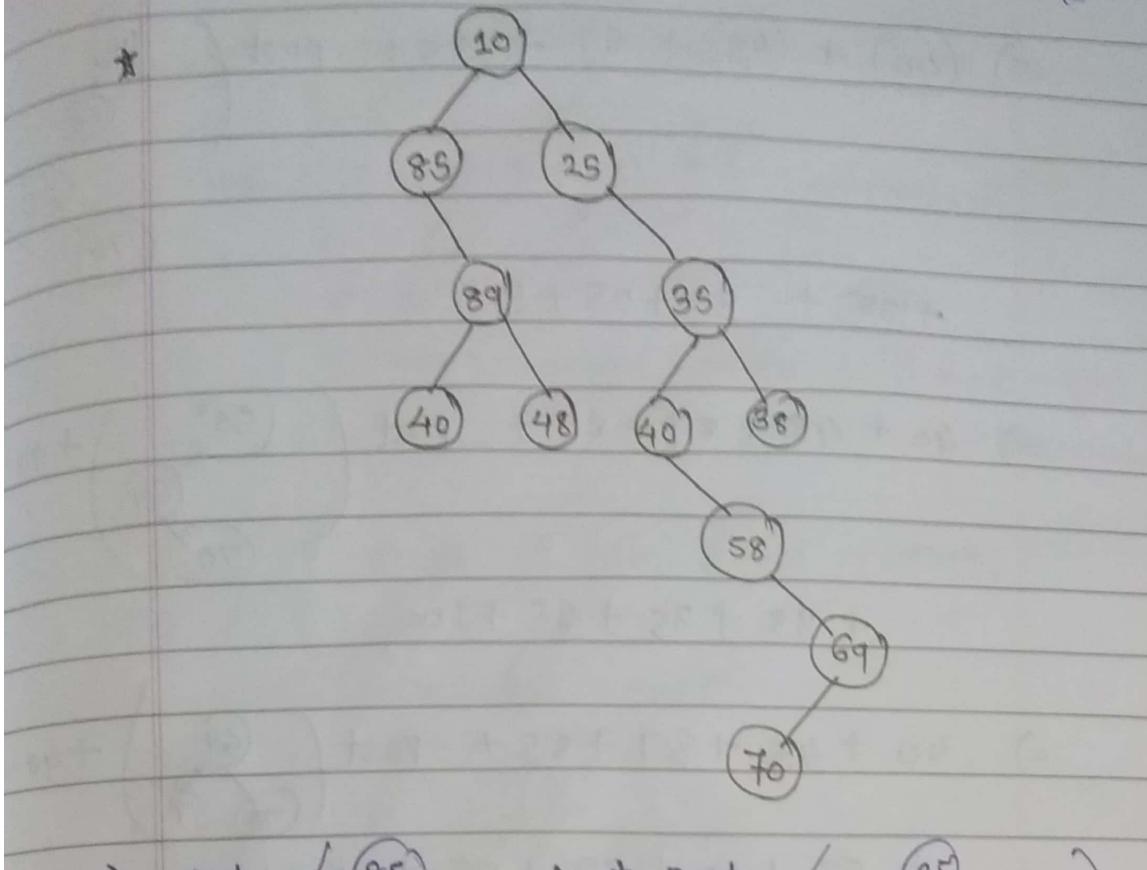
 printf ("% .d", root->data);

}

}

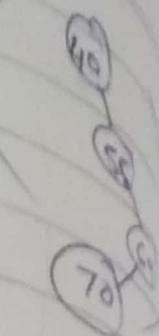
$$40 - 48 = 89 - 85 - 25 - 70 = 69 - 58 - 40 - 38$$

DATE: - 35 - 25
- 10



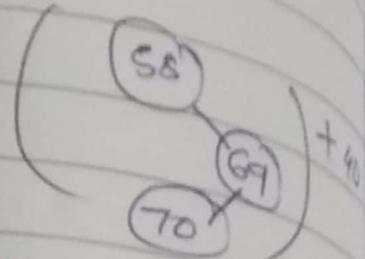
+ 25 + 10

$$\Rightarrow 40 + 48 + 89 + 85 + \text{post}$$



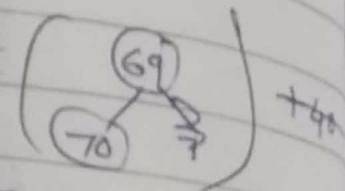
$$+ 38 + 35 + 25 + 10$$

$$\Rightarrow 40 + 48 + 89 + 85 + \text{post}$$



$$+ 38 + 35 + 25 + 10$$

$$\Rightarrow 40 + 48 + 89 + 85 + \text{post}$$



$$+ 58 + 40 + 38 + 35 + 25 + 10$$

$$\Rightarrow 40 + 4$$

$$\Rightarrow 40 - 48 - 89 - 85 - 70 - 69 - 58 - 40 \\ - 38 - 35 - 25 - 10 .$$

Non-Recursive Approach

```
void postorder (struct node *root)
```

```
{ struct node *previous = NULL;
```

```
do
```

```
while (root != NULL)
```

```
{
```

```

push (s, root);
root = root->left;
}

while (root == NULL && !emptystack(s))
{
    root = top(s);
    if (root->right == NULL || root->right
        == previous)
    {
        printf ("%d", root->data);
        pop (s);
        previous = root;
        root = NULL;
    }
    else
        root = root->right;
}
while (!emptystack(s));
delete (s);
}

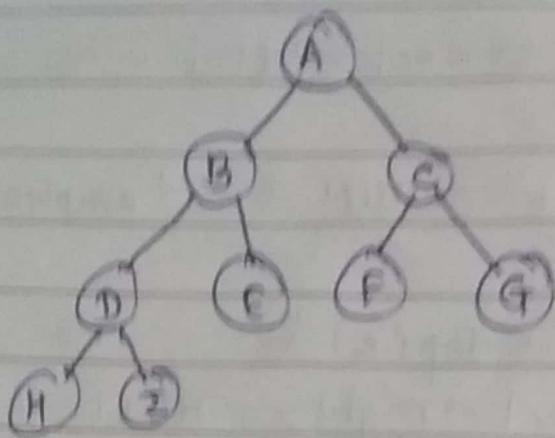
```

* Construct binary trees

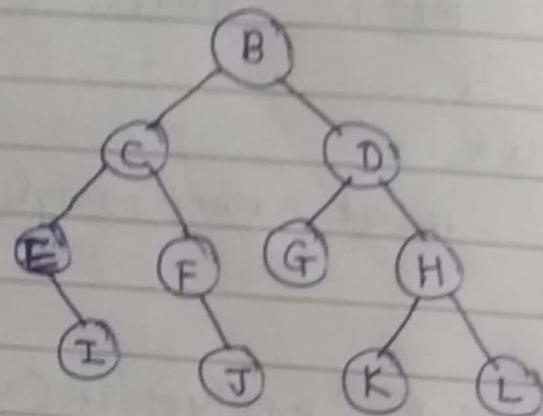
① Pre - A B D H I E C F G



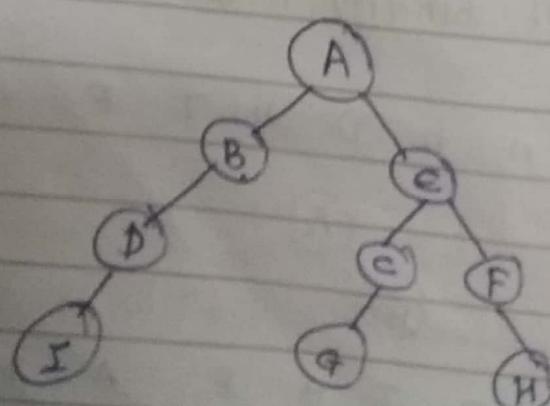
In - H D I B E A P C G



② post → I E J F C G K L H D
 In → E I C F J B G D K H L



③ pre - A B D I E C G F H
 post - I D B G C H F E A
 PER ↙
 LEP ↘



In pre order & post order, trees are not unique

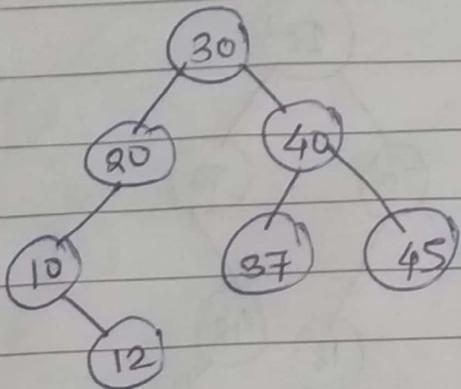
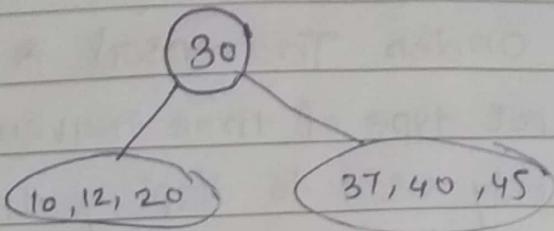
PAGE NO.	
DATE	/ /

