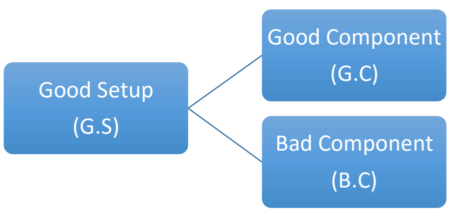
Question 1

A previous survey shows that a machine making plastic components is correctly set up for the day's production on 95% of days. On days when it is set up correctly, 98% of the components produced are good. If the machine is not set up correctly, only 40% of the components produced are good. On a particular day, the machine is set up and the first component produced is found to be good. What is the probability that the machine is set up correctly?

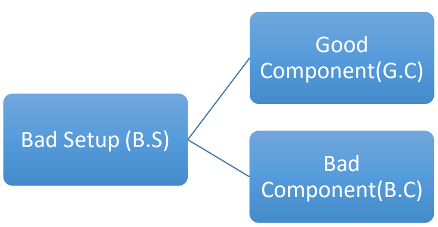
<u>*Answer:*</u>

Here we are using probability tree

<u>CONDITION A</u>



<u>CONDITION B</u>



According to the question we have
G.S = 0.95, G.C =0.98, B.C = 0.02

Condition A
As it is said that the machine is set up correctly (G.S) 95% and when it is set up correctly (G.C) 98% of components are good which means 2% are Bad component (B.C)

Condition B
5%(B.S) is the chances that machine is not set up correctly and in such case only 40% components are good (G.C) and rest which is 60% (B.C) are bad.

Now,

P (Good Component) = P(GS.GC) + P(BS.GC)
= (0.95 * 0.98) + (0.05 * 0.40)
= 0.931 + 0.02
= 0.951

P(GS.GC) given Good Component = P(GC.GS) / P(GC.GS) + P(GC.BS) **(BAYES RULE)**
= 0.931 / 0.951
= 0.9789

Hence the probability of the machine being set up correct when the component first produced is good = 0.9789

Question 2

A wheel bearing factory rejects bearings if they are either oversize or undersize. The probability that a bearing is oversize is 0.004 while the probability that it is undersize is 0.01.

(a) Are oversize rejection and undersize rejection events mutually exclusive?

Answer:

Yes, events are mutually exclusive when one event occurs and the other can't. We can't have a condition where the bearing is undersized and oversized.

(b) What is the probability that a bearing selected at random is not within size limits?

P(Random not within size limits) = P(Under_Size) + P(Over_Size)
= 0.004 + 0.01
= 0.014

(c) What is the probability that a bearing selected at random is correctly sized?

Answer:

Total Probability = 1

Sizelimit Probability = P(Under_Size) + P(Over_Size)

Probability that a bearing selected at random = P(Rndm)
= 1 – (0.004 + 0.01)
= 0.986

Question 3

In Question 2, bearings are rejected if they are outside size limits. Incorrectly sized bearings are placed in a bin. If a bearing is selected at random from the rejects bin, what is the probability that it is undersize?

Answer:

All bearings in the bin are either Over Size or Under Size

P(Over_Size) = 0.004
P(Under_Size) = 0.01

Proportionally we have more Under Size than Over Size. In the long run the ratio of Over Size to Under Size is

0.004 / 0.001
= 4 / 10

We can come to a conclusion that out of every 14 bearings 4 are Under Size and 10 are Over Size

Now, P(Under_Size) = 0.01 / 0.01 + 0.004
= 10/14

Question 4

A company manufacturing disk drives has two production plants located in Singapore and China. The Singapore plant contributes 30% of output with the balance coming from China. Over a prolonged period, it is observed that 4% of the Singapore drives are faulty while 5% of the Chinese produced drives are faulty. All drives are returned to the company's headquarters in Dublin for testing and dispatch. What is the probability that a drive found faulty at this check comes from China?

Answer:

Let P(F) be Probability of a faulty drive

P(Production of China) = P(C)
P(Production of Singapore) = P(S)

Probability of Faulty drives from China P(F/C) = 5 / 100 = 0.05

Probability of Faulty dries from Singapore P(F/S) = 4/100 = 0.04

China 5% Faulty Drives	70% Production
Singapore 4% Faulty Drives	30% Production

Now, P(F) =(0.3 * 0.04)+ (0.7 * 0.05)
= 0.012 + 0.035
= 0.047
For the faulty drives from china P(C/F) = 0.7 * 0.05 / 0.047 (USING BAYES RULE)
= 0.745

Hence the probability that the faulty drive comes from china is 74.5%

Question 5

A TV game show involves picking a coloured ball from one of two boxes. A blindfolded contestant chooses a box and then picks a ball at random from the box. Box 1 contains 2 green balls and seven red balls. Box 2 contains 4 green and 3 red balls. If a contestant picks a red ball, what is the probability that the contestant chose from Box 1?

