HPCA project

Batch Sort et Merge Path Sort

AlGORITHM OVERVIEW

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PROBLEM TO SOLVE

How to sort efficiently an array of elements **using GPU** ?

variable names :

- A,B and M are array of int
- A,B are sorted
- We save our result in M
- M is not sorted on the initialisation

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Of two sorted array, A and B

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tree approach

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tree approach, batches and streams

01

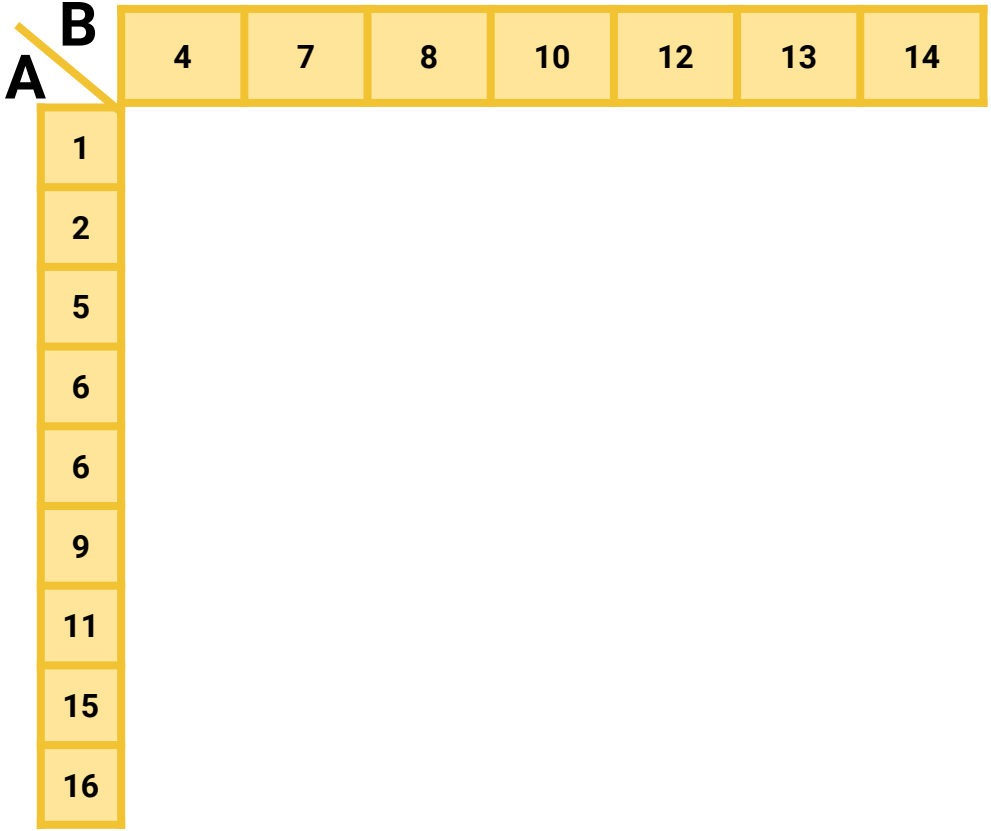
MERGE ON ONE BLOCK

ALGO mergeSmall_k

A

B

ALGO mergeSmall_k



ALGO mergeSmall_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

ALGO mergeSmall_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1

ALGO mergeSmall_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2

ALGO mergeSmall_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4

ALGO mergeSmall_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4	5

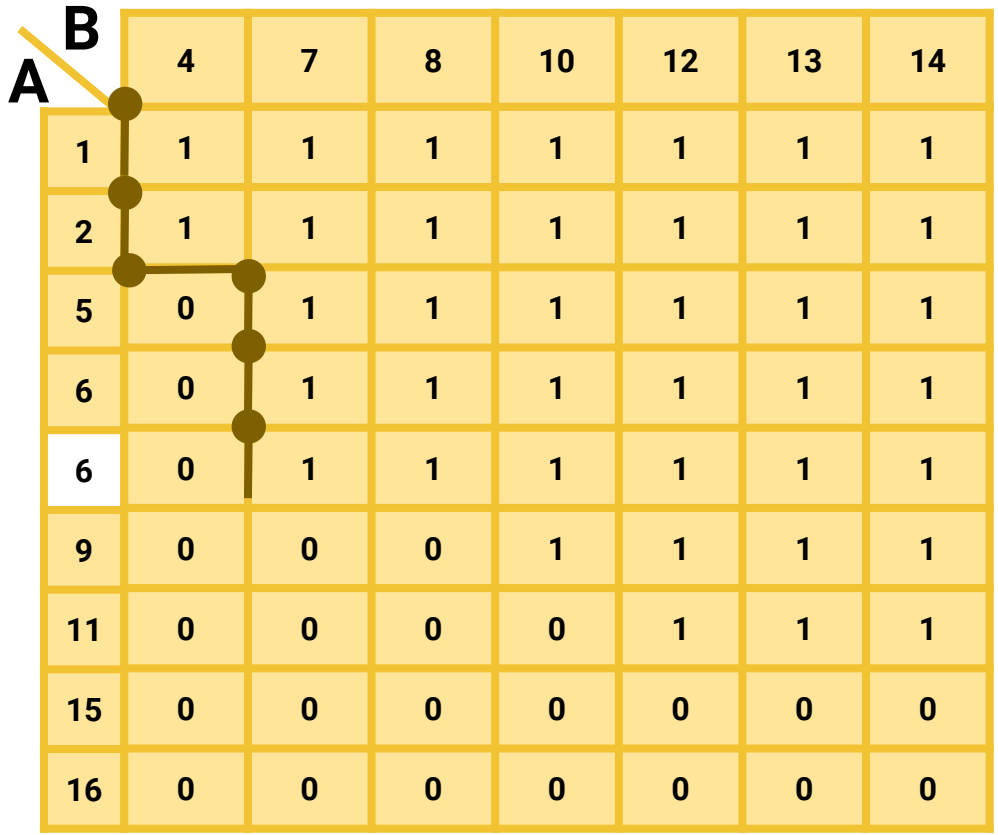
ALGO mergeSmall_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4	5	6

ALGO mergeSmall_k

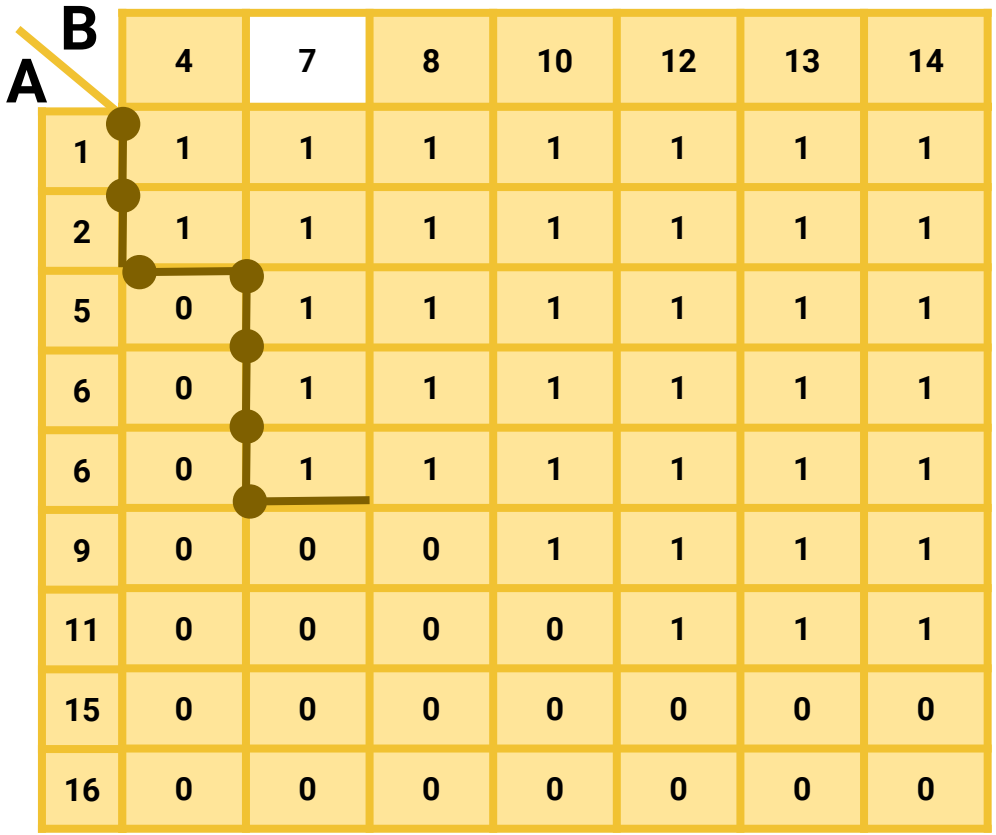


M

1	2	4	5	6

... 6

ALGO mergeSmall_k

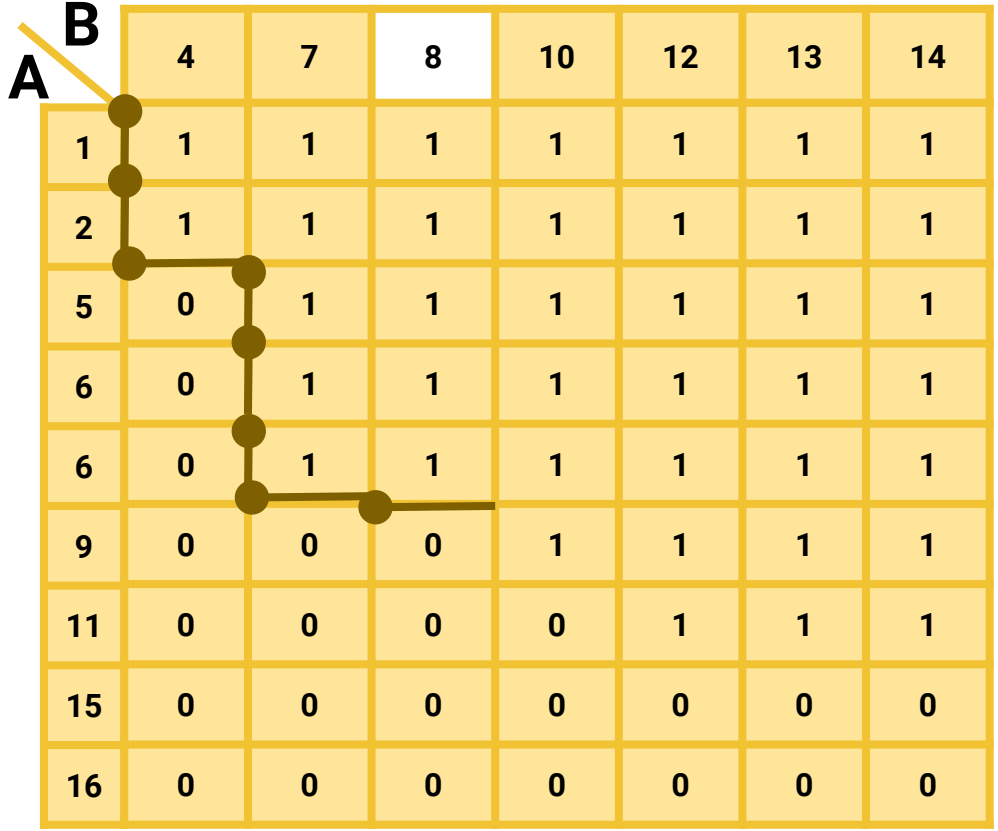


M

1	2	4	5	6

... | 6 | 7 |

ALGO mergeSmall_k

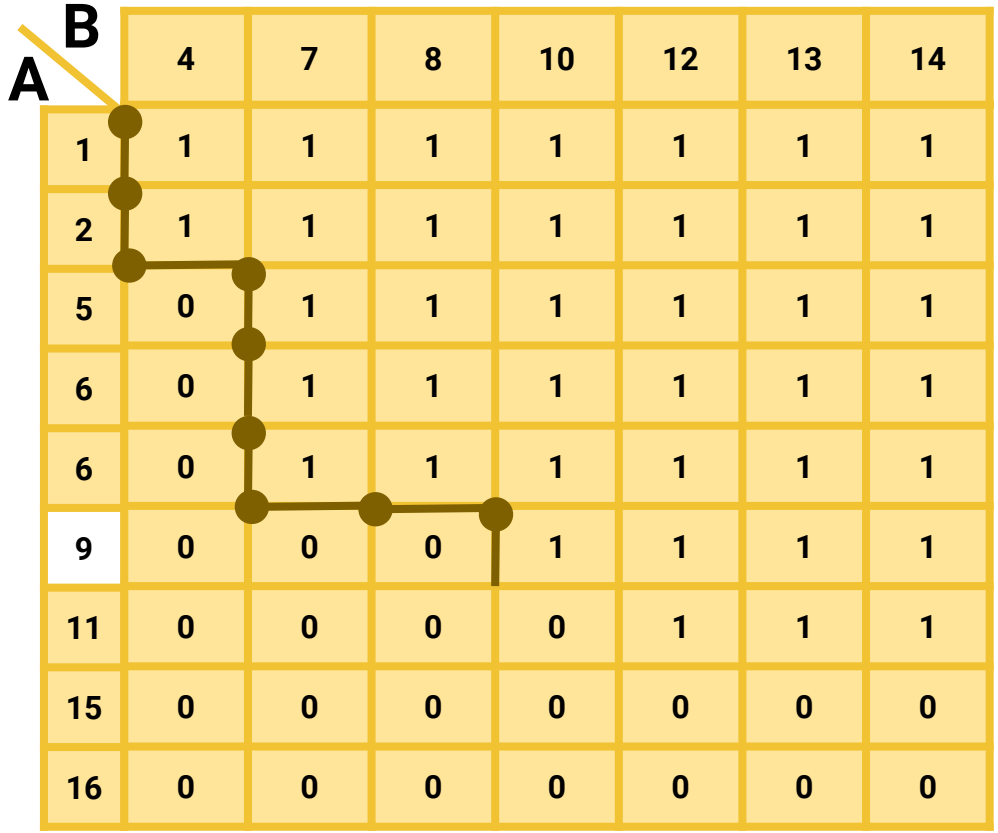


M

1	2	4	5	6

... | 6 | 7 | 8 |

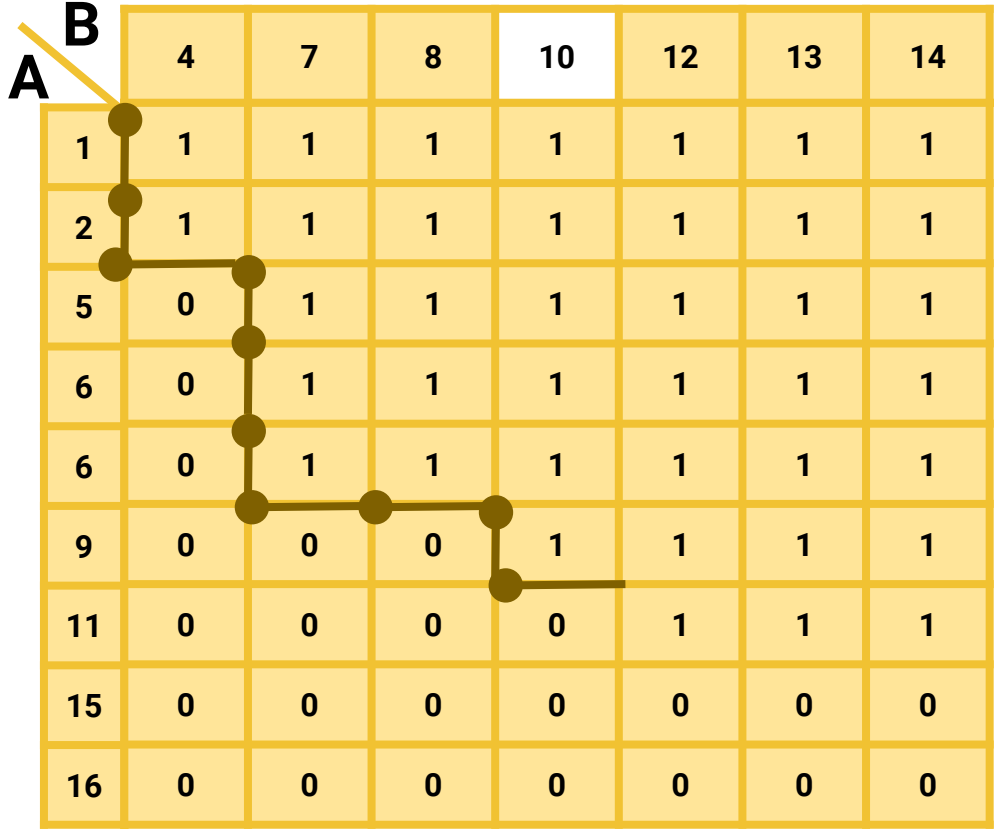
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	

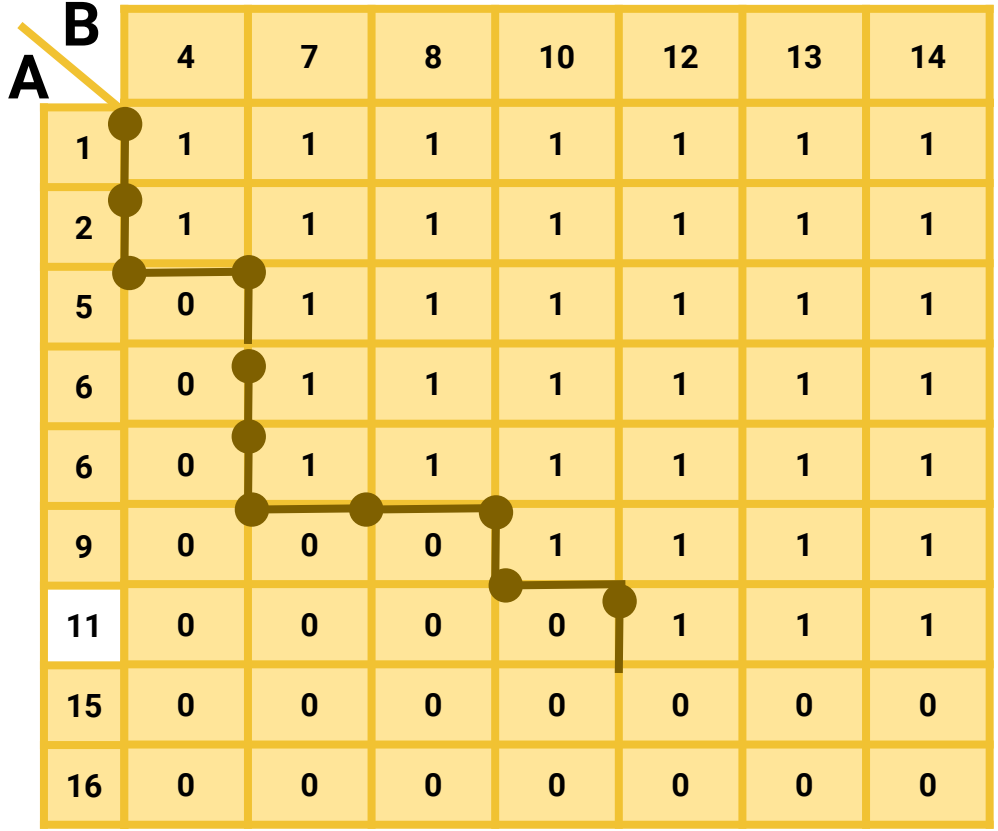
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10

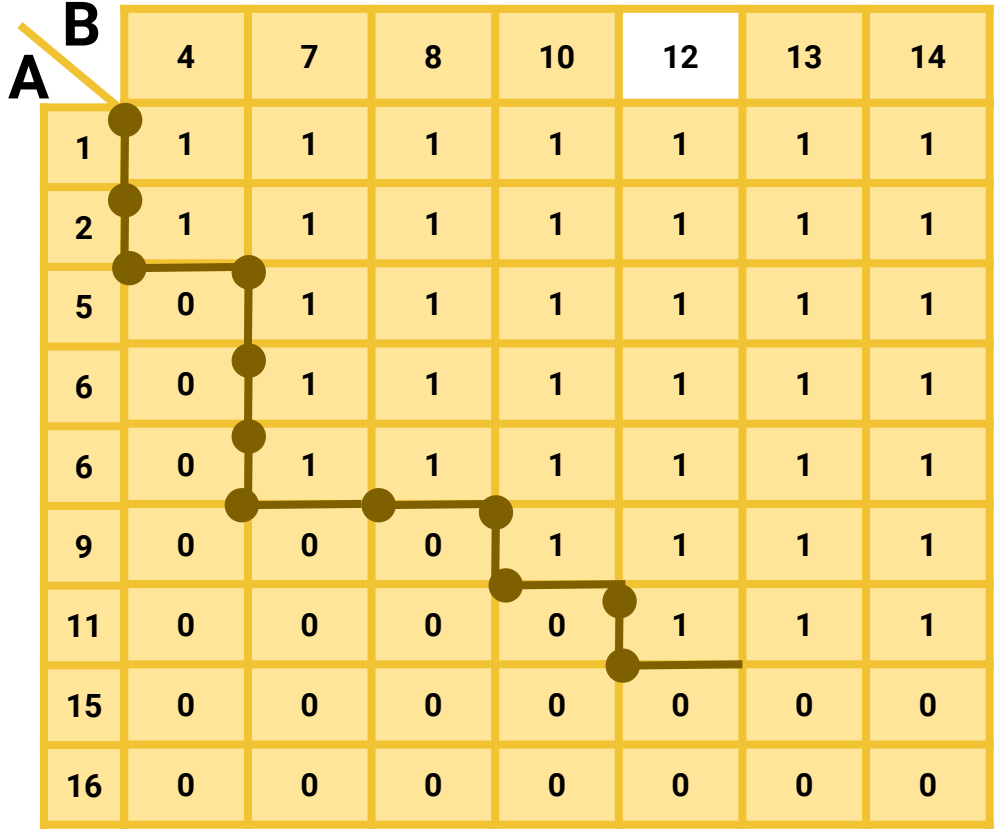
ALGO mergeSmall_k



M

	1	2	4	5	6
...	6	7	8	9	10
...	11				

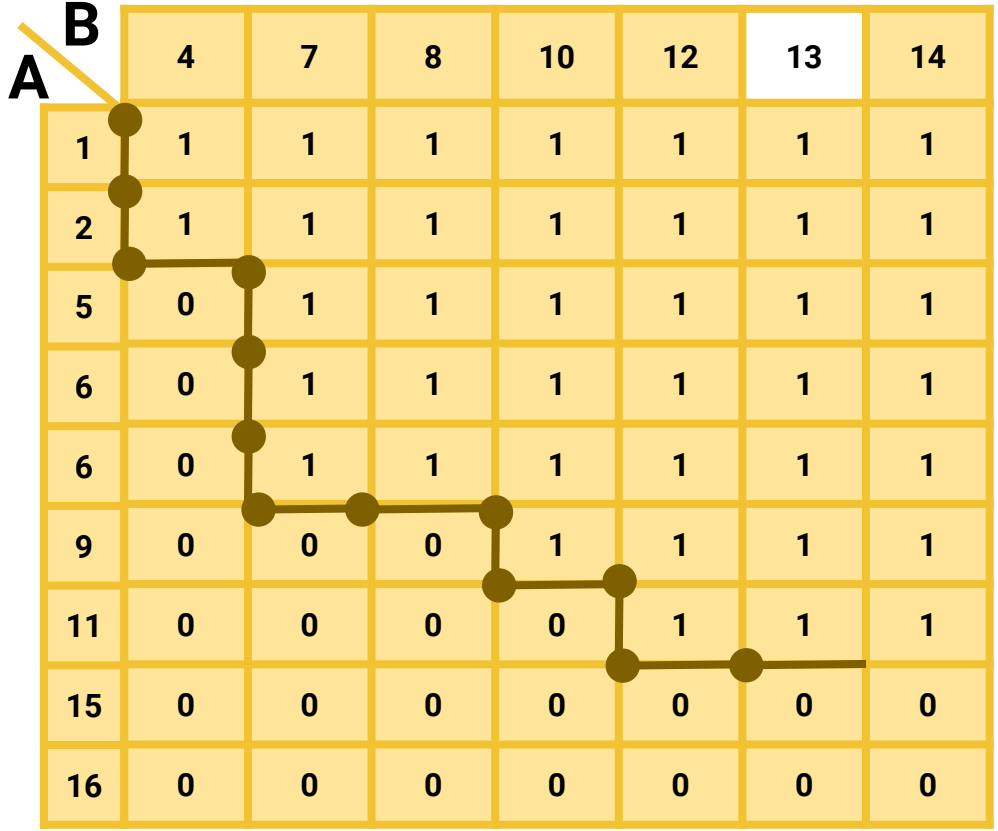
ALGO mergeSmall_k



M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12			

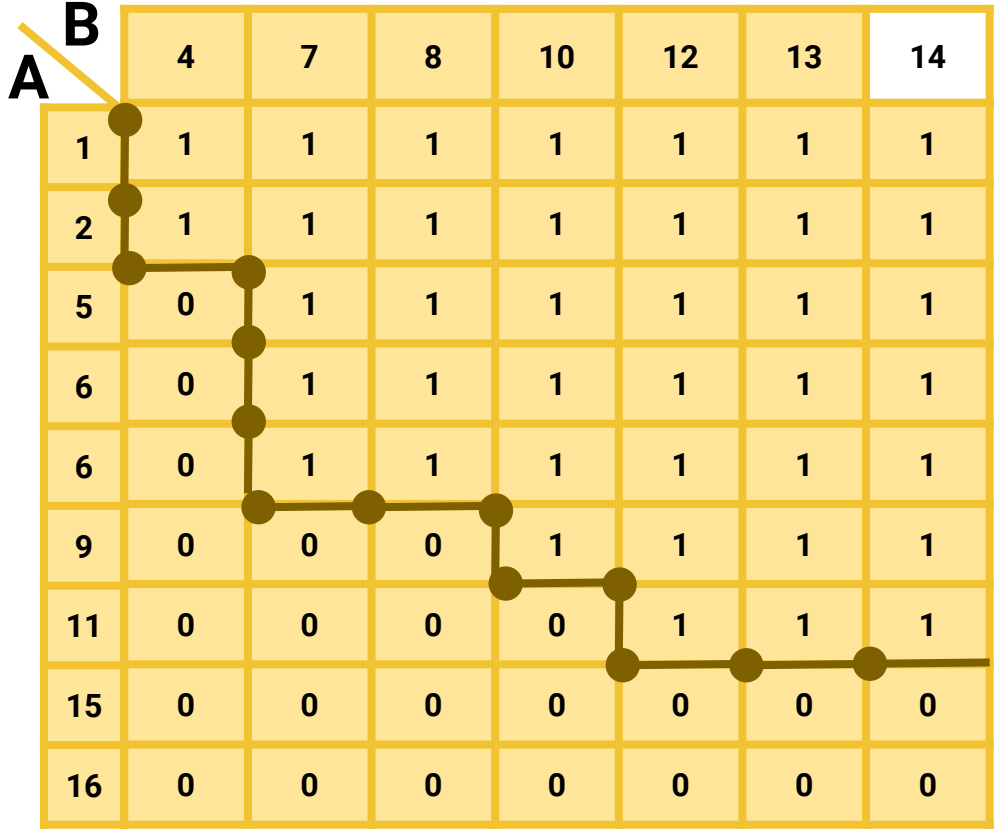
ALGO mergeSmall_k



M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13		

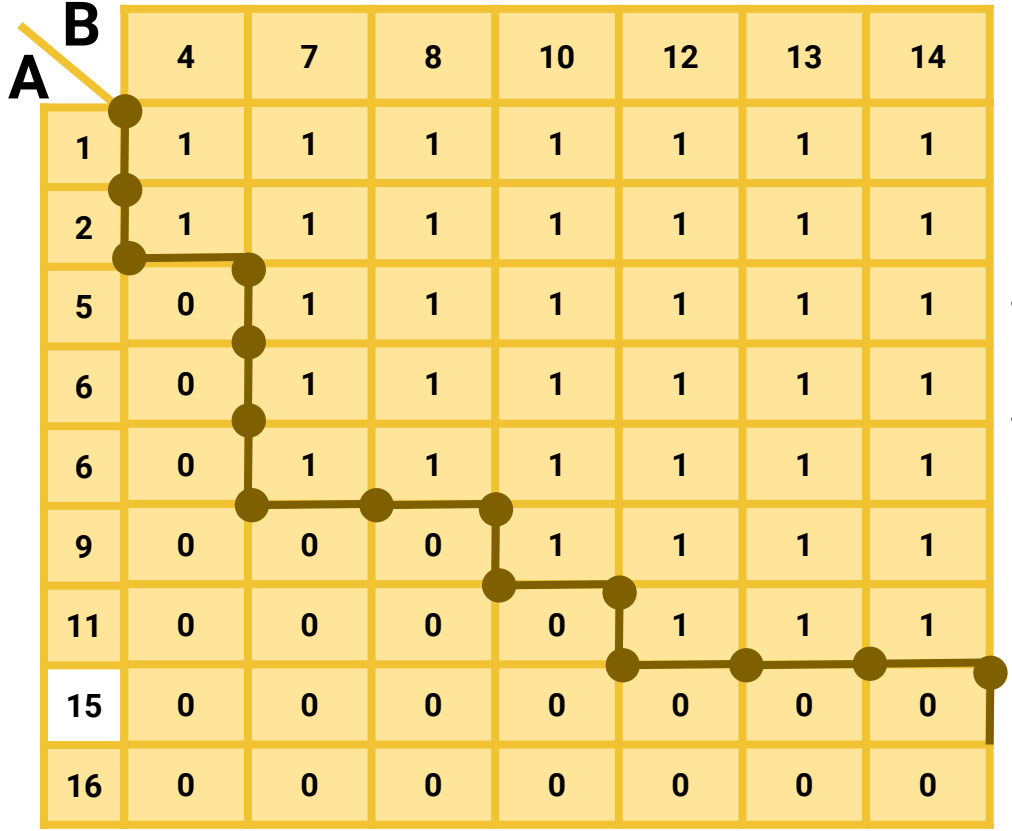
ALGO mergeSmall_k



M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13	14	

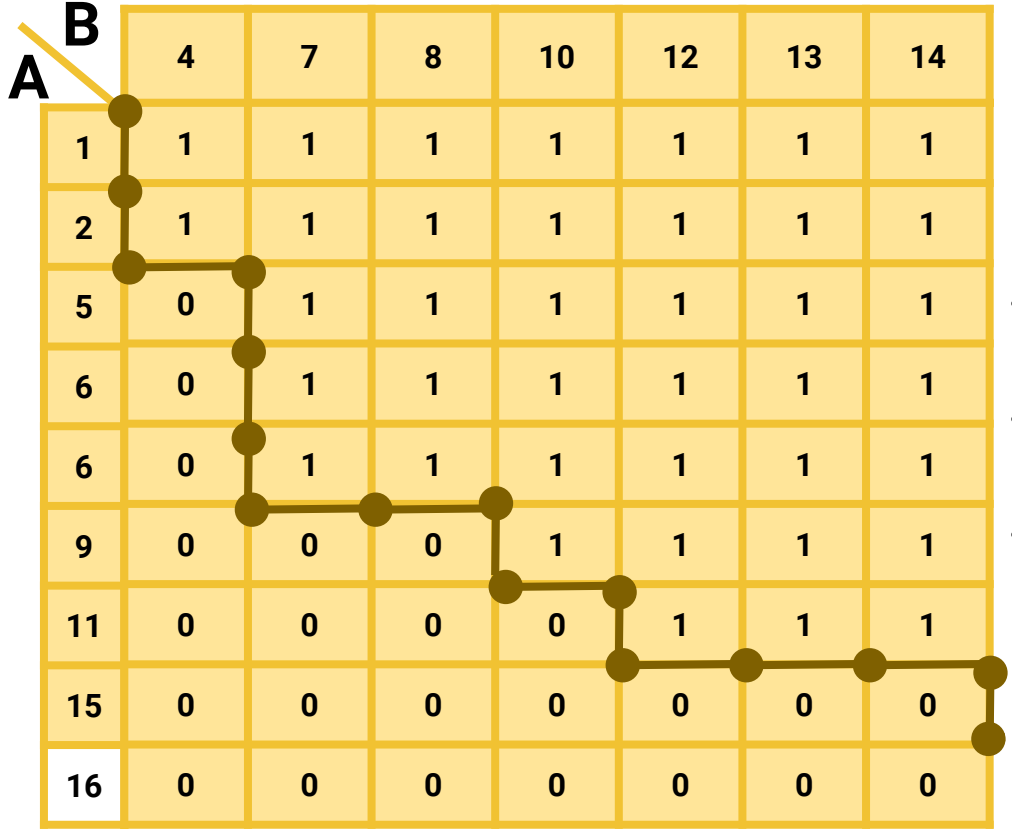
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11	12	13	14	15

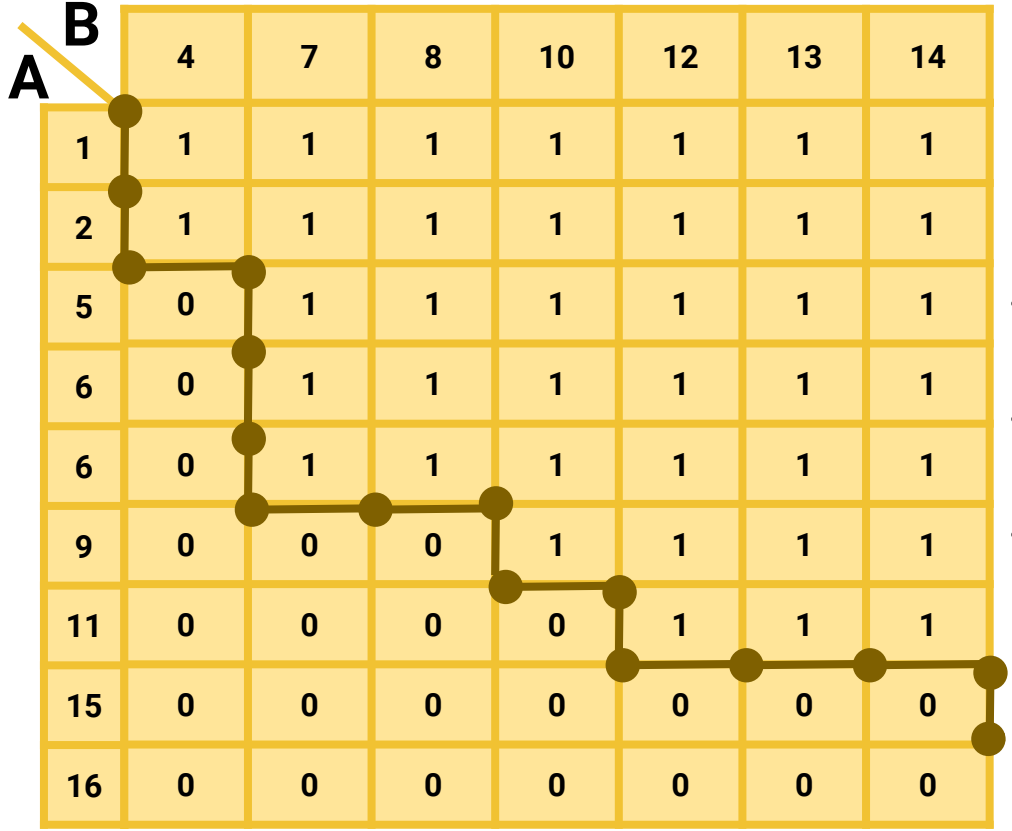
ALGO mergeSmall_k



M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13	14	15
...	16				

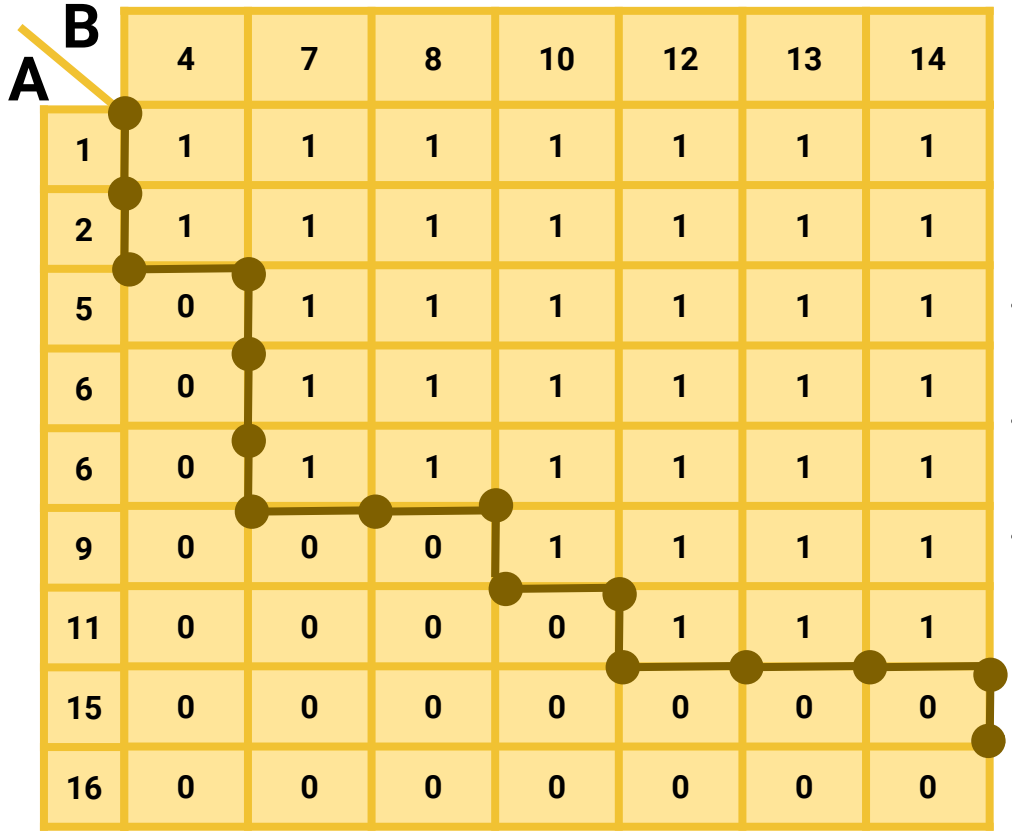
ALGO mergeSmall_k

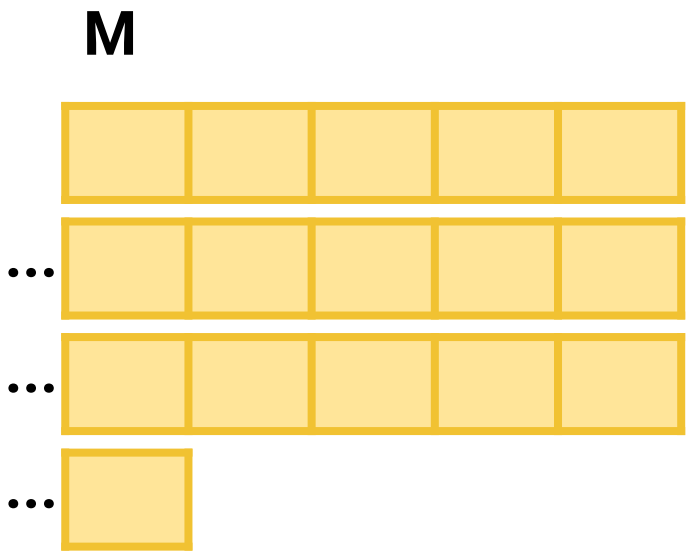


M

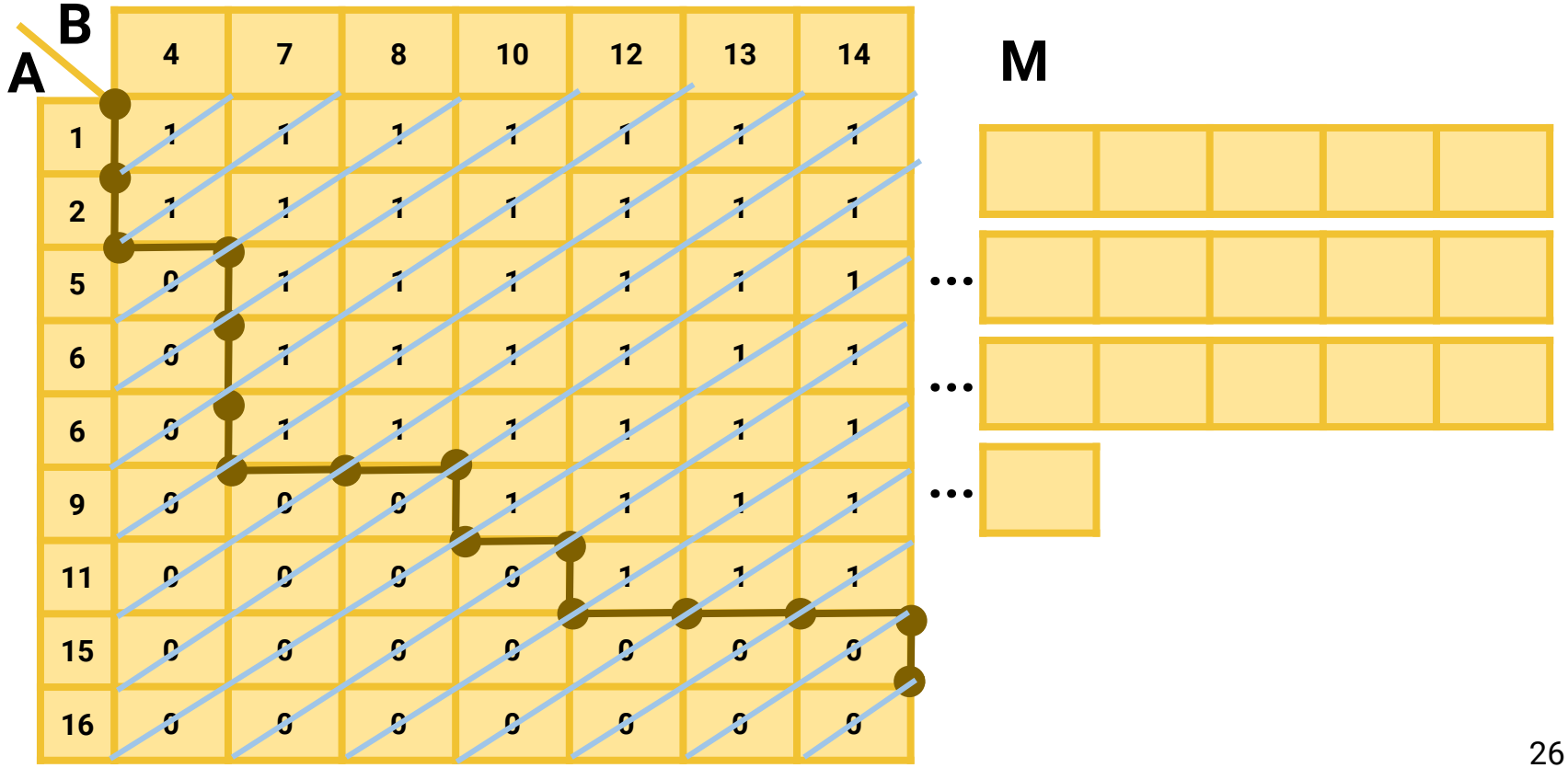
	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13	14	15
...	16				

