ADA4_1

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0.1 Física Computacional

0.1.1 Tarea 4: Diferenciación automática

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[227]:
```
using Plots
function rk4_system(f::Function,  ::Array, a::Real, b::Real, N::Int64)
    n1 = N + 1
    m = size( )[1]
    u = zeros(n1, m + 1)
    h = (b - a) / N
    u[1,1] = a
    u[1,2:end] =
    for i in 2:n1
        t = u[i-1,1]
        v = u[i-1,2:end]
        k1 = h * f(t, v)
        k2 = h * f(t + h / 2, v .+ k1 / 2)
        k3 = h * f(t + h / 2, v .+ k2 / 2)
        k4 = h * f(t + h, v .+ k3)
        u[i,2:end] = v .+ (k1 .+ 2 .* k2 .+ 2 .* k3 .+ k4) ./ 6
        u[i,1] = a + (i - 1) * h
    end
    return u
end
```

[227]:
```
rk4_system (generic function with 1 method)
```

0.1.2 Problema 1a

$$y'' + sin(xy'y) + 2x = 5$$

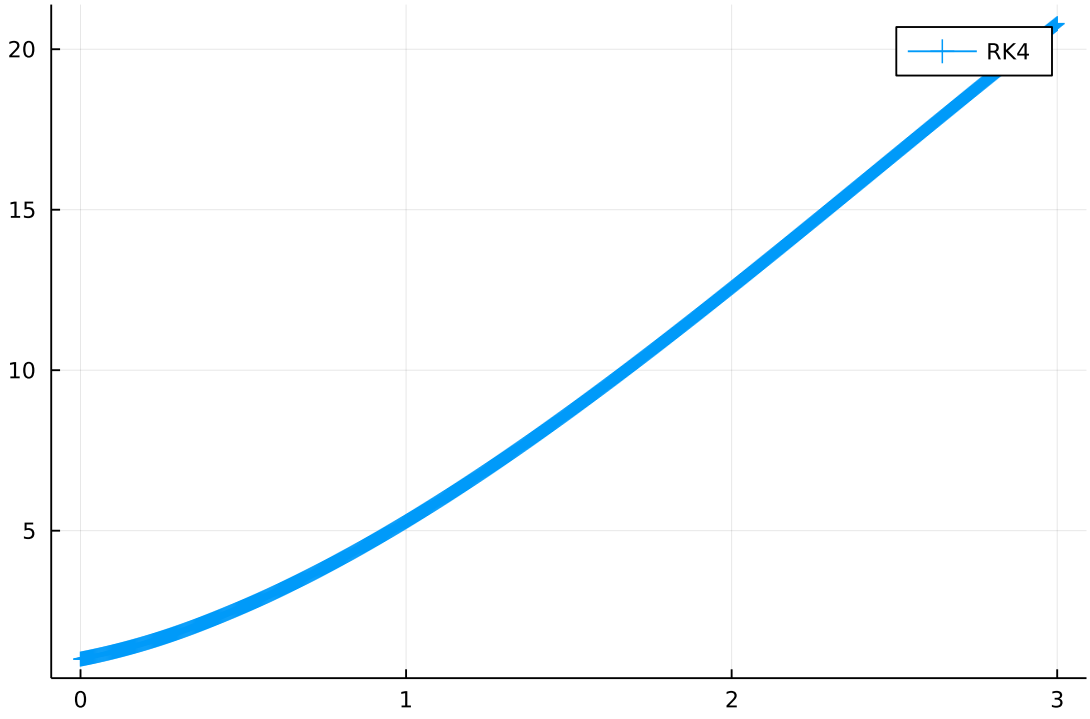
$$y_1 = y \quad y_2 = y'$$

$$y_1' = y_2$$

$$y_2' = 5 - 2x - sin(xy'y)$$

[228]:
```
f(x,y) = [ y[2] , 5 - 2*x  + sin( x*y[1]*y[2]) ]
init_con = [1.0,2.0]
a = 0
b = 3
n = 1000
sol = rk4_system(f,init_con,a,b,n)

plot(sol[:,1],sol[:,2],markershape=:+,label="RK4")
```

[228]:



0.1.3 Problema 1b

$$y^{(4)} + sin(xy^{(3)}) + e^{-x}y'' - xy = 0$$

$$y_1 = y \quad y_2 = y' \quad y_3 = y'' \quad y_4 = y'''$$

$$y_1' = y_2$$

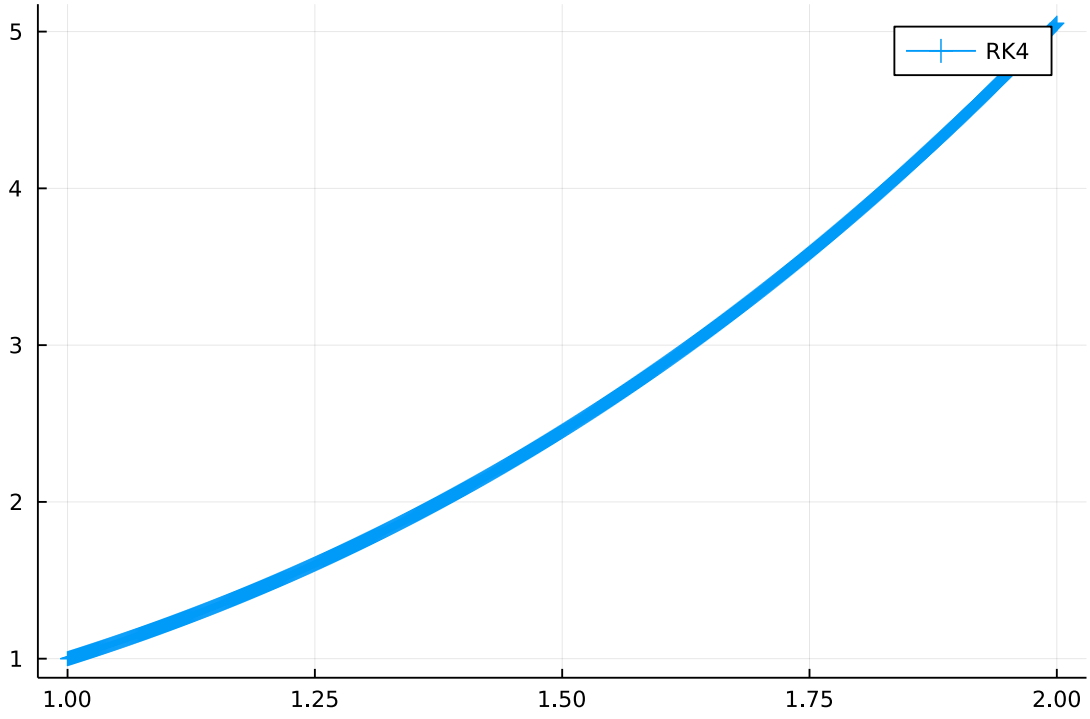
$$y_2' = y_3$$

$$y_3' = y_4$$

$$y_4' = xy_1 - y_2 - e^{-x}y_3 + sin(xy_4)$$

[87]:
```
f(x,y) = [ y[2] , y[3]  , y[4] , x*y[1] - y[2] - exp(-x)*y[3] + sin(x*y[4]) ]
init_con = [1.0,2.0,3.0,4.0]
a = 1
b = 2
n = 1000
sol = rk4_system(f,init_con,a,b,n)

plot(sol[:,1],sol[:,2],markershape=:+,label="RK4")
```

[87]:



0.1.4 Problema 2

$$f_1(t, \theta, \omega) = \omega$$

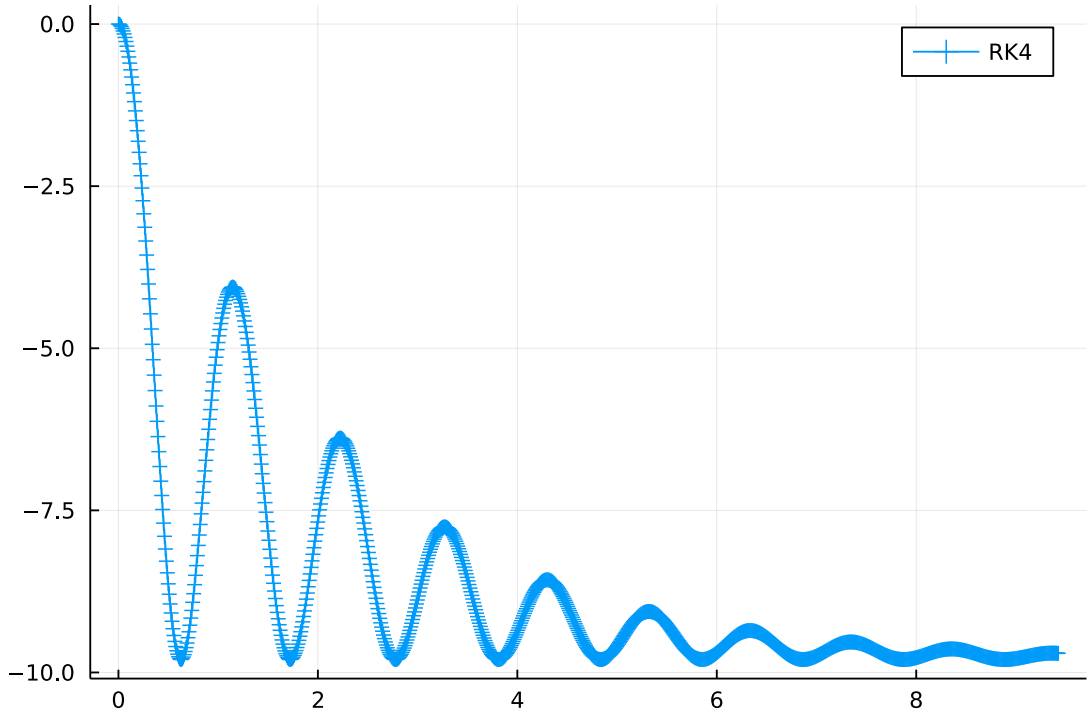
$$f_2(t, \theta, \omega) = -\frac{c}{m}\omega + \frac{g}{l}sin(\theta)$$