Answer:

Total Boxes = 2

So, probability of getting the ball from Box1 = ½

Now,
In Box 1

Red Ball = 7 Green Ball = 2

Probability of getting Red Ball from Box 1 = P(R/B1)

$$= \frac{(\frac{1}{2} * 7/9)}{(1/2 * 7/9) + (1/2 * 3/7)}$$

= 0.64

Here, we used the Bayes theorem which is P(A given B) = P(A) * P(B given A) / P(B)

Question 6

Show, with detailed working, how to find the mean, mode, median, interquartile range and standard deviation for the following set of data:

{29, 41, 6, 9, 32, 22, 40, 36}

Answer:

{6,9,22,29,32,36,40,41}

N = 8

∑X = 6 + 9 + 22 + 29 + 32 + 36 + 40 + 41
= 215

Mean = ∑X/ N
= 215 / 8
= 26.875

We don't have a repeating number so there is no **mode**

Median = 29 + 32 / 2
= 30.5

Interquartile Range
{6,9,22,29} | {32,36,40,41}
Q1 = 22 + 9 / 2 = 15.5
Q3 = 36 + 40 / 2 = 38

Interquartile range = Q3 – Q1

= 38 – 15.5

= 22.5

For Standard deviation

X	$X - \bar{X}$	$(X - \bar{X})^2$
6	-20.875	435.765
9	-17.875	319.515
22	-4.875	23.765
29	2.125	4.515
32	5.125	26.265
36	9.125	83.265
40	13.125	172.265
41	14.125	199.515

$\Sigma(x-\bar{x})^2 = 1264.87$

$N = 8$

$\bar{x} = 215/8$

$\bar{x} = 26.875$

Now, **Standard deviation** = √(E(X - MEAN)^2 / N)