ALGO mergeSmall_k

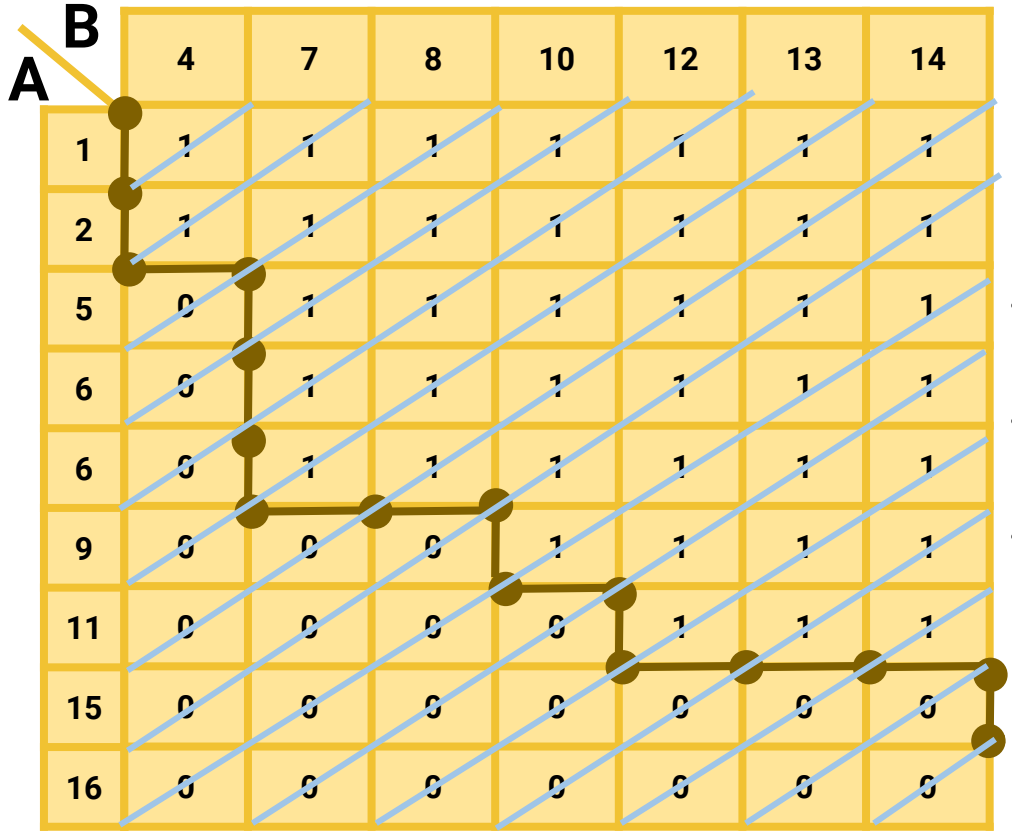


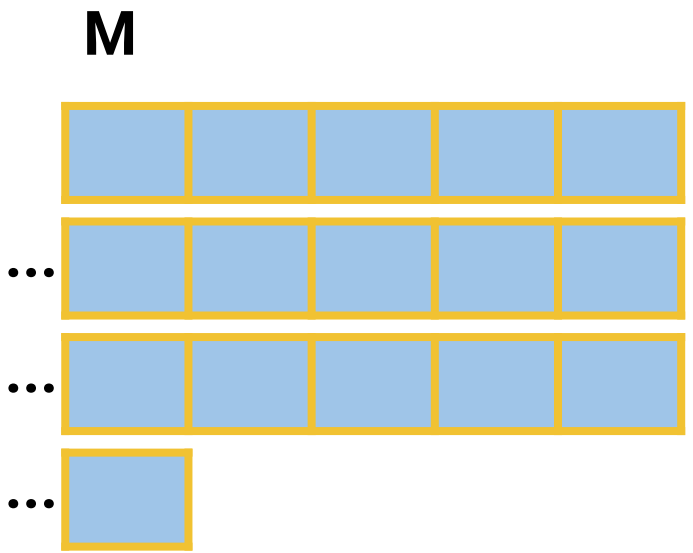


ALGO mergeSmall_k

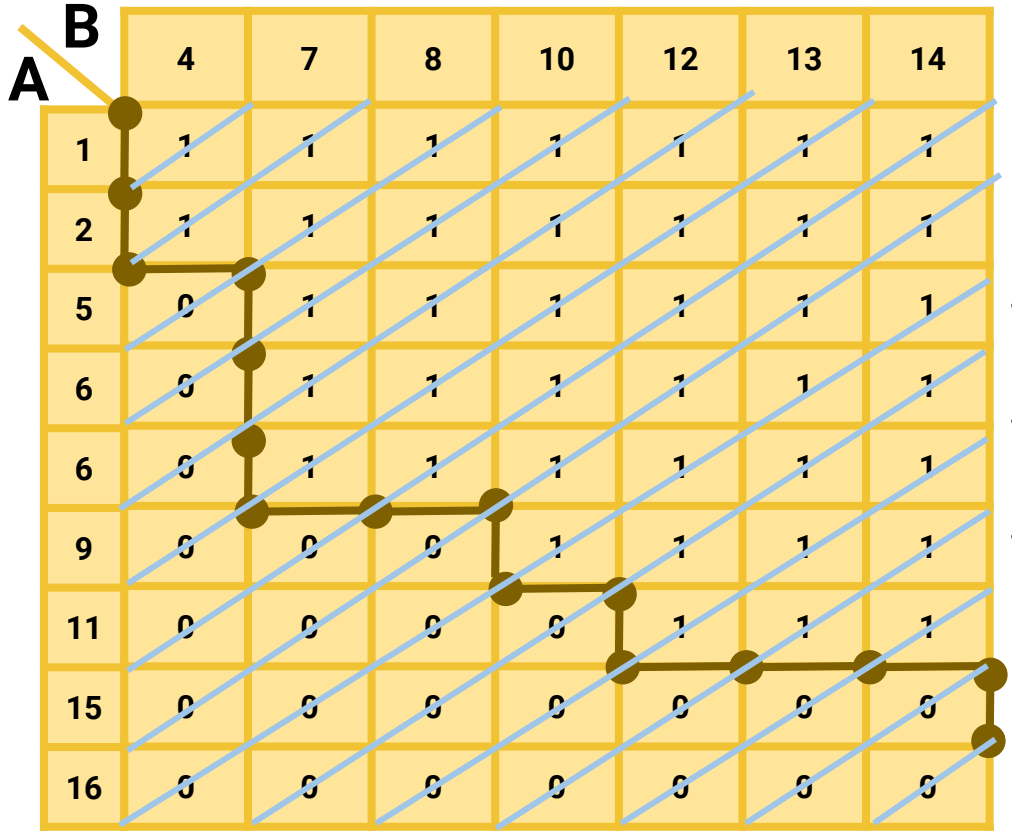


ALGO mergeSmall_k





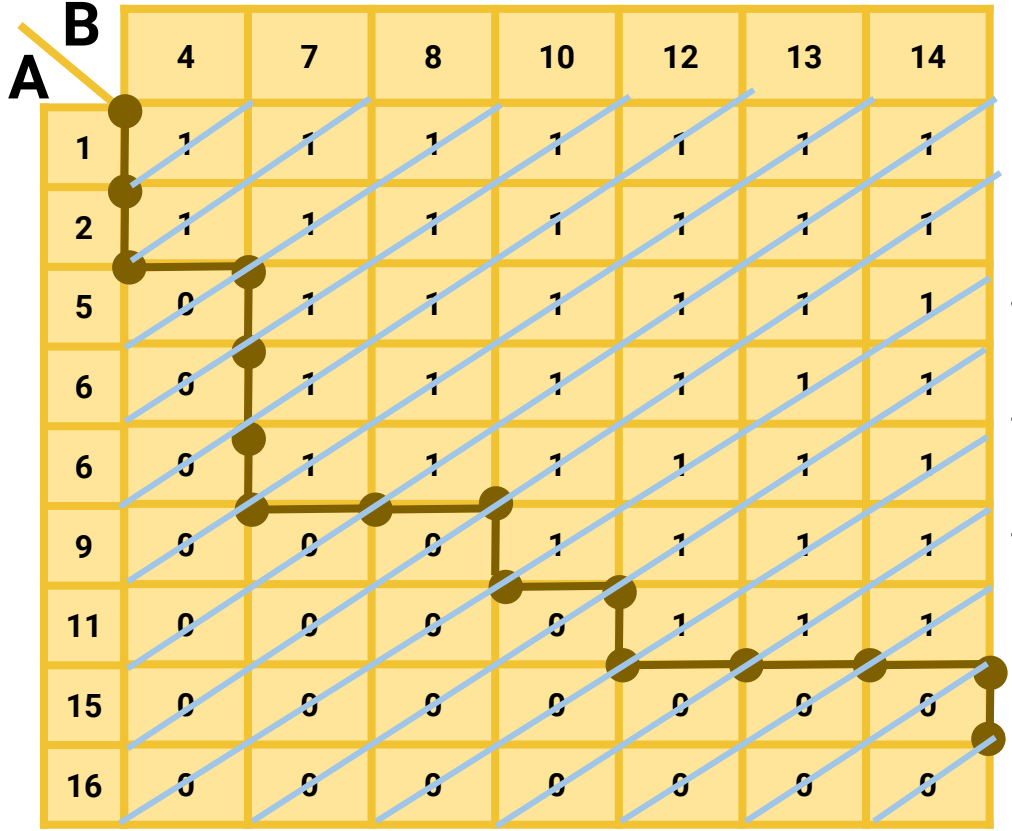
ALGO mergeSmall_k



M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13	14	15
...	16				

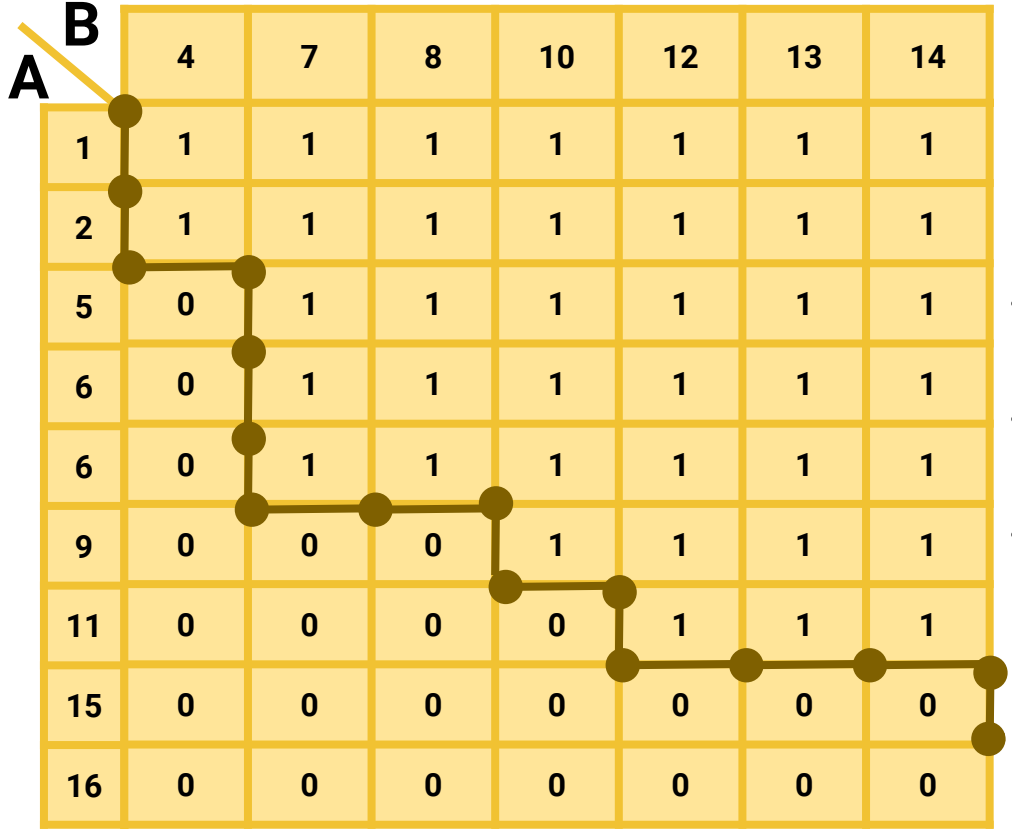
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11	12	13	14	15
16				

ALGO mergeSmall_k



M

1	2	4	5	6

…

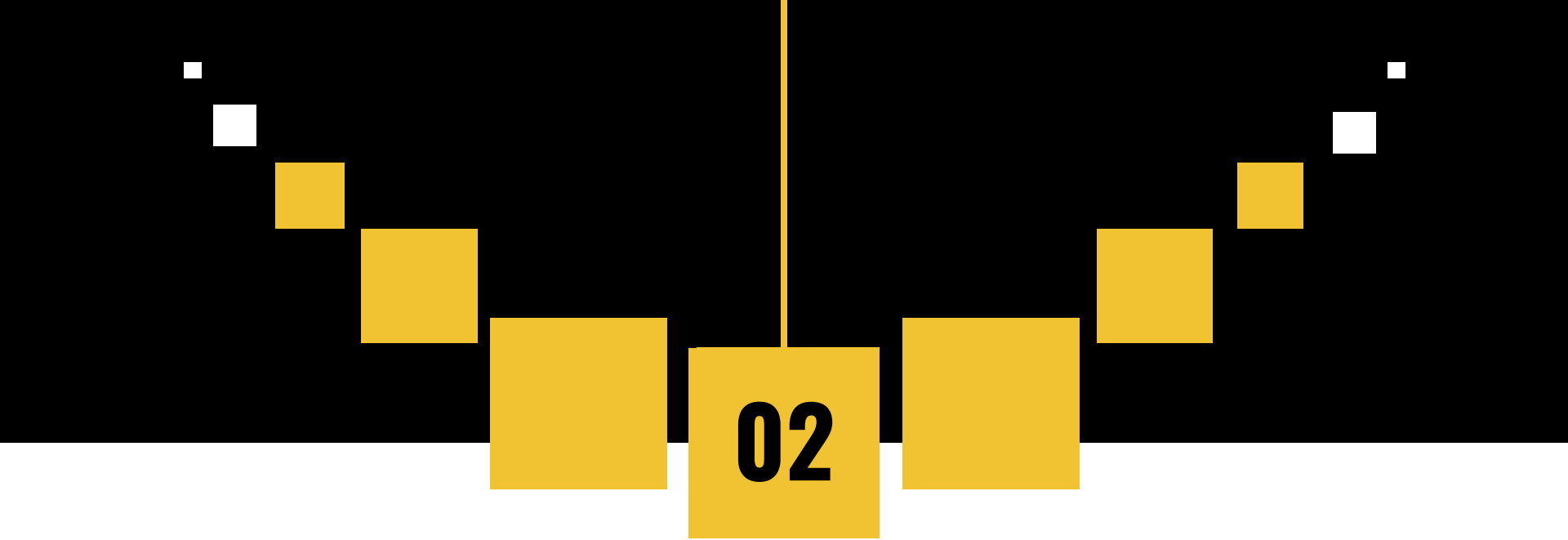
6	7	8	9	10

…

11	12	13	14	15

…

16



MERGE ON SEVERAL BLOCKS

ALGO pathBig_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1

ALGO pathBig_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1

ALGO pathBig_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1	0

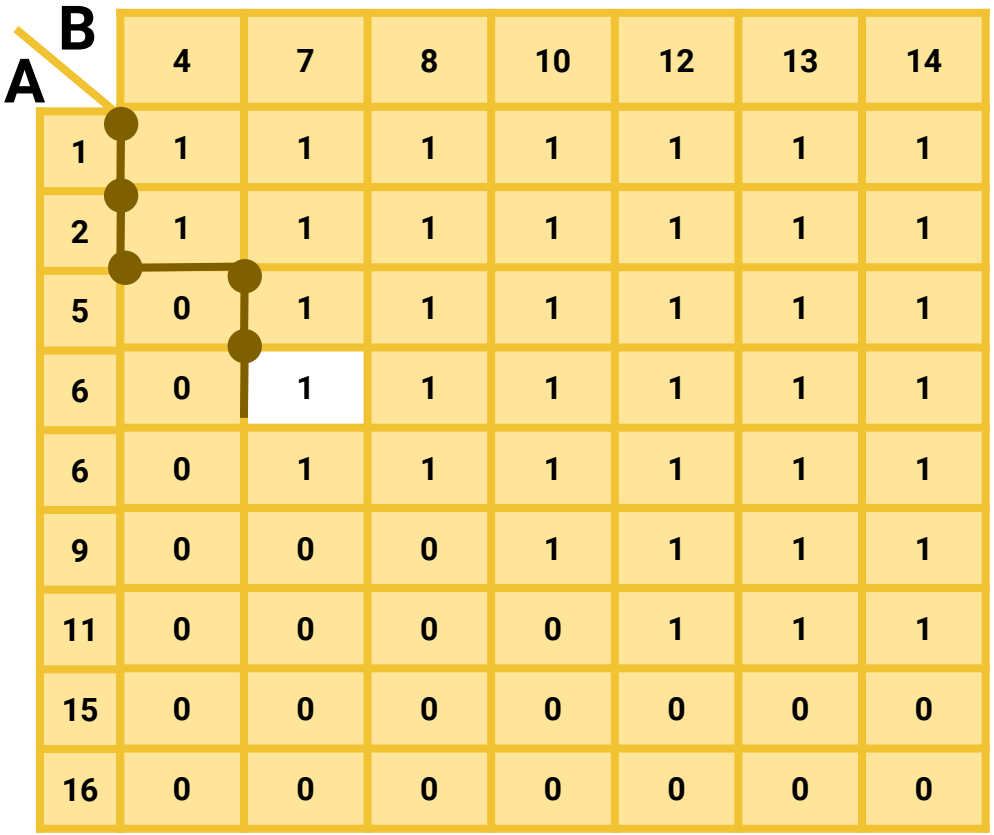
ALGO pathBig_k

A \ B	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1	0	1

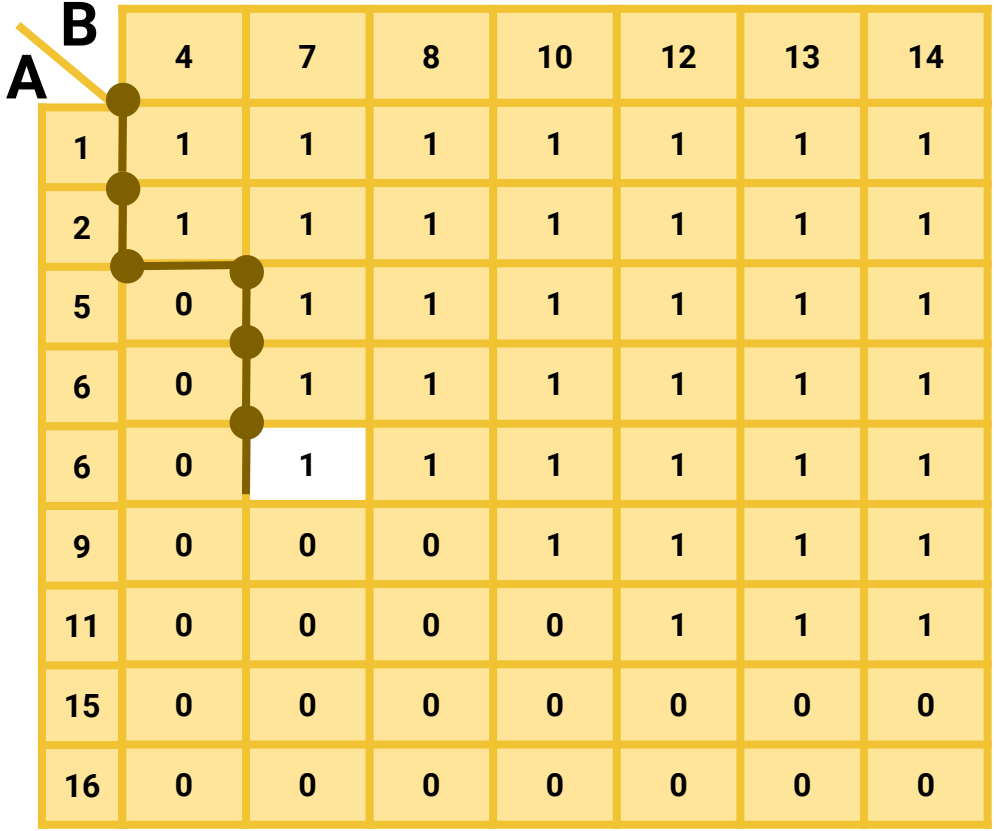
ALGO pathBig_k



Path

1	1	0	1	1

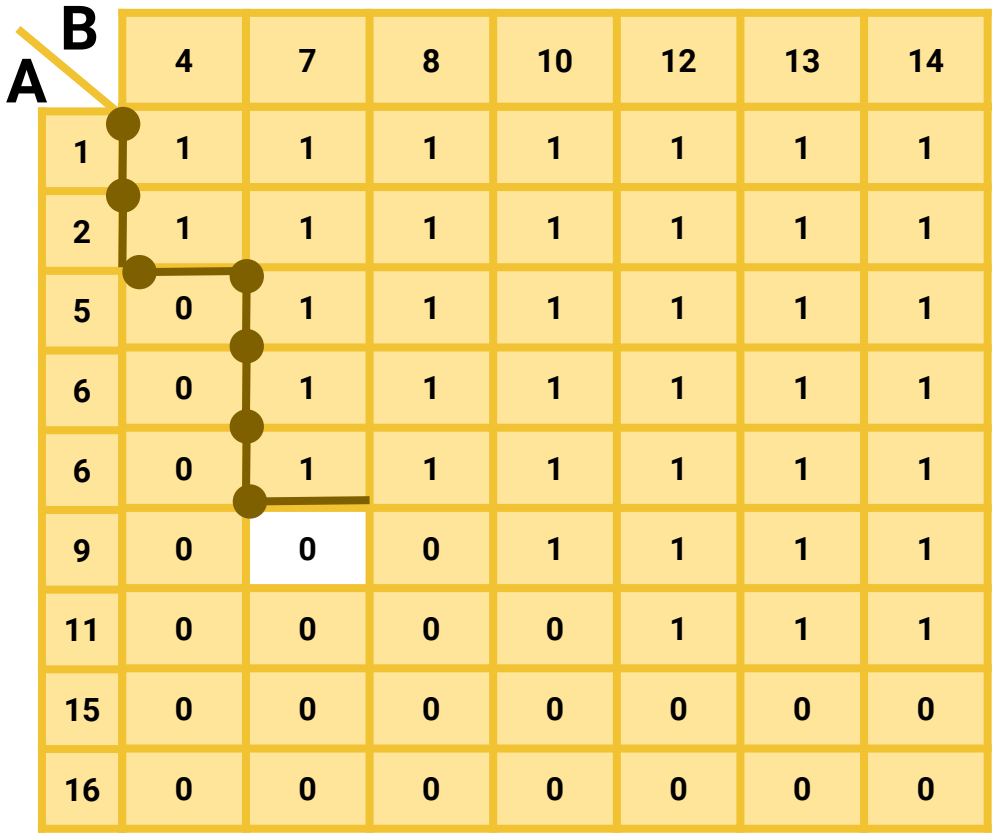
ALGO pathBig_k



Path

	1	1	0	1	1
...	1				

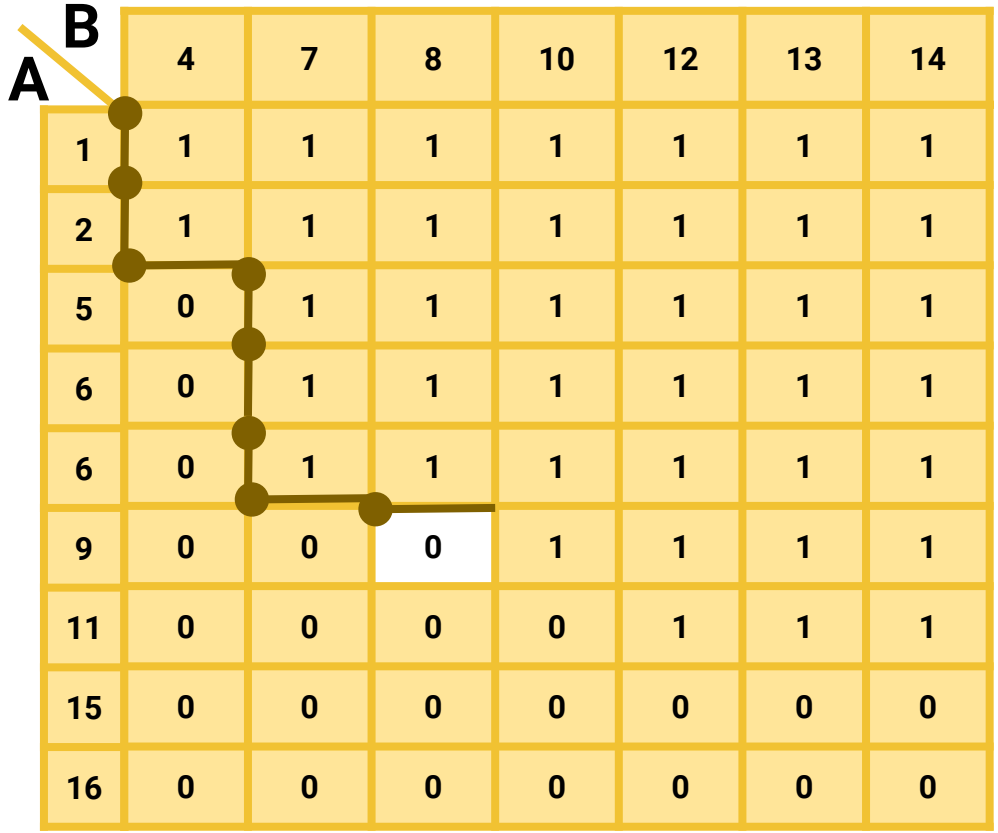
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0			

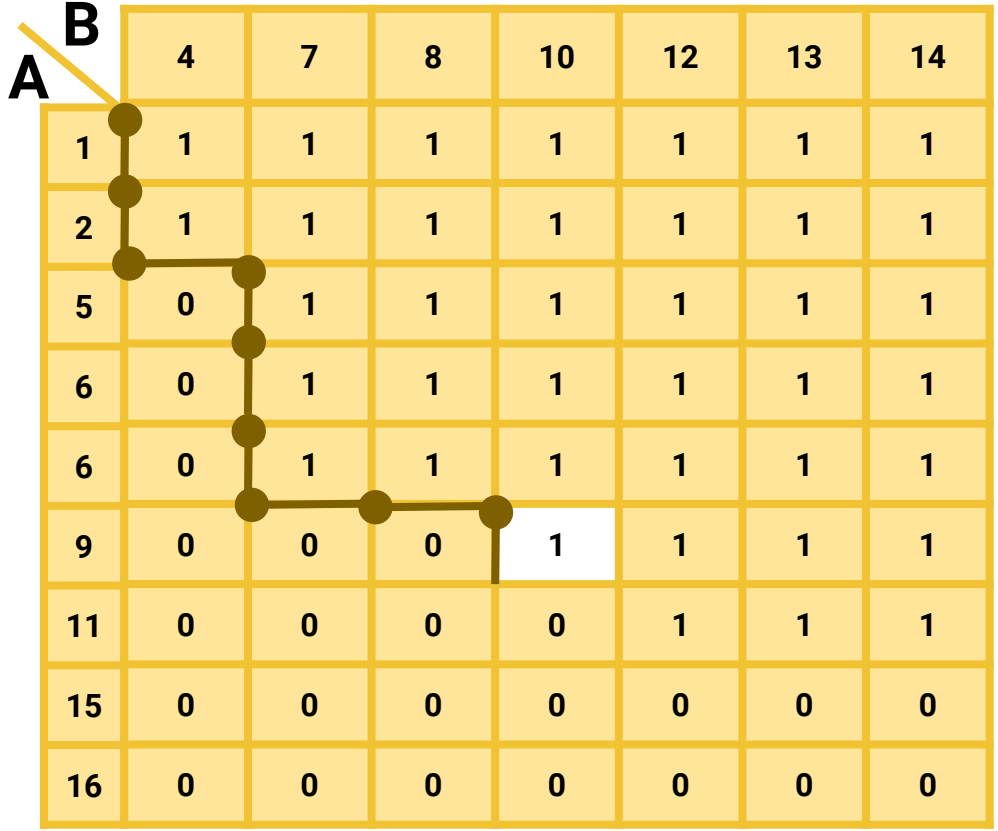
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0		

ALGO pathBig_k



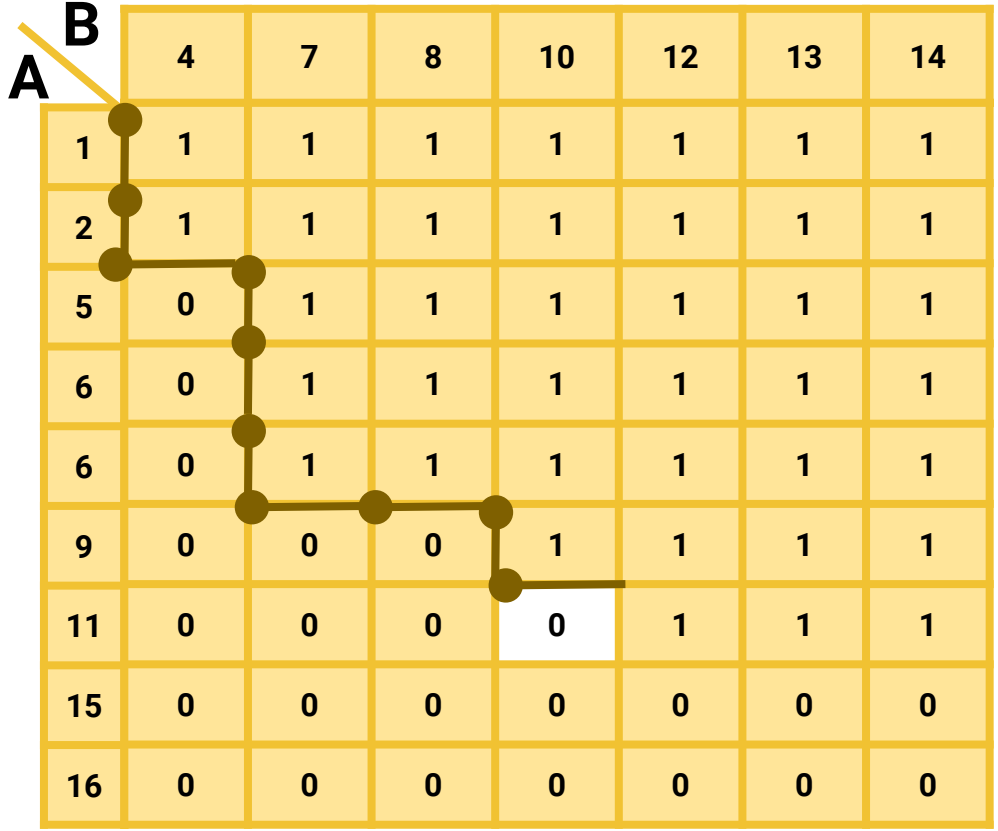
Path

1	1	0	1	1

…

1	0	0	1

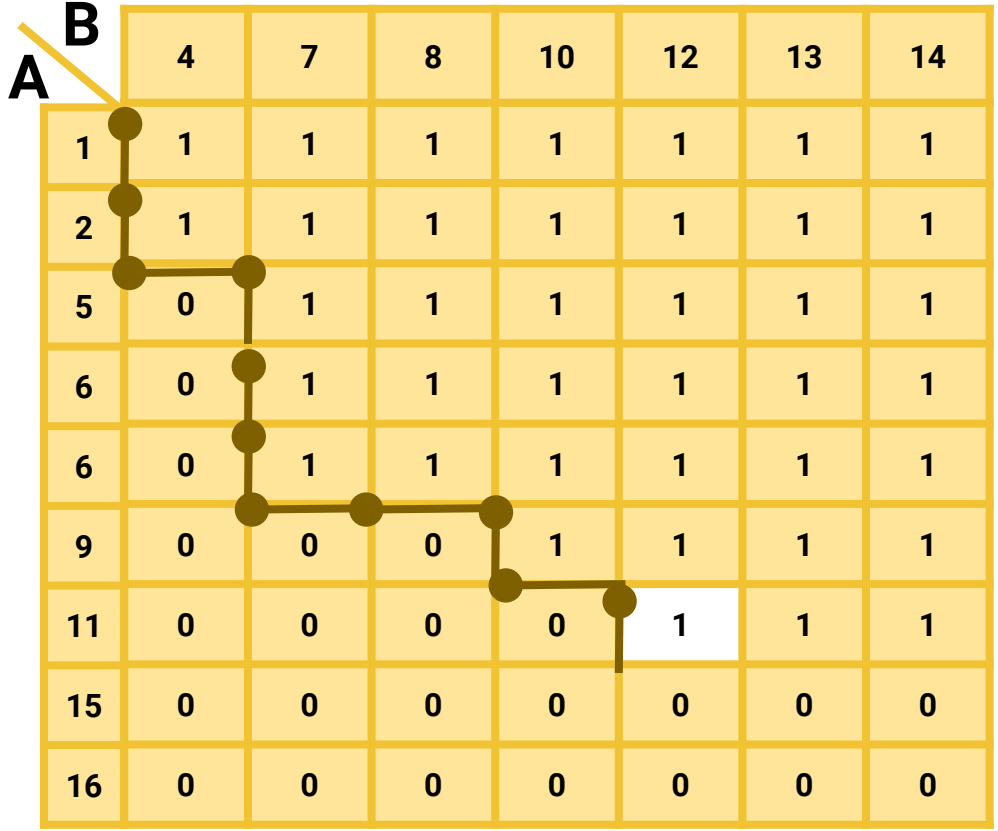
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0	1	0

