Financial Frictions, FX Reserves, and Exchange Rate Management in Emerging Economies

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Abstract

Emerging economies hold substantial amounts in foreign exchange (FX) reserves, and even those with considerable external debt hold significant reserves. This paper proposes a theory that can rationalize why net external debtor economies might choose to hold reserves instead of using them to reduce their external debt. Considering an environment of a small open economy with free international capital mobility but with frictions in financial markets, the model shows that a central bank optimally chooses to hold FX reserves while the household chooses to borrow from foreigners. This is because reserve operations influence the exchange rate; specifically, reserve accumulation induces an exchange rate depreciation, which dilutes the value of existing debt payments, thereby minimizing the resource loss for the economy. The paper also shows that the optimal reserve accumulation policy under commitment is time-inconsistent as the central bank wants to mitigate the household's debt burden. A time-consistent equilibrium features even more reserve accumulation than a commitment equilibrium. Finally, financial flow volatility creates an insurance motive for reserves to stabilize the exchange rate and deliver consumption smoothing.

Keywords: Foreign exchange reserves, financial frictions, FX interventions, exchange rate depreciation, time-consistent equilibrium.

JEL Classifications: E21, E44, E58, F31, F32, F34, G15.

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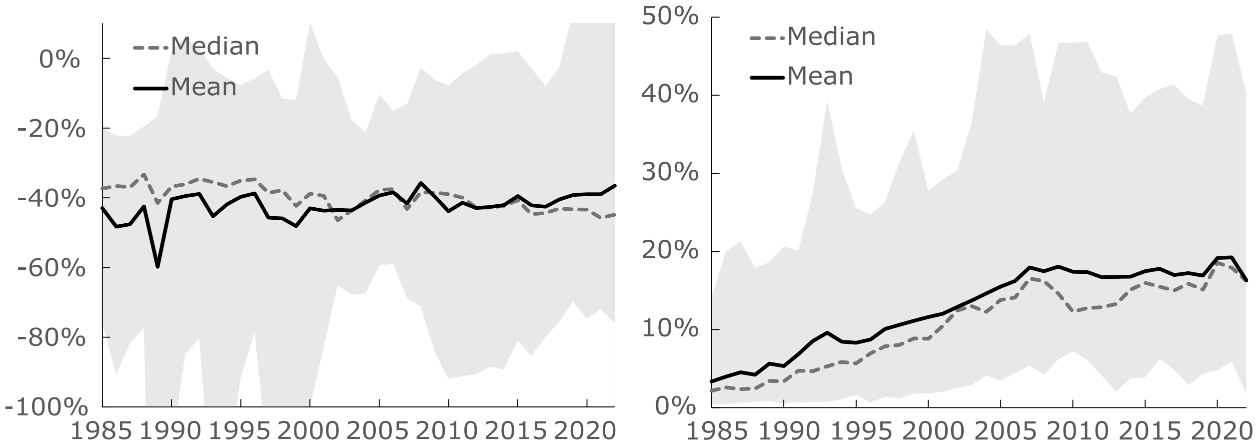
1 Introduction

It is a well-documented fact that central banks in emerging economies accumulated substantial amounts of foreign exchange reserves between the mid-1980s and mid-2000s. Currently, most emerging economies continue to hold reserves significantly higher than their advanced counterparts. Notably, however, even countries with considerable amounts of net external debt are holding significant reserves (Figure 1). This observation prompts the motivating question of this paper: why are net external debtors holding reserves? Typically, foreign exchange reserves are low-interest-bearing assets, while external debt carries an interest premium. Thus, it seems counterintuitive not to use these reserves to reduce the country's external debt. In this paper, we propose a theory that could rationalize this observation and explain why debtors might also hold reserves.

To motivate our theory, we relate this trend in reserves to two other empirical facts. First, this reserve accumulation has coincided with an increase in international capital mobility and simultaneous imposition of exchange rate controls in emerging economies. Second, countries that impose exchange rate controls hold significantly higher reserves than countries with free-floating currencies. These findings are discussed extensively by Ilzetzki, Reinhart, and Rogoff (2019) and we summarize them in Appendix A.

Guided by these facts, it is clear that reserve accumulation is related to the degree of exchange rate controls. The existing literature has proposed a variety of explanations linking them, the most common explanation being a mercantalist motive, i.e., countries build reserves to undervalue their exchange rate and maintain export competitiveness. While this motive might explain reserve accumulation by persistent surplus economies, it is less applicable to debtor economies. Moreover, it fails to account for the simultaneous trends of reserve accumulation and expansion of global financial markets.

We argue that a theory that relates reserve accumulation to exchange rate controls while simultaneously taking into account the role of global financial markets could more appropriately explain the reasons for a simultaneous debt-reserves portfolio. Not only does existing economic theory speak very little to this, but also it has traditionally argued against the effectiveness of exchange rate controls in the presence of free capital mobility (see, e.g., Backus and Kehoe (1989)). A Ricardian equivalence logic suggests that any attempt by the central bank to accumulate reserves would result in private agents borrowing more in the international markets thereby offsetting the impact



(a) IIP to GDP (excluding reserves) (b) Reserves to GDP

Notes: We consider a sample of 20 largest (by 2022 GDP) persistent deficit emerging economies i.e., those which are net external debtors (excluding reserves). For a list of countries in the sample, see Appendix A. In the left panel we show the mean and the median international investment position (IIP) of this sample and in the right panel we show the official foreign exchange reserves. The shaded region represents the maximum and minimum. In 2022, the median IIP was -44.86% and median reserves were 16.27% in this sample. Source: Computed using the External Wealth of Nations database (Milesi-Ferretti, 2022).

Figure 1: Net-external debtors holding reserves

of reserves with no resultant effect on the exchange rate. If this were the case, then an economy that holds both debt and reserves, would benefit from using all its reserves to decrease the debt.

However, recent literature has emphasized the significance of frictions in international financial markets. In the presence of such frictions, this equivalence result breaks and FX interventions can indeed be effective. Building on this framework, we propose a theory suggesting that in a world with free capital mobility but with financial frictions, it may not be optimal for a debtor economy to run down its reserves and decrease its debt.

Building on the international financial frictions literature, we present a model of a pure exchange small open economy (SOE) with free capital mobility. The model structure closely follows Gabaix and Maggiori (2015) and introduces frictions in international financial markets. First, the financial markets are segmented so that the SOE does not have direct access to international markets, but can only access domestic markets where agents trade in bonds denominated in domestic currency. The economy's link to the international financial markets is through financial intermediaries who can access both domestic and international markets and trade in bonds denominated in local currency as well as international currency. Secondly, there is a limited repayment commitment on the part of financial intermediaries: they can divert funds after taking asset positions and therefore face a credit constraint. Given these frictions, uncovered interest parity fails and the intermediaries' supply of funds in the domestic bonds market takes the form of a rule that depends positively on the appreciation rate of the local currency.

First, we emphasize the dual role of the exchange rate in this environment: it not only prices consumption for the household but is also a key asset price: an equilibrium variable that clears the domestic bonds market and adjusts in response to exogenous changes in financial flows- appreciating in times of booms and depreciating in times of disruptions.

Secondly, considering an economy that starts with both debt and reserves, we show that given the frictions above, a benevolent central bank that has the option to run down its reserves to zero and reduce the country's external debt may not find it optimal to do so. This is because such an intervention induces an exchange rate appreciation. On one hand, this appreciation allows the household to borrow lesser, but on the other hand, since debt is denominated in domestic currency, this appreciation is harmful: results in higher real payments on previous debt and therefore a larger resource loss. In other words, by choosing to hold reserves, while the central bank makes the household borrow more, it also means that the household pays a lower interest on existing debt- a dilution of existing debt payments induced by an exchange rate depreciation.

Third, given that the intermediaries' supply of funds is dependent on the appreciation rate, we show that the central bank's optimal reserve policy under commitment- one where it announces future exchange rates (or future reserve holdings), is time-inconsistent. We show that the central bank has an incentive to announce a higher exchange rate for the following period (and thus a lower appreciation rate) to mitigate the debt burden. However, when that future period arrives, it may find it sub-optimal to implement the announced plan, opting instead for a lower than announced exchange rate to allow for higher household consumption. Given this time inconsistency, we establish that if a central bank lacks commitment, in a time-consistent equilibrium, it will hold even more reserves than what it would under a commitment policy.

Finally, we quantitatively solve for a time-consistent equilibrium in the presence of exogenous shocks to financial flows. We show that the volatility in financial flows creates a motive for stabilizing the exchange rate: the central bank follows a leaning-against-the-wind reserve policy where it accumulates reserves in times of financial booms and runs down reserves in times of disruptions. While reserve accumulation in financially stable times induces an exchange rate depreciation, lowers household consumption, and increases household borrowing; in times of disruptions, reserves allow the central bank to decrease the extent of capital outflow led exchange rate depreciation, thereby providing higher consumption to the household. In essence, reserve accumulation is an insurance against volatile financial flows and allows for transferring resources from relatively abundant states to relatively scarce future states. We show that in the long-run equilibrium, the economy carries both debt and reserves and that the central bank's reserve policy delivers consumption smoothing.

The rest of this paper is organized as follows. Section 2 describes the related literature and discusses the salient features of this paper. Section 3 presents the model and the main analytical results of this paper. Section 4 contains a quantitative analysis of the model. Section 5 concludes.

2 Related Literature

This paper is primarily related to the literature studying motives for foreign exchange reserve accumulation. This has been the subject of an extensive empirical and theoretical literature going back to the 1980s. A variety of motives for reserve accumulation have been proposed, most commonly, mercantalist motives (Aizenman & Lee, 2007), growth and capital accumulation motives (Benigno, Fornaro, & Wolf, 2022; Korinek & Servén, 2016), precautionary and self-insurance motives (Durdu, Mendoza, & Terrones, 2009; Gourinchas & Obstfeld, 2012), and safe-asset motives (Caballero, Farhi, & Gourinchas, 2017). More recently, studies have linked international reserves to macroprudential policy (Arce, Bengui, & Bianchi, 2022; Davis, Devereux, & Yu, 2023; Jeanne & Sandri, 2023), lender of last resort policies (Bocola & Lorenzoni, 2020), and sovereign default risk hedging (Barbosa-Alves, Bianchi, & Sosa-Padilla, 2024; Bianchi & Sosa-Padilla, 2023).

This paper is also related to the literature on reserves and exchange rate controls that goes back to Krugman (1979). A benchmark result on the ineffectiveness of FX interventions was established by Backus and Kehoe (1989). More recently, the literature has highlighted the role of frictions in financial markets that breaks the benchmark ineffectiveness result. Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021) discuss the role of financial frictions in general equilibrium models of exchange rate determination and have emphasized the significance of the financial sector in influencing the exchange rates. Amador, Bianchi, Bocola, and Perri (2020) highlight the role of FX interventions at the zero lower bound in an environment with limits to international arbitrage. Fanelli and Straub (2021) discuss optimal exchange rate policies in an environment with distributional consequences. Bianchi and Lorenzoni (2022) discuss and compare capital controls and exchange rate controls in the presence of frictions and aggregate demand externalities. Itskhoki and Mukhin (2023) characterize the linkages between exchange rate and monetary policies in the presence of nominal rigidities. Basu, Boz, Gopinath, Roch, and Unsal (2020) discuss the interactions between exchange rate, capital control, macroprudential, and monetary policy instruments in an environment with financial, trade, and housing frictions and characterize optimal policies.

This paper has a few salient features, vis-à-vis the existing literature. First, we acknowledge that are multiple reasons that a country might hold reserves and/or manage its exchange rates. There is no one model or motive that could uniformly explain the trend in reserves across countries. We focus specifically on external debtor economies and provide microfoundations for an optimal portfolio with both debt and reserves. Our model speaks to why a debtor would want to hold reserves even though reserves are low-interest bearing and debt has a premium.

Secondly, following the arguments made by Ilzetzki, Reinhart, and Rogoff (2019), our model emphasizes and takes into account the fact that the trend in reserves has coincided with the expansion of global financial markets and imposition of exchange rate controls. While existing literature has underappreciated this fact, our model validates that a lack of capital controls has introduced an additional motive for reserve accumulation, especially for debtor economies: debt dilution vs financial intermediation costs.

Third, we emphasize the asset price role of the exchange rate- in our model, the exchange rate not only prices consumption, but also is the key variable that clears the bond markets. This asset price role and the forward looking nature of agents in our model creates a time-inconsistency problem: for a central bank that lacks commitment, it's plans for the future exchange rate may not be consistent with it's future actions. We extensively discuss the source of this time-inconsistency and compare optimal policies under commitment and lack of commitment.

Fourth, with a few exceptions, much of the literature on reserves has remained silent on the need for policymakers to intervene and accumulate reserves. In other words, there is no difference between household and government/central bank accumulation of reserves. While in our model we do nor directly allow the household to accumulate reserves, it is evident, that if allowed, such reserve accumulation would be suboptimal because the household does not internalize the general equilibrium effects of the exchange rate on debt payments and borrowing. In other words, this paper puts forward a motive for accumulation of reserves by public agents rather than private.

Finally, given that in our model exchange rate also prices consumption, we illustrate that frictions in international capital markets have introduced an additional precautionary motive for reserve accumulation: consumption smoothing in the presence of volatile financial flows.

3 Model

This section presents a dynamic general equilibrium model of a pure exchange small open economy (SOE) with free capital mobility but facing a financial intermediation friction. The model consists of three types of agents- a representative consumer, international financial intermediaries and a central bank. Time is discrete and denoted by t. The agents are infinitely lived.

First, we describe the environment and the optimization problems of the agents. We derive the competitive equilibrium conditions and set up the optimal policy problem. Finally, we discuss the optimal policy and analytically illustrate the main results of this paper.

3.1 Representative Consumer

The representative consumer is infinitely lived and consumes two goods- an internationally tradeable consumption good: c_t, and a domestic non-tradeable good: m_t. Each period, the consumer receives a fixed endowment of the two goods: y and m^s, respectively. The consumer also receives a transfer T_t from the central bank.

The non-tradeable good, m, is the domestic unit of account and its price is normalized to 1 every period [1]. Let p_t denote the relative price of the consumption good, c_t, in units of the domestic numeraire. The tradeable consumption good is assumed to be the world numeraire, its world price, $p_t^* = 1$. The law of one price holds in the tradeable good market: $p_t = e_t p_t^*$; where e_t is the exchange rate expressed in units of the domestic numeraire.

In addition to consuming the two goods, the consumer also participates in a competitive domestic bonds market where it can save or borrow using one-period risk-free bonds denominated in units of the domestic numeraire: $\tilde{b}_{t+1}$. $\tilde{b}_{t+1} > 0$ implies saving whereas $\tilde{b}_{t+1} < 0$ implies borrowing from the domestic bonds market. The consumer is subject to a no-Ponzi-games constraint. Let R_{t+1} denote the gross risk-free interest rate in this market. Moreover, the bonds markets are segmented: while the consumer can access the domestic bonds market, it does not have direct access to the international bonds market.

