

HPCA project

Batch Sort et Merge Path Sort

ALGORITHM OVERVIEW



PROBLEM TO SOLVE

How to sort efficiently an array of elements **using GPU** ?

variable names :

- ❖ A,B and M are array of int
- ❖ A,B are sorted
- ❖ We save our result in M
- ❖ M is not sorted on the initialisation

TABLES OF CONTENTS

01

MERGE ON ONE BLOCK

Of two sorted array,
A and B

02

MERGE ON SEVERAL BLOCKS

Of two sorted array,
A and B

03

“MERGE SORT” ON GPU

tree approach

04

SORT WITH BATCHES

tree approach, batches
and streams



01

MERGE ON ONE BLOCK

ALGO mergeSmall_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

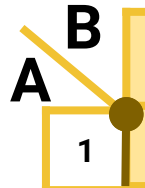
ALGO mergeSmall_k

A \ B							
	4	7	8	10	12	13	14
1							
2							
5							
6							
6							
9							
11							
15							
16							

ALGO mergeSmall_k

A \ B							
	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

ALGO mergeSmall_k



	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1

ALGO mergeSmall_k

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2
---	---

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4
---	---	---

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4	5
---	---	---	---

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4	5	6
---	---	---	---	---

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4	5	6
---	---	---	---	---

...

6

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

1	2	4	5	6
---	---	---	---	---

...

6	7
---	---

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

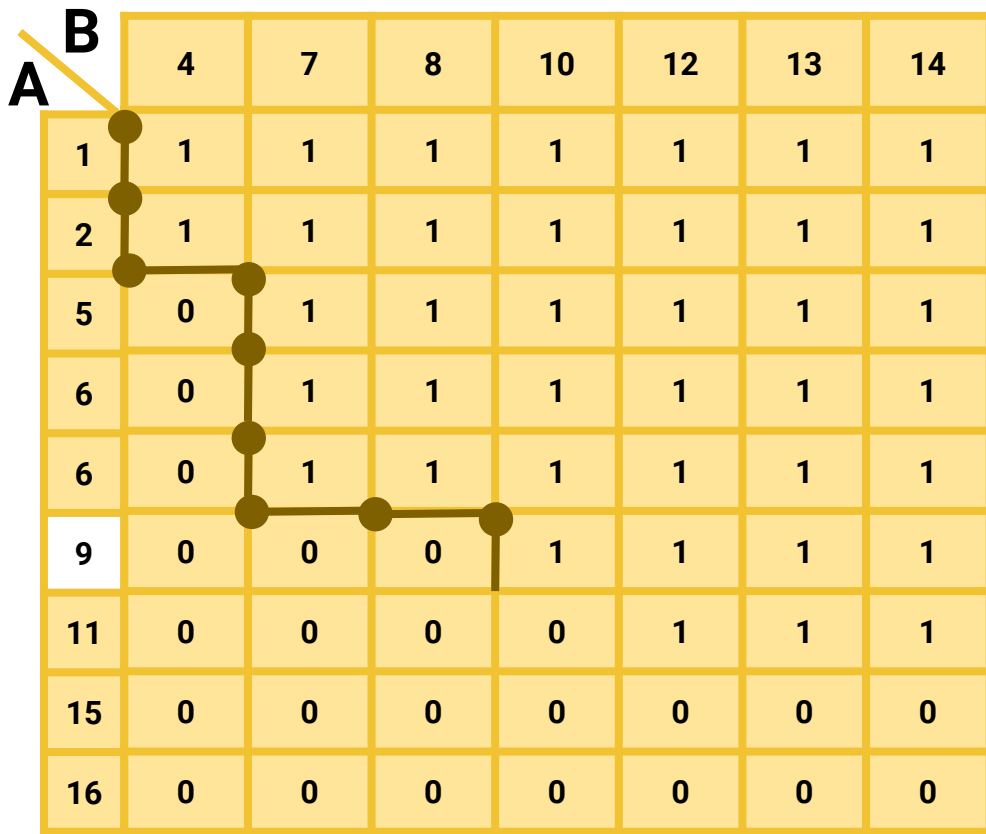
M

1	2	4	5	6
---	---	---	---	---

...

6	7	8
---	---	---

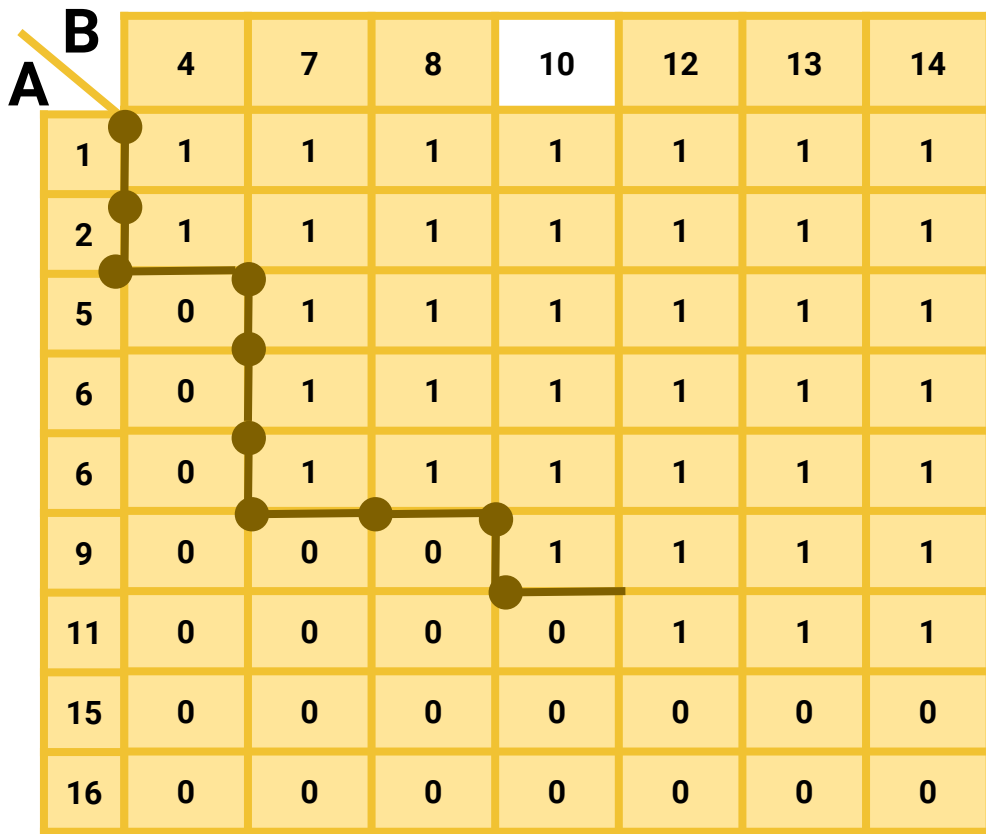
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	

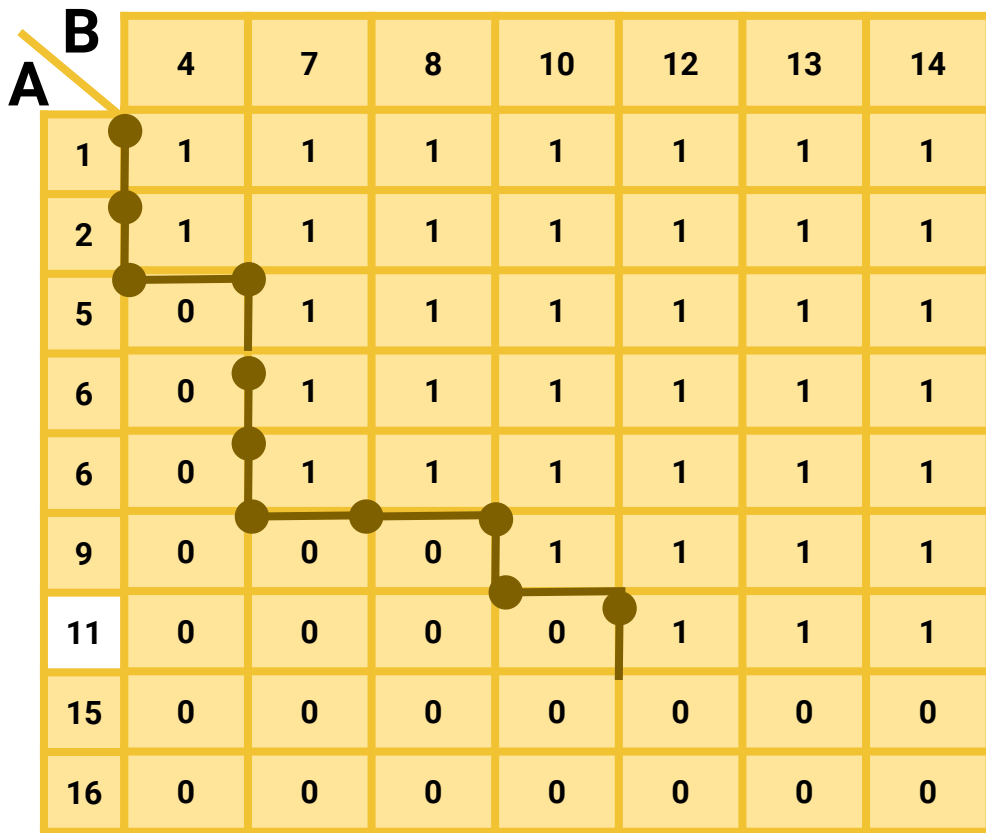
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10

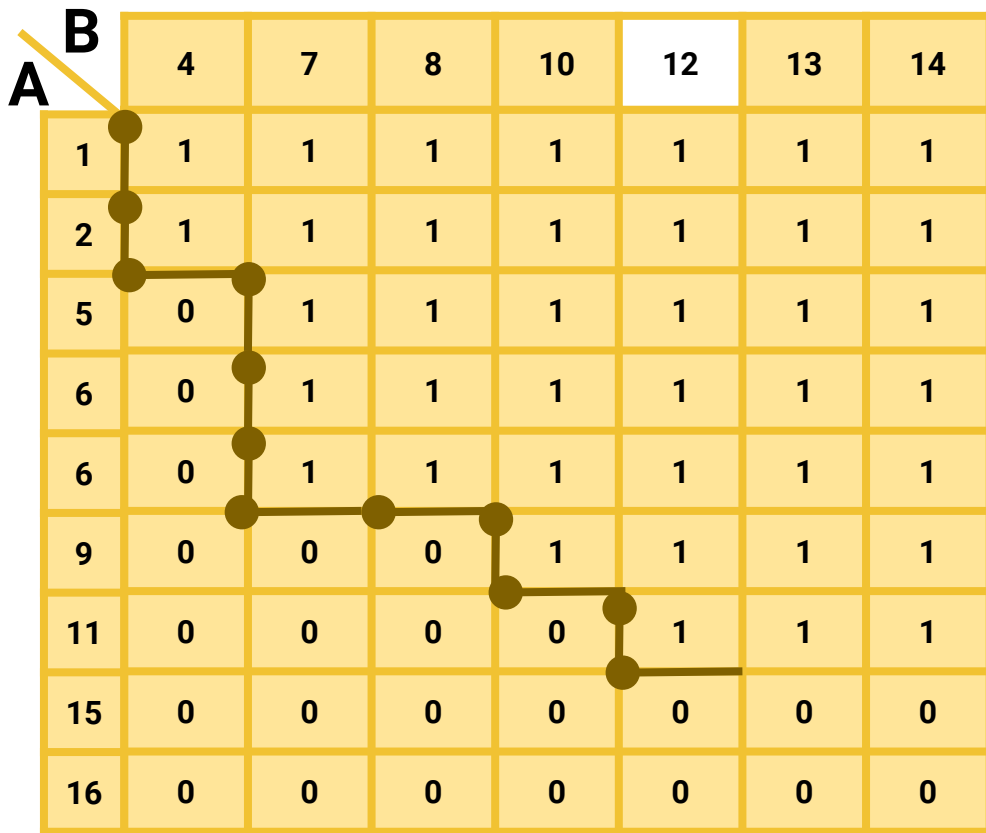
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11				

ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11	12			

ALGO mergeSmall_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13		

ALGO mergeSmall_k

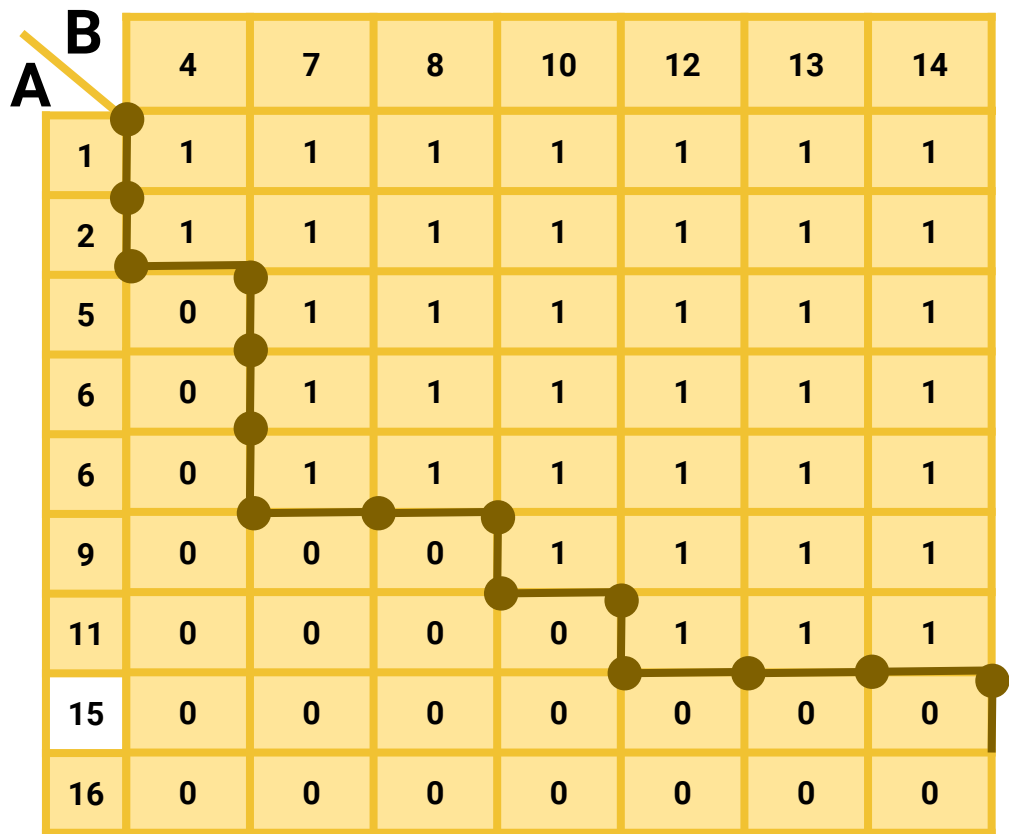
A **B**

		4	7	8	10	12	13	14
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1	1
6	0	1	1	1	1	1	1	1
6	0	1	1	1	1	1	1	1
9	0	0	0	1	1	1	1	1
11	0	0	0	0	1	1	1	1
15	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0

M

	1	2	4	5	6
...	6	7	8	9	10
...	11	12	13	14	

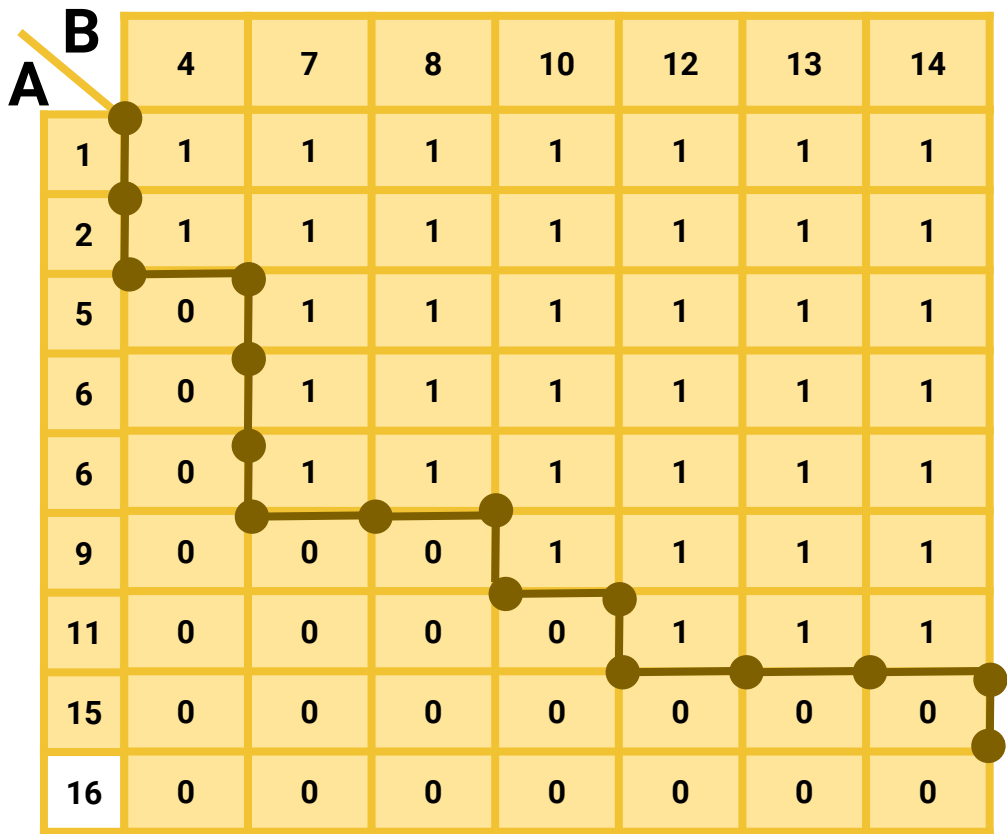
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11	12	13	14	15

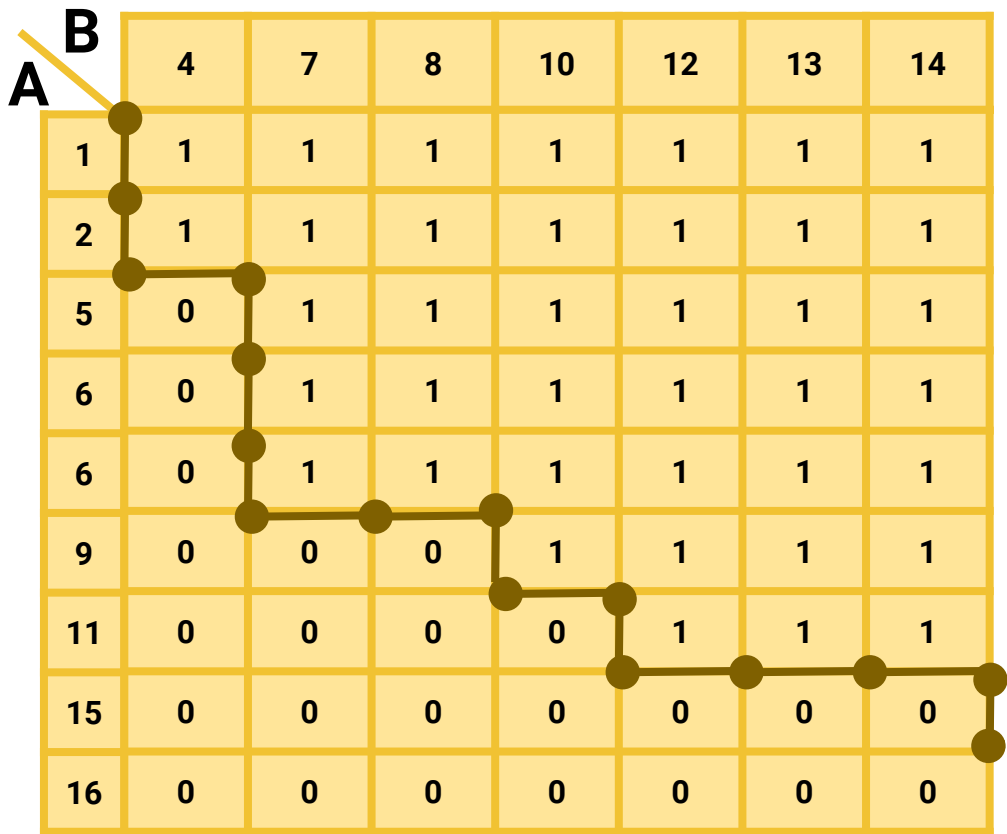
ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11	12	13	14	15
16				