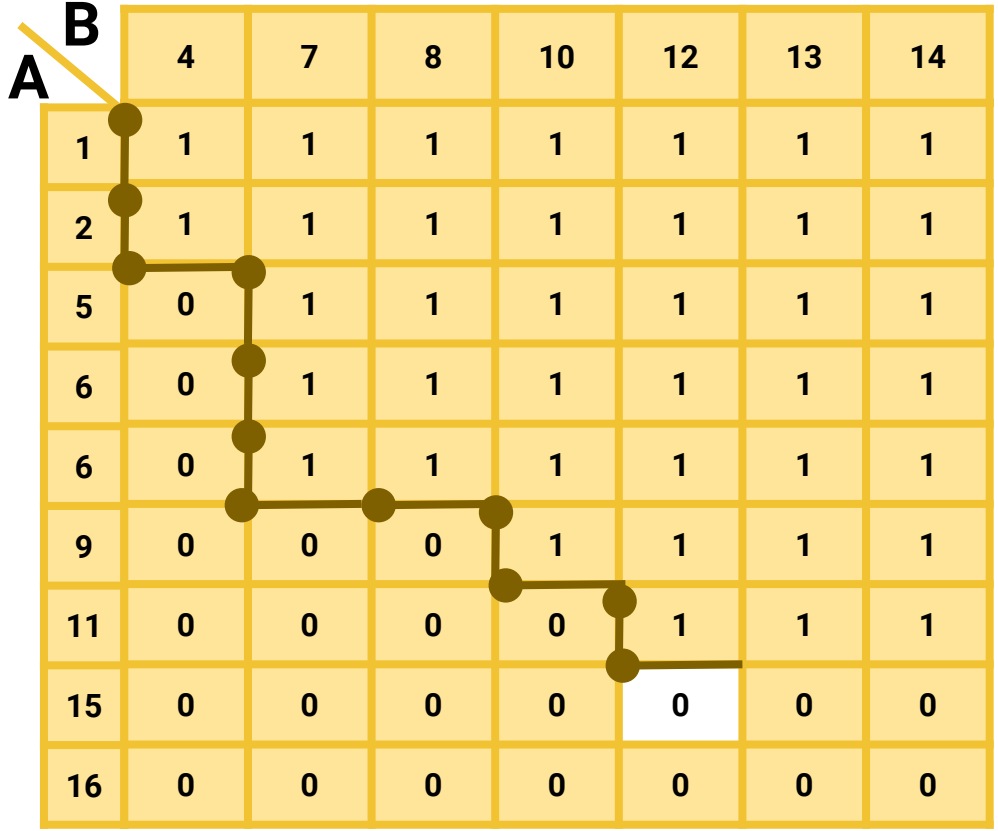
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0	1	0
...	1				

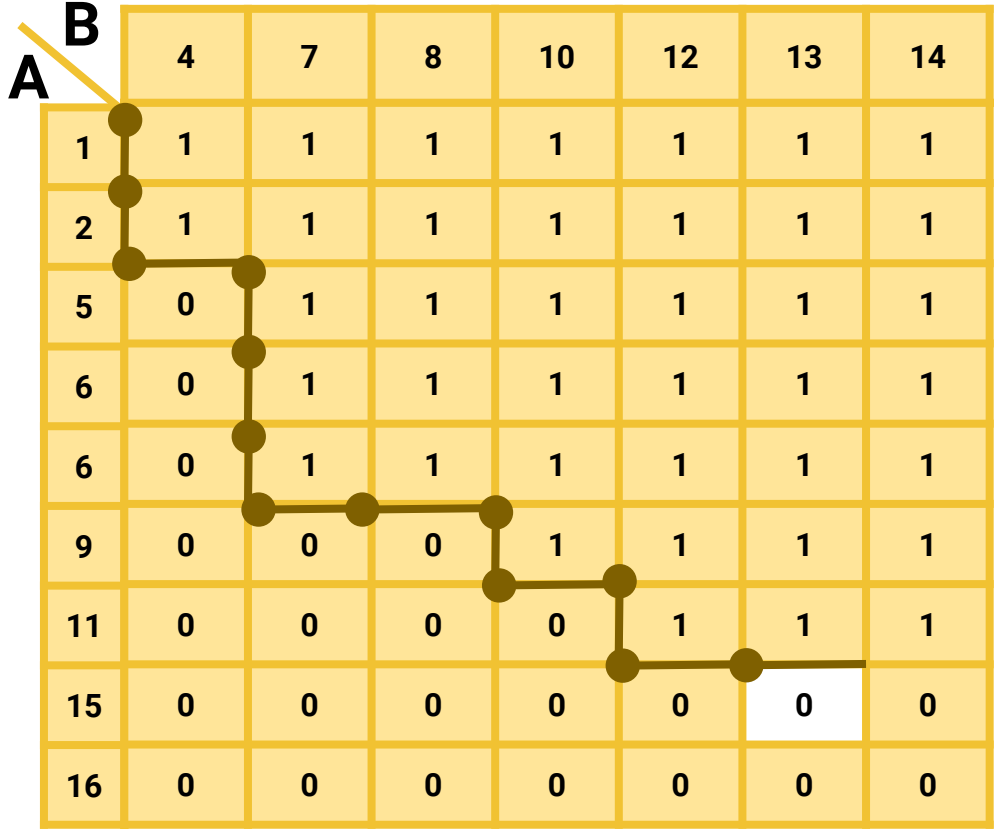
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0	1	0
...	1	0			

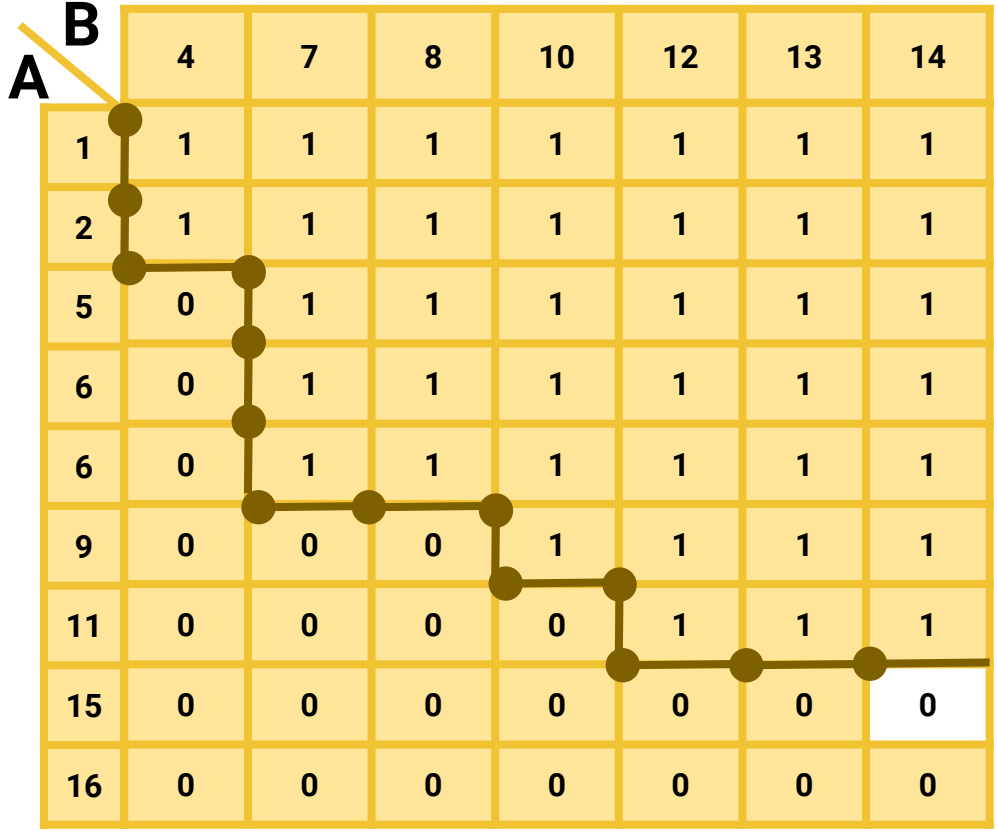
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0	1	0
...	1	0	0		

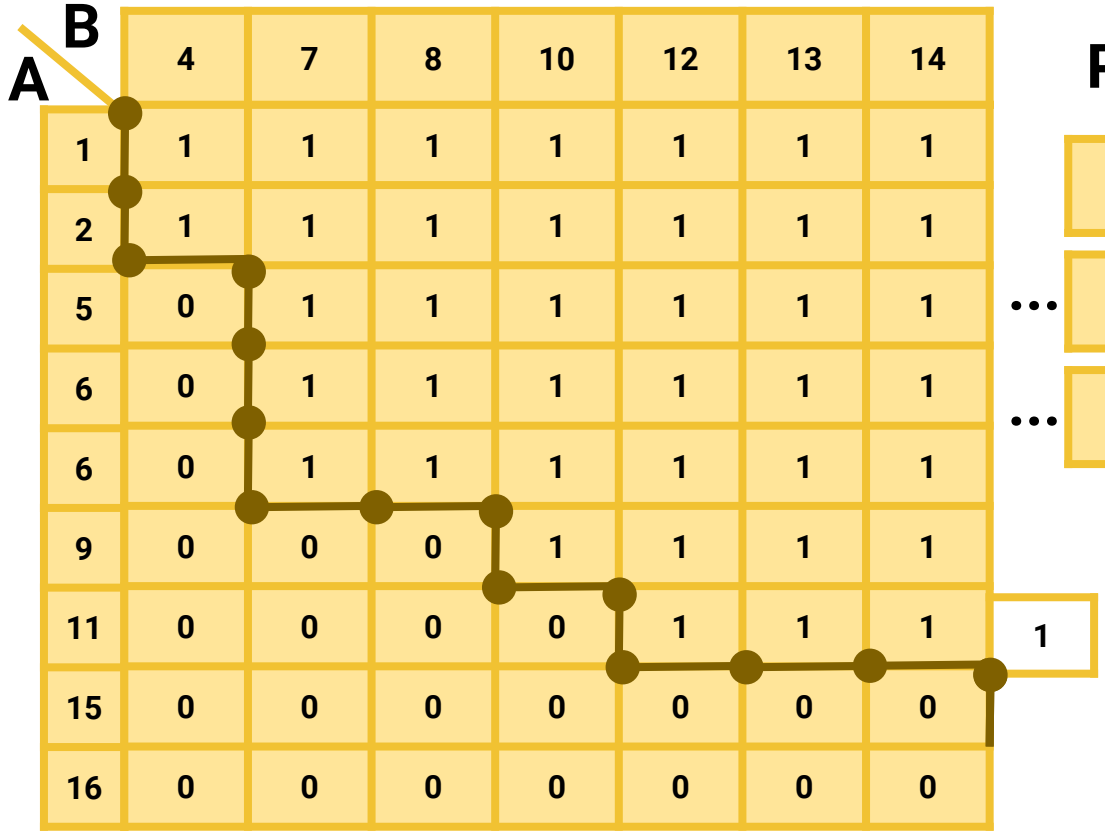
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0	1	0
...	1	0	0	0	

