GARCH models in portfolio optimization of the most traded stocks in Etoro exchange platform

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Abstract
This study investigates the triple of stocks that are the most occurring in Etoro newbies holdings. The core of the top 10 stocks consists of high tech companies stocks, AAPL, GOOG, TSLA are modelled. [1] Weekly closing prices, from 2016-04-20 to 2021-05-15, are gathered from NASDAQ were studied. Efficient portfolio weight distribution is examined by two GARCH models: ARMA-GARCH and ARMA-DCC-GARCH, where returns are modelled with Autoregressive Moving Average (ARMA) together with volatility by Generalized Autoregressive Conditional Heteroskedastic GARCH (or Dynamic Conditional Correlation GARCH, known as DCC-GARCH) respectively, and compared with strategy "Diversify portfolio with the uniform weights". The core idea of optimization hides in investors desire to have maximum payout with a lowest plausible risk, hence in this case multivariate DCC-GARCH model have shown a better performance, a model with higher returns mean and variance ratio that univariate GARCH.

Keywords
ARMA — GARCH — Multivariate GARCH models — Portfolio optimization — Mean-VaR efficiency — Value-at-Risk — Technology stocks

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Contents

Introduction 1
1 Methods 1
1.1 GARCH models 1
1.2 Portfolio optimization 3
2 Analysis and results 3
2.1 Data, descriptive statistics 3
2.2 ARMA-GARCH portfolio optimization 3
2.3 ARMA-DCC-GARCH portfolio optimization 4
2.4 Results comparison 5
3 Discussion and conclusions 5
References 6

Introduction

In recent years, from 2019, number of newbie investors surged to commission-free brokerages like "Etoro" for solely of fast financial gains. Having no prior knowledge whatsoever to make efficient portfolio, they tend to choose the most popular tech stocks as well as the most mainstream like: Alphabet Inc Class C (GOOG), Tesla Motors (TSLA), Apple (AAPL) and so on, however for simplicity of research, only mentioned will be included in analysis of portfolio optimization. Another credible aspect of why newcomers pick technology market might be of its long lasting bull run, that might reassemble situation of dot-com Bubble in 00's. [2]

It is well-known that diversification, portfolio spreading in-between many sectors, and uniform distribution of portfolio helps against immense plunges, high risk-variance. Nevertheless this strategy is valid only with normality assumption, hence for non-gaussian stock portfolio requires more sophisticated and robust methods to be applied. To deal with non-normality and varying variance we employ univariate ARMA-GARCH and multivariate ARMA-DCC-GARCH. Another point to stress out is that even if investors monitor volatility by following specific indices, for instance VIX index, or others possible market benchmarks, they will miss the efficiency of portfolio.

In this study, the core object is Mean-VaR ratio, since it gives out the best returns on average with the lowest risk. Obviously this ratio is dependent on how well we measure volatility hence different volatility measuring methods will output distinct results and suggested weights for portfolio optimization.

Objectives of the study The objective of this paper is to approach GARCH, ARMA-GARCH, ARMA-DCC-GARCH methods to estimate the best weights distribution inbetween portfolio and finally compare it with the uniformly diversified portfolio efficiency.

1. Methods

1.1 GARCH models

No methods were available for the variance modeling before the introduction of autoregressive conditionally heteroscedastic (ARCH) process introduced by Engle (1982). The ARCH model has been investigated and generalized by several authors, including Bollerslev (1986) and Gourie´roux (1997).

ARCH and GARCH models are applicable to a range of time series analyses, especially when researcher suspects a break-point of variability in time-series, however implementation in finance have been especially successful. Investing decisions are generally based upon the trade-off between risk and return. The econometric analysis of risk is an part of asset pricing, portfolio optimization and risk management. Hence ARCH/GARCH models became the standard tool to analyse volatility, specify how information can help forecast the mean and variance of the return.

The assumption of equal weights seems unrealistic, because the more recent events should be more significant and therefore should have higher weights. Furthermore the assumption of zero weights for observations more than one month old is also unattractive. ARCH/GARCH models solves this problem of residuals, it has declining weights that never go completely to zero. The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance. [3]

Main characteristic of model is that the variance of the current error term to be function of the error terms of the past time period. Model can be defined as stochastic process ε:

$$\varepsilon_t = \sigma_t z_t$$
$$z_t \sim i.i.d$$

where z_t is white noise process and also the series σ_t^2 is modeled as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_p \sigma_{t-p}^2$$

where p is the order of the GARCH term σ^2 and q is the order of the ARCH term ε^2. In this study only small p, q parameters are considered, higher order GARCH model is applicable for long period data, for instance decades. [4]

Multivariate GARCH models While modelling volatility of the returns understanding a co-movement of stocks returns is of great practical importance because no asset is fully independent of others. [5] It is therefore crucial to extend the univariate GARCH to multivariate GARCH models as commodities pricing also depends on the covariance of the assets in a portfolio. In short, risk has to be managed with asset allocation - updating optimal portfolio weights if it is suspected that volatility had a break-point. Lets consider DCC-GARCH model.