LPR *

Inorder - 10, 12, 20, 30, 37, 40, 45

PLR

pre order - 30, 20, 10, 12, 40, 37, 45

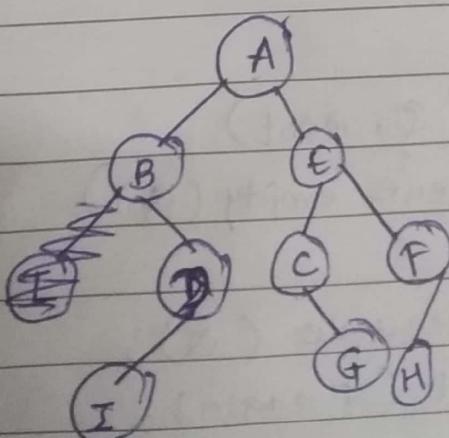


LPR *

Inorder - B, I, D, A, C, G, E, H, F

LRP

postorder - I, D, B, G, C, H, F, E, A

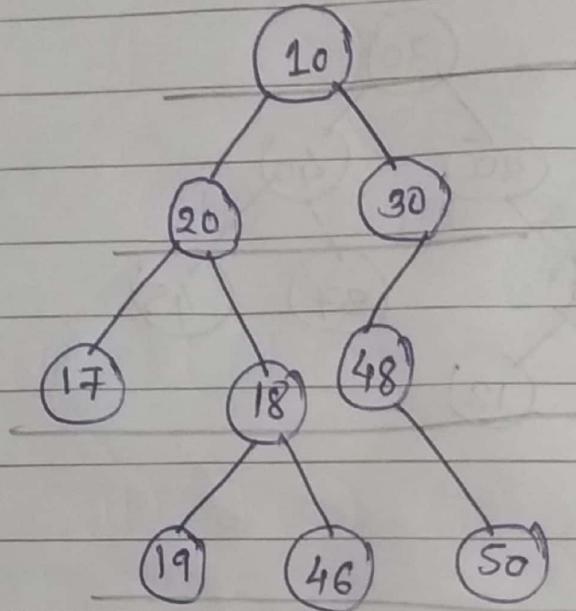


10, 18, 30, 4, 16, 19, 18, 17

* Level Order Traversal *

(It is not type of tree traversal.

It is only used to display tree level wise.



=> 10, 20, 30, 17, 18, 48, 19, 46, 50

+insert

enqueue (Q, root)

while (!queue empty (Q))

{

p = dequeue (Q);

print (p->data);

if (p->left != NULL)

enqueue (Q, p->left)

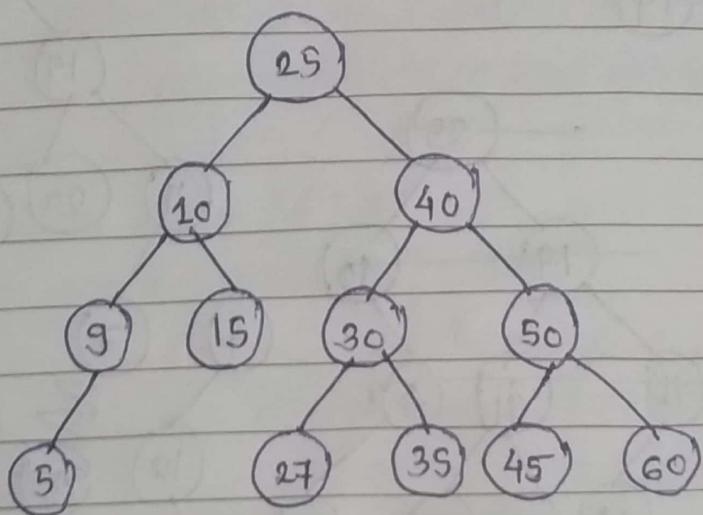
if (p->right != NULL)

enqueue (Q, p->right);

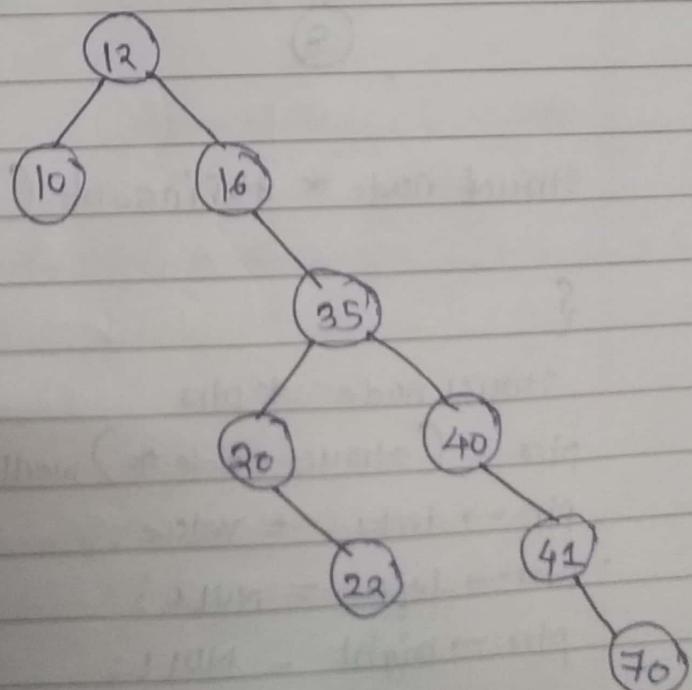
}

* BST (Binary Search Tree) *

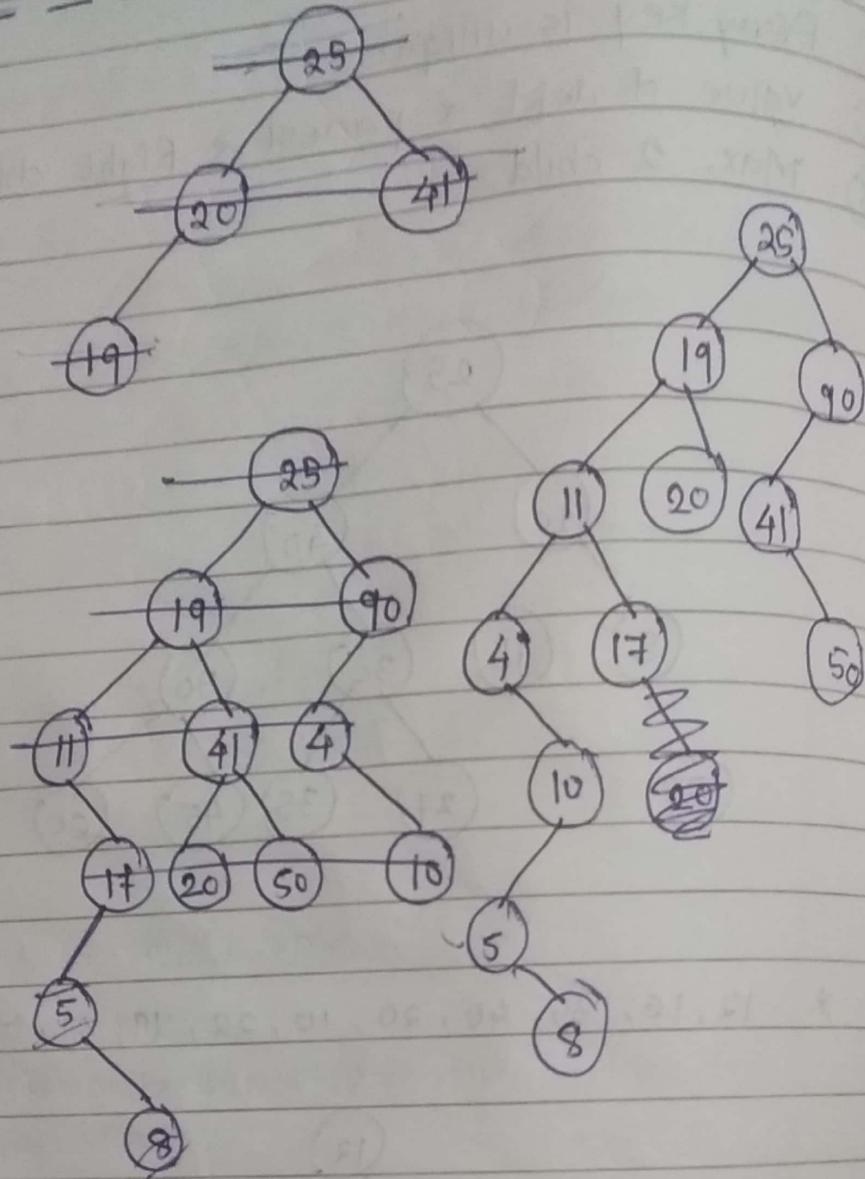
- (1) Every key is unique.
- (2) value of left < parent < Right child
- (3) Max. 2 child



* 12, 16, 35, 40, 20, 10, 22, 19, 41, 70



* 25, 90, 19, 11, 41, 4, 17, 20, 10, 50, 5, 8



struct node * BSTinsert (struct node * root,
int val)