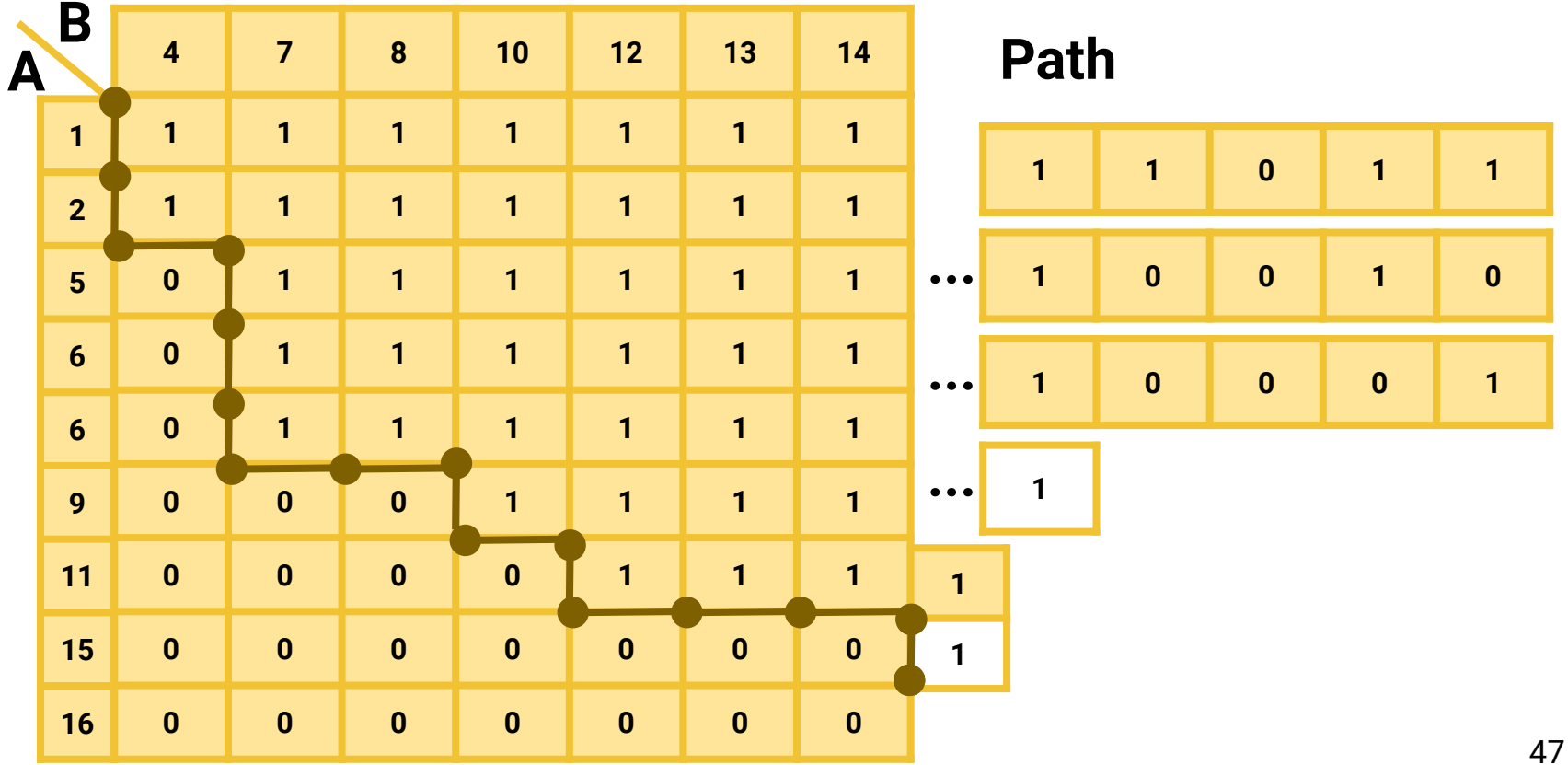
ALGO mergeSmall_k



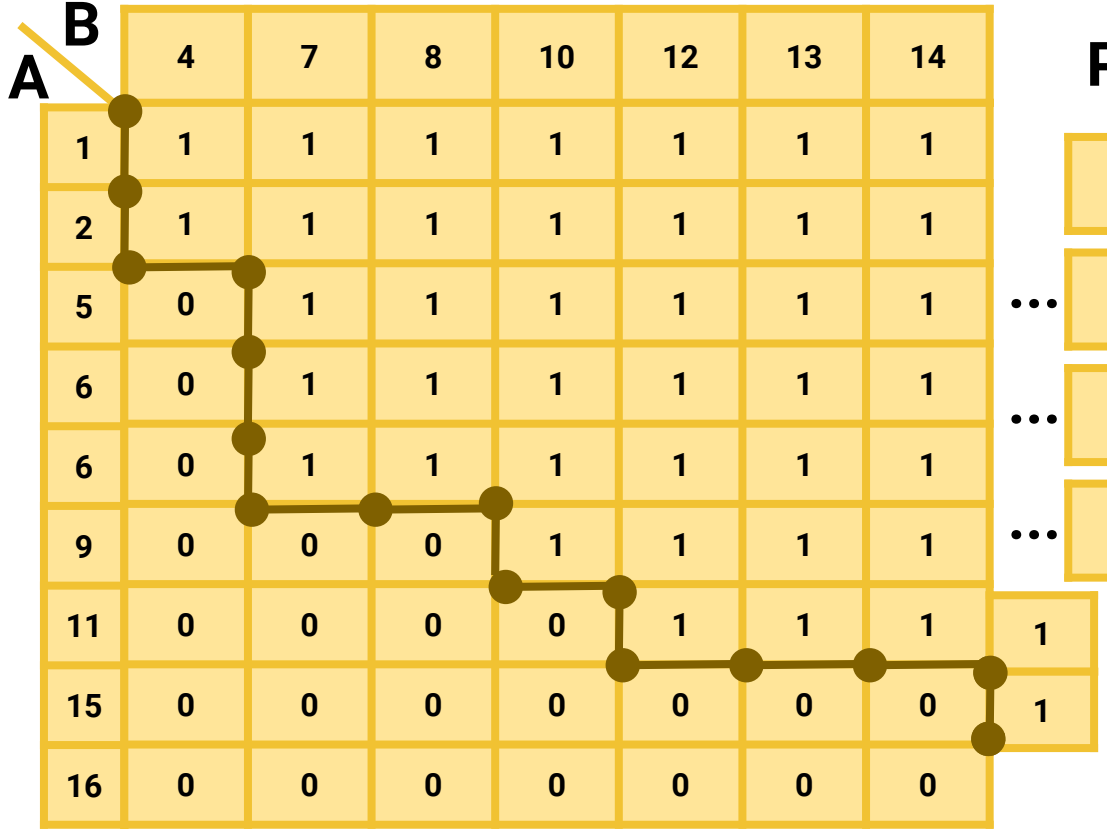
Path

	1	1	0	1	1
...	1	0	0	1	0
...	1	0	0	0	1

ALGO pathBig_k



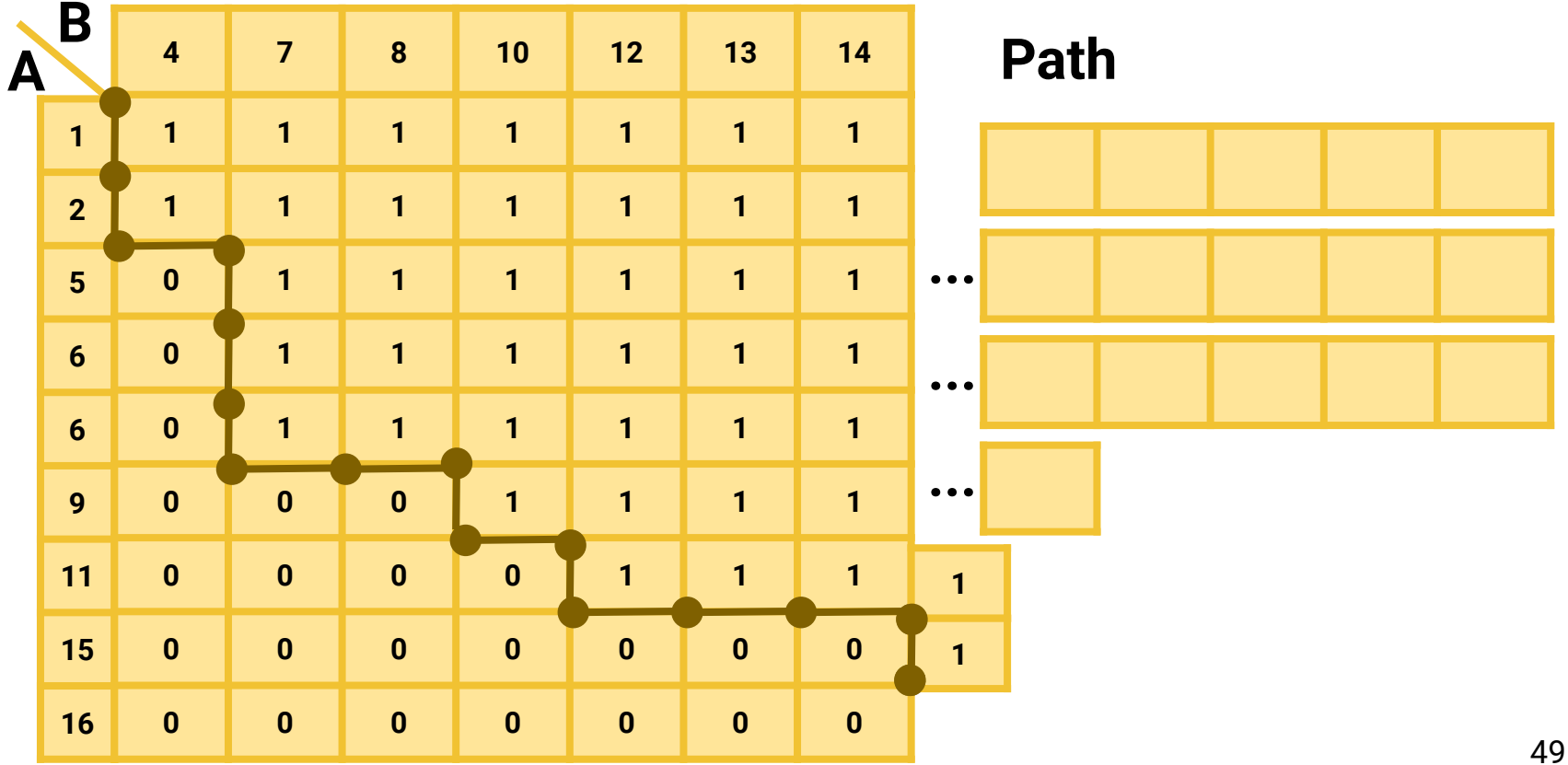
ALGO pathBig_k



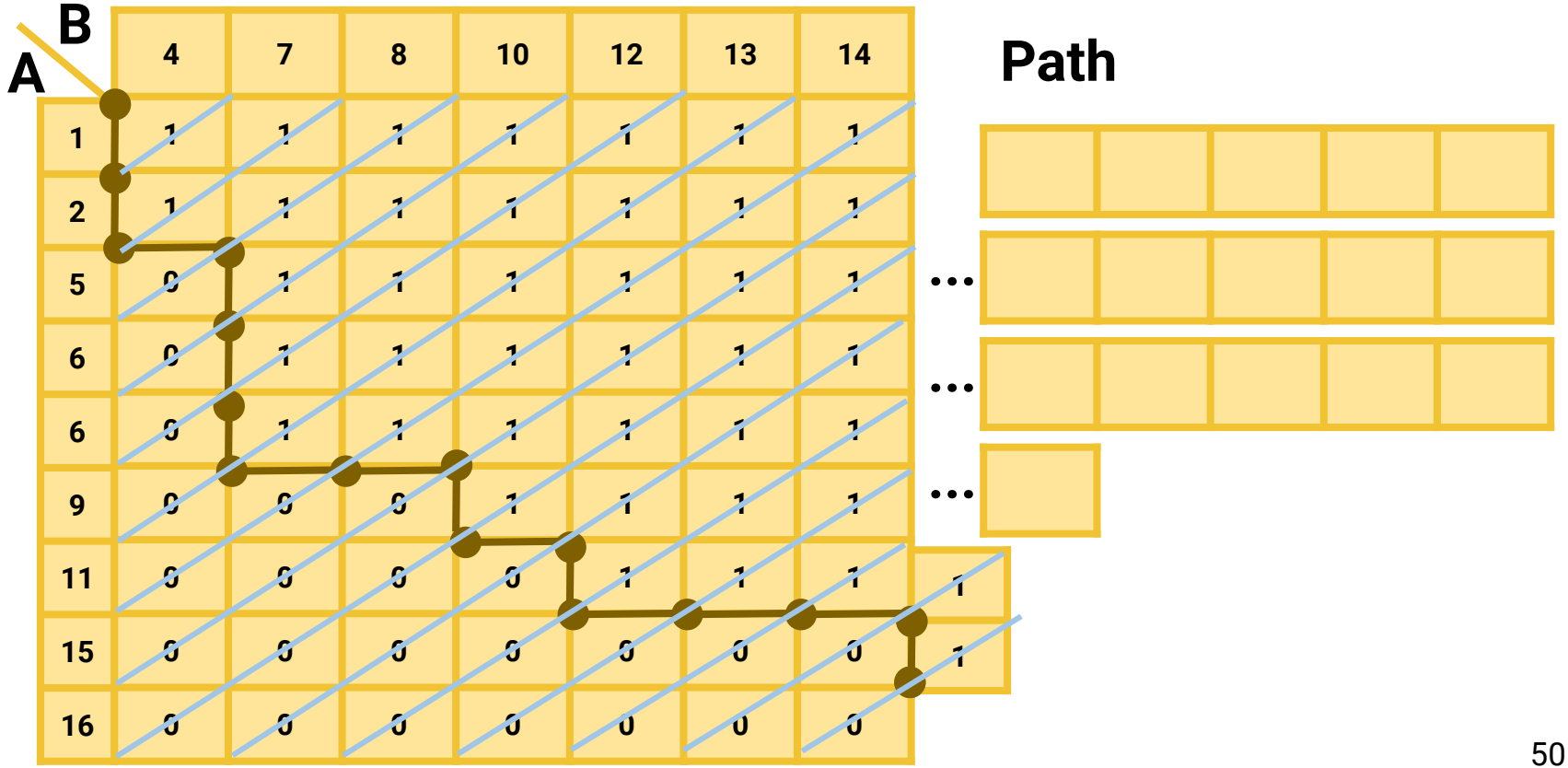
Path

	1	1	0	1	1
...	1	0	0	1	0
...	1	0	0	0	1
...	1				
	1				
	1				

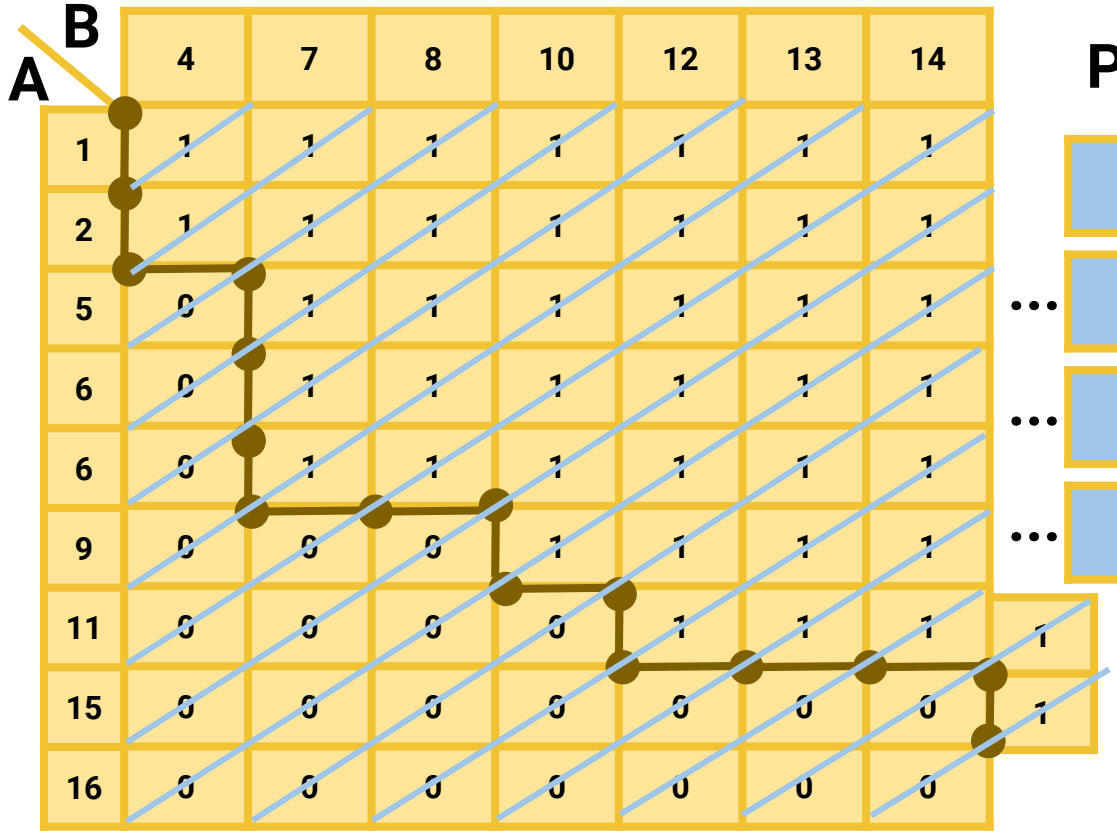
ALGO pathBig_k

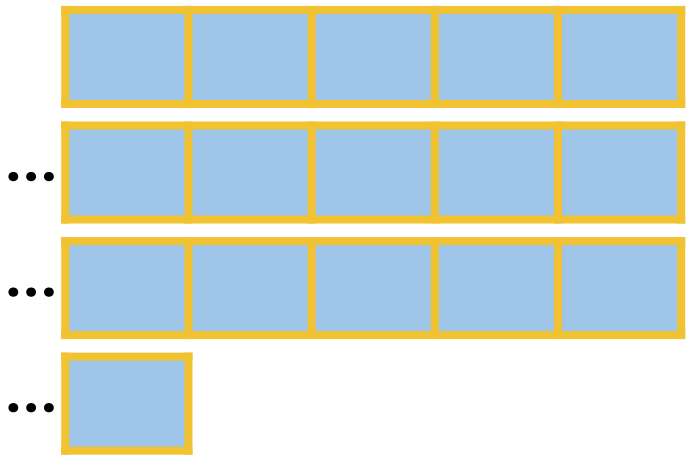


ALGO pathBig_k

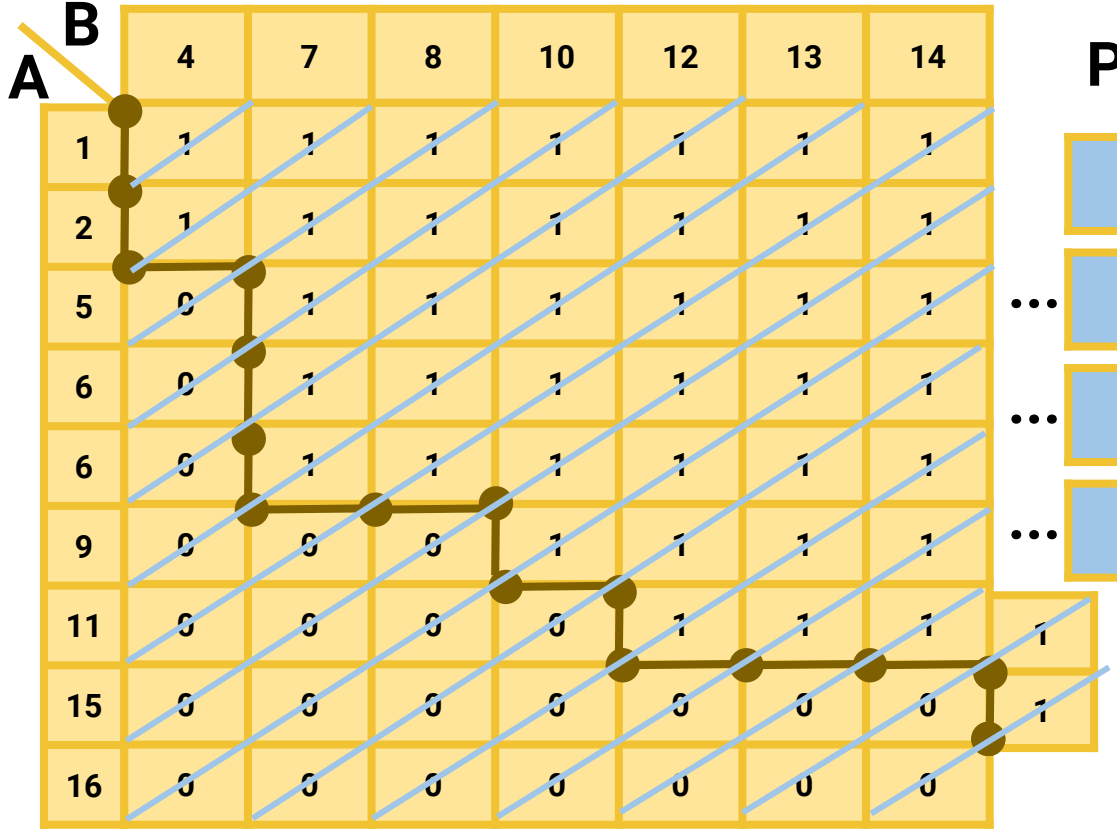


ALGO pathBig_k





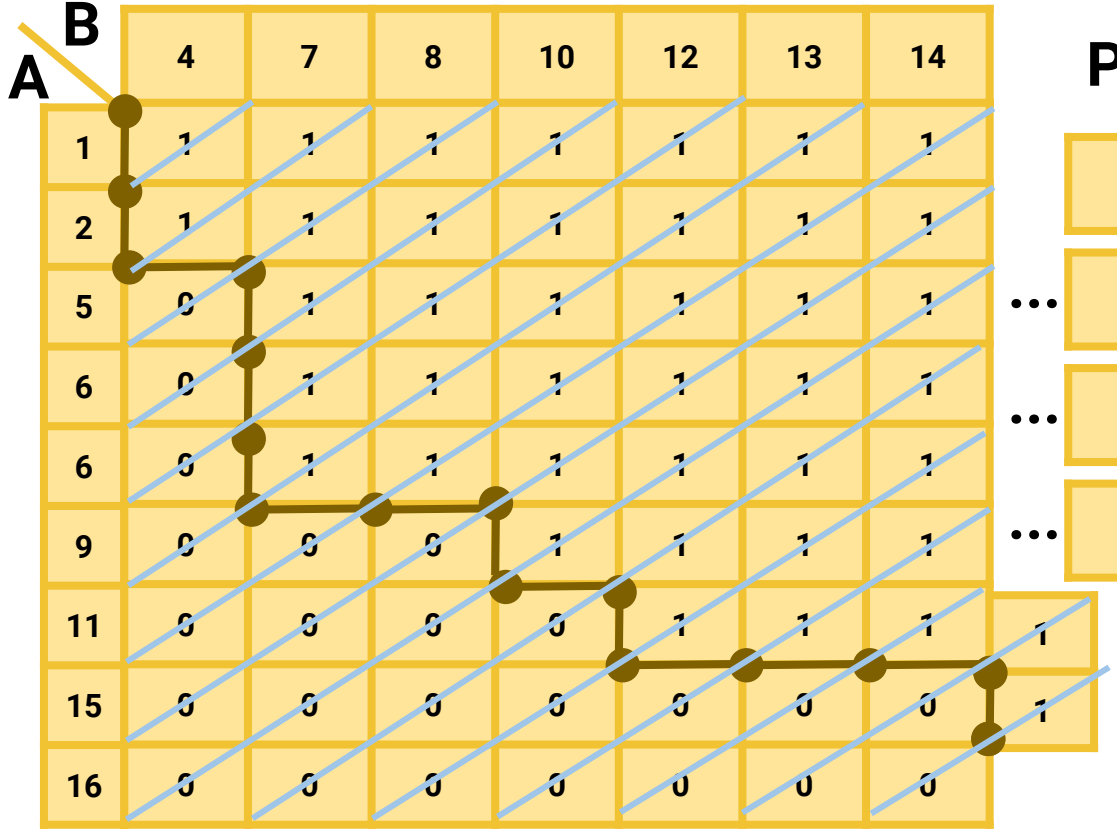
ALGO pathBig_k



Path

	1	1	0	1	1
...	1	0	0	1	0
...	1	0	0	0	1
...	1				

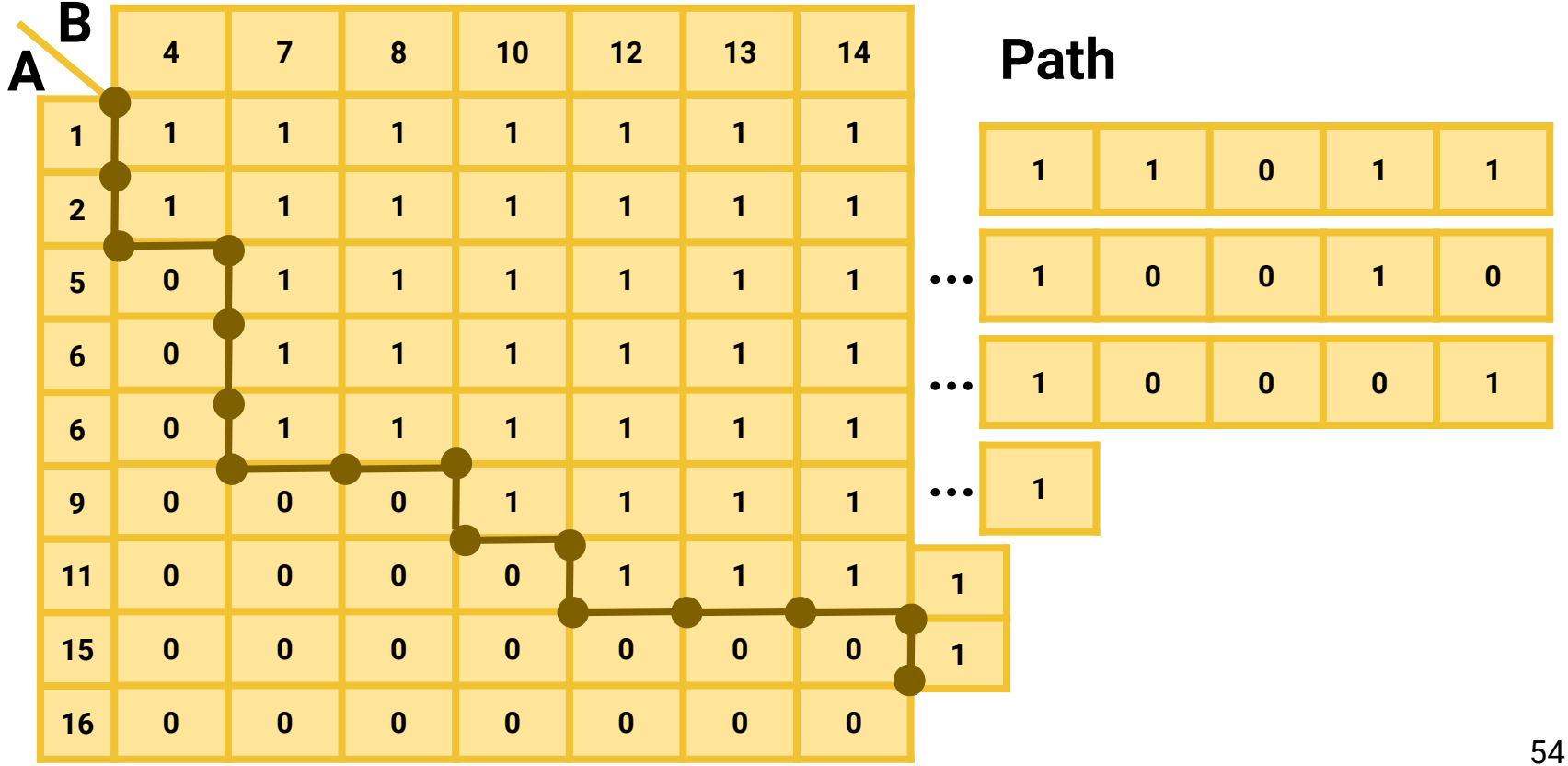
ALGO pathBig_k



Path

1	1	0	1	1
1	0	0	1	0
1	0	0	0	1
1				

ALGO pathBig_k



ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4

ALGO mergeBig_k

A

B

Path

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10	11

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10	11	12

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10	11	12	13

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14

ALGO mergeBig_k

A

B

Path

M

ALGO mergeBig_k

A

B

Path

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16

B

4	7	8	10	12	13	14

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16

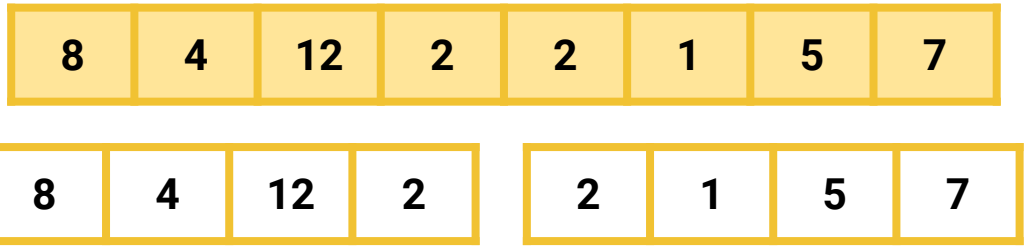
03

“MERGE SORT” ON GPU

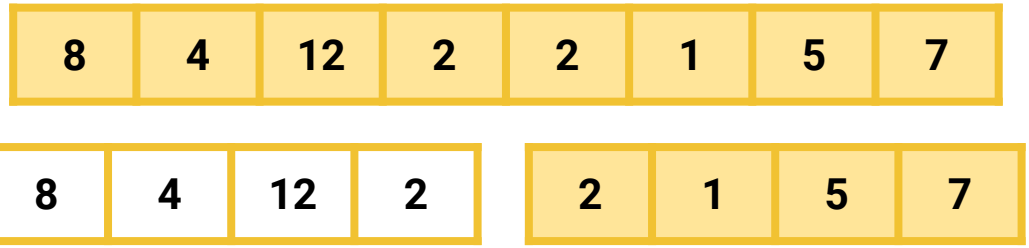
MERGE SORT ON CPU

MERGE SORT ON CPU

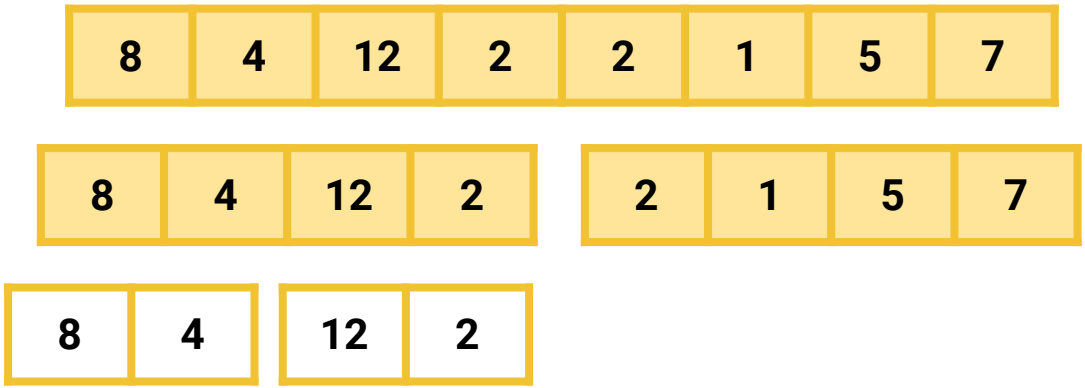
MERGE SORT ON CPU



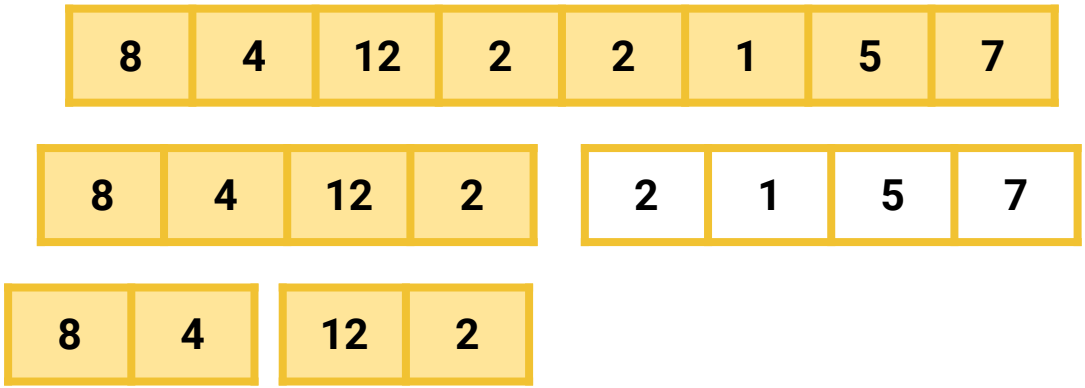
MERGE SORT ON CPU



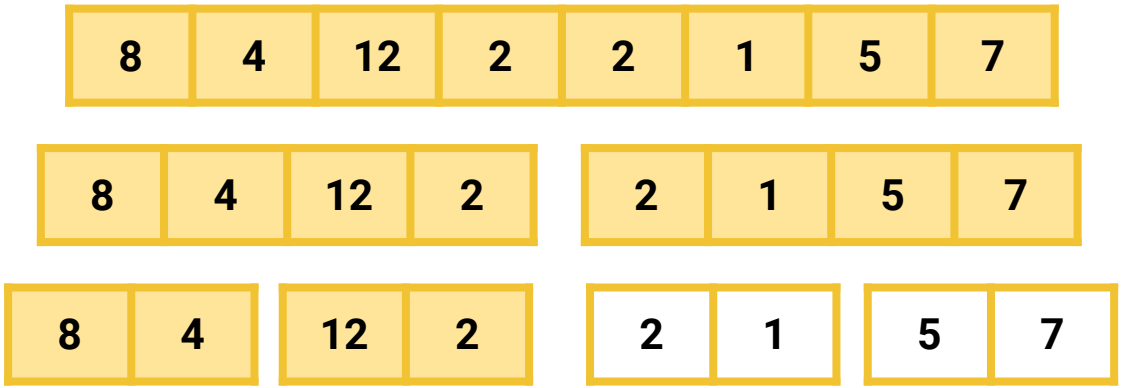
MERGE SORT ON CPU



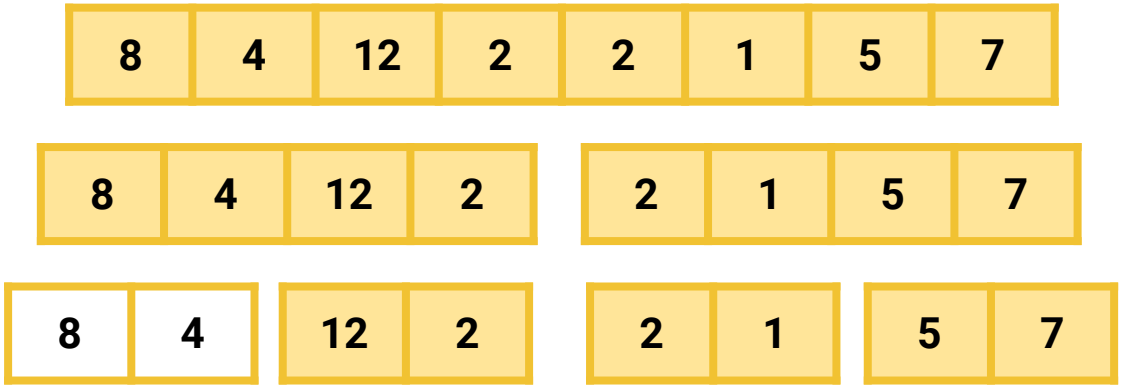
MERGE SORT ON CPU



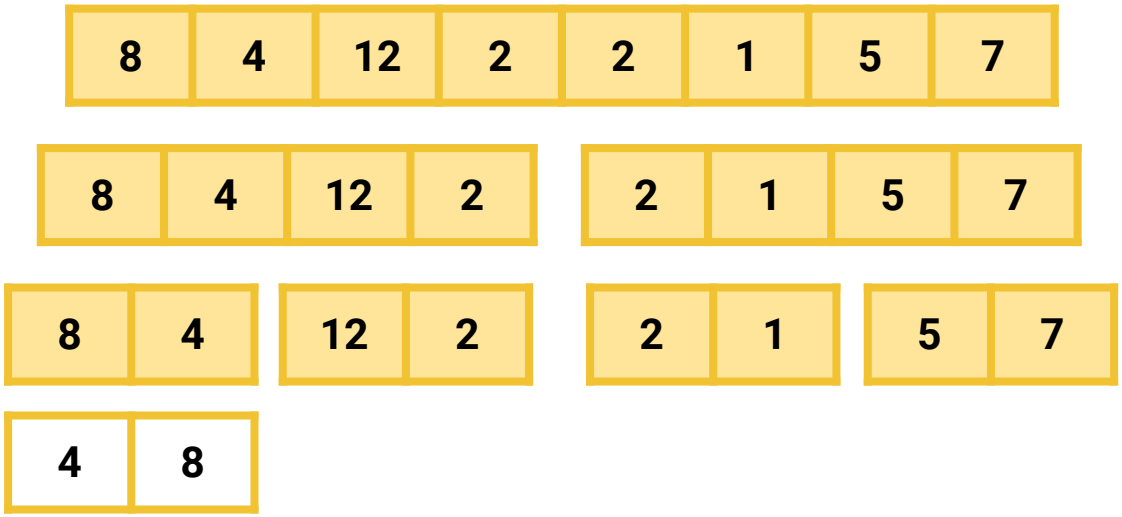
MERGE SORT ON CPU



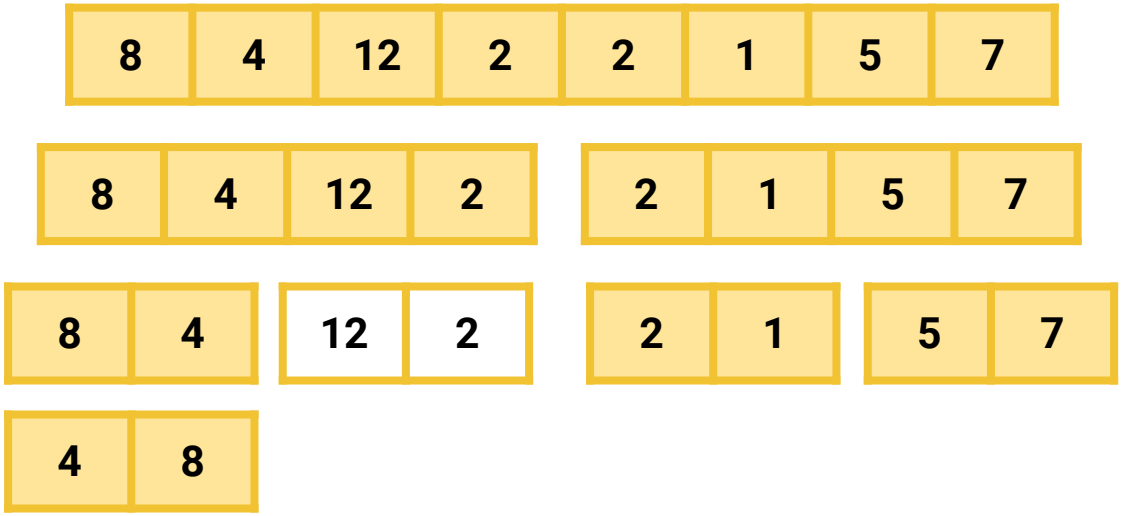
MERGE SORT ON CPU



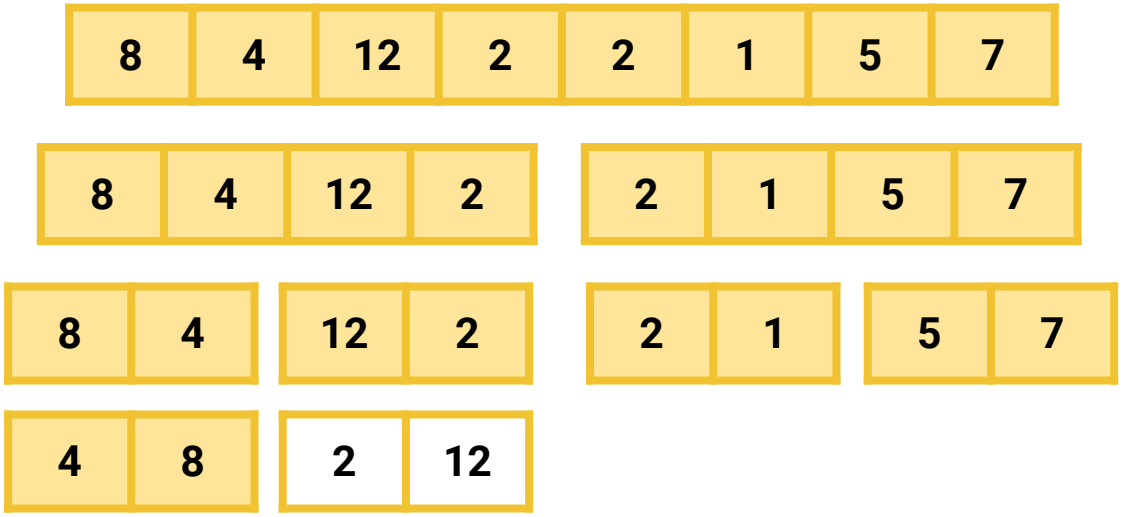
MERGE SORT ON CPU



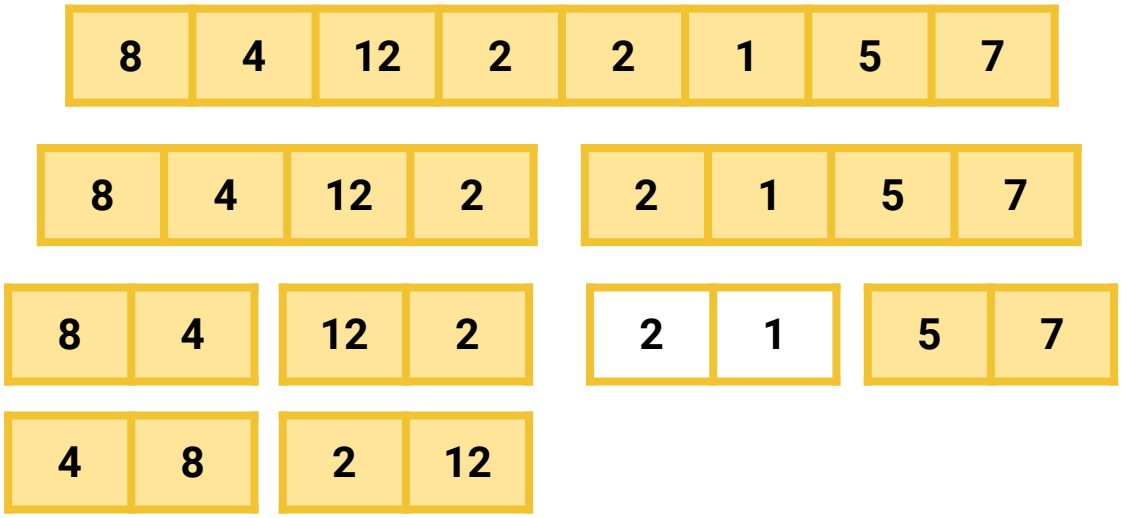
MERGE SORT ON CPU



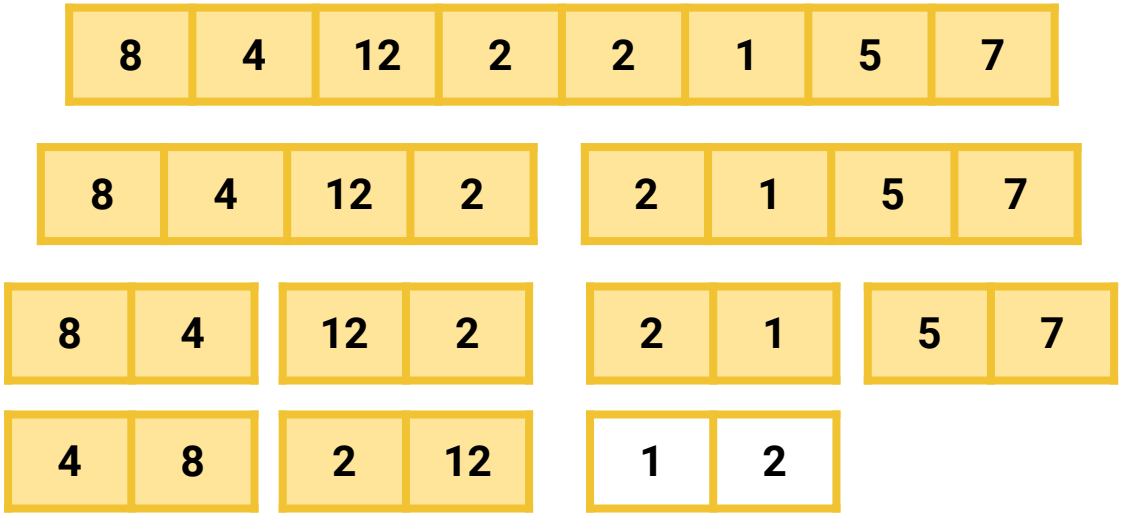
MERGE SORT ON CPU



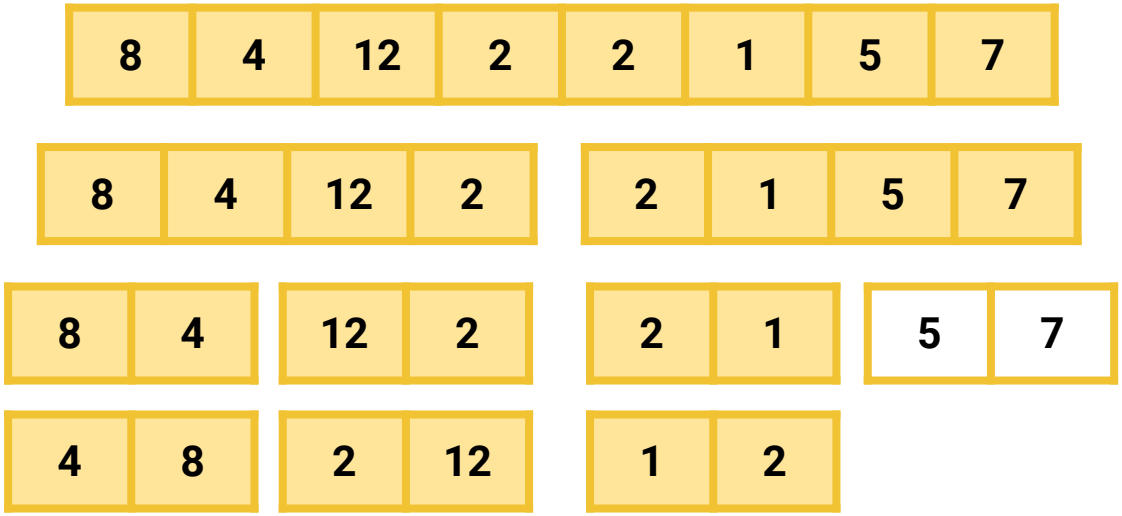
MERGE SORT ON CPU



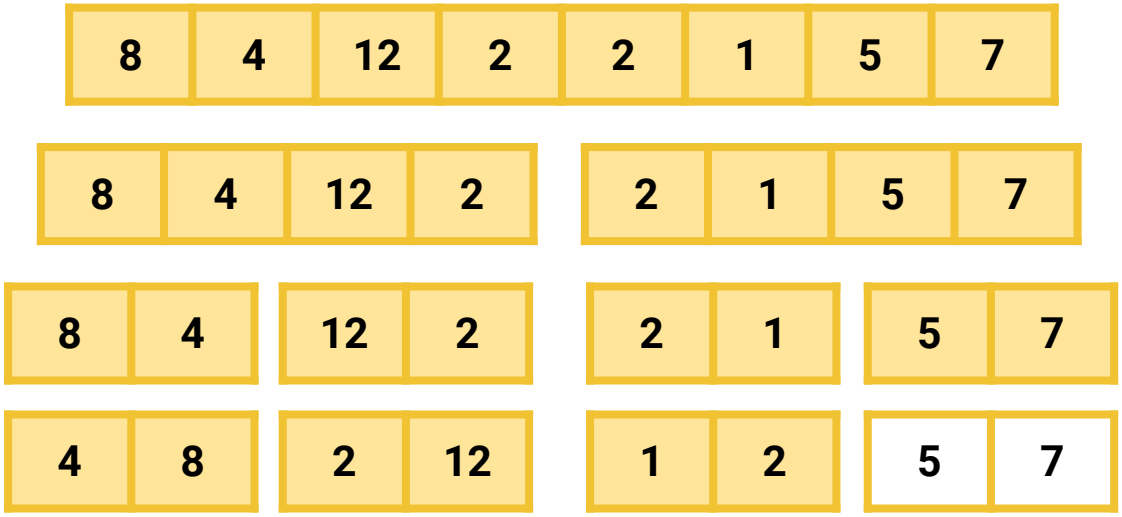
MERGE SORT ON CPU



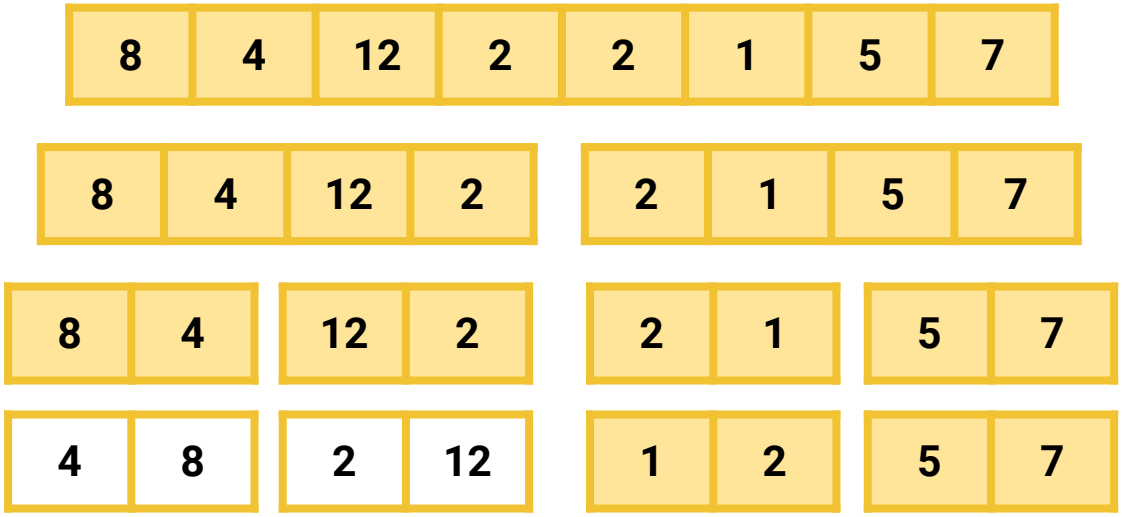
MERGE SORT ON CPU



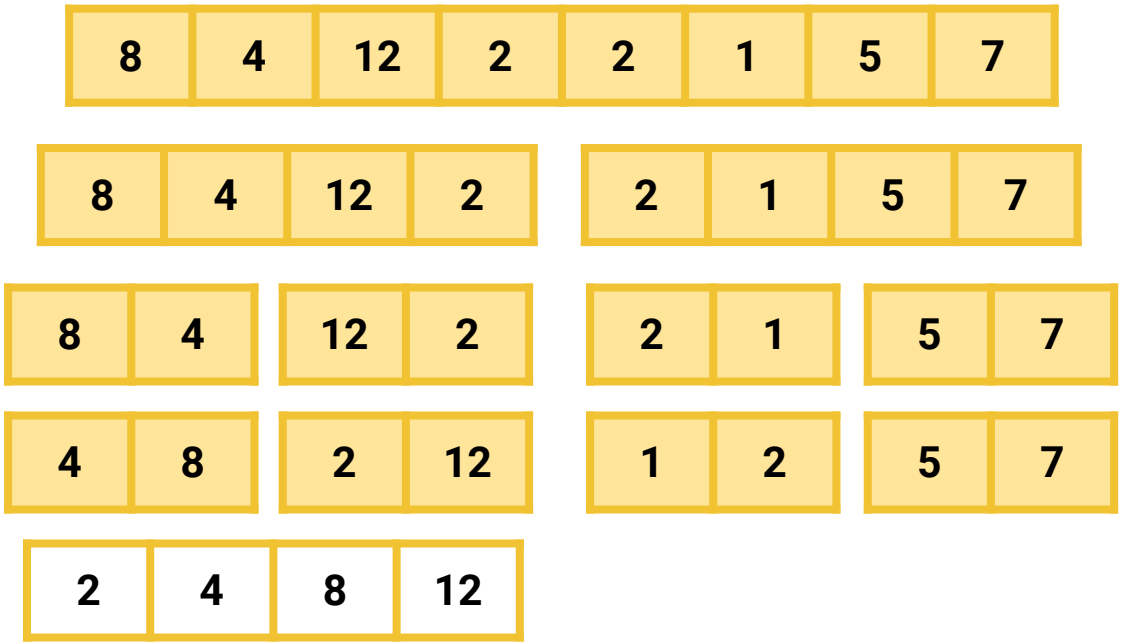
MERGE SORT ON CPU



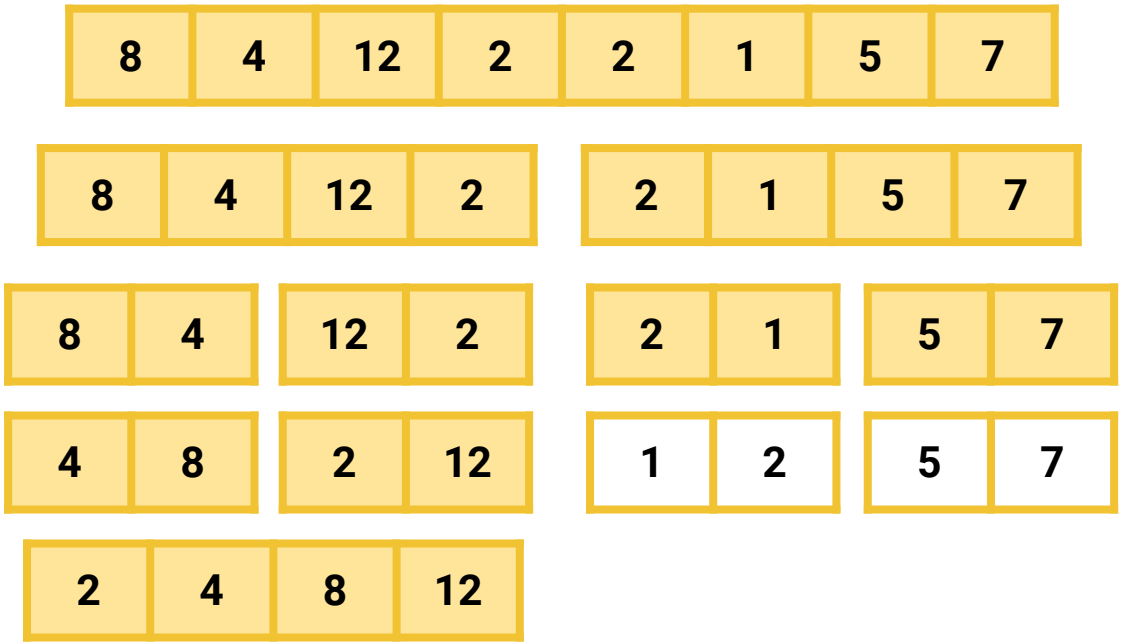
MERGE SORT ON CPU



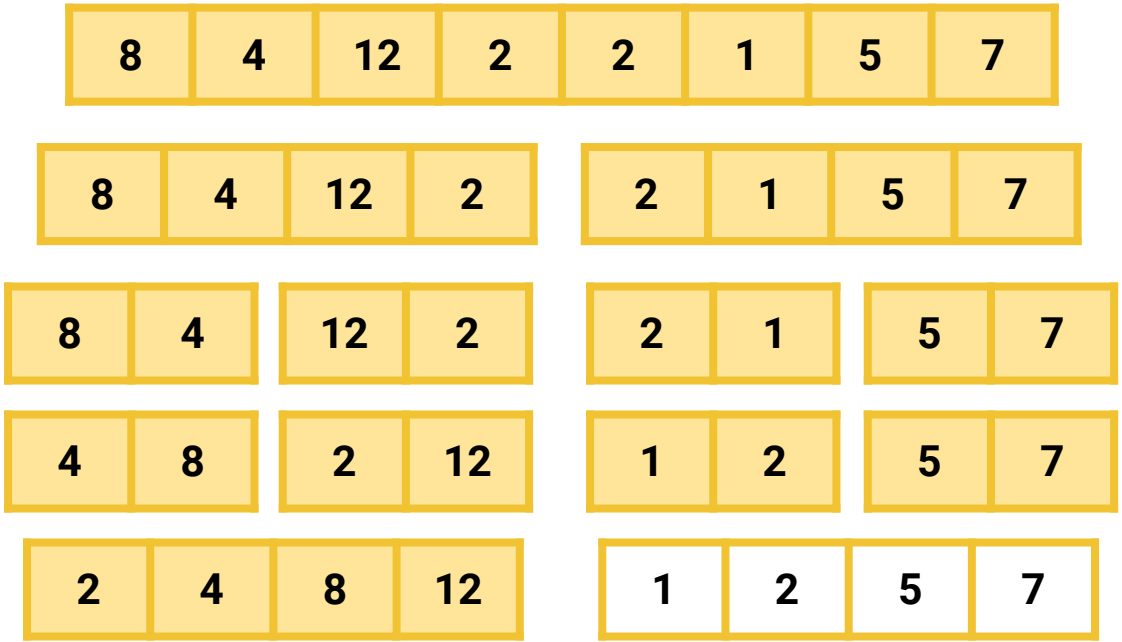
MERGE SORT ON CPU