= √1264.87 / 8

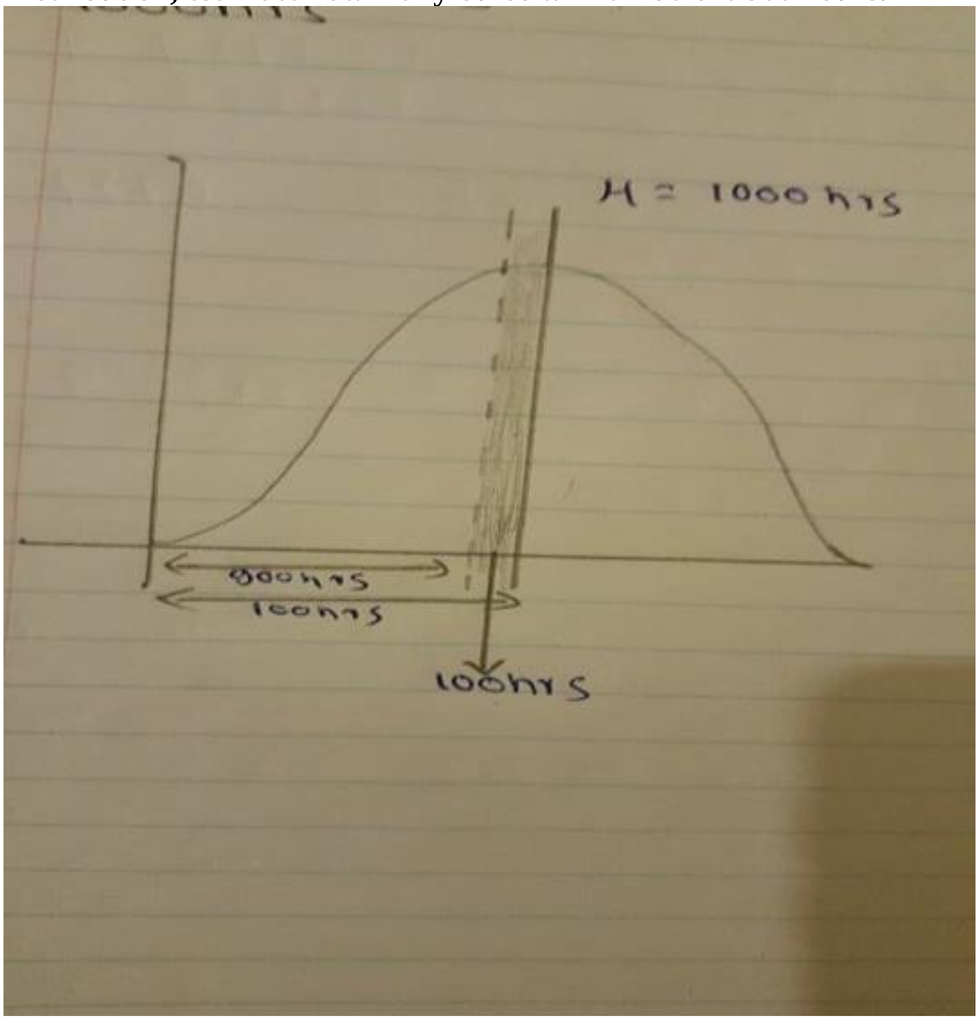
= √155.75

= 12.479

Question 7

A study of data collected at a light bulb factory shows that a batch of 6000 light bulbs have

a mean life of 1000 hours with a standard deviation of 75 hours. Assuming a Normal Distribution, estimate how many bulbs will fail before 900 hours.



We have,

u = 1000 hrs

Standard deviation = 75 hrs

Let's first find out the no of bulbs that fails in between 900 and 1000 hours i.e. 100hrs

= 100 / 75

= 1.33 standard deviation

Now, by looking in our table we get 1.33 = 0.4082

Bulb that fails before 900 hours = (0.5 – 0.4082) * 6000

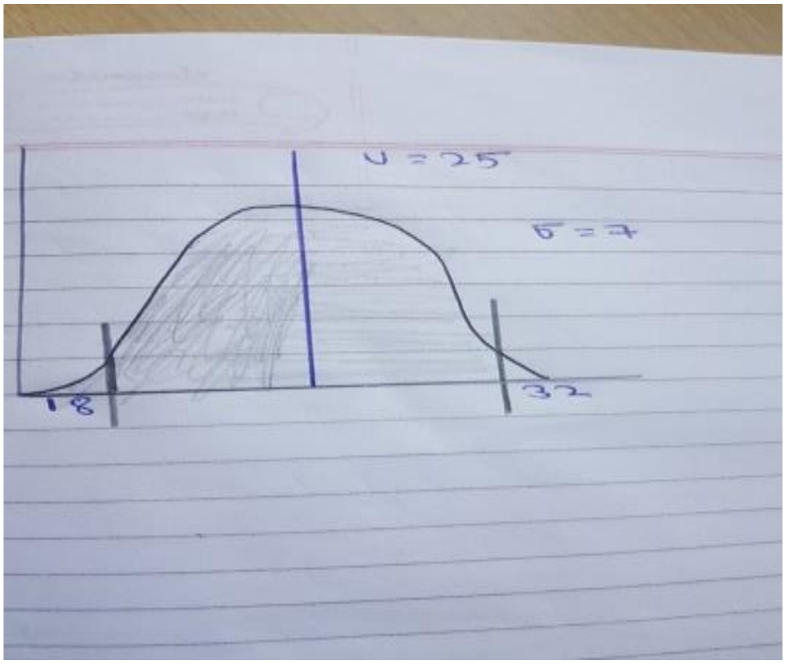
= 550.8 bulbs

Above since we are looking at half side of our distribution we have 0.5 which is from 0 to 1000 bulbs and we need to look for bulbs that fails before 900 hrs for which we have calculated the standard deviation of 100 bulbs and subtracting them and multiplying by the total no of bulbs we got the value.

Question 8

If x is normally distributed with u = 25 and s = 7 determine the probability that a randomly selected x lies between 18 and 32.

Answer:



We have

mean (u) = 25

standard deviation = 7

(18 <= x >= 32) = ?

Z1 = (x - u) / s.d

= (18 - 25) / 7

= -1

Z2 = (x - u) / s.d

= (32 - 25) / 7

= 1

Now, by having a look on our table we know that 1.00 is 0.3413

For both sides = 0.3413 + 0.3413

= 0.6826

The probability of randomly selected x to be between 18 and 32 is 68%.