{

 struct node * ptr;

 ptr = (struct node *) malloc (sizeof (struct node));

 ptr->data = value;

 ptr->left = NULL;

 ptr->right = NULL;

Program to find max value from BST.

max val	
max	/ /

if (root == NULL)

{

root = ptr;

}

else

{

nodeptr = root;

if (nodeptr->data < val)

nodeptr = nodeptr->right;

else

nodeptr = root;

while (nodeptr != NULL)

{ parent = nodeptr;

if (nodeptr->data < val)

nodeptr = nodeptr->right;

else

nodeptr = nodeptr->left;

if (val < parent->data)

parent->left = ptr;

else

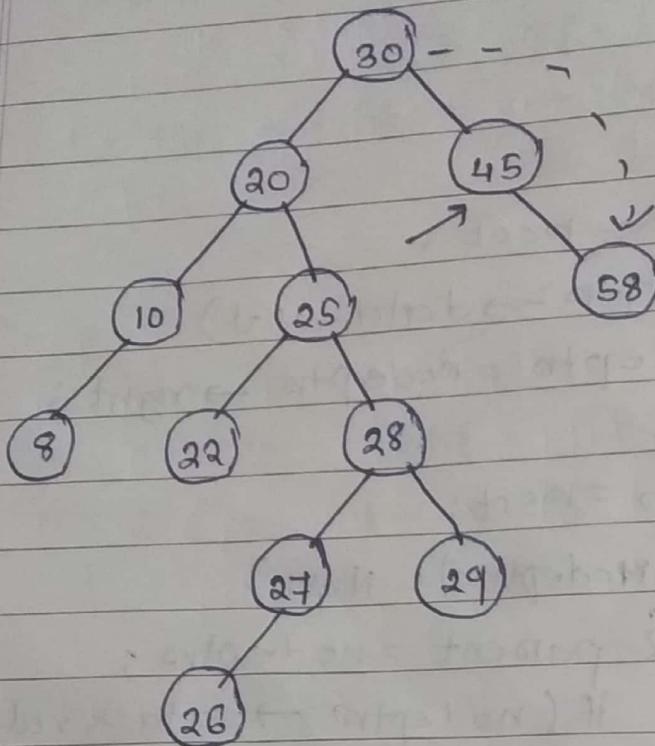
parent->right = ptr;

}

return (root);

}

* Deletion from BST *



- ① Tree is empty or not.
- ② Search value
- ③ If present check node has one child / two child.
- ④ If one child []
- ⑤ If two child []

Non-Recursive Approach.

```
struct node * delete BST ( struct node *root,  
                           int val )
```

```
struct node * cur, * parent;  
if (root == NULL)
```

{

```

printf ("Tree is empty");
return (root);
}
cur = root;
while (cur->data != val && cur != NULL)
{
    parent = cur;
    if (val ->
        if (val < cur->data)
            cur = cur->left;
        else if (val > cur->data)
            cur = cur->right;
    }
    if (cur == NULL)
    {
        printf ("value is not present");
        return ;
    }
    if (cur->left == NULL)
        ptr = cur->right;
    else if (cur->right == NULL)
        ptr = cur->left;
    else
    {
        suc = cur->right;
        psuc = cur;
        while (suc->left != NULL)
        {
            psuc = suc;
            suc = suc->left;
        }
        psuc->left = suc;
        suc->left = NULL;
    }
}

```

psuc → parent of successor

PAGE NO.	/ /
DATE	/ /

else

{

suc → left = cur → left ;

psuc → left = suc → right ;

suc → right = cur → right ;

}

ptr = suc ;

}

if (parent → right == cur)

parent → right = ptr ;

else if (parent → left == cur)

parent → left = ptr ;

return

return (root) ;

}

* Recursive Approach *

struct node * delete (struct node * root,

int val)

{

struct node * temp ;

if (root == NULL)

printf ("No element ");

if (val < root → data)

root → left = delete (root → left, val);

else if (val > root → data)

root → right = delete (root → right, val);

else {

if ($\text{root} \rightarrow \text{left} \neq \text{NULL}$ & & $\text{root} \rightarrow \text{right} \neq \text{NULL}$)
 {

 temp = findmax ($\text{root} \rightarrow \text{left}$);

 root \rightarrow data = temp \rightarrow data;

 root \rightarrow left = delete ($\text{root} \rightarrow \text{left}$,
 temp \rightarrow data);

 }

~~else~~ else

{

temp = root;

if ($\text{root} \rightarrow \text{left} == \text{NULL}$)

 root = root \rightarrow right;

if ($\text{root} \rightarrow \text{right} == \text{NULL}$)

 root = root \rightarrow left;

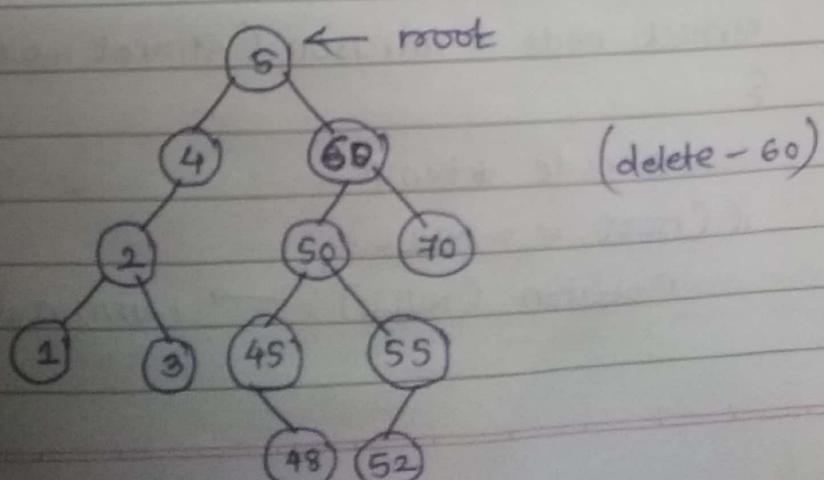
free (temp);

}

return (root);

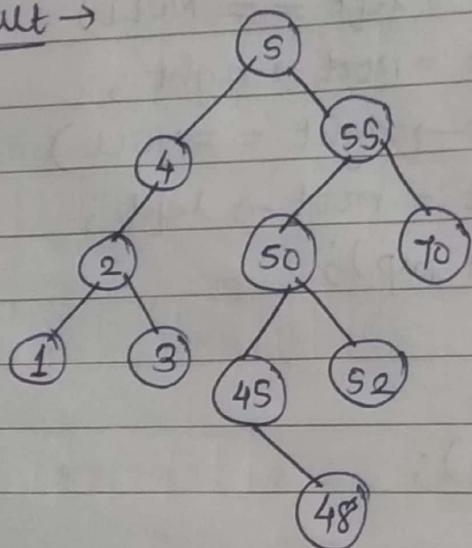
}

Explanation



- ① root = 5, val = 60
 $s \rightarrow \text{right} = \text{delete}(60, 60)$
- ② root = 60, val = 60
 $\text{temp} = \text{findmax}(60 \rightarrow \text{left})$
 $\text{temp} = 55$
 $55 \rightarrow \text{left} = \text{delete}(50, 55)$
- ③ root = 50, val = 55 \Rightarrow $50 \rightarrow \text{right} = \text{delete}(55, 55)$
- ④ root = 55, val = 55
root = 52

Result →



* Recursive code for mirror image of tree *

struct node * mirror (struct node * root)

{

 struct node *temp;

 if (root == NULL)

 return (NULL); \rightarrow termination case.

else {

temp = root;

reaching to leaf node [mirrors (root → left);
mirrors (root → right);

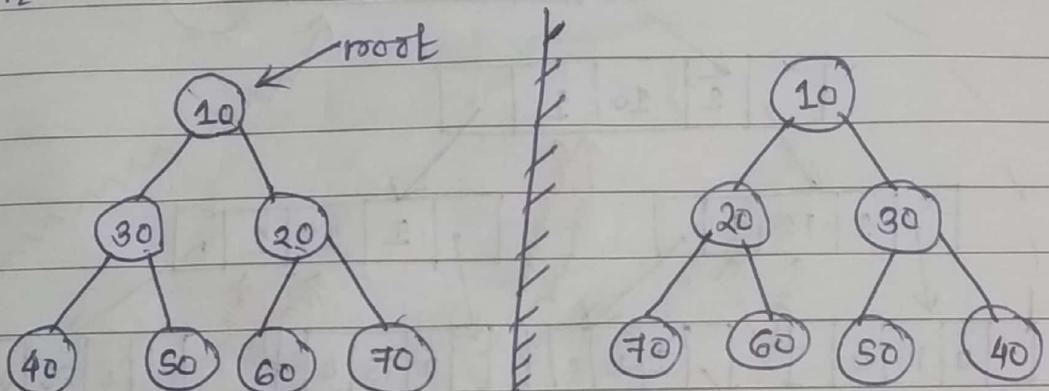
swapping left and right [temp = root → left;
root → left = root → right;
root → right = temp;

}

return (root);