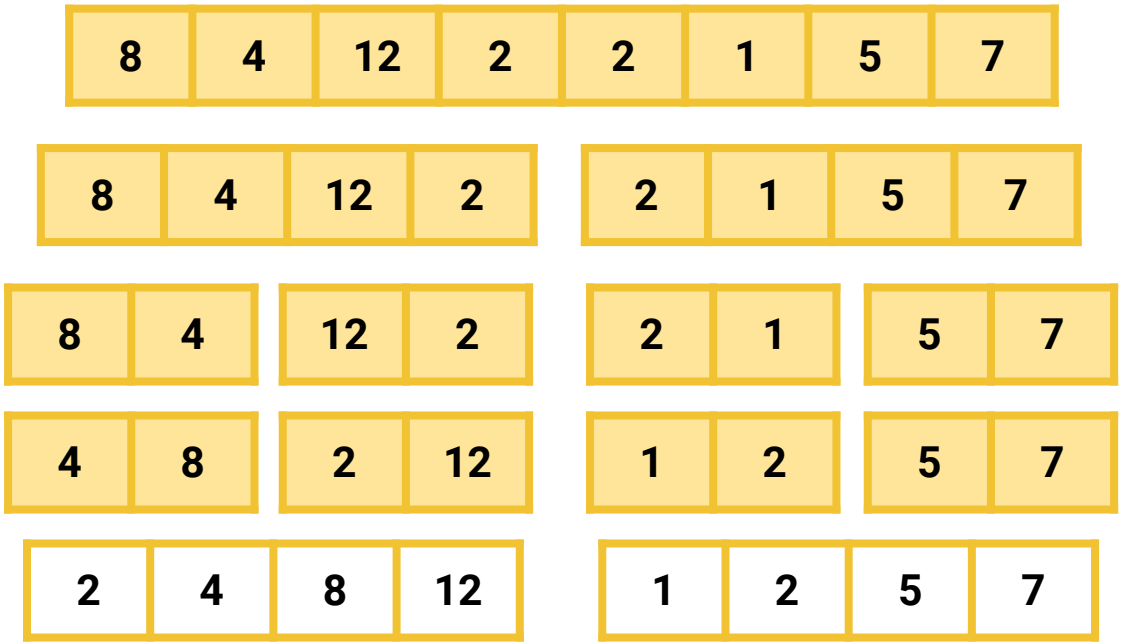
MERGE SORT ON CPU



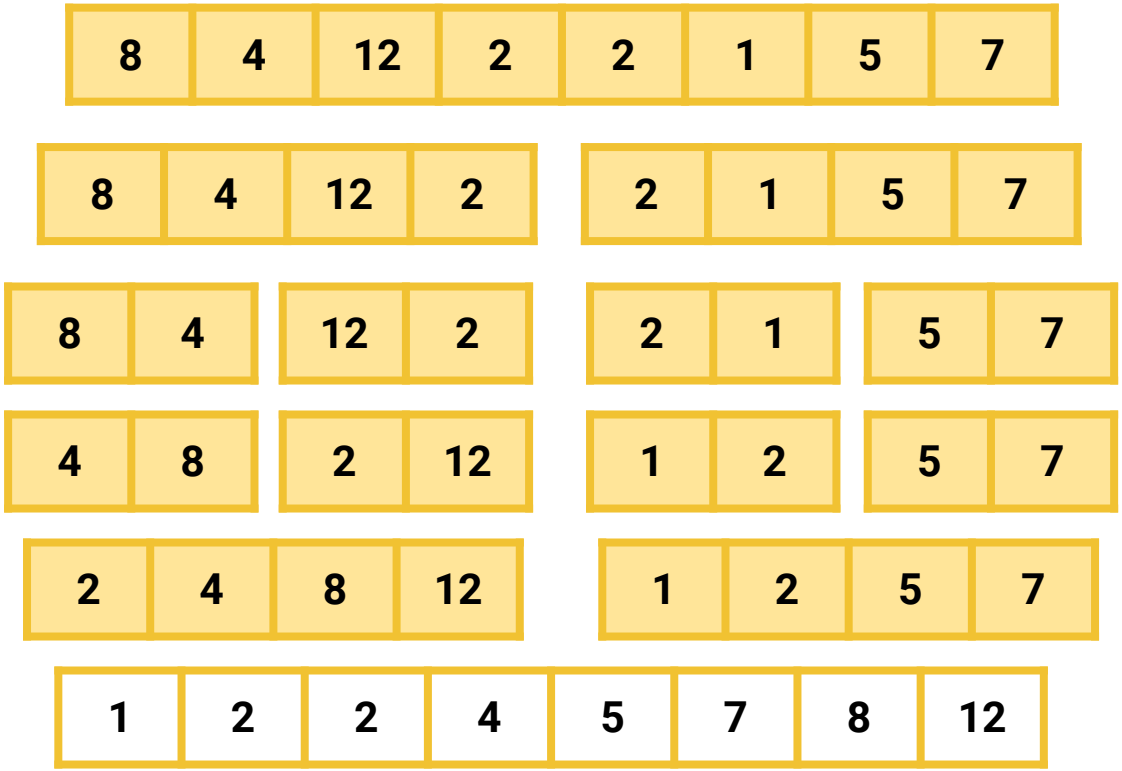
MERGE SORT ON CPU



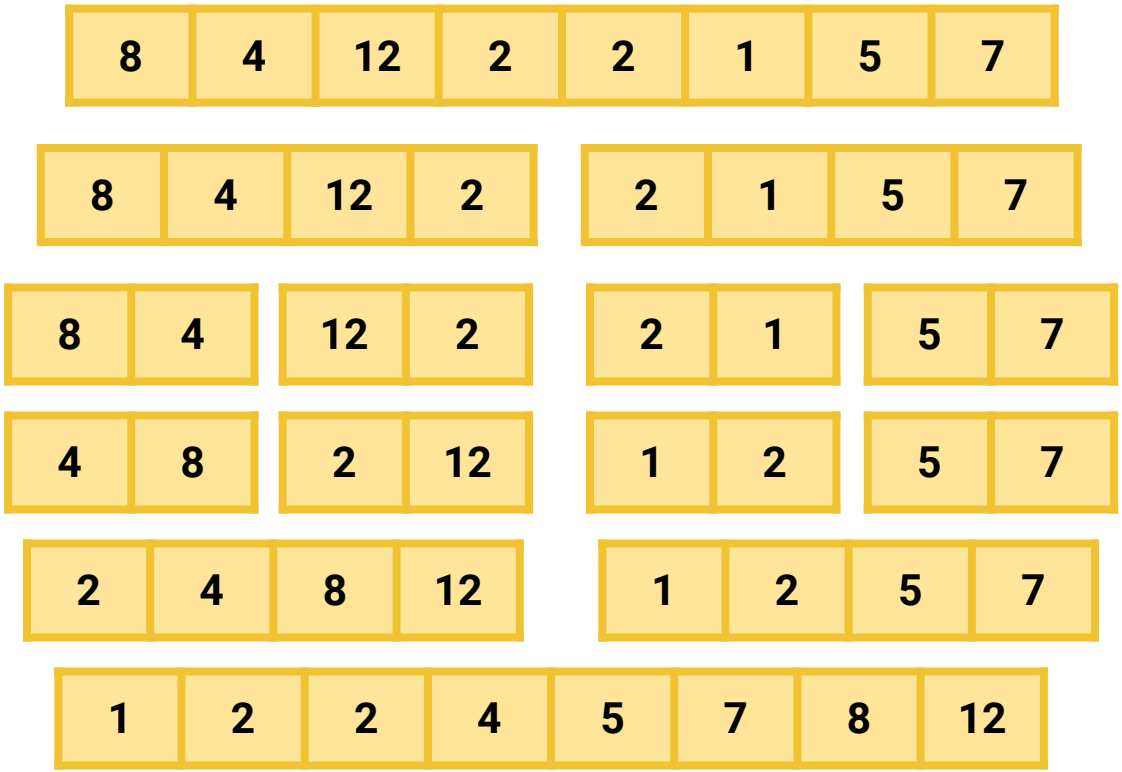
MERGE SORT ON CPU



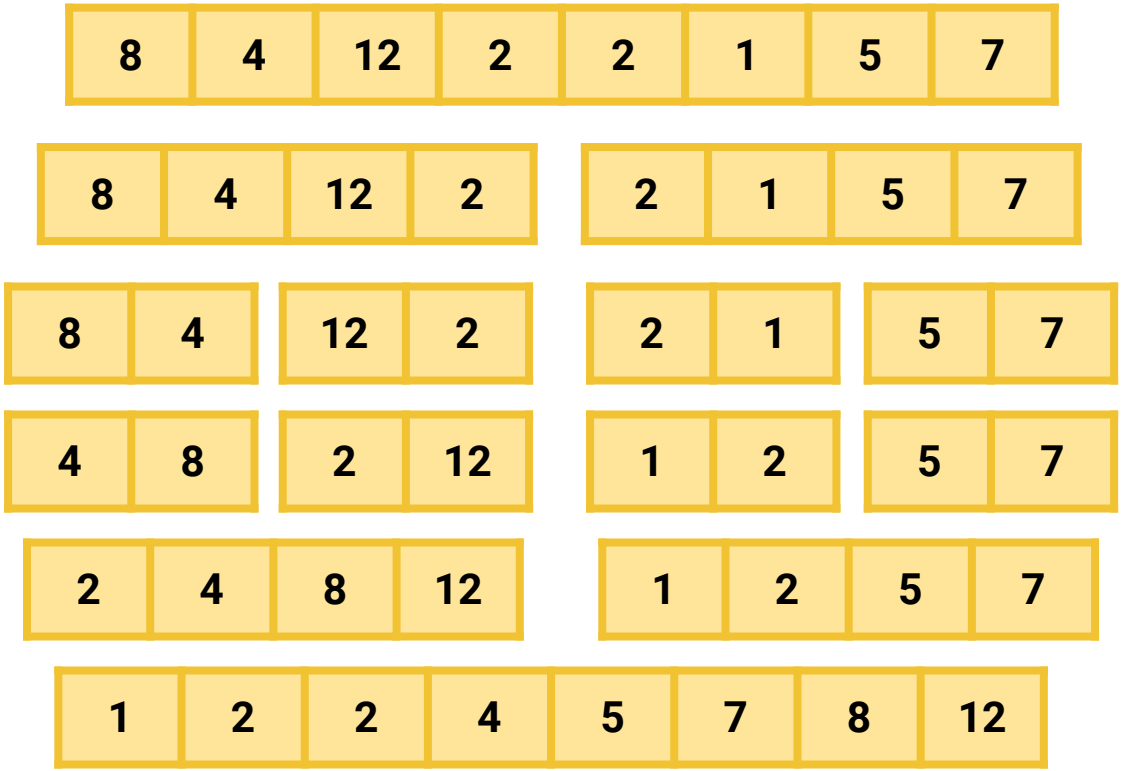
MERGE SORT ON CPU



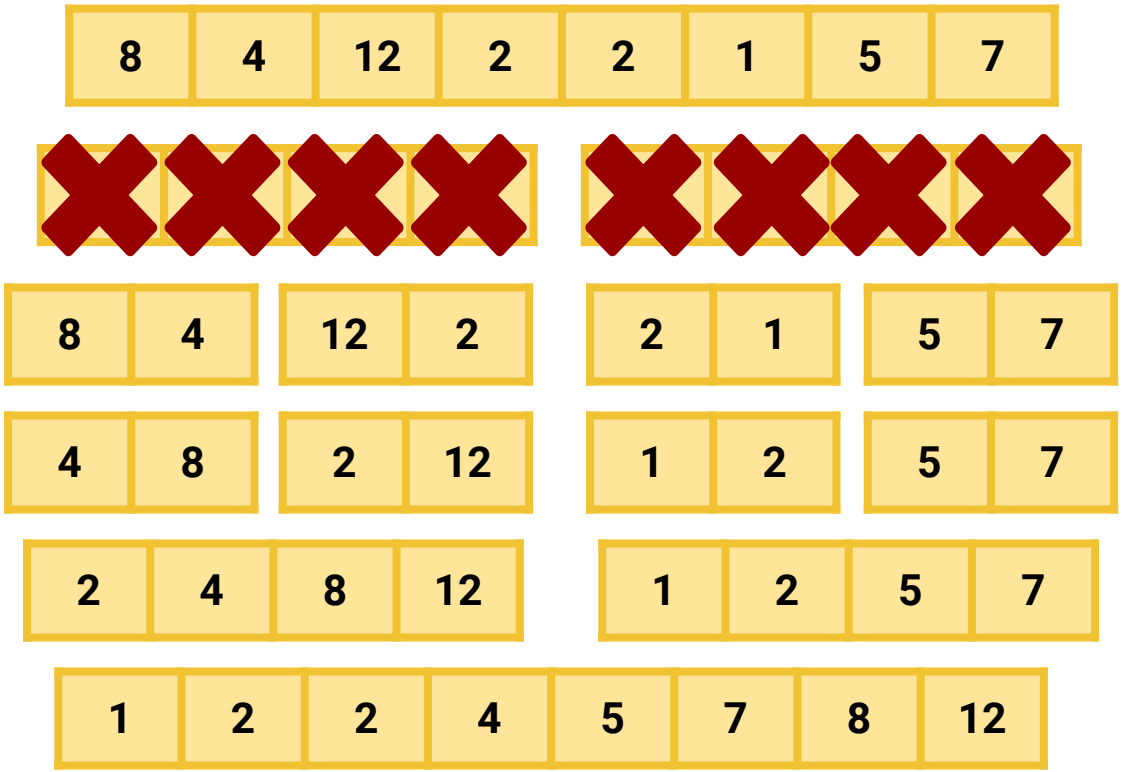
MERGE SORT ON CPU



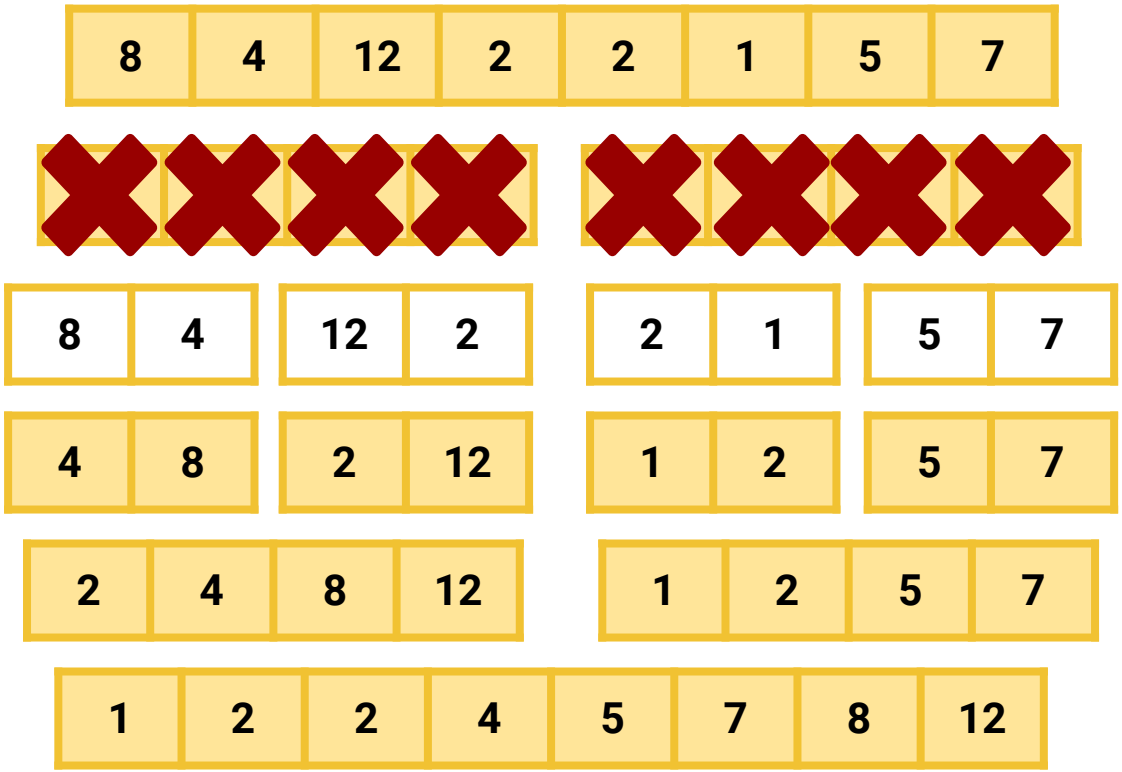
“MERGE SORT” ON GPU



“MERGE SORT” ON GPU



“MERGE SORT” ON GPU



“MERGE SORT” ON GPU

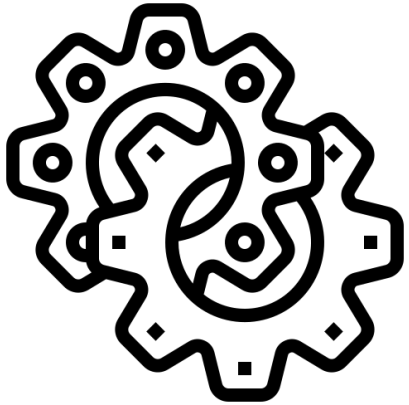
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

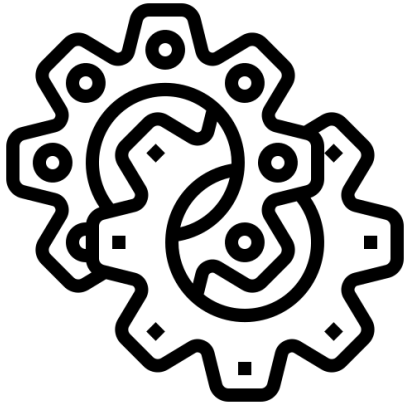
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

NOTATIONS

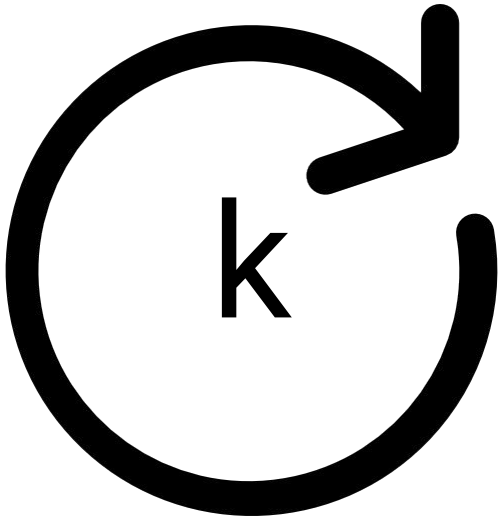


NOTATIONS

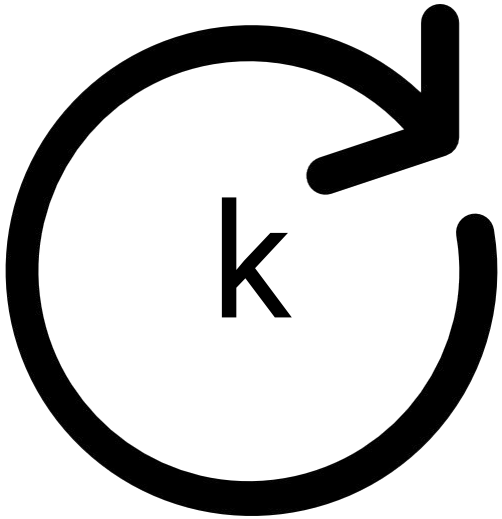


pathBig_k(A,B)
+
mergeBig_k(A,B)

NOTATIONS



NOTATIONS



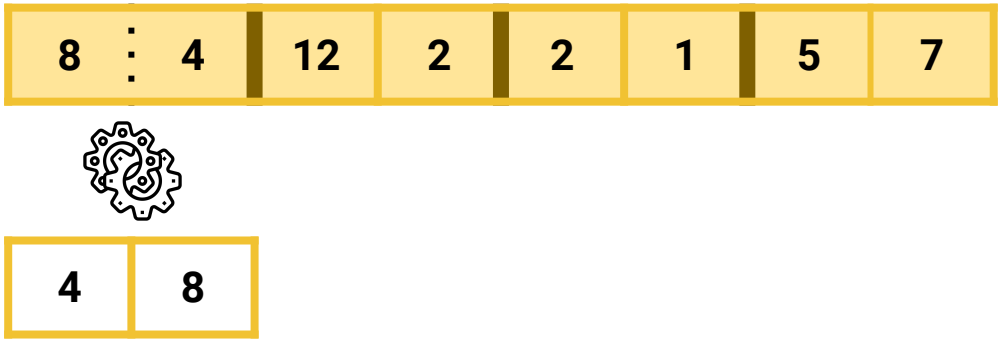
k loop tour

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

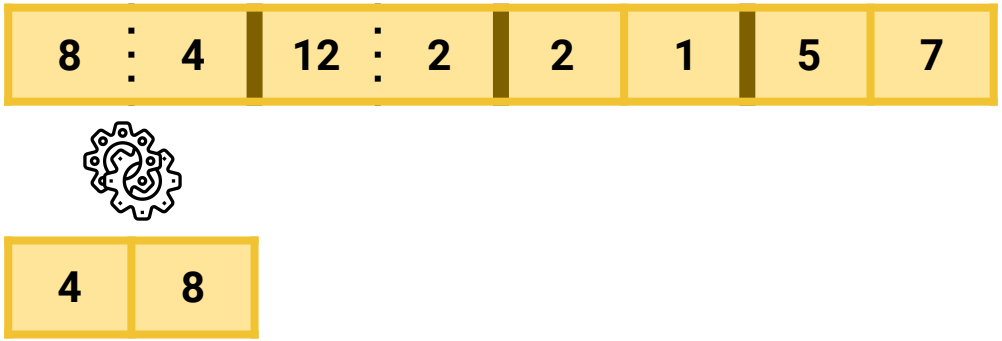
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



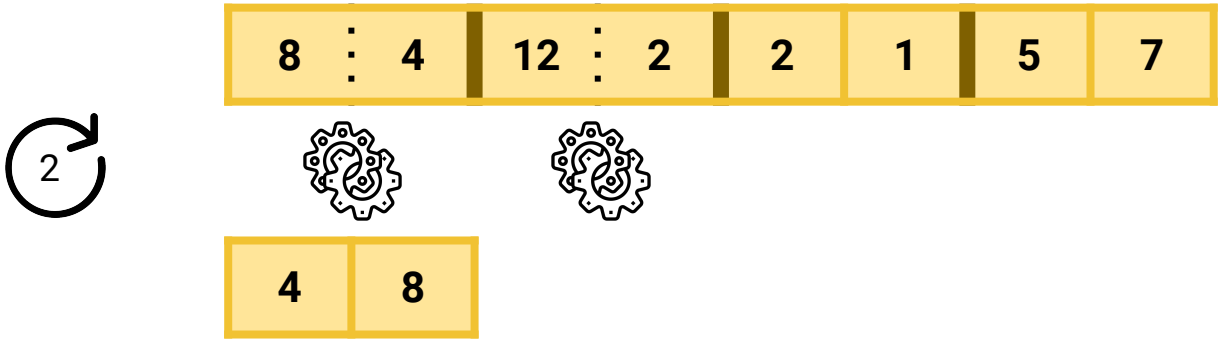
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



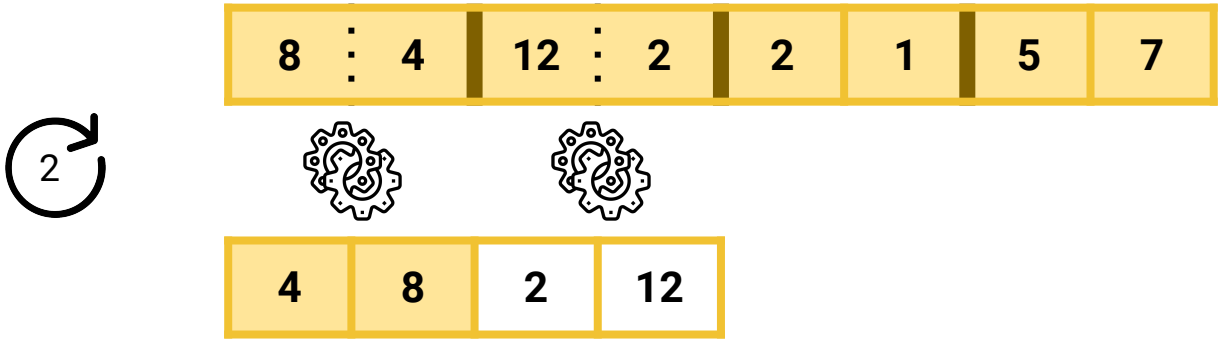
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



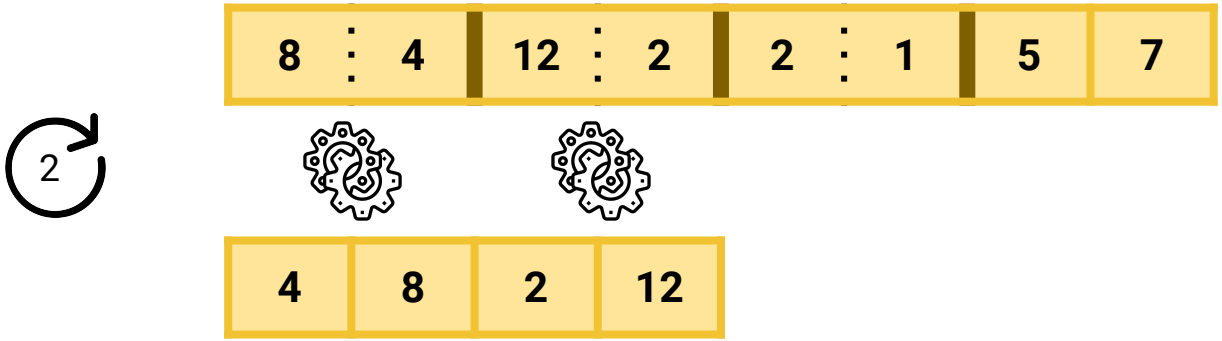
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



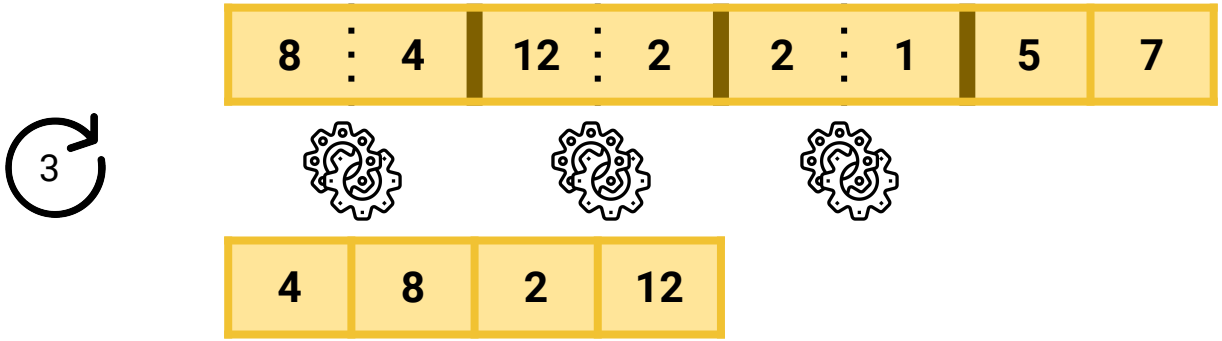
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



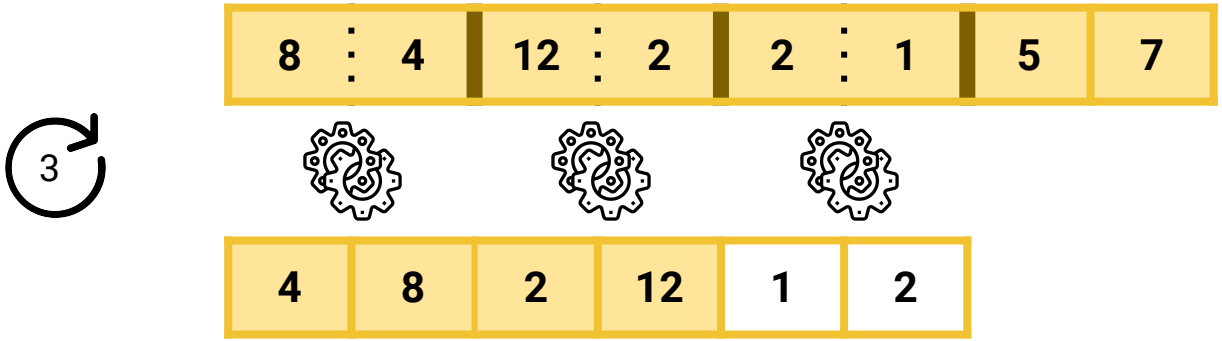
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



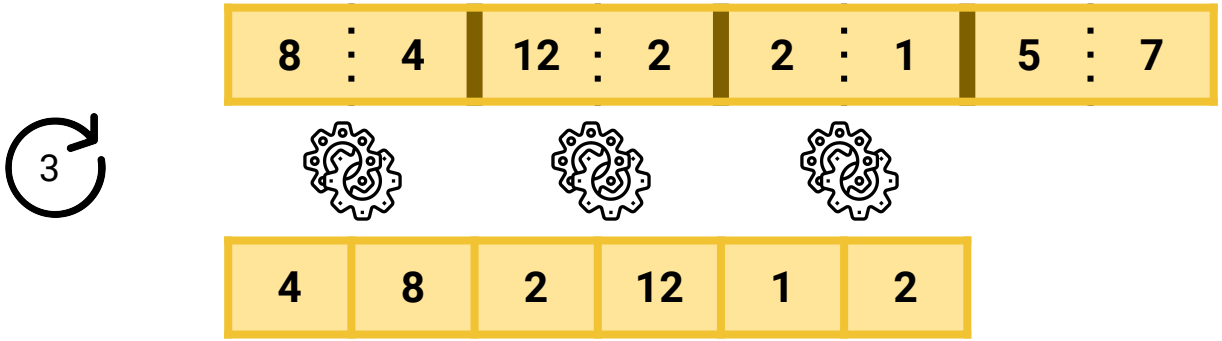
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



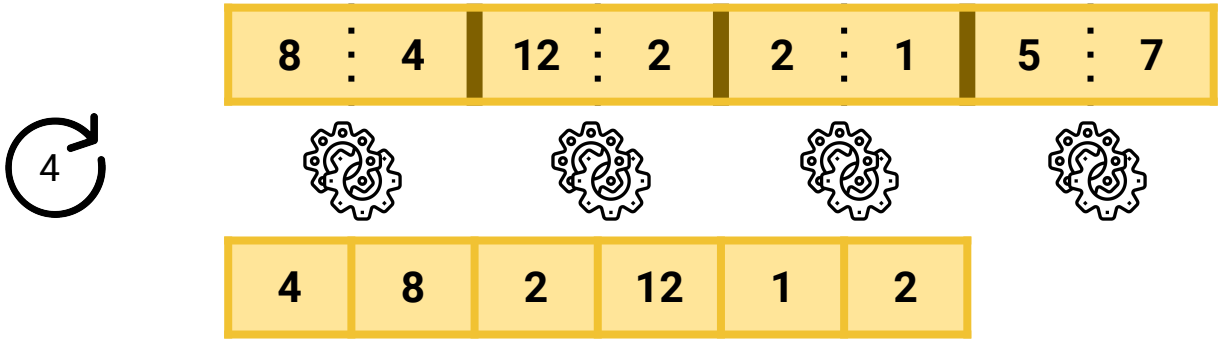
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



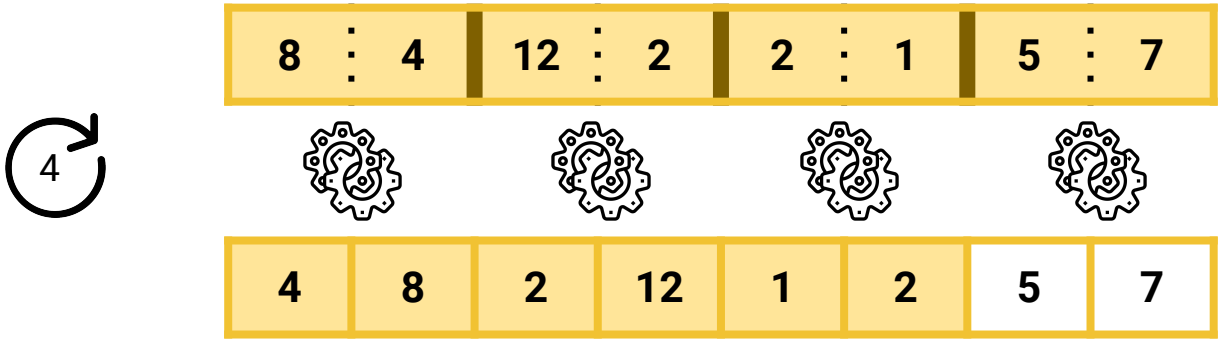
slice_size = 2
number_of_slices = 4

“MERGE SORT” ON GPU



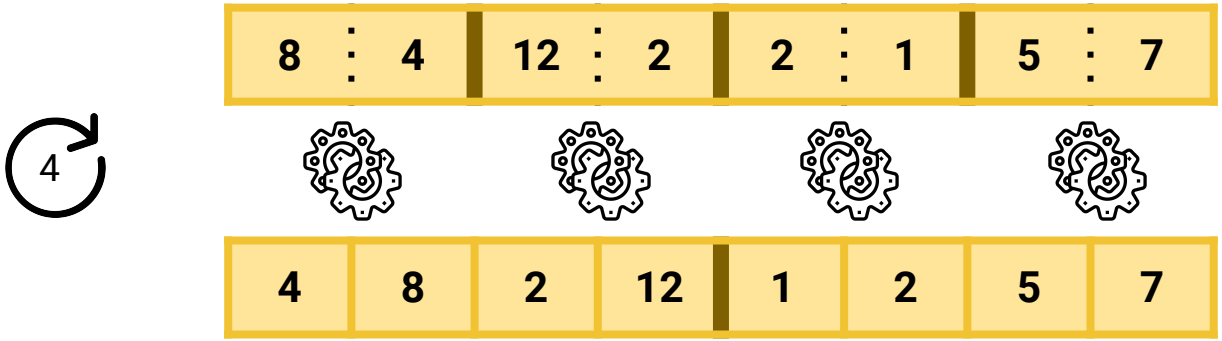
slice_size = 2
number_of_slices = 4

“MERGE SORT” ON GPU



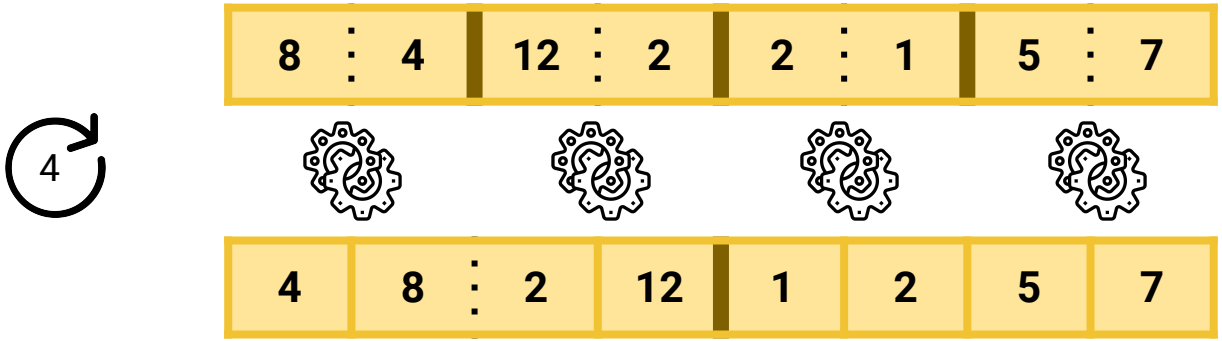
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



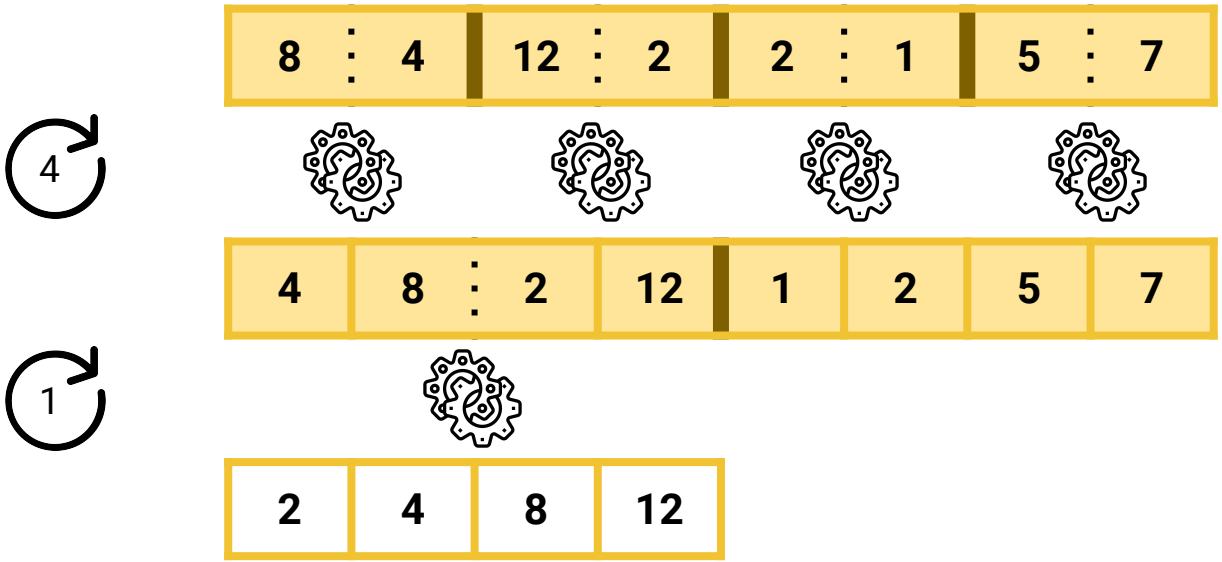
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



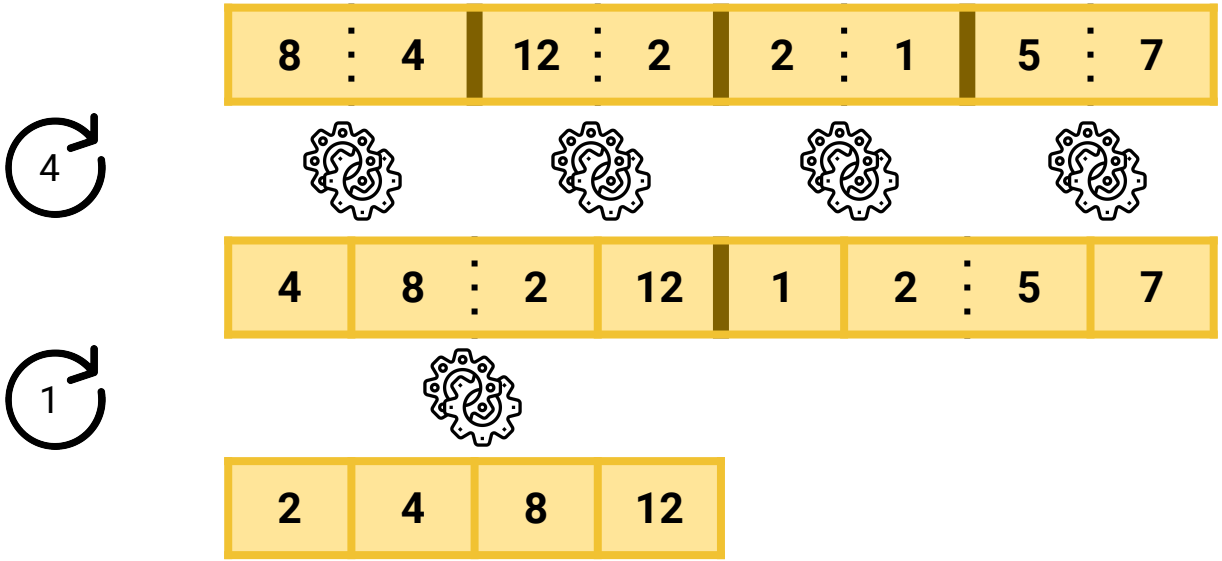
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



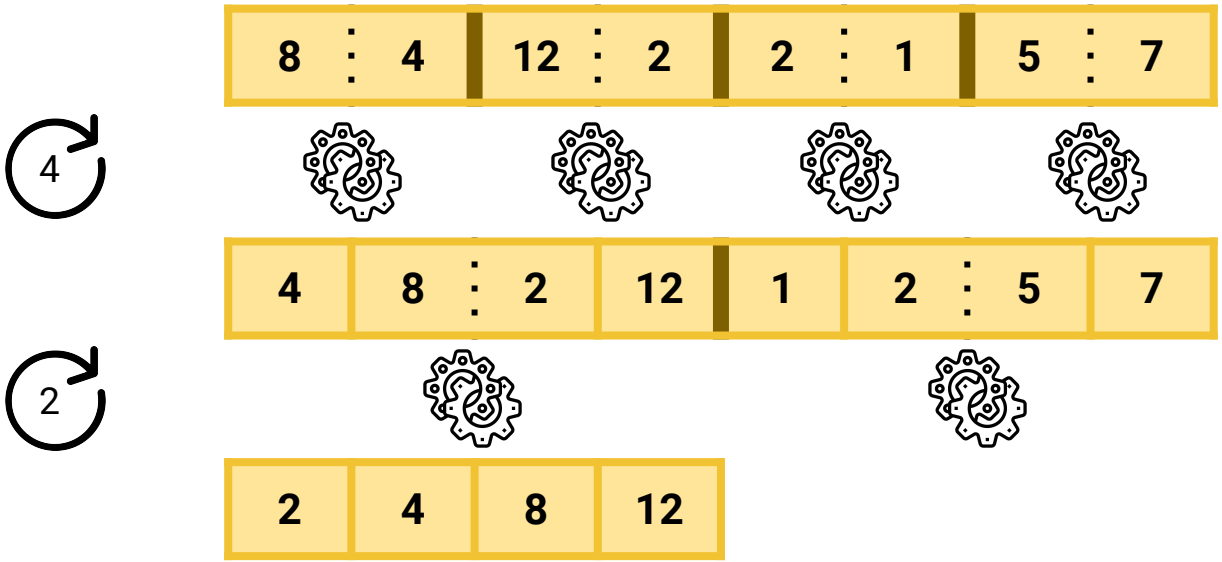
slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



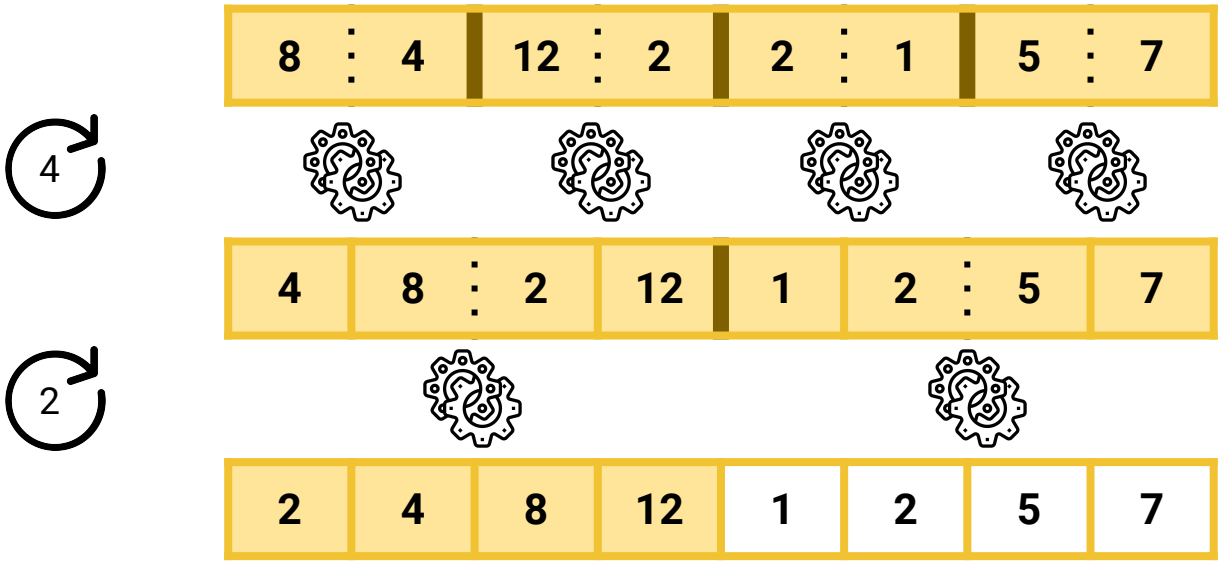
slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