DCC-GARCH However, Tse and Yu (1999) has proven that the constant correlation is not valid when the estimation process is multivariate. The pair-wise correlations are also time-varying and they need to be modeled to produce consistent errors. The challenging problem of non-constant correlation is solved by the dynamic conditional correlation GARCH (DCC-GARCH), proposed by Engle (2001).

Method has two core steps, first is to find conditional standard deviations through the univariate GARCH and second to model the time varying correlations relying on lagged values of residuals and covariance matrices. After that, conditional covariance matrix could be found by using conditional standard deviations and dynamic correlations. [6]

Further derivations will be taken from [7] paper, let define the DCC-GARCH. DCC-GARCH assumes that returns of stocks in portfolio are normally distributed with mean zero and covariance matrix C_t

$$x_t \sim N(0, C_t)$$

Combining conditional standard deviation and dynamic correlation matrix we can obtain the conditional covariance matrix. Let S_t be a $1 \times n$ vector of conditional standard deviation modeled by univariate GARCH process such that

$$s_t^2 \alpha_0 + \sum_{i=1}^n \alpha_i e_{t-i}^2 + \sum_{i=1}^n \beta_i \sigma_{t-i}^2$$

where $e_{t-i} = \sigma_{t-i} \xi_{t-1}$ and $\xi_{t-1} \sim N(0,1)$.

Next, time varying corellation matrix has to be obtained, we will employ model proposed by Engle (2002) for the time varying covariance such that

$$\mathrm{K}_t = (1 - \sum_{i=1}^p \alpha_i - \sum_{i=1}^q \beta_j)\overline{K} + \sum_{j=1}^p \alpha_i (e_{t-i} e_{t-i}^{'}) + \sum_{j=1}^q \beta_j K_{t-j}$$

Subject to $\alpha_i \geq 0$, $\beta_i \geq 0$, $\sum_{i=1}^p \alpha_i - \sum_{j=1}^q \beta_j < 1$

From the this equation, it is obvious that the GARCH process is employed to model time varying covariance matrices since it depends on historical data. $\overline{K}$ Represents the unconditional covariance and it is initially obtained by estimating the sample covariance. We forecast K_t by the lagged covariance's(K_{t-1}). To estimating conditional covariance matrix, a univariate GARCH is applied to model the conditional standard deviations of each asset in the porfolio.

The log likelihood function can be written as below

$$L = -\frac{1}{2} \sum_{t=1}^T (klog(2\pi) + log(|S_t M_t S_t|) + r_t^{'} S_t^{-1} M_t^{-1} S_t^{-1} r_r = -\frac{1}{2} \sum_{t=1}^T (klog(2\pi) + log(|S_t|) + log(|R_t|) + e_t^{'} M_t^{-1} e_r$$

and is used to find estimators of model.

Next, a optimum estimators that maximize the log likeligood functon is found. Later, covariance matrix series is generated by relying on initial unconditional covariance matrix. Then new covariance matrix for next time point is generated by previous one and standardized residuals as in simple univariate GARCH process. Hence, univariate GARCH process is employed to extract time varying positive definite correlation matrices from that covariance matrix series such that

$$M_t = (\sqrt{K_t^d})^{-1} K_t (\sqrt{K_t^d})^{-1}$$

here K_t^d refers to the diagonal matrix of variance which are obtained from the diagonal items of K_t as following:

$$K_t^d = \begin{bmatrix} \mathrm{S}_{11}^2 & 0 & \cdots & 0 \\ 0 & \mathrm{S}_{22}^2 & \cdots & 0 \\ & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \mathrm{S}_{nn}^2 \end{bmatrix}$$

The matrix notation for time-varying correlation indicates that the way of calculating correlations such as dividing covariance by standard deviations extracted from the covariance matrix where

$$\rho_{i,j,t} = \frac{s_{i,j,t}}{\sqrt{s_{it}^2 S_{jj}^2}}$$

Finally, conditional covariance matrix is obtained by Hadamard product of the matrix of conditional standard deviations and time varying correlation matrix such that

$$C_t = S_t^{'} S_t M_t$$

This methodology gives the conditional covariance matrix for each data points of the calibration period. To find the conditional covariance matrix that is used to optimize weights of assets in the portfolio one week conditional standard deviations of assets and their dynamics correlation matrix is forecasted.

