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Engineering Structures 27 (2005) 1465–1475
ENGINEERING STRUCTURES
www.elsevier.com/locate/engstruct

Flexural behaviour of stainless steel beams

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Received 8 March 2004; received in revised form 14 March 2005; accepted 14 April 2005
Available online 26 May 2005

Abstract

The present investigation studies the effect of the material non-linearity on the deflection calculation of stainless steel beams. A comparative analysis of experimental results and those obtained by means of different analytical methods is presented. This comparative analysis includes experimental results on deflection obtained from several tested stainless steel beams. The methods studied are the simplified method proposed in Eurocode 3, Part 1-4, ENV-1993-1-4: Design of Steel Structures. General Rules–Supplementary Rules for Stainless Steel, and different methods based on a numerical model that takes into account the material non-linearity. Finally, a new method for calculation of deflections considering the material non-linearity is proposed. It is based on an analytical expression of the moment–curvature relationship for stainless steel cross-sections.

Keywords: Stainless steel beams; Deflections; Serviceability limit state; Material non-linearity

1. Introduction

Stainless steel members have been increasingly used for structural applications in recent years due to high corrosion resistance, ease of maintenance, pleasing appearance and improved fire resistant. Nevertheless, the use of stainless steel as a resistant material is still limited, mainly due to its high cost of fabrication.

It is well known that the design of steel members is mostly governed by serviceability requirements—deflection control and vibration—and those efficient and economic designs may only be achieved if the methods used to obtain the structural response of the member consider an accurate material modelling.

Although the basis of design of stainless steel structures may be similar to that of carbon steel, a different specification for the design of stainless steel structures is necessary because the mechanical properties are significantly different from those of carbon steel. Stainless steel exhibits a non-linear stress–strain relationship, even for low stress magnitudes and a relatively low proportional limit.

The stainless steel material model should consider both aspects, obtaining all the benefits derived from taking into account strain hardening and, therefore, not leading to conservative designs.

Most of the design guides permit us to represent the stress–strain curve by using the Ramberg–Osgood equation:

$$\varepsilon = \frac{\sigma}{E_0} + 0.002\left(\frac{\sigma}{\sigma_{02}}\right)^n \quad (1)$$

where ε is the strain, σ the stress, E_0 the Young's modulus, n a constant related with the non-linearity degree, and σ_{02} is the yield stress determined as the 0.2% proof stress.

The non-linear stainless steel stress–strain relationship influences the flexural behaviour of stainless steel beams and makes the use of non-linear procedures necessary to determine deflections in stainless steel beams.

There are many different types of stainless steel. They were each developed to meet specific needs such as higher corrosion resistance, and improved mechanical properties: higher strength, hardness or ductility, metallurgical stability

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doi:10.1016/j.engstruct.2005.04.008

Notation

d	deflection
d_l	linear deflection
d_{li}	linear integrated deflection
d_{mc}	integrated moment curvature deflection
E_{secant}	secant modulus
E_0	Young's modulus
h	depth
I	second moment of area
I1	elastic deflection
I2	plastic deflection
l	span length
K_σ	experimental factor
M	bending moment
M_{max}	maximum bending moment
M_{02}	moment when the maximum tension stress reaches the yield stress (σ_{02})
R	elastic rigidity
W_{eff}	elastic section modulus
χ	curvature
χ_p	plastic curvature for M_{02}
χ_{02}	curvature for M_{02} strain
ε_p	plastic strain at yield stress (0.2%)
ε_e	elastic strain
ε_{02}	total strain at yield stress
σ	stress
σ_e	elastic stress
σ_{02}	yield stress

under the influence of welding heat, and in special cases improved machinability. Not all types of stainless steels are suitable for structural applications, particularly where welding is contemplated.

In fact, all the stainless steels known fall into five basic groups classified according to their metallurgical structure: these are the austenitic, ferritic, martensitic, duplex and precipitation-hardening groups. The austenitic stainless steels and the duplex (austenitic–ferritic) stainless steels are generally the more useful groups for structural applications. Table 1 shows the correlation between different designations for these stainless steels.

Pioneering research on the design of stainless steel structures and the calculation of deflections was conducted by Johnson and Winter [1] and Wang et al. [2]. These works represent the basis of the first standard related to the use of stainless steel in structural applications (AISI, 1974) [3] and proposed a simple method, based on the use of the secant modulus, for the determination of the inelastic deflection in stainless steel beams. This method is nowadays accepted for most of the standards.

Eurocode 3, Part 1-4 [4] proposes that deflections should be estimated using the secant modulus of elasticity determined taking account of the stresses in the member under the load combination for the relevant serviceability limit state.

In this work a study of the flexural behaviour of stainless steel beams is carried out, mainly focusing on the calculation of deflections. Experimental deflection results obtained from several tested stainless steel beams are presented. Such results are analysed and compared to the results derived from a numerical model that takes into account the material non-linearity. Moreover, a comparative analysis between those results and the results obtained according to Eurocode 3 is carried out.

This comparative analysis shows that stainless steel beam deflections obtained by the application of the simplified method derived from Eurocode 3, Part 1-4 [4], that considers a unique value of the secant modulus of elasticity along the structural element, may lead to overestimated deflections.

A new method for deflection calculation considering the material non-linearity is proposed in this paper. This method is based on an analytical expression of the moment–curvature relationship for stainless steel cross-sections.

The moment–curvature relationship for stainless steel cross-sections is fitted by an expression similar to the Ramberg–Osgood equation (1). In the same way as such an equation, an approximated analytical expression for the moment–curvature relationship is obtained as the addition of a plastic component to the elastic curvature. The parameter related to the non-linearity of the moment–curvature relationship is referred to the coefficient n of non-linearity of the Ramberg–Osgood equation (1). Lastly a direct integration procedure of the curvature law allows us to determine the deflection of stainless steel members considering the actual mechanical properties of the material. This method for determining deflections in stainless steel members may be applied to other structural members whose material modelling may be represented by the Ramberg–Osgood equation.

