Statistics Part 2: Assignment Questions

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1. What is hypothesis testing in statistics?

Hypothesis testing is a fundamental concept in inferential statistics used to make decisions or draw conclusions about a population based on sample data. It involves:

- Setting up two competing hypotheses: the **null hypothesis (H_0)** and the **alternative hypothesis (H_a)**.
- Using sample evidence to determine whether there is enough evidence to reject the null hypothesis in favor of the alternative.
- Calculating a **test statistic** and comparing it to a **critical value** or a **p-value** to make a decision.

2. What is the null hypothesis, and how does it differ from the alternative hypothesis?

- **Null Hypothesis (H_0):** A statement of no effect, no difference, or no relationship. It is the default assumption that any observed difference is due to random chance. *Example: The population mean is equal to a specific value.*
- **Alternative Hypothesis (H_a or H_1):** A statement that contradicts the null hypothesis, proposing there is an effect, difference, or relationship. This is what the researcher aims to support. *Example: The population mean is not equal to, greater than, or less than a specific value.*

Primary Difference: The null hypothesis represents the status quo (baseline), while the alternative hypothesis represents the change or effect being investigated.

3. What is the significance level in hypothesis testing, and why is it important?

The significance level, denoted by alpha (α), is the probability of rejecting the null hypothesis when it is actually true (**Type I error**). Common values include 0.05 (5%), 0.01, and 0.10.

Importance: It sets the threshold for how strong the sample evidence must be. A smaller α (e.g., 0.01) requires stronger evidence, reducing the risk of a Type I error but increasing the risk of a Type II error.

4. What does a P-value represent in hypothesis testing?

The P-value represents the probability of obtaining test results at least as extreme as the results observed, assuming the null hypothesis is correct. A small P-value indicates the observed data is unlikely to have occurred under the null hypothesis.

5. How do you interpret the P-value in hypothesis testing?

Interpretation is based on comparing the P-value to the significance level (α):

- **If P-value $\le \alpha$:** Reject the null hypothesis. There is statistically significant evidence for the alternative hypothesis.
- **If P-value $> \alpha$:** Fail to reject the null hypothesis. There is insufficient evidence to support the alternative hypothesis.

6. What are Type 1 and Type 2 errors in hypothesis testing?

- **Type I Error:** Rejecting a true null hypothesis (probability = α).
- **Type II Error:** Failing to reject a false null hypothesis (probability = β).

Decision	**H0 is True**	**H0 is False**
Reject H_0	Type I Error	Correct Decision
Fail to Reject H_0	Correct Decision	Type II Error

7. What is the difference between a one-tailed and a two-tailed test?

- **One-Tailed Test:** Used when the alternative hypothesis specifies a **direction** (e.g., testing if a drug *increases* a metric). The rejection region is in only one tail of the distribution.
- **Two-Tailed Test:** Used when the alternative hypothesis does not specify a direction (e.g., testing if a drug has a *different* effect). The rejection region is split between both tails.

8. What is the Z-test, and when is it used?

The Z-test determines if two population means are different when **variances are known** and the **sample size is large** ($n > 30$). It is based on the standard normal distribution.

Uses:

- Comparing a sample mean to a known population mean.
- Comparing the means of two samples.

9. How do you calculate the Z-score, and what does it represent?

The Z-score is calculated as:

$$Z = \frac{(\bar{x} - \mu)}{(\frac{\sigma}{\sqrt{n}})}$$

Where:

- $\bar{x}$ = sample mean, μ = population mean, σ = population standard deviation, n = sample size.

Representation: It measures how many standard deviations a sample mean is from the population mean, indicating the strength of evidence against H_0.

10. What is the T-distribution, and when should it be used?

The T-distribution (Student's t-distribution) is similar to the normal distribution but has **heavier tails**. It is used when the sample size is small ($n < 30$) and the population standard deviation is **unknown**. It approaches the normal distribution as n increases.

11. What is the difference between a Z-test and a T-test?

Feature	**Z-test**	**T-test**
Pop. Standard Deviation	Known	Unknown
Sample Size	Large ($n > 30$)	Small ($n < 30$)
Distribution	Standard Normal	T-distribution

12. What is the T-test, and how is it used?

The T-test compares the means of groups when the population standard deviation is unknown.

Types:

- **One-sample T-test:** Compares a sample mean to a hypothesized population mean.
- **Independent two-sample T-test:** Compares means of two independent groups.
- **Paired sample T-test:** Compares means of the same group at different times/conditions.

13. What is the relationship between Z-test and T-test?

The T-test converges to the Z-test as the sample size (n) increases. For large samples ($n > 30$), the results of both tests are very similar.

14. What is a confidence interval, and how is it used?

A confidence interval is a range of values likely to contain the true population parameter. A **95% confidence interval** means that if you sampled many times, 95% of those intervals would contain the true mean.

- **Interpretation:** If a hypothesized value falls outside the interval, it provides evidence against the null hypothesis.

15. What is the margin of error, and how does it affect the confidence interval?

The margin of error is the half-width of the confidence interval, representing random sampling error.

- **Higher confidence level** $\rightarrow$ Larger margin of error (wider interval).
- **Larger sample size** $\rightarrow$ Smaller margin of error (narrower interval).
- **Higher variability** $\rightarrow$ Larger margin of error.

16. How is Bayes' Theorem used in statistics?

Bayes' Theorem updates the probability of an event based on prior knowledge:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Significance: It is the foundation of **Bayesian inference**, where probabilities are updated as new evidence becomes available. It is used in machine learning, medical diagnosis, and spam filtering.

17. What is the Chi-square distribution, and when is it used?

A continuous distribution used for:

- **Goodness-of-fit tests:** Checking if data fits a specific distribution.
- **Tests of independence:** Checking associations between categorical variables.
- **Tests for variance:** Testing hypotheses about population variance.

18. What is the Chi-square goodness of fit test?

It determines if an observed frequency distribution differs from a theoretical one.

Steps:

1. State H_0 and H_a.
2. Calculate expected frequencies (E).
3. Calculate the statistic: $\chi^2 = \sum \frac{(O - E)^2}{E}$.
4. Compare to critical value from the χ^2 table.

19. What is the F-distribution, and when is it used?

A continuous distribution used in **ANOVA** and F-tests. It is used for:

- Comparing variances of two populations.
- Comparing means of three or more groups.

20. What is an ANOVA test, and what are its assumptions?

ANOVA (Analysis of Variance) compares means of two or more groups.

Assumptions:

- **Independence:** Observations are independent.
- **Normality:** Data in each group is approximately normally distributed.
- **Homogeneity of variances:** Group variances are equal.

21. What are the different types of ANOVA tests?

- **One-Way ANOVA:** Compares means based on one independent variable.
- **Two-Way ANOVA:** Studies the effect of two independent variables and their interaction.
- **MANOVA:** Used when there is more than one dependent variable.

22. What is the F-test, and how does it relate to hypothesis testing?

The F-test uses the F-distribution. In ANOVA, it is the ratio of variances:

$$F = \frac{\text{Variance between groups}}{\text{Variance within groups}}$$

If the calculated F-statistic is larger than the critical F-value, the null hypothesis (that all means are equal) is rejected.