Let β denote the consumers discount factor, σ be the risk aversion parameter and ω be the relative

[1] m_t is a good with an endowment every period. It has the unit of account role but is not a store of value.

utility preference parameter for the tradeable consumption good. For simplicity, we assume that the consumer's preferences are separable and homothetic in the two goods. The consumer's optimization problem is described by (1) and involves maximizing expected life-time utility subject to the budget constraints (and a no-Ponzi-games constraint).

$$\max_{\{c_t, m_t, \tilde{b}_{t+1}\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\omega \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{m_t^{1-\sigma}}{1-\sigma} \right) \qquad \text{s.t.}$$
$$e_t c_t + m_t + \tilde{b}_{t+1} = e_t y + R_t \tilde{b}_t + m^s + T_t \tag{1}$$

The optimality conditions for (1) are given by an an Euler equation, (2), an intratemporal relation between c_t and m_t, (3) and a transversality condition, (4).

$$c_t{}^{-\sigma} = \beta R_{t+1} \mathbb{E}_t \left(\frac{e_t}{e_{t+1}} c_{t+1}{}^{-\sigma} \right) \tag{2}$$

$$m_t^{\sigma} = \frac{1}{\omega} e_t c_t{}^{\sigma} \tag{3}$$

$$\lim_{J \to \infty} \beta^J \mathbb{E}_t \left(c_{t+J}^{-\sigma} \frac{\tilde{b}_{t+J}}{e_{t+J}} \right) = 0 \quad \forall t \tag{4}$$

3.2 Financial Intermediaries

The world is populated by a unit mass of identical financial intermediaries who intermediate borrowing and lending transactions in the financial markets. The intermediaries are assumed to be owned outside the SOE. The intermediaries have access to an international bonds market where they trade using one-period risk-free bonds denominated in units of the world numeraire, i.e., the tradeable consumption good: q^*_{t+1}. Let R^* denote the gross interest rate in the international bonds market. This rate is assumed to be fixed, and crucially, we assume $R^* < \beta^{-1}$.

While intermediaries have access to the international bonds market, the SOE has free capital mobility, so that they also have unrestricted access to the domestic bonds market. Let $\tilde{q}_{t+1}$ denote the asset position of the intermediaries in the domestic bonds market denominated in units of the domestic numeraire.

A positive asset position in either market denotes saving whereas a negative position denotes borrowing. The intermediaries take offsetting positions and face a balance sheet constraint, (5).

$$\frac{\tilde{q}_{t+1}}{e_t} + q^*_{t+1} = 0 \tag{5}$$

The intermediaries are risk neutral and seek to maximize their expected return, $\mathbb{E}_t \nu_{t+1}$, from their asset positions:

$$\mathbb{E}_t \nu_{t+1} = \mathbb{E}_t \left[\frac{e_t}{e_{t+1}} R_{t+1} - R^* \right] \frac{\tilde{q}_{t+1}}{e_t} \tag{6}$$

We now describe the key financial friction in this environment. Following Gabaix and Maggiori (2015), the financial friction takes the form of a limited repayment commitment on the part of the intermediaries. Every period, immediately after taking their asset positions, an intermediary can divert a portion $\Gamma_t \left| \frac{\tilde{q}_{t+1}}{e_t} \right|$, with $\Gamma_t > 0$, of the the position it intermediates: $\left| \frac{\tilde{q}_{t+1}}{e_t} \right|$. If the intermediary diverts the funds, the firm gets unwounded and the lenders recover a portion $\left(1 - \Gamma_t \left| \frac{\tilde{q}_{t+1}}{e_t} \right|\right)$ of their claims $\left| \frac{\tilde{q}_{t+1}}{e_t} \right|$. Since the lenders anticipate the incentives of the intermediary to divert funds, the intermediary is subject to a credit constraint such that expected discounted returns from the intermediation business are weakly higher than the returns earned by diverting the funds. This constraint is described by (7)[2].

$$\frac{1}{R^*} \mathbb{E}_t \nu_{t+1} \geqslant \Gamma_t \left| \frac{\tilde{q}_{t+1}}{e_t} \right| \times \left| \frac{\tilde{q}_{t+1}}{e_t} \right| \tag{7}$$

The intermediary's constrained optimization problem is then given by (8).

$$\max_{\tilde{q}_{t+1}} \frac{1}{R^*} \mathbb{E}_t \left[\frac{e_t}{e_{t+1}} R_{t+1} - R^* \right] \frac{\tilde{q}_{t+1}}{e_t} \qquad \text{s.t. to (7)} \tag{8}$$

Since, the objective in (8) is linear in $\tilde{q}_{t+1}$ and the constraint (7) is convex in $\tilde{q}_{t+1}$, the constraint,(7), always binds at the optimum. The solution to (8) is then given by a supply of funds rule, (9).

$$\frac{\tilde{q}_{t+1}}{e_t} = \frac{1}{\Gamma_t} \left[\frac{R_{t+1}}{R^*} \mathbb{E}_t \left(\frac{e_t}{e_{t+1}} \right) - 1 \right] \tag{9}$$

First note that given Γ_t and the interest differential, $\frac{R^*}{R_{t+1}}$, the intermediary's saving is a linearly increasing function of the expected appreciation rate, $\mathbb{E}_t \left(\frac{e_t}{e_{t+1}} \right)$, with $\tilde{q}_{t+1} = 0 \Leftrightarrow \mathbb{E}_t \left(\frac{e_t}{e_{t+1}} \right) = \frac{R^*}{R_{t+1}}$. Secondly, the key variable in this equation is Γ_t which is implicitly a measure of friction in the financial markets. $\Gamma_t = 0$ is the frictionless model where the solution to the optimization problem gives us the uncovered interest parity condition[3]. For any finite value of $\Gamma_t > 0$ uncovered interest parity fails. In other words, $\Gamma_t > 0$ drives a wedge between the interest differential, $\frac{R^*}{R_{t+1}}$ and the expected appreciation rate, $\mathbb{E}_t \left(\frac{e_t}{e_{t+1}} \right)$. The supply of funds by the intermediaries is an increasing

[2] For further discussion on such limited commitment constraints and their microfoundations see Gabaix and Maggiori (2015) and the references therein.

[3] If $\Gamma_t = 0$ intermediaries are simply a veil, the solution to their optimization problem gives us the UIP condition: $\frac{R^*}{R_{t+1}} = \mathbb{E}_t \left(\frac{e_t}{e_{t+1}} \right)$

function of this wedge: given the interest differential, higher the expected appreciation rate, higher is the amount that intermediaries are willing to save in the domestic bonds market. Moreover, the supply is also a function of Γ_t: an increase in frictions is equivalent to financial disruptions where the intermediaries are willing to lend smaller amounts. As we will illustrate, it turns out that the expected appreciation rate, $\mathbb{E}_t\left(\frac{e_t}{e_{t+1}}\right)$ is not only an essential equilibrium variable, but is also the key variable that adjusts in response to exogenous shocks to Γ_t.

In what follows, we set up and describe the competitive equilibrium in a deterministic as well as a stochastic setting. In the deterministic setting, we assume $\Gamma_t = \Gamma > 0, \forall t$ and in the stochastic setting, we assume $\Gamma_t > 0$ follows an exogenous Markov switching regime.

3.3 Central bank

Since the SOE is assumed to have free capital mobility, the central bank does not have access to any distortionary capital taxation instrument. It only has access to lump-sump transfers/taxes T_t denominated in units of the domestic numeraire. However, the central bank can bypass the intermediaries and directly access the international bonds market, where it can *save* in one-period risk-free foreign assets denominated in units of the tradeable consumption good, henceforth called reserves: a_{t+1}. The central bank faces a period-by-period budget constraint, (10).

$$e_t a_{t+1} + T_t = e_t R^* a_t \tag{10}$$

It is crucial to note here that while the central bank can save in foreign assets, borrowing directly through the international bonds market is not an option[4]: $a_{t+1} \geqslant 0$.

3.4 Competitive Equilibrium

Given $(a_0, \tilde{b}_0)$, central bank policy $\{T_t, a_{t+1}\}$, and an exogenous stochastic process for Γ_t, a competitive equilibrium is given by stochastic sequences of the exchange rate, $\{e_t\}$, the interest rate on domestic bonds, $\{R_{t+1}\}$, consumption of the two goods, $\{c_t, m_t\}$ and asset positions in the domestic bonds market $\{\tilde{b}_{t+1}, \tilde{q}_{t+1}\}$ so that,

- Given the exchange rates and interest rates, the representative consumer solves its optimization problem (1).

[4]If the central bank can borrow directly from the international bonds market, the intermediation friction is irrelevant. Moreover this constraint reflects a real world reality where central banks have zero to negligible access to borrowed reserves. See the discussion in Davis, Devereux, and Yu (2023).

- Given the exchange rates and interest rates, the intermediaries solve their optimization problem, (8).

- The central bank's budget constraint, (10) holds every period.

- The domestic non-tradeable and the domestic bonds markets clear every period:

$$m_t = m^s \tag{11}$$

$$\tilde{b}_{t+1} + \tilde{q}_{t+1} = 0 \tag{12}$$

We now derive the competitive equilibrium conditions. Since the endowment for the domestic non-tradeable good, m^s, is assumed to be constant, for simplicity and without loss of generality, we normalize $m^s = 1$. Using this, we obtain two equilibrium conditions. First, using (11) and (3) we get:

$$e_t = \omega c_t^{-\sigma} \tag{13}$$

Equation (13) is a crucial equilibrium condition that establishes a relation between the exchange rate, e_t, and the consumption of the tradeable good c_t. This equation is essentially equivalent to a constant-elasticity-of-demand rule and suggests that an exchange rate depreciation implies that the consumer is willing to consume a smaller amount of the tradeable good.

Secondly, using (13) and the consumer's Euler equation, (2), we get:

$$R_{t+1} = \beta^{-1} \quad \forall t \tag{14}$$

In other words, since the endowment for the domestic non-tradeable good is constant, and the domestic bonds are denominated in units of this good, the interest rate on domestic bonds is also pinned down and equal to the inverse of the discount factor.

Combining the consumer's budget constraint in (1) with the central bank budget constraint, (10), and market clearing condition, (11), gives us a resource constraint for the tradeable consumption good, henceforth called the balance of payments (BoP) condition, (15).

$$e_t c_t + \tilde{b}_{t+1} + e_t a_{t+1} = e_t y + R_t \tilde{b}_t + e_t R^* a_t \tag{15}$$

Let $\frac{\tilde{b}_{t+1}}{e_t} \equiv b_{t+1}$ and $\frac{\tilde{q}_{t+1}}{e_t} \equiv q_{t+1}$ i.e., private agents' savings expressed in units of the tradeable consumption good. Using (12) and (13) the BoP condition can be expressed in units of the tradeable consumption good, (16).

$$c_t - q_{t+1} + a_{t+1} = y + R^* a_t - R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma} q_t \tag{16}$$

The constraint (16) suggests that the 'real' interest rate on external debt, q_t, is given by $R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma}$. where $\left(\frac{c_t}{c_{t-1}}\right)^{\sigma}$ denotes the appreciation rate.

Secondly, using (13), the intermediaries' supply of assets, (9), can be effectively expressed as a distorted Euler equation, (17).

$$q_{t+1} = \frac{1}{\Gamma_t}\left[\frac{R}{R^*}\mathbb{E}_t\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \tag{17}$$

Finally, the transversality condition can be expressed as below:

$$\lim_{J\to\infty} \beta^J \mathbb{E}_t(c_{t+J}^{-\sigma} q_{t+J}) = 0 \quad \forall t \tag{18}$$

Given (a_0, q_0, c_{-1}), given a reserve accumulation policy of the central bank $\{a_{t+1}\}$, given an exogenous stochastic process for Γ_t, and given a no-Ponzi-games condition, the competitive equilibrium is summarized by a system of two stochastic difference equations in $\{c_t, q_{t+1}\}$: (16), (17) and a transversality condition, (18).

The competitive equilibrium conditions underscore the dual role of the exchange rate in this environment. First, the exchange rate prices consumption for the household as illustrated by (13). Second, it is also the key asset price that adjusts to clear the domestic bonds market. Effectively, this gives us conditions (16) and (17). Not only the value of existing debt payments depend on the appreciation rate, as illustrated by (16), but also, the external borrowing is also a function of the expected appreciation rate, as illustrated by (17).

3.5 Deterministic Dynamics: Laissez-faire

To better illustrate the competitive equilibrium dynamics and the determination of the equilibrium exchange rate, this section describes the solution of a laissez-faire equilibrium in a deterministic setting. In other words, assume, there is no uncertainty: $\Gamma_t = \Gamma > 0, \forall t$ and that the central bank follows a free-floating exchange rate regime so that it does not accumulate or hold reserves: $a_{t+1} = 0, \forall t$. Using deterministic versions of (16) and (17), a laissez-faire equilibrium is summarized by a system of two difference equations in $\{c_t, q_{t+1}\}$ (19), (20), and a transversality condition, (21).

$$c_t - q_{t+1} = y - R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma} q_t \tag{19}$$

$$q_{t+1} = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \tag{20}$$

$$\lim_{t\to\infty} \beta^t (c_t^{-\sigma} q_{t+1}) = 0 \tag{21}$$

The steady state of this system is given by $(\bar{c}, \bar{q})$:

$$\bar{c} = y - (R-1)\bar{q}; \qquad \bar{q} = \frac{1}{\Gamma}\left[\frac{R}{R^*} - 1\right]$$

Since we have that $R^* < \beta^{-1} = R$ and $\Gamma > 0$, then $\bar{q} > 0$, i.e., the SOE carries a positive debt in the long-run equilibrium. To understand the dynamics of debt and consumption away from the long-run equilibrium we consider a phase diagram in the (q, c) space. Let $\Delta q = 0$ represent the zero-change locus for q and $\Delta c = 0$ represent the zero-change locus for c. Using (19) and (20) the loci can be expressed as below:

$$\Delta q = 0: \quad c = y + q\left(1 - R^*(1 + \Gamma q)\right)$$
$$\Delta c = 0: \quad q = \frac{1}{\Gamma}\left[\frac{R}{R^*} - 1\right]$$

The left panel of Figure 2 presents the phase diagram depicting these zero-change loci. The intersection of these loci signifies the long-run equilibrium $E_0 = (\bar{q}, \bar{c})$. The top-right quadrant has trajectories where consumption and debt explode. Such paths cannot constitute an equilibrium as they violate the no-ponzi-games constraint. Similarly, the trajectories in the bottom-left quadrant cannot constitute an equilibrium as they involve the household accumulating wealth without consuming anything asymptotically, thereby violating the transversality condition. However, the top-left and the bottom-right quadrants have a unique saddle path that asymptotically converges to the long-run equilibrium. If the household begins with low levels of debt (or high saving) it consumes more in the initial periods by borrowing and builds debt over time. As debt starts building up, higher debt payments cause consumption to decrease and the economy approaches the long-run equilibrium. Conversely, if the household starts with very high levels of debt, it initially lowers consumption to alleviate its debt burden. As debt decreases over time, the household can gradually increase its consumption, eventually approaching the long-run equilibrium.