}

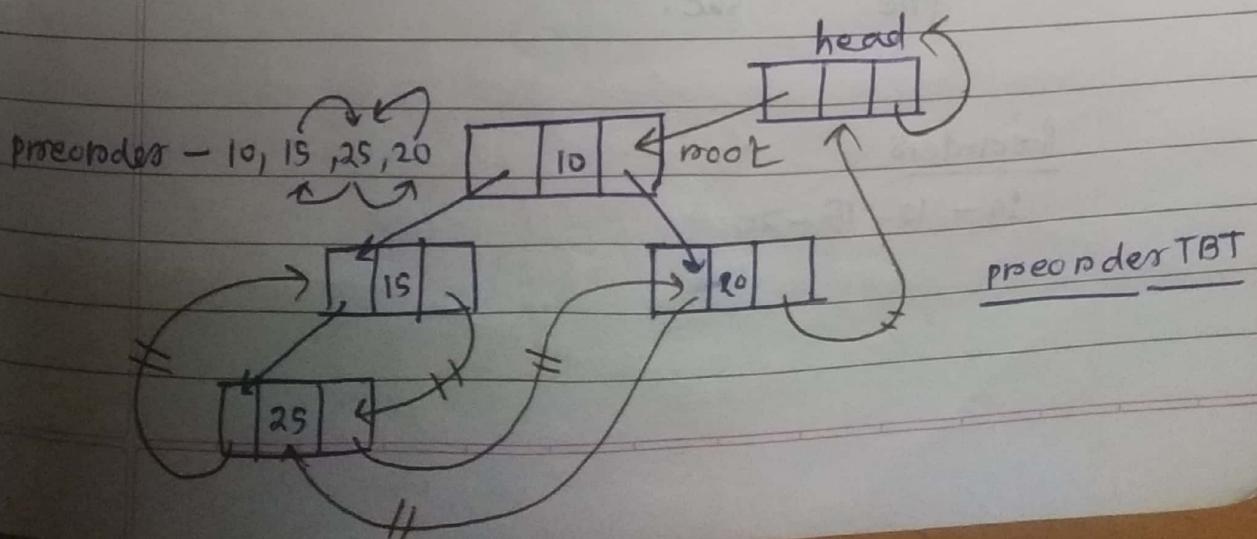
2

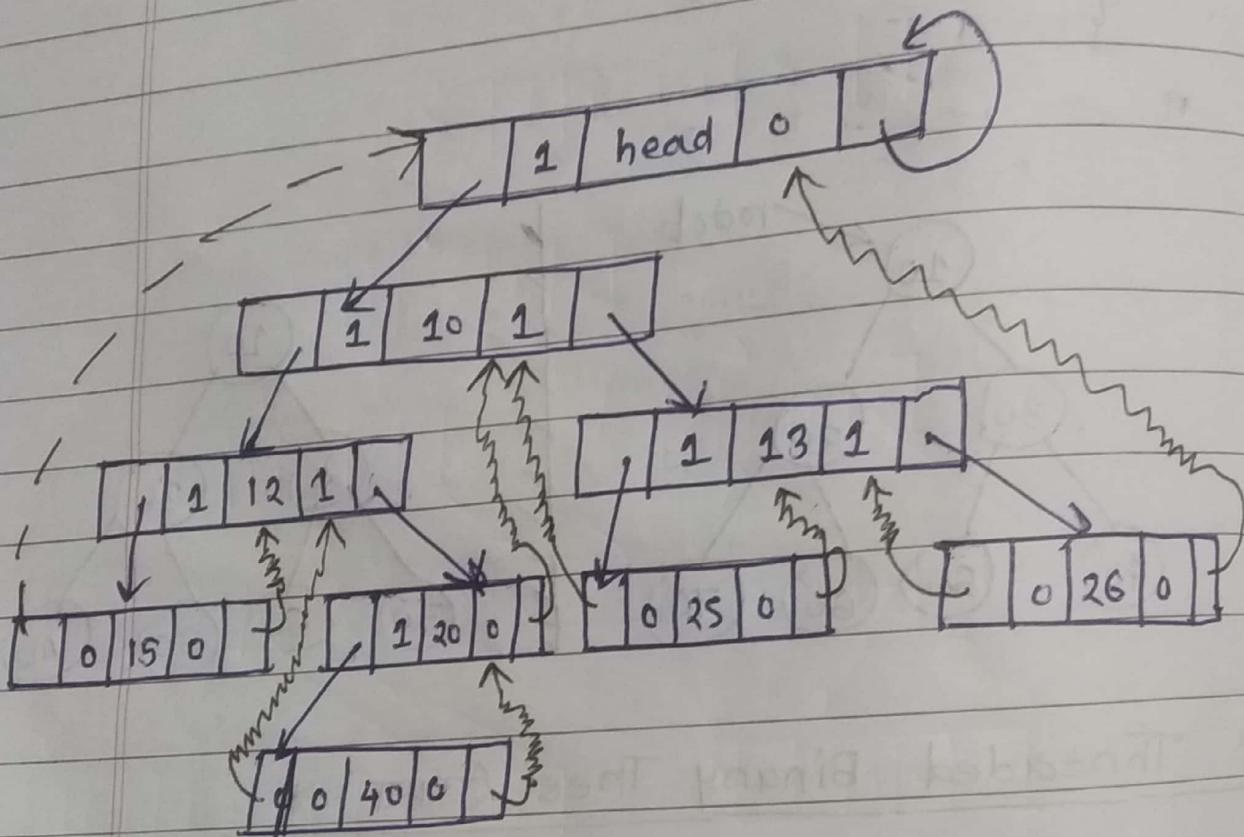
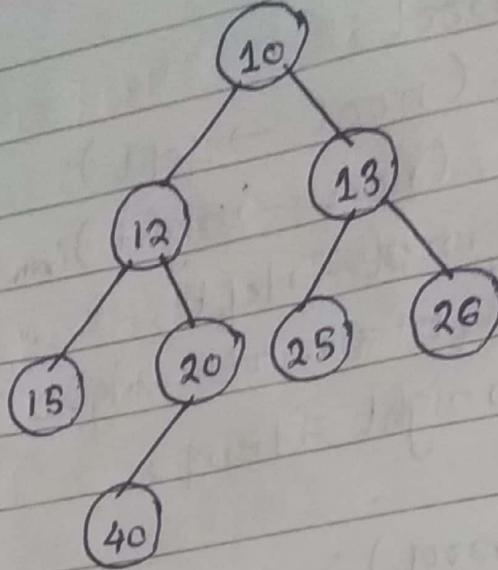


* Threaded Binary Tree (TBT) *

* A tree of 'n' nodes have 'n+1' null links.

* 'n+1' links are utilized links.





Inorder Traversal

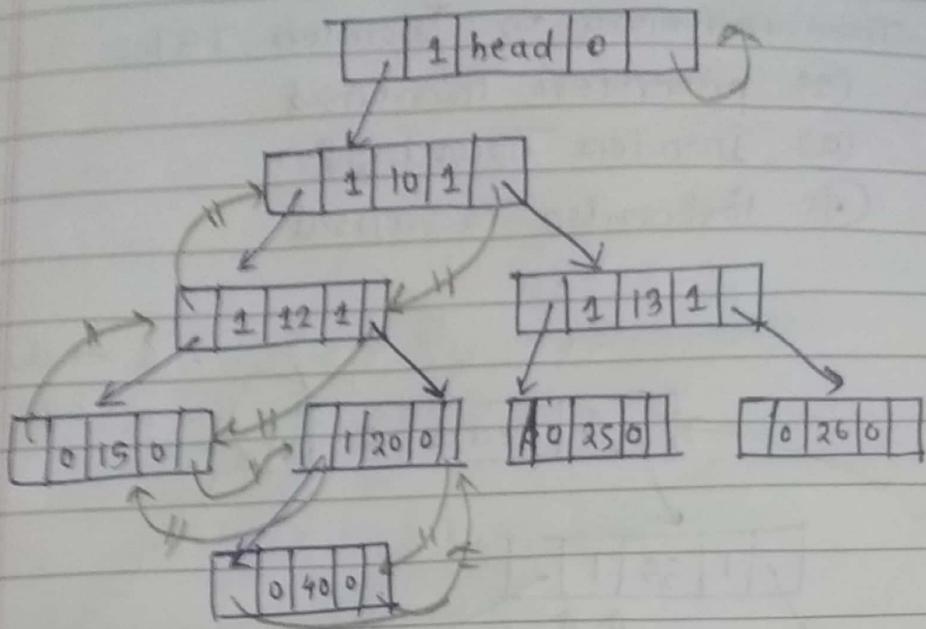
- 15 - 12 - 40 - 20 - 10 - 25 - 13 - 26 -
 pre suc.