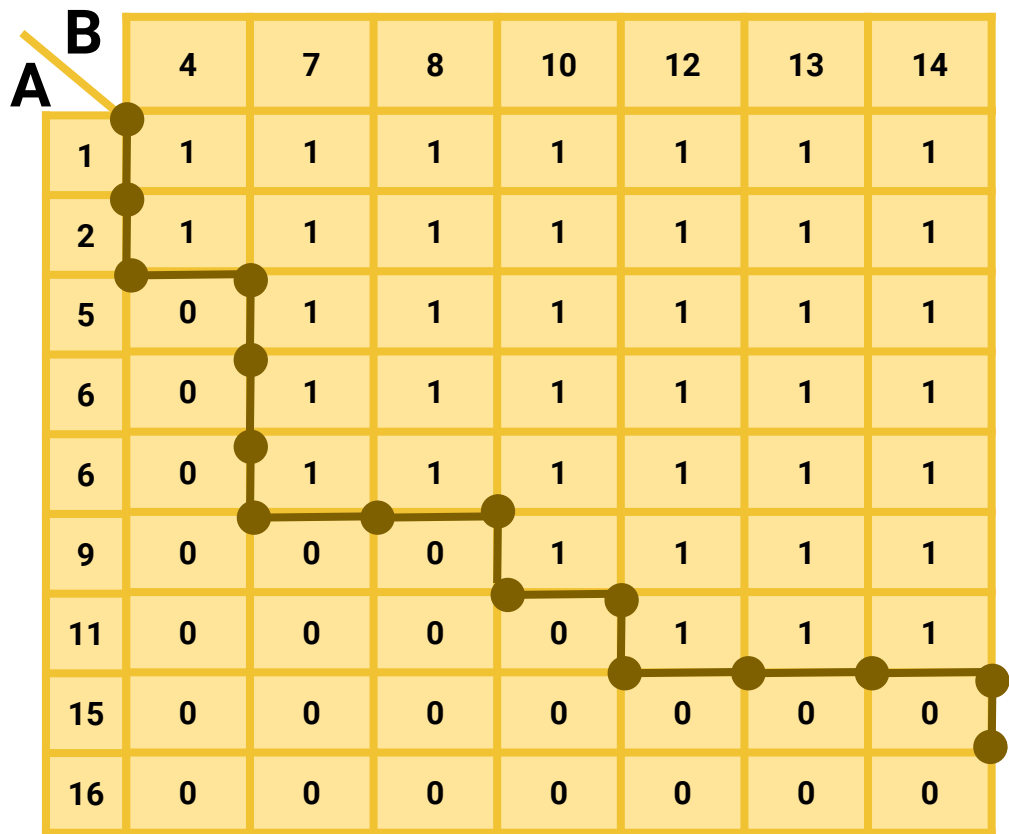
ALGO mergeSmall_k



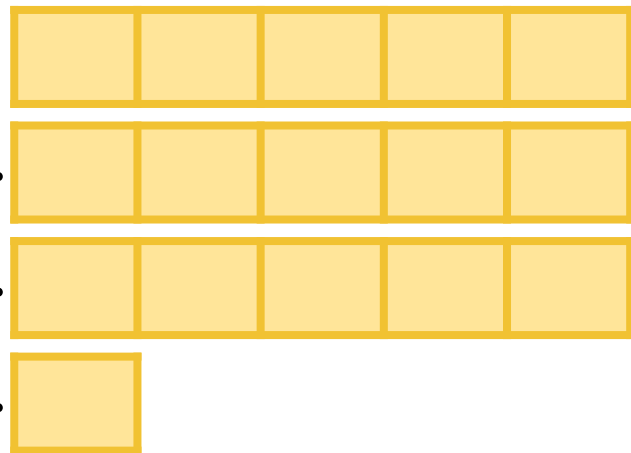
M

1	2	4	5	6
6	7	8	9	10
11	12	13	14	15
16				

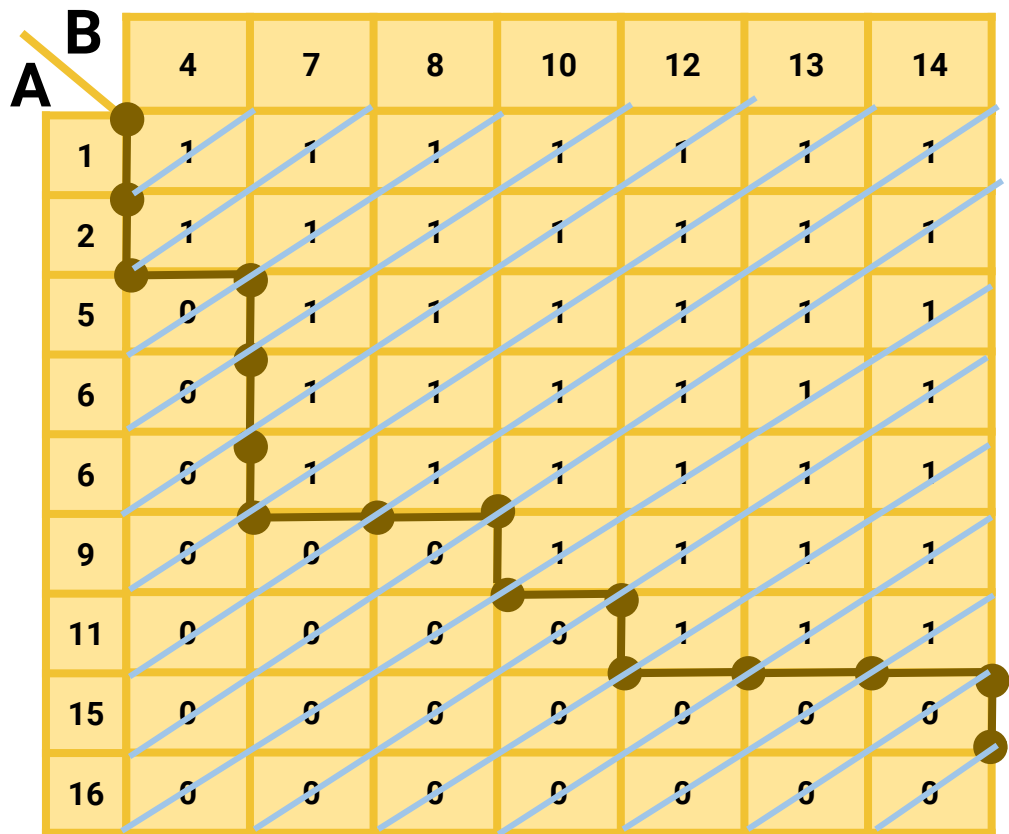
ALGO mergeSmall_k



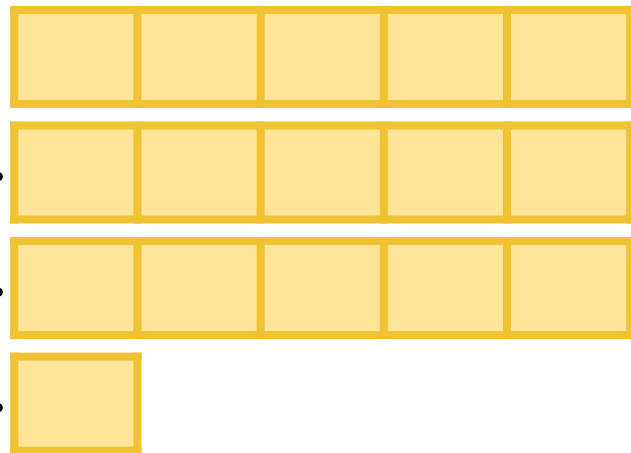
M



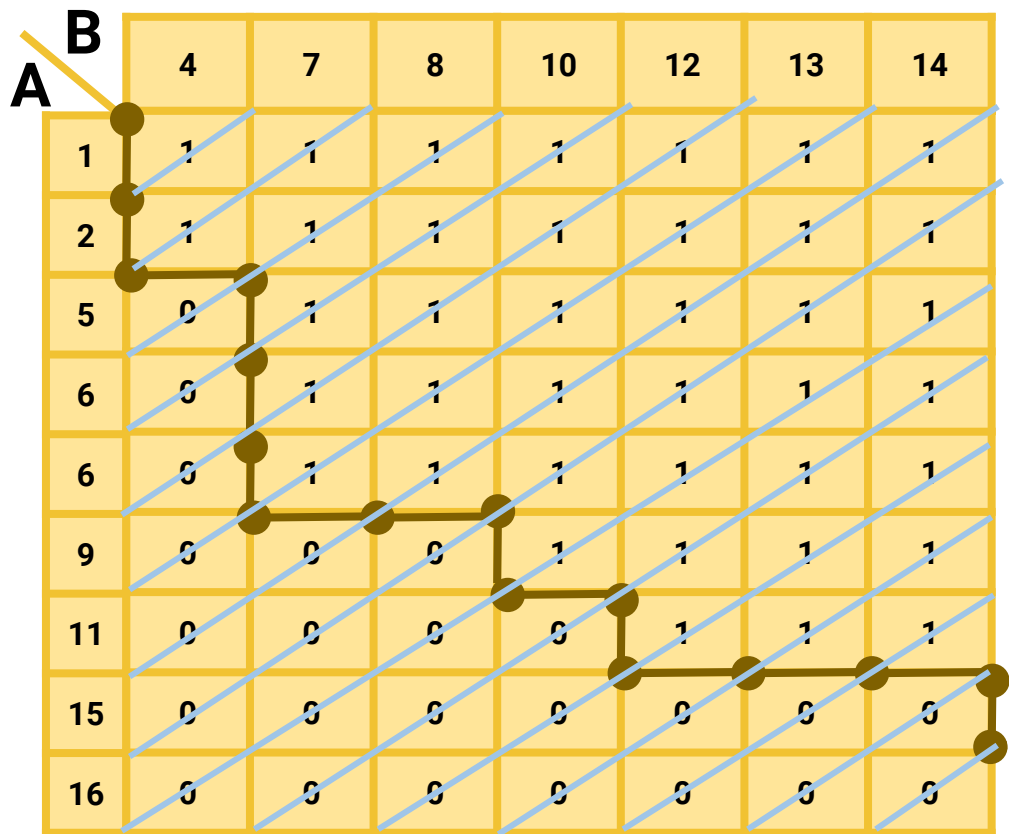
ALGO mergeSmall_k



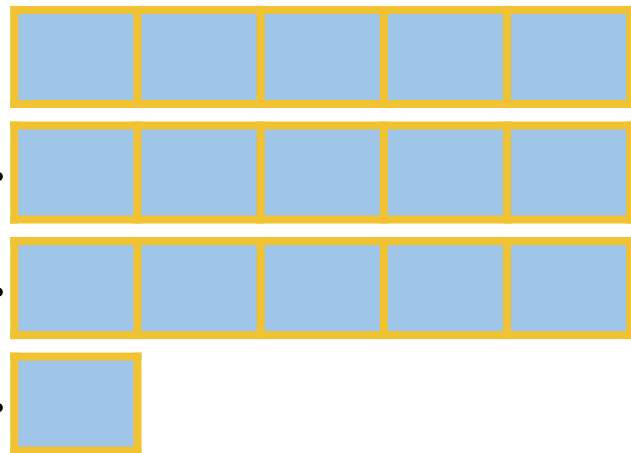
M



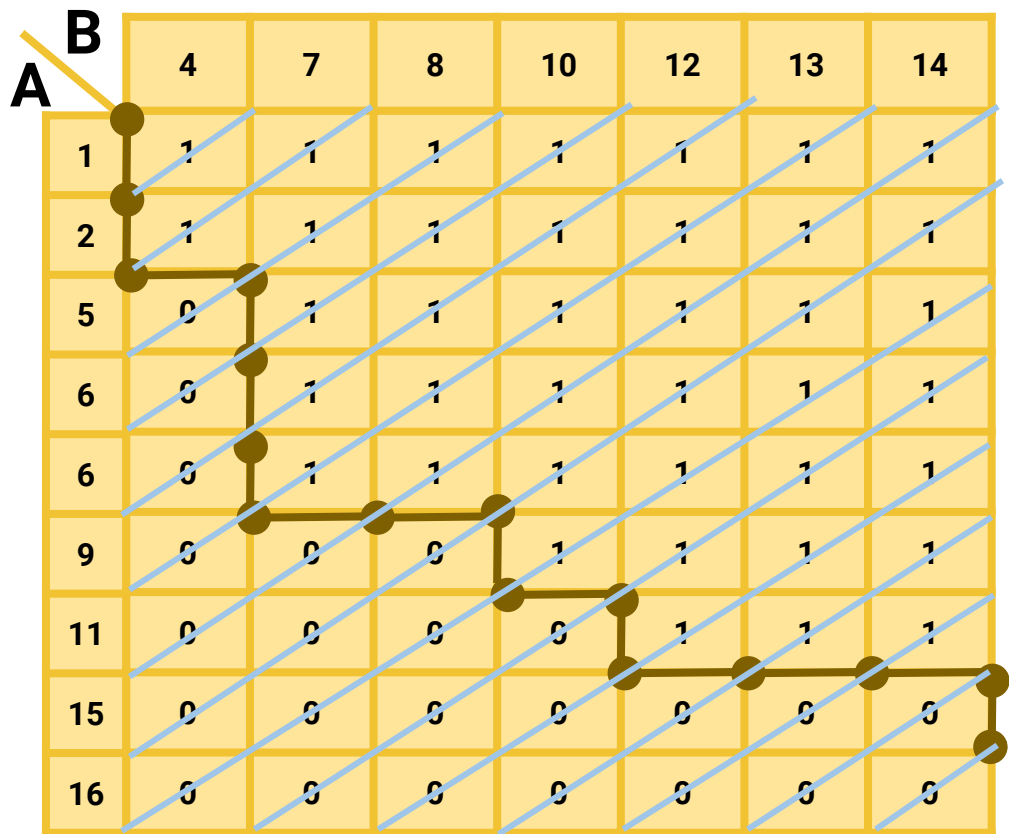
ALGO mergeSmall_k



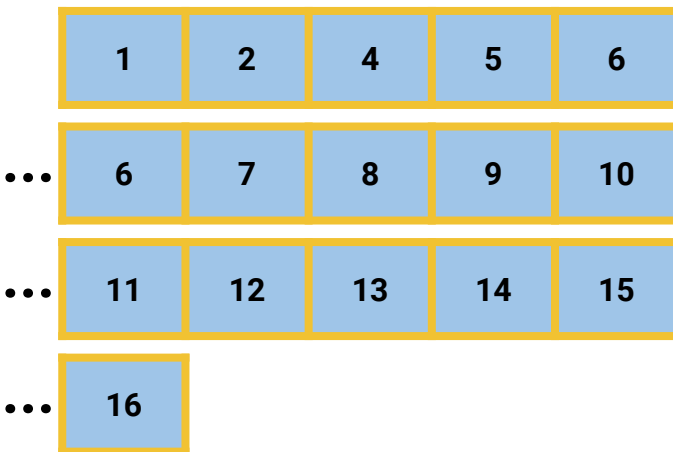
M



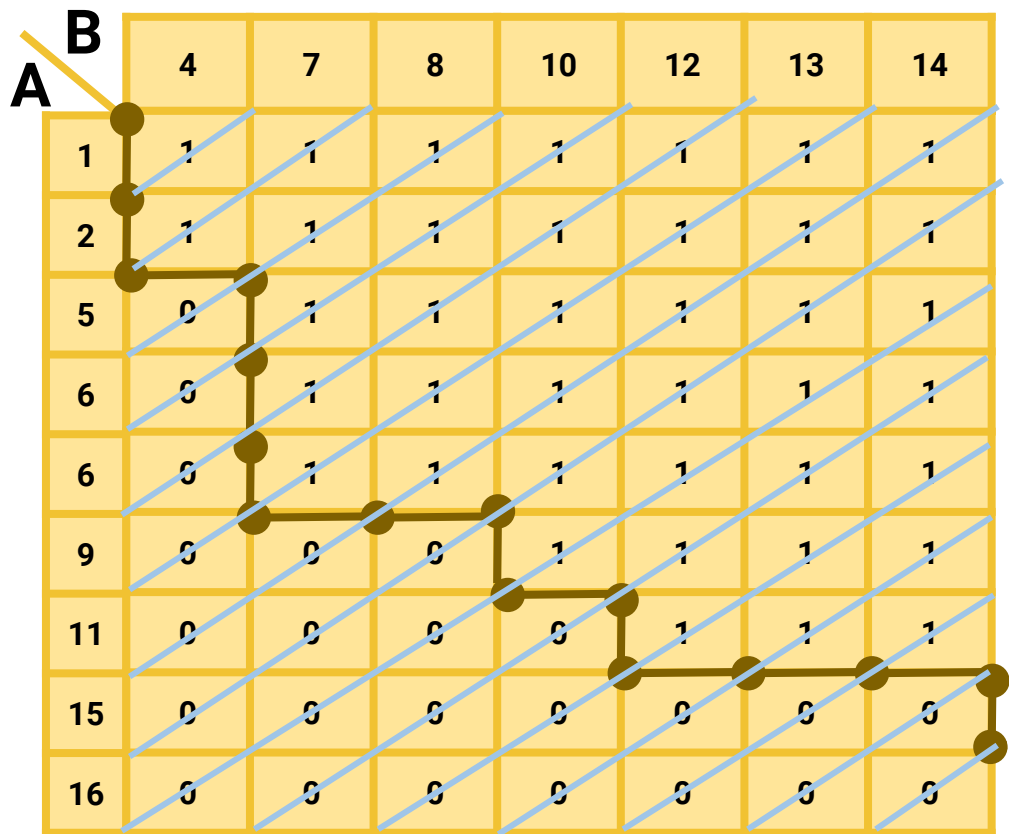
ALGO mergeSmall_k



M



ALGO mergeSmall_k

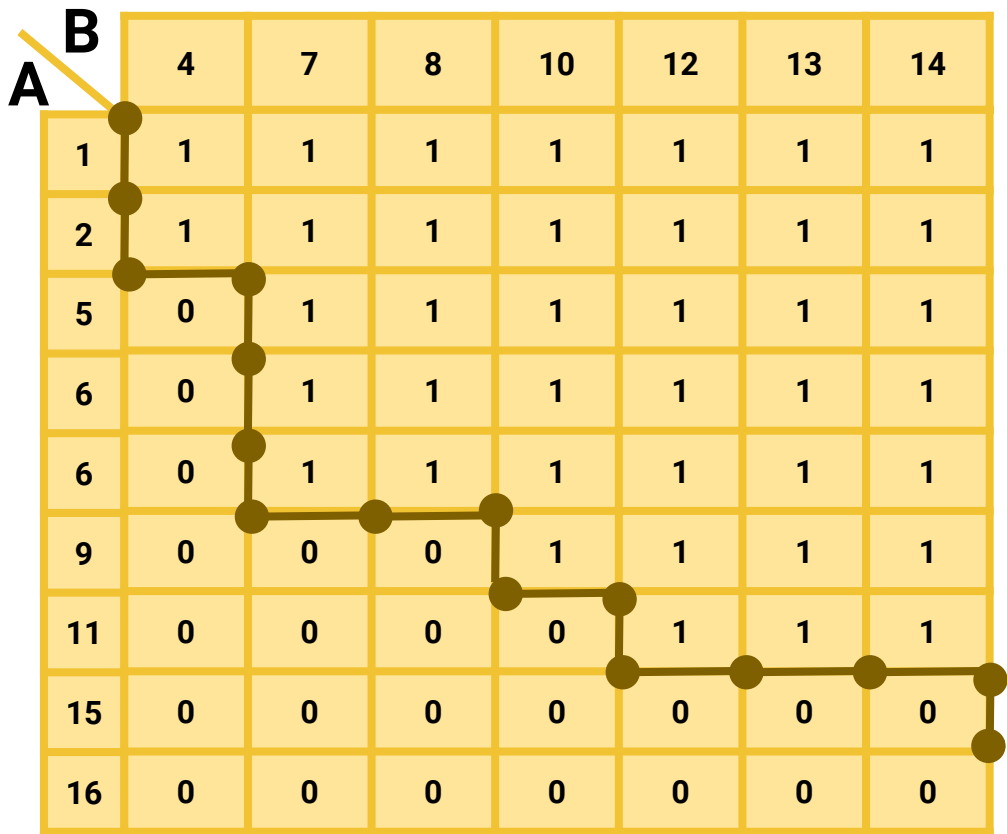


M

1	2	4	5	6
---	---	---	---	---

...				
6	7	8	9	10
...				
11	12	13	14	15
...				
16				

ALGO mergeSmall_k



M

1	2	4	5	6
6	7	8	9	10
11	12	13	14	15
16				

A decorative graphic at the top of the slide. It features a central vertical yellow line. On either side of this line, there are several yellow squares of varying sizes arranged in a stepped, descending pattern towards the center. The squares are set against a black background.

02

MERGE ON SEVERAL BLOCKS

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1

ALGO pathBig_k

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1
---	---

ALGO pathBig_k

A \ B		4	7	8	10	12	13	14
1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1	1
6	0	1	1	1	1	1	1	1
6	0	1	1	1	1	1	1	1
9	0	0	0	1	1	1	1	1
11	0	0	0	0	1	1	1	1
15	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0

Path

1	1	0
---	---	---

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1	0	1
---	---	---	---

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1	0	1	1
---	---	---	---	---

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

A path is highlighted with black dots and lines, starting from cell (1,4) and ending at cell (6,7). The path consists of the following cells: (1,4), (2,4), (5,4), (5,5), (6,5), (6,6), (6,7).

Path

1	1	0	1	1
---	---	---	---	---

...

1

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

The path starts at cell (1,4) and ends at cell (9,2). The path is marked with black dots and a black line. The path consists of the following cells: (1,4), (2,4), (3,4), (3,5), (4,5), (5,5), (6,5), (7,5), (8,5), (9,5), (9,4), (9,3), (9,2).

Path

1	1	0	1	1
---	---	---	---	---

...

1	0
---	---

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

The path starts at cell (1,4) and ends at cell (9,8). The path consists of the following cells: (1,4), (2,4), (5,4), (5,5), (6,5), (6,6), (9,6), (9,7), (9,8). The cell (9,8) is highlighted in white.

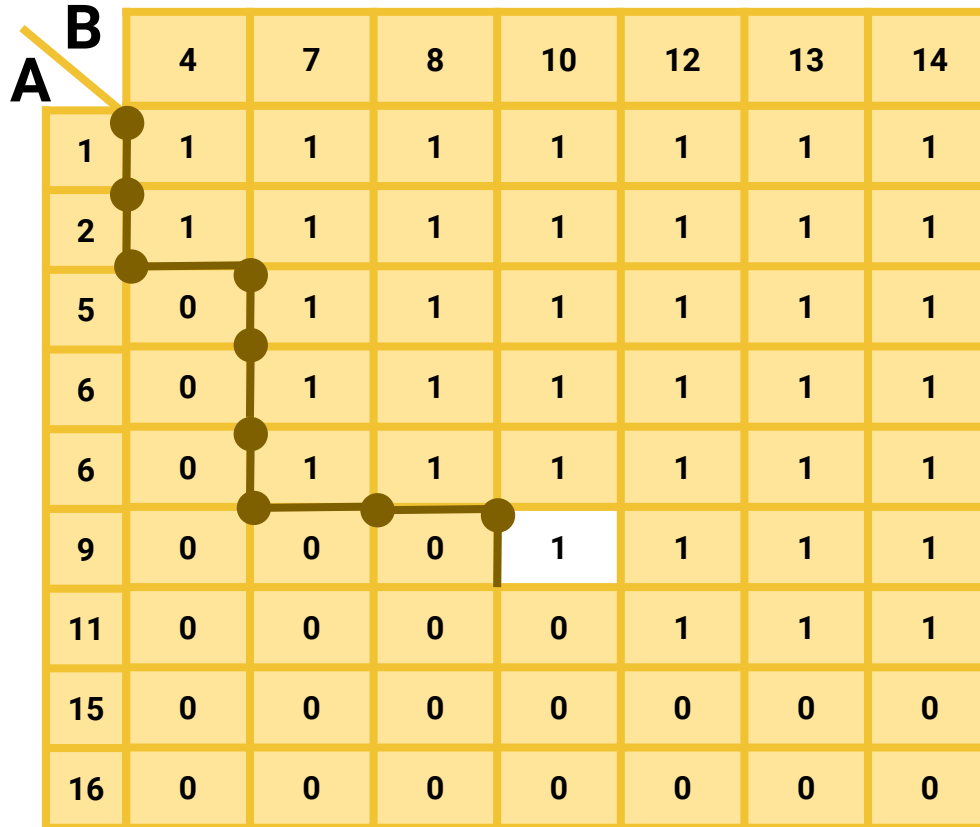
Path

1	1	0	1	1
---	---	---	---	---

...