Next, we demonstrate the adjustment and dynamics of the equilibrium exchange rate in response

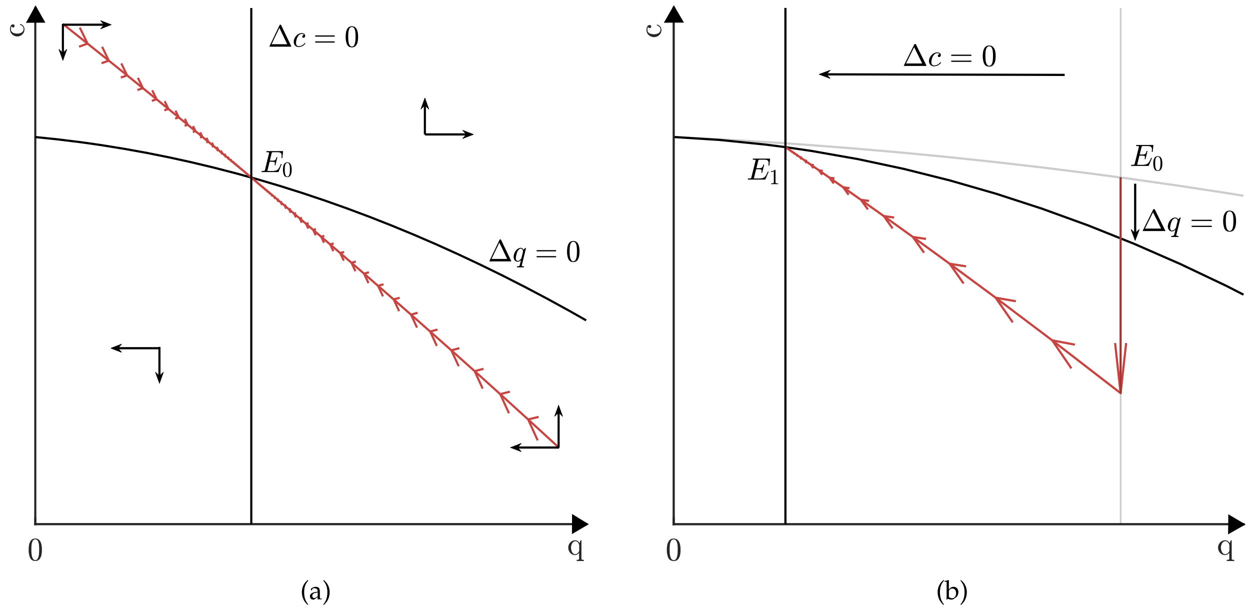


Figure 2: Phase diagram analysis

to exogenous shocks. The right panel of Figure 2 illustrates the short-run and long-run impact of an unanticipated financial shock: a permanent increase in Γ. This increase is interpreted as a tightening of the credit constraint faced by the intermediaries, constituting a financial disruption. The shock induces a shift in both the loci and the new long-run equilibrium shifts from E_0 to E_1. From (13), noting the inverse relation between consumption and the exchange rate, we observe that on impact, a financial disruption induces an exchange rate depreciation and a credit contraction. In the longer run the exchange rate appreciates towards its new long-run equilibrium. Analogously, a decrease in Γ, interpreted as a loosening of the credit constraint faced by the intermediaries-a financial boom- induces an exchange rate appreciation and a surge in capital inflows.

This analysis highlights the asset price role of exchange rates: the exchange rate adjusts to ensure that the financial markets clear. Furthermore, periods of financial booms and busts are accompanied by corresponding exchange rate appreciations or depreciations in response to adjustments in financial flows.

3.6 Optimal Reserves: A Ramsey Problem

With an understanding of competitive equilibrium dynamics, we proceed to set up the policy problem and characterize the optimal reserve accumulation policy for the central bank. We assume that the central bank is benevolent and describe a Ramsey equilibrium, i.e., the central bank chooses a reserve policy that maximizes the expected lifetime utility of the consumer, subject to the BoP constraint, (16), and the supply of funds, (17). The Ramsey equilibrium is given by the solution to (22).

$$\begin{gathered}\max_{\{c_t,q_{t+1},a_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\omega {c_t}^{1-\sigma}}{1-\sigma} \quad \text{s.t.} \\ c_t - q_{t+1} + a_{t+1} = y + R^* a_t - R\left(\frac{c_t}{c_{t-1}}\right)^{\sigma} q_t \\ q_{t+1} = \frac{1}{\Gamma_t}\left[\frac{R}{R^*}\mathbb{E}_t\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \\ a_{t+1} \geqslant 0\end{gathered} \tag{22}$$

The solution to this problem helps us address the motivation question of this paper: why a country might simultaneously hold both debt and reserves. We claim that a Ramsey equilibrium in this environment could indeed feature a portfolio with both debt and reserves held together and that the financial frictions play a key role in determining the optimal portfolio. To further examine this claim, it is helpful to analyze a two-period version of this policy problem in a deterministic environment, which we consider next.

3.6.1 Two-period problem

In this sub-section we consider a two-period deterministic version of the policy problem (22). Let $t = 1, 2$, with the initial state (a_1, q_1, c_0) given at $t = 1$. We assume that the economy begins with some positive levels of debt and reserves at $t = 1$: $a_1, q_1 > 0$[5]. After substituting the asset supply equation into the BoP constraints, the policy problem is given by (23).

$$\begin{gathered}\max_{\{c_1,c_2,a_2\}} \frac{\omega c_1^{1-\sigma}}{1-\sigma} + \beta\frac{\omega c_2^{1-\sigma}}{1-\sigma} \qquad \text{s.t.} \\ c_1 - \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] + a_2 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1 \\ c_2 = y + R^* a_2 - R\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \\ a_2 \geqslant 0\end{gathered} \tag{23}$$

[5] $q_1, a_1 > 0$ is the relevant state for answering the motivating question of this paper.

Furthermore, the two BoP constraints in (23) can be combined together by substituting out a_2 to obtain an intertemporal resource constraint, (24).

$$c_1 + \frac{c_2}{R^*} + \underbrace{\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right]^2}_{B} = y + \frac{y}{R^*} + R^* a_1 - \underbrace{R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1}_{A} \tag{24}$$

The non-negativity constraint on a_2 can be expressed in terms of c_1, c_2 as well, which gives us (25).

$$a_2 = \frac{c_2 - y}{R^*} + \frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \geqslant 0 \tag{25}$$

One can observe that the problem (23) looks very similar to the standard two-period consumption savings problem. The intertemporal resource constraint, (24), is also very similar to the usual intertemporal budget constraint that appears in the consumption-savings problem, except for two peculiar expressions in this equation- labelled A and B.

First, the expression labelled A, $R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1$, signifies the value of debt payments from previously existing debt. Since the consumer borrows in units of the domestic numeraire, the 'real' debt payments, i.e., payments in units of the tradeable consumption good, are dependent on the appreciation rate of the currency, $\left(\frac{c_1}{c_0}\right)^{\sigma}$, so that the 'real' interest rate is $R\left(\frac{c_1}{c_0}\right)^{\sigma}$. With c_0 given at $t = 1$, higher consumption in period-1, c_1, also means that the consumer pays a higher interest on previous debt. Intuitively, if debt is denominated in local currency, an exchange rate appreciation results in higher payments in units of the international currency. Similarly, lower consumption in period-1 results in low interest payments. In other words, an exchange rate depreciation lowers the burden of domestic currency denominated debt.

Second, the expression labelled B, $\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right]^2$, which is also equal to Γq_2^2, represents a quadratic resource cost associated with the use of the financial intermediation technology. The quadratic term signifies that only the magnitude matters, i.e., it does not matter whether the financial intermediation technology is used for borrowing or saving, but what matters is the extent to which it is used. Said differently, this expression, B, represents a resource cost associated with borrowing or saving through the intermediaries- in the form of profits made by them. The magnitude of this cost depends on the interest parity wedge, i.e., how much $\left(\frac{c_2}{c_1}\right)^{\sigma} \lessgtr \frac{R^*}{R}$ with no cost if and only if $q_2 = 0$ i.e., $\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{R^*}{R}$.

Panel (a) of Figure 3 plots (24) in the (c_1, c_2) plane and illustrates how the expressions A and B alter the usual downward sloping budget constraint that shows up in the basic consumption-savings

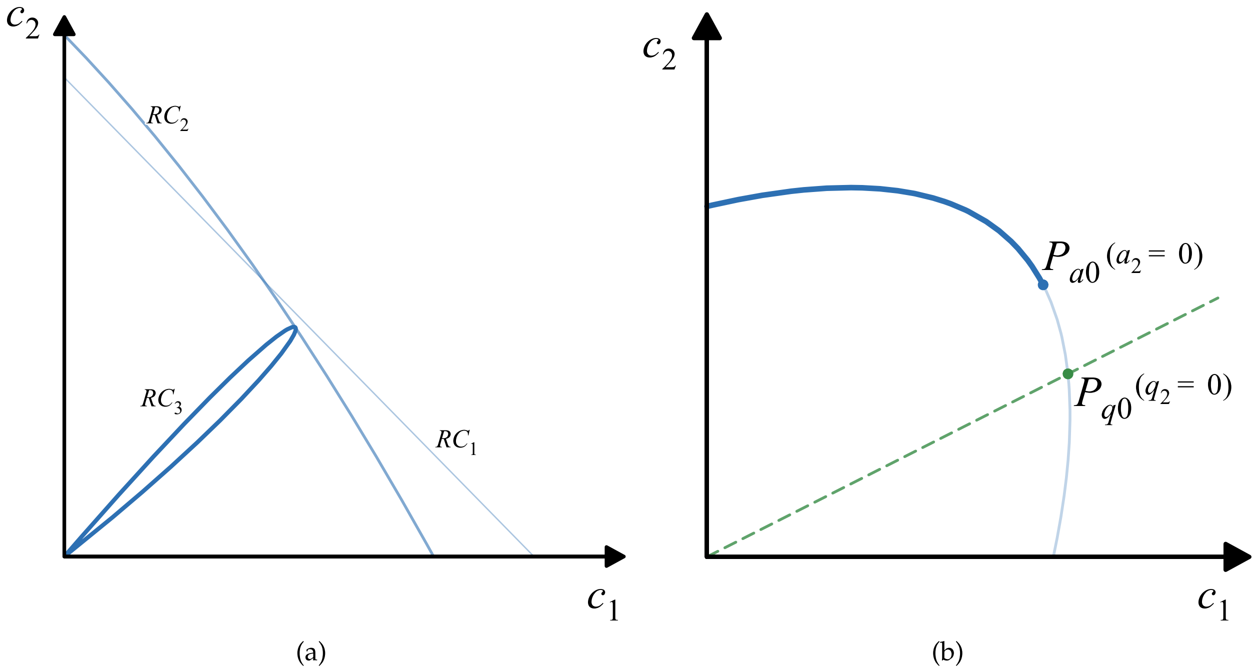


Figure 3: Intertemporal Resource Constraint

problem. First, if we ignore expression B and the term 'c_1' in expression A, we get the usual downward sloping constraint labelled RC_1 in panel (a) of Figure 3. Secondly, after incorporating the entire expression A, but still ignoring expression B, we get RC_2. The inward tilt along the c_1-axis illustrates the loss in resources due to higher debt payments being made when c_1 is higher; whereas the outward tilt along the c_2-axis illustrates that by lowering c_1 resources are saved by making smaller debt payments which allows higher consumption in period-2. Finally, if we also incorporate expression B, the intertemporal constraint is depicted by RC_3 in panel (a) of Figure 3. The peculiar petal shape [6] of this constraint reflects the effect of expression B- the quadratic loss in resources, i.e., the intermediation costs, as $\frac{c_2}{c_1} \to 0$ or $\frac{c_2}{c_1} \to \infty$. It is also noteworthy that RC_3 is tangent to RC_2 at the point where the loss is 0, i.e., $\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{R^*}{R}$.

In terms of our model, each point on RC_3 represents a competitive equilibrium which would prevail for some choice of a_2 by the central bank. Again relating this problem to the standard consumption-savings problem, the equivalent of an 'endowment' point here is the point on RC_3 where $a_2 = 0$, i.e., the non-negativity constraint (25) binds. Focusing only on the frontier of the petal shaped RC_3, this point is shown in panel (b) of the Figure 3 labeled P_{a0}. The points to the left of P_{a0}, shown by a darker shade, represent the set of feasible competitive equilibria which can

[6]The exact shape of RC_3 may or may not be a perfect petal as illustrated in Figure 3, the shape and convexity depends on the parameters- R, R^*, Γ, σ.

be reached by the central bank choosing $a_2 > 0$. In other words, by increasing a_2 we move left to right along this constraint. The lighter shaded region on the constraint, i.e., the region to the right of P_{a0}, represents infeasible competitive equilibria since they require $a_2 < 0$.

Since q_2 is endogenous in $\frac{c_2}{c_1}$, as asserted by the asset supply equation, every point on RC_3 also corresponds to a unique implied value for q_2. The panel (b) of Figure 3 shows the point where the intermediation loss is 0, i.e., $q_2 = 0$, no borrowing or saving with the intermediaries. This point is labelled P_{q0}. To the right of P_{q0} we have $\left(\frac{c_2}{c_1}\right)^{\sigma} < \frac{R^*}{R} \Leftrightarrow q_2 < 0$ i.e., positive saving with the intermediaries. Conversely, to the left of P_{q0}, there is positive borrowing through the intermediaries: $\left(\frac{c_2}{c_1}\right)^{\sigma} > \frac{R^*}{R} \Leftrightarrow q_2 > 0$. Where exactly P_{a0} and P_{q0} lie relative to each other depends on the initial state (a_1, q_1, c_0). In this Figure, P_{q0} lies in the infeasible region. In other words, even if the central bank runs down it's reserves to 0, and chooses to be at P_{a0}, the competitive equilibrium involves the consumer borrowing a positive amount through the intermediaries, given the initial state.

A movement along this constraint, accomplished by altering a_2, illustrates the effectiveness of FX interventions in this environment. As the central bank increases a_2 by moving leftwards from the point P_{a0}, it is able to alter the competitive equilibrium choices of c_1, c_2, q_2 made by the consumer and the intermediaries. Intuitively, as the central bank builds reserves, it induces the consumer to borrow more to offset this additional borrowing. However, as implied by the intermediaries' asset supply equation, the intermediaries are only willing to lend more if and only if they are offered a higher appreciation rate, i.e., the exchange rate must depreciate at $t = 1$. Such an exchange rate depreciation lowers c_1 and also decreases the 'real' value of previous debt payments (the expression A in (24)). Moreover, because this depreciation increases the appreciation rate of the currency, it brings in additional borrowing through the intermediaries and therefore also increases the intermediation resource costs (the expression B in (24)). It is noteworthy that due to the exchange rate depreciation, the increase in reserves is not met by a one-for-one increase in borrowing. Only a part of these reserves are financed through additional borrowing, the remaining is financed through a reduction in c_1, and dilution of previous debt payments. The key insight is that given the financial frictions, reserve operations have general equilibrium effects- the central bank is able to manipulate the exchange rate and thereby alter consumption, borrowing, and value of debt payments.

To clarify further, it is useful to compare this outcome to the frictionless case, i.e., $\Gamma = 0$, where reserve operations would be ineffective. To see this, observe that in the absence of the friction, the intermediary asset supply is perfectly elastic, i.e., the interest parity condition holds, which gives us the frictionless Euler equation:

$$\left(\frac{c_2}{c_1}\right)^{\sigma} = \frac{R^*}{R}$$

And the intertemporal constraint is given as below (same as (24) but with expression B=0):

$$c_1 + \frac{c_2}{R^*} = y + \frac{y}{R^*} + R^* a_1 - Rc_1^{\sigma} \frac{q_1}{c_0^{\sigma}}$$

The frictionless equilibrium is thus characterized by a unique net foreign asset position $(a_2 - q_2)$. In other words, any change in a_2 would be met by a one-for-one offsetting change in q_2 such that $(a_2 - q_2)$ does not change and hence no resultant change in c_1, c_2. Intuitively, due to a perfectly elastic supply of assets by the intermediaries, a version of Ricardian equivalence holds- in response to the central bank accumulating reserves, the consumer borrows more, one-for-one, so that the net foreign asset position does not change. This corresponds to the benchmark ineffectiveness of FX interventions result illustrated by Backus and Kehoe (1989).

Coming back to a world with financial frictions ($\Gamma > 0$), as mentioned previously, an increase in reserves is met by an exchange rate depreciation which lowers c_1 and dilutes previous debt payments in addition to an increase in borrowing. While dilution saves resources, the increase in q_2 also increases the size of the intermediation costs (the expression B in (24)). The net effect on c_2 depends on the relative size of the intermediation costs and the extent of dilution. The central bank therefore faces a tradeoff. By building more and more reserves, on one hand it saves resources by diluting previous debt payments, whereas on the other hand, the 'premium' associated with the increase in borrowing through intermediaries also leads to a resource loss. The optimal reserve choice is thus such that the marginal gain from dilution is equal to the marginal loss associated with the intermediation leakage.