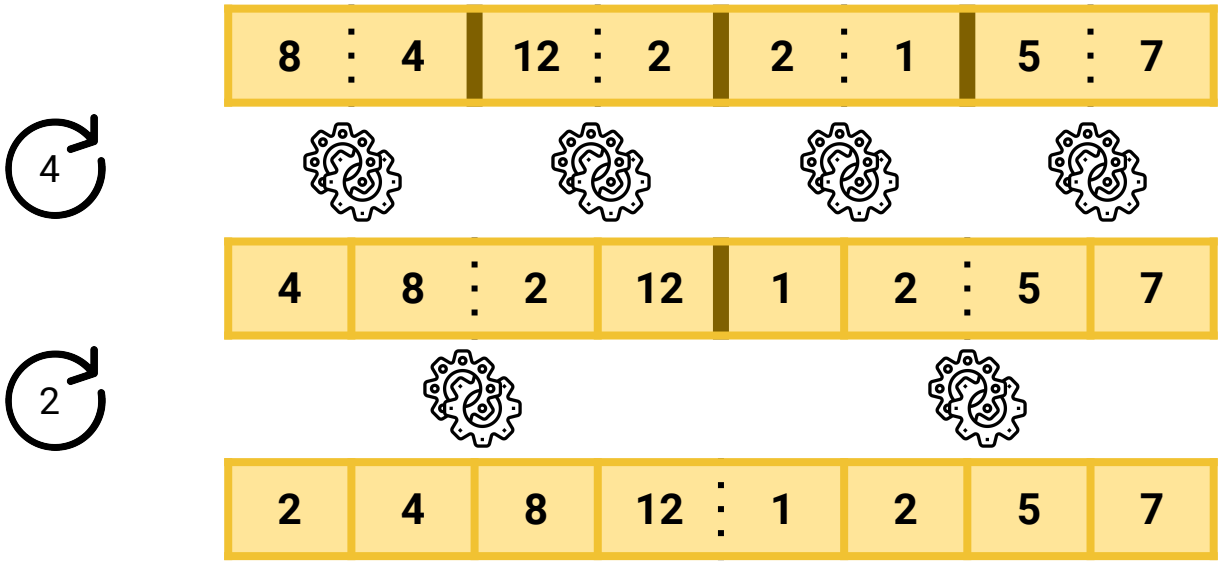
slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



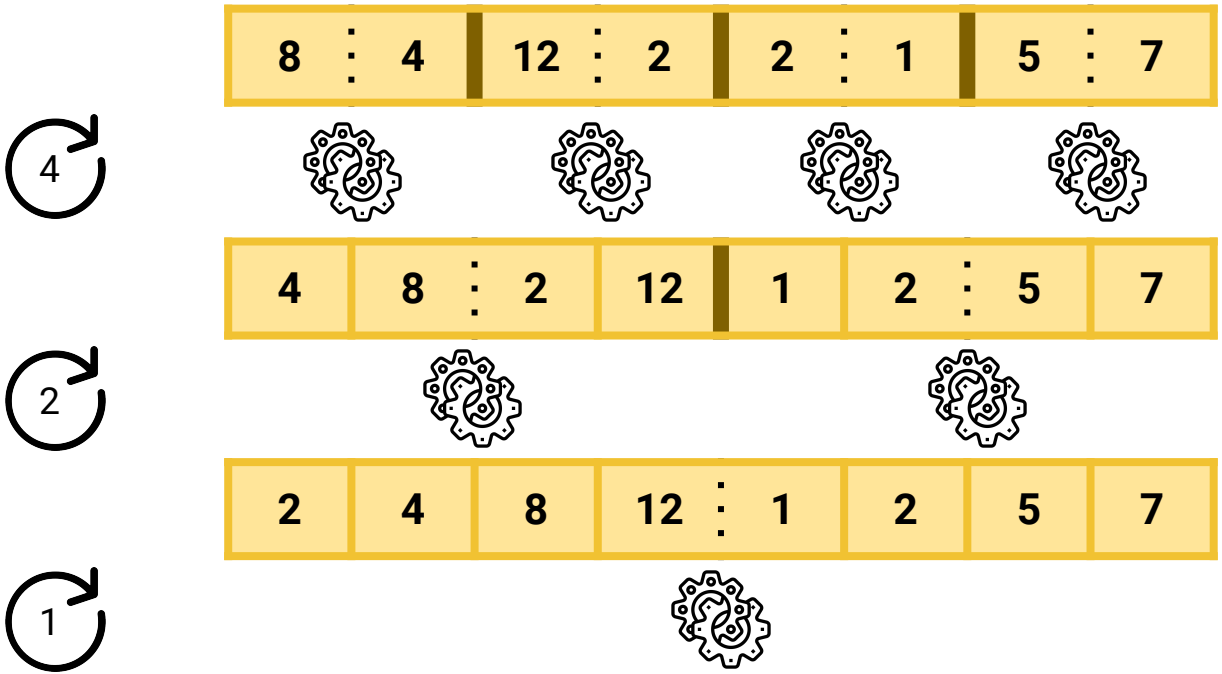
“MERGE SORT” ON GPU

slice_size = 8
number_of_slices = 1



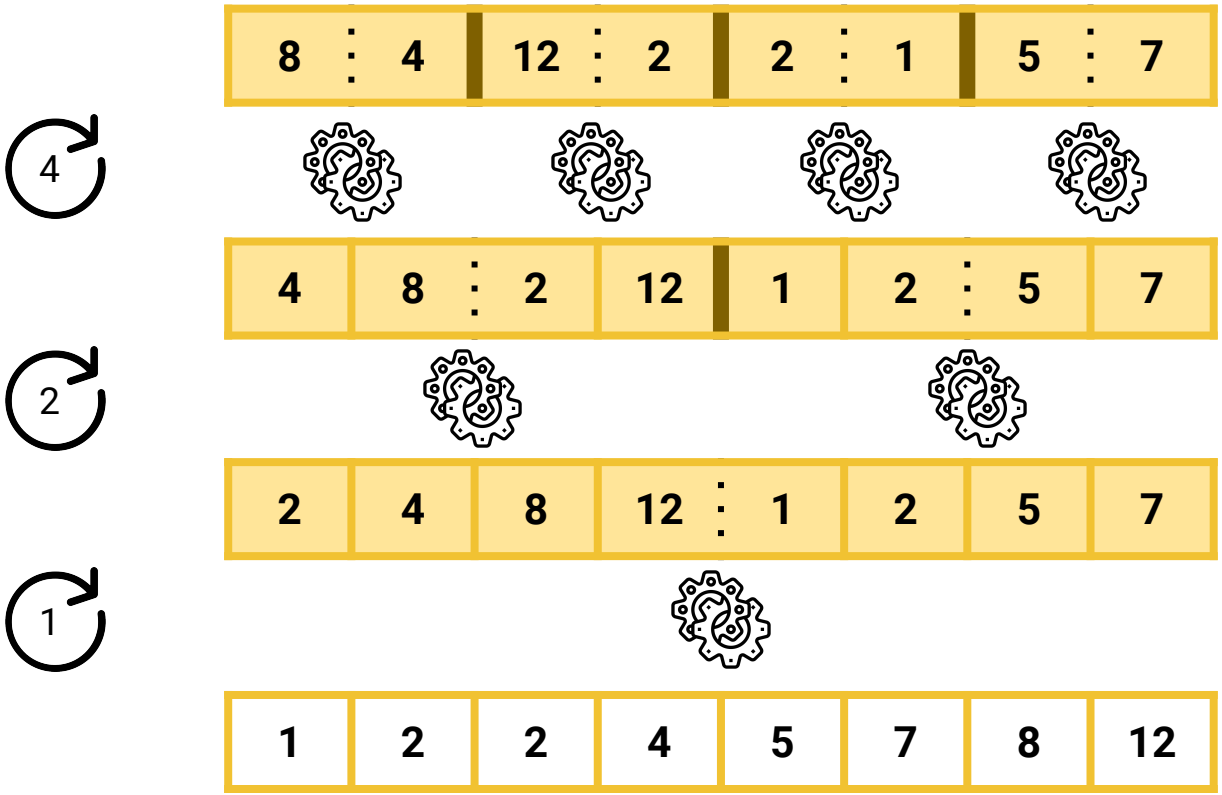
slice_size = 8
number_of_slices = 1

“MERGE SORT” ON GPU

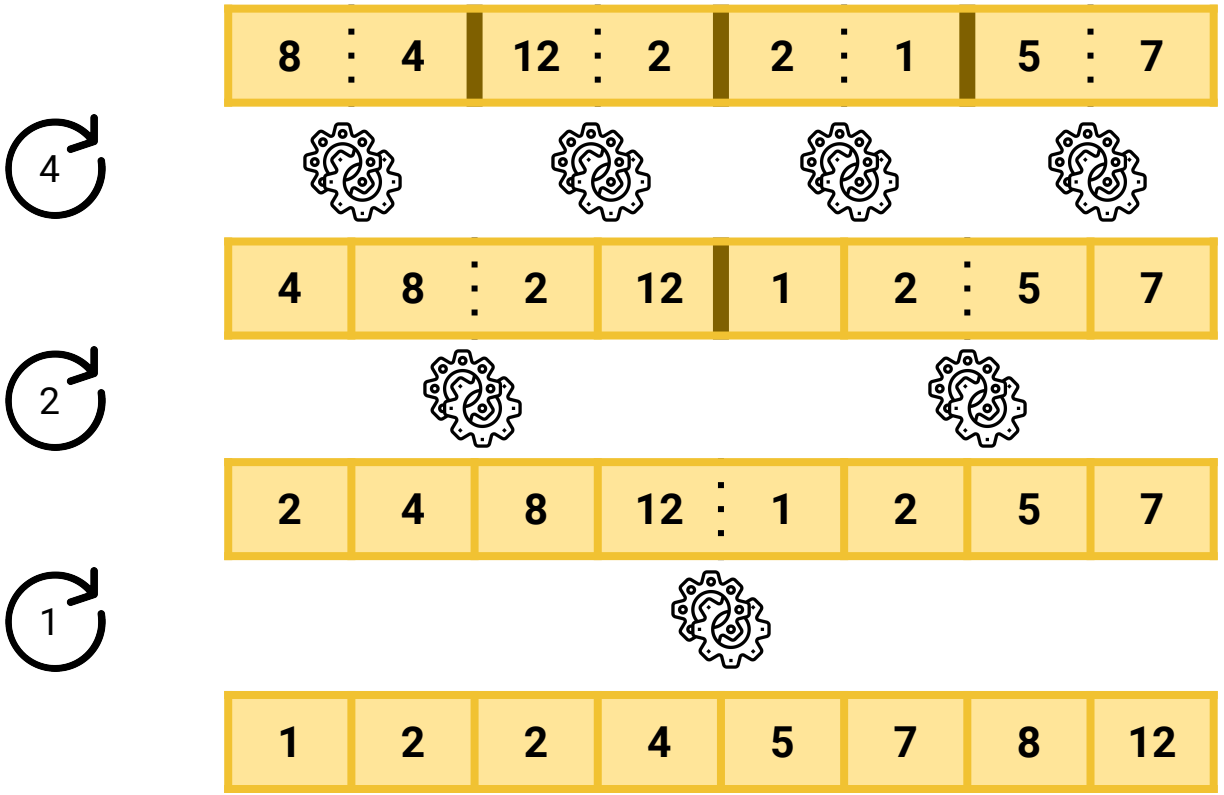


slice_size = 8
number_of_slices = 1

“MERGE SORT” ON GPU



“MERGE SORT” ON GPU



Ideal Case

Size of M is an exponent of 2

Every slices are full

More Often

M as any size

Some slices are not full

“MERGE SORT” ON GPU

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = ~~4~~

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**

Remainder of 1 element

DEFINITION : REMAINDER

We call “remainder” an incomplete slice in our cutting

DEFINITION : REMAINDER

We call “remainder” an incomplete slice in our cutting

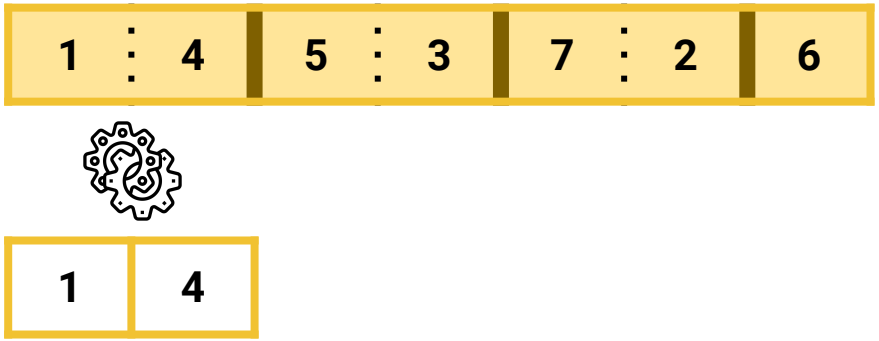
N.B : If there is a remainder, it’s always and only on the last slice

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**

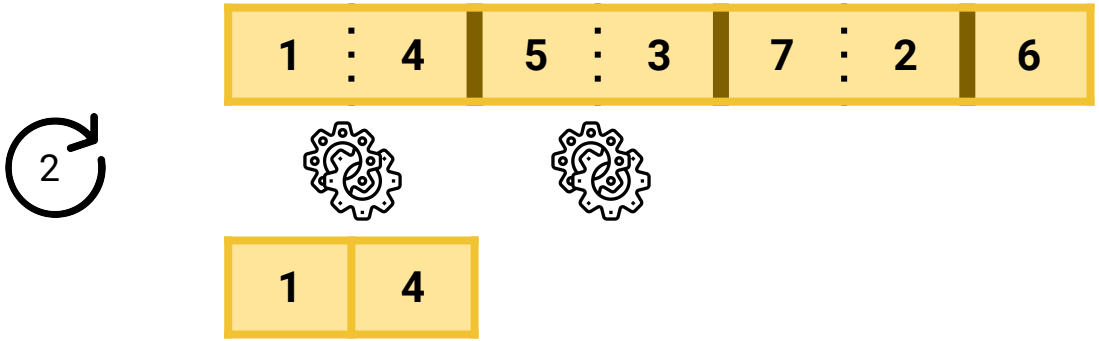
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**



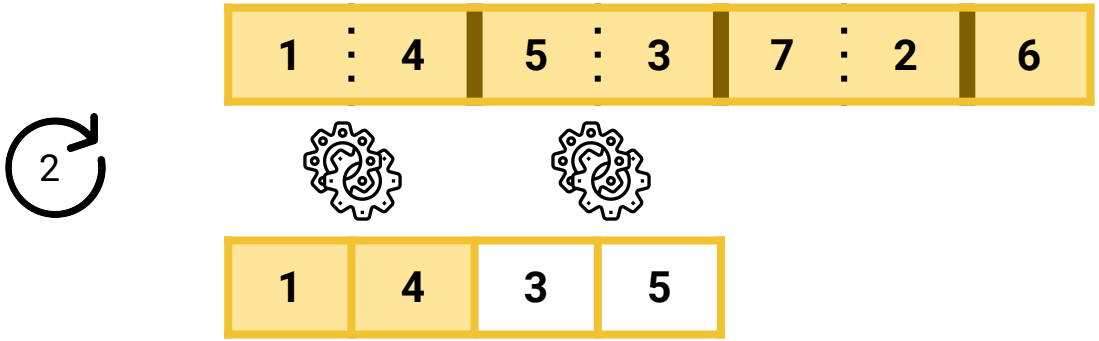
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**



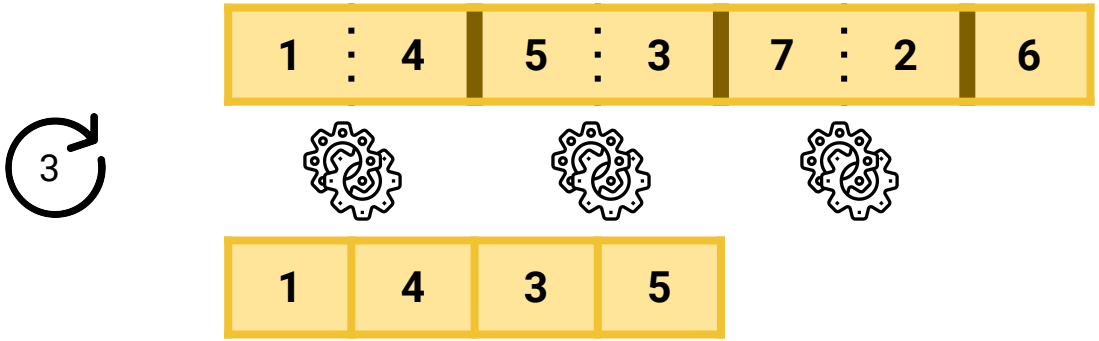
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**



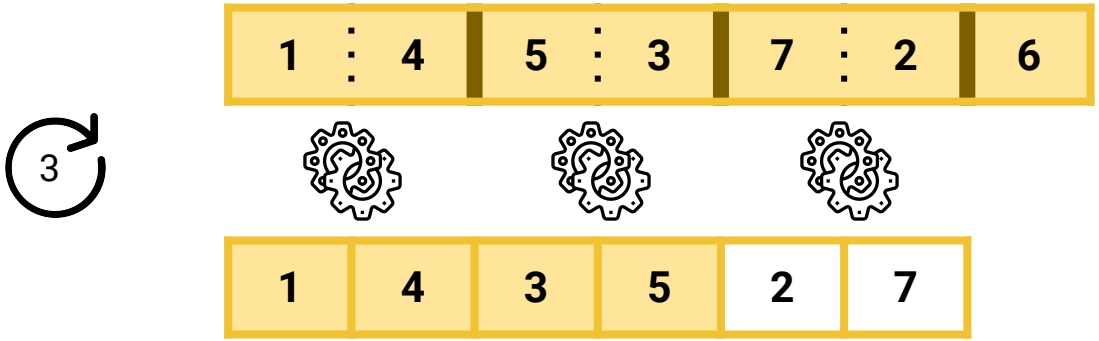
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**



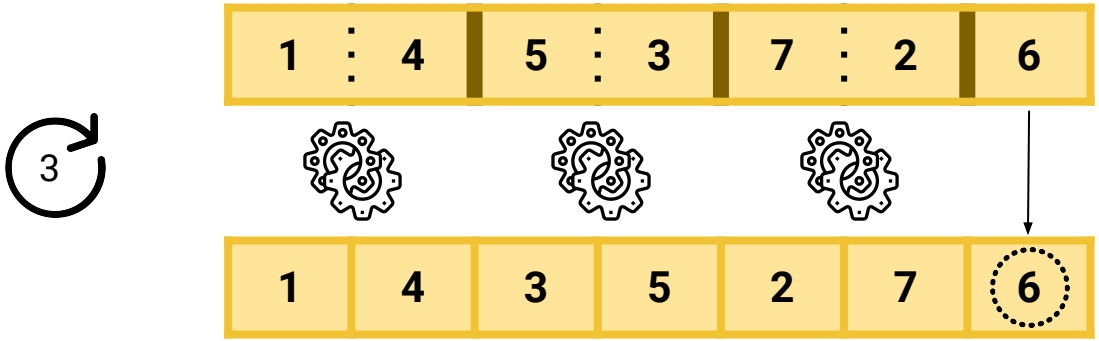
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**



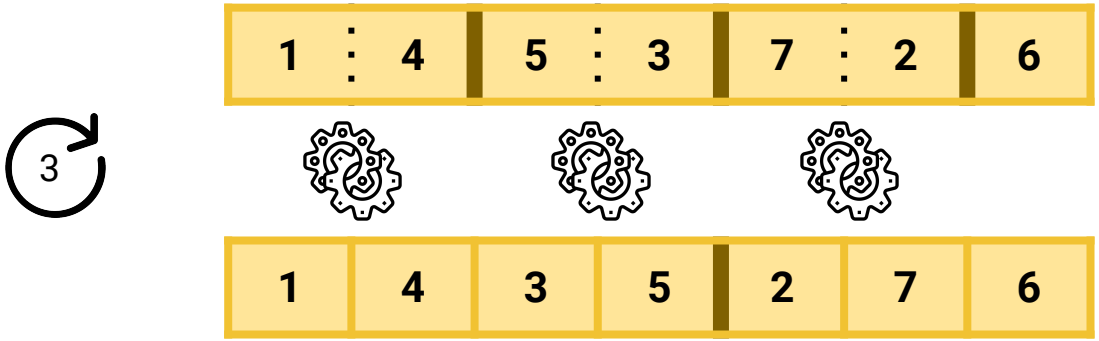
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = **3**



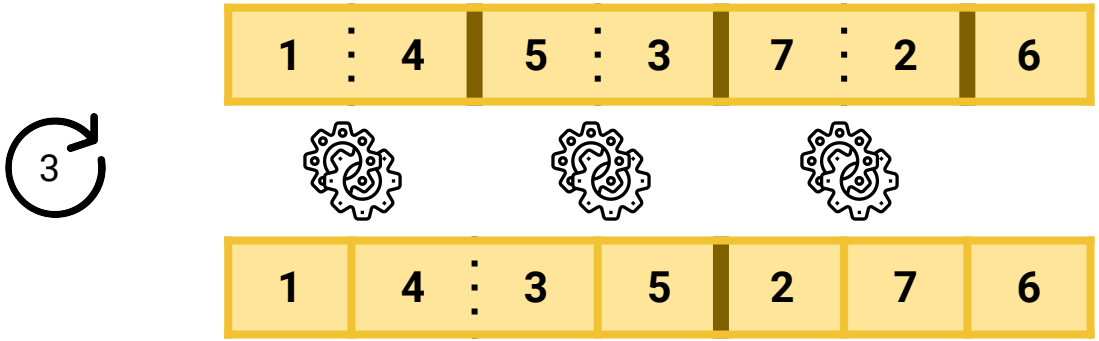
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



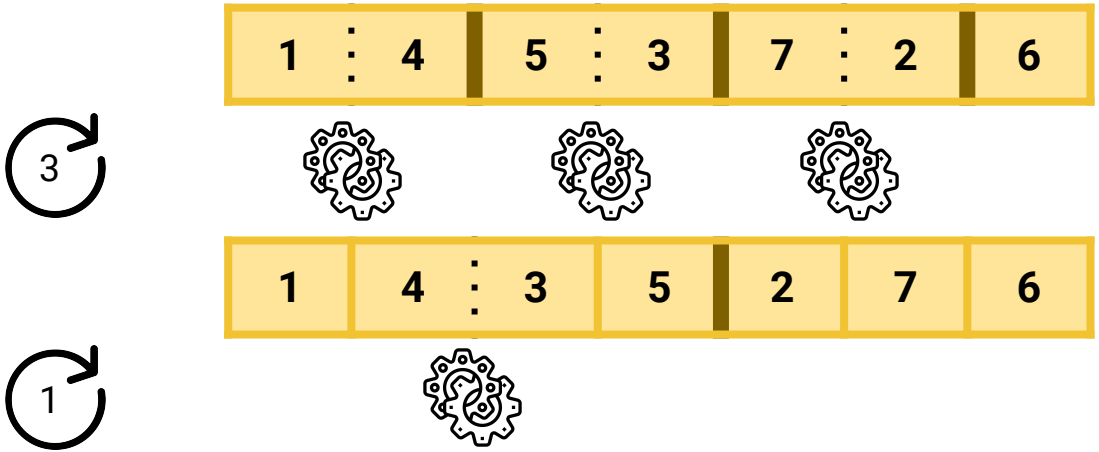
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = **1**



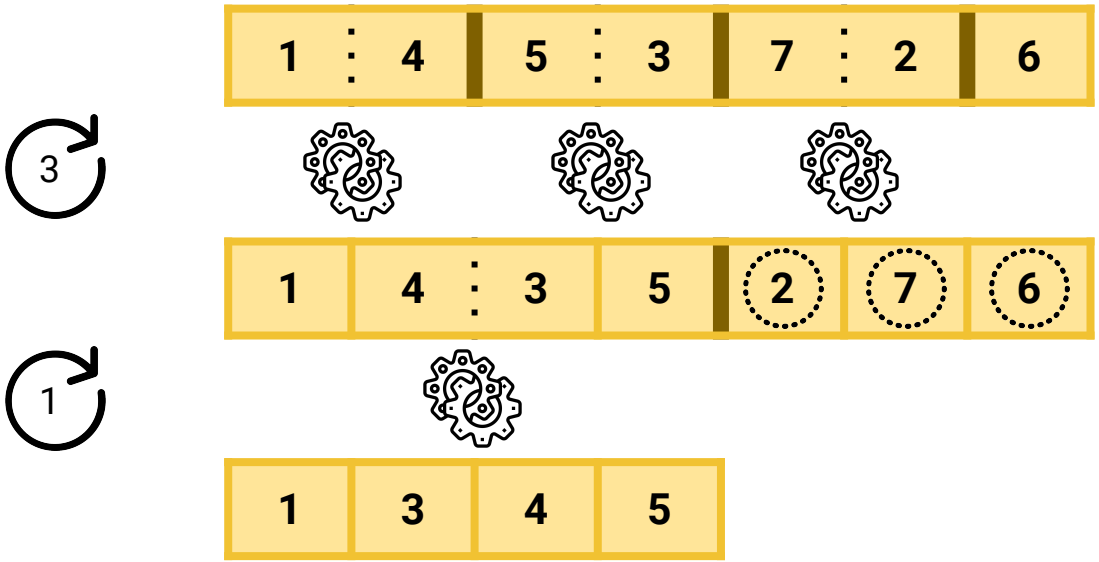
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = **1**



“MERGE SORT” ON GPU

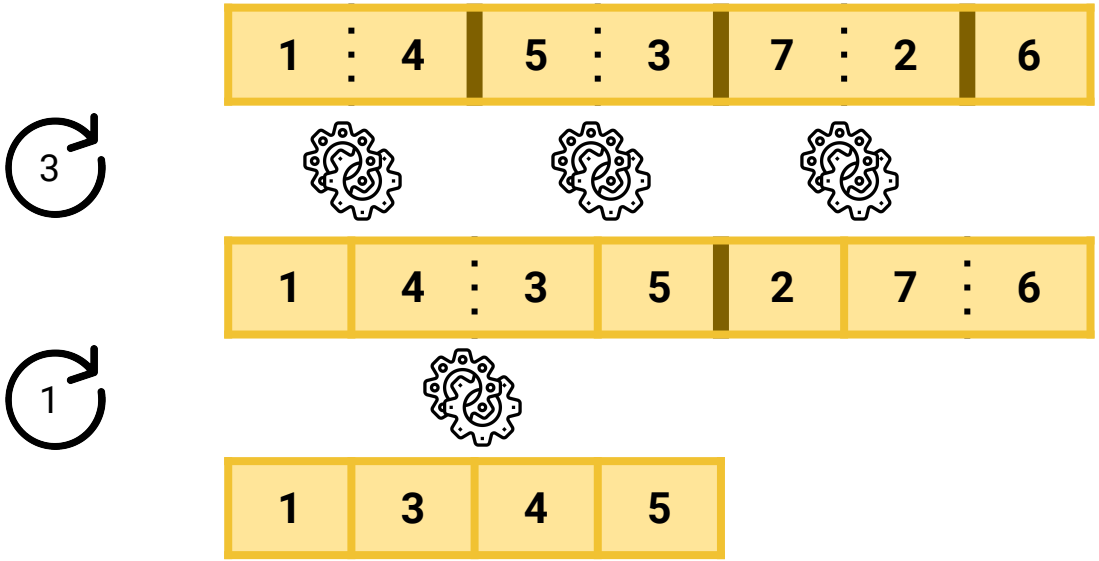
slice_size = 4
number_of_slices = **1**



Remainder of 3 elements

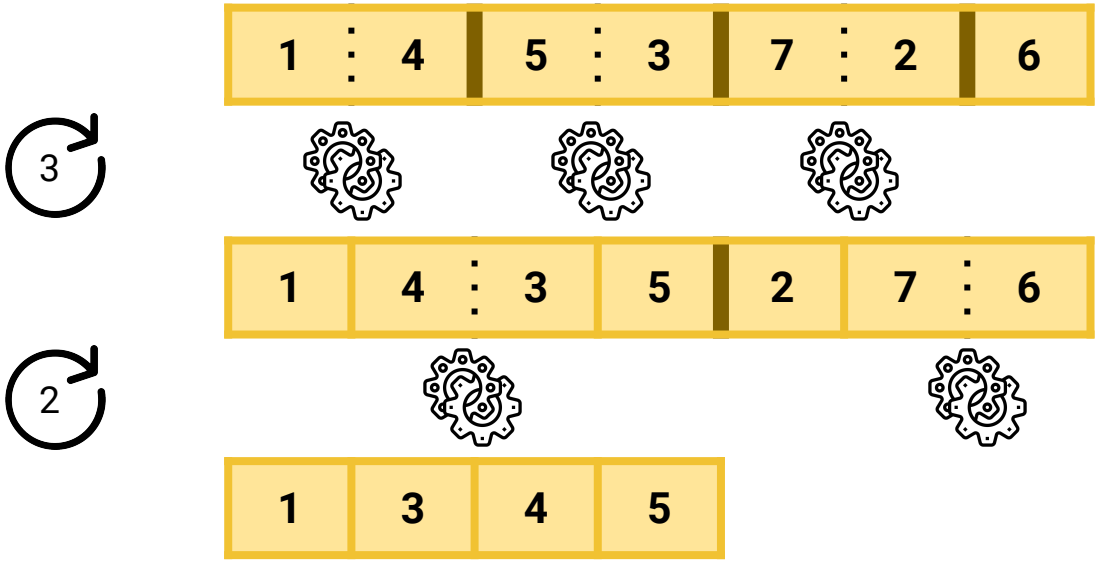
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = **1**



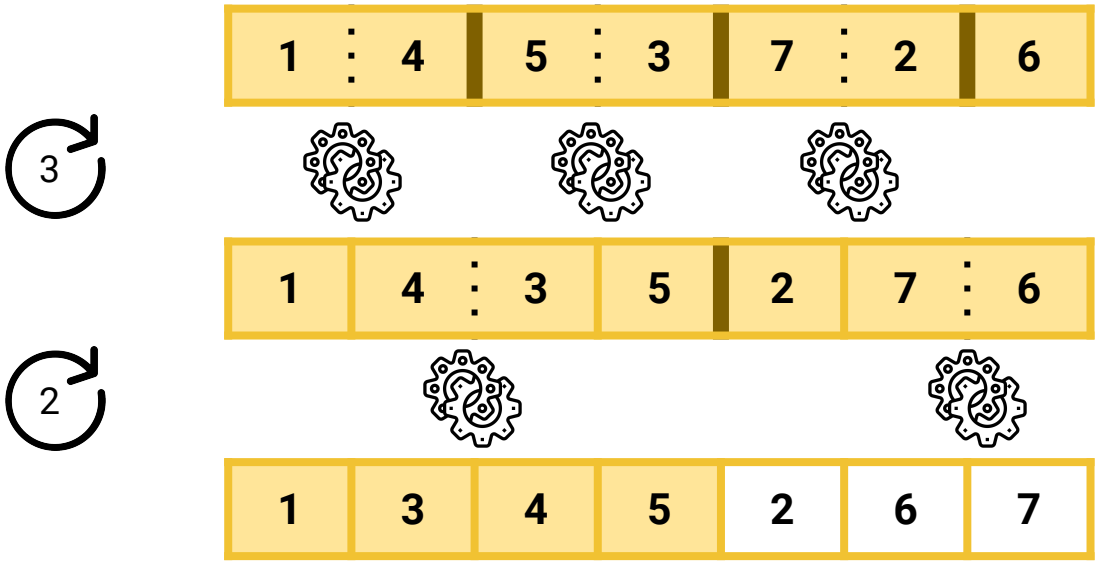
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = **1**