1	0	0
---	---	---

ALGO pathBig_k



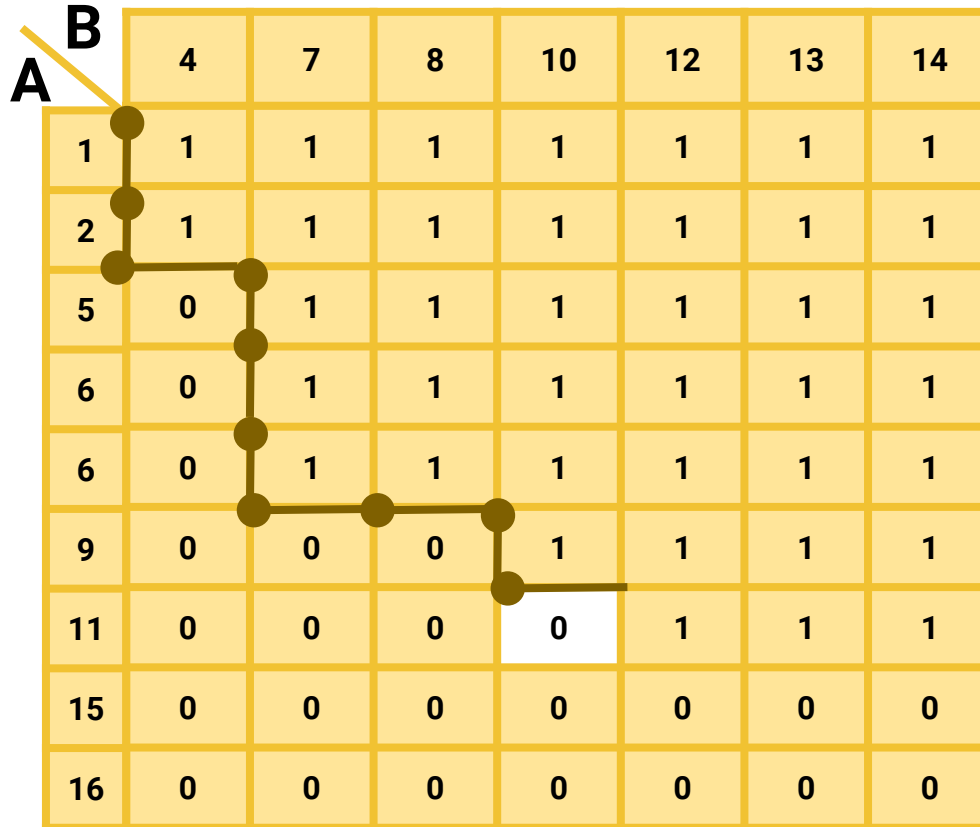
Path

1	1	0	1	1
---	---	---	---	---

...

1	0	0	1
---	---	---	---

ALGO pathBig_k



Path

1	1	0	1	1	
•	1	0	0	1	0

ALGO pathBig_k

A **B**

	4	7	8	10	12	13	14
1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1
5	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
6	0	1	1	1	1	1	1
9	0	0	0	1	1	1	1
11	0	0	0	0	1	1	1
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0

Path

1	1	0	1	1
---	---	---	---	---

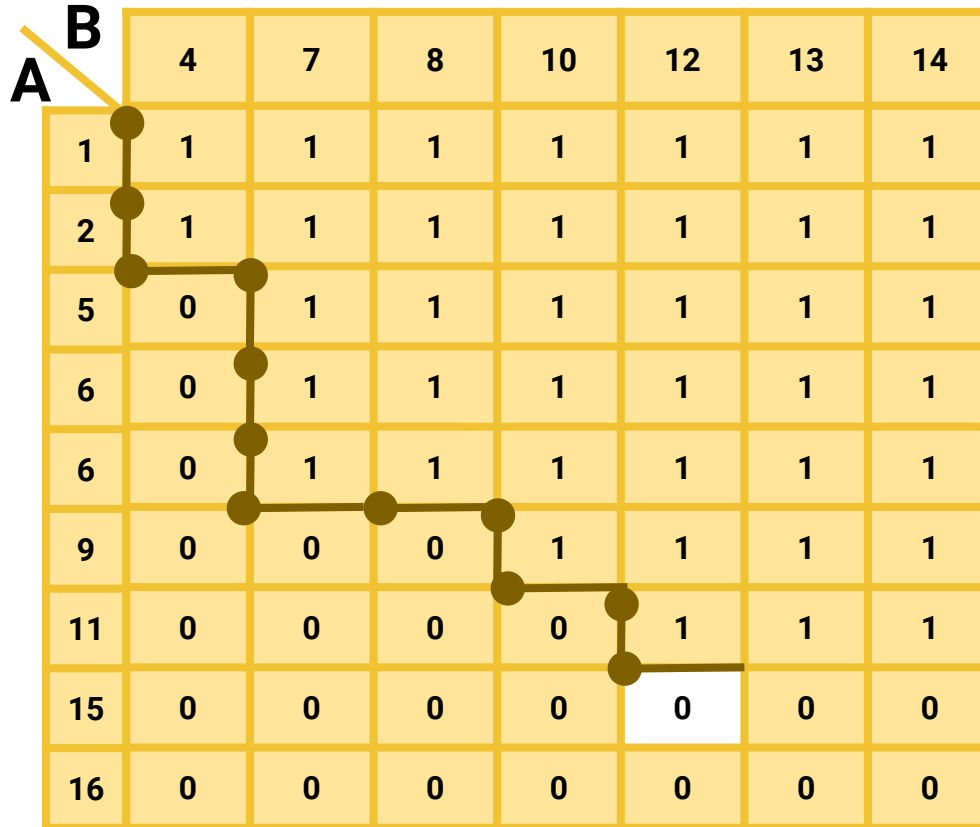
...

1	0	0	1	0
---	---	---	---	---

...

1

ALGO pathBig_k



Path

1	1	0	1	1
---	---	---	---	---

...

1	0	0	1	0
---	---	---	---	---

...

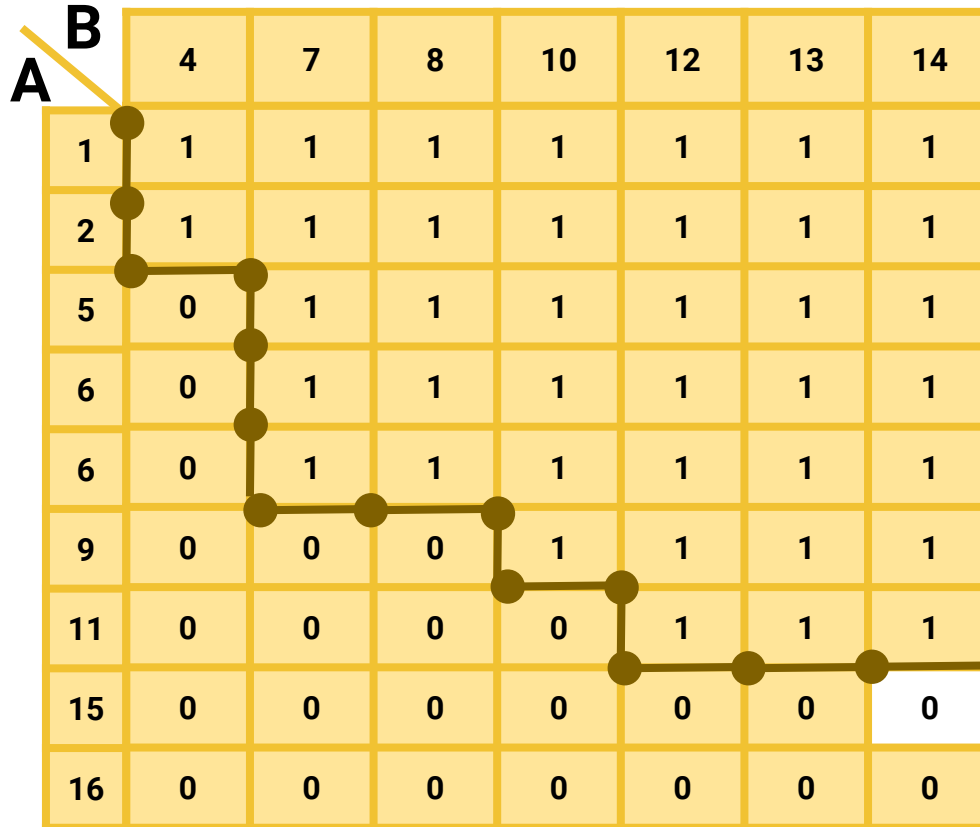
1	0
---	---



...

...

ALGO pathBig_k



Path

1	1	0	1	1
---	---	---	---	---

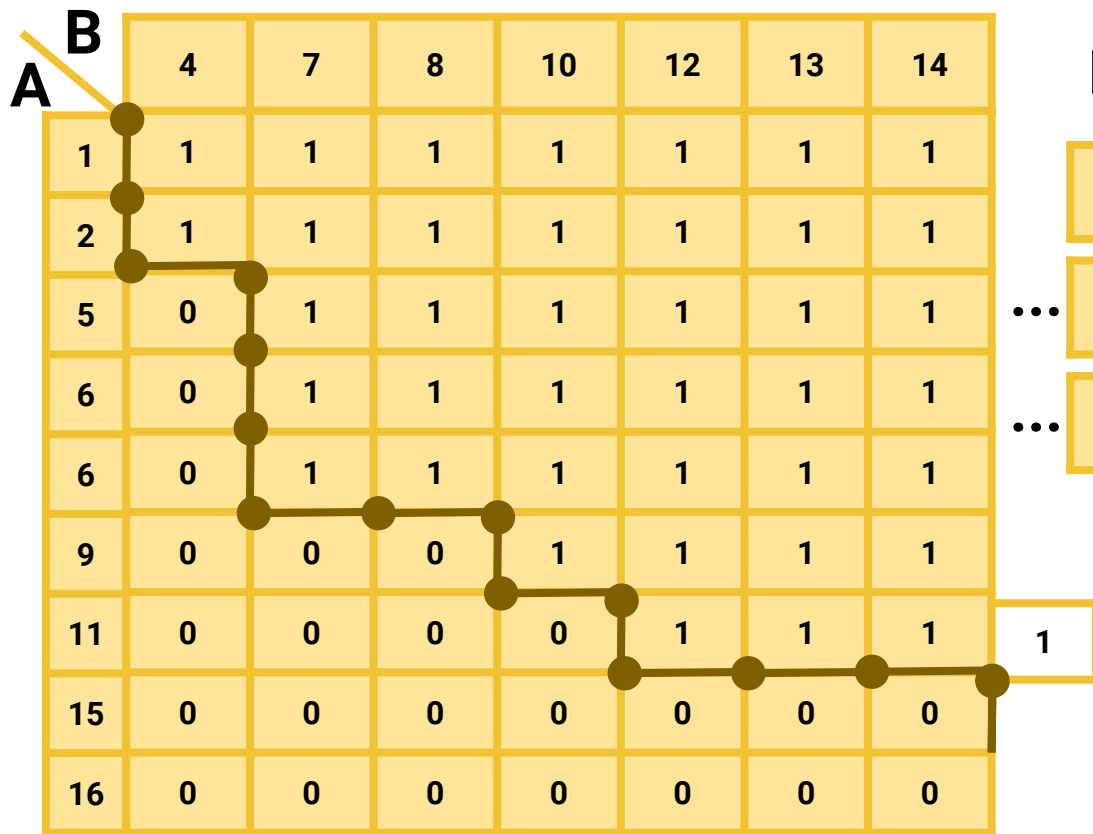
...

1	0	0	1	0
---	---	---	---	---

...

1	0	0	0
---	---	---	---

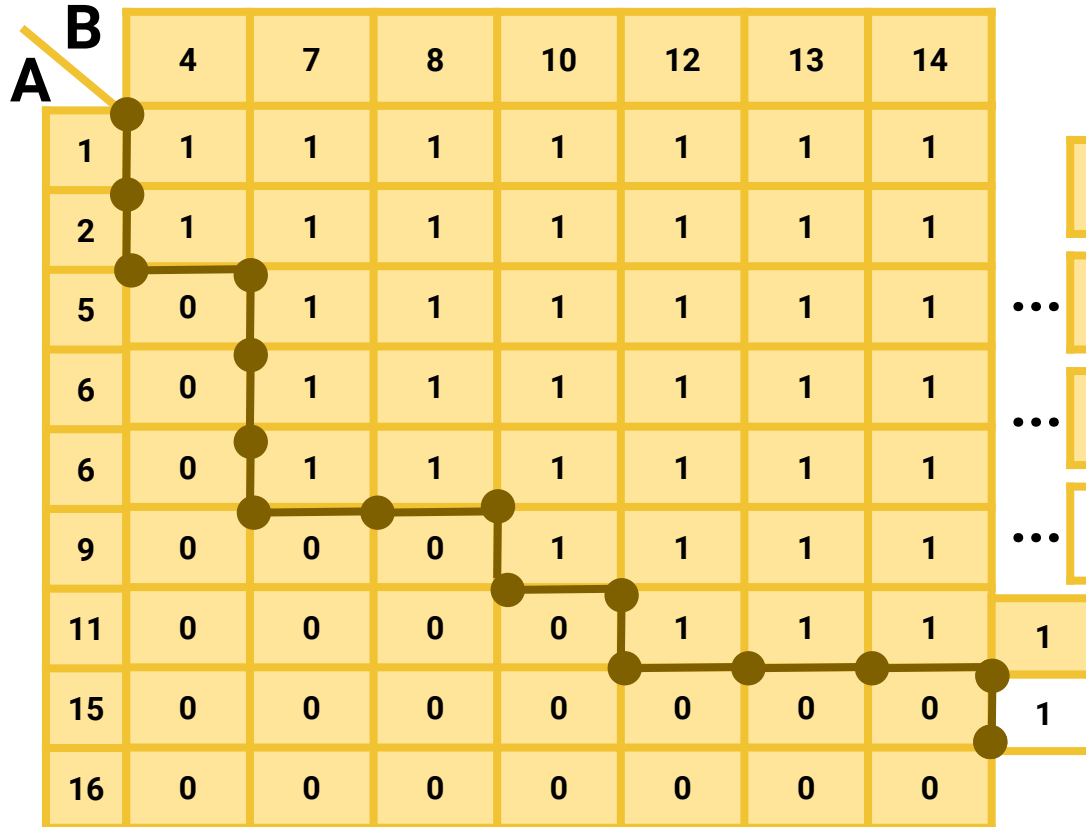
ALGO mergeSmall_k



Path

1	1	0	1	1
1	0	0	1	0
1	0	0	0	1

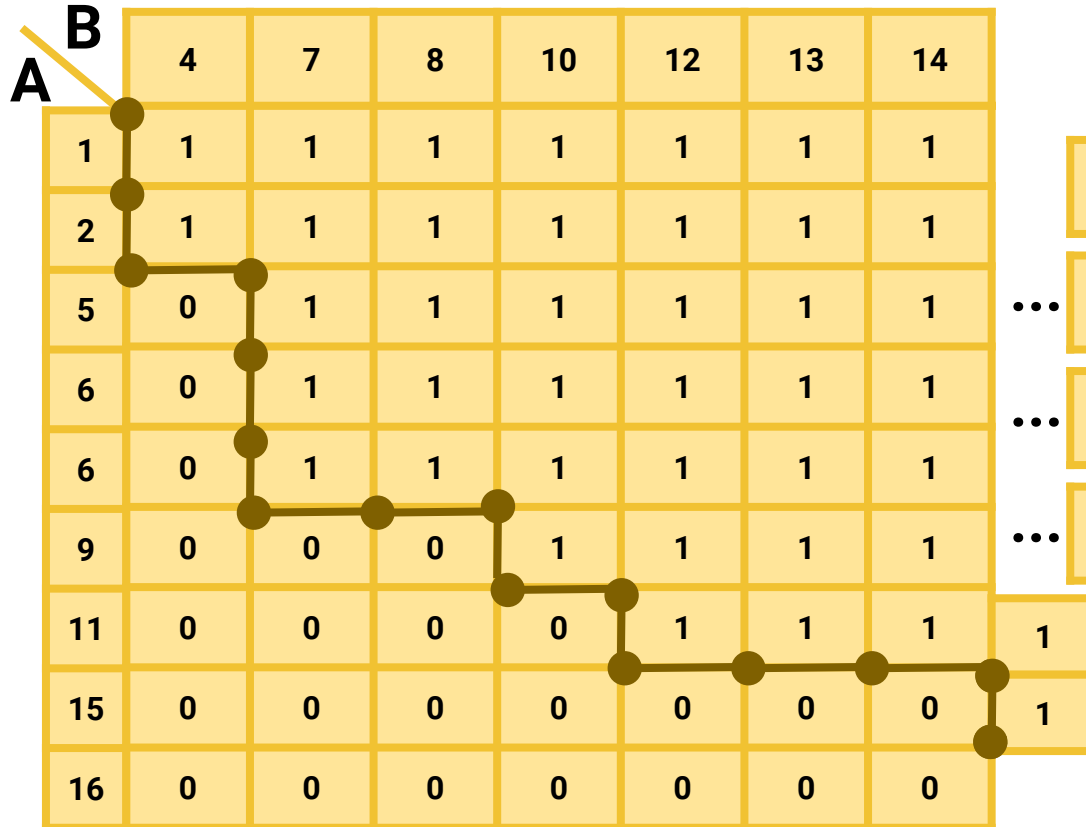
ALGO pathBig_k



Path

1	1	0	1	1
1	0	0	1	0
1	0	0	0	1
1				

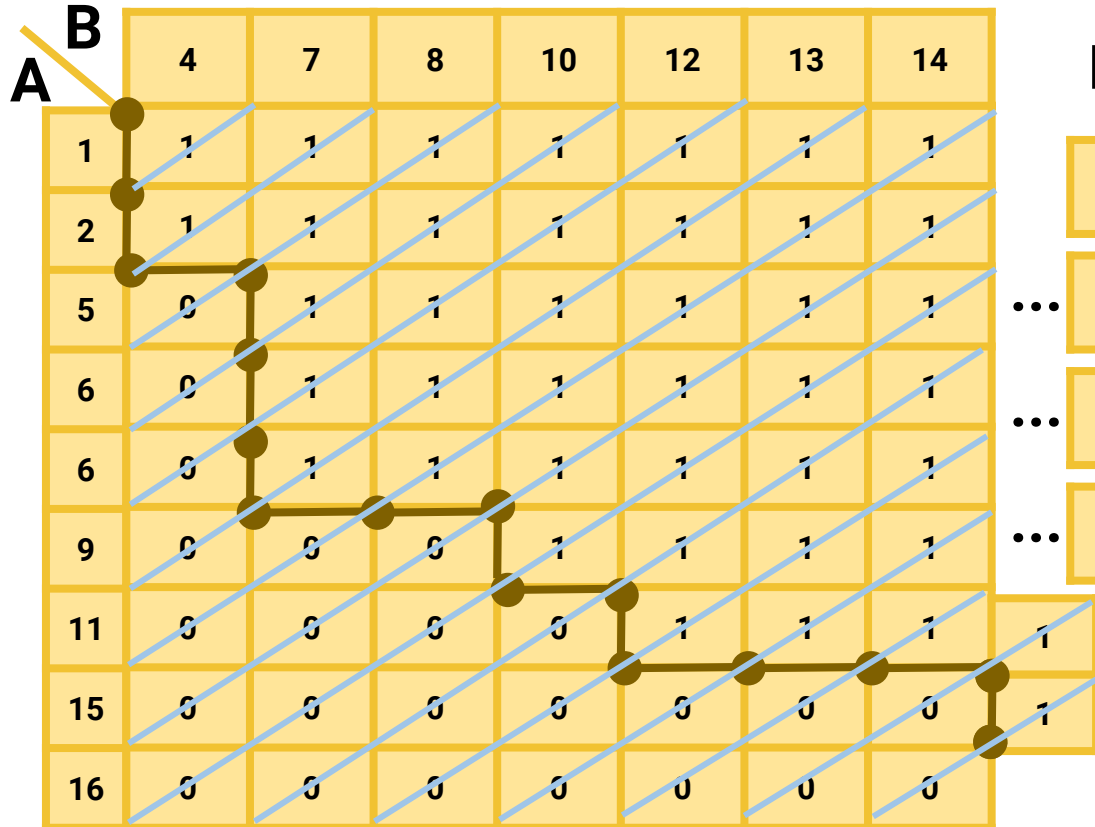
ALGO pathBig_k



Path

...				
...				
...				

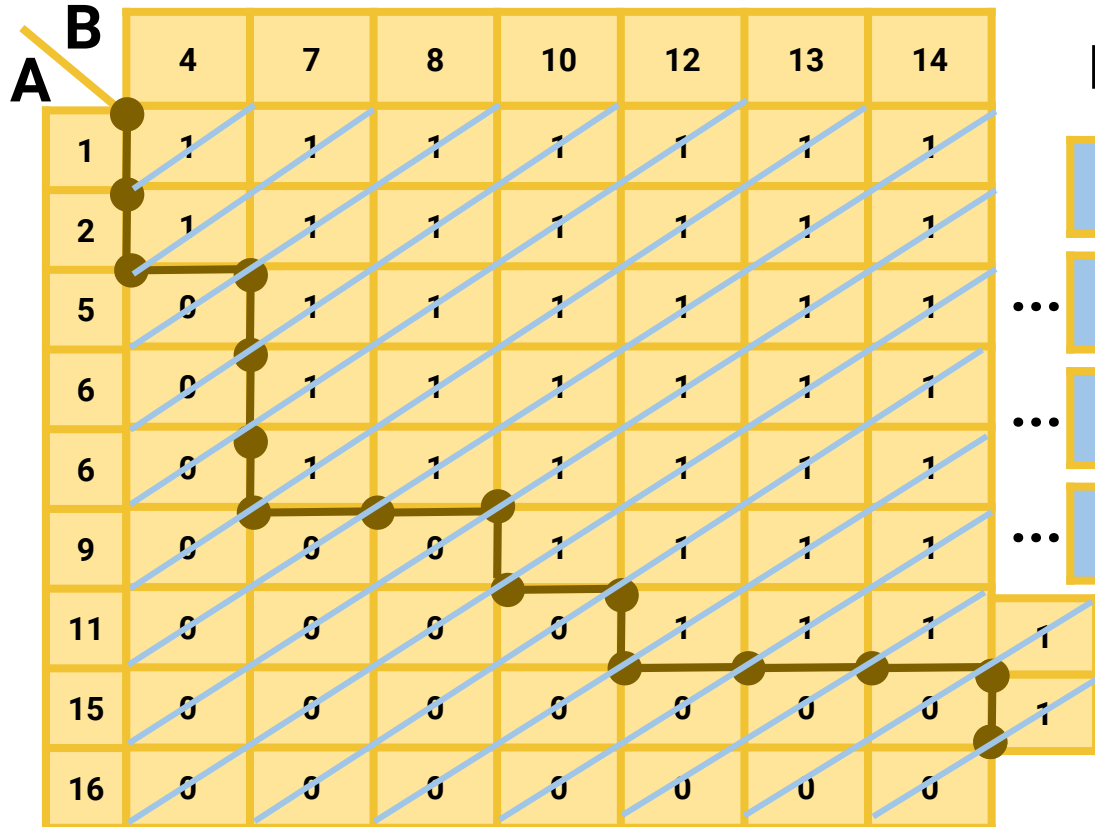
ALGO pathBig_k



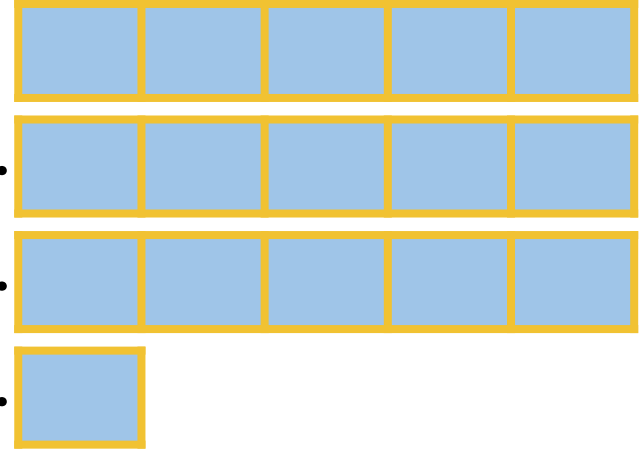
Path

...				
...				
...				