1.2 Portfolio optimization

The modern portfolio theory has been trying to determine how an investor might allocate assets among the possible investments options, however the simplest strategy to uniformly distribute weights between assets is not optimal under non-normally distributed stocks returns.

Let denote

$$r_p = \sum_{i=1}^{n} w_i r_{it}$$

as the return of portfolio at the time t, where r_{it} is a return of stock i at the time t and w_i is weight of stock i in portfolio, also $\sum_{i=1}^{N} w = 1$, $0 < w_i < 1$ for $\forall$ i. Define $\mu^{'} = (\mu_1, ..., \mu_N)$ to be the mean vector and $\mathbf{w}^{'} = (w_1, ..., w_N)$ to be the weight vector with property of $\mathbf{e}^{'}\mathbf{w} = 1$, where e is ones' vector

Value-at-risk Banks and other financial institutions employ the concept of value-at-risk (VaR) as a way to measure the risks faced by their portfolios at the worst possible scenario. VaR can be computed based solely on estimation of the quantile of the forecast distribution or simple historical empirical data. [3] This idea has been proposed by Engle and Manganelli (2001).

VaR definition VaR of an portfolio is based on standard normal distribution and is calculated using the equation:

$$Var_p = -W_0\{\mathbf{w}^{'}\mu + Z_\alpha(\mathbf{w}^{'}\Omega\mathbf{w})^{1/2}\}$$

where W_0 the number of assets in portfolio, Z_α the percentile of standard normal distribution of the significance level α, we will use $\alpha = 0.05$

Efficiency of a portfolio by mean-VaR criterion Portfolio with weights $\mathbf{w}^*$ is called Mean-VaR efficient if there is no other portfolio weight allocation with $\mu_p \leq \mu_p^*$ and $VaR_p \leq VaR_p^*$. To optimize portfolio, we denote τ - investors tolerance factor that works as parameter in objective:

$$Maximize\{\mathbf{2\tau\mu_p - VaR_p}\}$$

with $\tau \geq 0$; now if we substitute VaR as defined above we obtain

$$Maximize\{2(\tau+1)\mathbf{w}^{'}\mu_{\mathbf{p}} + Z_\alpha(\mathbf{w}^{'}\Omega\mathbf{w})^{1/2}\}$$

Subject to $\mathbf{w}^{'}\mathbf{e} = 1$ and $\mathbf{w}^{'} > 0$. Using Lagrangian function and Kuhn-Tucker theorem it can be shown that optimal weights is defined by

$$\mathbf{w}^* = \frac{(2\tau+1)\Omega^{-1}\mu + \lambda\Omega^{-1}\mathbf{e}}{(2\tau+1)\mathbf{e}^{'}\Omega^{-1}\mu + \lambda e^{'}\Omega^{-1}\mathbf{e}}$$

2. Analysis and results

2.1 Data, descriptive statistics

Data from 2016-04-20 to 2021-05-15 were gathered from NASDAQ to analyse the problem. From Table 1 it can be seen that TSLA has the lowest min value -0.26 that represents approximately -26% weekly loss in value while it also has the highest max return of 0.32 representing approximately gain of 32 %. The mean parameter of TSLA asset is the highest again, with 1.2% weekly increase, it can be called high risk- high gain holding. We should notice that every stock distribution is far from Gaussian, that is $N(0,1)$, hence uniformly distributed weights won't give us the optimized portfolio. To add up, all stocks have positive kurtosis in excess of 3, that indicates us about stoks heaver tails, againt contradicting gaussianity, however we an conclude it is roughly normally distributed - VaR should behave decently.

stock	APPL	GOOG	TSLA
min	-0.1753067	-0.1208546	-0.2586126
max	0.1473304	0.1428634	0.3156894
mean	0.006692162	0.00478272	0.012790449
sd	0.03925115	0.03357286	0.08357147
skewness	-0.257358	0.1058368	0.5519998
kurtosis	2.828163	1.824926	1.747306

Table 1. Descriptive statistics of chosen stocks

2.2 ARMA-GARCH portfolio optimization

In Tables 2, 3 it can be observed what ARMA, GARCH models fit the best for each stock by the Akaike Information Criterion (AIC): ARMA(2,2) and GARCH(1,1) for AAPL, ARMA(1,1) and GARCH(2,1) for GOOG, ARMA(1,1) and GARCH(1,2) for TSLA.