Preorders

10 - 12 - 15 - 20

10 - 12 - 15 - 20 - 40 - 13 - 25 - 26

Preorder TBT



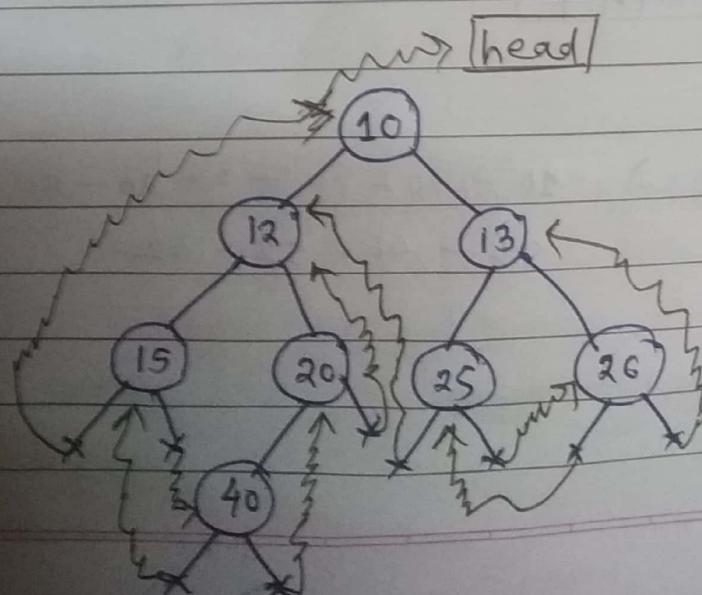
Link/ Thread	L Bit	Data	R Bit	Link/ thread
--------------	-------	------	-------	--------------

L bit / R bit \Rightarrow 1 \Rightarrow dlink to child

L bit / R bit \Rightarrow 0 \Rightarrow Threading.

Postorder

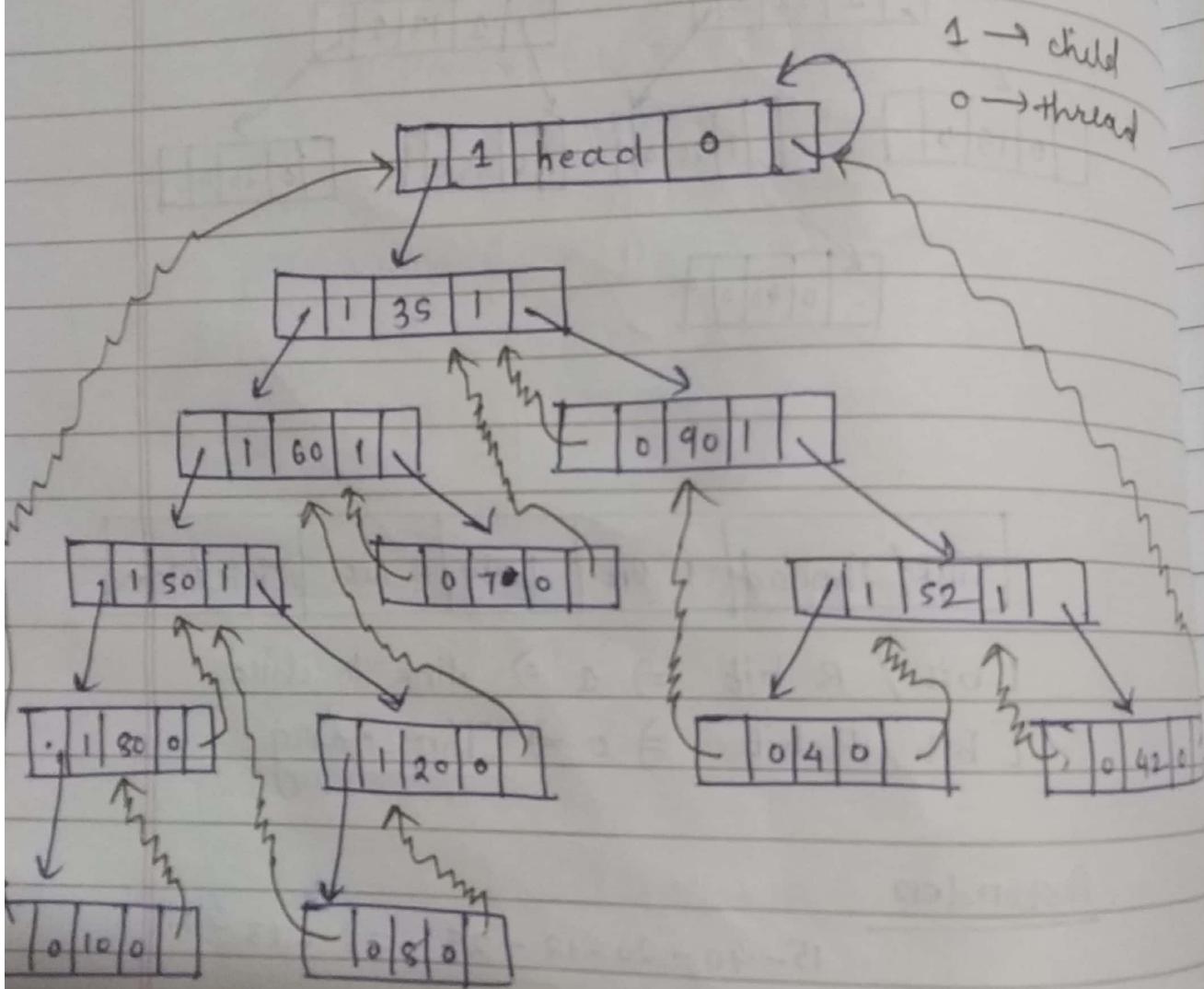
15 - 40 - 20 - 12 - 25 - 26 - 13 - 10



* Inorder TBT *

Tree traversal on Inorder TBT :-

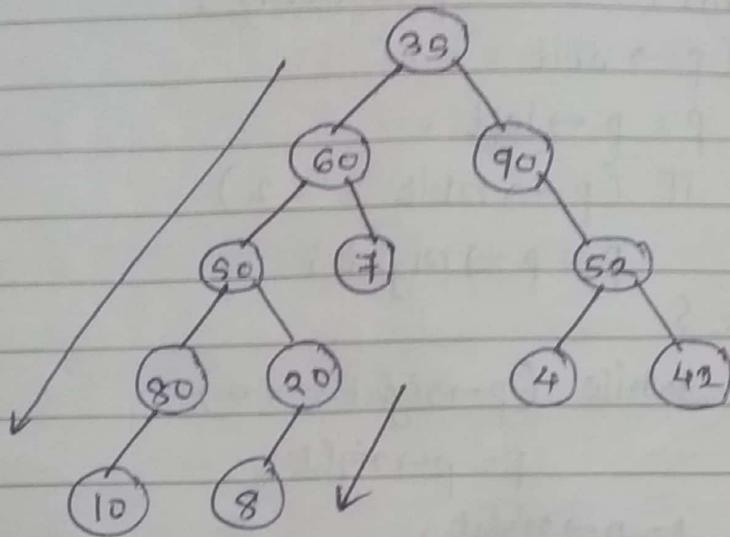
- ① preorders Traversal
- ② Inorders Traversal
- ③ Postorders Traversal



Inorders \Rightarrow 10 - 80 - 50 - 8 - 20 - 60 - 7 - 35 -
90 - 41 - 52 - 42

(1) Preorders Traversal on Inorder TBT
Parent → Left - Right

35 - 60 - 50 - 80 - 10 - 20 - 8 - 7 - 90 - 52 -
41 - 42



Algorithm

1) if 1 → child if 0 → thread. Go back

Algorithm

while (node is not head)

{

 display node

 if (left child parent)

 goto left node

 else if (right child parent)

 goto right

 else (till not getting parent child)

 goto right

}

```

void preorders ( struct node *head)
{
    struct node *p;
    p = head -> left;
    while ( p != head )
        {
            printf ( "%d", p-> data );
            if ( p-> lbit == 1 )
                p = p-> left;
            else if ( p-> rbit == 1 )
                p = p-> right;
            else
                {
                    while ( p-> right == 0 )
                        p = p-> right;
                }
        }
}

```

② Inorders Traversal On Inorders TBT

Left - Parent - Right

10 - 80 - 80 - 8 - 20 - 60 - 7 - 35 - 90 - 4 - 52 - 42

~~② if : child = 1~~

~~if many people are present in the entries then you go for skip it~~

```

void inorders ( struct node *head)
{

```

struct node *T;