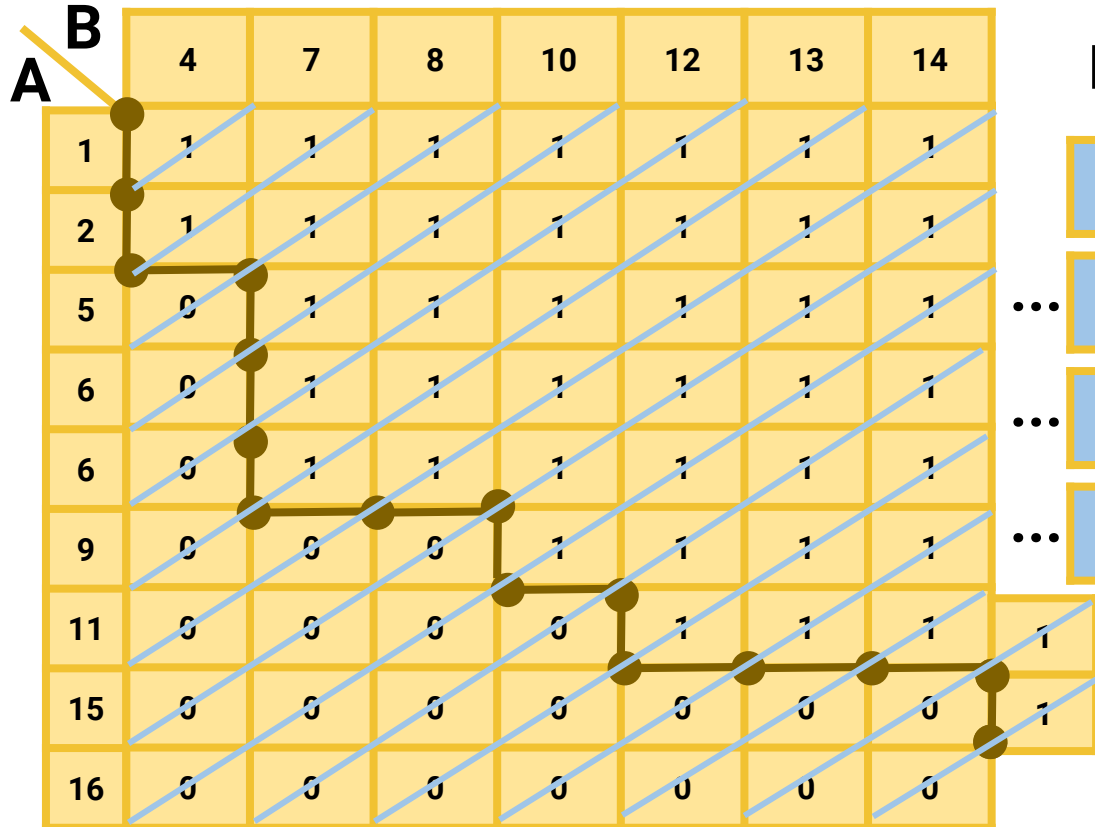
ALGO pathBig_k



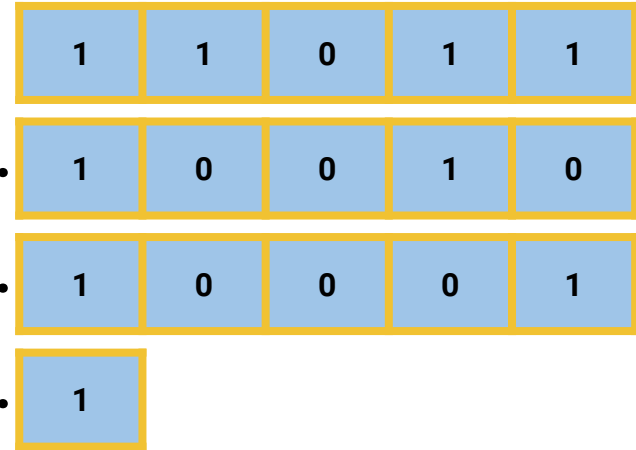
Path



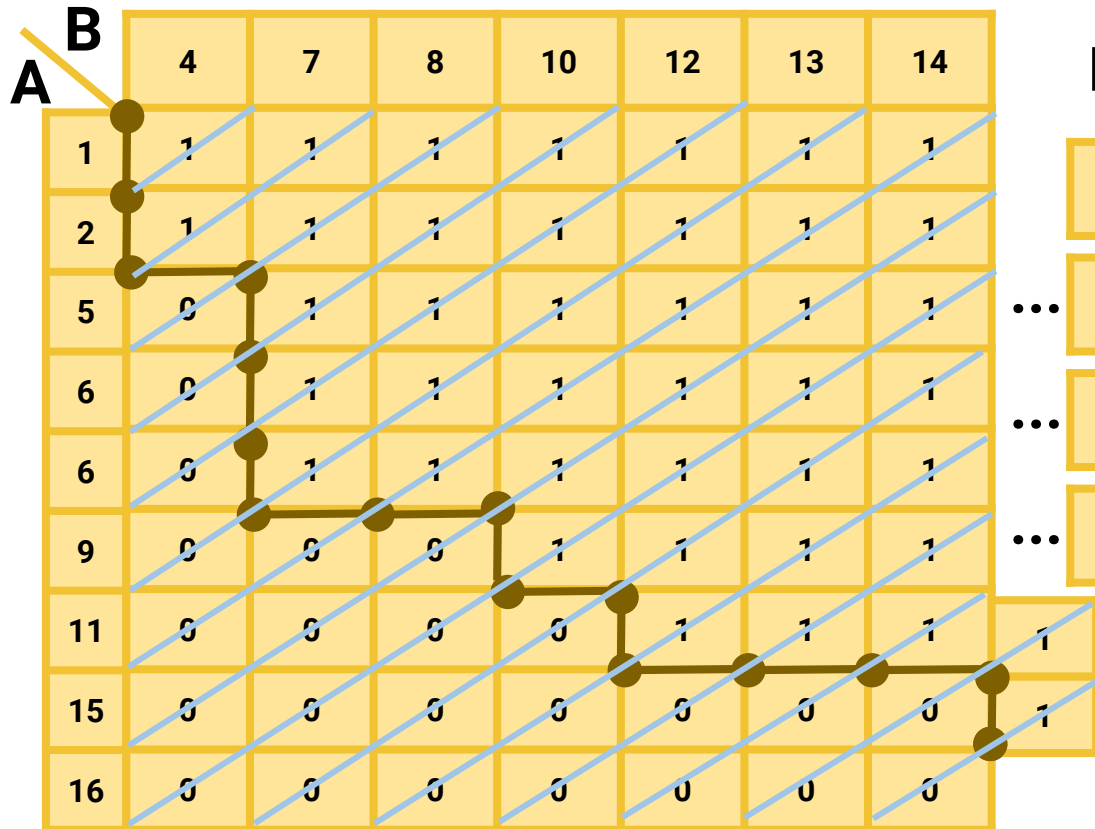
ALGO pathBig_k



Path



ALGO pathBig_k



Path

1	1	0	1	1
---	---	---	---	---

1	0	0	1	0
---	---	---	---	---

...

1	0	0	0	1
---	---	---	---	---

...

1

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2
---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4
---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5
---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6
---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6
---	---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

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M

1	2	4	5	6	6	7
---	---	---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

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M

1	2	4	5	6	6	7	8
---	---	---	---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9
---	---	---	---	---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11
---	---	---	---	---	---	---	---	---	----	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12
---	---	---	---	---	---	---	---	---	----	----	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12	13
---	---	---	---	---	---	---	---	---	----	----	----	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14
---	---	---	---	---	---	---	---	---	----	----	----	----	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16 ₇₁
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	------------------

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16 ₇₂
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	------------------

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16 ₇₃
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	------------------

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
0	1	0	2	4	5	1	2	6	3	7	4	5	6	8	9

M

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
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M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----

ALGO mergeBig_k

A

1	2	5	6	6	9	11	15	16
---	---	---	---	---	---	----	----	----

B

4	7	8	10	12	13	14
---	---	---	----	----	----	----

Path

1	1	0	1	1	1	0	0	1	0	1	0	0	0	1	1
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M

1	2	4	5	6	6	7	8	9	10	11	12	13	14	15	16
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----



03

“MERGE SORT” ON GPU

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

MERGE SORT ON CPU

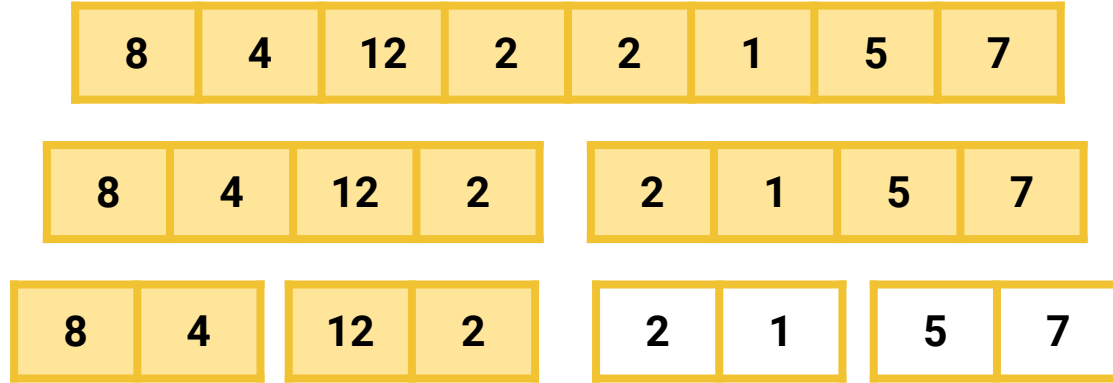
8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

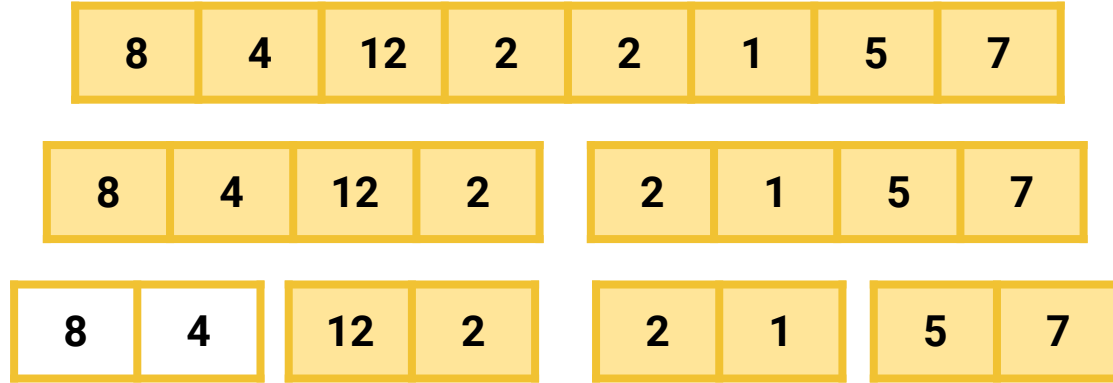
2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

MERGE SORT ON CPU



MERGE SORT ON CPU



MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4
---	---

12	2
----	---

2	1
---	---

5	7
---	---

4	8
---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

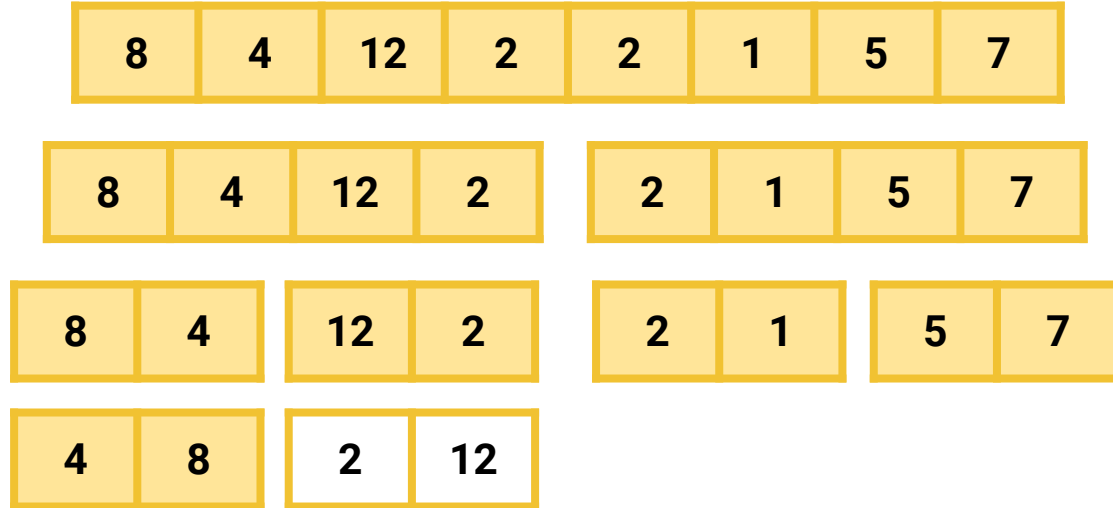
2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8
---	---

MERGE SORT ON CPU



MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

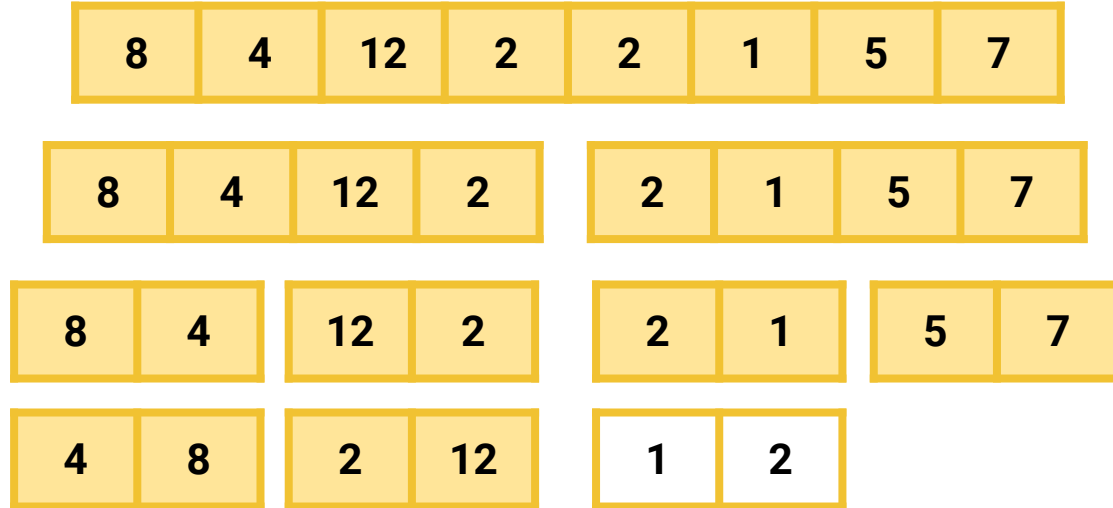
2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

MERGE SORT ON CPU



MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1
---	---

5	7
---	---

4	8
---	---

2	12
---	----

1	2
---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

1	2	5	7
---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

1	2	5	7
---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

1	2	5	7
---	---	---	---

2	4	8	12
---	---	---	----

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

1	2	5	7
---	---	---	---

2	4	8	12
---	---	---	----

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

1	2	5	7
---	---	---	---

2	4	8	12
---	---	---	----

1	2	5	7
---	---	---	---

MERGE SORT ON CPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
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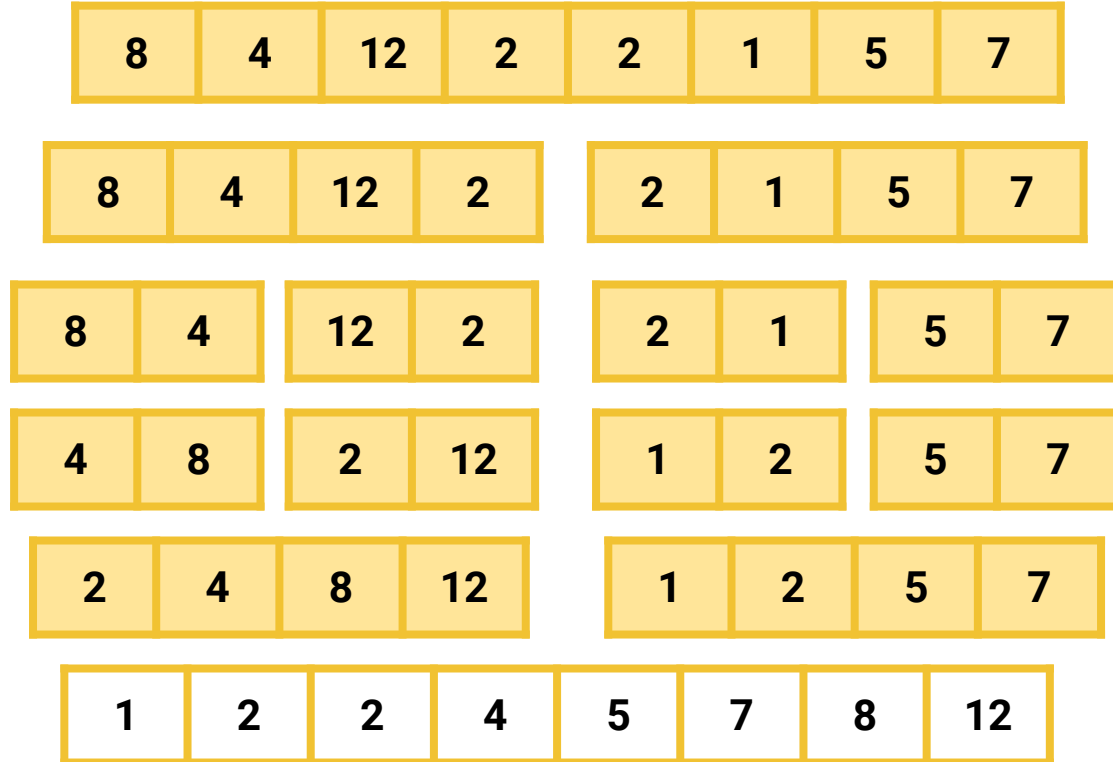
4	8	2	12
---	---	---	----