2. Experimental programme

In order to quantify the influence of the material non-linearity effects on the flexural behaviour of stainless steel beams, an experimental programme was conducted.

The main goal of this experimental investigation was to study the flexural behaviour of stainless steel beams and to obtain the maximum deflection under different load levels, especially near service conditions. All tests were carried out under displacement control by the imposition of a vertical displacement. More detailed aspects related to this experimental and numerical investigation on deflections calculation in stainless steel structures can be found in [5] and in [6].

2.1. Tested beams: Geometry and material properties

Six simply supported and six continuous stainless steel beams with square and rectangular hollow sections (SHS

Table 1
Correlation between stainless steel designations

Micro-structure	Steel grade to Eurocode3, Part 1-4 Nominal strength class	Steel grade to EN 10088		UK	USA
		Name	No.	BSI	AISI
		X5CrNi18-10	1.4301	304 S 15	
				304 S 16	
				304 S 31	304
Austenitic	S220	X2CrNi19-11	1.4306	304 S 11	304 L
		X2CrNi18-9	1.4307	–	–
		X6CrNiTi18-10	1.4541	321 S 31	321
		X5CrNi Mo17-12-2	1.4401	316 S 31	316
		X2CrNiMo17-12-2	1.4404	316 S 11	316 L
Austenitic	S240	X2CrNiMo17-12-3	1.4432	–	
		X2CrNiMo18-14-3	1.4435	316 S 13	316 L
		X1NiCrMoCu25-20-5	1.4539	–	904 L
		X6CrNiMoTi17-12-2	1.4571	320 S 31	316 Ti
Austenitic–ferritic	S480	X2CrNiMoN22-5-3	1.4462	Duplex 2205	–

Table 2
Dimensions of the stainless steel tested beams

Tested beam	Total depth (mm)	Flange width (mm)	Flange and web thickness (mm)	Span length (mm)	Total length (mm)
SHS 80 × 80 (simply supported)	80	80	3	1800	2000
RHS 80 × 120 (simply supported)	120	80	4	2800	3000
H 100 × 100 (simply supported)	100	100	8	2400	2600
SHS 80 × 80c (continuous)	80	80	3	1800	3800
RHS 80 × 120c (continuous)	120	80	4	2800	5800
H 100 × 100c (continuous)	100	100	8	2250	4700

Nominal cross-section dimensions.

and RHS) and H cross-sections were tested. The main characteristics and dimensions of the cross-sections of the tested beams are shown in Table 2. Notice that two beams of each type were tested.

The span length of the beams analysed is great enough to reproduce the beam behaviour. The depth/span length ratio is about 1/20 and 1/25 for all tested beams.

In order to compare experimental results to those derived by the numerical analysis and other analytical formulations it is absolutely necessary to know the material properties. The stainless steel properties of the tested beams were obtained by the steel producer according to ASTM [7]. Fig. 1 shows the experimental stress–strain relationships, for the stainless steel tested beams, and the basic material properties.

It may be appreciated that differences between the values of the mechanical properties obtained in the specimen tests for the materials and the values proposed by Eurocode 3, Part 1-4 [4] for the annealed properties are very significant.

2.2. Instrumentation

The tested simply supported beams were subjected to a concentrated load at mid-span and the continuous beams to two concentrated loads close to the mid-span.

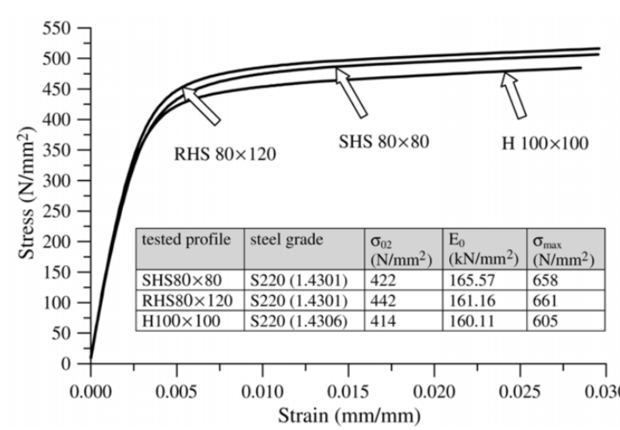

Fig. 1. Stainless steel test results.

Linear displacement transducers were used to measure beam deflections, and unidirectional strain gauges to measure longitudinal strains. In addition, load cells are located at the support sections of the continuous beams in order to know the reaction forces to control possible deviations of the jacking force during the test. In Fig. 2 the two typical instrumentation schemes used in this experimental programme are represented.