T = head -> left;

```

while ( T->lbit == 1 )
    T = T->left;
while ( T != head )
{
    printf (" %d", T->data);
    T = inorder_succ(T);
}

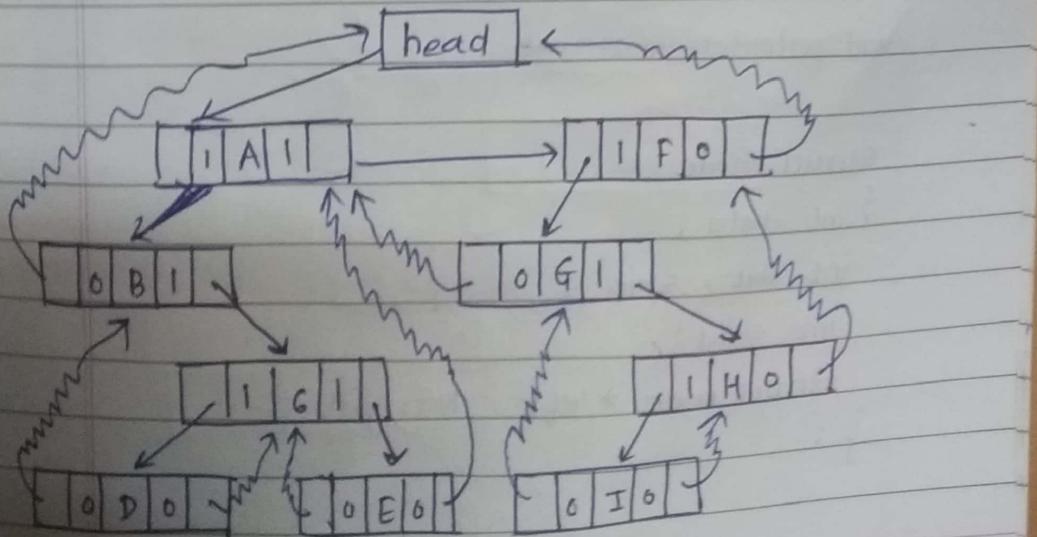
```

```

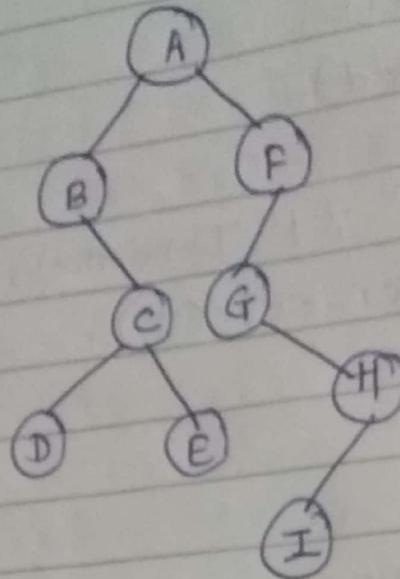
struct node * inorder_succ ( struct node *T )
{
    if ( T->lbit == 0 )
        return ( T->right );
    T = T->right;
    while ( T->lbit == 1 )
        T = T->left;
    return ( T );
}

```

* Postorder Traversal on Inorder TBT *

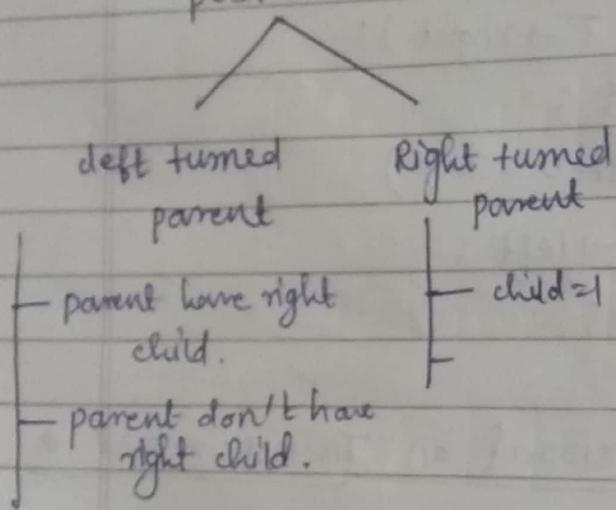


L - R - P



D - B - C - B - I - H
G - F - A.

- ① Find out 1st node to display.
postorder succ.



child = 1 → right
child = 0 → left.

wid postorder (struct

```
struct node  
{ int data ;  
    int lbit , rbit ;  
    int child ;  
    struct node *left , *right ;  
};
```

void post_traversal (struct node *head)

{ struct node *T = head → left ; }

while (T → lbit == 1 || T → rbit == 1),
 {

 if (T → lbit == 1)

 T = T → left ;

 else

 T = T → right ;

}

 while (T != head)

 { printf ("%.d", T → data) ;

 T = post_succ (T) ;

}

}

struct node * postor_succ (struct Node *T)

{

 if (T → child == 1)

 {

 while (T → lbit == 1)

 T = T → left ;

 return (T → left) ;

}

 else

 { if (T → rbit == 1)

 Reach to
 parent ←

 T = T → right ;

 T = T → right ;

 if (T → rbit

 if (T → rbit == 0)

 return (T) ;

] — no right child .

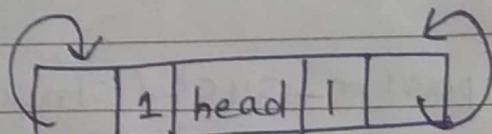
 + else { T = T → right ;

Return
1st node
Right
subtree.

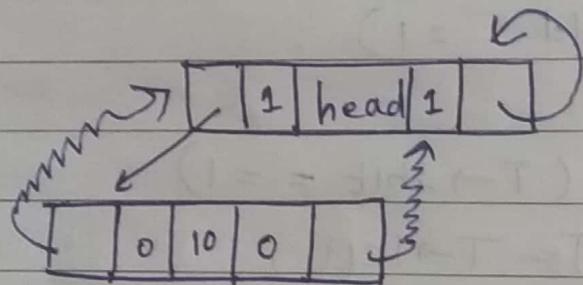
```
while ( T->lbit == 1 || T->rbit == 1 )  
{  
    if ( T->lbit == 1 )  
        T = T->left ;  
    else  
        T = T->right ;  
}  
return (T);  
}
```

* Creating Inorder TBT and Inserting Node in
Inorder TBT *

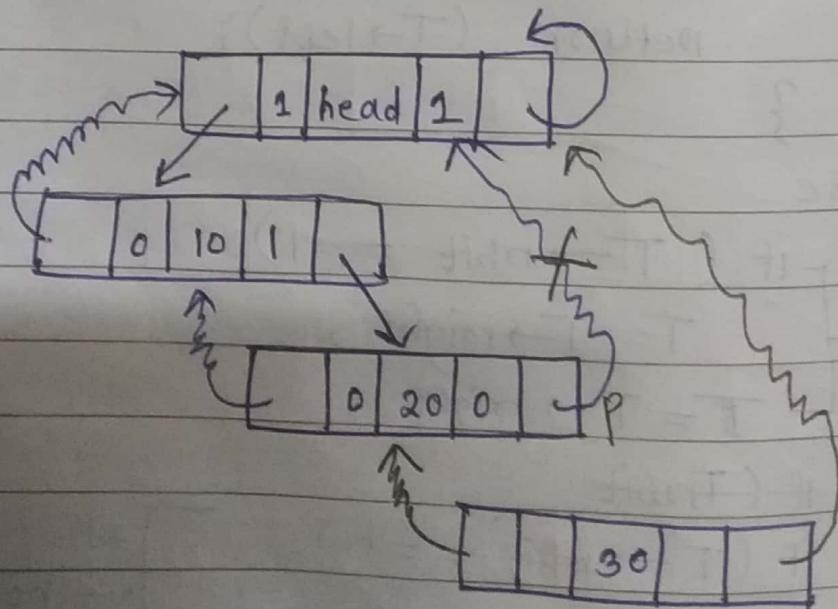
①



②



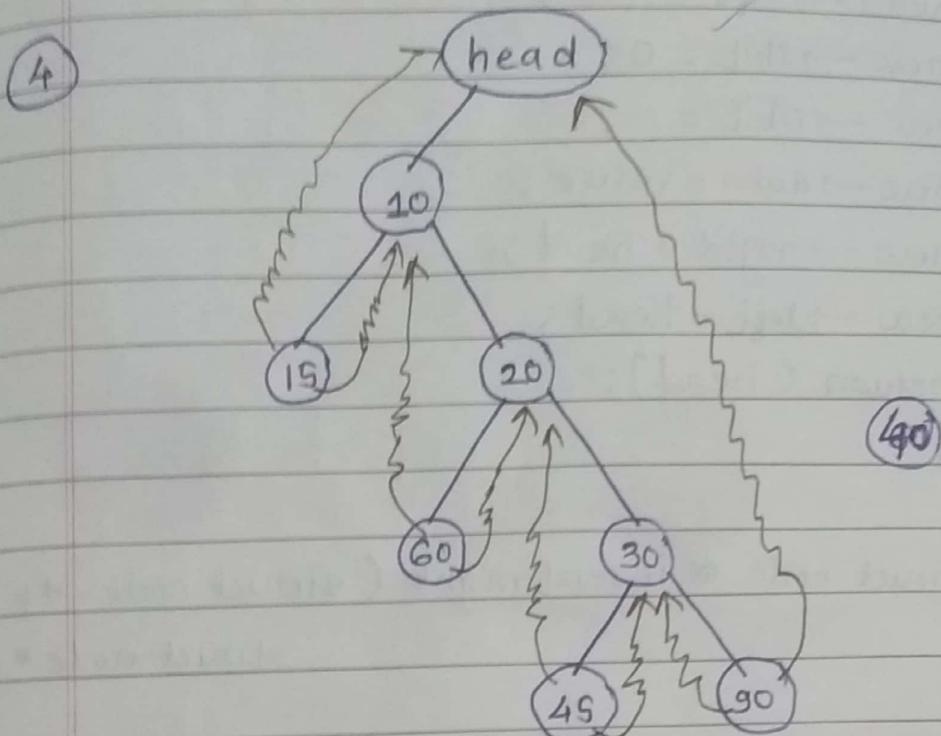
③



struct Node

```

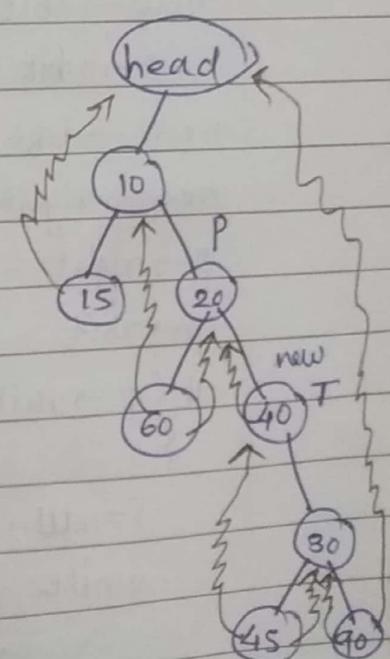
{ int data;
  int lbit, rbit;
  struct node *left, *right;
}
```



