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COP 4630

Problem 1: Conjunctive Normal Form (4 pts)

Convert the following logical sentences to CNF, showing each step (include the name of each logical equivalence as you use it, referring to Figure 7.11)

a) **$((A \land B) \Rightarrow C) \land (\neg A \Rightarrow C) \land ((C \land D) \Rightarrow \neg E)$**

b) **$(\neg(A \land B) \lor (C \Longrightarrow D)) \Leftrightarrow (D \land A)$**

c) **$(A \lor B \lor C) \Longrightarrow (B \land C \land D)$**

d) **$(\neg A \Longrightarrow \neg B) \Longrightarrow ((C \Longrightarrow D) \land \neg(D \lor F))$**

A-D in the PDF

Problem 2: Resolution (4 pts)

Consider the following knowledge base, pre-translated into conjunctive normal form.

1. $\neg A \lor \neg B \lor \neg C$
2. $C \lor \neg D \lor G$
3. $D \lor F$
4. $A \lor \neg C$
5. $B \lor \neg E$
6. $C \lor \neg F$
7. E

   In the PDF

Prove, via resolution, G. Show your steps.

Problem 3: Truth Table (4 pts)

<u>Propositions:</u>

D: The car has two doors.
S: The car is a sports car.
E: The car is expensive.
C: The car is a convertible.

a) Based on the propositions above, take the following statements and convert them into propositional logic:

- “The car, if has two doors, is a sports car."

  $(D) \Rightarrow (S)$

- "If the car is a sports car or convertible, it is also expensive."

  $(S \lor C) \Rightarrow E$

- "The car is not expensive if it is old.”

  $O \Rightarrow \neg E$

b) Assume that the car is convertible. Construct a truth table (similar to the one shown in Figure 7.9 in the textbook) to show all cases where all statements from part (a) are true given that the car is convertible. Can we prove that the car is a sports car?

Convertible?	SportsCar?	Old?	HasTwoDoors?	Expensive?	All statements true?
1	0	0	0	0	0
1	0	0	0	1	1
1	0	0	1	0	0
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	0
1	1	0	0	0	0

1	1	0	0	1	1
1	1	0	1	0	0
1	1	0	1	1	1
1	1	1	0	0	0

Yes, it could be proven by having the truth table. In the beginning shows that its not a sports car but later on it shows that it is.

Problem 4: FOL Translation (8 pts)

Translate the following natural language statements into first-order logic (FOL) expressions.

1. "Every student loves logic."

   $\forall x\ (Student(x) \rightarrow Loves(x, Logic))$

2. "A person's mother's brother is that person's uncle."

   $\forall x\ (Person(x) \rightarrow \forall y\ (Mother(y, x) \rightarrow \exists z\ (Brother(z, y) \land Uncle(z, x))))$

3. "Every child has a mother."

   $\forall x\ (\mathrm{Child}(x) \rightarrow \exists y\ (\mathrm{Mother}(y, x)))$

4. "All birds can fly, except for penguins."

   $\forall x\ (Bird(x) \land \neg Penguin(x) \rightarrow CanFly(x))$

5. "Someone in the room is a spy."

   $\exists x\ (InRoom(x) \land Spy(x))$

6. "Every person who loves someone hates anyone who loves everyone."

   $\forall x\ (\exists y\ (Loves(x, y)) \rightarrow \forall z\ (\exists w\ (Loves(z, w)) \rightarrow Hates(x, z)))$

7. "No one can win the race without training."

   $\forall x\ (WinRace(x) \rightarrow Trained(x))$

8. "There is a tallest mountain."