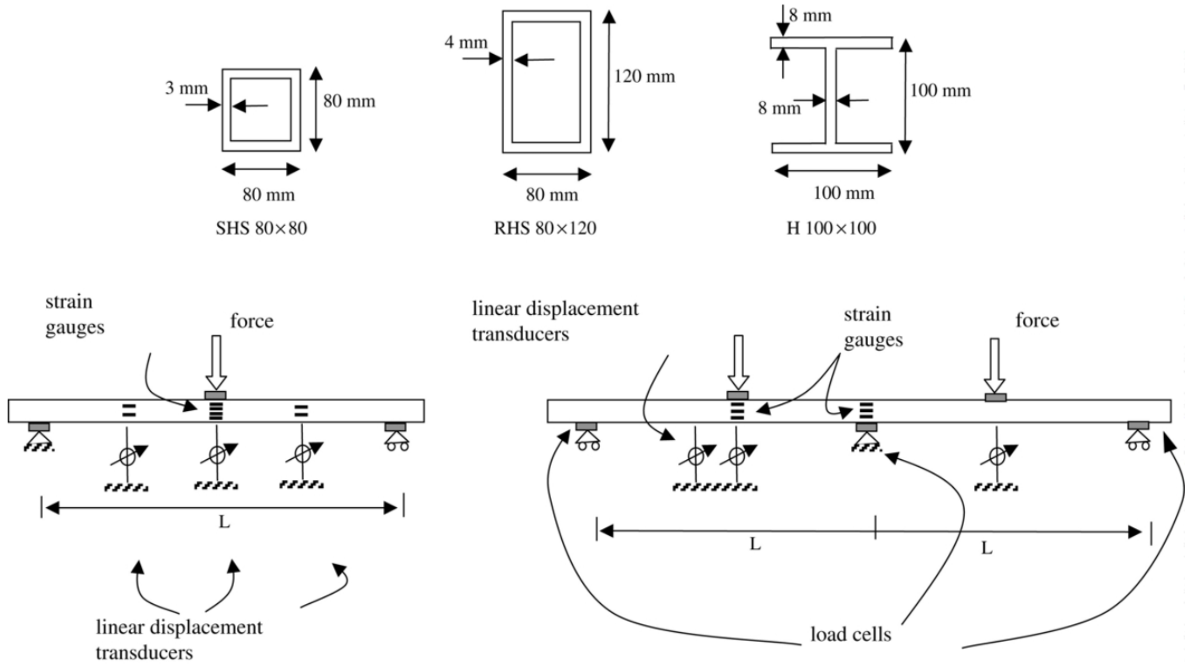

Fig. 2. Instrumentation, cross-section and loading schemes of the tested beams.

2.3. Experimental results

Experimental deflections were determined by using linear displacement transducers in all tested stainless beams. Fig. 3 shows the experimental load–deflection curves obtained with the transducers 1 and 3 (ldt1 and ldt3) located at 1/4 cross-sections, and with the transducer 2 (ldt2) located at mid-span for the SHS 80 × 80 simply supported beam.

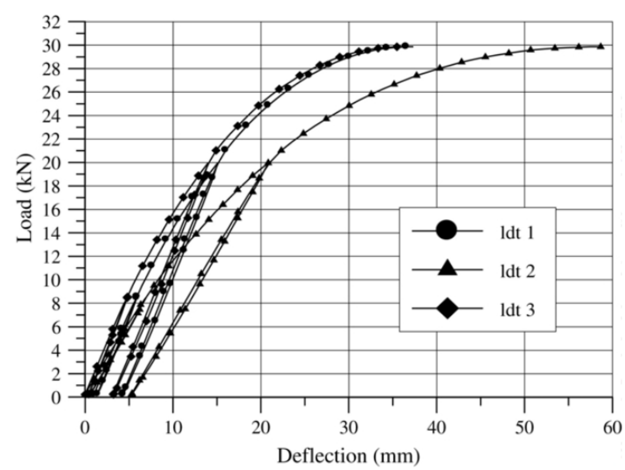

Fig. 3. Experimental load–deflection curves for the SHS 80 × 80 simply supported beam.

3. Methods for determining deflections

Deflections obtained by the experimental tests were compared to those estimated using the simplified methods proposed in Eurocode 3, Part 1-4 [4], other approximate methods based on the use of adjusted coefficients [8,9], and the deflections obtained by using the numerical model in the FE-code Abaqus [10].

3.1. Simplified methods

3.1.1. Eurocode 3, Part 1-4

Eurocode 3, Part 1-4 [4] states that the effects of the non-linear stress–strain curve of the material and the effective cross-section should be considered in estimating deflections in stainless steel beams. This code also proposes to evaluate deflections using the secant modulus of elasticity E_{secant}, determined taking into account stresses in the member under the load combination for the relevant serviceability limit state.

Two different linear simplified methods of calculating deflections in stainless steel beams were used in this study in order to evaluate the variation of Young's modulus along the length of a stainless steel beam and within the cross-section.

First of all deflections are estimated using linear expressions with a unique value of the secant modulus of elasticity and neglecting the variation of this modulus along the length of the element. This secant modulus is determined also with linear expressions and considering the maximum stresses of the structural member. These deflections are called linear deflections.

The second method determines linear deflections like the previously mentioned one, but now considering the variation of the modulus of elasticity along the length of the element. These deflections are called integrated linear deflections.

3.1.2. Approximate methods

Some authors [8,9] propose other simple approximate methods for determining deflections of stainless steel beams. These methods use different factors to take account of the moment distribution along the beam and the stress distribution of the cross-section.

The method proposed by Rasmussen and Hancock [8] is an approximate explicit non-linear method based upon the simplified method described by Johnson and Winter [1]. The maximum deflection is expressed as the linear-elastic deflection using the secant modulus obtained as the average of the secant modulus calculated at the extreme fibers in tension and compression at the cross-section of maximum bending moment. It was assumed that the stress at the extreme fibers could be determined using

$$\sigma = K_\sigma \frac{M_{\max}}{W_{\text{eff}}} \tag{2}$$

where $M_{\max}$ is the maximum bending moment, W_{eff} the elastic section modulus and K_σ an experimental factor less than or equal to unity. Using some test results and a finite element analysis, they propose $K_\sigma = 2/3$ for SHS simply supported stainless steel beams and $K_\sigma = 1/2$ for SHS continuous stainless steel beams.