Tensión

$$T(t) = mgcos\theta(t)$$

[242]:
```
m = 1
l = 1
g = 9.8
c = 0.5

f(x,y) = [ y[2] , -(c/m)*y[2] + (g/l)*sin(y[1]) ]
init_con = [ /2,0.0]
a = 0
b = 3
n = 1000

sol = rk4_system(f,init_con,a,b,n)

plot(sol[:,1],m*g*cos.(sol[:,2]),markershape=:+,label="RK4")
```

[242]:

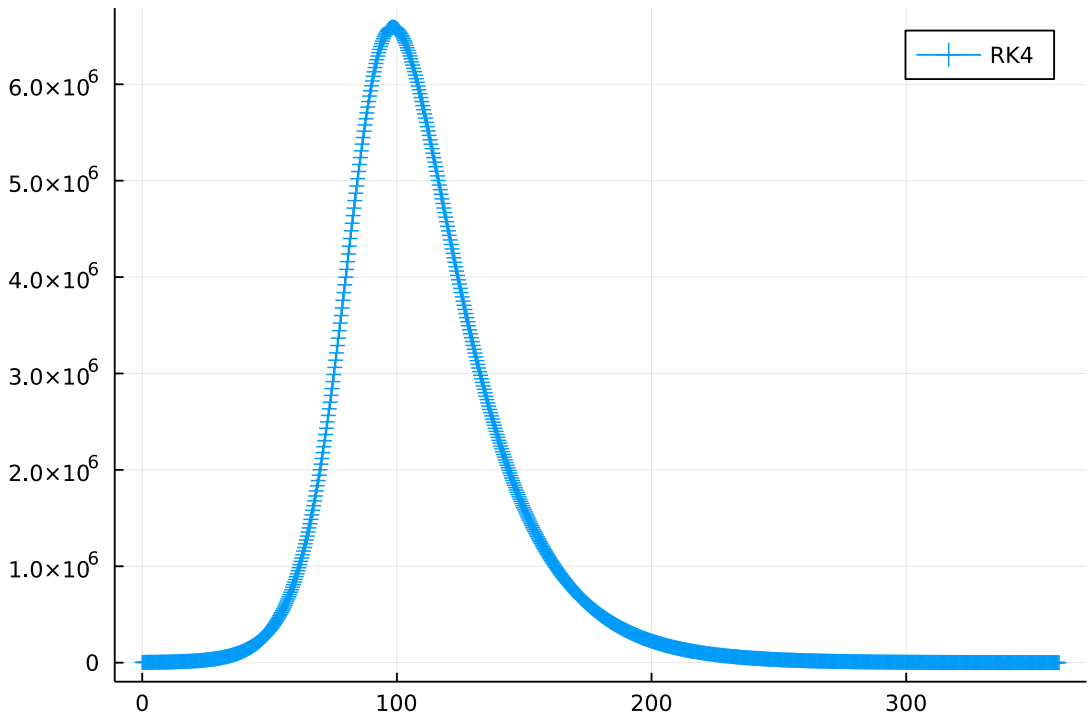


0.1.5 Problema 3

[219]:
```
 = 0.2;
e = 0.3;
q = 0;
j = 0.1;
```

```
 = 0.000034;
 = 408.09;
p = 0;
k1 = 0.1;
k2 = 0.125;
d1 = 0.0079;
d2 = 0.0068;
 1 = 0.0337;
 2  = 0.0386;
tf = 360;
B1 = 1;
B2 = 1;
B3 = 1;
B4 = 1;
C1 = 300;
C2 = 600;

f(x,y) =
[   - y[1]*( *y[4] +  e* *y[2] +  q* *y[3] +  j* *y[5])/(␣
 ↪y[1]+y[2]+y[3]+y[4]+y[5]+y[6] ) -  *y[1],
p + y[1]*( *y[4] +  e* *y[2] +  q* *y[3] +  j* *y[5])/(␣
 ↪y[1]+y[2]+y[3]+y[4]+y[5]+y[6] ) - (k1+ )*y[2] ,
-(k2+ )*y[3] ,
k1*y[2] - (d1 +  1 +  )*y[4],
k2*y[3] - (d2 +  2 +  )*y[5] ,
 1*y[4] +  2*y[5] -  *y[6] ]


init_con = [12000000,1565,292,695,326,20]
a = 0
b = 360
n = 1000

sol = rk4_system(f,init_con,a,b,n);

plot(sol[:,1],sol[:,3]+sol[:,4]+sol[:,5]+sol[:,6],markershape=:+,label="RK4")
```

[219]:



0.2 Problema 4

0.2.1 Números duales

$$d = f_0 + \epsilon f_1 \qquad \epsilon \neq 0 \quad \epsilon^2 = 0$$

[220]:
```
struct dual1
    f0
    f1
end
```

[222]:
```
# Operadores
import Base: + , - , * , ^
x::dual1 + y::dual1  = dual1(x.f0 + y.f0, x.f1 + y.f1)
x::dual1 - y::dual1  = dual1(x.f0 - y.f0, x.f1 - y.f1)
x::dual1 * y::dual1  = dual1(x.f0 * y.f0, x.f1 * y.f0 + x.f0 * y.f1)
x::dual1 ^ y::dual1 = dual1(x.f0 .^ y.f0, (x.f0 .^ (y.f0-1)) .* (y.f1.*log(x.f0␣