Applied above mentioned models to our portfolio stocks expected means and variances we get:

GARCH(p,q)	APPL	GOOG	TSLA
1,1	-3.712498	-4.007539	-2.268982
1,2	-3.712145	-4.002347	-2.270
2,1	-3.713453	-3.989129	-2.239022

Table 2. GARCH model evaluation by AIC

ARMA(p,q)	APPL	GOOG	TSLA
1,1	957.1536	-1043.385	-563.0891
1,2	-956.2539	-1038.476	-561.0999
2,1	-956.111	-1038.533	-556.4874
2,2	-963.2662	-1040.676	-560.886

Table 3. ARMA model evaluation by AIC

Apple

$$r_t = 0.1795 r_{t-1} + 0.33803 r_{t-2} + 0.037228\varepsilon_t + 0.08414\varepsilon_{t-1}$$
$$\sigma_t^2 = 0.14095 + 0.13944\sigma_{t-1}^2 + 0.21034\sigma_{t-2}^2 + 0.19094 r_{t-1}$$

Google

$$r_t = 0.4939 + 0.1021 r_{t-1} + 0.33803 r_{t-2} + 0.1206\varepsilon_t$$
$$\sigma_t^2 = 0.497 + 0.3553\sigma_{t-1}^2 + 0.363 r_{t-1}$$

Tesla

$$r_t = 0.1848 + 0.1075 r_{t-1} + 0.9842\varepsilon_t$$
$$\sigma_t^2 = 0.3308 + 0.248\sigma_{t-1}^2 + 0.2483\sigma_{t-2}^2 + 0.1952 r_{t-1}$$

Estimated mean and standard deviation Those metrics are shown in Table 4 and will be used in further calculations.

	Mean	Variance
AAPL	0.00668995	0.08576258
GOOG	0.00478102	0.03083618
TSLA	0.01277561	0.03083618

Table 4. Mean and variance from ARMA, GARCH models

ARMA-GARCH model Summarizing obtained models and other metrics like mean:

$$\mu = \begin{bmatrix} 0.006689952 \\ 0.004781020 \\ 0.012775613 \end{bmatrix} \text{ also } e = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

The variance for ARMA-GARCH model is calculated out of standardized residuals, and matrix defined as Ω, see the matrix and its inverse below.

$$\Omega = \begin{bmatrix} 0.0014805585 & 0.0003390505 & 0.0031381797 \\ 0.0003390505 & 0.0011095859 & 0.0007551476 \\ 0.0031381797 & 0.0007551476 & 0.0067734325 \end{bmatrix}$$

$$\Omega^{-1} = \begin{bmatrix} 37731.1104 & 397.9473 & -17525.4589 \\ 397.9473 & 979.4291 & -293.5652 \\ -17525.4589 & -293.5652 & 8300.0346 \end{bmatrix}$$

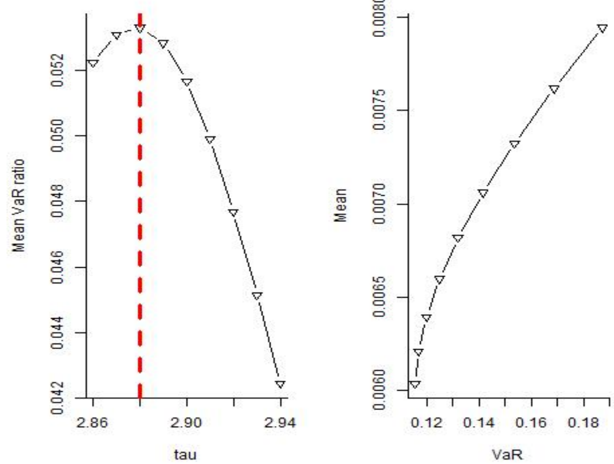

Figure 1. ARMA-GARCH optimization

Model findings ARMA-GARCH model suggested weights for corresponding tolerance level parameter τ are displayed in Table 5, where w_1, w_2, w_3 are weights of AAPL, GOOG, TSLA respectively given $\alpha = 0.05$. In Table 6, tolerance level varies between $2.86 \leq \tau \leq 2.94$ and is associated with mean, Value-at-Risk, ratio between those. It is interesting that $\tau = 2.94$ produces the biggest weekly return of 0.794% with VaR of 18.7% and the minimum mean of 0.6% is with tolerance level $tau = 2.86$ with VaR of 11.5%, however the efficient portfolio condition is satisfied with the highest mean for the lowest VaR, in other words, with tolerance level that contains the highest mean-VaR ratio of all, in this case $\tau = 2.88$. Tolerance level, mean-VaR behaviours can be checked in Figure 1.