1	2	5	7
---	---	---	---

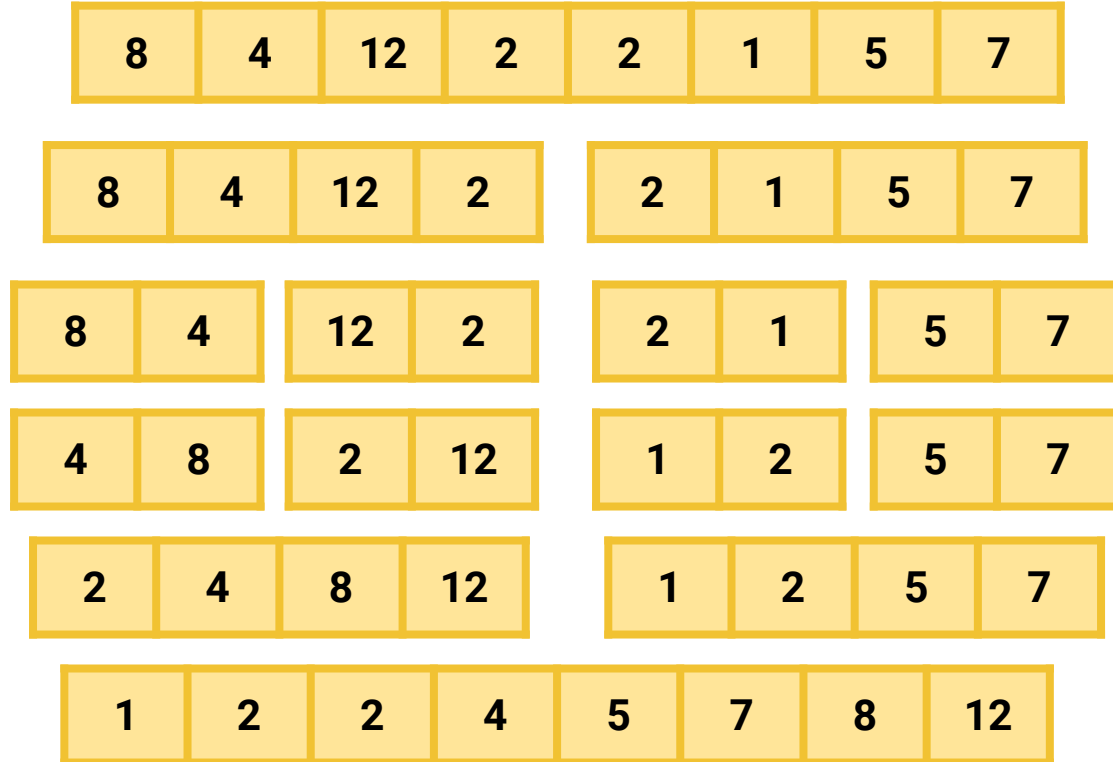
2	4	8	12
---	---	---	----

1	2	5	7
---	---	---	---

MERGE SORT ON CPU



MERGE SORT ON CPU



“MERGE SORT” ON GPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

8	4	12	2
---	---	----	---

2	1	5	7
---	---	---	---

4	8	2	12
---	---	---	----

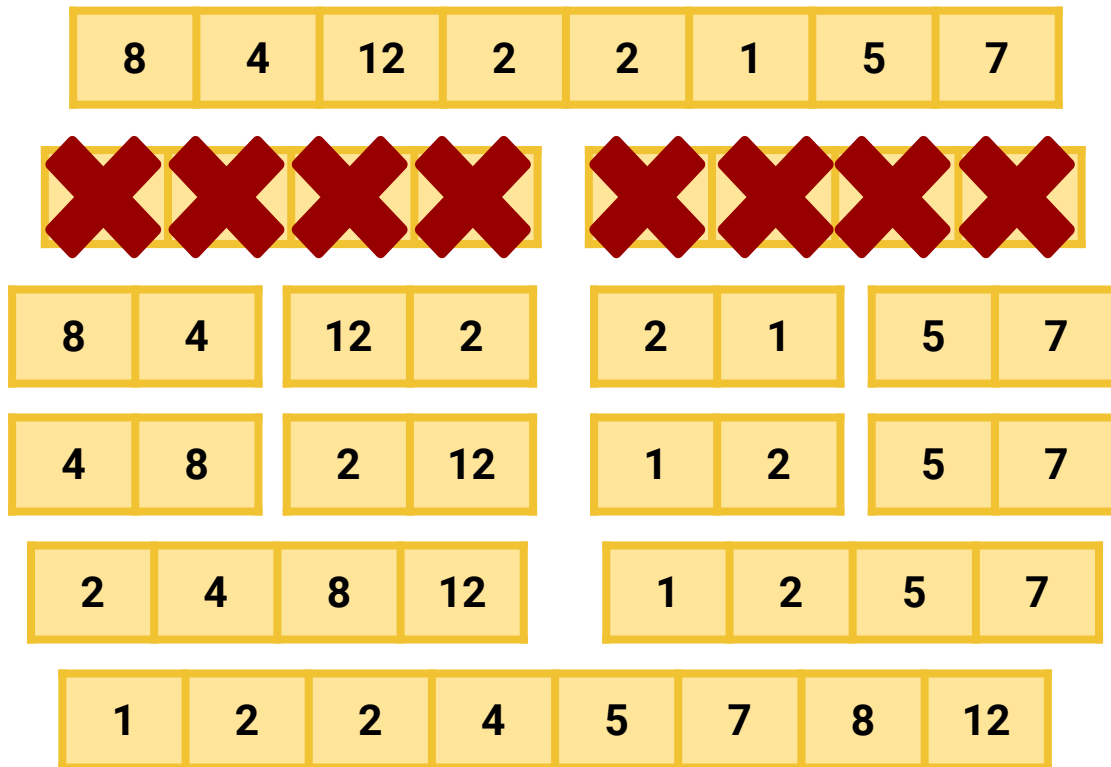
1	2	5	7
---	---	---	---

2	4	8	12
---	---	---	----

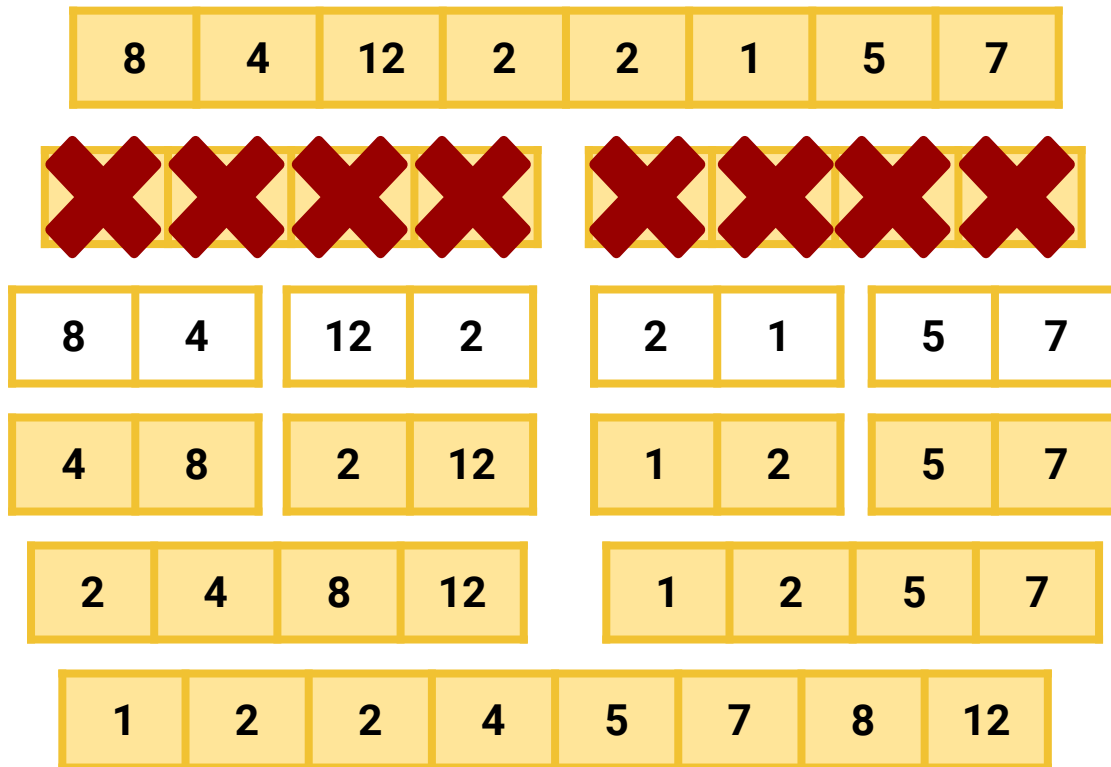
1	2	5	7
---	---	---	---

1	2	2	4	5	7	8	12
---	---	---	---	---	---	---	----

“MERGE SORT” ON GPU



“MERGE SORT” ON GPU



“MERGE SORT” ON GPU

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

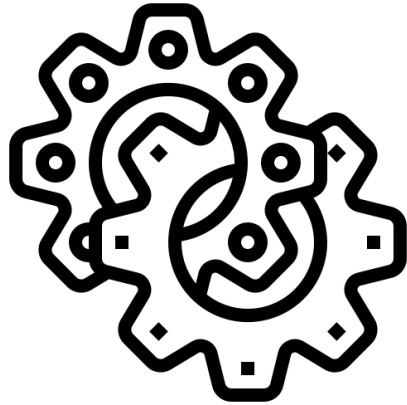
8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

“MERGE SORT” ON GPU

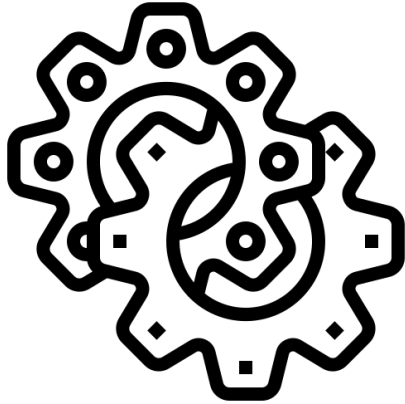
slice_size = 2
number_of_slices = 4

8	:	4	12	2	2	1	5	7
---	---	---	----	---	---	---	---	---

NOTATIONS

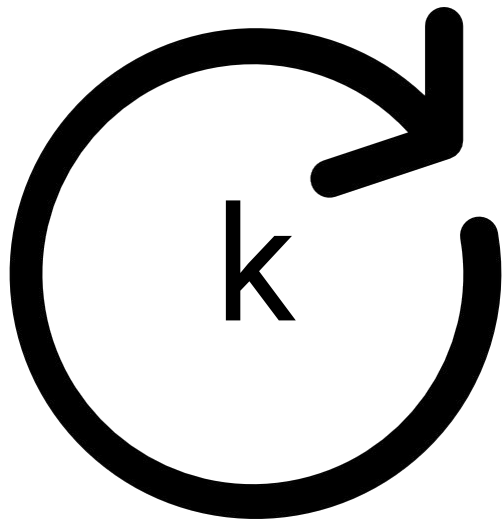


NOTATIONS

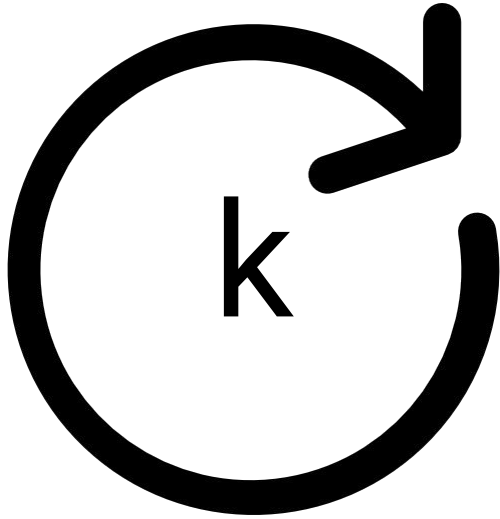


$\text{pathBig}_k(A,B)$
+
 $\text{mergeBig}_k(A,B)$

NOTATIONS



NOTATIONS



k loop tour

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



8	:	4	12	2	2	1	5	7
---	---	---	----	---	---	---	---	---



4	8
---	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



8	:	4		12	:	2		2		1		5		7
---	---	---	--	----	---	---	--	---	--	---	--	---	--	---



4	8
---	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



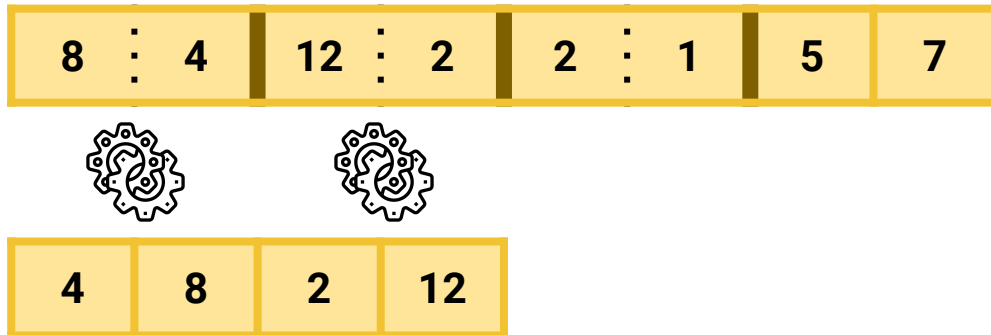
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



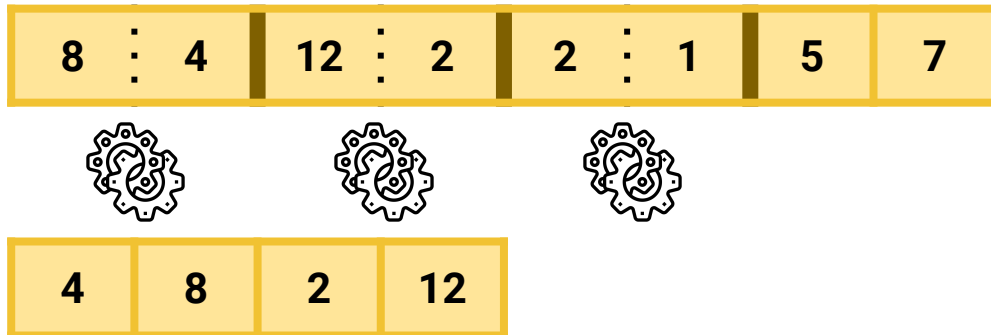
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



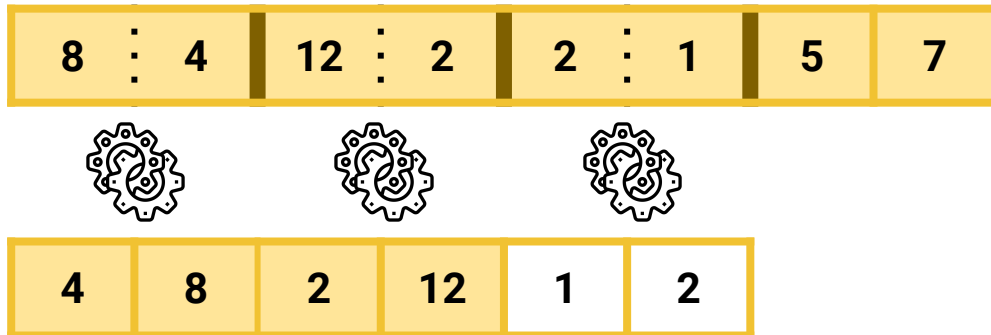
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



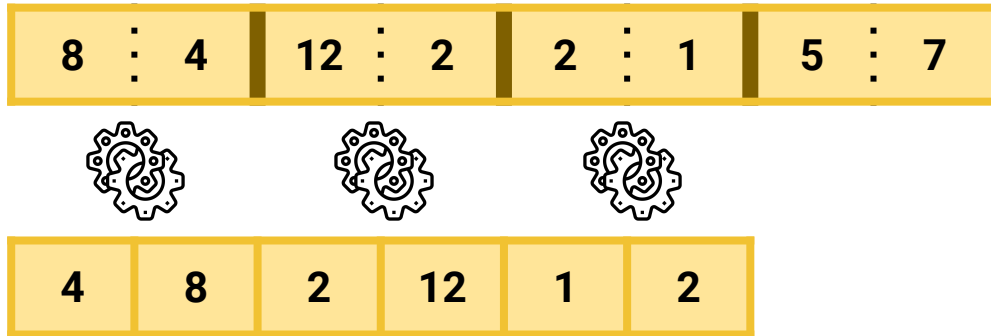
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



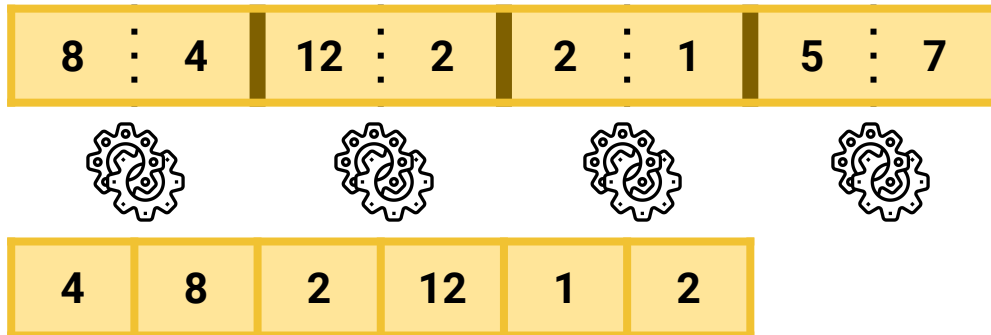
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



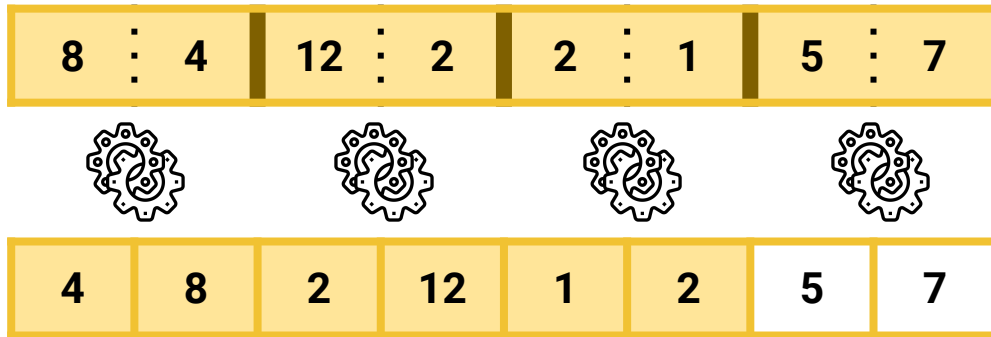
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



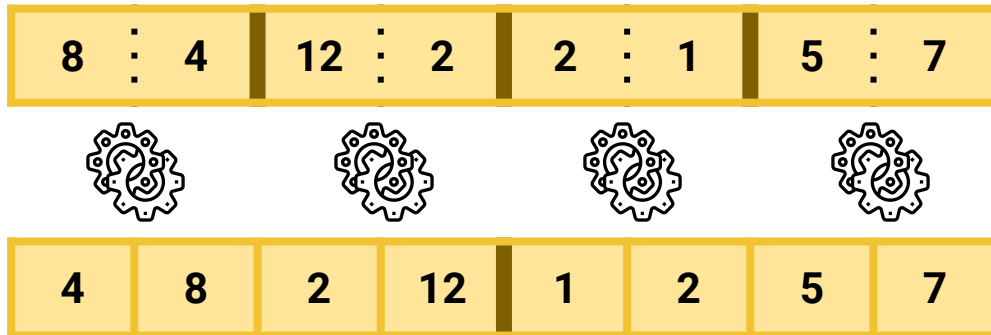
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4



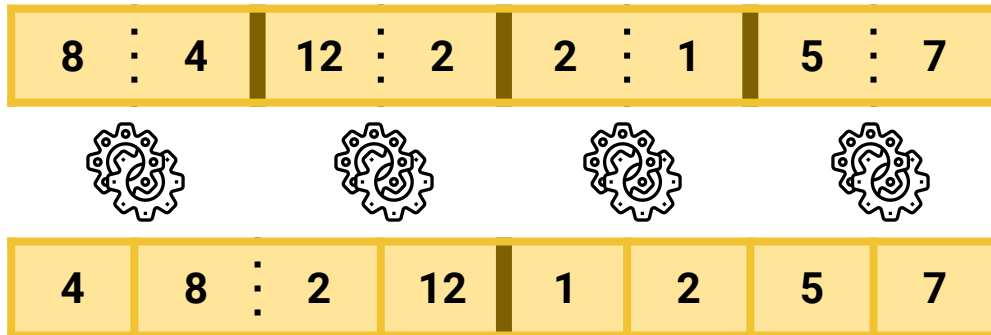
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



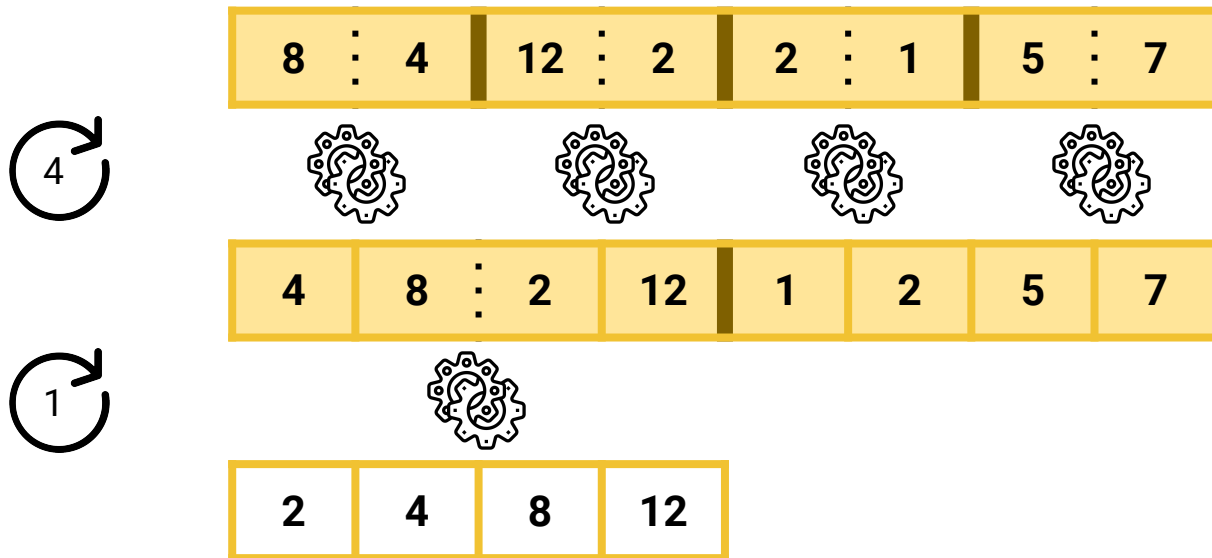
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



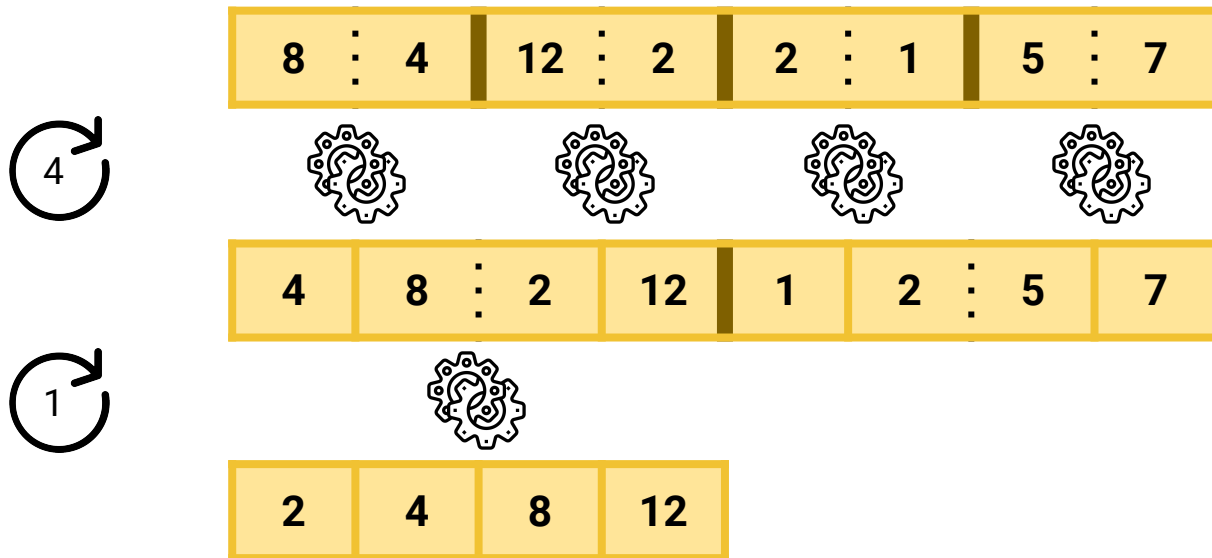
slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



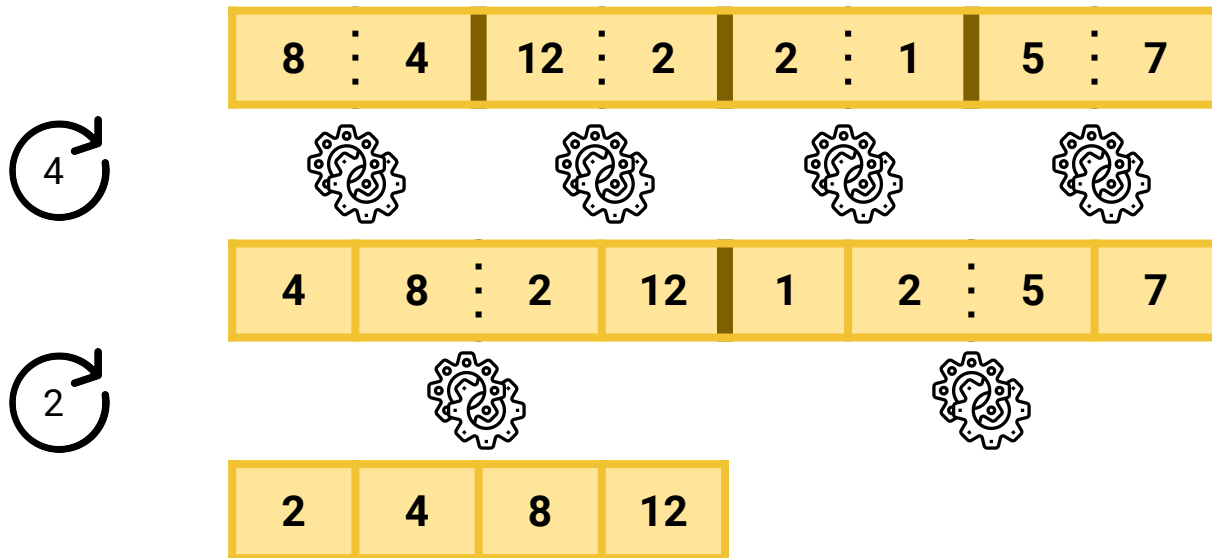
slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



