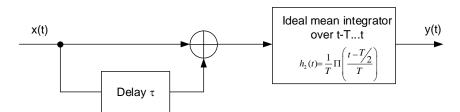
## COMMUNICATION THEORY, Class Exercise 2, Fall 2023

1. Derive the impulse response h(t) and the frequency response or transfer function H(f) of the following LTI system, and investigate how the results vary depending on the values of the delay  $\tau$  and the integrating window T.



2. An LTI transmission channel has the following frequency response for  $f_{MHz} \ge 0$ , where K = 3 and  $t_d = 0.04$  µs. Note that here the frequency domain is expressed in MHz.

$$H(f_{MHz}) = \begin{cases} Ke^{-j20\pi t_d} & f_{MHz} < 10 \text{ MHz} \\ K\left|\cos\left[\frac{6\pi}{70}(f_{MHz} - 10)\right]\right|e^{-j2\pi f_{MHz}t_d} & 10 \text{ MHz} \le f_{MHz} < 30 \text{ MHz} \end{cases}$$

$$K\left|\cos\left(\frac{6\pi}{7}\right)\right|e^{-j2\pi f_{MHz}t_d} & f_{MHz} \ge 30 \text{ MHz} \end{cases}$$

- a) Assuming the channel impulse is real-valued, find the frequency response at  $f_{MHz} < 0$
- b) Determine at which frequencies the channel causes either amplitude or phase linear distortion, and whether it causes any non-linear distortion
- 3. A sinusoidal signal  $a \cdot \cos(2\pi f_0 t)$  with frequency  $f_0 = 1$  kHz and amplitude a either 0.5 V or 1.0 V is used as the input x(t) of a non-linear device, which is described by the following input-output response:

$$y(t) = 12x(t) - 3x^2(t) + x^3(t)$$

The output signal y(t) is then filtered by an RC-lowpass filter described by it frequency response  $H_{RC}(f) = \frac{1}{1+j2\pi fRC}$ , with parameters R=1 k $\Omega$ , C=0.500 mF.

- a) Find which harmonics of the original frequency  $f_0$  appear in the output signal y(t)
- b) Calculate the distortion coefficients associated to the harmonics and the total harmonic distortion factor (THD) of the filtered output signal, as per the following definitions:
  - distortion coefficients  $d_2 = \left| A_{2f_0} \right| / \left| A_{f_0} \right|$ ,  $d_3 = \left| A_{3f_0} \right| / \left| A_{f_0} \right|$ , ... with  $A_f$  denoting the amplitude of a frequency component f
  - total harmonic distortion factor  $THD = \sqrt{d_2^2 + d_3^2 + ...}$