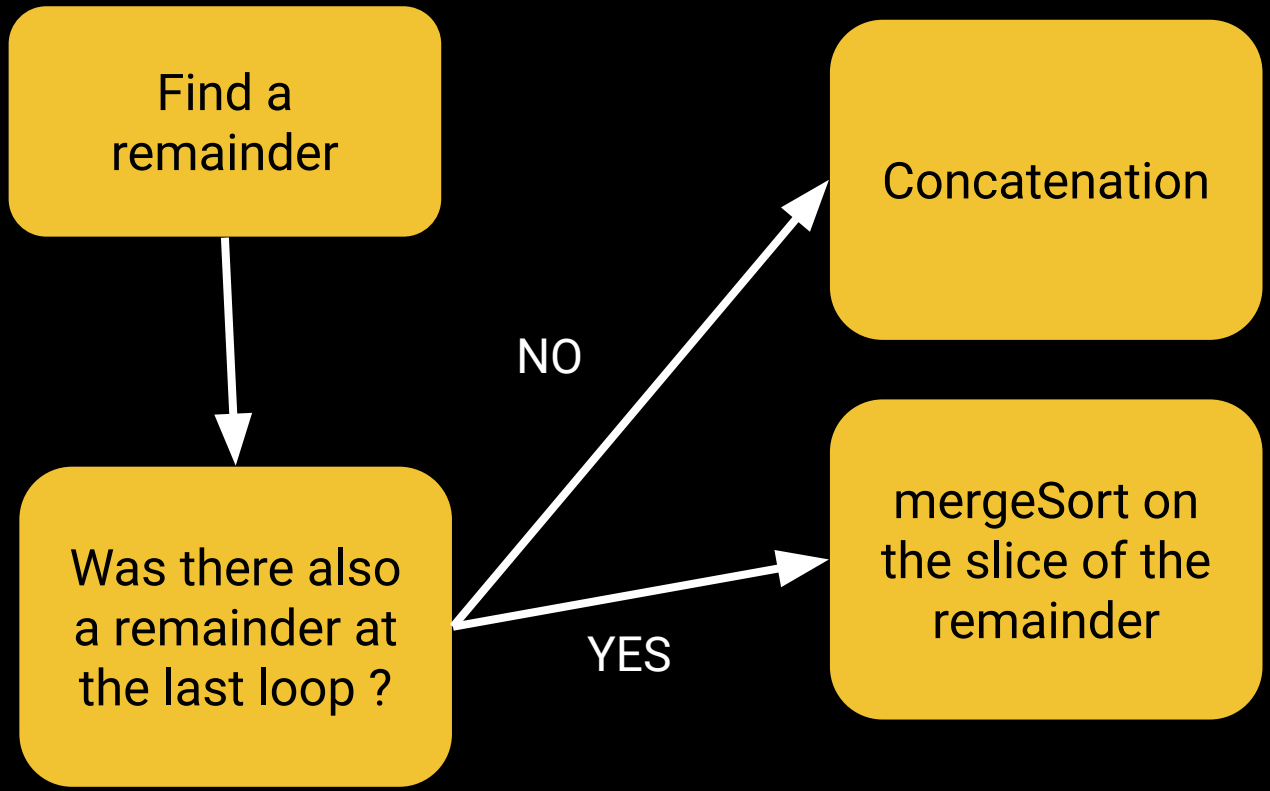


slice_size = 4
number_of_slices = **1**

“MERGE SORT” ON GPU

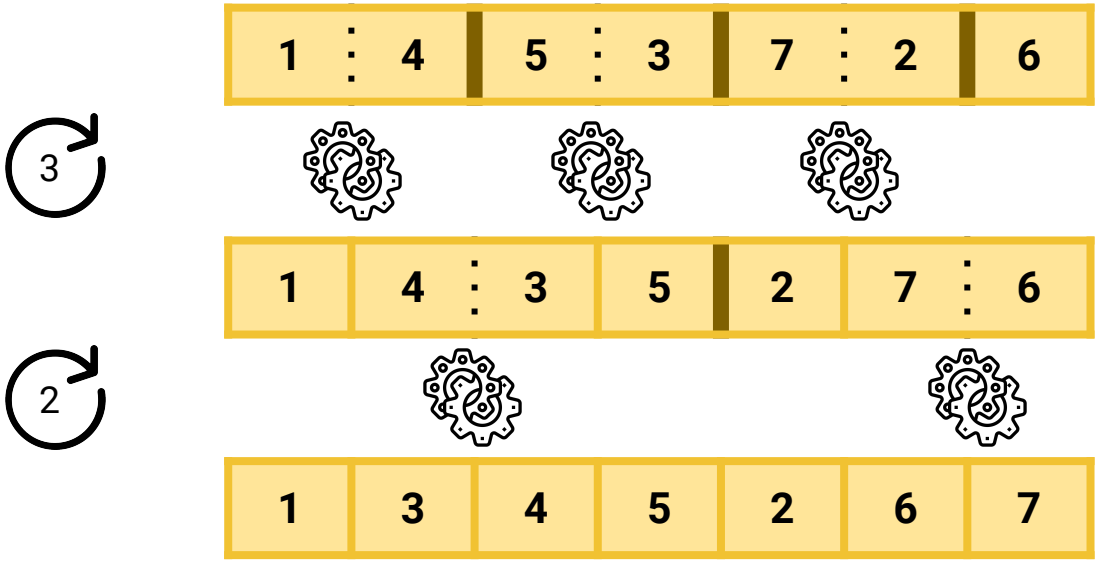


MANAGING THE REMAINDERS



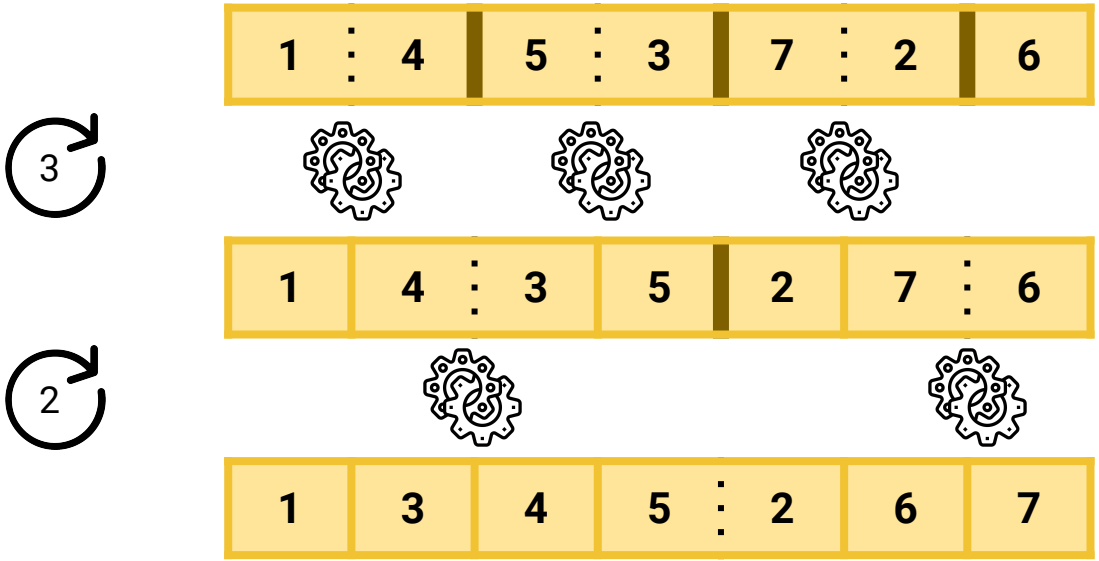
“MERGE SORT” ON GPU

slice_size = 8
number_of_slices =



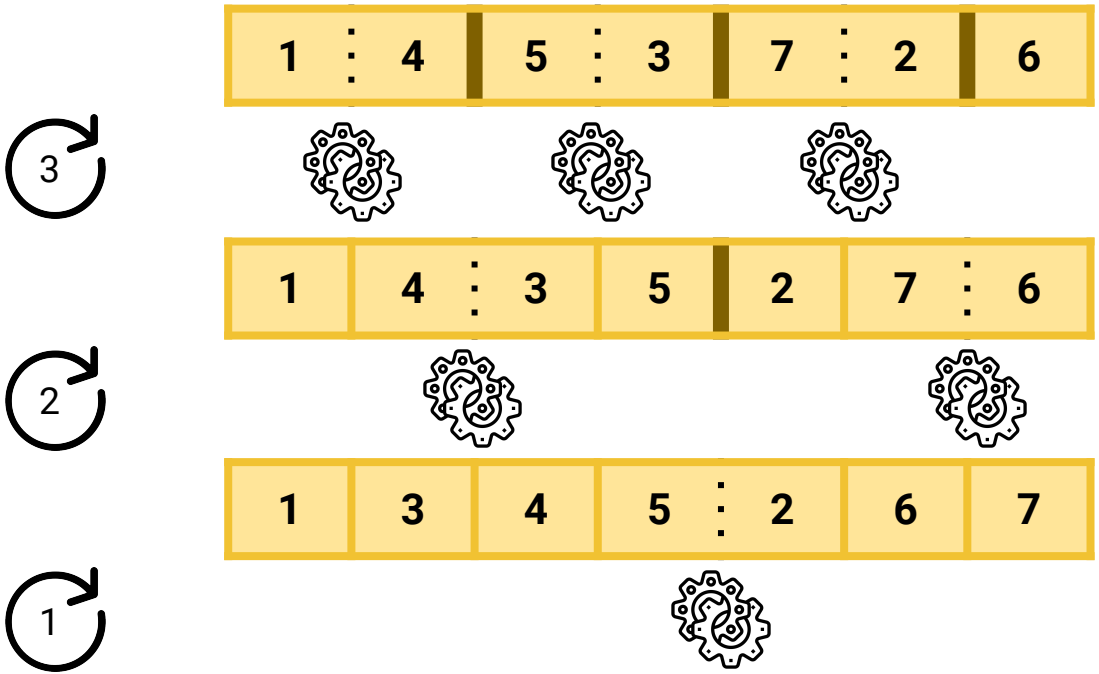
“MERGE SORT” ON GPU

slice_size = 8
number_of_slices = **1**



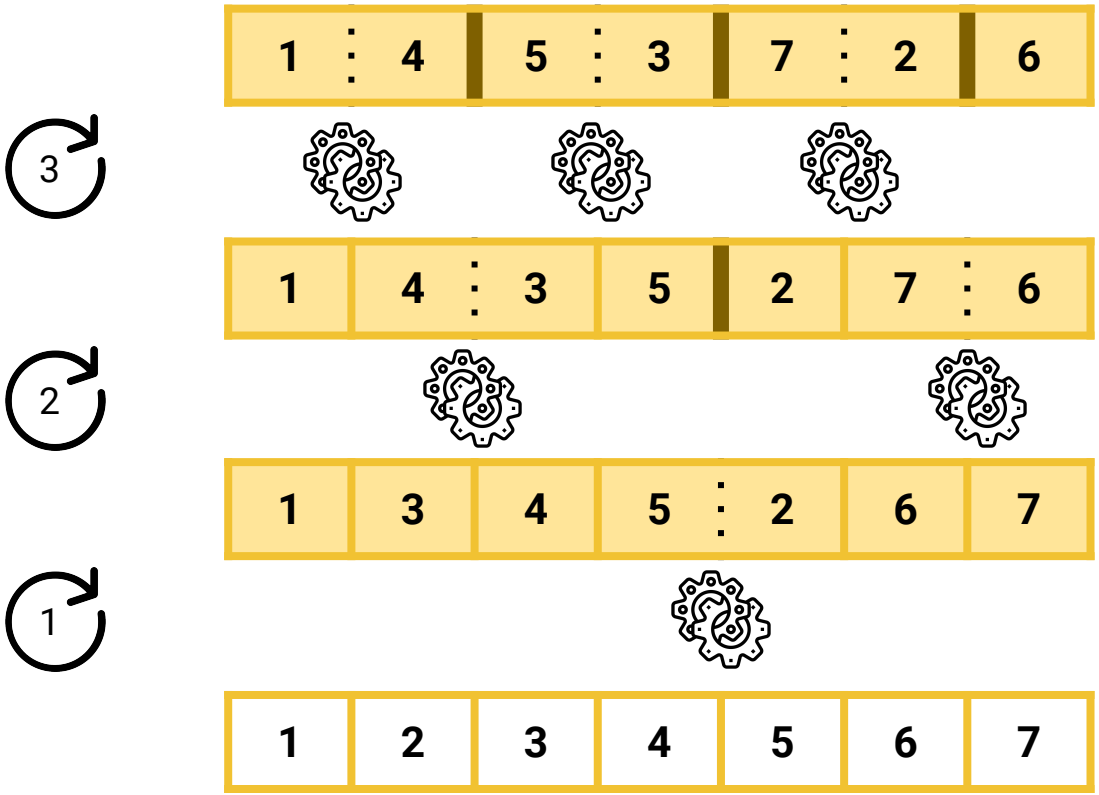
slice_size = 8
number_of_slices = **1**

“MERGE SORT” ON GPU

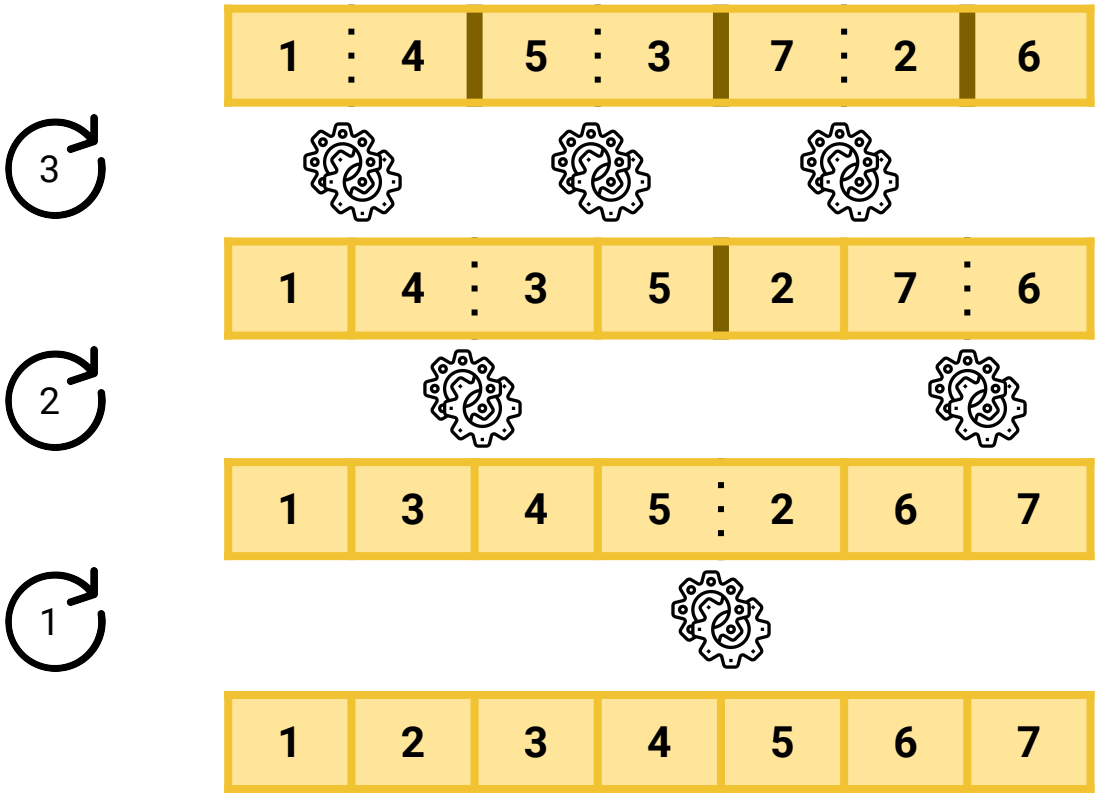


slice_size = 8
number_of_slices = **1**

“MERGE SORT” ON GPU



“MERGE SORT” ON GPU



“MERGE SORT” ON GPU

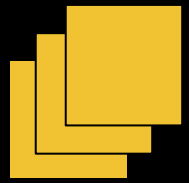
```
slice_size = 1
number_of_slices = 1
reste_prec_size = 0
reste_size = 0
```

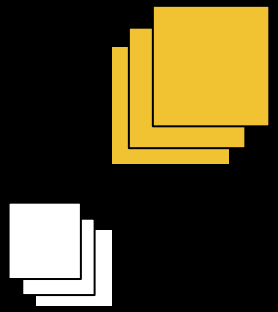
```
while (number_of_slices > 0)
        slice_size = 2*slice_size
        number_of_slices = size_M/slice_size
        reste_prec_size = reste_size
        reste_size = size_M%slice_size

        for (i=0, i<number_of_slices, i++)
                sort(M, slice_size, i)

        if (reste_prec_size!=0 && reste_size!=0)

                sort(M, reste_size + reste_prec_size, number_of_slices)
```



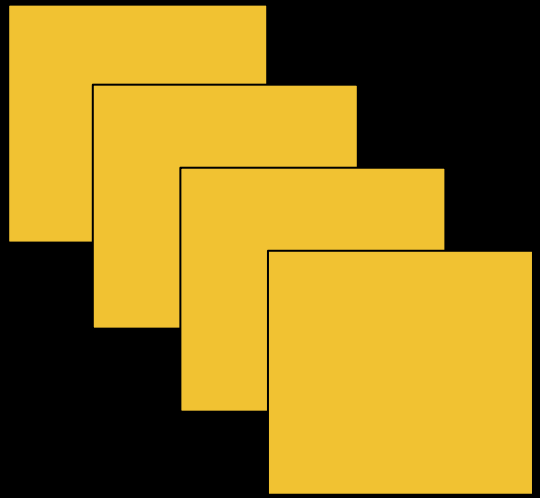


04

SORT WITH BATCHES

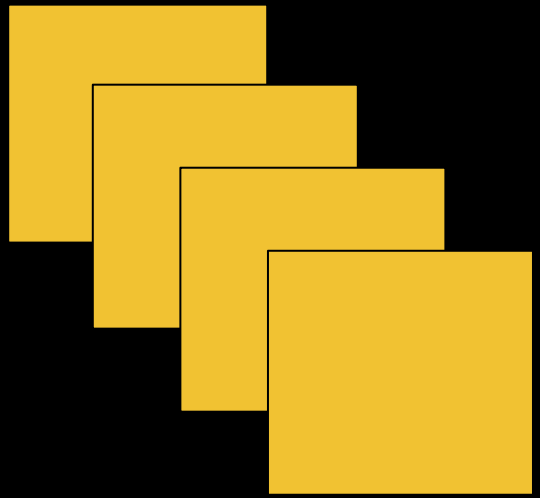
DEFINITION : BATCH

Sub array with at the most 1024 elements

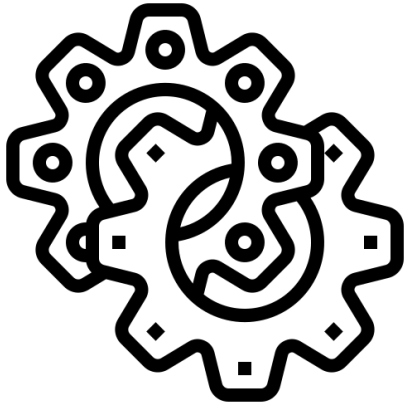


DEFINITION : BATCH

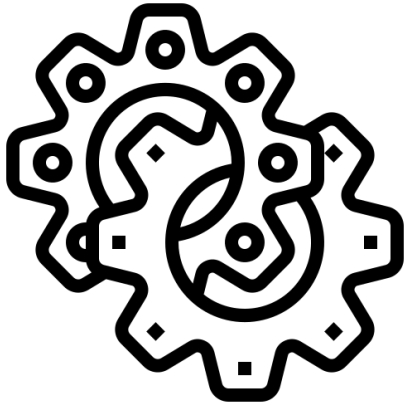
Sub array with at the most 1024 elements
(ie the maximum number of threads on one block)



NOTATIONS

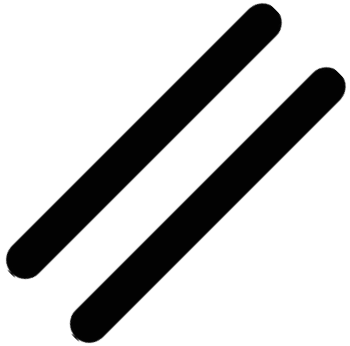


NOTATIONS

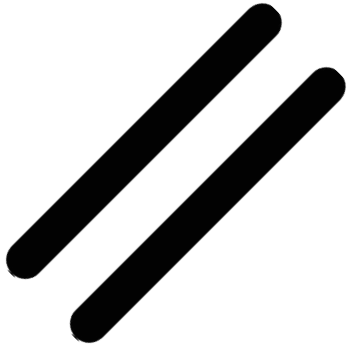


mergeSmallBatch_k(A,B)

NOTATIONS



NOTATIONS



In parallel

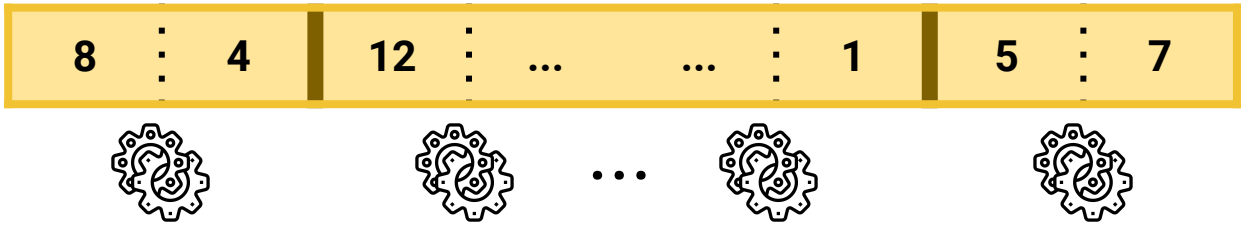
SORT ON ONE BATCH

SORT ON ONE BATCH

SORT ON ONE BATCH

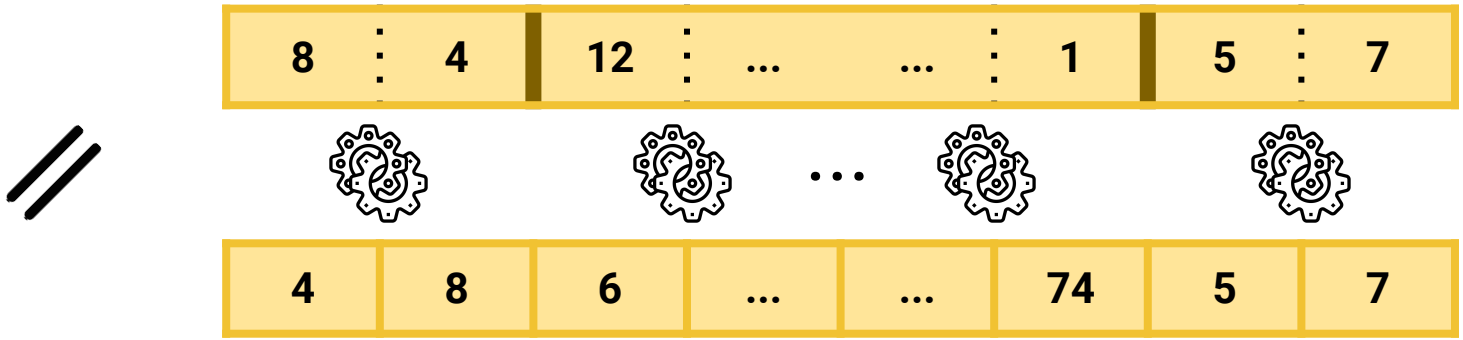
SORT ON ONE BATCH

SORT ON ONE BATCH

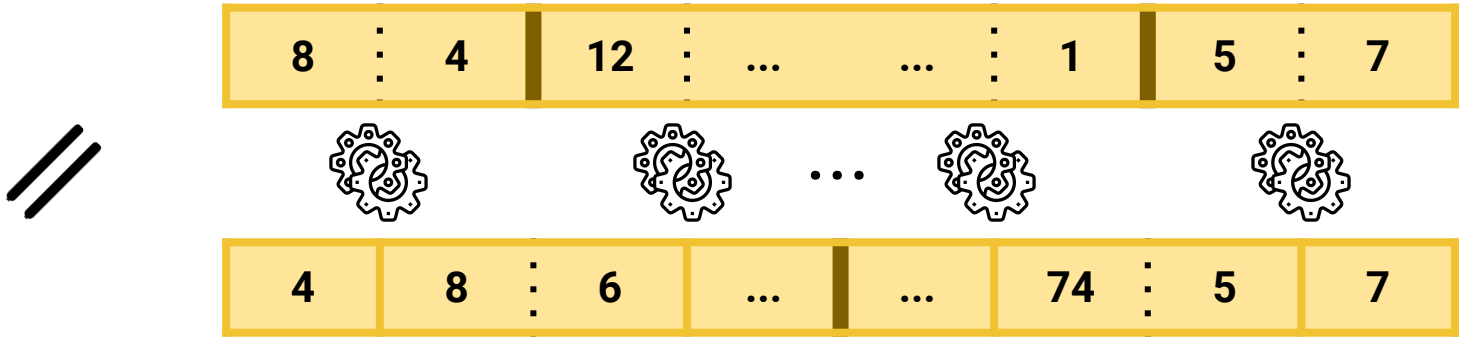


SORT ON ONE BATCH

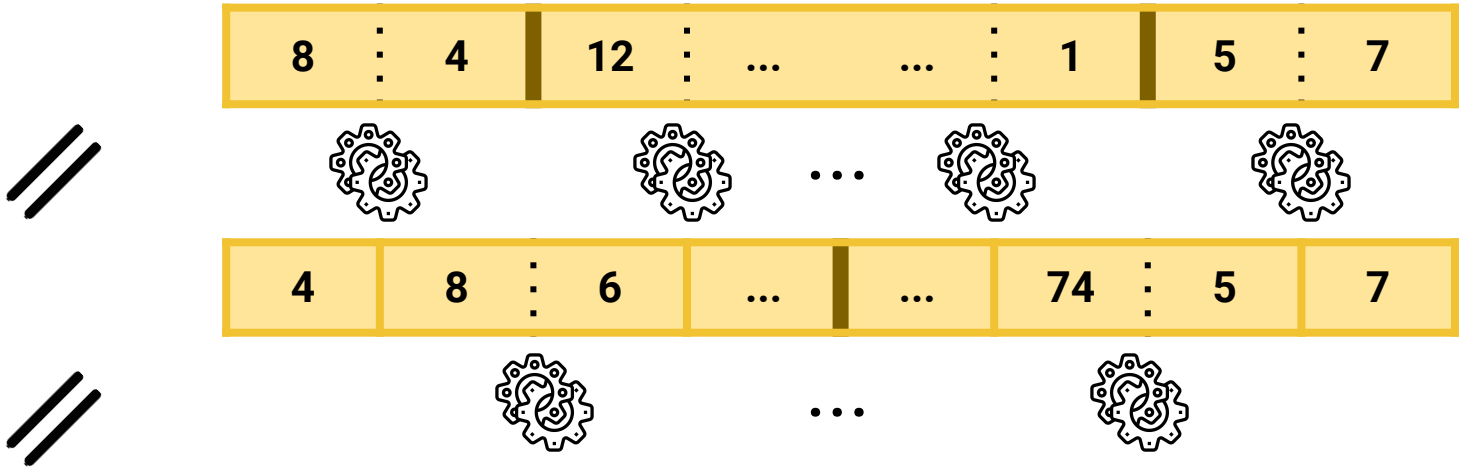
SORT ON ONE BATCH



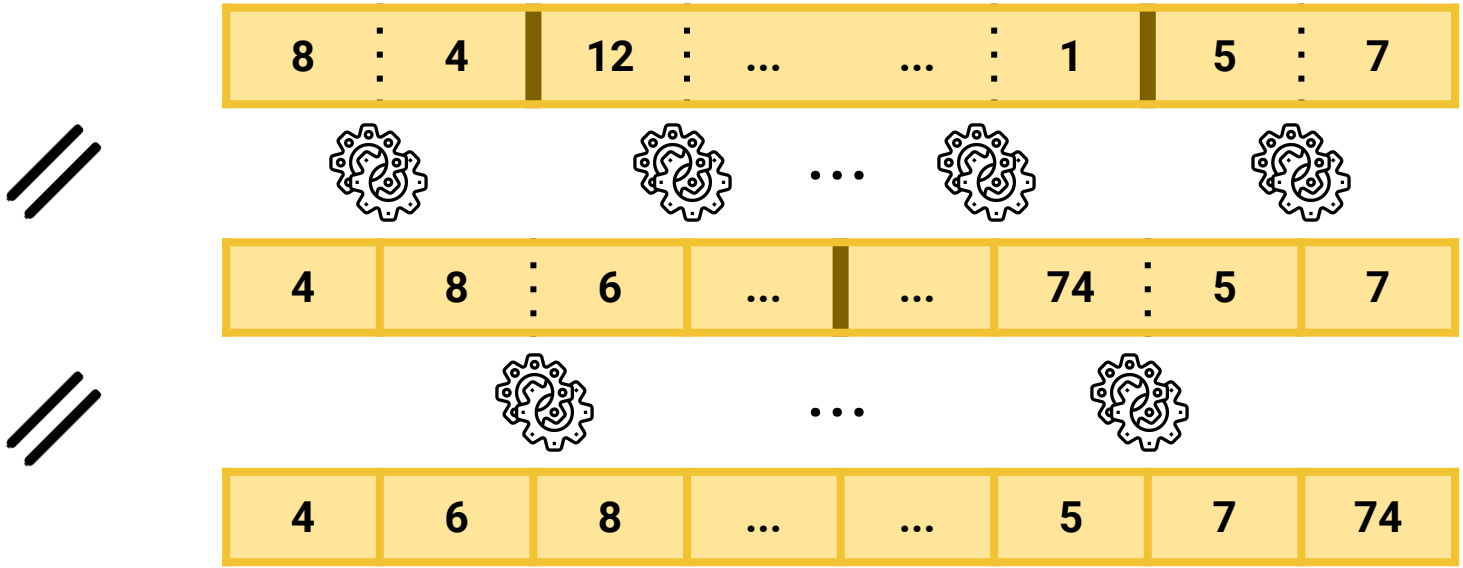
SORT ON ONE BATCH



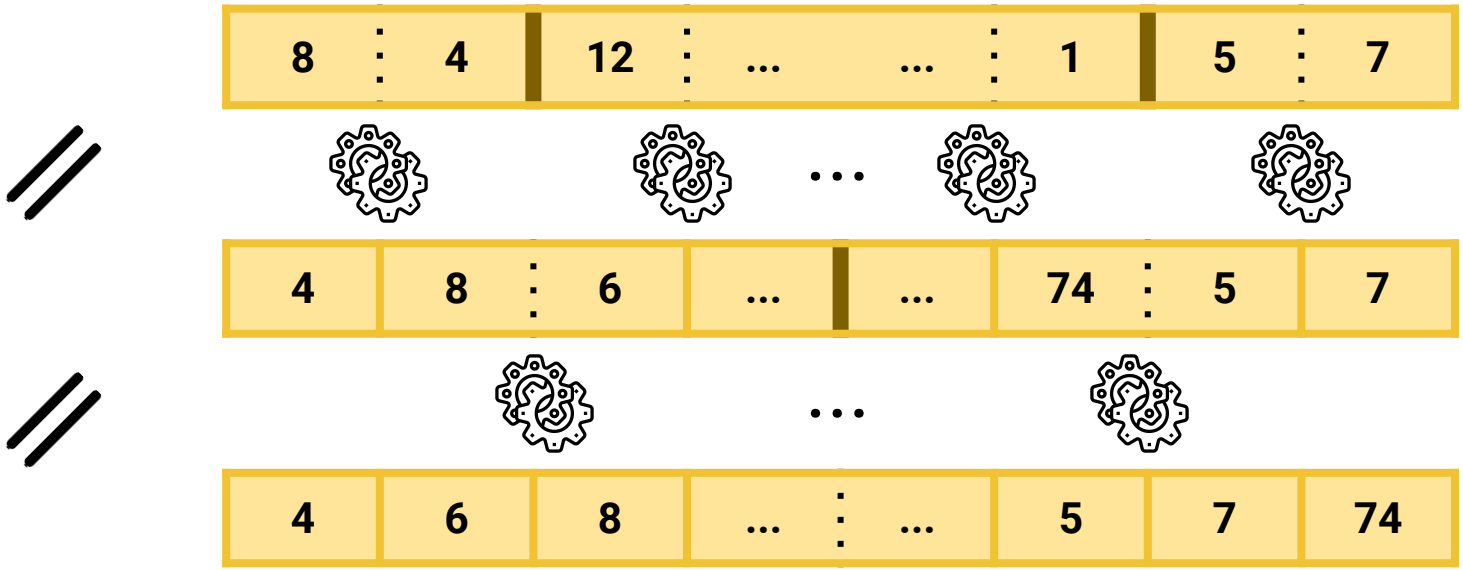
SORT ON ONE BATCH



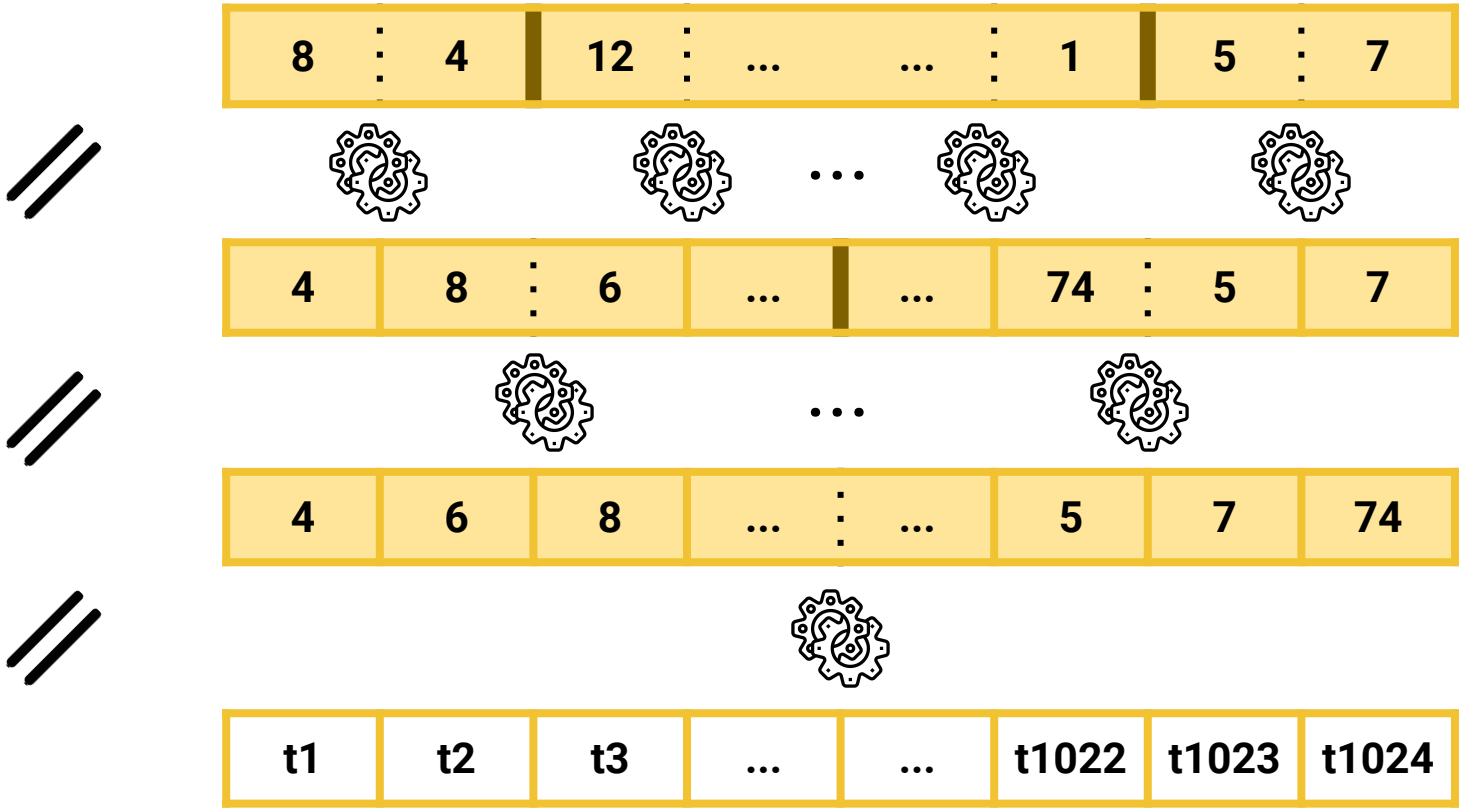
SORT ON ONE BATCH



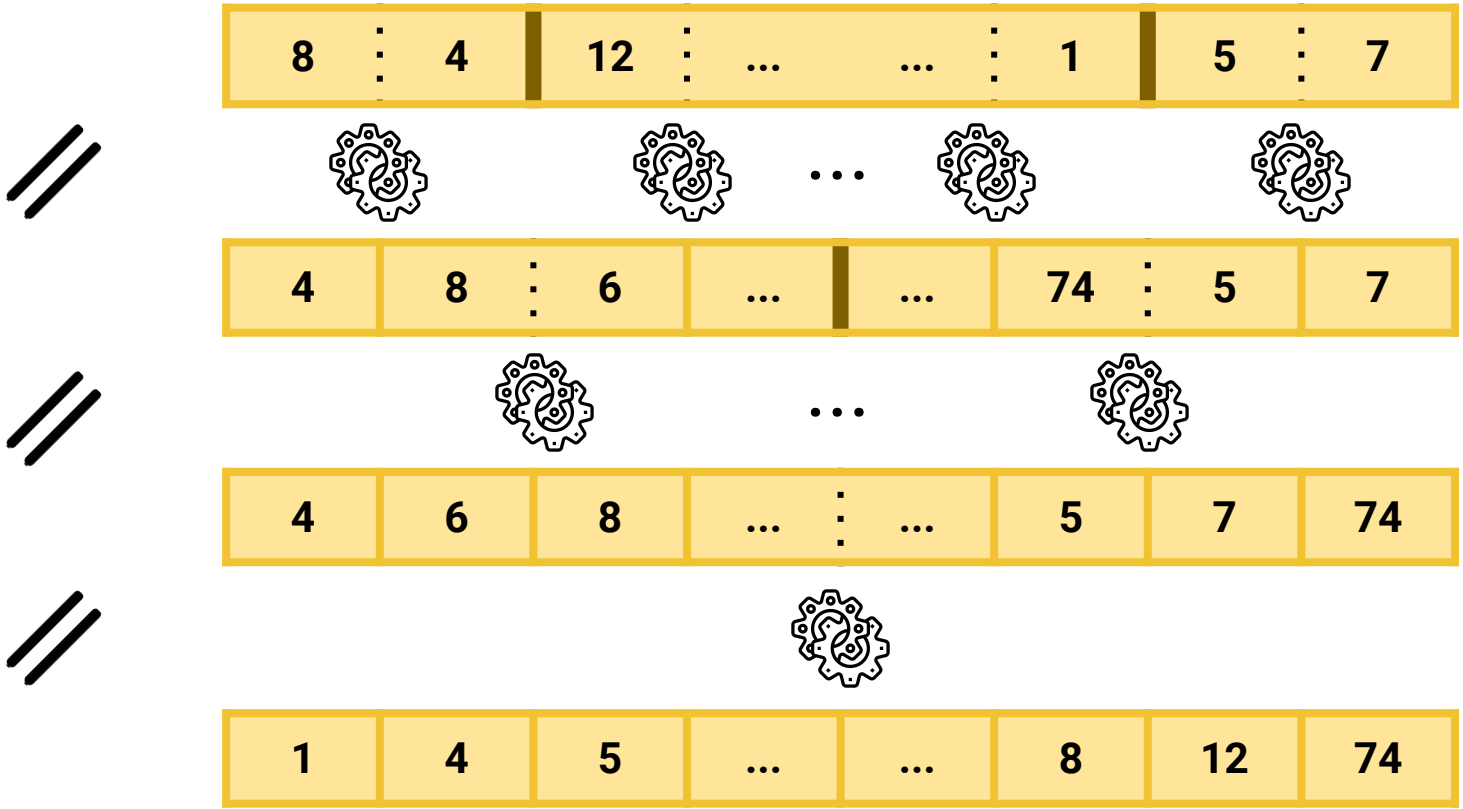
SORT ON ONE BATCH



SORT ON ONE BATCH



SORT ON ONE BATCH



SMALL SORT

M

SMALL SORT

M

Batches of 1024 elements

SMALL SORT

M

Batches of 1024 elements

SMALL SORT

M

...	...	...	4	...	...	...	26	...	...	...	17	...	...	...	2

Batches of 1024 elements

SMALL SORT

M

...	...	...	4	...	...	...	26	...	...	...	17	...	...	...	2

Batches of 1024 elements

SMALL SORT

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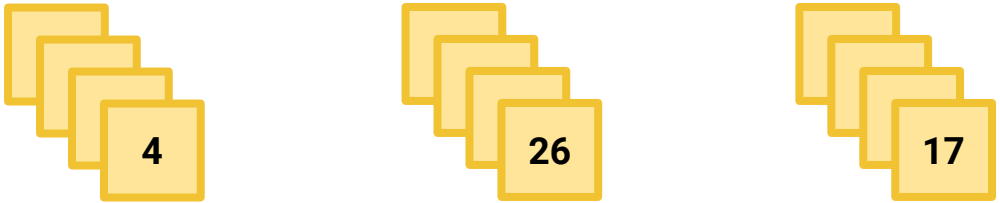
Batches of 1024 elements



SMALL SORT

M

...	...	...	4	...	...	...	26	...	...	...	17	...	...	...	2

Batches of 1024 elements

SMALL SORT

M

Batches of 1024 elements

We sort each batch in parallel

SMALL SORT

M

Batches of 1024 elements

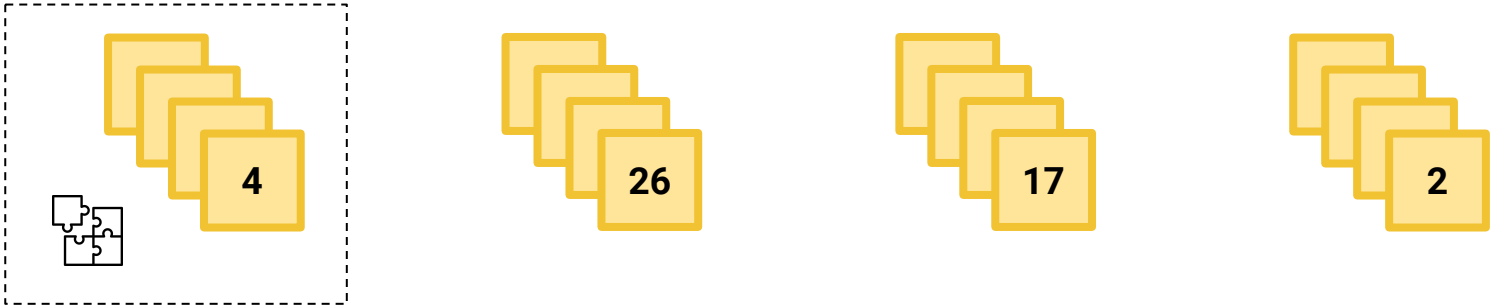
We sort each batch in parallel on one block