Some other tests on RHS stainless steel beams were performed by Talja and Salmi [9]. The displacements measured from the bending tests were used to verify the determination of the deflections of the stainless steel beams. The maximum deflection of the beams was calculated with the method proposed by Rasmussen and Hancock [8]. The Talja and Salmi [9] report concluded that when the ideal modulus of elasticity is used in the Rasmussen and Hancock [8] method, the deflections are underestimated. If the modulus of elasticity is reduced by a factor of 0.85 the deflections are quite accurate below the service load.

These methods have been used in this study and the results derived from them have been compared to the other results.

3.2. Numerical method

The non-linear stainless steel stress–strain relationship influences the flexural behaviour of stainless steel beams and it is difficult to obtain inelastic deflections. A numerical model based on the finite element method is very suitable to analyse the beams' behaviour, calibrate it against the experimental test and calculate non-linear deflections in stainless steel beams.

The finite element code Abaqus [10] has been used to observe the behaviour described by theoretical models and to study and optimise the recording of information during the tests.

In the structures modelled in this study the length is significantly greater than the other two dimensions and the longitudinal stress is most important, so beam elements have been used for the modelling.

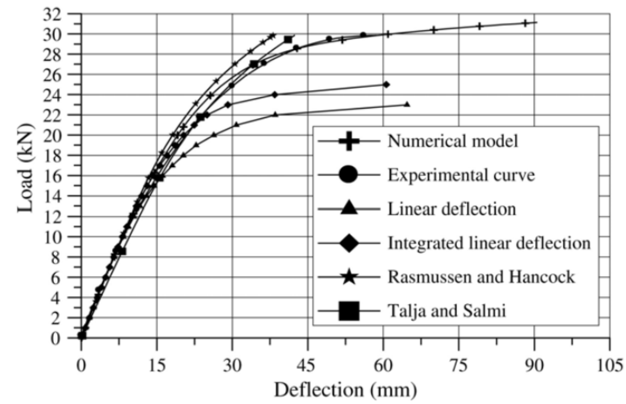

Fig. 4. Load–deflection curves for the SHS 80 × 80 simply supported beam.

In this case Euler–Bernoulli beam elements have been used. These elements do not allow for transverse shear deformation; plane sections initially normal to the beam axis remain plane and normal to the beam axis. They should be used only to model slender beams: the beam's cross-sectional dimensions should be small compared to distances along the axis. The Euler–Bernoulli beam elements use cubic interpolation functions and are written for small-strain, large-rotation analysis. The through thickness integration is accomplished with Simpson's rule of order five.

The material has been modelled as a von Mises material with isotropic hardening. The stress–strain relation used in the numerical simulation has been determined from the information provided from the steel producer. Elastic properties used were a Young's modulus of 2×10^5 MPa and a Poisson's ratio of 0.3. The analysis involves a static problem with a non-linear material and the Newton–Raphson method is used.

A more detailed description of elements, constitutive equations and resolution procedures can be found in the Abaqus manuals [10].

3.3. Comparative analysis

Deflections obtained by the experimental tests have been compared to those estimated using the numerical model Abaqus, the deflections obtained by using the simplified methods proposed in Eurocode 3, Part 1-4 [4] and the results of the application of the approximate methods proposed by Rasmussen and Hancock [8] and Talja and Salmi [9] (Fig. 4).

In Fig. 4 it is noticed that the numerical model results agree with the experimental results. As for the results obtained from Eurocode 3, Part 1-4, it can be pointed out that the load–deflection curve that takes into account the variation of the secant modulus along the length of the element (integrated linear deflection) is the most appropriate one to compare with the experimental results. The other one (linear deflection), obtained by using the minimum value of the secant modulus of elasticity, is close to the experimental curve until 60% of the load value on which the maximum stress in the beam reaches the yield stress

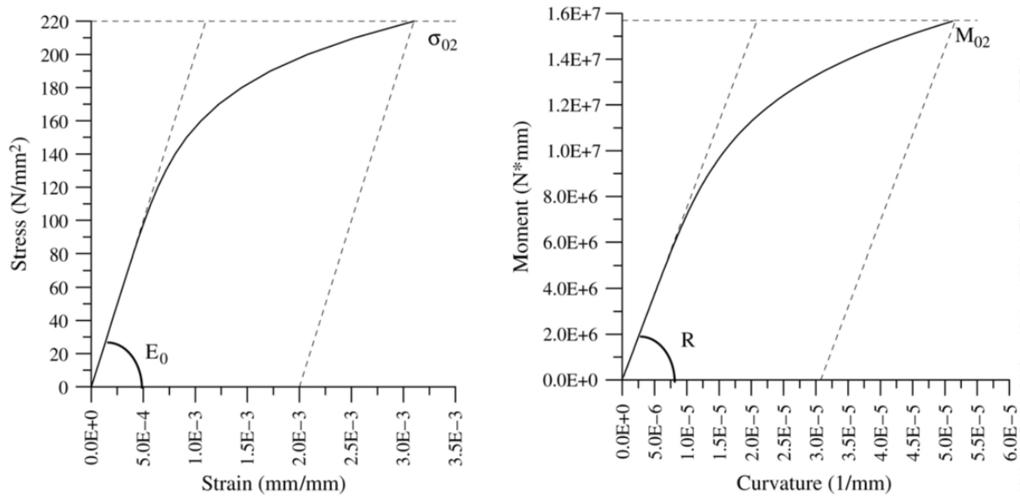

Fig. 5. Relationships σ–ε and M–χ for a S220 (1.4301) stainless steel.

(load $=$ 21.44 kN). For upper load levels, differences between two curves increase because the Eurocode 3, Part 1-4 method does not consider the variation of elasticity modulus along the beam and within the cross-section. Differences between the experimental curves and the integrated linear deflection curve over the load value on which the maximum stress in the beam reaches the yield stress (load $=$ 21.44 kN) are due to the fact that the integrated linear method does not consider the variation of the modulus of elasticity within the cross-section.