2.3 ARMA-DCC-GARCH portfolio optimization

For multivariate volatility analysis we will consider only GARCH(1,1) model since higher parameters rarely improve model, especially when we have short period of study. [8] The covariance matrix from the ARMA-DCC-GARCH model is bit different since it is time-varying, also the mean vector is the same as for ARMA-GARCH model.

$$\Omega = \begin{bmatrix} 0.0010064054 & 0.0004490263 & 0.0010046245 \\ 0.0004490263 & 0.0009508702 & 0.0006607072 \\ 0.0010046245 & 0.0006607072 & 0.0079728296 \end{bmatrix}$$

$$\Omega^{-1} = \begin{bmatrix} 1368.8936 & -558.74887 & -126.18539 \\ -558.7489 & 1343.99277 & -40.97076 \\ -126.1854 & -40.97076 & 144.72134 \end{bmatrix}$$

Model findings DCC-ARMA-GARCH model suggested weights for corresponding tolerance level parameter τ are displayed in Table 7, where w_1, w_2, w_3 are weights of AAPL, GOOG, TSLA respectively given $\alpha = 0.05$. In Table 8, tolerance level varies between $0.7 \leq \tau \leq 3.1$ and is associated with

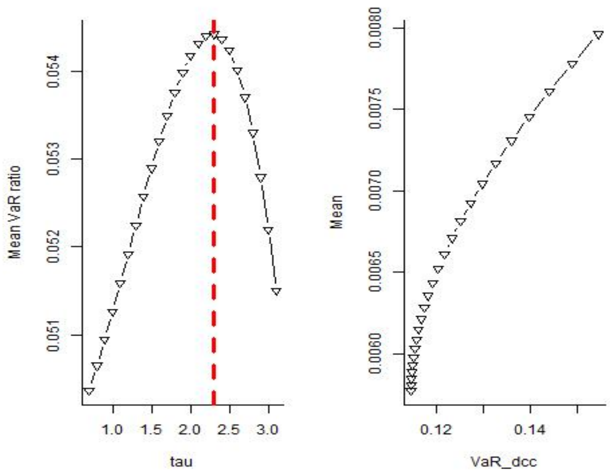

Figure 2. DCC-ARMA-GARCH optimization

mean, Value-at-Risk, ratio between those. It is interesting that $\tau = 3.1$ produces the biggest weekly return of 0.796% with VaR of 15.4% and the minimum mean of 0.577% is with tolerance level $tau = 0.7$ with VaR of 11.5%, however the efficient portfolio condition is satisfied with the highest mean for the lowest VaR, in other words, with tolerance level that contains the highest mean-VaR ratio of all, in this case $\tau = 2.3$. Tolerance level, mean-VaR behaviours can be checked in Figure 2.

2.4 Results comparison

Univariate ARMA-GARCH model has shown distribution with mean 0.006395988, VaR 0.1199766 and ratio of mentioned estimates ratio equal to 0.05331028, on the other hand, multivariate DCC-ARMA-GARCH model displayed significantly better results: mean 0.006814356, VaR 0.1252245 and ratio of 0.0544171, also the uniformly distributed portfolio accomplished mean of 0.008082195 with VaR 0.1714759, where ratio is 00.04713312. We can observe that the multivariate GARCH model has the most efficient holdings, even when fully diversified portfolio has higher weekly return by 18.6% however mean VaR ratio is 13.4% lower, hence investors portfolio is by way more stable in DCC-ARMA-GARCH model case.

3. Discussion and conclusions

Descriptive statistics revealed that the stocks distribution does not follow gausality and ARMA-GARCH, ARMA-DCC-GARCH models are good to fit those heavy tailed stock data and produce estimates for mean and variance of studied assets.

After model application it were found that the optimal portfolio with DCC-ARMA-GARCH model is superior in this case, because it involves overall market comovement information.