SMALL SORT

M

...	...	...	4	...	...	...	26	...	...	...	17	...	...	...	2

Batches of 1024 elements



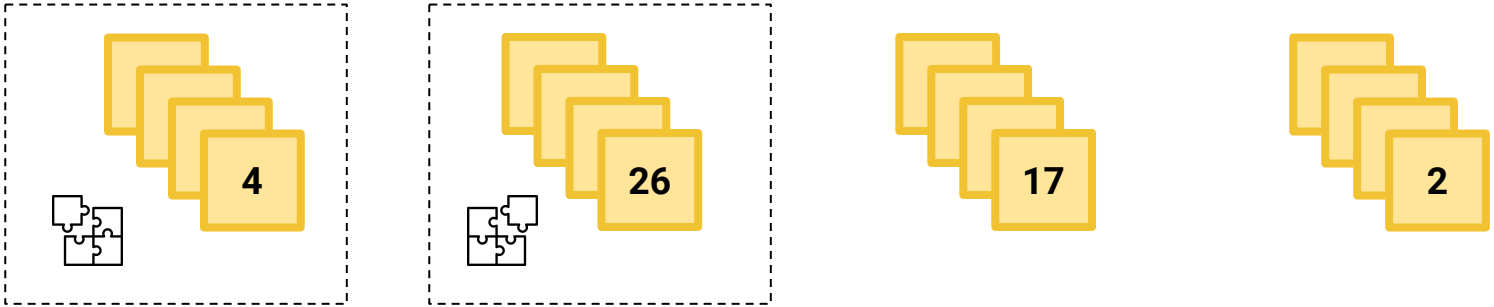
We sort each batch in parallel on one block

SMALL SORT

M

...	...	...	4	...	...	...	26	...	...	...	17	...	...	...	2

Batches of 1024 elements



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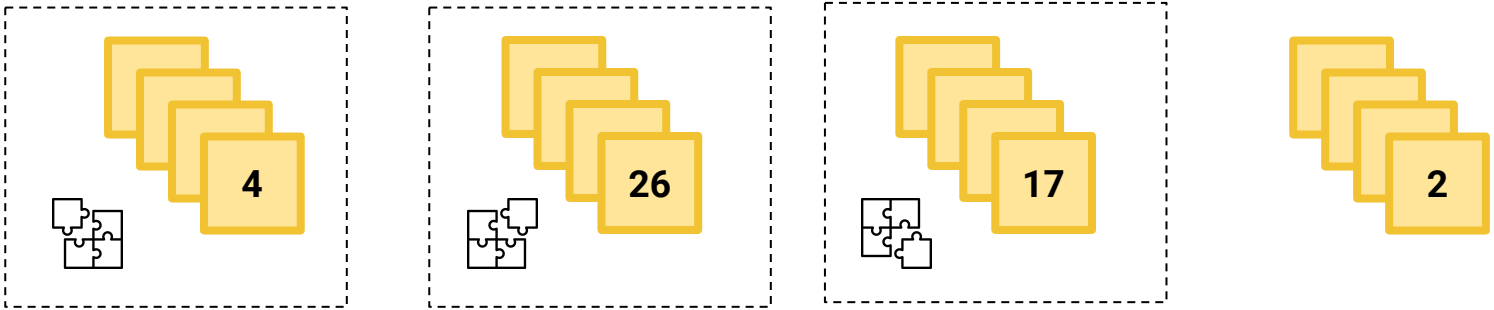
SMALL SORT

M

Batches of 1024 elements



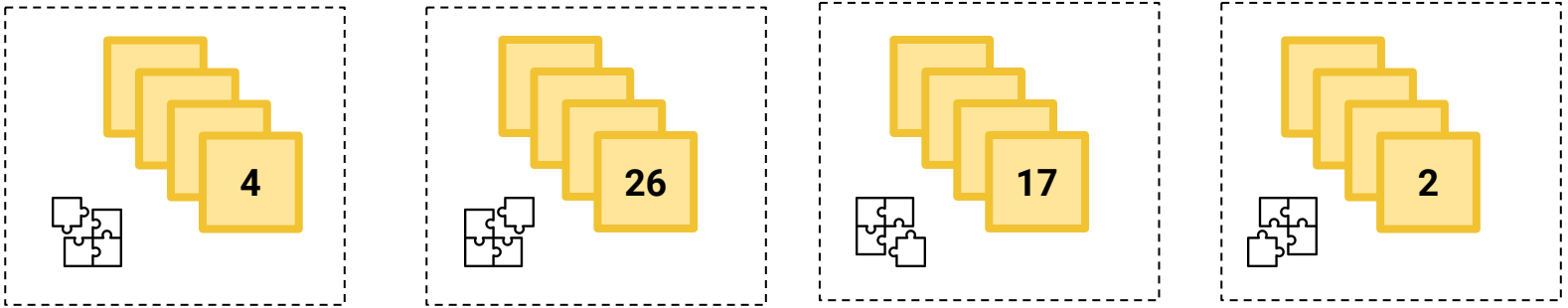
We sort each batch in parallel on one block

SMALL SORT

M

...	...	...	4	...	...	...	26	...	...	...	17	...	...	...	2

Batches of 1024 elements



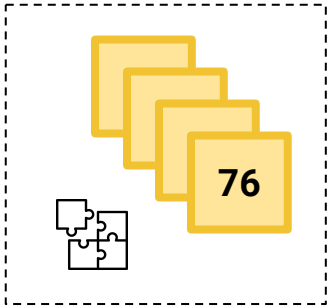
We sort each batch in parallel on one block

BIG SORT

Batches of 1024 sorted elements

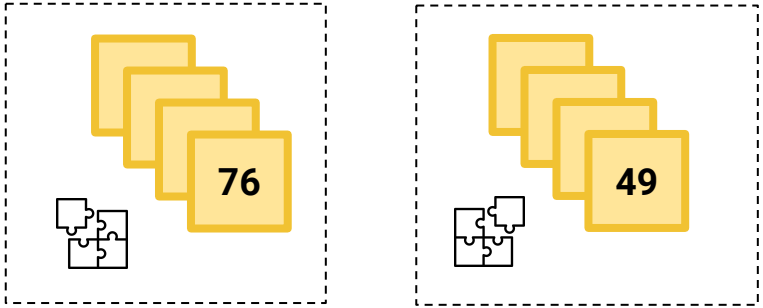
BIG SORT

Batches of 1024 sorted elements



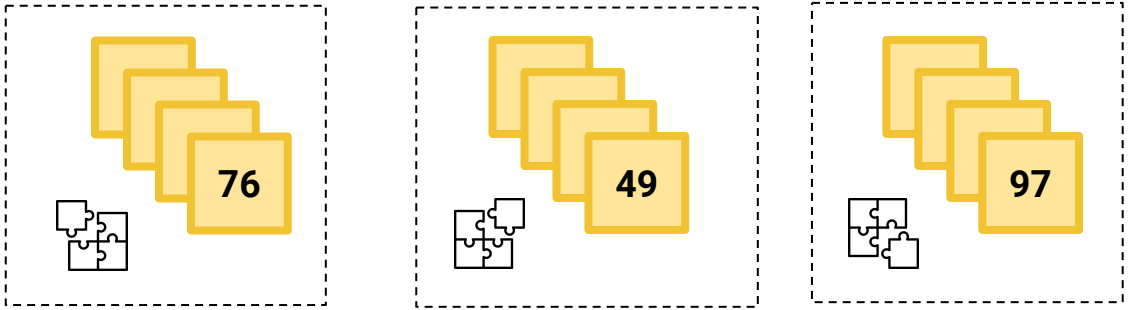
BIG SORT

Batches of 1024 sorted elements



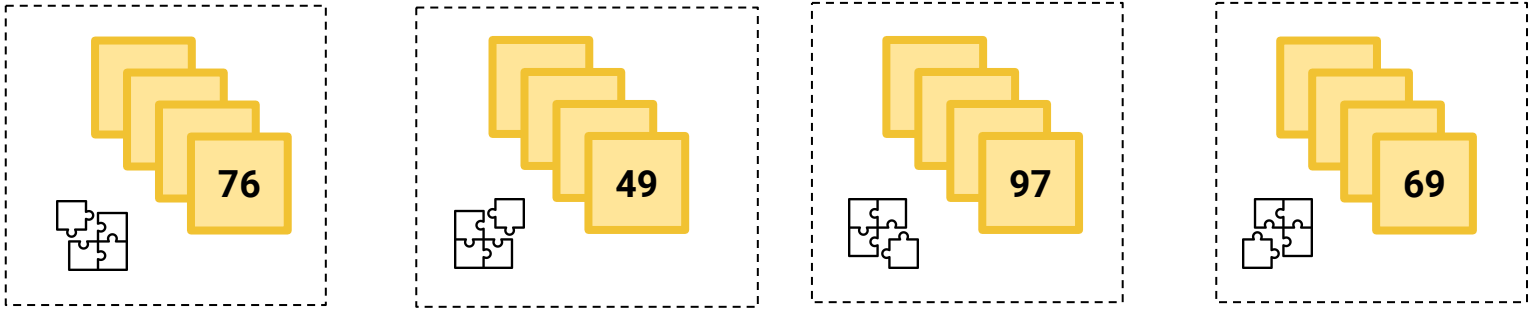
BIG SORT

Batches of 1024 sorted elements



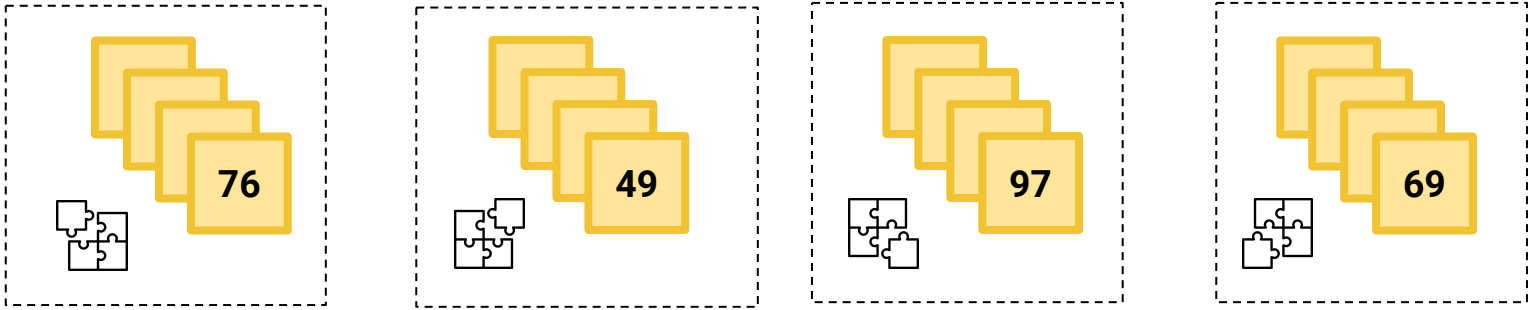
BIG SORT

Batches of 1024 sorted elements



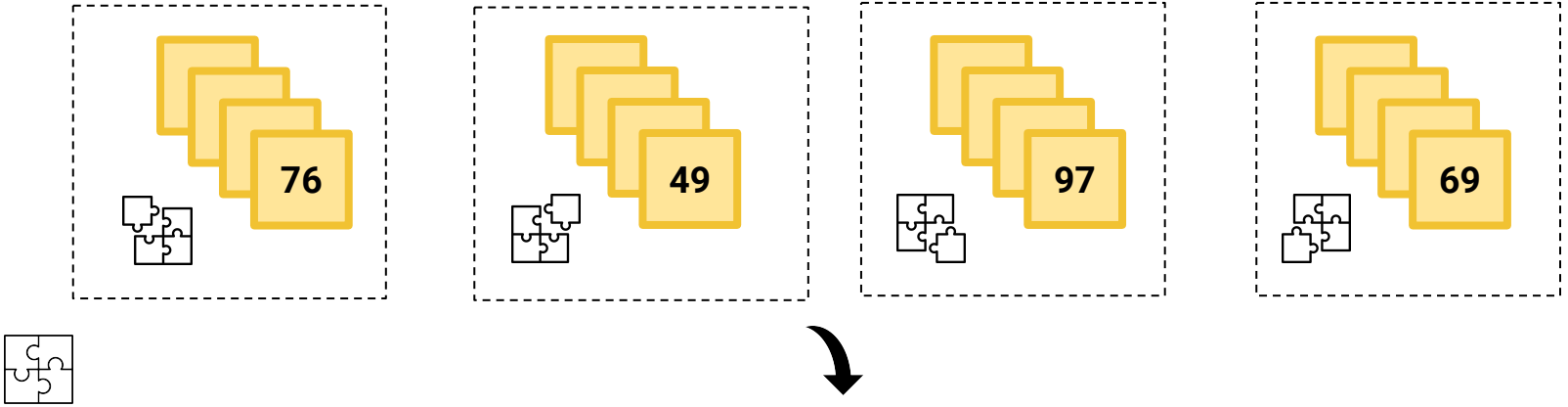
BIG SORT

Batches of 1024 sorted elements



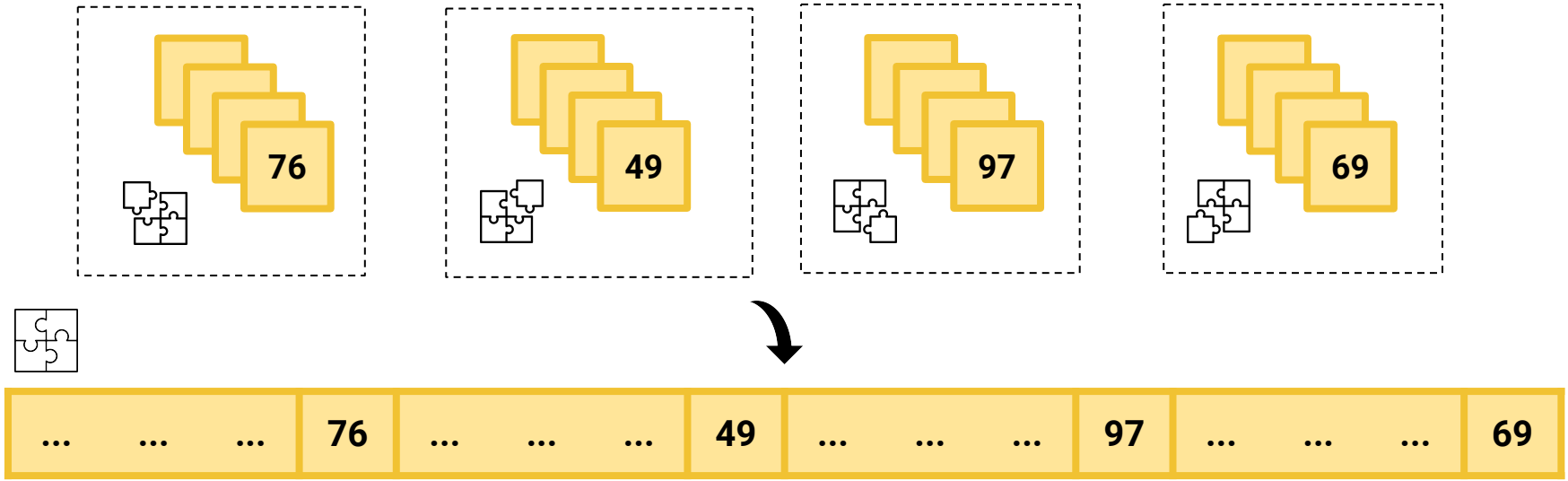
BIG SORT

Batches of 1024 sorted elements



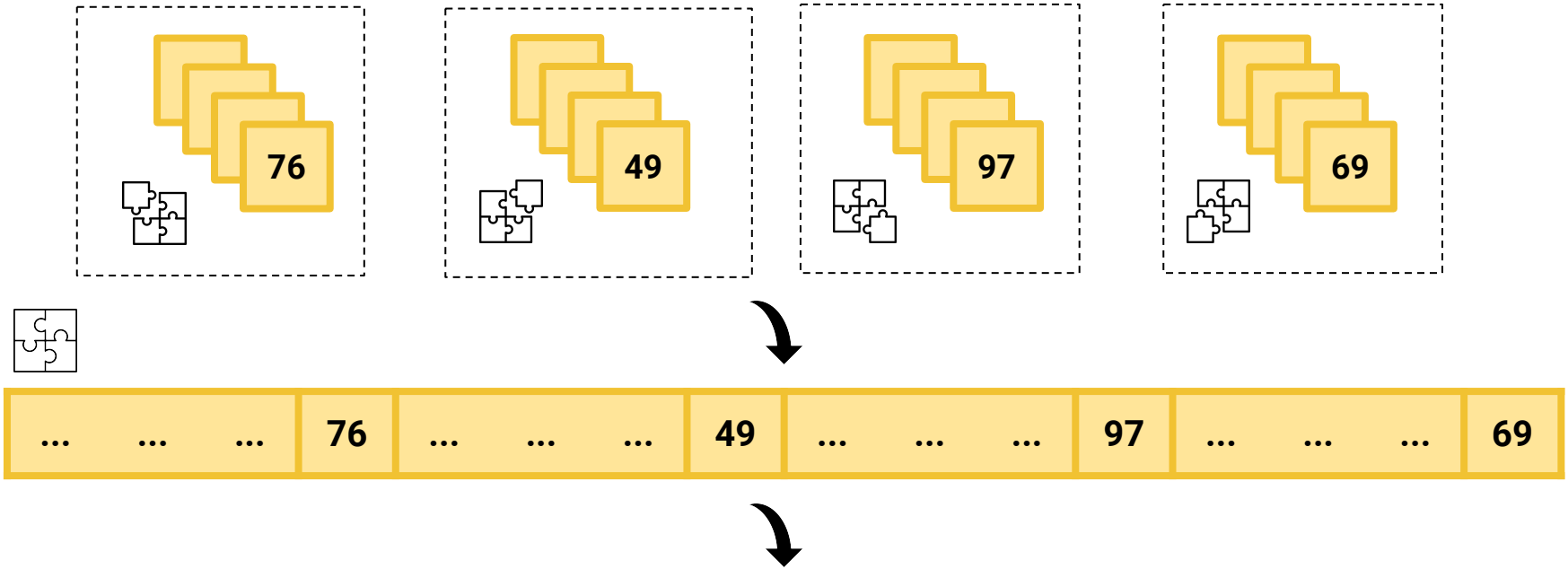
BIG SORT

Batches of 1024 sorted elements



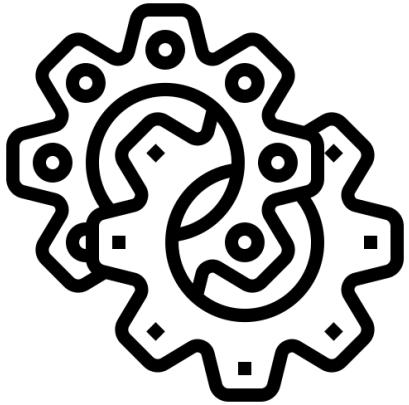
BIG SORT

Batches of 1024 sorted elements

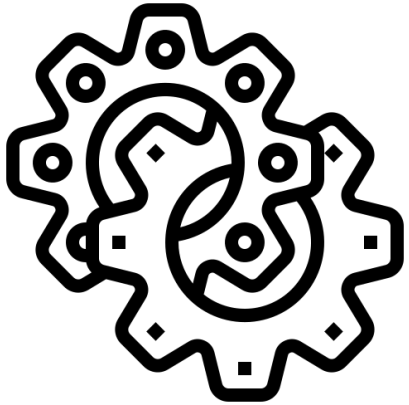


We sort the new reconstitued array

NOTATIONS



NOTATIONS



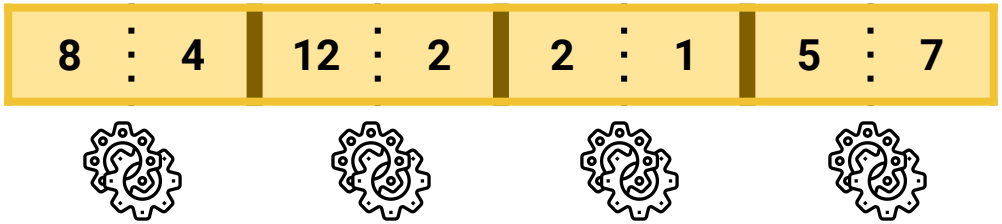
pathBig_k(A,B)
+
mergeBig_k(A,B)

GPU SORT

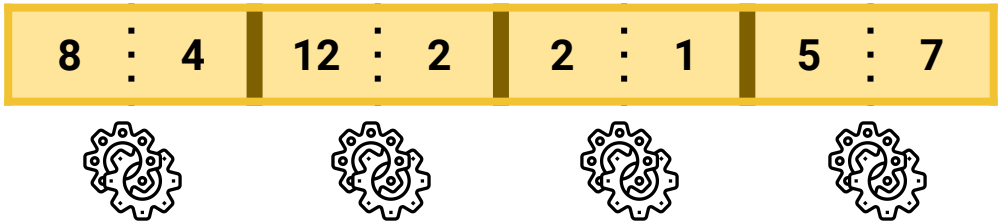
GPU SORT

GPU SORT

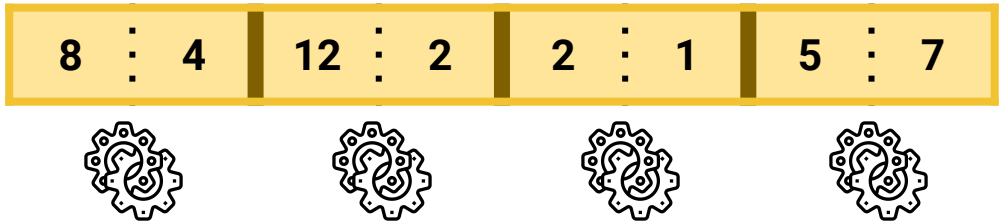
GPU SORT



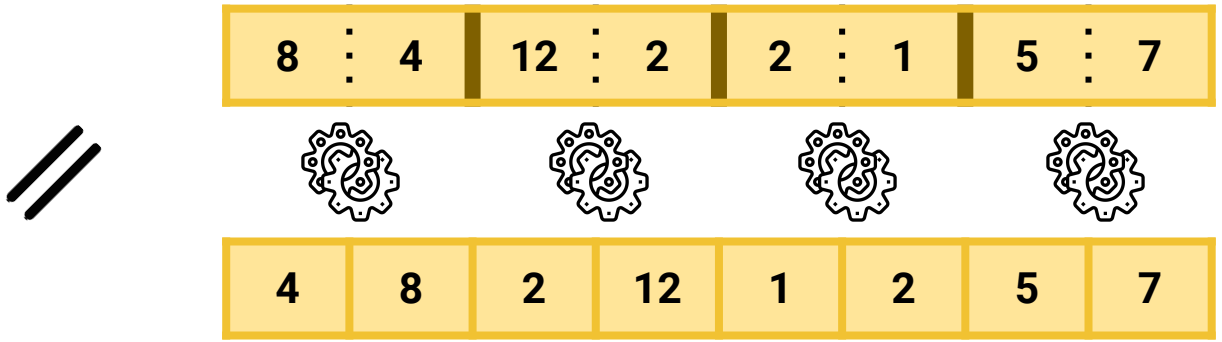
GPU SORT WITH STREAM



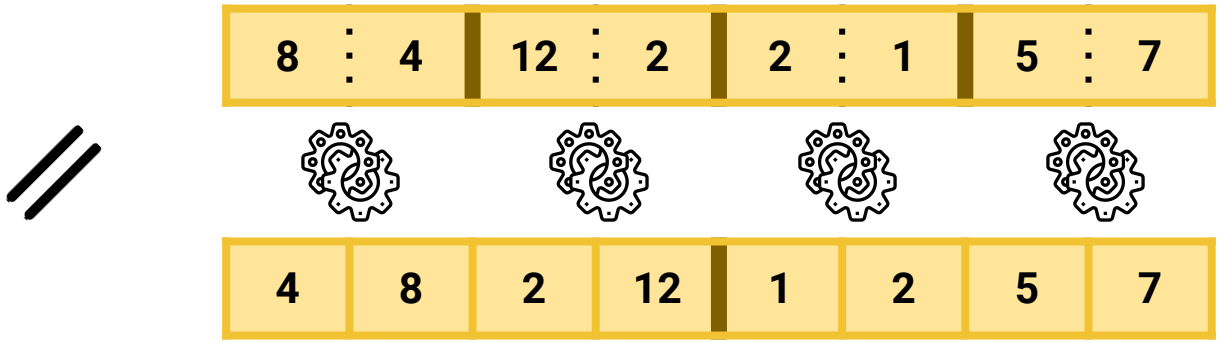
GPU SORT WITH STREAM



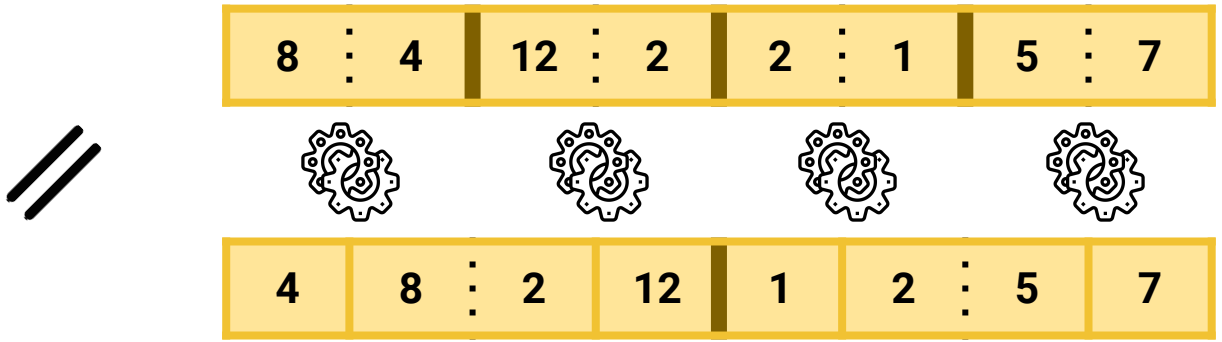
GPU SORT WITH STREAM



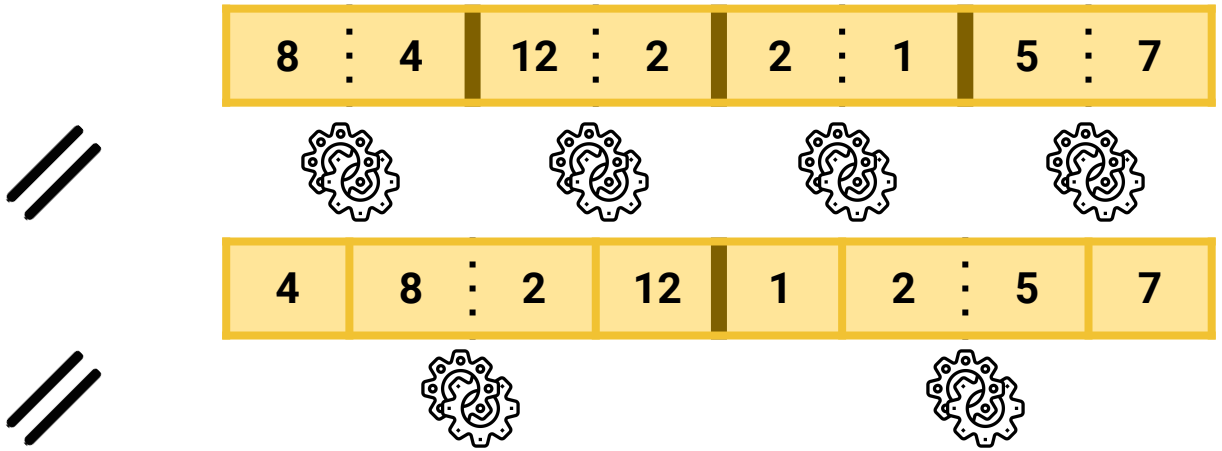
GPU SORT WITH STREAM



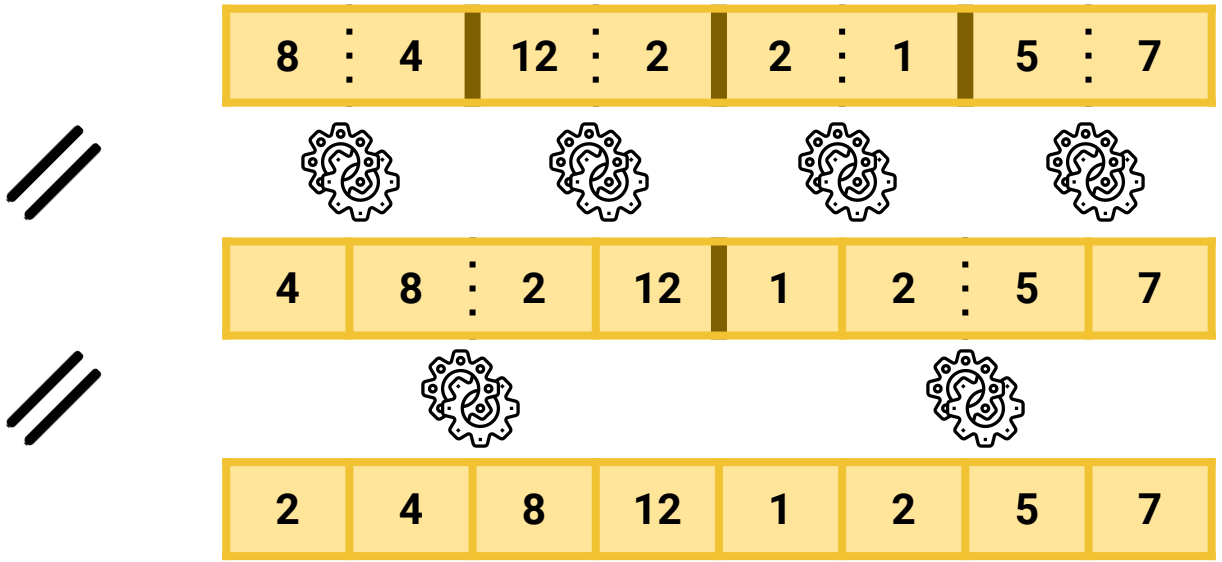
GPU SORT WITH STREAM



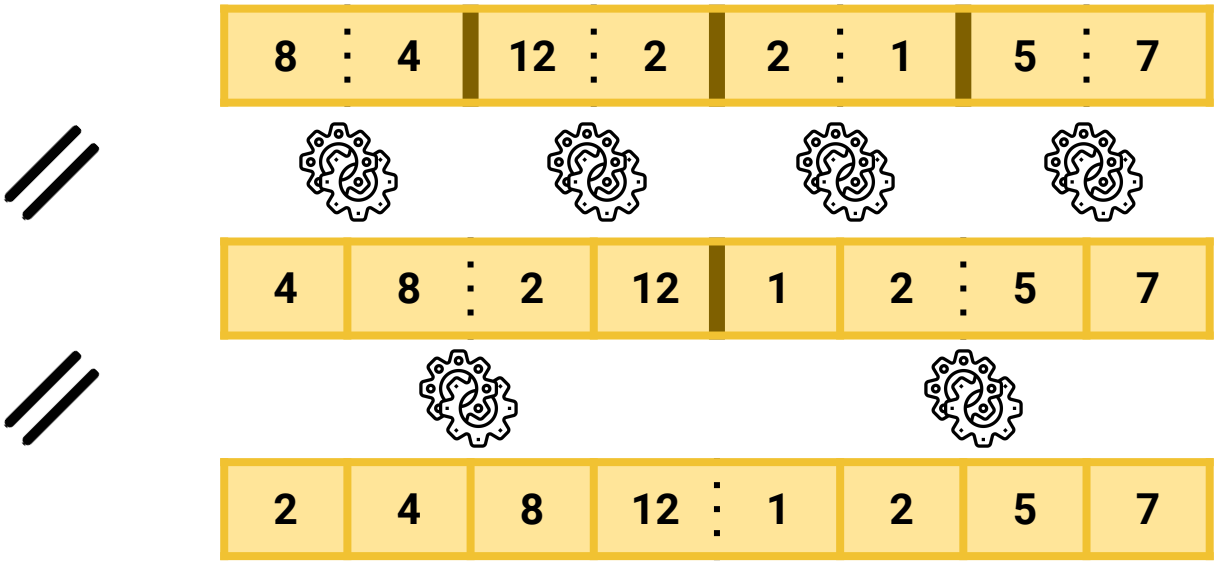
GPU SORT WITH STREAM



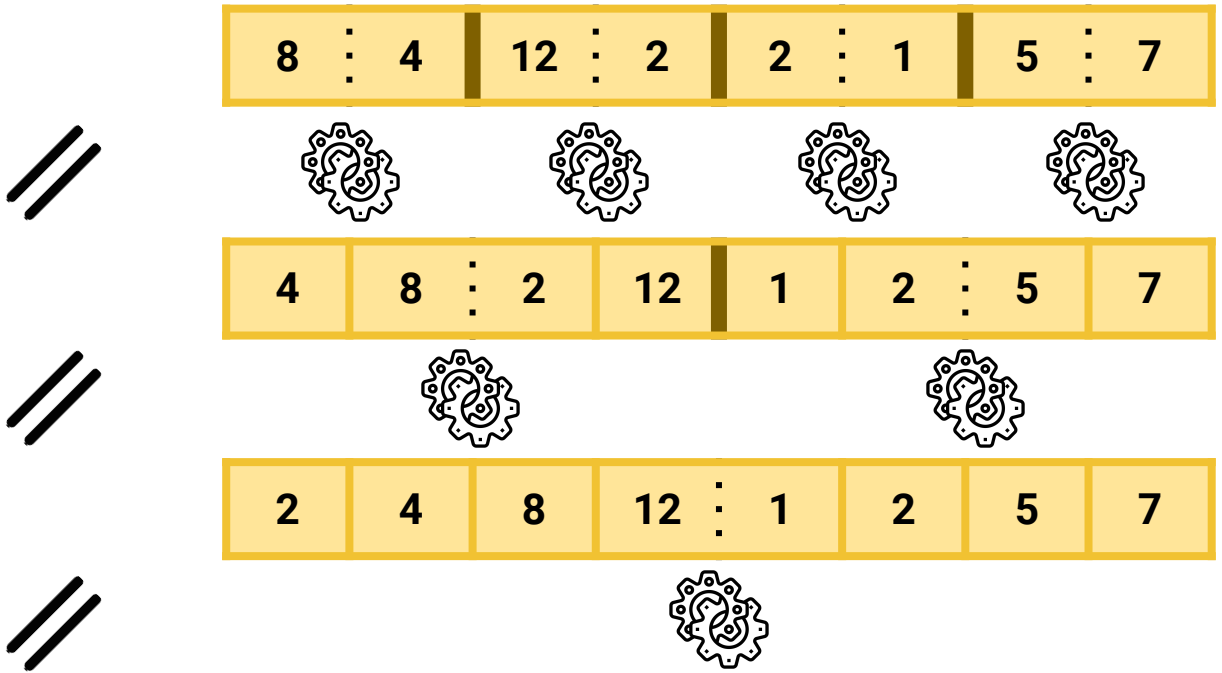
GPU SORT WITH STREAM



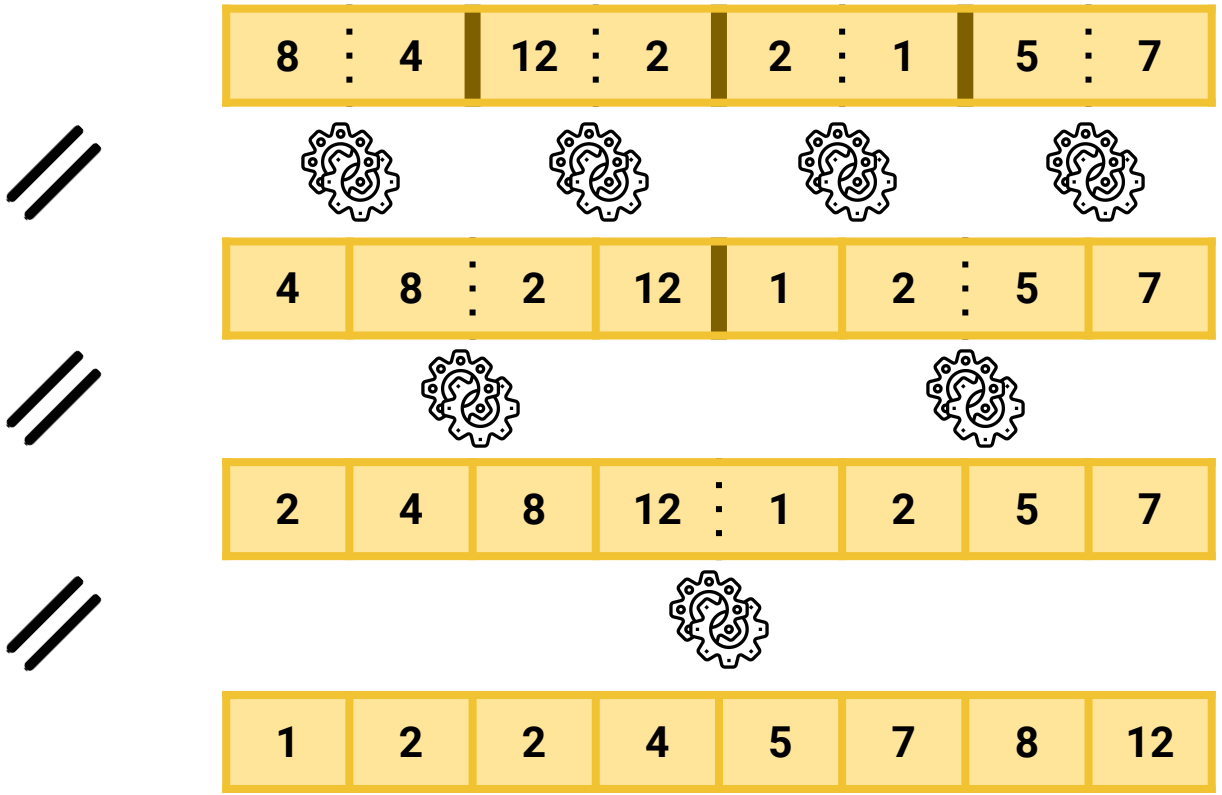
GPU SORT WITH STREAM



GPU SORT WITH STREAM



GPU SORT WITH STREAM



SMALL SORT

mergeSmall_Batch_k
Batches on shared memory
Very Fast
Impossible if batch_size > 1024

SMALL SORT

mergeSmall_Batch_k
Batches on shared memory
Very Fast
Impossible if batch_size > 1024

BIG SORT

pathBig_k and mergeBig_k
Streams
Global Memory

OUR GITHUB :

https://github.com/saulnierS/main5_hpc.git