Demonstrating this choice, panel (a) of Figure 4 shows the optimal solution where the indifference curve is tangent to the intertemporal resource constraint (24) at the point P^*. The optimal choice features a positive reserve position $a_2 > 0$ and a positive debt position $q_2 > 0$. This brings us back to the question we raised earlier: why does a country hold both external debt and reserves? Why is it not optimal to run down the reserves to pay off the debt? In other words why is the point P_{a0} not optimal? The answer lies in the fact that while running down the reserves will allow the country to borrow lesser, it also results in an exchange rate appreciation which increases the 'real' payments on previous domestic currency denominated debt. The resource loss associated

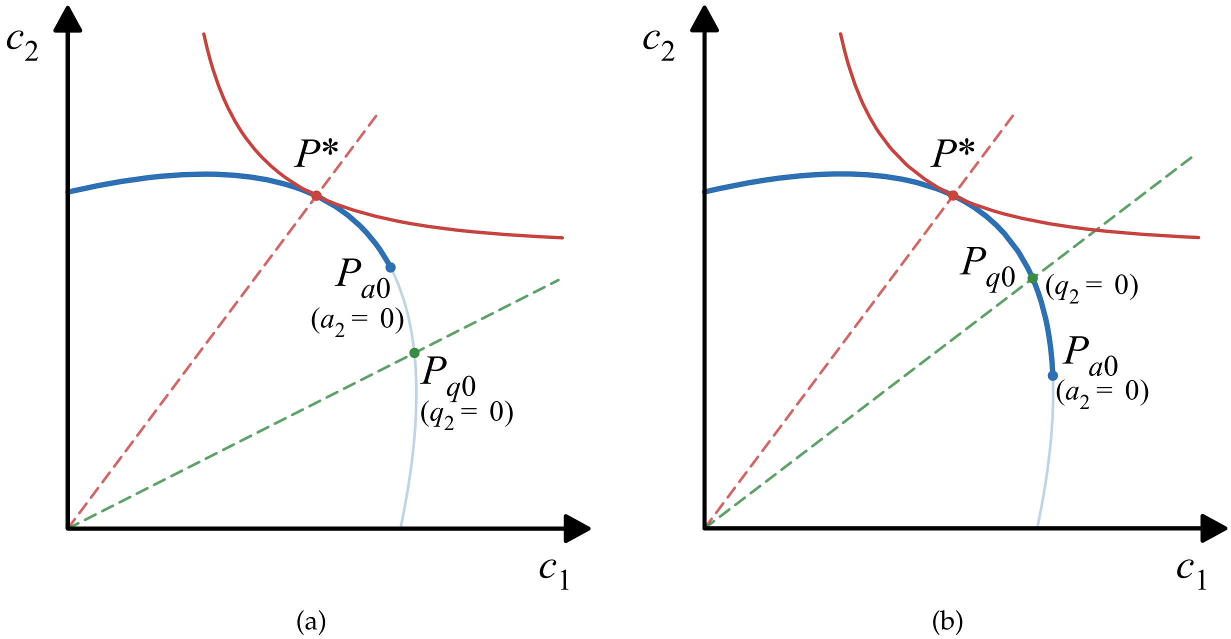


Figure 4: Optimal Portfolio

with this appreciation exceeds the gain generated by lower borrowing. Said differently, by holding reserves, the central bank is equating the marginal benefits of debt dilution to the marginal costs of additional household borrowing. This result is formally stated in Lemma 1. This Lemma establishes the conditions under which P_{a0} is not optimal and P^* lies to the left to P_{a0}.

Lemma 1. *Suppose that* $c_0 > 0, a_1 > 0, q_1 > 0$ *and suppose there exists* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *that satisfies the constraints of* (23) *such that:*

$\hat{a}_2 = 0,$

$\hat{q}_2 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma} - 1\right] \geqslant 0$

$\hat{c}_2 = y - R^*(1 + \Gamma\hat{q}_2)\hat{q}_2$

$\hat{c}_1 - \hat{q}_2 = y + R^* a_1 - R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1$

If

$$R^* a_1 + (\sigma - 1)R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 > \left[\frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} - 1\right] y + \Gamma\hat{q}_2^2 + (2\sigma - 1)(\hat{q}_2 + \Gamma\hat{q}_2^2)\left(R\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} + 1\right)$$

then $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *is NOT a solution to* (23). *Furthermore, the optimal solution involves* $a_2 > 0$ *and* $q_2 > \hat{q}_2$.

Proof. See Appendix B.1. □

In panel (a) of Figure 4 P_{q0} lies in the infeasible region, i.e., not using the intermediation technology is not an option even if reserves are set to 0. However, depending on the initial state, it is possible that P_{q0} lies within the set of feasible competitive equilibria. This is illustrated in the panel (b) of Figure 4. The Figure shows that there exists a positive level of reserves that could eliminate the use of the intermediation technology and set $q_2 = 0$. At this point the intermediation leakage is minimized. However, Lemma 2 establishes that as long as previous debt $q_1 > 0$, choosing $q_2 = 0$ can never be optimal. In other words, at P_{q0} the welfare benefits from dilution always exceed the losses from marginally using the intermediation technology. In simpler words, the optimal point P^* always lies to the left of P_{q0}. This is illustrated in panel (b) of Figure 4.

Lemma 2. *Suppose that* $c_0 > 0, a_1 > 0, q_1 > 0$ *and suppose there exists* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *that satisfies the constraints of* (23) *such that:*

$\hat{q}_2 = 0,$

$\hat{c}_2 = \left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1$

$\hat{a}_2 = \frac{1}{R^*}\left(\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1 - y\right) \geqslant 0$

$\hat{c}_1 + \frac{1}{R^*}\left(\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1 - y\right) = y + R^* a_1 - R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1$

Since $q_1 > 0$*, then* $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ *is NOT a solution to* (23)*. Furthermore, the optimal solution involves* $q_2 > 0$ *and* $a_2 > \hat{a}_2$*.*

Proof. See Appendix B.2. □

The results of the two-period model can be generalized to the infinite horizon policy problem. Consider the deterministic version of the policy problem, (22) with $\Gamma_t = \Gamma, \forall t$. Suppose the economy begins in an initial state (q_0, a_0, c_{-1}) with $q_0 > 0$ and $a_0 > 0$. In this case, a unique portfolio of debt and reserves, $\{a_{t+1}, q_{t+1}\}$ characterizes the solution to this problem. The presence of this unique portfolio is due to the friction. For the sake of comparison, consider the frictionless case. Analogous to the two-period frictionless case discussed above, the solution is entirely characterized by the interest parity condition (frictionless Euler equation):

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \frac{R^*}{R}$$

And an intertemporal resource constraint:

$$\sum_{t=0}^{\infty} \frac{c_t}{{R^*}^t} = \sum_{t=0}^{\infty} \frac{y}{{R^*}^t} + R^* a_0 - R c_0^{\sigma} \frac{q_1}{c_{-1}^{\sigma}}$$

As a result, a unique NFA $(a_{t+1} - q_{t+1})$ characterizes the optimal asset position. Any choice of reserves would not be effective in altering the consumption sequence- Ricardian equivalence implies that reserve operations would be ineffective.

In the presence of the friction, however, this equivalence does not hold anymore and by choosing to hold reserves, the central bank is able to manipulate the exchange rate and therefore the equilibrium choices of consumption, borrowing, and value of debt payments. The central bank faces a tradeoff between dilution benefits from time-0 debt payments, and the intermediation costs associated with additional consumer borrowing in response to the central bank accumulating reserves ($a_{t+1} > 0$ for any $t \geqslant 0$). The optimal sequence of reserves, $\{a_{t+1}\}$, is such that the marginal benefits of diluting time-0 debt are equal to the marginal resource costs of the additional consumer borrowing. Since $R^* < \beta^{-1}$, as one would expect, the central bank eventually finds it optimal to run down the reserves to 0 over time[7]. Reserves decline monotonically to 0 and there exists a $T \geqslant 1$ (with strict inequality for some initial states) such that $a_t > 0$ for all $t < T$. Moreover, once the central bank has run down its reserves, the transition dynamics are identical to the laissez-faire dynamics discussed in section 3.5, i.e., the economy approaches a steady state. This result is formally summarized in Proposition 1.

Proposition 1. *Let* $R^* < R$ *and* $\lim_{t\to\infty} \beta^t q_{t+1} = 0$. *Suppose* $q_0 \geqslant 0, a_0 > 0$. *Consider the deterministic version of the policy problem* (22) *with* $\Gamma_t = \Gamma > 0, \forall t$. *The problem has a solution with a steady state limit* $(\bar{q}, \bar{c}, \bar{a})$*:*

- $q_{t+1} > 0, \forall t; \lim_{t\to\infty} q_{t+1} = \bar{q} = \frac{1}{\Gamma}\left[\frac{R}{R^*} - 1\right]$
- $\lim_{t\to\infty} c_t = \bar{c} = y - (R-1)\bar{q}$
- $\exists T \geqslant 1$ *(with strict inequality for some initial states* (q_0, a_0)*) such that* $a_t > 0, \forall t < T$ *and* $a_t = \bar{a} = 0, \forall t \geqslant T$.

Figures 12, 13 in Appendix C, show numerical simulations, illustrating Proposition 1.

Next, we continue to analyze the optimal policy problem and illustrate that the solution is time-inconsistent.

[7] This is analogous to the the usual infinite-horizon consumption-savings problem with $R^* < \beta^{-1}$ where any initial wealth $a_0 > 0$ is driven down to 0 (or in general, to the borrowing limit) over time: $a_0 \geqslant a_1 \geqslant a_2 ... \geqslant 0$.

3.7 Time inconsistency of the Ramsey optimal reserve policy

In this sub-section we continue to analyze the Ramsey optimal reserve policy that solves the deterministic version of (22). One way to think of the Ramsey planning problem is that at time-0 a Ramsey planner is followed by a sequence of continuation Ramsey planners at times $t = 1, 2, ...$ Consider a transformed version of the policy problem by denoting the net-foreign asset position in period t as $x_t = a_{t+1} - q_{t+1}$. A time-t continuation Ramsey planner takes (c_t, x_t) for $t \geqslant 1$ as state variables passed onto it by the time-(t-1) planner and is obligated to choose the gross asset positions a_{t+1} and q_{t+1} so that they sum up to the 'promised' net position x_t. The time-t planner also chooses (c_{t+1}, x_{t+1}) and passes them as state variables to the time-(t+1) planner. The time-t planner's objective is to maximize continuation utility subject to three constraints: a promise-keeping constraint, (26), a BoP constraint for period t+1, (27), and the intermediaries' asset supply equation, (28):

$$a_{t+1} - q_{t+1} = x_t \tag{26}$$

$$c_{t+1} + x_{t+1} = y + R^* a_{t+1} - R\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} q_{t+1} \tag{27}$$

$$q_{t+1} = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} - 1\right] \tag{28}$$

However, the time-0 planner faces a different set of constraints. It does not face (x_0, c_0) as state variables. Rather it faces (c_{-1}, a_0, q_0) as state variables and has the ability to choose (c_0, q_1, a_1) without any promise-keeping restriction. The time-0 planner chooses (c_0, q_1, a_1) as well as (c_1, x_1), which are passed as state-variables to the time-1 planner, subject to period-0 and period-1 BoP constraints,(29), (30), and a period-0 asset supply equation, (31):

$$c_0 + a_1 - q_1 = y + R^* a_0 - R\left(\frac{c_0}{c_{-1}}\right)^{\sigma} q_0 \tag{29}$$

$$c_1 + x_1 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1 \tag{30}$$

$$q_1 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] \tag{31}$$

The source of time-inconsistency is evident from the fact that while the time-t continuation planner is restricted by accepting (c_t, x_t) as state variables and faces a promise-keeping constraint, (26), the time-0 planner does not face these restrictions. The time-t continuation planner at every $t \geqslant 1$ potentially has an incentive to ignore the promise-keeping constraint and act like a time-0 planner. To better illustrate this problem and understand the direction of a potential deviation from commitment, we next consider a three-period version of the policy problem.

3.7.1 Three-period policy problem: Commitment vs Deviation

Consider the three-period deterministic version of (22). Let $t = 0, 1, 2$ with the initial state (a_0, q_0, c_{-1}) given at $t = 0$. Let $\Gamma_t = \Gamma, \forall t$. As before, we assume that the economy begins with some initial debt and reserves: $a_0, q_0 > 0$. After substituting the asset supply equations into the BoP constraints, the time-0 policy problem, i.e., the problem under commitment, is given by (32).

$$
\begin{gathered}
\max_{\{c_0,c_1,c_2,a_1,a_2\}} \frac{\omega c_0^{1-\sigma}}{1-\sigma} + \beta\frac{\omega c_1^{1-\sigma}}{1-\sigma} + \beta^2\frac{\omega c_2^{1-\sigma}}{1-\sigma} \quad \text{s.t.} \\
c_0 - \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] + a_1 = y + R^* a_0 - R c_0^{\sigma}\frac{q_0}{c_{-1}^{\sigma}} \\
c_1 - \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] + a_2 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right] \\
c_2 = y + R^* a_2 - R\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \\
a_1, a_2 \geqslant 0
\end{gathered}
\tag{32}
$$

Consider now a potential deviation from the commitment plan at $t = 1$, i.e., the planner is allowed to re-optimize and choose (c_1, c_2, a_2, q_2). The problem faced by the time-1 planner is identical to the two period problem discussed previously, (23). Comparing the $t = 1, 2$ BoP constraints of (32) with those of (23), the source of time-inconsistency is established: the time-1 planner takes q_1 as a state variable. In other words, while the period-0 asset supply equation $q_1 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1}{c_0}\right)^{\sigma} - 1\right]$ is a constraint for the time-0 planner, it is not a constraint for the time-1 planner. The time-0 planner internalizes that the choice of c_1 affects the quantity of borrowing received from the intermediaries at $t = 0$ and therefore the debt burden at $t = 1$. On the other hand, since time-1 planner takes q_1 as a given, from his perspective, the choice of c_1 does not affect the quantity of debt burden, q_1, at $t = 1$.

Let $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ denote the solution to (32), which we call the solution under 'commitment' as the planner is required to make this choice at $t = 0$ and then must commit to this solution at $t = 1$, i.e., it is not allowed to re-optimize at $t = 1$. Suppose further that the initial state (a_0, q_0, c_0)

is such that $a_1^c, a_2^c > 0$.

Now consider a hypothetical situation, in which, after having made the choices at $t = 0$, the planner is allowed to re-optimize at $t = 1$, by solving (23), taking (a_1^c, q_1^c, c_0^c) as state variables. Let the solution to (23) under this 'deviation' situation be given by $(c_1^D, c_2^D, a_2^D, q_2^D)$. How does c_1^c compare to c_1^D? We claim that if $a_1^c, a_2^c > 0$, then $c_1^D > c_1^c$.

The reason for this is that for a time-0 planner, increasing c_1 is more costly at the margin because he internalizes that increasing c_1 also increases the quantity of the debt burden, q_1. While increasing c_1 brings more borrowing through the intermediaries at $t = 0$, it also increases the debt burden at $t = 1$. As long as $a_1 > 0$ at the optimum, this additional borrowing is always more costly than beneficial, i.e., it is cheaper to raise resources at $t = 0$ by decreasing a_1 rather than increasing q_1 because of the intermediation resource costs associated with increase in q_1.[8].