The curves determined by the approximate method proposed by Rasmussen and Hancock [8] agree well with the experimental results for service loads, although the deflection values are slightly underestimated. Moreover, the factor K_σ should vary with load cases and bearing conditions.

Talja and Salmi [9] propose the reduction of the modulus of elasticity by a factor of 0.85 to avoid the underestimation of deflections obtained by the behind method. It is possible to observe a slight loss of rigidity in the obtained curve since the beginning of the test.

A more detailed analysis of the experimental, numerical and analytical results can be obtained in [6].

4. A new proposed method for determining deflections in stainless steel beams

In the previous section we have observed that linear simplified methods may lead to overestimates of the deflections in stainless steel beams. Based on previous results, it is obvious that a more accurate method for determining non-linear deflections in stainless steel beams should be provided.

Inelastic deflections in stainless steel beams can be determined by a direct integration procedure, with adjustments for the boundary conditions, using the moment–curvature relationship for stainless steel cross-sections. The necessary steps for determining these inelastic deflections from the moment–curvature relationship are explained in the following sections.

4.1. Moment–curvature relationship

In this section the moment–curvature relationship for stainless steel beams will be analysed in order to obtain a direct relation between the applied moment and the curvature of a cross-section.

Using the static equilibrium and compatibility equations it is possible to obtain numerically a direct relation between the curvature and the maximum deformation in the cross-section. However, the moment–curvature relationship for a stainless steel cross-section could be fitted by an expression similar to the Ramberg–Osgood equation (1).

As an example, the stress–strain relationship for the S220 (1.4301) stainless steel and the moment–curvature relationship for a rectangular beam are presented in Fig. 5.

In the same way as the Ramberg–Osgood equation, it is possible to obtain an approximate analytical expression for the moment–curvature relationship as the addition of a plastic component to the elastic curvature.

$$\chi = \frac{M}{R} + \chi_{\mathrm{p}} \left(\frac{M}{M_{02}} \right)^{m} \tag{3}$$

where $R = E_0 I$ is the elastic rigidity, χ_{p} the plastic curvature for M_{02}, m a constant related to the material non-linearity and M_{02} the applied moment when the maximum tension stress reaches the yield stress (σ_{02}).

These coefficients can be determined by analysing the behaviour of a cross-section subjected to a bending moment and releasing the applied bending moment (Fig. 6).

In the loading process the maximum strain ε_{02} when the maximum tension reaches the yield stress is

$$\varepsilon_{02} = \frac{\sigma_{02}}{E_0} + 0.002. \tag{4}$$

On releasing the applied moment the beam rebounds elastically. On this basis the strain can be determined as

$$\varepsilon_{\mathrm{e}} = \frac{M_{02} h}{2 E_0 I}. \tag{5}$$

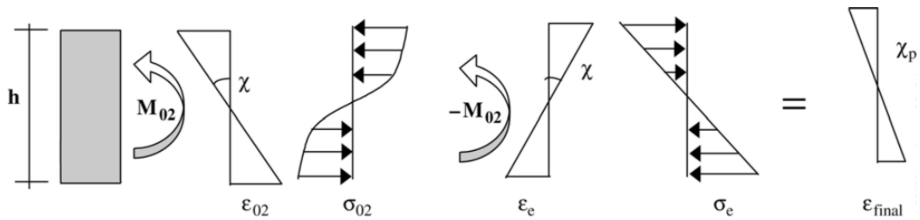

Fig. 6. Rectangular section subjected to a bending moment and releasing the applied moment.

So, the plastic strain is

$$\varepsilon_p = \varepsilon_{02} - \varepsilon_e = \frac{\sigma_{02}}{E_0} + 0.002 - \frac{M_{02}h}{2E_0 I} \tag{6}$$

and the plastic curvature χ_p can be determined as

$$\chi_p = \frac{2\,\varepsilon_p}{h} = \frac{2}{h}\left(\frac{\sigma_{02}}{E_0} + 0.002\right) - \frac{M_{02}}{E_0 I}. \tag{7}$$

In Eq. (3) the constant m related to the non-linearity has been adjusted numerically taking $m = n - 1$ where n is the coefficient in the Ramberg–Osgood equation.

There is not an explicit relationship to determine stresses from strains for stainless steel. Therefore M_{02} must be determined by using numerical integration of the stress distribution. So, the deformation that corresponds to the yield strength is imposed as the maximum strain. Assuming that plane sections remain plane during deformation it is possible to obtain the strain distribution along the cross-section and consequently, using the Ramberg–Osgood equation, the stress distribution. Finally, M_{02} is obtained by integrating this stress distribution.

Considering Eqs. (3) and (7) and assuming that $m = n - 1$, an explicit expression of the moment–curvature relationship for stainless steel cross-sections can be obtained.

$$\chi = \frac{M}{E_0 I} + \left(\frac{2}{h}\left(\frac{\sigma_{02}}{E_0} + 0.002\right) - \frac{M_{02}}{E_0 I}\right)\left(\frac{M}{M_{02}}\right)^{n-1}. \tag{8}$$

Once the moment–curvature relationship has been determined the deflection of a beam will be obtained by means of a direct integration procedure of the curvatures law along the beam considering the boundary conditions. Furthermore, the maximum strain in the cross-section can be determined from the curvature, assuming that plane sections remain plane during deformation. The maximum stress can be determined by using the Ramberg–Osgood equation.

4.2. Verification of the proposed moment–curvature relationship

The analytical formulation proposed to determine moment–curvature relationships for stainless steel cross-sections (Eq. (8)) is now compared to actual moment–curvature relationships in order to evaluate the accuracy of the formulation.