main ()

```

{
  struct node *head;
  head->lbit = 1;
  head->rbit = 1;
  head->left = head;
  head->right = head;
  head = create(head);
}
```



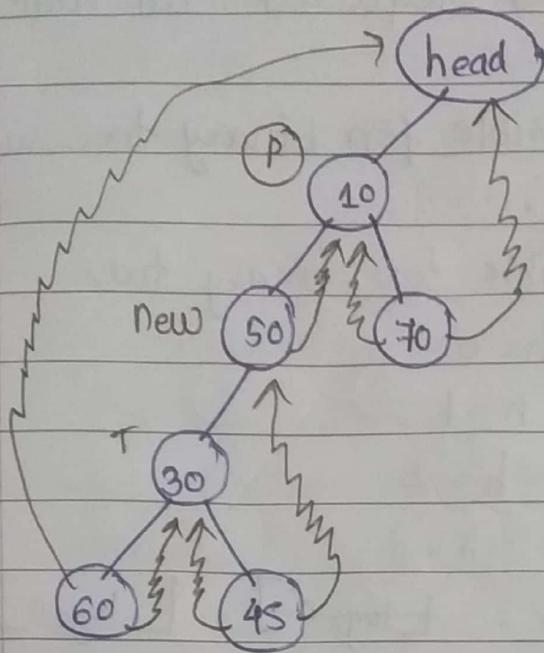
```
struct node * create ( struct node * head )  
{  
    int val ;  
    struct node * new ;  
    new = ( ) malloc ( ) ;  
    head -> left = new ;  
    new -> lbit = 0 ;  
    new -> rbit = 0 ;  
    new -> data = value ;  
    new -> right = head ;  
    new -> left = head ;  
    return ( head );  
}
```

`struct node * insert_might (struct node * p,`

{
new → lbit = 0 ;
new → rbit = p → rbit ;
new → left = p ;
new → right = p → right ;
p → right = new ;
p → rbit = 1 ;
if (p → rbit == 1)
{

```
T = root->right  
while (T->left == 1)  
    T = T->left;  
T->left = new;  
}  
}
```

$p \rightarrow \text{rabit} = 1;$
}



void insertLeft (struct node *p, struct node *new)
{

 struct node *T ;

 new \rightarrow rabit = 0;

 new \rightarrow lbit = p \rightarrow lbit ;

 new \rightarrow right = p;

 new \rightarrow left = p \rightarrow left ;

 p \rightarrow left = new ;

 if (p \rightarrow lbit == -1)

 T = new \rightarrow left ;

 while (T \rightarrow rabit == -1)

 T = T \rightarrow right ;

 T \rightarrow right = new ;

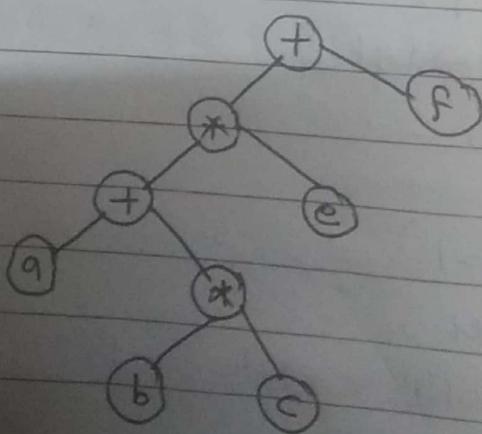
}

- Recursive approach is always best for traversal
- Time complexity for non-recursive approach over threaded memory binary tree is $O(n \log n)$
- more memory is required in non-recursive than recursive
- Max. height possible for binary tree with n nodes is ' $n - 1$ '.
- Min. height possible for binary tree
 - $n=1 \quad h=0$
 - $n=2, 3 \quad h=1$
 - $n=4, 5, 6, 7 \quad h=2$
 - $n=8, 9, 10, \dots, 15 \quad h=3$
- min height = $\lceil \log_2 n \rceil \quad \lfloor \log_2 n \rfloor$

* Application of Trees *

① Expression Tree :-

$$(a+b * c) * e + f$$



Page No.	
Date	

PLR

preorder - $+ * + a * b c e f$

LPR

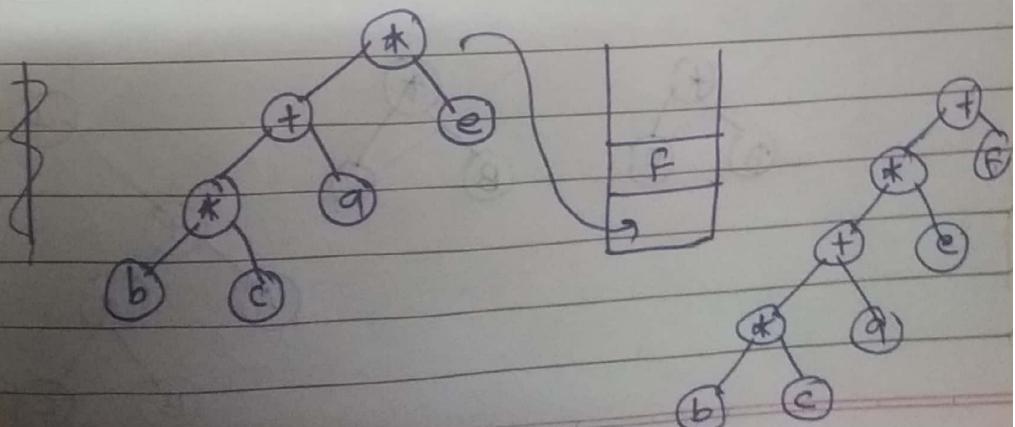
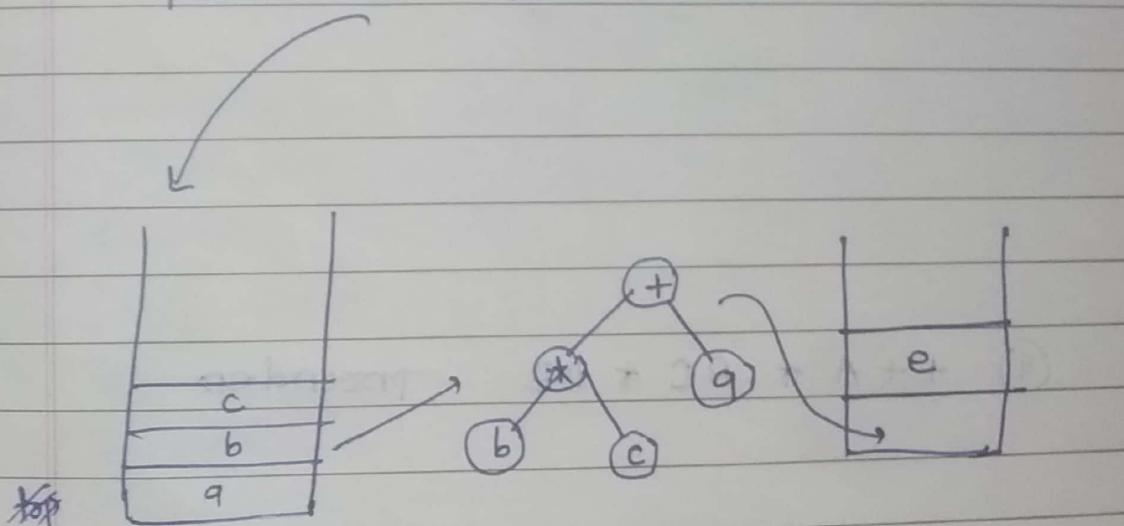
inorder - $a + b * c * e + f$

LRP

postorder - $a b c * + e * f +$ $(a + b * c) * e + f$ prefix = $+ * + a * b c e f$ (preorder)postfix = $a b c * + e * f +$ (postorder)

* If preorder & postorder is given, find expression tree.