   $\exists x\, (Mountain(x) \land \forall y\, (Mountain(y) \rightarrow Taller(x, y)))$

Ex: "All dogs are friendly.": $\forall x\, (Dog(x) \rightarrow Friendly(x))$

Part A:

$$((A \land B) \Rightarrow C) \land (\neg A \Rightarrow C) \land ((C \land D) \Rightarrow \neg E)$$

$$(\neg(A \land B) \lor C) \land (A \lor C) \land (\neg(C \land D) \lor \neg E)$$

Implication Elimination

$$((\neg A \lor \neg B) \lor C) \land (A \lor C) \land ((\neg C \lor \neg D) \lor \neg E)$$

De Morgan's Law

$$(\neg A \lor \neg B \lor C) \land (A \lor C) \land (\neg C \lor \neg D \lor \neg E)$$

Associativy Property

$$\boxed{\text{CNF: } (\neg A \lor \neg B \lor C) \land (A \lor C) \land (\neg C \lor \neg D \lor \neg E)}$$

Part B: $(\neg(A \land B) \lor (C \Rightarrow D)) \Leftrightarrow (D \land A)$

$$(\neg(A \land B) \lor (\neg C \lor D)) \Leftrightarrow (D \land A)$$

Implication Elimination

$$((\neg(A \land B) \lor (\neg C \lor D)) \Rightarrow (D \land A)) \land ((D \land A) \Rightarrow (\neg(A \land B) \lor (\neg C \lor D)))$$

Biconditional Elimination

$$\neg(\neg(A \land B) \lor (\neg C \lor D)) \lor (D \land A)$$

$$\neg(D \land A) \lor (\neg(A \land B) \lor (\neg C \lor D))$$

Implication Elimination

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Part B continued:

D) $\neg(\neg A \vee \neg B \vee \neg C \vee D) \vee (D \wedge A)$

De Morgan's Law

$(A \wedge B \wedge C \wedge \neg D) \vee (D \wedge A)$

Distributive property

$\boxed{\text{CNF: } (A \vee D) \wedge (B \vee D) \wedge (C \vee D) \wedge (\neg D \vee D)}$

Part C: $(A \vee B \vee C) \Rightarrow (B \wedge C \wedge D)$

$\neg(A \vee B \vee C) \vee (B \wedge C \wedge D)$

Implecation Elimination

$(\neg A \wedge \neg B \wedge \neg C) \vee (B \wedge C \wedge D)$

De Morgan's Law

$(\neg A \vee B) \wedge (\neg A \vee C) \wedge (\neg A \vee D) \wedge (\neg B \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D)$

Distributive property

$\boxed{\text{CNF: } (\neg A \vee B) \wedge (\neg A \vee C) \wedge (\neg A \vee D) \wedge (\neg B \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee D)}$

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D. $(\neg A \Rightarrow \neg B) \Rightarrow ((C \Rightarrow D) \land \neg(D \lor F))$

$A \lor \neg B \Rightarrow ((\neg C \lor D) \land (\neg D \land \neg F))$

Implication Elimination

$\neg(A \lor \neg B) \lor ((\neg C \lor D) \land (\neg D \land \neg F)$

Implication Elimination

$(\neg A \land B) \lor ((\neg C \lor D) \land (\neg D \land \neg F))$

De Morgan's Law

$((\neg A \lor \neg C \lor D) \land (\neg A \lor \neg D) \land (\neg A \lor \neg F)) \land ((B \lor \neg C \lor D)$
$\land (B \lor \neg D) \land (B \lor \neg F)$

Distributive Law

CNF: $\boxed{(\neg A \lor \neg C \lor D) \land (\neg A \lor \neg D) \land (\neg A \lor \neg F) \land (B \lor \neg C \lor D) \land (B \lor \neg D) \land (B \lor \neg F)}$

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2.

$C \lor \neg D \lor G$ $\quad$ G $\quad$ 2,3

1,3,6,4,5

$C \lor \neg D$ $\quad$ $\neg A \lor \neg B \lor \neg C$

$\neg A \lor \neg B \lor \neg D$ $\quad$ $D \lor F$

$\neg A \lor \neg B \lor F$ $\quad$ $C \lor \neg F$

$\neg A \lor \neg B \lor \neg F$ $\quad$ $A \lor \neg C$

$\neg B \lor \neg C \lor \neg F$ $\quad$ $B \lor \neg E$

$\neg C \lor \neg F \neg E$ $\quad$ E

$\neg C \lor \neg F$ $\quad$ $C \lor \neg F$

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