On the other hand, for a 'deviation' planner at $t = 1$ who takes q_1 as given, this marginal cost disappears: increasing c_1 does not increase the quantity of the debt burden, q_1, which was already chosen by the intermediaries and the consumer at $t = 0$[9]. Given that this additional marginal cost has disappeared and the marginal benefits have not changed, the time-1 deviation planner would like to increase c_1 above the 'announced' level by lowering the reserves below the 'announced' level, i.e., $c_1^D > c_1^C$ and $a_1^D < a_1^c$. This result is formally established in Lemma 3.

Lemma 3. *Let $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ be the solution to (32) given (a_0, q_0, c_{-1}). Suppose $q_1^c, a_1^c, a_2^c > 0$. Then at time $t = 1$, given (a_1^c, q_1^c, c_0^c), let $(c_1^D, c_2^D, a_2^D, q_2^D)$ be the solution to the two period sub-problem (23).*

Since $a_1^c, a_2^c, q_1^c > 0$, then $c_1^D > c_1^c$, $a_2^D < a_2^c$, $c_2^D < c_2^c$ and $q_2^D < q_2^c$.

Proof. See Appendix B.3 □

To summarize this idea in terms of our model, the central bank's reserve accumulation policy under commitment is time-inconsistent. At $t = 0$, the announced reserve policy (a_1^c, a_2^c) is such that offers a lower exchange rate appreciation rate between period 0 and 1, $\left(\frac{c_1^c}{c_0^c}\right)^\sigma$. This lower appreciation rate results in a smaller lending by intermediaries at $t = 0$ and hence a smaller debt burden at $t = 1$. At $t = 1$, given the debt burden, q_1^c, the central bank would find it optimal to reduce its reserves, a_2, below the 'announced' levels. This 'surprise' intervention induces an

[8]We discuss this intuition more formally with the help of first order conditions in Appendix B.3

[9]The choice of c_1 at $t = 1$ still affects the interest rate paid on q_1, i.e., $R\left(\frac{c_1}{c_0}\right)^\sigma$, but not the quantity itself.

exchange rate appreciation, thereby allowing the household to consume more: $c_1^D > c_1^c$ and also increases the ex-post appreciation rate[10] $\left(\frac{c_1^D}{c_0^c}\right)^{\sigma}$.

This line of reasoning applies to the infinite horizon problem as well. The result is formally stated in Proposition 2.

Proposition 2. *Let* $\{c_t, a_{t+1}, q_{t+1}\}_{t=0}^{\infty}$ *be the time-0 Ramsey plan that solves the deterministic version of* (22) *given* (a_0, q_0, c_{-1}). *Then at any* $T > 0$, *if* $a_T, a_{T+1}, q_T > 0$, *the continuation of the time-0 plan,* $\{c_t, q_{t+1}, a_{t+1}\}_{t=T}^{\infty}$, *is NOT a Ramsey plan that solves* (22) *given* $(a_t q_T, c_{T-1})$.

Figure 14 in Appendix C shows a numerical simulation illustrating Proposition 2.

Given the time-inconsistency of the Ramsey optimal reserve policy, it is imperative to discuss a time-consistent reserve accumulation policy. In the next section we continue our discussion of the three-period deterministic policy problem, but now assuming that the central bank lacks commitment, we describe a time-consistent equilibrium.

3.7.2 Three-period policy problem: Equilibrium under Lack of Commitment

The previous section established that the Ramsey optimal reserve accumulation policy is time-inconsistent. In other words, a central bank that lacks commitment would find it optimal to deviate from previously announced plans. Along the equilibrium path, the market participants would anticipate such deviations and internalize the possibility of a deviation in future. In this sub-section, we continue our discussion of the deterministic three-period policy problem, but now consider a time-consistent equilibrium. We solve for this equilibrium using backwards induction.

At $t = 1$, the policy problem is identical to the two-period problem (23) given (a_1, q_1, c_0) as state variables. Let $\mathcal{C}_1(a_1, q_1, c_0)$, $\mathcal{C}_2(a_1, q_1, c_0)$ and $\mathcal{A}_2(a_1, q_1, c_0)$ be the optimal decision rules that solve (23). At $t = 0$, the planner internalizes these decision rules to solve for (a_1, q_1, c_0). At $t = 0$ the policy problem is given by (33).

[10] It is useful at this stage to describe an analogy: the Coase conjecture (Coase, 1972) about a durable good monopolist who faces two types of consumers- high valuation and low valuation consumers over two time periods. If the high value consumers are patient enough they can wait until period 2 to buy the good. Considering this, the monopolist would like to 'announce' that he would commit to a high price for the good in both the periods. The high value consumers will find it optimal to buy the good in period 1. In period 2, since the high value consumers have already bought this durable good, the only remaining consumers in the market are low value consumers. The monopolist no longer finds its optimal to commit to the previously announced high price in period 2 and would like to deviate to a lower price to serve the low value consumers.

$$\max_{\{c_0,a_1,q_1\}} \frac{\omega c_0^{1-\sigma}}{1-\sigma} + \beta \frac{\omega C_1(c_0,a_1,q_1)^{1-\sigma}}{1-\sigma} + \beta^2 \frac{\omega C_2(c_0,a_1,q_1)^{1-\sigma}}{1-\sigma} \quad \text{s.t.}$$

$$c_0 - q_1 + a_1 = y + R^* a_0 - R\left(\frac{c_0}{c_{-1}}\right)^{\sigma} q_0 \tag{33}$$

$$q_1 = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{C_1(c_0,a_1,q_1)}{c_0}\right)^{\sigma} - 1\right]$$

The period-0 asset supply equation in (33) suggests that intermediaries internalize that the central bank lacks commitment and take future policy, $C_1(.)$, as a given when lending at $t = 0$. Similarly, the planner's objective also takes future policies as given. Let $(c_0^l, c_1^l, c_2^l, a_1^l, a_2^l, q_1^l, q_2^l)$ be the time-consistent equilibrium solution to this problem given (a_0, q_0, c_{-1}). How does this solution compare to the solution to (32)? Specifically, how does a_1^c compare to a_1^l? We claim that if $a_1^c, a_2^c, q_1^c > 0$ then $a_1^l > a_1^c$.

Let us start with the solution to (32), $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$. First observe that the solution to (32) is no longer feasible as it violates the asset supply equation constraint for (33). This follows from Lemma 3 where we established that $c_1^D \equiv C_1(a_1^c, q_1^c, c_0^c) > c_1^c$, i.e., at $t = 1$, given (a_1^c, q_1^c, c_0^c), the central bank would like to deviate to a higher level of consumption by lowering reserves below the 'announced' level. At $t = 0$, the intermediaries will anticipate this deviation and will want to lend more in response to this deviation. In other words, the asset supply equation constraint in (33) is violated:

$$q_1^c < \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{C_1(c_0^c, a_1^c, q_1^c)}{c_0^c}\right)^{\sigma} - 1\right]$$

Therefore, the solution to (32) in no longer feasible for the time-consistent problem. Intuitively, we think of this as the domestic bonds market clearing condition getting violated with the supply of funds exceeding the demand, i.e., there is excess lending by the intermediaries at $t = 0$ in response to the anticipated deviation in future. This result is formally stated in Lemma 4.

Lemma 4. *Let* $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ *be the solution to* (32) *given* (a_0, q_0, c_{-1}). *Let* $a_1^c, a_2^c, q_1^c > 0$. *Then* (c_0^c, a_1^c, q_1^c) *is NOT a solution to* (33) *given* (a_0, q_0, c_{-1}).

Proof. See Appendix B.4 □

Given that the intermediaries anticipate the central bank to deviate at $t = 1$, they are willing to lend more at $t = 0$. This creates a situation of excess supply of funds: intermediaries are willing to lend more than what the consumer is willing to borrow at the current exchange rate. To restore the equilibrium, the exchange rate must appreciate at $t = 0$, i.e., even if the central bank does not change its reserve position, a_1^c, c_0 will increase to absorb the excess lending and clear the bond market. However, such an exchange rate appreciation will also increase the debt payments on previous debt, $R\left(\frac{c_0}{c_{-1}}\right)^{\sigma} q_0$. This creates a motive for central bank to respond by increasing its reserve position to prevent excessive appreciation. In other words, the excess lending by the intermediaries is partially absorbed through an exchange rate appreciation and partially by the central bank increasing it's reserves. Therefore in the time-consistent equilibrium, we have $c_0^l > c_0^c$ and $a_1^l > a_1^c$. We formally summarize this result in Proposition 3.

Proposition 3. *Given* (a_0, q_0, c_{-1})*, let* $(c_0^c, c_1^c, c_2^c, a_1^c, a_2^c, q_1^c, q_2^c)$ *be the solution to* (32)*. Suppose* $a_1^c > 0, a_2^c, q_1^c > 0$*. Let* $(c_0^l, a_1^l, q_1^l, q_2^l)$ *solve* (33) *given* (a_0, q_0, c_{-1})*. Then* $a_1^l > a_1^c$ *and* $c_0^l > c_0^c$*.*

Figure 15 in Appendix C shows a numerical example comparing the equilibrium solutions under commitment and lack of commitment, thereby illustrating Proposition 3, in particular, $a_1^l > a_1^c$. The argument outlined above applies to the infinite horizon model as well. In the next section, we define a time-consistent equilibrium for the infinite horizon policy problem and provide a quantitative solution to the equilibrium in the presence of shocks to financial flows.

4 Quantitative Analysis

In this section we present a quantitative analysis of our model. First, we define a time-consistent equilibrium for the infinite horizon stochastic policy problem. Second, using a test calibration, we present a solution to the time-consistent equilibrium, illustrating the decision rules of the central bank. Third, using a simulation, we compare the long-run equilibrium with a laissez-faire equilibrium. We illustrate that in the presence of financial shocks, reserves provide an insurance against volatile exchange rates. The optimal policy is to lean against the wind and accumulate reserves in financially stable times and run down reserves in times of disruptions. In the long-run equilibrium, the economy carries both debt and reserves simultaneously.

4.1 A Time-Consistent Policy Problem

In order to define a time-consistent equilibrium, it is useful to setup a recursive policy problem. For the sake of computational ease, it is also useful to setup the problem with the exchange rate e as the choice variable rather rather than consumption. Let the endogenous state variables be given by the reserve position at the beginning of a period, a, and the existing debt denominated in domestic currency, $\tilde{q}$.

We assume that Γ is the exogenous state variable and follows a Markov switching regime. As discussed in sub-section 3.5, a lower value of Γ corresponds to periods of financial boom, a loosening of the credit constraint faced by the intermediaries which results in increased capital inflows. Conversely, a higher value of Γ corresponds to periods of financial disruption, due to a tightening of the intermediation credit constraint, the economy experiences a capital flight.

Let $V(.)$ denote the value function associated with the consumer's utility function and let $\mathcal{E}(.)$ denote the exchange rate policy function. The control variables are choices of reserves, a', domestic currency denominated borrowing, $\tilde{q}'$, and the exchange rate, e. The central bank's recursive policy problem is given by (34).

$$
\begin{aligned}
V\big(a, \tilde{q}, \Gamma\big) = \max_{a', \tilde{q}', e} & \left\{ \omega^{\frac{1}{\sigma}} \frac{e^{1-\frac{1}{\sigma}}}{1-\sigma} + \beta \mathbb{E}_{\Gamma'|\Gamma}\Big[V\big(a', \tilde{q}', \Gamma'\big)\Big] \right\} \qquad \text{s.t.} \\
& \omega^{\frac{1}{\sigma}} e^{1-\frac{1}{\sigma}} - \tilde{q}' + e a' = e y - R\tilde{q} + e R^* a \\
& \tilde{q}' = \frac{1}{\Gamma}\left[\frac{R}{R^*}\mathbb{E}_{\Gamma'|\Gamma}\left(\frac{e^2}{\mathcal{E}(a', \tilde{q}', \Gamma')}\right) - e\right] \\
& a' \geqslant 0
\end{aligned} \tag{34}
$$

First, we have substituted out c in terms of e in the objective function using (13). The first constraint is the BoP equation (15) where we have also substituted out c in terms of e and substituted $(\tilde{b}, \tilde{b}')$ in terms of $(\tilde{q}, \tilde{q}')$ using (12). The second constraint is the intermediaries' asset supply equation (9). The future exchange rate in this equation is denoted by the future optimal policy $\mathcal{E}(.)$, reflecting the fact that the central bank lacks commitment and the intermediaries internalize this, i.e., they take future exchange rate policy as a given while making their lending decisions. Next, we define a time-consistent (Markov) equilibrium.

4.1.1 A Time-Consistent (Markov) Equilibrium

A time consistent (Markov) equilibrium is defined by optimal exchange rate and borrowing decision rules $\big(\mathcal{E}(.), \tilde{\mathcal{Q}}(.)\big)$, a reserve accumulation rule $\mathcal{A}(.)$, and a value function $V(.)$ such that:

- Given a conjectured future exchange rate policy, $\mathcal{E}(.)$, and a reserve accumulation rule $\mathcal{A}(.)$, $\big(\mathcal{E}(.), \tilde{\mathcal{Q}}(.)\big)$ constitute a competitive equilibrium.

- Given conjectured future exchange rate policy, $\mathcal{E}(.)$, and the competitive equilibrium, $\left(\mathcal{E}(.), \tilde{Q}(.)\right)$, the central bank follows a reserve accumulation rule $\mathcal{A}(.)$ such that $V(.)$ attains a maximum.

- The conjectured future exchange rate policy is indeed correct: $\mathcal{E}(.)$.

4.2 Calibration

We solve (34) using a test calibration, with parameter values not significantly different from standard choices in literature. A period in the model corresponds to a year. The domestic interest rate is assumed to be 4% ($R = \beta^{-1} = 1.04$). Since it is crucial that $R^* < R$, we set the world interest rate at 2% ($R^* = 1.02$). The risk aversion parameter, $\sigma = 2$ and the tradeable good preference parameter, $\omega = 0.3$. The tradeable good endowment is normalized, $y = 1$. The long-run equilibrium value of Γ is set as $\bar{\Gamma} = 0.05$. This implies that in the deterministic steady-state, we have $\bar{c} = 0.9843$ and $\bar{q} = 0.392$.

We assume that $\ln \Gamma$ follows an $AR(1)$ structure:

$$\ln \Gamma_{t+1} = (1 - \phi) \ln \bar{\Gamma} + \phi \ln \Gamma_{t+1} + \epsilon_{t+1} \qquad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon)$$

Assuming that the financial shock is persistent, we set $\phi = 0.95$ and $\sigma_\epsilon = 0.5$. The $AR(1)$ process is discretized to a three-state Markov chain using the Rowenhorst method.

4.3 Results

We solve (34) using value function iteration. In this section we present the optimal decision rules and the associated long-run moments for a simulated economy. We establish that in the long-run equilibrium the economy carries both debt and reserves simultaneously. In other words, the optimal exchange rate policy is a managed float wherein the central bank actively uses reserves to manage the exchange rate. We compare this equilibrium to a laissez-faire outcome where the exchange rate is allowed to freely float.

Figures 5, 6, 7 depict the policy functions for reserves, borrowing, and exchange rate respectively. In each of the figures, the left panel shows how the corresponding variable varies with the beginning of period reserves for a given level of debt. Similarly, the right panel shows how the corresponding variable varies with debt, for a given level of initial reserves. In both panels of all figures, we plot the policy functions for the lowest and the highest state of Γ.