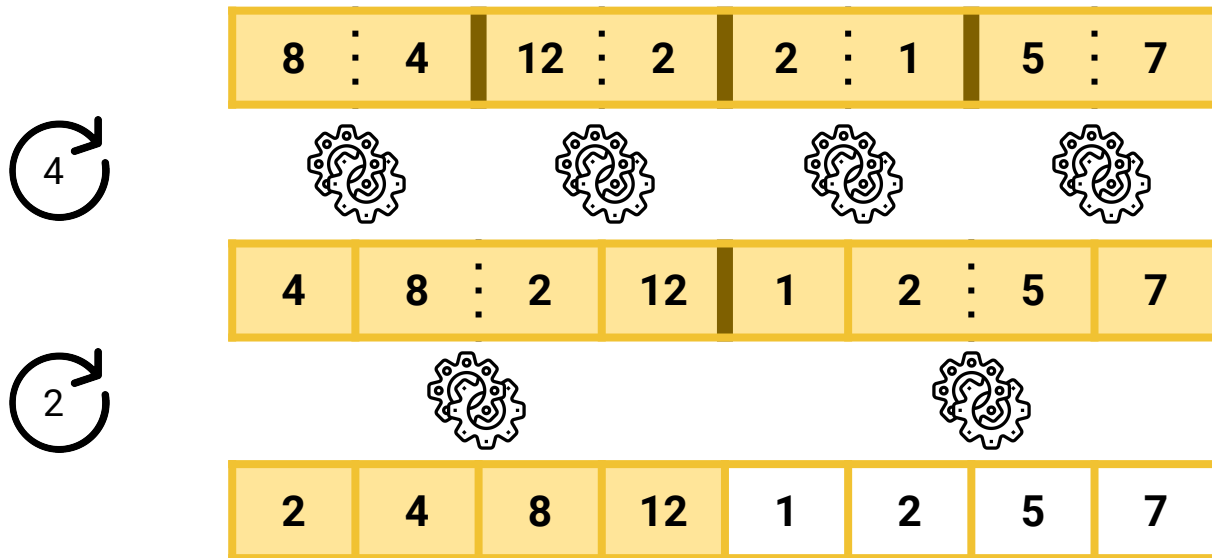
slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



slice_size = 4
number_of_slices = 2

“MERGE SORT” ON GPU



“MERGE SORT” ON GPU

slice_size = 8
number_of_slices = 1

4

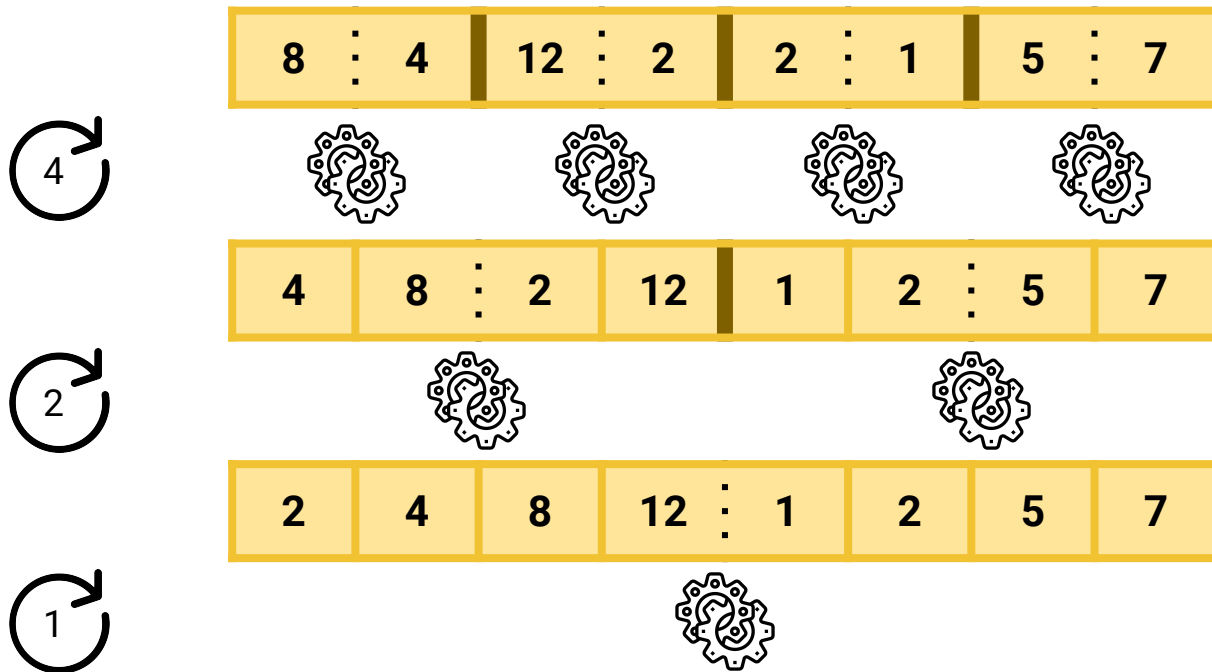


2



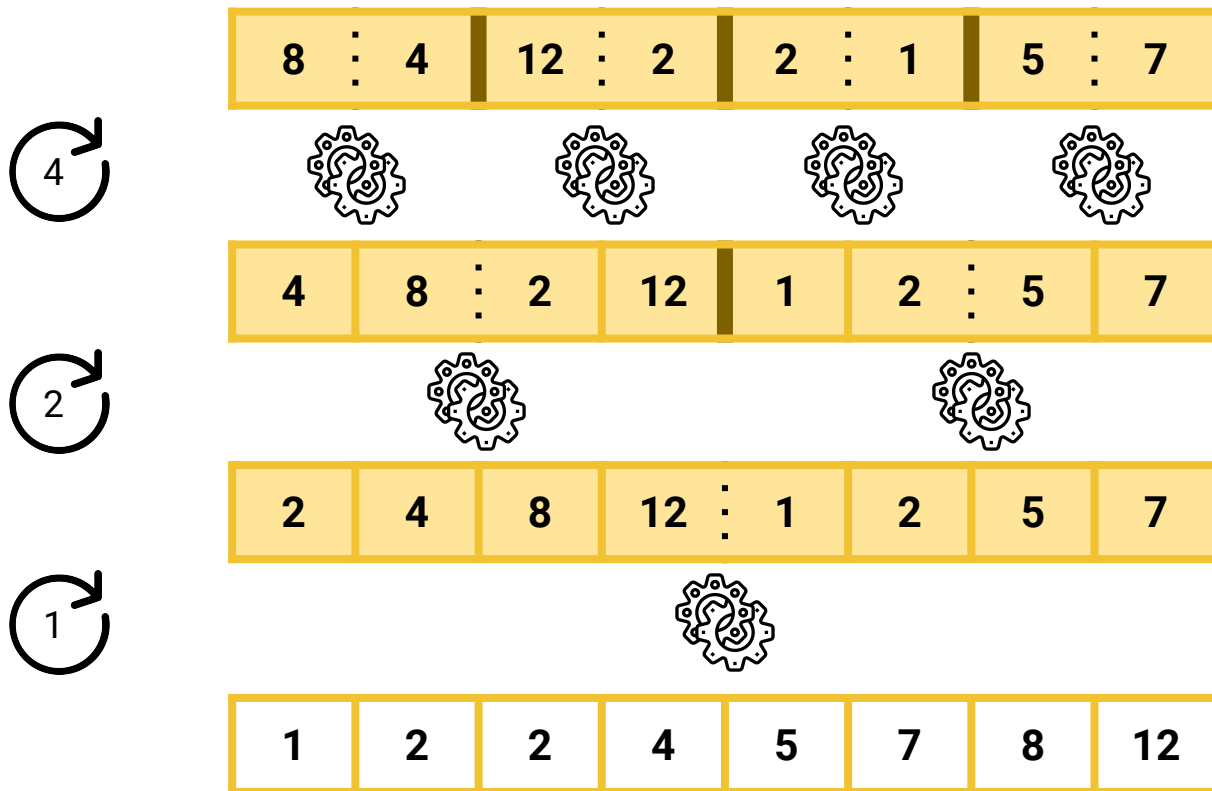
slice_size = 8
number_of_slices = 1

“MERGE SORT” ON GPU

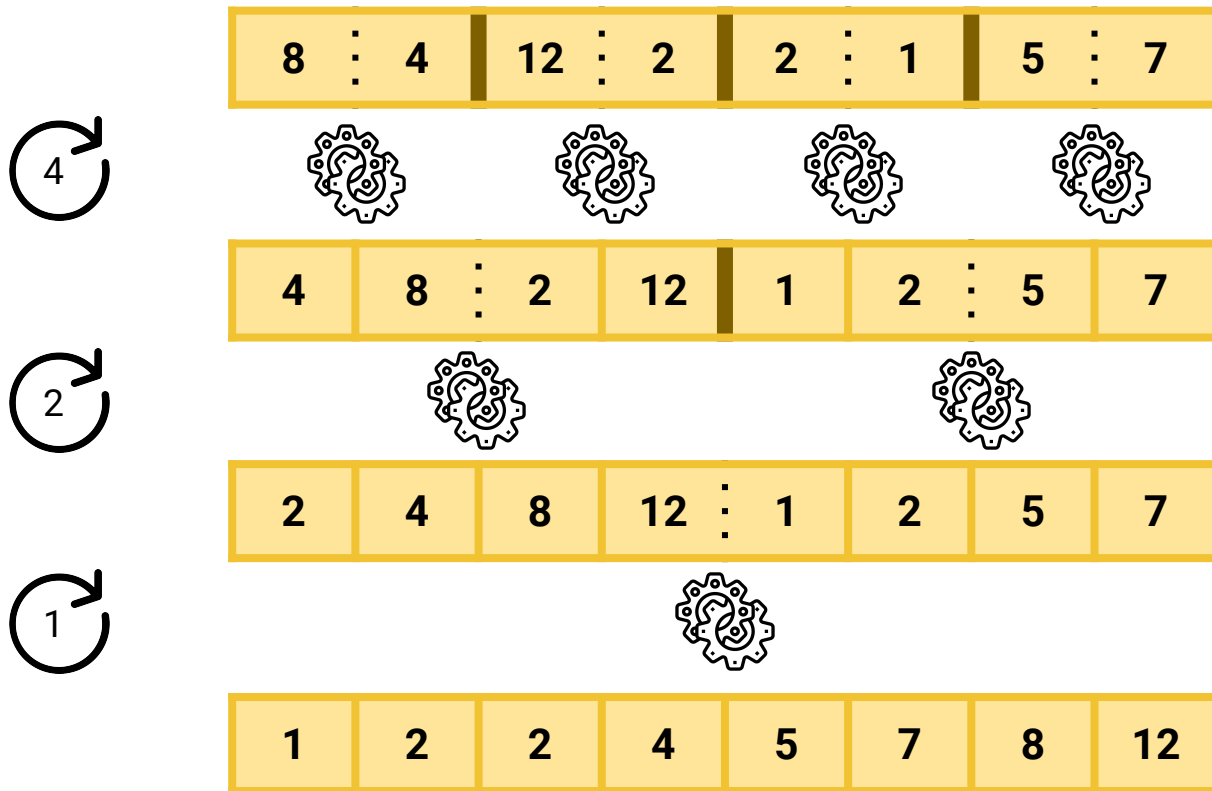


slice_size = 8
number_of_slices = 1

“MERGE SORT” ON GPU



“MERGE SORT” ON GPU



Ideal Case

Size of M is an exponent of 2

Every slices are full

More Often

M as any size

Some slices are not full

“MERGE SORT” ON GPU

1	4	5	3	7	2	6
---	---	---	---	---	---	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

1	4	5	3	7	2	6
---	---	---	---	---	---	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 4

1	4	5	3	7	2	6
---	---	---	---	---	---	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3

1	:	4		5	:	3		7	:	2		6
---	---	---	--	---	---	---	--	---	---	---	--	---

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



Remainder of 1
element

DEFINITION : REMAINDER

We call “remainder” an incomplete slice in our cutting

DEFINITION : REMAINDER

We call “remainder” an incomplete slice in our cutting

N.B : If there is a remainder, it's always and only on the last slice

“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



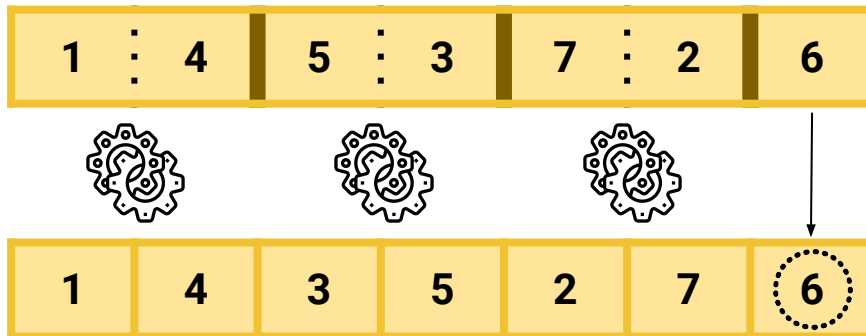
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



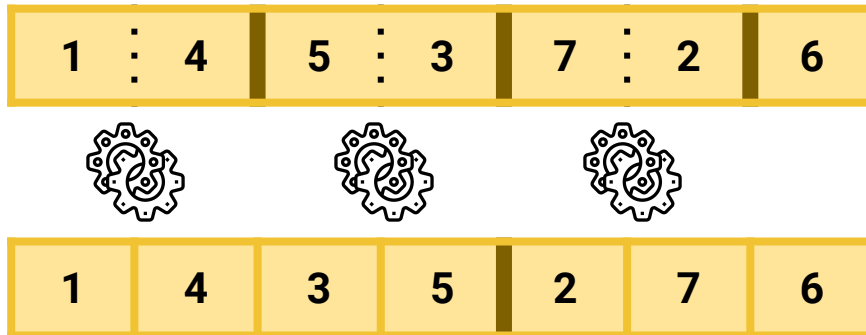
“MERGE SORT” ON GPU

slice_size = 2
number_of_slices = 3



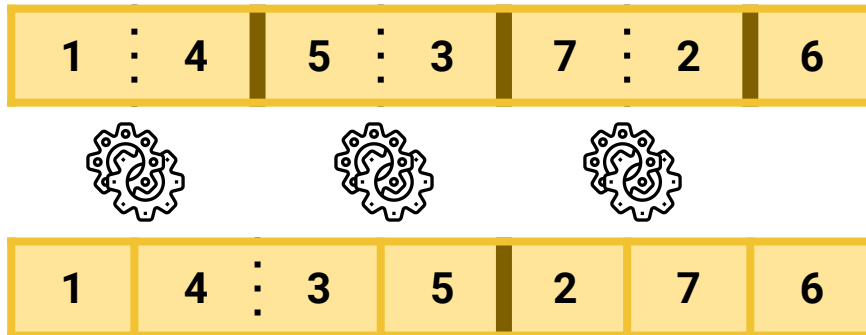
“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 2



“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 1



“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 1



“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 1



Remainder of 3
elements

“MERGE SORT” ON GPU

slice_size = 4
number_of_slices = 1



slice_size = 4
number_of_slices = 1

“MERGE SORT” ON GPU

3



2



slice_size = 4
number_of_slices = 1

“MERGE SORT” ON GPU

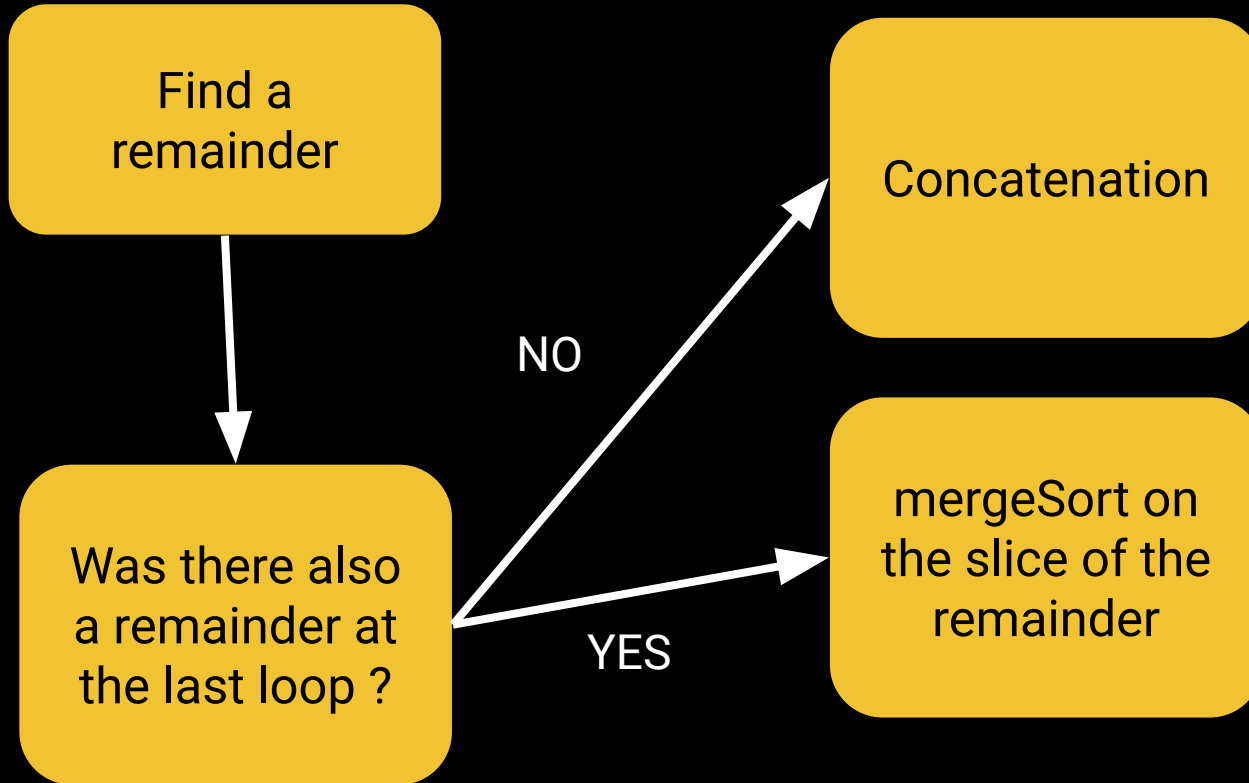
3



2



MANAGING THE REMAINDERS



“MERGE SORT” ON GPU

slice_size = 8
number_of_slices =



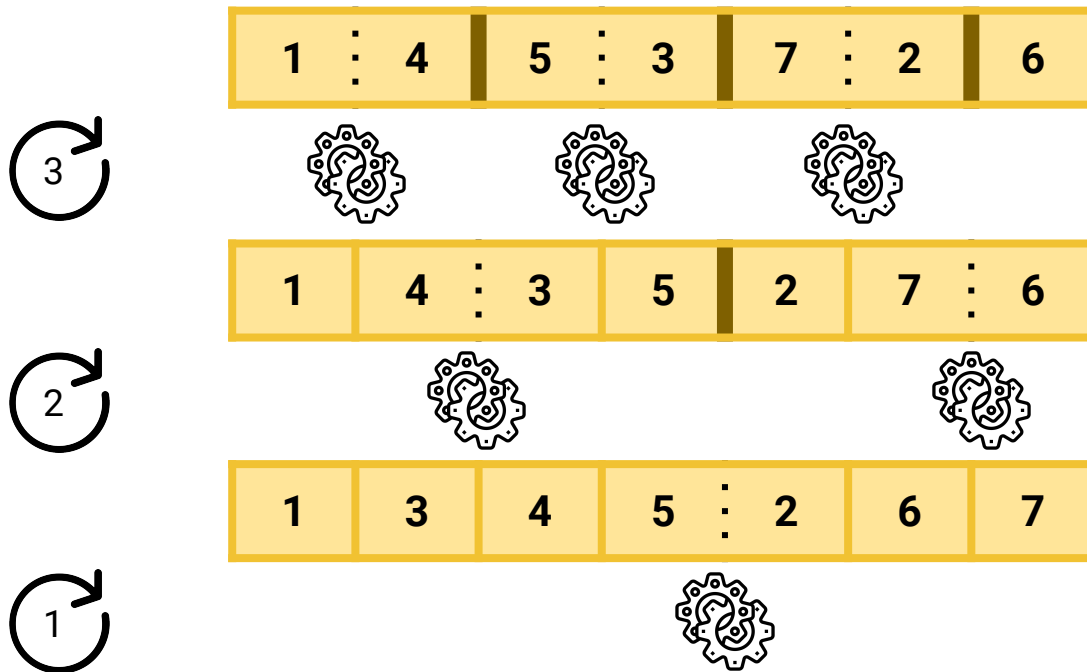
“MERGE SORT” ON GPU

slice_size = 8
number_of_slices = 1



slice_size = 8
number_of_slices = 1

“MERGE SORT” ON GPU



slice_size = 8
number_of_slices = 1

“MERGE SORT” ON GPU

3



2



1



“MERGE SORT” ON GPU

3



2



1



“MERGE SORT” ON GPU

```
slice_size = 1  
number_of_slices = 1  
reste_prec_size = 0  
reste_size = 0
```

```
while (number_of_slices > 0)  
    slice_size = 2*slice_size  
    number_of_slices = size_M/slice_size  
    reste_prec_size = reste_size  
    reste_size = size_M%slice_size  
  
    for (i=0, i<number_of_slices, i++)  
        sort(M, slice_size, i)  
  
    if (reste_prec_size!=0 && reste_size!=0)  
  
        sort(M, reste_size + reste_prec_size, number_of_slices)
```