Analysis could have been improved by including way more assets in the portfolio or stressing it more on non-constant mean and variance stocks since the DCC GARCH is base on this assumption. Another idea, where this study could develop is in stock choice, to create a crawling bot to gather the most common combinations of portfolio existing in Etoro exchange platform to make this paper more familiar to investing newbies that are coming to the investment world.

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Appendix

tau	w1	w2	w3
2.86	0.5582182	0.4179413	0.0238405
2.87	0.51250788	0.4311863	0.0563058
2.88	0.4630316	0.4455226	0.0914458
2.89	0.40929987	0.4610919	0.1296082
2.9	0.35073451	0.4780619	0.1712036
2.91	0.28664753	0.4966318	0.2167207
2.92	0.21621379	0.5170407	0.2667455
2.93	0.138435	0.5395779	0.3219871
2.94	0.05209186	0.5645968	0.3833114

Table 5. ARMA-GARCH weights per tolerance level

tau	mean	VaR	mean VaR ratio
2.86	0.006037216	0.1155459	0.05224949
2.87	0.006209505	0.1169535	0.05309378
2.88	0.006395988	0.1199766	0.05331028
2.89	0.00659851	0.1248916	0.05283388
2.9	0.006819252	0.1319704	0.05167257
2.91	0.007060805	0.1414766	0.04990792
2.92	0.00732628	0.1536708	0.04767517
2.93	0.007619439	0.1688249	0.0451322
2.94	0.007944879	0.1872455	0.04243027

Table 6. ARMA-GARCH weights per tolerance level

tau	w1	w2	w3
0.7	0.5078868	0.48918522	0.002928024
0.8	0.5117453	0.4819148	0.00633992
0.9	0.5159648	0.4739641	0.010071053
1.0	0.5205587	0.46530813	0.01413316
1.1	0.5255416	0.45591912	0.018539273
1.2	0.5309299	0.4457663	0.02330383
1.3	0.5367416	0.43481563	0.028442804
1.4	0.5429966	0.42302951	0.033973841
1.5	0.5497172	0.41036641	0.039916431
1.6	0.5569274	0.39678048	0.046292093
1.7	0.5646543	0.38222108	0.053124587
1.8	0.5729276	0.36663227	0.060440164
1.9	0.5817799	0.34995223	0.068267843
2.0	0.5912478	0.3321125	0.07663974
2.1	0.6013713	0.31303728	0.08559144
2.2	0.6121952	0.2926424	0.09516243
2.3	0.6237691	0.27083434	0.105396601
2.4	0.6361482	0.24750895	0.116342836
2.5	0.6493943	0.22254996	0.12805569
2.6	0.6635765	0.19582733	0.140596195
2.7	0.678772	0.16719517	0.154032806
2.8	0.695068	0.13648944	0.168442518
2.9	0.7125628	0.10352501	0.183912196
3.0	0.7313675	0.06809237	0.200540168
3.1	0.7516084	0.0299535	0.218438132

Table 7. DCC-ARMA-GARCH weights per tolerance level

tau	mean	VaR	mean VaR ratio
0.7	0.00577395	0.1146458	0.05036337
0.8	0.005808592	0.1146827	0.05064923
0.9	0.005846476	0.1147496	0.05094986
1.0	0.00588772	0.1148537	0.05126277
1.1	0.005932457	0.1150034	0.05158505
1.2	0.005980834	0.1152081	0.05191333
1.3	0.006033012	0.1154782	0.05224374
1.4	0.006089171	0.1158256	0.05257188
1.5	0.006149508	0.1162637	0.05289274
1.6	0.006214243	0.1168074	0.05320077
1.7	0.006283616	0.1174732	0.05348979
1.8	0.006357894	0.1182797	0.05375306
1.9	0.006437372	0.1192474	0.05398334
2.0	0.006522375	0.1203991	0.05417294
2.1	0.006613266	0.1217602	0.05431384
2.2	0.006710444	0.1233585	0.05439793
2.3	0.006814356	0.1252245	0.0544171
2.4	0.006925497	0.1273922	0.05436359
2.5	0.007044423	0.1298985	0.05423022
2.6	0.007171752	0.132784	0.05401067
2.7	0.007308179	0.1360932	0.05369983
2.8	0.007454487	0.1398748	0.05329401
2.9	0.007611557	0.1441823	0.05279122
3.0	0.007780388	0.1490745	0.05219128
3.1	0.007962113	0.1546162	0.05149598

Table 8. DCC-ARMA-GARCH weights per tolerance level