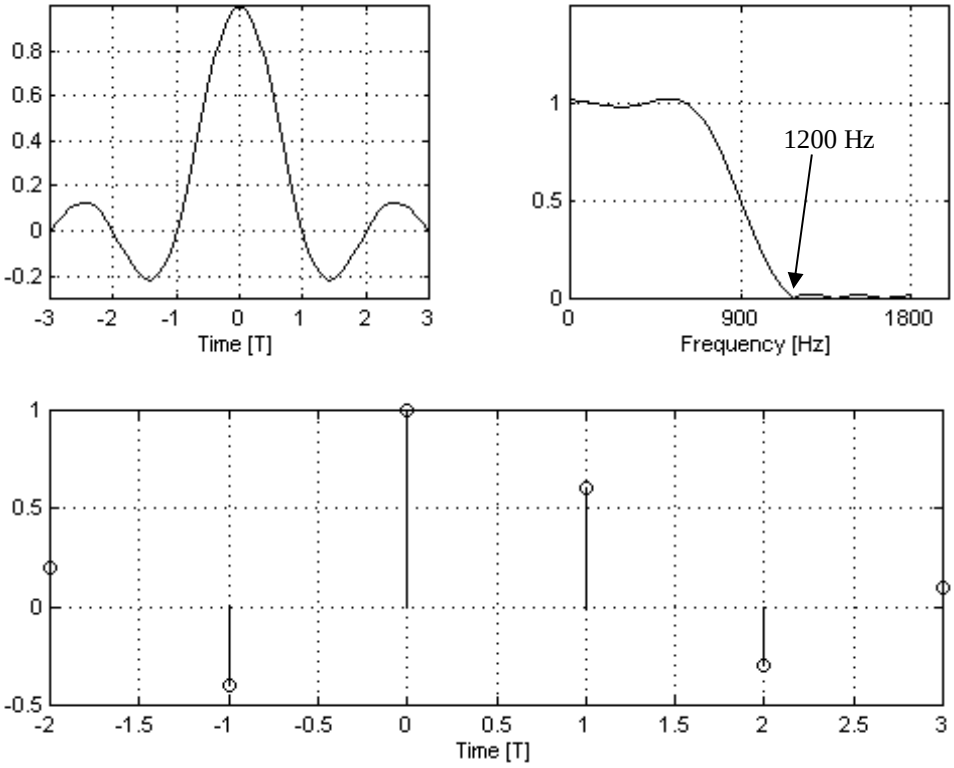
COMMUNICATION THEORY, Exercise 8, Fall 2023

1. A PAM system uses raised-cosine pulses with time-domain shape and spectrum shown below to transmit binary symbols ±1 over a bandlimited channel at symbol rate $R_s = 1/T$ sps.

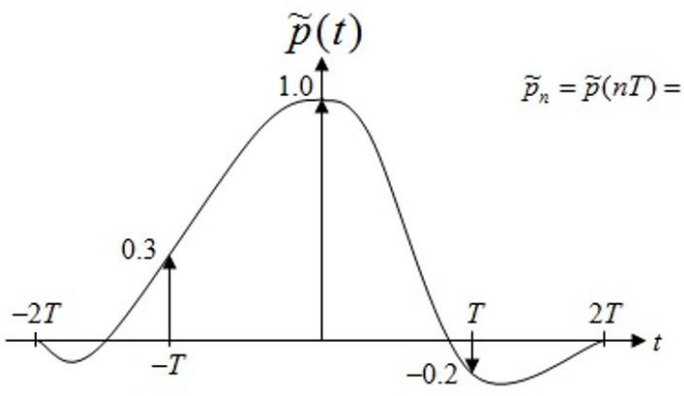
   The transmission channel distorts the pulses so that the sampled values of the received pulse are [0.2 -0.4 1 0.6 -0.3 0.1] as shown in the lower picture.



   a) Find the symbol rate R_s and the roll-off factor α of the raised-cosine pulse.

   b) Find the symbol sequences which cause the largest inter-symbol interference (ISI), and its probability of occurrence, assuming equally probable and independent symbols.

2. A transmission channel distorts a PAM signal so that the original pulse $p(t)$ is distorted into the received pulse $\tilde{p}(t)$ represented in the following picture.



$$\tilde{p}_n = \tilde{p}(nT) = \begin{cases} 0.3 & n = -1 \\ 1.0 & n = 0 \\ -0.2 & n = 1 \\ 0 & otherwise \end{cases}$$

To remove as much inter-symbol interference (ISI) as possible at the receiver, an equalizer can be applied after the sampling of the signal; with the equalizer implemented as a digital filter, its discrete-time impulse response can generally be represented using the zeta-transform:

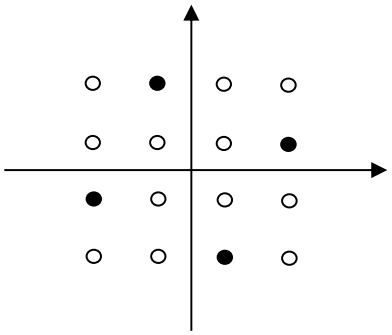
$$C(z) = c_{-N} z^{N} + c_{-N+1} z^{N-1} + \ldots + c_0 + \ldots + c_{N-1} z^{-N+1} + c_N z^{-N}$$

Design the equalizer according to the zero-forcing (ZF) principle, by finding the required digital filter coefficients c_n in these two cases:

a) 3-taps equalizer, with zeta-transform $C(z) = c_{-1} z^{1} + c_0 + c_1 z^{-1}$

b) 5-taps equalizer, with zeta-transform $C(z) = c_{-2} z^{2} + c_{-1} z^{1} + c_0 + c_1 z^{-1} + c_2 z^{-2}$

3. Consider the following 16-QAM signal constellation:



A digital transmission system is designed with an option to dynamically switch between using the whole 16-QAM constellation and using only a subset of the symbols (represented by the blackened points in the picture), depending on the channel conditions.

a) Assuming the symbol rate R_s is fixed, find the bit rate R_b in the two cases.

b) Show that, when only the symbols subset is transmitted, the resulting signal is equivalent to a 4-PSK signal with the same transmitted power as the whole 16-QAM constellation.