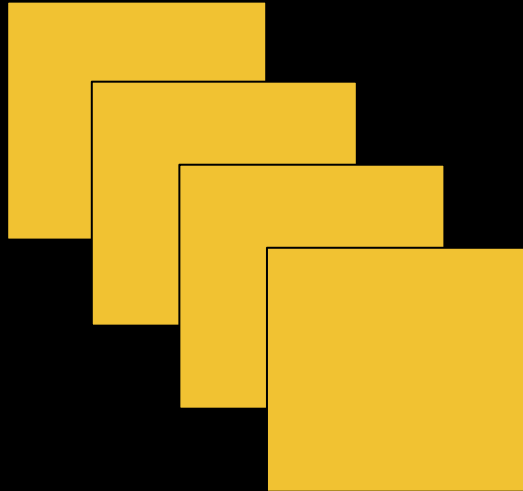


04

SORT WITH BATCHES

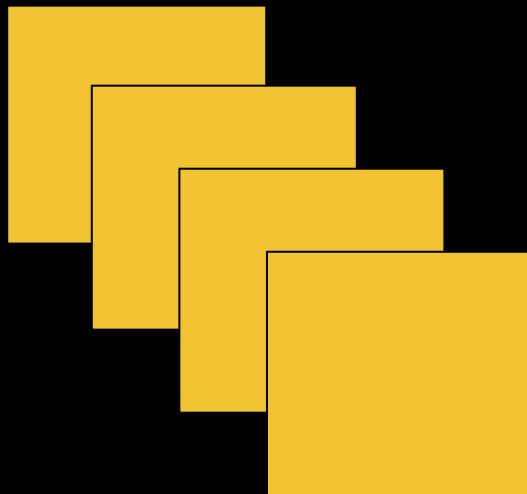
DEFINITION : BATCH

Sub array with at the most 1024 elements

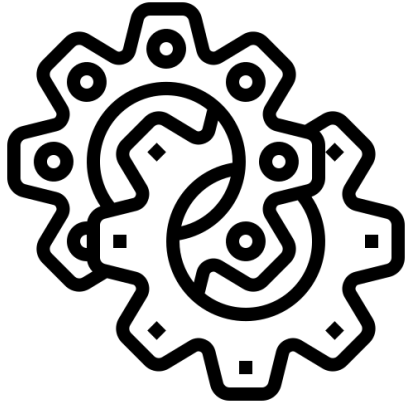


DEFINITION : BATCH

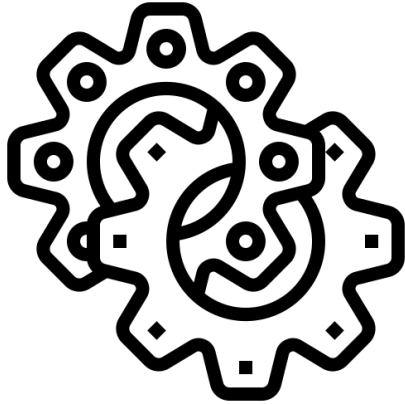
Sub array with at the most 1024 elements
(ie the maximum number of threads on one block)



NOTATIONS

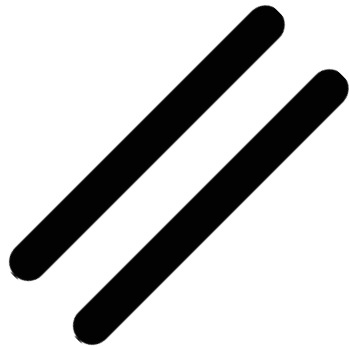


NOTATIONS

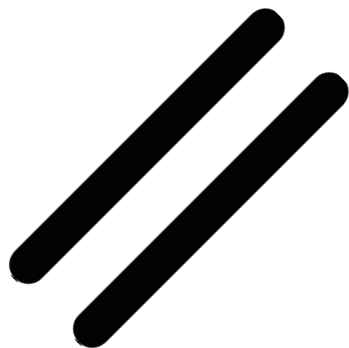


`mergeSmallBatch_k(A,B)`

NOTATIONS



NOTATIONS



In parallel

SORT ON ONE BATCH

8	4	12	1	5	7
---	---	----	-----	-----	---	---	---

SORT ON ONE BATCH

t1	t2	t3	t1022	t1023	t1024
----	----	----	-----	-----	-------	-------	-------

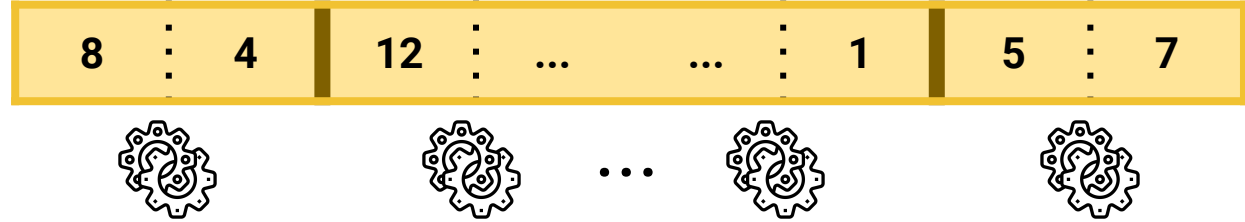
SORT ON ONE BATCH

8	4	12	1	5	7
---	---	----	-----	-----	---	---	---

SORT ON ONE BATCH

8	:	4		12	:	:	1		5	:	7
---	---	---	--	----	---	-----	--	-----	---	---	--	---	---	---

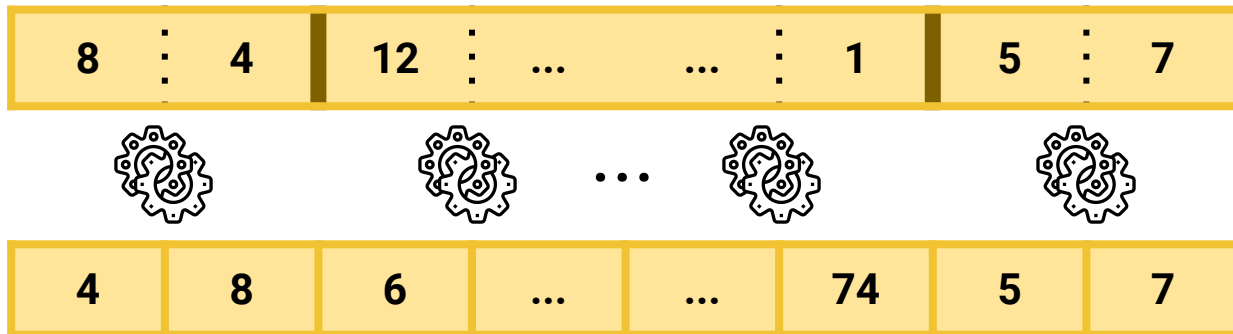
SORT ON ONE BATCH



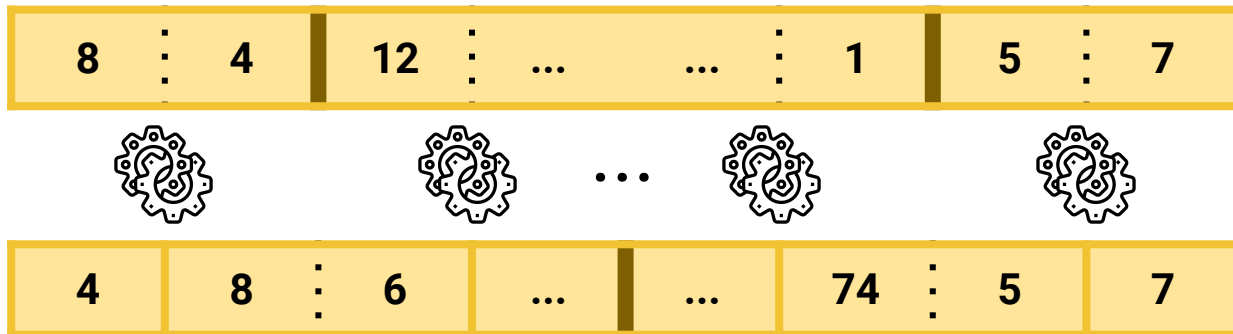
SORT ON ONE BATCH



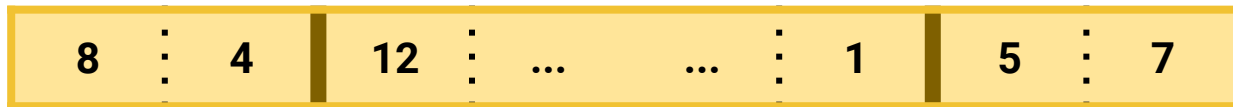
SORT ON ONE BATCH



SORT ON ONE BATCH



SORT ON ONE BATCH



...



...



SORT ON ONE BATCH



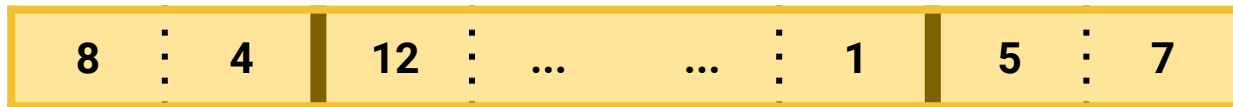
...



...



SORT ON ONE BATCH



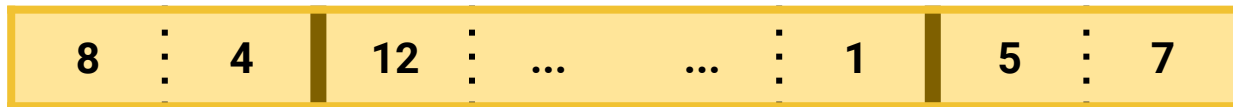
...



...



SORT ON ONE BATCH



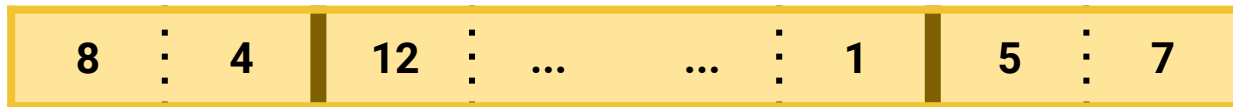
...



...



SORT ON ONE BATCH



...



...



SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

Batches of 1024 elements

SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

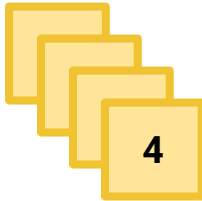
Batches of 1024 elements ↘

SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

Batches of 1024 elements ↘



SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

Batches of 1024 elements



SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

Batches of 1024 elements



SMALL SORT

M

...	4	26	17	2
-----	-----	-----	---	-----	-----	-----	----	-----	-----	-----	----	-----	-----	-----	---

Batches of 1024 elements



SMALL SORT

M



Batches of 1024 elements



We sort each batch in parallel

SMALL SORT

M



Batches of 1024 elements



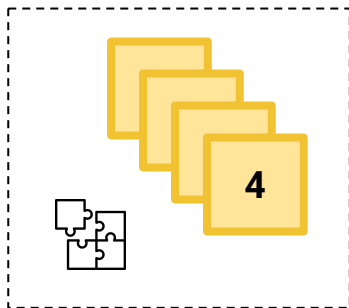
We sort each batch in parallel on one block

SMALL SORT

M



Batches of 1024 elements



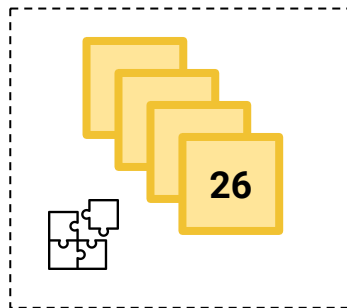
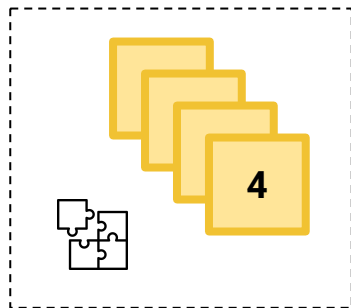
We sort each batch in parallel on one block

SMALL SORT

M



Batches of 1024 elements



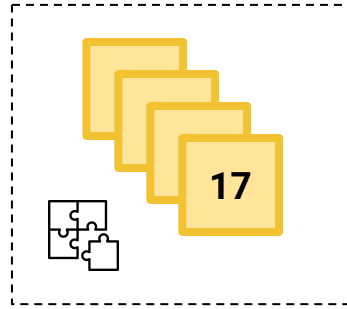
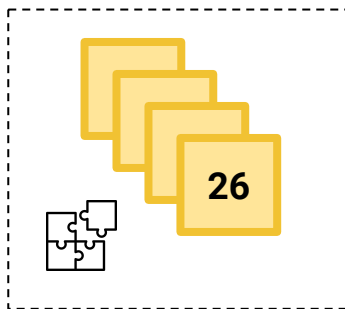
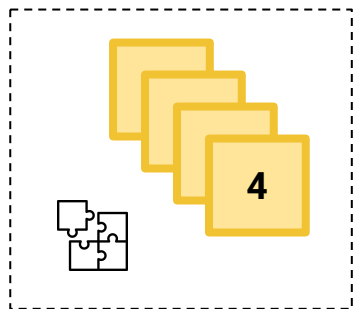
We sort each batch in parallel on one block

SMALL SORT

M



Batches of 1024 elements



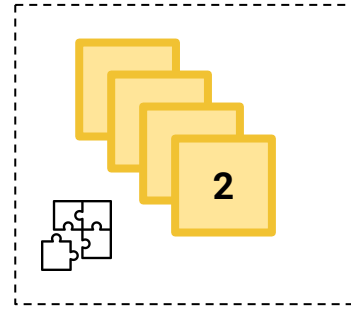
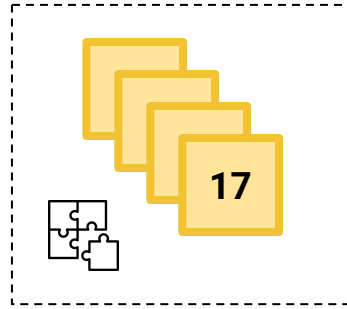
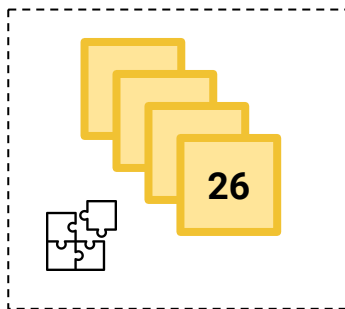
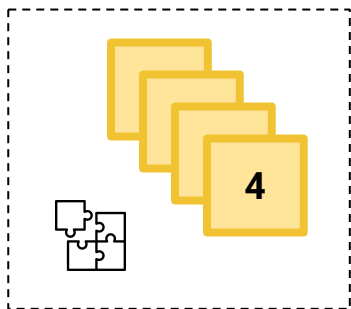
We sort each batch in parallel on one block

SMALL SORT

M



Batches of 1024 elements



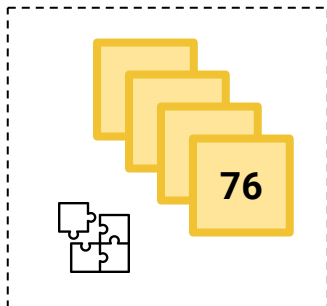
We sort each batch in parallel on one block

BIG SORT

Batches of 1024 sorted elements

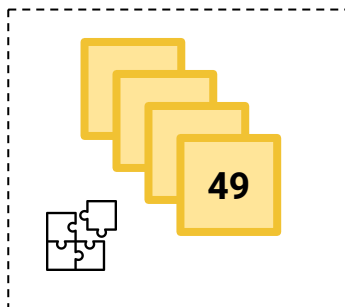
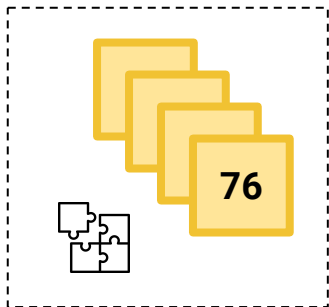
BIG SORT

Batches of 1024 sorted elements



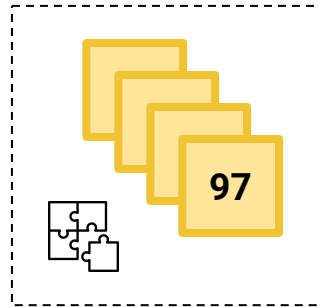
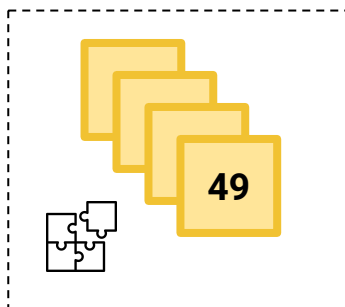
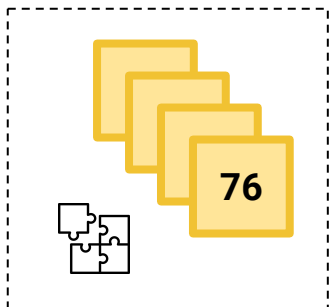
BIG SORT

Batches of 1024 sorted elements



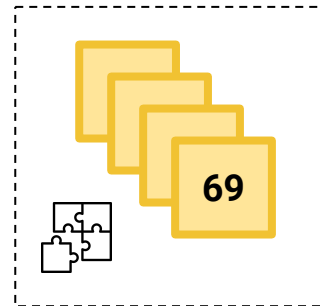
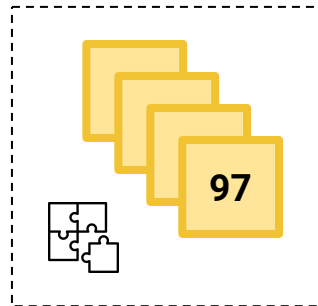
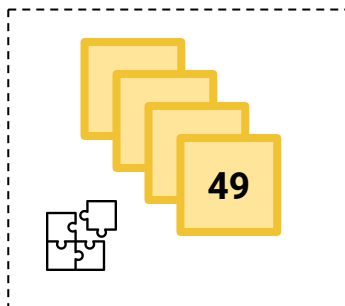
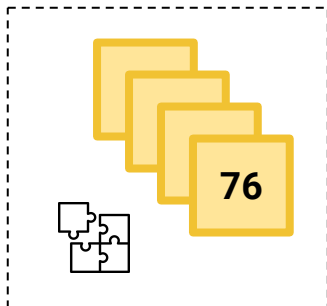
BIG SORT

Batches of 1024 sorted elements



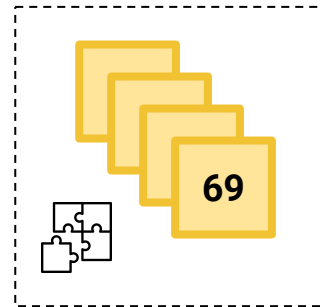
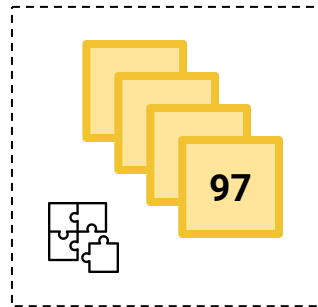
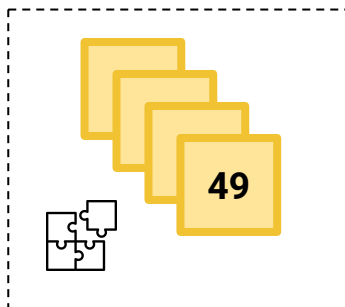
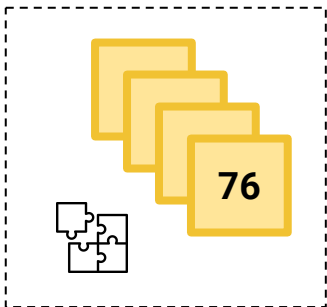
BIG SORT

Batches of 1024 sorted elements



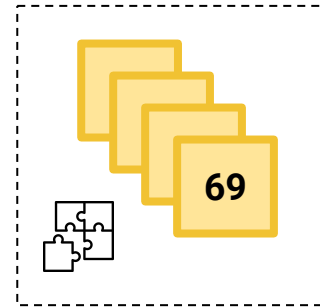
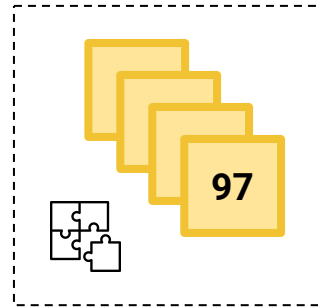
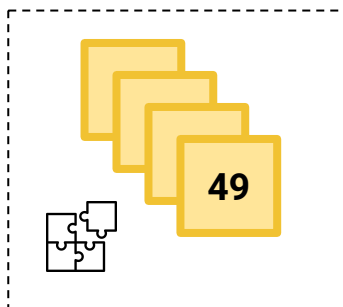
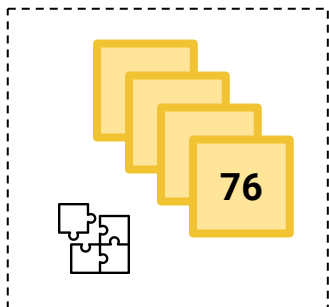
BIG SORT

Batches of 1024 sorted elements



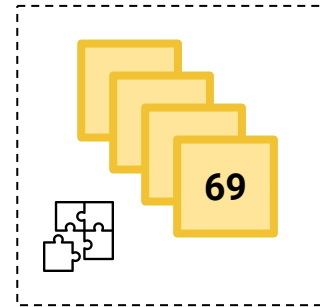
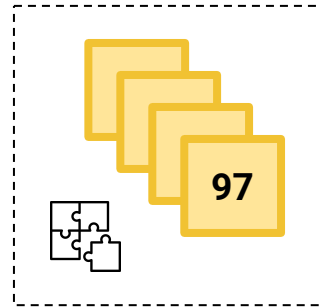
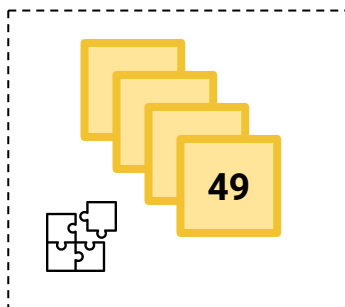
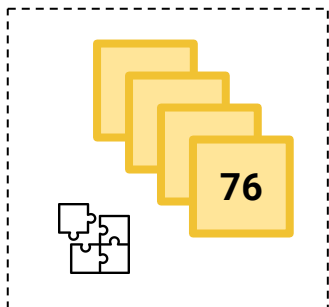
BIG SORT