Figure 5 illustrates a *leaning-against-the-wind* reserve accumulation policy: the central bank builds reserves ($a' \geqslant a$) in financially stable times, i.e., periods when Γ is low and the economy receives higher financial inflows. Whereas, in periods of financial disruption, when Γ is higher, the central bank finds it optimal to run down the reserves ($a' < a$). The amount of reserve accumulation/decumulation is also affected by the beginning of period debt: a low debt economy accumulates more reserves, whereas there is little to no accumulation in a high debt economy. In other words, reserve accumulation becomes more costly as the debt levels increase. The left panel of Figure 6 shows that borrowing is a decreasing function of the beginning of period reserves: more reserves allow for smaller borrowing. The right panel shows that the economy accumulates debt when the initial debt levels are lower whereas at higher initial debt levels, it is optimal to run down the debt. The exchange rate policy function illustrated in Figure 7 shows that starting a period with more reserves allows for a lower exchange rate (and higher consumption), whereas higher existing debt is associated with a higher exchange rate and hence lower consumption. The figures also show the impact of Γ on optimal choices: an increase in Γ leads to reduction in reserves and borrowing and an exchange rate depreciation.

We observe that in the presence of financial shocks, reserves are equivalent to a precautionary saving. In other words, they act as an insurance against the volatility in consumption that results from exchange rate movements created by the financial shocks. As illustrated in section 3.5, an increase in Γ results in an exchange rate depreciation (lower consumption) and conversely

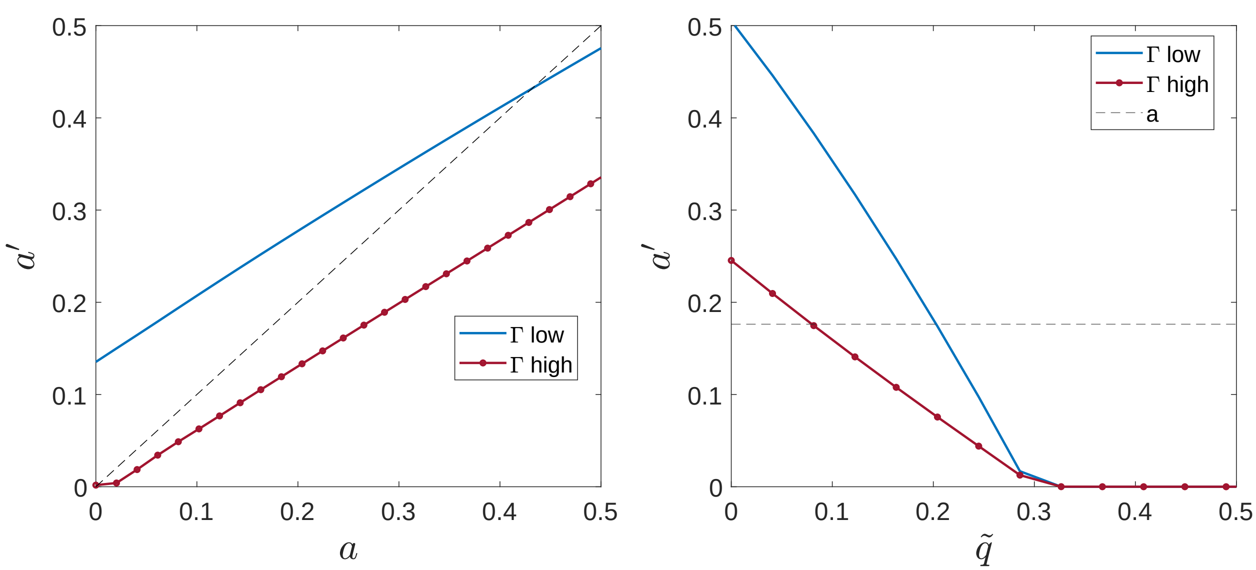


Figure 5: Policy Function- Reserves

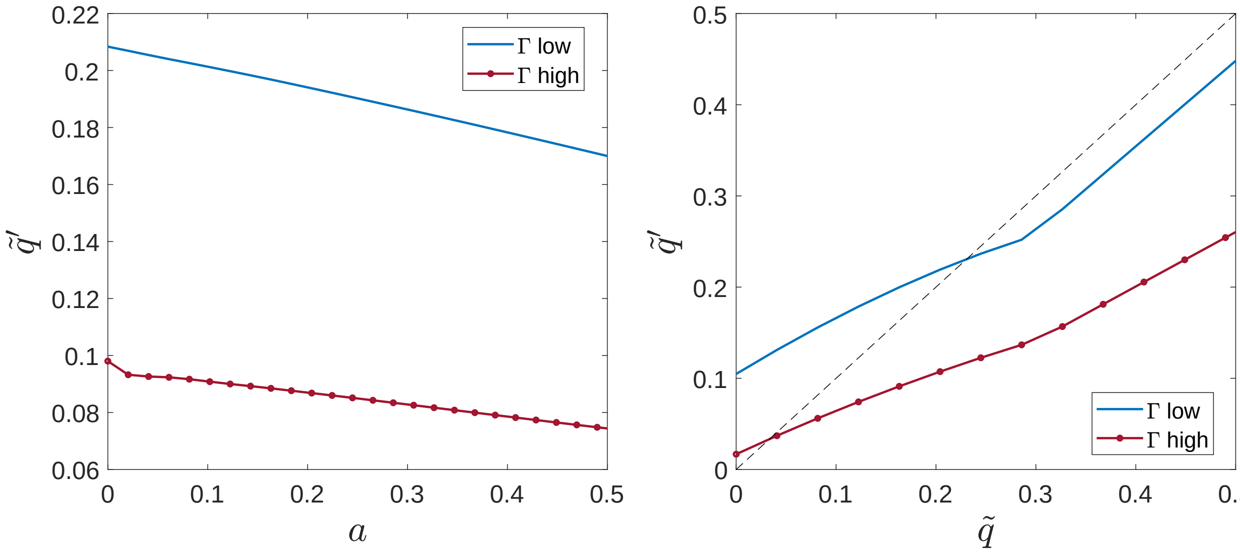


Figure 6: Policy Function- Debt

a decrease leads to an appreciation (higher consumption). Accumulation of reserves in times when Γ is lower is then equivalent to buying an insurance to mitigate the degree of exchange rate depreciation induced by a potential increase in Γ in the future. In other words, by running down reserves, the central bank can decrease the extent of capital outflow led depreciation during times of financial disruptions, thereby providing higher consumption.

However, the optimal insurance, i.e., the reserve accumulation in 'good' times, is influenced by two

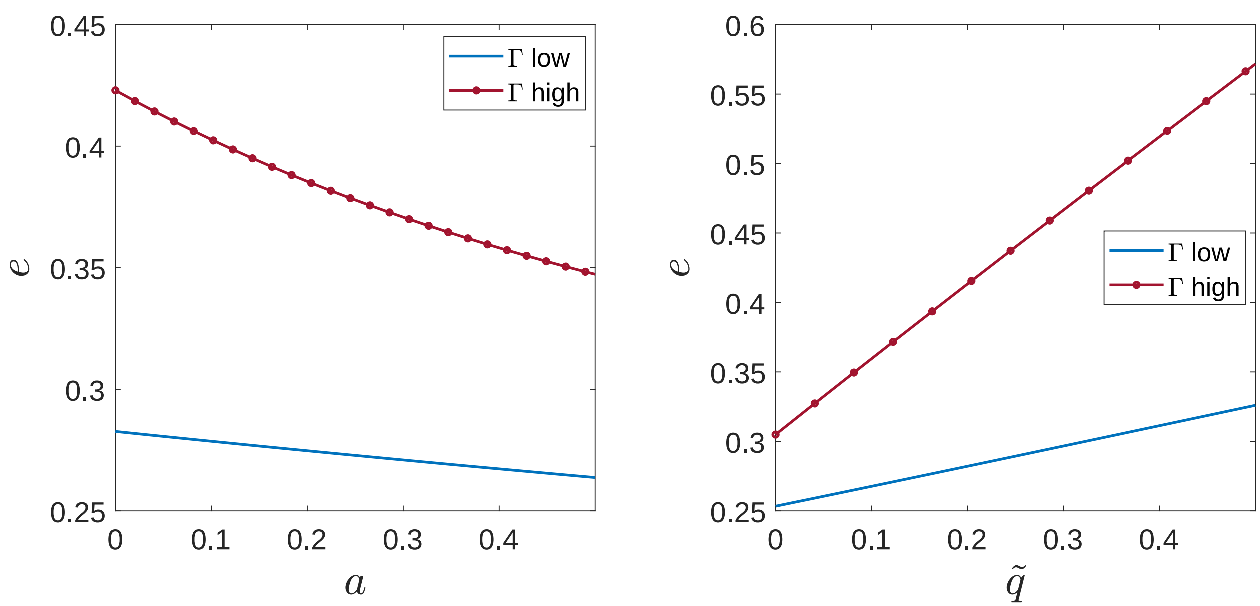


Figure 7: Policy Function- Exchange Rate

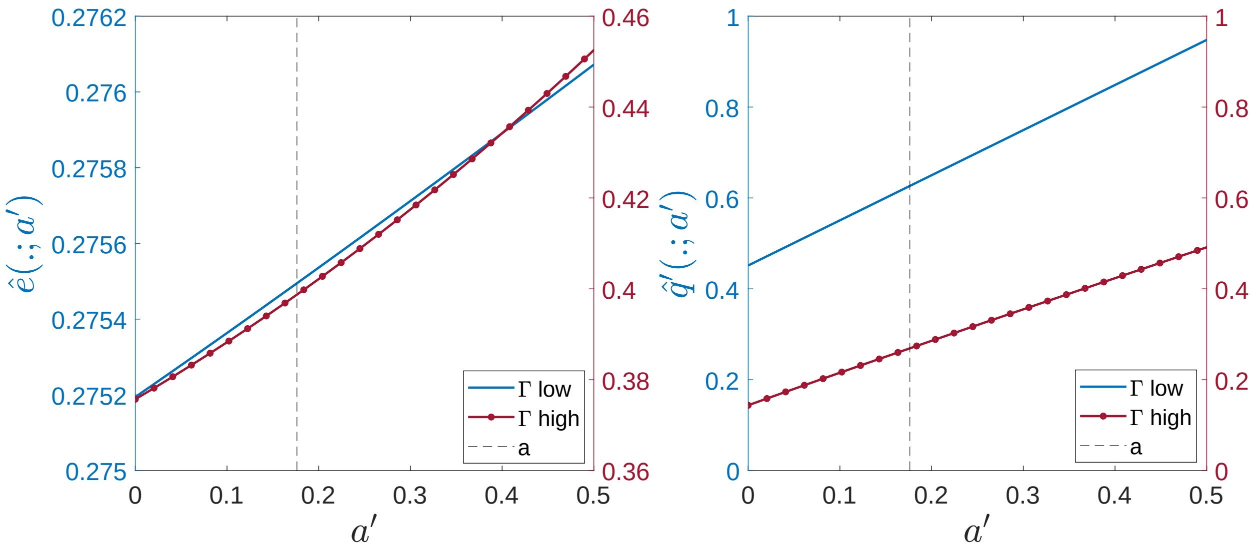


Figure 8: Competitive equilibrium variation with reserves

costs: exchange rate depreciation and higher debt. We illustrated these costs using the two-period model in section 3.6.1. The same is true for the infinite horizon model. To elucidate further, we show how the choice of reserves alters the competitive equilibrium. Fix an arbitrary choice of a', then the competitive equilibrium can be described by two implicit functions $\hat{e}(a, \tilde{q}, \Gamma; a')$ and $\hat{\tilde{q}}'(a, \tilde{q}, \Gamma; a')$ that satisfy the BoP constraint and the asset supply equation in (34)[11]. In Figure 8 we plot $\hat{e}(.)$ and real debt, $\frac{\hat{\tilde{q}}'(.)}{\hat{e}(.)}$, with respect to a' for a given initial state $(a, \tilde{q})$. First, we observe, that reserve accumulation leads to an exchange rate depreciation and hence results in a lower consumption. Second, when the central bank builds reserves, consumers respond by borrowing more, thereby increasing the future debt burden. Analogous to the two-period model, an exchange rate depreciation in equilibrium results in a lower consumption and dilution of previous debt payments, and as a result, borrowing and reserves do not increase one-for-one. Considering these two costs, the optimal choice of reserves is thus such that the costs are equated to the marginal insurance benefits of reserves- ability to appreciate the exchange rate in times of financial disruptions thereby providing additional resources for consumption. This reasoning is similar to the insurance role of savings in the standard consumption-savings problem with incomplete markets: reserves transfer resources from relatively abundant current states to future states where resources are relatively scarce.

Secondly, while it is clear that FX interventions are effective in this environment, it is also note-

[11]Note that given the lack of commitment, these functions take as given, the conjectured future exchange rate policy $\mathcal{E}(.)$.

worthy that the effectiveness of the interventions are crucially dependent on Γ. As the left panel of Figure 8 shows, accumulating reserves in times when Γ is higher is more costly as it induces a stronger exchange rate depreciation vis-à-vis times when Γ is lower. Conversely, with higher Γ, a smaller intervention can induce a sharper response in exchange rates. In other words, the steepness of the intermediary supply rule which is dependent on the degree of friction in financial markets, Γ, is also a key determinant of the effectiveness of FX interventions.

Finally, we simulate the model using the policy functions and present the long-run moments in Table 1. The table shows the long-run average levels of debt, reserves, and consumption as percentages of the tradeable good endowment. For the sake of comparison, we also show the moments for a laissez-faire outcome- one where the reserves are always 0. In the long-run time-consistent equilibrium, the economy carries both debt and reserves simultaneously, i.e., the central bank actively manages the exchange rate using reserves thereby influencing the levels of debt and consumption. In the laissez-faire outcome, the economy carries a moderately smaller amount of debt and consumption is also lower on average. It is interesting to note that reserves do not have a significant effect on the long-run average debt levels. While reserves induce higher debt levels on average, the debt does not increase one-for-one with reserves. This is because, in periods when the central bank builds reserves, borrowing is higher than laissez-faire levels due to exchange rate depreciation; but in periods where it runs down reserves, an exchange rate appreciation results in lower borrowing than laissez-faire levels. Similarly, while FX intervention results in higher average consumption, it also useful to compare the long-run equilibrium distribution of consumption with the laissez-faire distribution. In Figure 9 we plot the consumption densities comparing the laissez-faire outcome with the optimal outcome. The use of reserves for transferring resources from abundant states to relatively scarcer states is evident from the fact that the consumption density is higher for higher levels of consumption under the intervention equilibrium vis-à-vis the laissez-faire outcome.

To conclude, we have illustrated in this section that in the presence of financial shocks, the optimal exchange rate policy is a managed float, i.e, the central bank actively manages the exchange rate

	Reserves	'Real' Debt	Consumption
Reserve Policy	17.63	49.43	98.69
Laissez-faire	-	48.97	98.21

Table 1: Long-run averages

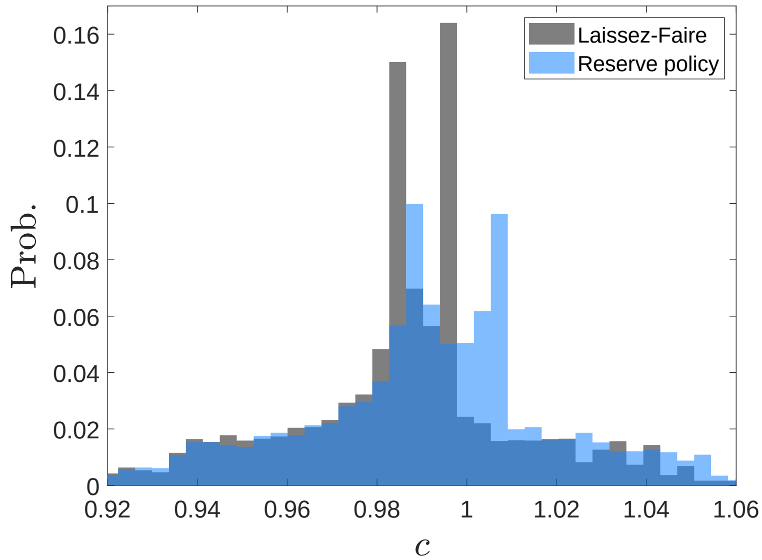


Figure 9: Consumption Density: Reserve Policy vs Laissez-faire

using reserves. This result sheds light on why a free floating exchange rate may not be optimal and offers a potential explanation to the 'Fear of Floating' phenomenon in emerging economies (Calvo & Reinhart, 2002). Secondly, the expansion of global financial markets and financial flow volatility provides a rationale for increasing exchange rate controls in emerging economies (Ilzetzki, Reinhart, & Rogoff, 2019). In the presence of financial frictions and shocks, the volatility in exchange rate, and consequently consumption, creates an exchange rate stability motive, and therefore an additional motive to accumulate reserves.