A simple numerical program has been developed to determine the actual moment–curvature relationship. For each cross-section, and different load levels, a plane deformation is imposed and the corresponding stress distribution is calculated to obtain the applied moment by the integration of the stress distribution.

As an example the M–χ relationships for two different cross-sections are presented in Fig. 7. The first one corresponds to a square hollow section (80 × 80 × 5) with an S220 (1.4301) stainless steel, and the other one corresponds to a rectangular hollow section (80 × 120 × 5) with an S240 (1.4401) stainless steel. Table 3 shows the main characteristics for determining the M–χ relationships.

Table 3
Main characteristics for determining M–χ relationships

Stainless steel	σ_{02} (N/mm^2)	E_0 (N/mm^2)	n
S220 (1.4301)	220	200 000	6.5
S240 (1.4401)	240	200 000	7
Section	**Flange width (mm)**	**Depth (mm)**	**Thickness (mm)**
80 × 80 × 5	80	80	5
80 × 120 × 5	80	120	5

In this figure, relationships are presented in a non-dimensional form M/M_{02}–χ/χ_{02} in order to get a better comparison. All curves present very little differences between the numerical relationship and the analytical one.

In order to evaluate these differences, a more extensive analysis has been made. The cross-sections used in the study are rectangular hollow sections and H sections with different dimensions. The study has been performed with the most common stainless steels used in practice, S220 (1.4301), S240 (1.4401) and S480 (1.4462).

During this study a wide range of stainless steel cross-sections have been analysed and it has been possible to assert that the differences between the numerical and analytical relationships are less than 5%.

4.3. Integration of curvatures: Determination of deflections

An analytical expression for the moment–curvature relationship for stainless steel cross-sections has been presented above (Eq. (8)). Inelastic deflections in stainless steel beams can be determined by a direct integration procedure of the curvatures law, with adjustments for the boundary conditions.

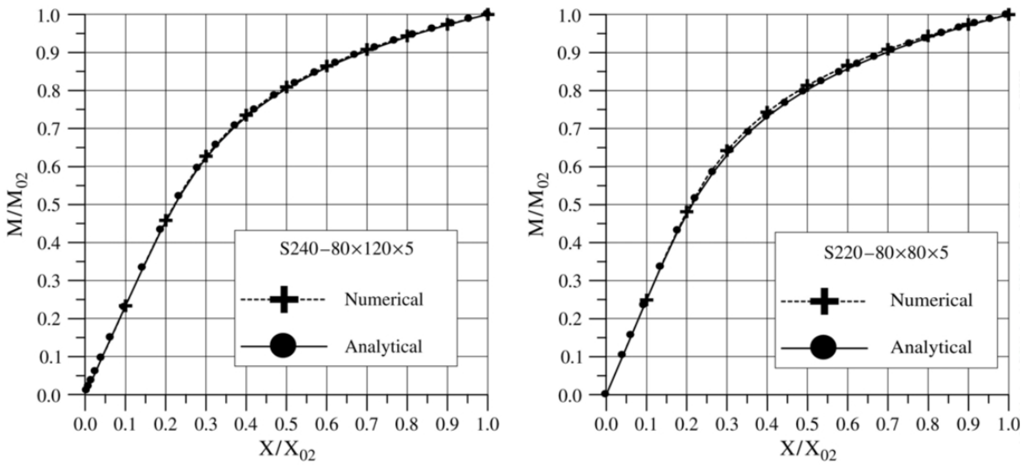

Fig. 7. Moment–curvature relationships.

In some statically determinate beams with simple loading states it is possible to obtain analytical expressions to determine inelastic deflections. In the case of simply supported beams with length l, and a symmetric bending moment law, the maximum deflection is

$$
\begin{aligned}
d &= \int_0^{l/2} \chi(x) x \, \mathrm{d}x \\
&= \underbrace{\int_0^{l/2} \frac{M(x)x}{E_0 I} \mathrm{d}x}_{\mathrm{I1}} + \underbrace{\int_0^{l/2} \chi_{\mathrm{p}} \left(\frac{M(x)}{M_{02}}\right)^{n-1} x \, \mathrm{d}x}_{\mathrm{I2}}
\end{aligned} \tag{9}
$$

x being the longitudinal coordinate of the beam. In this equation I1 is the elastic deflection and I2 is the plastic contribution that depends on $M(x)$ and on the plastic curvature χ_{p}.

In the Appendix, analytical expressions of the integrals I1 and I2 for simply supported beams subjected to different loading schemes are shown.

The integration procedure discussed above for obtaining the deflections of the beams is generally applicable, but only in the simplest cases is it possible to obtain an analytical expression of the deflections. For other type of beams it is necessary to make adjustments for the boundary conditions that make the analytical integration more difficult. In these cases the deflections should be determined by a numerical integration procedure with a simple program, using the moment–curvature relationship.

5. Analysis of results

In order to evaluate the result in deflections using the moment–curvature relationship, a comparative analysis in simply supported beams with different loading conditions and different cross-sections has been made.

Four different methods are used to determine deflections. First, deflections have been determined by a numerical model in the finite element code Abaqus [10] taking into account the material non-linearity. In the second method deflections are estimated by using linear expressions with a unique value of the secant modulus of elasticity and neglecting the variation of this modulus along the length of the element (linear deflection, d_{l}). This secant modulus is determined also with linear expressions and considering the maximum stresses of the structural member. The third method determines linear deflections like the previously mentioned one but now considering the variation of the modulus of elasticity along the length of the element (integrated linear deflection, d_{li}). The last two methods are the simplified methods proposed in Eurocode 3, Part 1-4 and presented in Section 3.1. For the fourth method, deflections are determined by integrating the moment–curvature relationship in each cross-section (integrated moment curvature deflection, d_{mc}).