↪).*x.f0 + y.f0.*x.f1) )
x::dual1 / y::dual1 = dual1(x.f0 / y.f0, (x.f1*y.f0 - y.f1*x.f0) / y.f0^2)

#Funciones
import Base:  acos, acosh, asin, asinh, atan , atanh, sin, cos
acos(x::dual1) = dual1(acos(x.f0), -x.f1 ./ (sqrt(1-(x.f0)^2)))
acosh(x::dual1) = dual1(acosh(x.f0), x.f1 ./ (sqrt((x.f0)^2-1)))
```

```
asin(x::dual1) = dual1(asin(x.f0), x.f1 ./ (sqrt(1-(x.f0)^2)))
asinh(x::dual1) = dual1(asinh(x.f0), x.f1 ./ (sqrt((x.f0)^2+1)))
atan(x::dual1) = dual1(atan(x.f0), x.f1 ./ (1+(x.f0)^2))
atanh(x::dual1) = dual1(atanh(x.f0), x.f1 ./ (1-(x.f0)^2))
sin(x::dual1) = dual1(sin(x.f0), x.f1.*cos(x.f0))
cos(x::dual1) = dual1(cos(x.f0), -sin(x.f0) .* x.f1)

import Base: cosh, sinh, tan, tanh, exp, log, sqrt, abs
cosh(x::dual1) = dual1(cosh(x.f0), x.f1.*sinh(x.f0))
sinh(x::dual1) = dual1(sinh(x.f0), x.f1.*cosh(x.f0))
tan(x::dual1) = dual1(tan(x.f0), x.f1 ./ ((cos(x.f0))^2))
tanh(x::dual1) = dual1(tan(x.f0), x.f1 ./ ((cos(x.f0))^2))
exp(x::dual1) = dual1(exp(x.f0), x.f1.*exp(x.f0))
log(x::dual1) = dual1(log(x.f0), x.f1 ./ x.f0)
sqrt(x::dual1) = dual1(sqrt(x.f0), x.f1 ./ (2*sqrt(x.f0)))
abs(x::dual1) = dual1(abs(x.f0), (x.f0./abs(x.f0)).*x.f1)
```

[222]:
```
abs (generic function with 17 methods)
```

[223]:
```
a = dual1(1,1)
b = dual1(1,2)
c = dual1(2,2)
```

[223]:
```
dual1(2, 2)
```

[224]:
```
(a + c)*b
```

[224]:
```
dual1(3, 9)
```

[225]:
```
a^b
```

[225]:
```
dual1(1, 1.0)
```

[226]:
```
sinh(a)
```

[226]:
```
dual1(1.1752011936438014, 1.5430806348152437)
```