5 Conclusion

This paper has explored the widespread trend of emerging economy central banks holding significant amounts of foreign exchange reserves, a phenomenon observed even in net external debtor economies. We have proposed a theory that could rationalize why debtor economies might hold a portfolio with both debt and reserves simultaneously. Focusing on the fact that the trend in reserves has coincided with the expansion of global financial flows and imposition of exchange rate controls, our model proposes a motive for reserve accumulation that links it to exchange rate controls and external debt in the absence of capital controls. A key ingredient in our model is the presence of frictions in international financial markets.

First, we presented a general equilibrium model of a small open economy. We highlighted the dual role of the exchange rate in our economy: it not only prices consumption for the household but is also an asset price that adjusts to clear the bond markets. Secondly, considering an economy that begins with previous domestic currency denominated external debt and foreign reserves, we illustrated that a central bank may not find it optimal to run down its reserves. While running down the reserves allows the economy to borrow lesser, it also induces an exchange rate appreciation. Such an appreciation is harmful as results in a higher real interest rate on previous domestic currency debt. Therefore by holding reserves, while on one hand the central bank is making the households borrow more, on the other hand the induced exchange rate depreciation dilutes the value of previous debt payments. Given this tradeoff, the optimal portfolio could indeed feature both debt and reserves simultaneously.

Third, given that exchange rate has an asset price role and that the agents are forward looking, we illustrated a time-inconsistency problem in the central bank's optimal reserve policy. We showed that in order to mitigate the household's external debt burden, the central bank has an incentive to announce a high exchange rate for future periods. However when the said future period arrives, it is sub-optimal to implement the announced exchange rate. Instead, it would like to deviate to a lower exchange rate to provide higher consumption to the household. Given this time-inconsistency problem we showed that a central bank that lacks commitment would hold even more reserves in a time-consistent equilibrium vis-à-vis a commitment equilibrium.

Finally, we quantitatively solved for a time-consistent equilibrium in the presence of shocks to financial flows. Because these shocks result in exchange rate volatility, we illustrated that this creates an insurance motive for reserves: the central bank builds reserves in times financial stability. While this induces an exchange rate depreciation and increases the household's debt, in times of financial disruptions, reserves allow the central bank to appreciate the exchange rate thereby supporting smoother consumption.

In this paper we have not tackled the normative question of whether capital controls should be preferred to reserves, consistent with empirical facts, we have taken absence of capital controls as a given. We have also not considered the role and interaction of monetary policy with an exchange rate policy. Finally, we have ignored foreign currency debt, sovereign debt and default risk, and a domestic production sector with foreign investment. Further research is needed in studying the interactions of financial frictions with these aspects and characterizing the optimal policy mix.

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Appendix

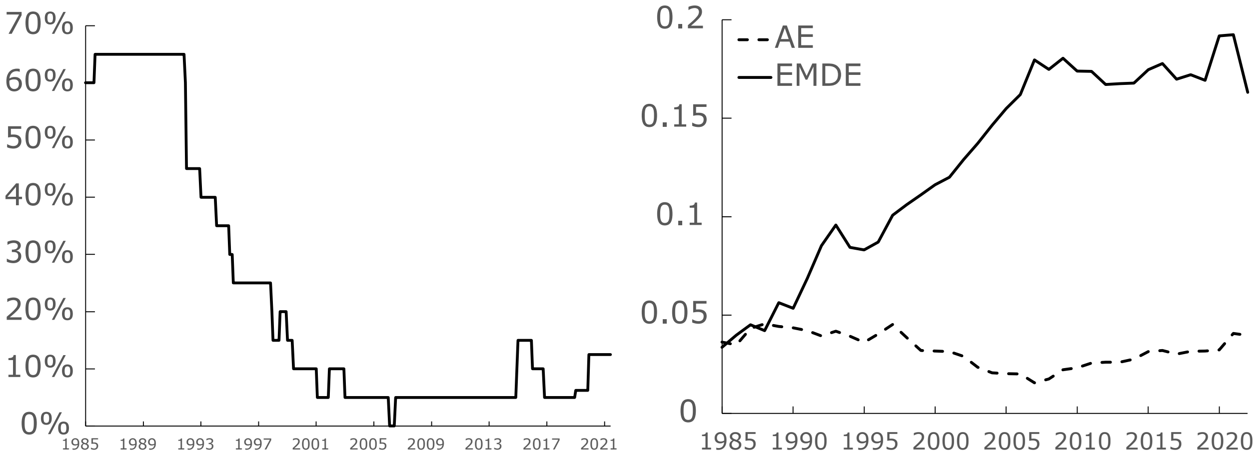
A Insights from Data

To provide context to our theory, we summarize and relate a few well-established empirical facts. First, foreign exchange reserves held by central banks in emerging economies have experienced a secular upward trend starting mid 1980s; and reserve holdings continue to be relatively high. Secondly, even net external debtor economies (excluding reserves) are also holding significant amounts in reserves. By contrast, most advanced economies did not jump on this reserve building trend. Third, while reserves grew rapidly, emerging economies simultaneously eased capital controls but enforced exchange rate controls. By contrast, most advanced economy currencies are free floating. Most of these findings are discussed extensively by Ilzetzki, Reinhart, and Rogoff (2019).

We consider a sample of 20 largest (by 2022 GDP) emerging economies which are also persistent deficit economies and typically external debtors. The countries in the sample are: Argentina, Bangladesh, Brazil, Colombia, Egypt, India, Indonesia, Korea; Republic of, Malaysia, Mexico, Nigeria, Pakistan, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, and Vietnam. In the left panel of Figure 1 we show the mean and the median international investment position (IIP) of this sample and in the right panel we show the official foreign exchange reserves (excluding gold and SDRs). These charts highlight that net external debtor economies have accumulated significant FX reserves: in 2022, the median IIP was -44.86% and median reserves were 16.27% in this sample.

In the left panel of Figure 10 we show the IRR capital controls index (Ilzetzki, Reinhart, & Rogoff, 2019) for this sample of countries. It is evident that the trend in reserves is accompanied with a simultaneous trend in easing of capital controls.

For the sake of comparison, we also consider a sample of 10 of the 12 largest advanced economies (by 2022 GDP), namely, Australia, Belgium, Canada, France, Germany, Italy, Netherlands, Spain, United Kingdom and United States. Notably Japan and Switzerland are excluded. The criteria for selection of these countries is that not only are the currencies of these countries globally dominant, but also they are the most freely-floating currencies in the world. In the right panel of Figure 10 we compare the reserve accumulation by the emerging economies sample with this sample of advanced economies. It is clear that free-floating currency advanced economies had negligible

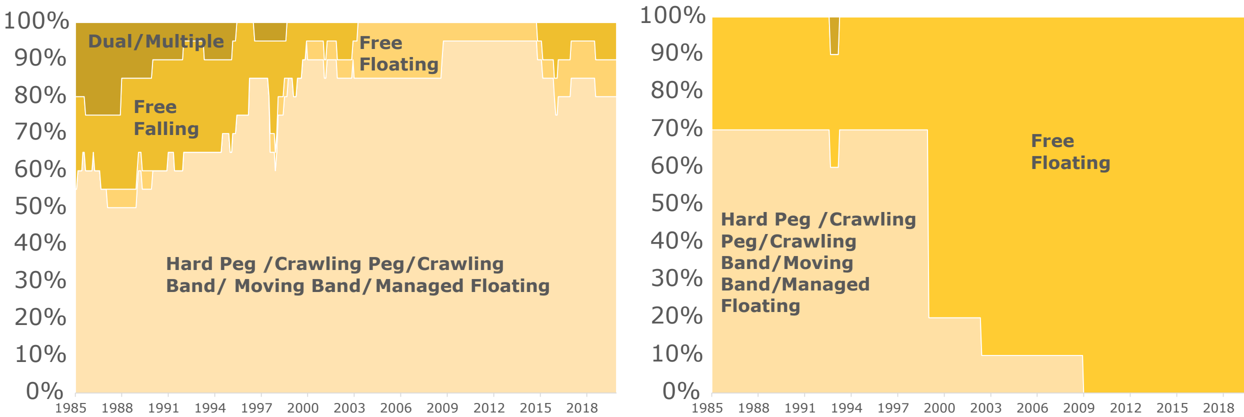


(a) Easing of Capital Controls

(b) (Mean) Reserves to GDP: EMDE vs AE

Notes: The left panel shows the IRR capital controls index for the sample of EMDEs. Source: Computed using the IRR Unified Market Analysis database (Ilzetzki, Reinhart, & Rogoff, 2019). The right panel shows the mean Reserves to GDP in the sample of EMDEs and AEs. Source: Computed using the External Wealth of Nations database (Milesi-Ferretti, 2022).

Figure 10: Reserves and Capital Controls



(a) Exchange rate regimes: EMDE

(b) Exchange rate regimes: AE

Notes: The left panel shows the exchange rate regimes for the sample of EMDEs and the right panel shows the regimes for the sample of AEs. Source: Computed using the IRR's coarse classification (Ilzetzki, Reinhart, & Rogoff, 2019).

Figure 11: Exchange rate regimes: EMDE vs AE

reserve accumulation compared to the emerging economies.

Finally, in the left and right panels of Figure 11 we compare the evolution of exchange rate regimes in these emerging and advanced economies. The regime classification follows Ilzetzki, Reinhart, and Rogoff (2019). While the advanced economies in our sample are typically free-floaters, a 'Fear of Floating' phenomenon (Calvo & Reinhart, 2002) is evident in the emerging economies: they have historically imposed stronger exchange rate controls and continue to do so.

We draw two main conclusions from this analysis. First, the trend is reserves is accompanied by a simultaneous trend in easing of capital controls and imposition of exchange rate controls. Second, Countries which impose exchange rate controls have accumulated significantly higher reserves. These facts constitute the key motivating factors for our theory.

B Proofs

B.1 Proof of Lemma 1

Proof. It suffices to establish the condition under which a perturbation to $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ leads to a welfare gain. Consider a perturbation given by:

$q_2 = \hat{q}_2 + \Delta = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\right]$, where $\Delta > 0$

$c_2 = y + R^* a_2 - R^*(1 + \Gamma(\hat{q}_2 + \Delta))(\hat{q}_2 + \Delta)$

$c_1 + a_2 - (\hat{q}_2 + \Delta) = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1$

Differentiating wrt Δ:

$\frac{dc_1}{d\Delta} + \frac{da_2}{d\Delta} - 1 + R\sigma c_1^{\sigma-1}\frac{q_1}{c_0^{\sigma}}\frac{dc_1}{d\Delta} = 0$

$\frac{dc_2}{d\Delta} = R^*\left(\frac{da_2}{d\Delta} - (1 + 2\Gamma\Delta + 2\Gamma\hat{q}_2)\right)$

$1 = \sigma\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\left[\frac{1}{c_2}\frac{dc_2}{d\Delta} - \frac{1}{c_1}\frac{dc_1}{d\Delta}\right]$

Solving the above system of equations gives us:

$$\frac{da_2}{d\Delta} = \frac{\frac{1}{\hat{\sigma}} + (1 + 2\Gamma\Delta + 2\Gamma\hat{q}_2)\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}{\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}$$

$$\frac{dc_2}{d\Delta} = R^*\left(\frac{\frac{1}{\hat{\sigma}} - \frac{1}{c_1\kappa}(2\Gamma\Delta + 2\Gamma\hat{q}_2)}{\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}\right)$$

$$\frac{dc_1}{d\Delta} = -\frac{1}{\kappa}\left(\frac{\frac{1}{\hat{\sigma}} + (2\Gamma\Delta + 2\Gamma\hat{q}_2)\frac{R^*}{c_2}}{\frac{R^*}{c_2} + \frac{1}{c_1\kappa}}\right)$$

where $\kappa = 1 + \sigma R c_1^{\sigma-1}\frac{q_1}{c_0^{\sigma}}$ and $\hat{\sigma} = \sigma\left(\hat{q}_2 + \Delta + \frac{1}{\Gamma}\right) = \sigma\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}$.

Since $q_1 > 0$, then $\hat{\kappa} = 1 + \sigma R\hat{c}_1^{\sigma-1}\frac{q_1}{c_0^{\sigma}} > 0$.

Since $\hat{q}_2 \geqslant 0$, then $\hat{\sigma} > 0$.

Let $\hat{D} \equiv \frac{R^*}{\hat{c}_2} + \frac{1}{\hat{c}_1\hat{\kappa}}$ where $\hat{D} > 0$.