The numerical model considers the actual constitutive equation of the stainless steel, considering the material non-linearity in deflection calculations, as has been mentioned above, so the deflections calculated by the numerical model are understood as actual deflections.

Fig. 8 shows the load–deflection curves in the cases analysed: concentrated load, distributed load and applied bending moments. In this figure, “actual yield stress” line means the load value on which the maximum stress in the beam reaches the yield stress.

Analysing these curves it is possible to highlight the benefit of using the moment–curvature relationship for determining inelastic deflections in stainless steel beams. The linear simplified methods show important differences compared to the numerical solution as the stress level increases.

The analysis of the behaviour of a stainless steel beam subjected to different bending moments shows that stainless steel beam deflections obtained by using one value of the secant modulus of elasticity along the structural element, determined by linear theory, may lead to overestimated deflections. Indeed, as the maximum linear stress is higher than the maximum actual stress then the corresponding secant modulus is lower, and deflections are increased.

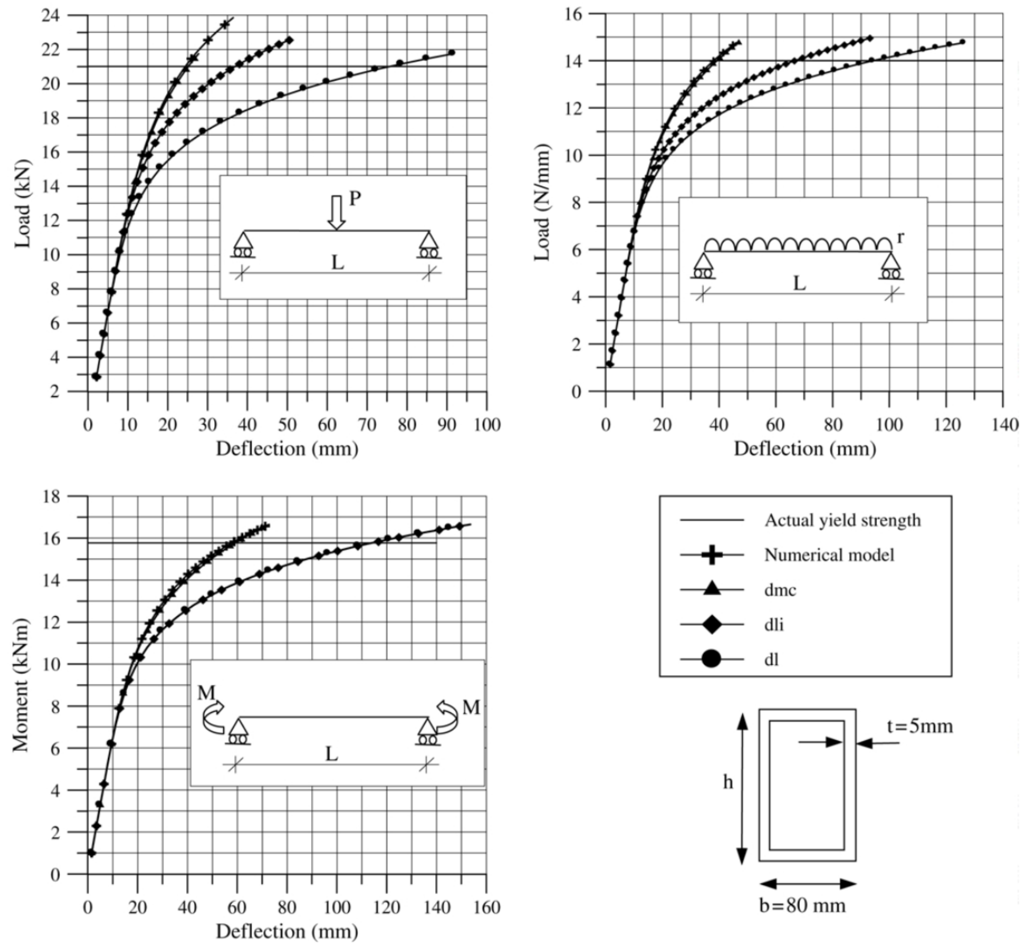

Fig. 8. Load–deflection curves in the analysed cases for a S220 (1.4301) stainless steel.

Nevertheless, superposition does not apply in inelastic problems since deflections are not linearly related to the applied forces. In statically indeterminate beams the bending moment law depends on the material non-linearity, and the redistribution of bending moments needs to be taken into account. As a consequence of this, in these cases some different solutions are required to calculate deflections.

A deflection calculation analysis for continuous beams is also presented here. The study is similar to the one presented above for simply supported beams, although in this case the redistribution of internal forces due to material non-linearity effects is considered.

Deflections are also calculated by using linear expressions with a unique value of the secant modulus of elasticity corresponding to the intermediate support cross-section (maximum negative bending moment, d_{lneg}) and the value of the secant modulus of elasticity corresponding to the loaded cross-section (maximum positive bending moment, d_{lpos}).

Fig. 9 shows, as an example, the results for a continuous beam subjected to concentrated loads at mid-span, with rectangular hollow section and S220 (1.4301) stainless steel.

We can see in this figure that, as it happens for statically determinate beams, the linear determination of deflections produces large errors, even for stresses beyond yield strength. The use of a unique value of the secant modulus corresponding to the cross-section subjected to maximum bending moment (positive or negative) may lead to overestimated deflections. In any case, the better approximation is the one derived from the integration of the curvatures determined by the analytical moment–curvature relationship.

6. Conclusions

In the present work a study of the effect of the material non-linearity on the deflection calculation of stainless steel beams has been done. Experimental results derived from several tested stainless steel beams have been compared to the results derived from a numerical model and with some linear simplified methods proposed in codes.

