Exact Results in $\mathcal{N} = 4$ Super Yang-Mills

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Abstract

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Contents

1 Introduction

In this section we give a brief history of the developments in the subject and an overview for the thesis.

1.1 Brief history of the subject

QCD is hard, $\mathcal{N} = 4$ is easier. From Maldacena to Zarembo, Bethe ansatz, asyptotics, strings, algebraic curves, matching. Finally TBA, $\mathbf{P}\mu$. Of course still a lot of things to do.

1.2 Thesis overview

Maybe a nice picture for the structure of the thesis.

2 $\mathcal{N} = 4$ super Yang-Mills

Here we describe the theory that is the main interest of the thesis.

2.1 The theory and its action

Write the action, maybe also show dimensional reduction from 10d. Talk about observables: traces, Wilson lines.

Planar limit.

2.2 Symmetry

Talk about conformal symmetry, write down the algebra ? Oscillator representation ? Discuss subgroups of psu(2,2—4), closed sectors.

Since it's a CFT we want to find 2pt and 3pt function. The spectral problem.

2.3 Perturbation theory

Take a simple operator, e.g. Konishi and calculate the anomalous dimension using perturbation theory ?

3 String description: AdS/CFT

Here we talk about the alternative description of the theory as strings moving in AdS.

3.1 Motivation

Planar diagrams are string interactions.

3.2 String theory and the duality

Give details of the string theory, what are the parameters on both sides, how they match up. What are the limits. Anomalous dimensions match string state energies.

3.3 First tests: BMN, GKP, FT

Describe these limits, give first eveidence for the duality. Is this where one finds the first strong coupling coefficient to Konishi ?

4 Integrability

In this section we dive into the magical world of integrability.

4.1 Overview

Give picture summarizing all techniques and their ranges of applicability.

4.2 One loop at weak coupling

Roughly rederive the Minahan/Zarembo result.

4.2.1 $\mathfrak{su}(2)$ sector

Give the su(2) Hamiltonian, example states and energies.

4.2.2 $\mathfrak{sl}(2)$ sector

Same with sl(2).

4.3 Higher loops

Short example of a two loop Hamiltonian, perturbative corrections for the states found above with contact terms.

4.4 Asymptotic solution

Generalize su(2) BAE to all loops, maybe give a simple example.

4.4.1 A glimpse ahead: the slope function

Derive slope from ABA.

4.5 Strong coupling and the algebraic curve construction

Describe flat connections, monodromies, sheets etc.

4.6 Classical solutions

All finite gap solutions can be described this way.

4.6.1 BMN string

Give explicit solution.

4.6.2 Folded string

Something similar.

4.7 Quantization and semi-classics

Describe the quantization procedure. Derive next coefficient for Konishi.

4.8 Short strings

Combine with slope, derive next coeffient for Konishi.

4.9 Full solution to the spectral problem

Here we finally give the complete solution.

4.9.1 The full theory

Mention nested BAE, full psu(2,2—4) spin chain without going into much detail.

4.9.2 Finite length

Deprecated approches: TBA, Y-system.

4.9.3 The $\mathbf{P}\mu$ system

Define $\mathbf{P}\mu$ as if it was an axiom.

5 Exact results

Exact results are rare and important.

5.1 Folded string

Mention Frolov numerics. Volin's 8(9) ? loops with $\mathbf{P}\mu$.

5.2 Cusped Wilson line

Bremstahlung result from $\mathbf{P}\mu$.

5.2.1 Classical limit

Find the curve, matrix models.

5.3 Revisiting the slope function

Derive slope from $\mathbf{P}\mu$.

5.4 The curvature function

Derive curvature from $\mathbf{P}\mu$.

5.4.1 Weak coupling expansion

Mention weak coupling and how it matches ABA.

5.4.2 Strong coupling expansion

Mention strong coupling, be amazed how it matches string theory.

5.5 Update on short strings

Combine semiclassics with curvature and finally derive three-loop Konishi coefficient.

6 Conclusions

Conclude with a tearful and heroic description about the journey of Konishi through the land of integrability - from weak to strong coupling.

References

[1] J. M. Maldacena, "The Large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].