Simplify further to obtain:

$$\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} = \frac{R^*}{\hat{\kappa}\hat{D}\hat{\sigma}}\left(\frac{\hat{c}_1\hat{\kappa} - 2\Gamma\hat{q}_2\hat{\sigma}}{\hat{c}_1}\right)$$

$$\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} = -\frac{1}{\hat{\kappa}\hat{D}\hat{\sigma}}\frac{\hat{c}_1}{\hat{c}_2}\left(\frac{\hat{c}_2 + R^* 2\Gamma \hat{q}_2\hat{\sigma}}{\hat{c}_1}\right)$$

Define welfare as:

$$W = \frac{\omega}{1-\sigma}\left(c_1^{1-\sigma} + \beta c_2^{1-\sigma}\right)$$

Change in welfare can be expressed as:

$$\frac{dW}{d\Delta} = \omega c_2^{-\sigma}\left(\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{dc_1}{d\Delta} + \frac{1}{R}\frac{dc_2}{d\Delta}\right)$$

$$\begin{aligned}\left.\frac{dW}{d\Delta}\right|_{\Delta=0} &= \omega\hat{c}_2^{-\sigma}\left(\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma}\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} + \frac{1}{R}\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0}\right)\\ &= \frac{\omega\hat{c}_2^{-\sigma}}{\hat{\kappa}\hat{D}\hat{\sigma}}\frac{1}{\hat{c}_1}\frac{R^*}{R}\left(\hat{c}_1\hat{\kappa} - 2\Gamma\hat{q}_2\hat{\sigma} - \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(\hat{c}_2 + R^* 2\Gamma\hat{q}_2\hat{\sigma})\right)\end{aligned}$$

Then:

$$\begin{aligned} & \left.\frac{dW}{d\Delta}\right|_{\Delta=0} > 0\\ \Leftrightarrow\quad & \left(\hat{c}_1\hat{\kappa} - 2\Gamma\hat{q}_2\hat{\sigma} - \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(\hat{c}_2 + R^* 2\Gamma\hat{q}_2\hat{\sigma})\right) > 0\\ \Leftrightarrow\quad & \hat{c}_1 + \sigma R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 - 2\sigma(\hat{q}_2 + \Gamma\hat{q}_2^2) > \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(\hat{c}_2 + R^* 2\sigma(\hat{q}_2 + \Gamma\hat{q}_2^2))\\ \Leftrightarrow\quad & y + R^* a_1 + (\sigma-1)R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 - 2\sigma(\hat{q}_2 + \Gamma\hat{q}_2^2) + \hat{q}_2 > \frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1}(y + (2\sigma-1)R^*(\hat{q}_2 + \Gamma\hat{q}_2^2))\\ \Leftrightarrow\quad & R^* a_1 + (\sigma-1)R\left(\frac{\hat{c}_1}{c_0}\right)^{\sigma} q_1 > \left[\frac{R}{R^*}\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} - 1\right] y + \Gamma\hat{q}_2^2 + (2\sigma-1)(\hat{q}_2 + \Gamma\hat{q}_2^2)\left(R\left(\frac{\hat{c}_2}{\hat{c}_1}\right)^{\sigma-1} + 1\right)\end{aligned}$$

□

B.2 Proof of Lemma 2

Proof. It suffices to establish the condition under which a perturbation to $(\hat{c}_1, \hat{c}_2, \hat{q}_2, \hat{a}_2)$ leads to a welfare gain. Consider a perturbation given by:

$q_2 = \hat{q}_2 + \Delta$, where $\Delta > 0$

$c_2 = \left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} c_1$

$a_2 = \frac{1}{R^*}\left(\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} c_1 - y\right) + \Delta(1+\Gamma\Delta)$

$c_1 + \frac{1}{R^*}\left(\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} c_1 - y\right) + \Gamma\Delta^2 = y + R^* a_1 - R\left(\frac{c_1}{c_0}\right)^{\sigma} q_1$

Differentiating wrt Δ:

$\frac{dc_2}{d\Delta} = \frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}-1} c_1 + \left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}}\frac{dc_1}{d\Delta}$

$\frac{da_2}{d\Delta} = \frac{1}{R^*}\frac{dc_2}{d\Delta} + 1 + 2\Gamma\Delta$
$\frac{dc_1}{d\Delta}\left(1 + \frac{1}{R^*}\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} + c_1^{\sigma-1}\sigma R\frac{q_1}{c_0^\sigma}\right) = -\left(2\Gamma\Delta + \frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}-1}\frac{c_1}{R^*}\right)$
which gives us:

$$\frac{dc_1}{d\Delta} = -\frac{\left(2\Gamma\Delta + \frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}-1}\frac{c_1}{R^*}\right)}{1 + \frac{1}{R^*}\left(\frac{R^*}{R}(1+\Gamma\Delta)\right)^{\frac{1}{\sigma}} + c_1^{\sigma-1}\sigma R\frac{q_1}{c_0^\sigma}}$$

$$\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} = -\frac{\frac{1}{\sigma}\frac{R^*}{R}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}-1}\frac{\hat{c}_1}{R^*}}{1 + \frac{1}{R^*}\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} + \hat{c}_1^{\sigma-1}\sigma R\frac{q_1}{c_0^\sigma}} = -\frac{\frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\frac{\hat{c}_1}{R^*}}{\frac{1}{\hat{c}_1}\left(\hat{c}_1 + \frac{\hat{c}_1}{R^*}\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^\sigma}\right)} < 0$$

since $q_1, c_0 > 0$.

Further simplification implies:

$$\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} = -\frac{\frac{1}{\sigma}\Gamma\hat{c}_1\frac{\hat{c}_2}{R^*}}{\left(\hat{c}_1 + \frac{\hat{c}_2}{R^*} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^\sigma}\right)}$$

Similarly,

$$\begin{aligned}\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} &= \frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\hat{c}_1 + \left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} \\ &= \frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}}\hat{c}_1\left(\frac{\hat{c}_1 + \sigma R\hat{c}_1^{\sigma}\frac{q_1}{c_0^\sigma}}{\hat{c}_1 + \frac{\hat{c}_2}{R^*} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^\sigma}}\right) > 0\end{aligned}$$

since $\frac{q_1}{c_0^\sigma} > 0$.

Using these we get:

$$\left.\frac{da_2}{d\Delta}\right|_{\Delta=0} = \frac{1}{R^*}\left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} + 1 > 0$$

Define welfare as:

$$W = \frac{\omega}{1-\sigma}\left(c_1^{1-\sigma} + \beta c_2^{1-\sigma}\right)$$

Change in welfare can be expressed as:

$$\frac{dW}{d\Delta} = \omega c_2^{-\sigma}\left(\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{dc_1}{d\Delta} + \frac{1}{R}\frac{dc_2}{d\Delta}\right)$$

$$\left.\frac{dW}{d\Delta}\right|_{\Delta=0} = \omega\hat{c}_2^{-\sigma}\frac{1}{R}\left(R^*\left.\frac{dc_1}{d\Delta}\right|_{\Delta=0} + \left.\frac{dc_2}{d\Delta}\right|_{\Delta=0}\right)$$

$$\left.\frac{dW}{d\Delta}\right|_{\Delta=0} > 0$$

$$\Leftrightarrow \quad \left.\frac{dc_2}{d\Delta}\right|_{\Delta=0} > -R^* \left.\frac{dc_1}{d\Delta}\right|_{\Delta=0}$$

$$\Leftrightarrow \quad \frac{1}{\sigma}\Gamma\left(\frac{R^*}{R}\right)^{\frac{1}{\sigma}} \hat{c}_1 \left(\frac{\hat{c}_1 + \sigma R \hat{c}_1^{\sigma}\frac{q_1}{c_0^{\sigma}}}{\hat{c}_1 + \frac{\hat{c}_2}{R^*} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^{\sigma}}}\right) > R^* \frac{\frac{1}{\sigma}\Gamma\hat{c}_1\frac{\hat{c}_2}{R^*}}{\left(\hat{c}_1 + \frac{\hat{c}_2}{R^*} + \hat{c}_1^{\sigma}\sigma R\frac{q_1}{c_0^{\sigma}}\right)}$$

$$\Leftrightarrow \quad \left(\hat{c}_1 + \sigma R \hat{c}_1^{\sigma}\frac{q_1}{c_0^{\sigma}}\right) > \hat{c}_1$$

$$\Leftrightarrow \quad q_1 > 0$$

□

B.3 Proof (Sketch) of Lemma 3

Proof. (Sketch) Consider the problem (32). The First order conditions for a_1, c_1, with Lagrange multipliers $(\lambda_0, \lambda_1, \lambda_2)$ on the BoP constraints and (ν_1, ν_2) on the non-negativity constraints are given below. FOCs for c_0, c_2, a_2 are omitted here for conciseness.

a_1:

$$\lambda_0 = \beta R^* \lambda_1 + \nu_1 \tag{35}$$

c_1:

$$\begin{aligned} &\omega c_1^{-\sigma} - \lambda_1 - \lambda_1 \frac{\sigma}{c_1} R \frac{c_1^{\sigma}}{c_0^{\sigma}} \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1^{\sigma}}{c_0^{\sigma}}\right) - 1\right] \\ &-\lambda_1 \frac{\sigma}{c_1}\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} \\ &+\lambda_2 \frac{\sigma}{c_1}\left(\frac{c_2}{c_1}\right)^{\sigma}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma} - 1\right] \\ &+\lambda_2 \frac{\sigma}{c_1}\left(\frac{c_2}{c_1}\right)^{\sigma}\left(\frac{1}{\Gamma}\frac{R}{R^*}\left(\frac{c_2}{c_1}\right)^{\sigma}\right) \\ &\underbrace{-R\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1}\left(\frac{c_1}{c_0}\right)^{\sigma}\left(\lambda_1\left(\frac{c_1}{c_0}\right)^{\sigma} - \lambda_0\right)}_{A} \end{aligned} \tag{36}$$

The expression labelled A in (36) represents the net marginal effect of c_1 on the quantity of borrowing in period 1, q_1. The marginal benefit of additional borrowing at $t = 0$ is given by $R\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1}\left(\frac{c_1}{c_0}\right)^{\sigma}\lambda_0$. However additional borrowing also increases the debt burden at $t = 1$. The marginal cost associated with this additional debt burden is given by $-\lambda_1 R\left(\frac{c_1}{c_0}\right)^{\sigma}\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1}\left(\frac{c_1}{c_0}\right)^{\sigma}$. The net marginal cost is thus given by the expression A. To show that the net effect is indeed costly at the margin, we need to establish that A<0 at the optimum.

Since $a_1^c > 0$ by assumption, $\nu_1 = 0$. Substitute (35) into the above expression to get:

$$\begin{aligned} A &= -R\frac{1}{\Gamma}\frac{R}{R^*}\frac{\sigma}{c_1^c}\left(\frac{c_1^c}{c_0^c}\right)^\sigma\left(\lambda_1^c\left(\frac{c_1^c}{c_0^c}\right)^\sigma - \frac{R^*}{R}\lambda_1^c\right) \\ &= -\lambda_1^c R\left(\frac{c_1^c}{c_0^c}\right)^\sigma \frac{\sigma}{c_1^c}\frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1^c}{c_0^c}\right)^\sigma - 1\right] \\ &= -\lambda_1^c R\left(\frac{c_1^c}{c_0^c}\right)^\sigma \frac{\sigma}{c_1^c} q_1^c < 0 \end{aligned}$$

Since $q_1^c > 0$ by assumption and the Lagrange multiplier on the BoP constraint, $\lambda_1^c > 0$ then $A < 0$. The idea here is that as long as $a_1 > 0$ it is always cheaper to raise resources by reducing a_1 rather than by increasing q_1 due to the resource losses involves in using the intermediation technology. Therefore, the net effect of a marginal increase in c_1 on q_1 is costly at the margin.

Now consider the first order condition for c_1 for the two-period sub-problem (23). The FOC is identical to (36), except for the fact that the expression A now disappears because q_1 is a state variable in (23): changing c_1 has no effect on q_1. Intuitively, for a time-0 planner there is an additional marginal cost associated with increasing c_1 whereas for the time-1 planner this cost has disappeared. A lower marginal cost implies that the time-1 planner would like to deviate to a higher level of consumption, $c_1^D > c_1^c$. Given our discussion on the two period model, in section 3.6.1, this deviation implies a movement along the IRC from left to right, i.e, $a_2^D < a_2^c$, $q_2^D < q_2^c$ and $c_2^D < c_2^c$. □

B.4 Proof of Lemma 4

Proof. From Lemma 3, we know that given $a_1^c, a_2^c, q_1^c > 0$ the solution to the two period problem (23) is given by:

$$c_1^D \equiv \mathcal{C}_1(c_0^c, a_1^c, q_1^c) > c_1^c$$

But since:

$$q_1^c = \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{c_1^c}{c_0^c}\right)^\sigma - 1\right]$$

Then:

$$q_1^c < \frac{1}{\Gamma}\left[\frac{R}{R^*}\left(\frac{\mathcal{C}_1(c_0^c, a_1^c, q_1^c)}{c_0^c}\right)^\sigma - 1\right]$$

Therefore (c_0^c, a_1^c, q_1^c) violates the asset supply equation constraint for (33) and is thus infeasible. □

C Numerical Examples

In this section, we present numerical simulations illustrating Propositions 1, 2 and 3. The parameters used in these simulations are described in Section 4.2.

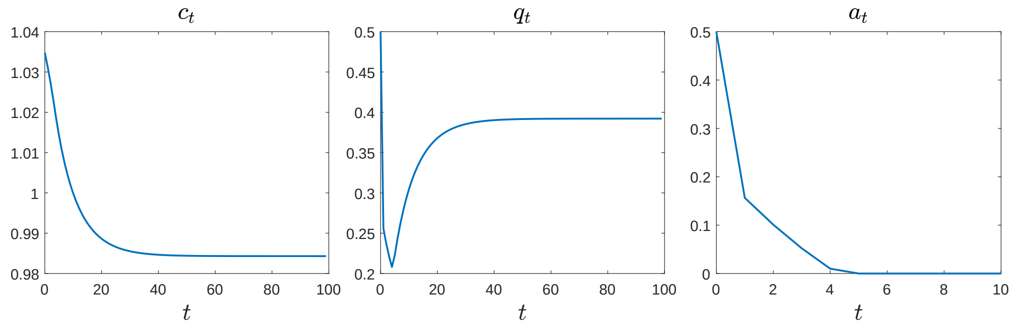


Figure 12: Proposition 1 illustrated- Example 1

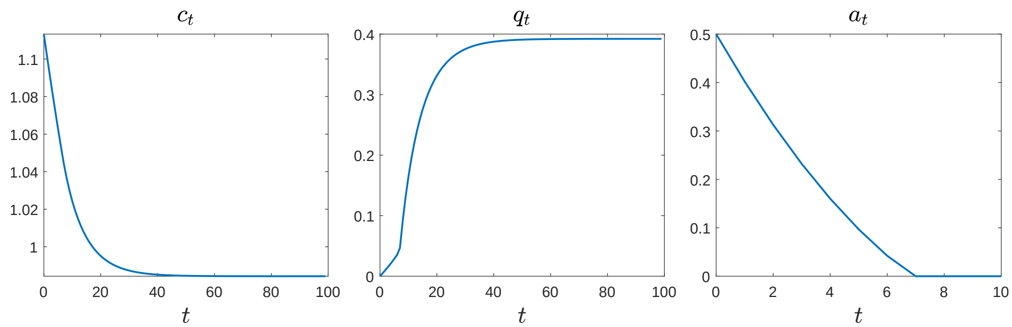


Figure 13: Proposition 1 illustrated- Example 2

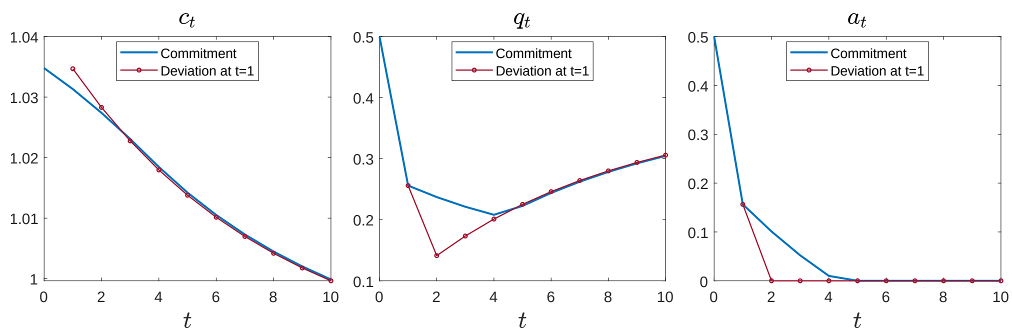


Figure 14: Proposition 2 illustrated

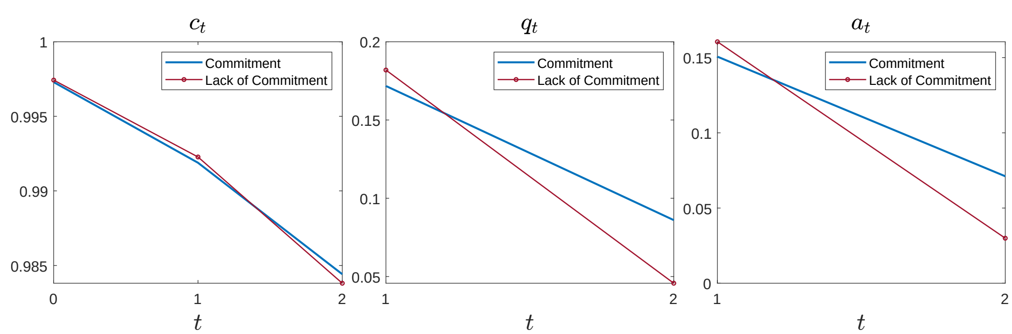


Figure 15: Proposition 3 illustrated