COMMUNICATION THEORY, Exercise 4, Fall 2023

1. How does the concept of *white* noise differs from the concept of *Gaussian* noise?

2. Consider a complex oscillator signal $x(t) = Ae^{j(2\pi f_0 t + \phi(t))}$, with a time-dependent phase noise $\phi(t)$ modeled as a WSS (wide-sense stationary) stochastic process with zero mean and with small enough values so that $\forall t,\ \phi(t)$ << 1 radians.

   a) prove whether the signal x(t) itself is a WSS stochastic process or not

   b) derive the autocorrelation and spectral density functions of $x(t)$

3. Consider the thermal noise power over a 1kΩ resistor at a temperature T = 29K given by:

$$\sigma_v^2 = \frac{2(\pi kT)^2}{3h} R$$

   a) prove that the mathematical model of the spectral density $G_v(f) = \dfrac{2Rh|f|}{e^{h|f|/kT} - 1}$ for thermal noise derived from physics can be approximated as:

$$G_v(f) \approx 2RkT\left(1 - \frac{h|f|}{2kT}\right), \text{ when } h|f| << kT$$

   b) Find how much of the total noise power is located within the first GHz of frequency

   [*Note:* $k=1.37*10^{-23}$ is the Boltzman constant and $h=6.62*10^{-34}$ is the Planck constant]

4. Prove that the equivalent noise bandwidth B_N of an n-th order Butterworth low-pass filter can be written as $B_N = \dfrac{\pi B}{2n\sin(\pi/2n)}$, where B is the corresponding 3 dB bandwidth, and show what happens to B_N when the filter order n is increased indefinitely ($n \to \infty$).

5. Assume that just AWGN noise is fed through either one of the following LTI systems:

   a) an ideal mean integrator, with integration interval T

   b) a 2^{nd}-order Butterworth filter, with amplitude response $|H(f)| = (1 + (f/B)^4)^{-1/2}$

   Determine the power spectrum and the average power at the output of either filter.

   Find also the value of the integration interval T so that the two filters have the same equivalent noise bandwidth.