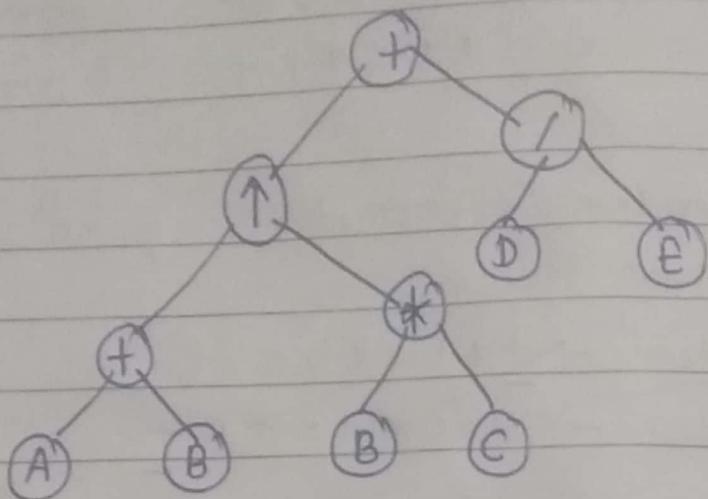
~~preorder - $+ * + a * b c e f$~~ RLR
~~postorder - $a b c * + e * f +$~~ LRP

preorder - $+ * + a * b c e f$ postorder - $a b c * + e * f +$ 

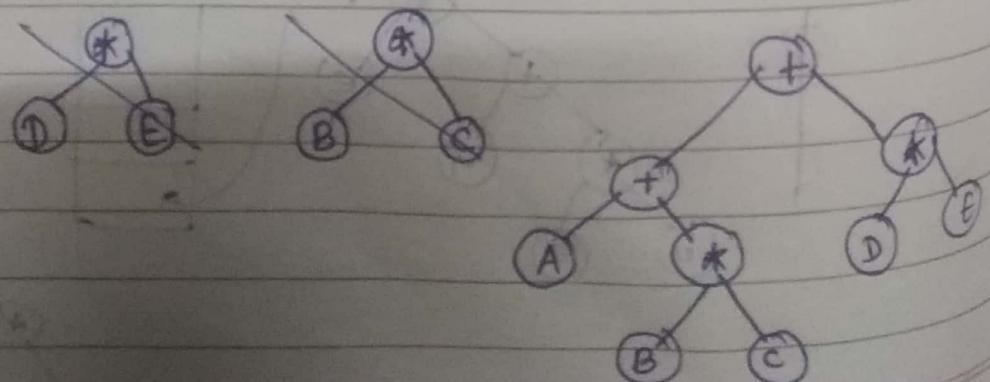
$\uparrow \rightarrow$ highest preference

① $(A+B) \uparrow (B * C) + D / E$

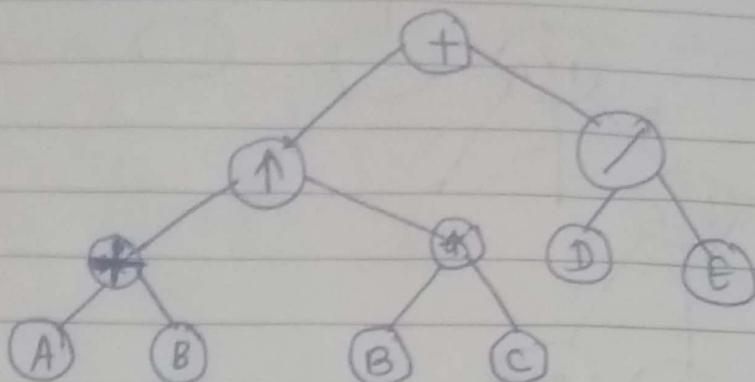
Inorder



② ++ A * BC * DE preorders

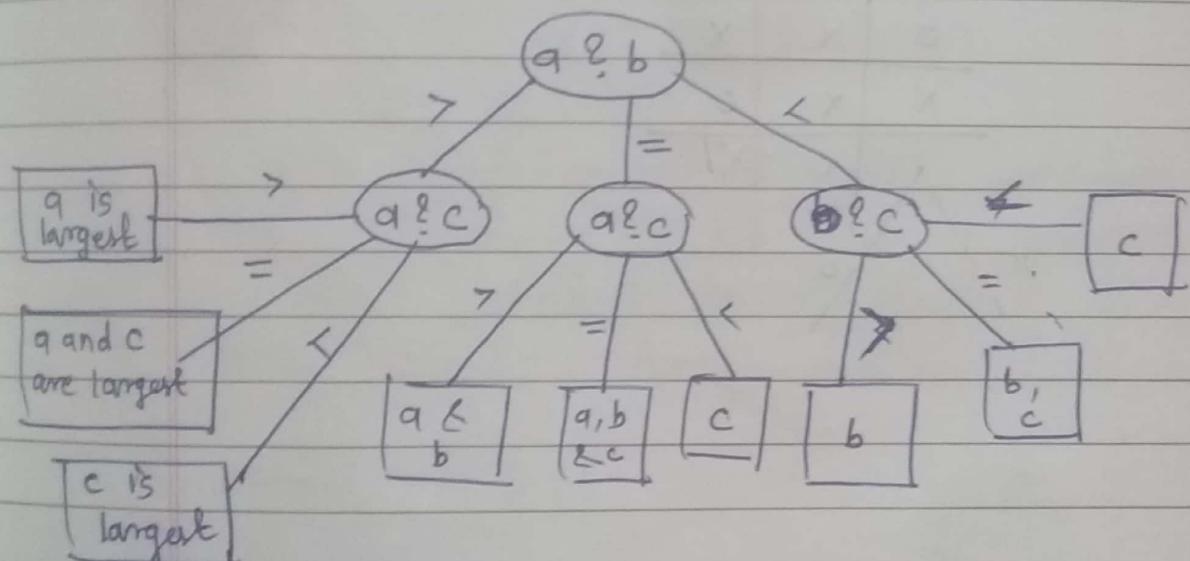


③ $AB + BC \xrightarrow{*} \uparrow DE / +$
postorder



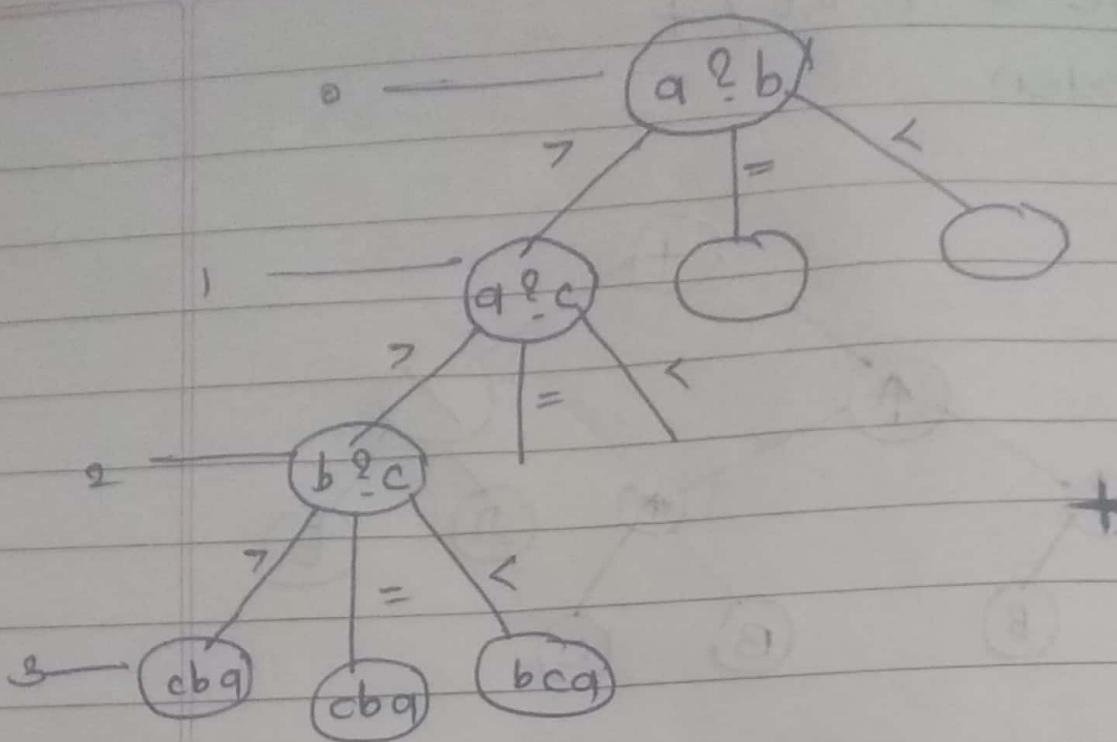
* Decision Tree *

Largest



Decision tree to sort a, b, c in ascending orders.

a	b	c
a	c	b
b	c	a
b	a	c
c	a	b
c	b	a



* Game Tree *

o	x	x
x	x	o
	o	x