Batches of 1024 sorted elements



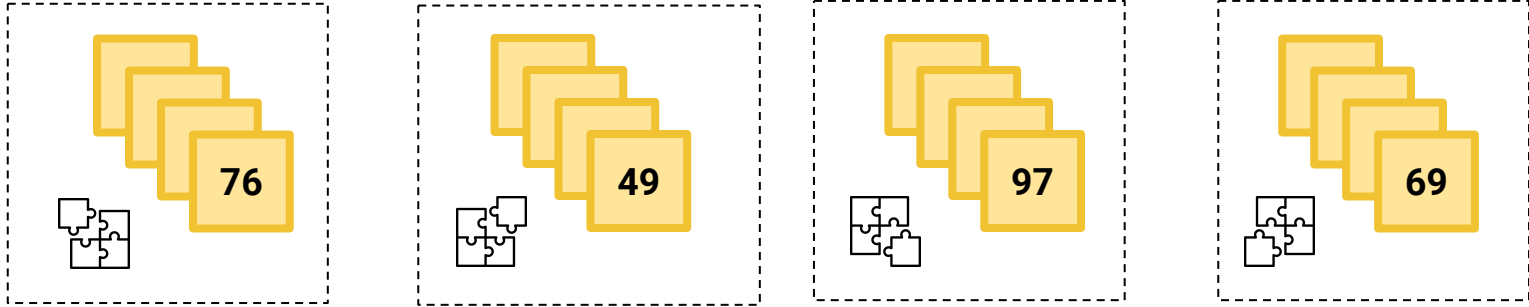
BIG SORT

Batches of 1024 sorted elements



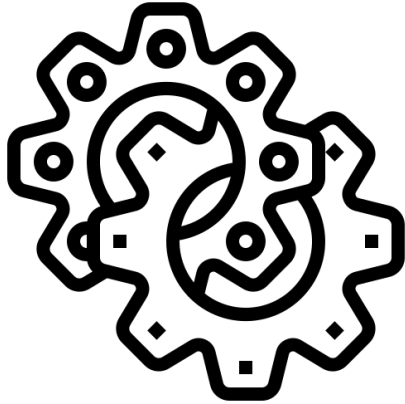
BIG SORT

Batches of 1024 sorted elements

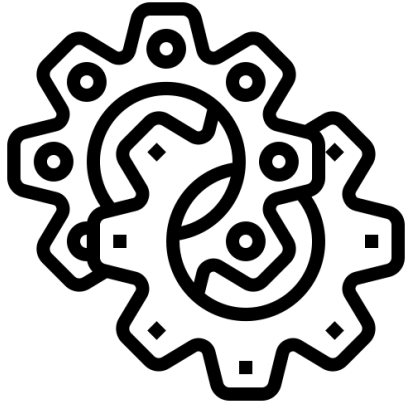


We sort the new reconstituted array

NOTATIONS



NOTATIONS



$\text{pathBig}_k(A,B)$
+
 $\text{mergeBig}_k(A,B)$

GPU SORT

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

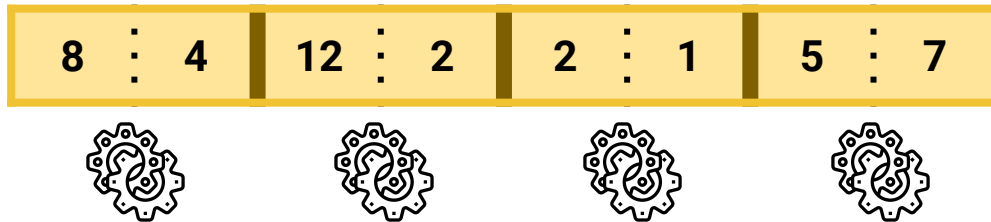
GPU SORT

8	4	12	2	2	1	5	7
---	---	----	---	---	---	---	---

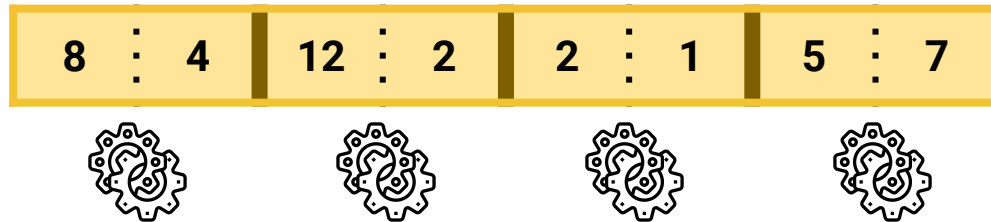
GPU SORT

8	:	4		12	:	2		2	:	1		5	:	7
---	---	---	--	----	---	---	--	---	---	---	--	---	---	---

GPU SORT



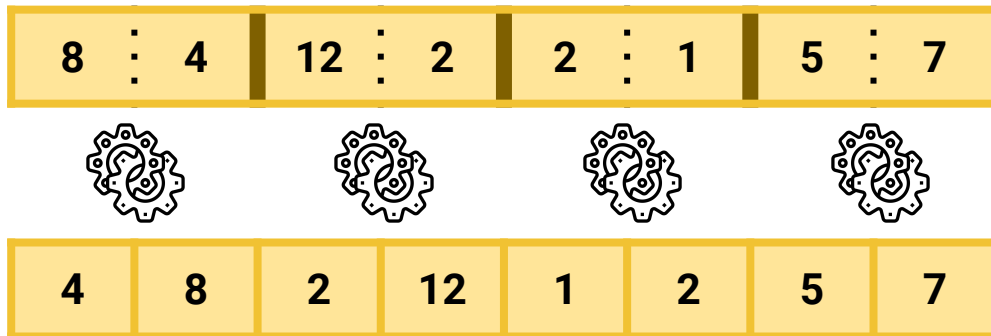
GPU SORT WITH STREAM



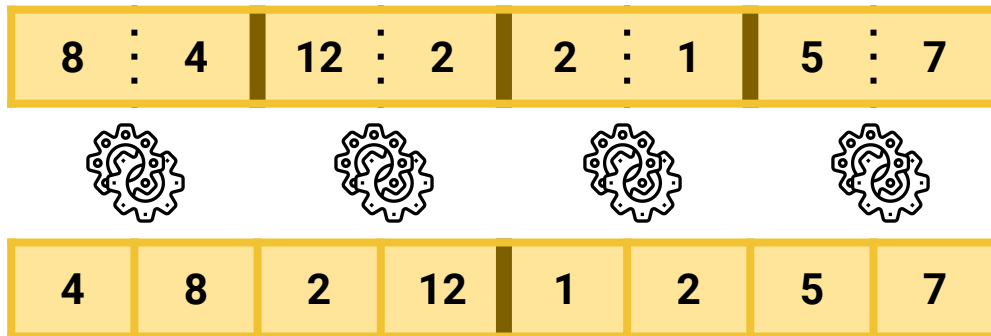
GPU SORT WITH STREAM



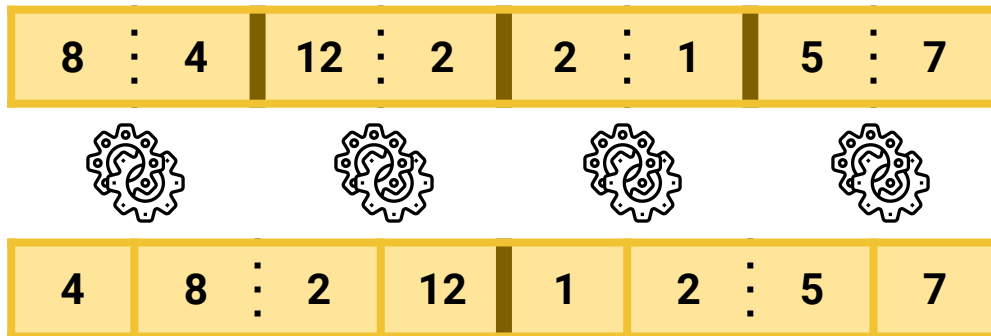
GPU SORT WITH STREAM



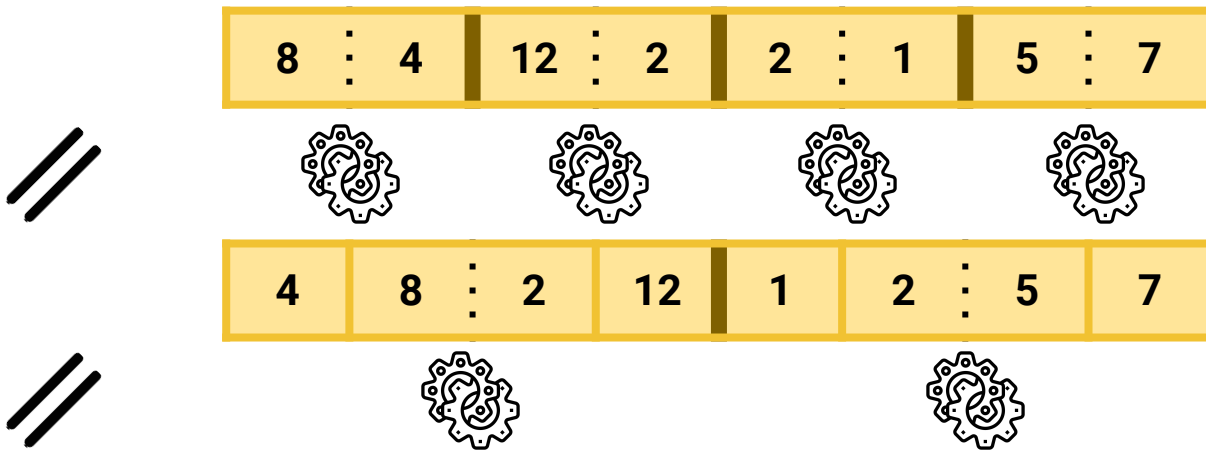
GPU SORT WITH STREAM



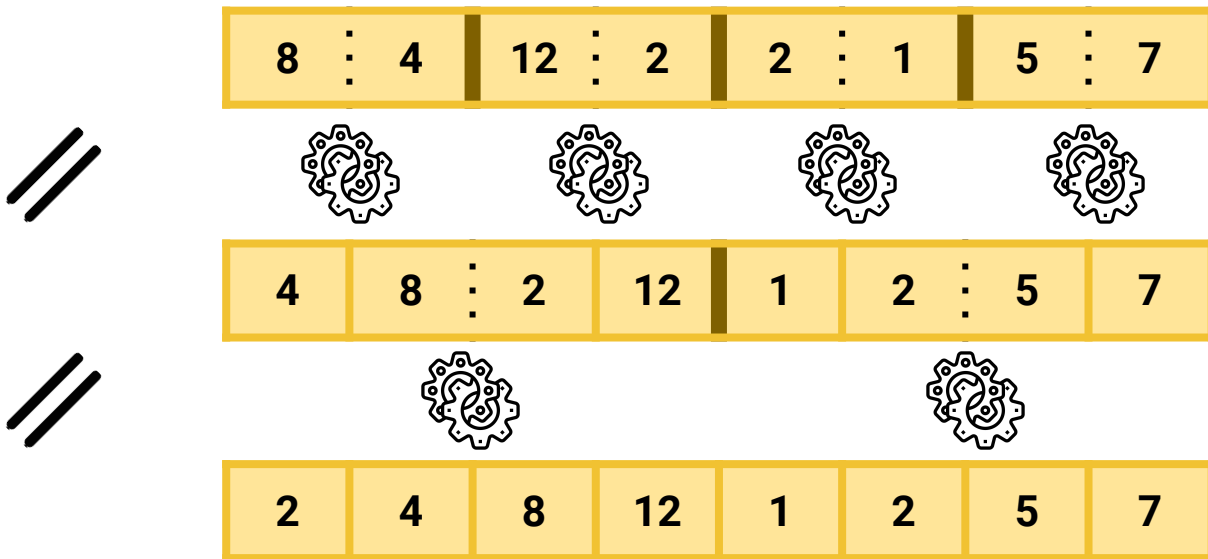
GPU SORT WITH STREAM



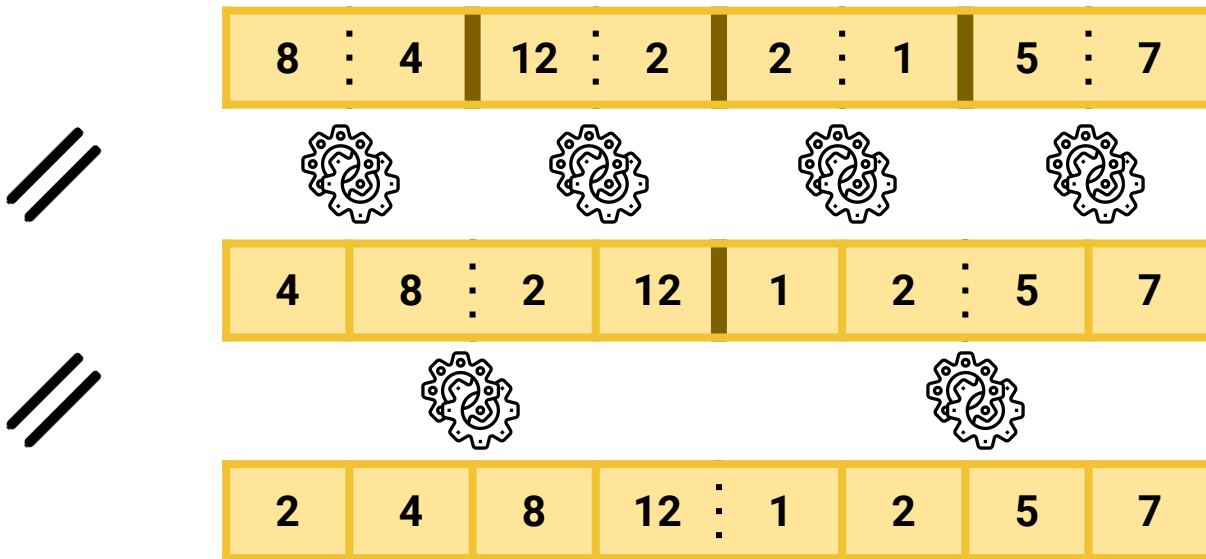
GPU SORT WITH STREAM



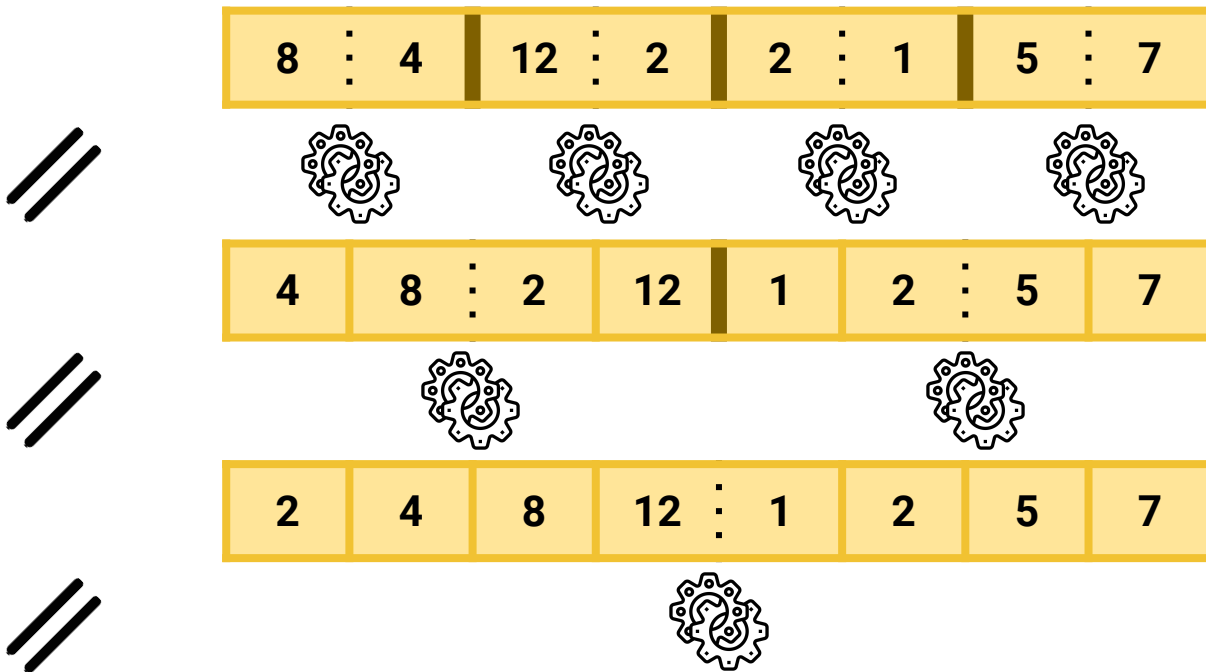
GPU SORT WITH STREAM



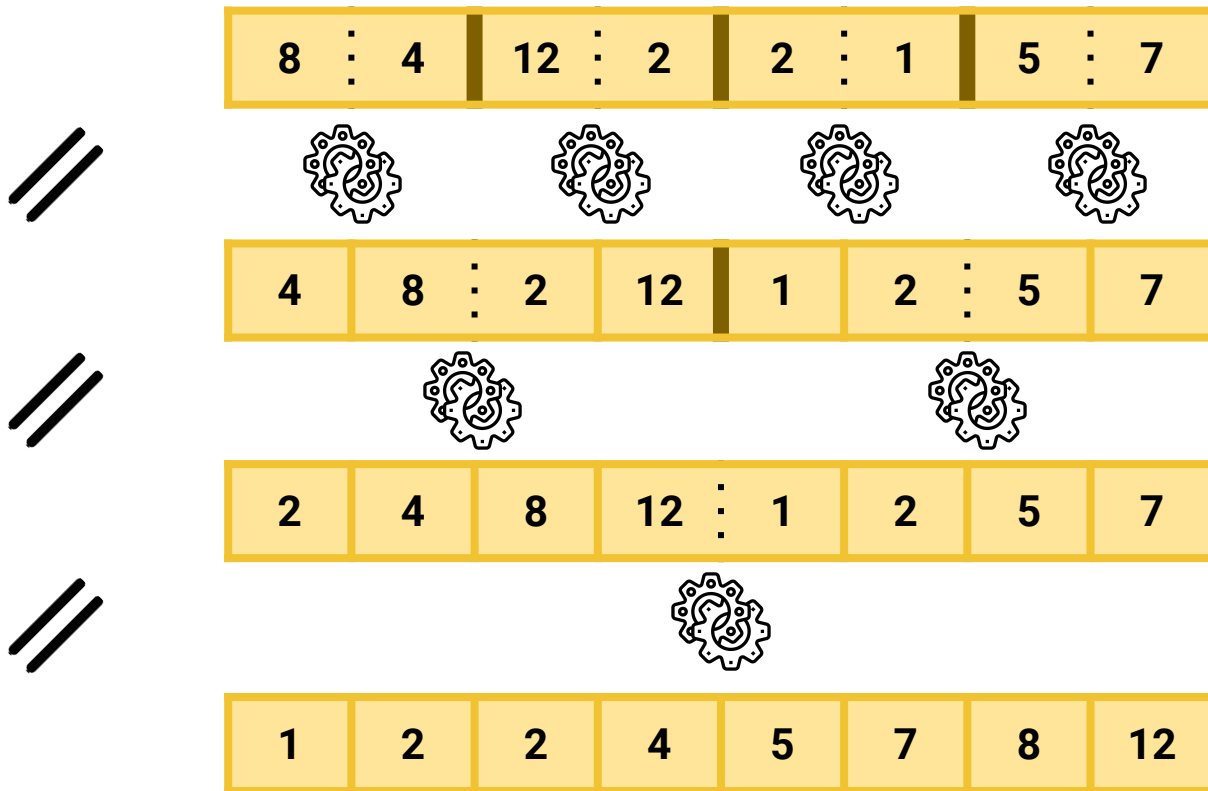
GPU SORT WITH STREAM

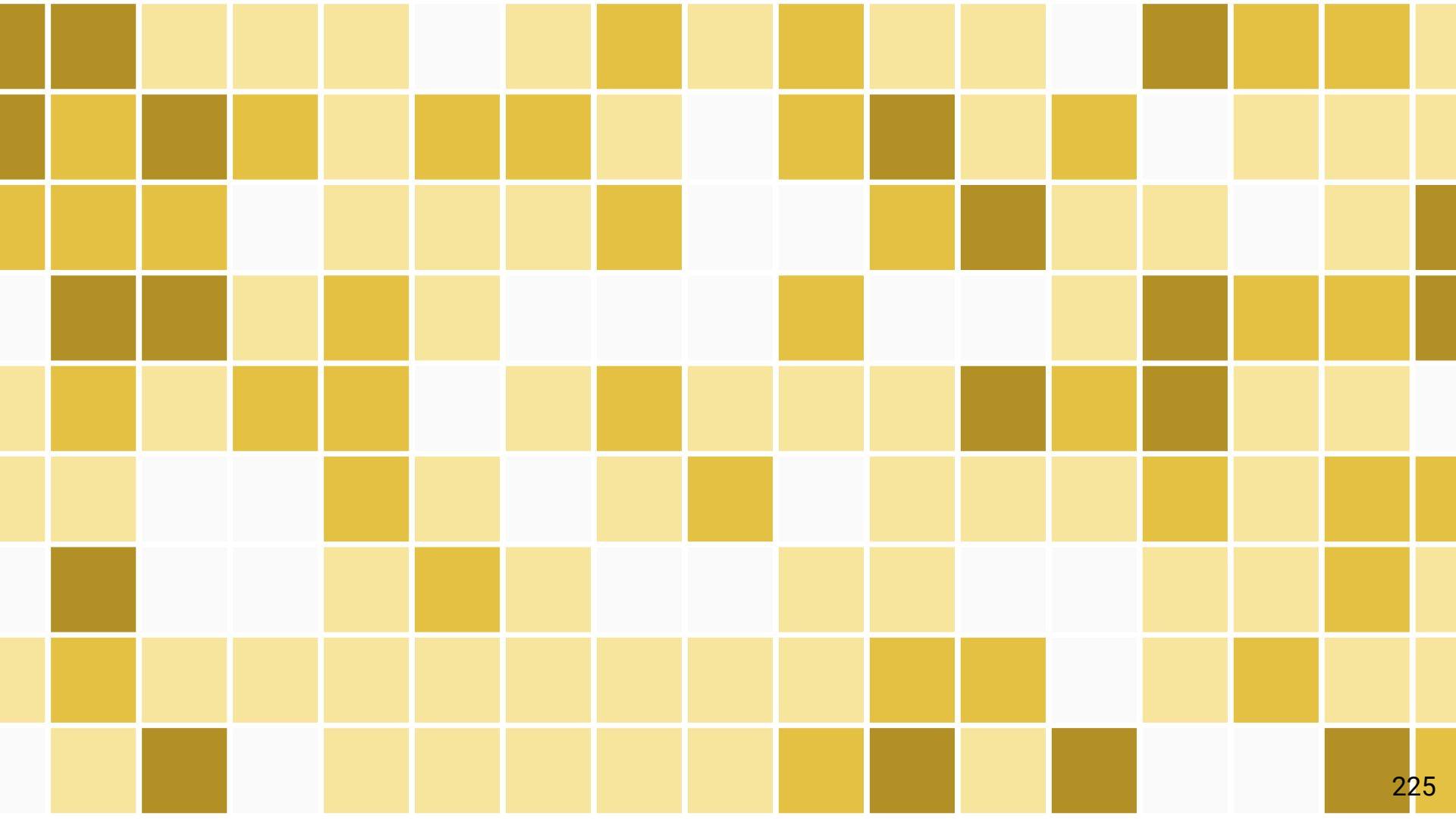


GPU SORT WITH STREAM



GPU SORT WITH STREAM





SMALL SORT

mergeSmall_Batch_k

Batches on shared memory

Very Fast

Impossible if batch_size > 1024

SMALL SORT

`mergeSmall_Batch_k`

Batches on shared memory

Very Fast

Impossible if `batch_size > 1024`

BIG SORT

`pathBig_k` and `mergeBig_k`

Streams

Global Memory



OUR GITHUB :



https://github.com/saulnierS/main5_hpc.git