The analysis of the differences in determining deflections allows us to conclude that linear simplified methods that propose calculating deflections by using a unique value of the secant modulus may lead to overestimated deflections.

A new method for deflection calculation considering the material non-linearity and based on an analytical expression

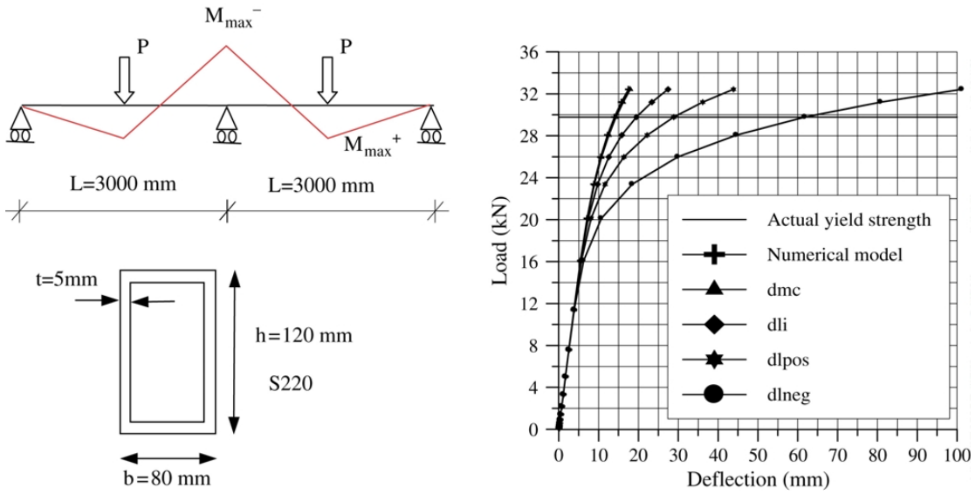

Fig. 9. Load–deflection curves in continuous beams for a S220 (1.4301) stainless steel.

of the moment–curvature relationship for stainless steel cross-sections is proposed.

An explicit expression of the moment–curvature relationship for stainless steel cross-sections has been presented (Eq. (8)):

$$\chi = \frac{M}{E_0 I} + \left(\frac{2}{h}\left(\frac{\sigma_{02}}{E_0} + 0.002\right) - \frac{M_{02}}{E_0 I}\right)\left(\frac{M}{M_{02}}\right)^{n-1} \quad (8)$$

where $R = E_0 I$ is the elastic rigidity and χ_p is the plastic curvature for M_{02} that can be determined assuming that on releasing the applied moment the beam rebounds elastically.

Inelastic deflections in stainless steel beams can be determined by a direct integration procedure of the moment–curvature relationship, with adjustments for the boundary conditions.

Acknowledgements

This study was carried out with financial support provided by the Spanish Ministry of Education and Culture as a part of Research Project PB95-0772. This Project is continued by Research Project MAT 2000-1000 with financial support from the Spanish Ministry of Science and Technology.

Appendix. Analytical expressions of the integrals I1 and I2 for simply supported beams subjected to different loading schemes

In the case of simply supported beams with length l, and a symmetric bending moment law, the maximum deflection is

$$d = \int_0^{l/2} \chi(x) x \, \mathrm{d}x = \underbrace{\int_0^{l/2} \frac{M(x)x}{E_0 I} \mathrm{d}x}_{\mathrm{I1}} + \underbrace{\int_0^{l/2} \chi_p \left(\frac{M(x)}{M_{02}}\right)^{n-1} x \, \mathrm{d}x}_{\mathrm{I2}}.$$

(a) Concentrated load at mid-span: $M(x) = Px/2$

$$\mathrm{I1} = \frac{Pl^3}{48 E_0 I}$$

$$\mathrm{I2} = \int_0^{l/2} \chi_p \left(\frac{Px}{2M_{02}}\right)^{n-1} x \, \mathrm{d}x = \chi_p \left(\frac{P}{2M_{02}}\right)^{n-1} \int_0^{l/2} x^n \mathrm{d}x = \chi_p \left(\frac{P}{2M_{02}}\right)^{n-1} \left(\frac{(l/2)^{n+1}}{n+1}\right).$$

(b) Constant bending moments: $M(x) = M$

$$\mathrm{I1} = \frac{Ml^2}{8 E_0 I}$$

$$\mathrm{I2} = \int_0^{l/2} \chi_p \left(\frac{M}{M_{02}}\right)^{n-1} x \, \mathrm{d}x = \chi_p \left(\frac{M}{M_{02}}\right)^{n-1} \frac{l^2}{8}.$$

(c) Uniformly distributed load: $M(x) = px(l-x)/2$

$$\mathrm{I1} = \frac{5pl^4}{384 E_0 I}$$

$$\mathrm{I2} = \int_0^{l/2} \chi_p \left(\frac{px(l-x)}{2M_{02}}\right)^{n-1} x \, \mathrm{d}x = \chi_p \left(\frac{p}{2M_{02}}\right)^{n-1} \int_0^{l/2} \underbrace{x^n (l-x)^{n-1} \mathrm{d}x}_{\mathrm{I2}'}.$$

There is no direct solution to determine I2′ so the way to obtain an analytical expression is by changing the integration variable ($y = x/l$) to make the integral length independent. Finally it has been numerically fitted to an exponential function.

$$\mathrm{I2}' = \int_0^{l/2} x^n (l-x)^{n-1} \mathrm{d}x = \int_0^{1/2} (ly)^n (l-ly)^{n-1} l \, \mathrm{d}y = l^{2n} \int_0^{1/2} y^n (1-y)^{n-1} \mathrm{d}y \approx l^{2n} * 0.1 * \mathrm{e}^{-1.45(n-1)}.$$

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