Simulation of the Evolution of ^{137}Cs Human Contamination due to the Chernobyl Incident Using Markov Chains in MATLAB

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Abstract

Markov chains were applied to simulate the time evolution of the ^{137}Cs released in the atmosphere as a result of the incident in Chernobyl nuclear power plant in 1986. The different possible paths that the radioactive elements could take to eventually reach humans were considered. The main source of contamination for humans was found to be the consumption of contaminated vegetables.

Introduction

In 1986 the number four reactor in the Chernobyl nuclear power plant suffered a meltdown. This is the worst nuclear disaster in history in both cost and casualties. It is one, of only two nuclear energy accidents rated at 7, the maximum severity, on the International Nuclear Event Scale. A United Nations committee found that to date fewer than 100 deaths have resulted from the fallout. However, robust estimates predict about 9,000 to 16,000 related fatalities across all of Europe when considering the coming decades. The nuclear clean-up of the site is scheduled for completion in 2065.

The incident of Chernobyl predominantly released the isotope ^{131}I to the atmosphere. However, due to its small half-life, around 8 days, the predominant isotope was the ^{137}Cs . About 27 kg of this isotope were released into the atmosphere and its effects are still felt. [1]

^{137}Cs has a half-life period of about 30 years, it is enough time to contaminate the environment for a generation. The isotope emits beta particles as it decays.[1] [2]. The decay rate λ of any nucleus can be given by its half-life τ according to

$$\lambda = \frac{\ln 2}{\tau} \tag{1}$$

In fact, most of the ^{137}Cs released to the atmosphere in 1986 has been deposited on the ground and, consequently, dispersed to the living beings. This has provoked an intense study of many organizations through all the globe[2] to find out its potential consequences in the last 40 years.[3]

The ISRN of France has estimated that 80000 ^{137}Cs TBq were released into the environment.[1] Many studies have shown that the destination of this cesium went from the atmosphere to the ground and inhaled by living beings such as animals, humans or vegetables. Since the human body can be affected by this isotope causing burns, risk of cancer, acute radiation sickness, etc., this has led an area of study focus in the trail of the remnants of the ^{137}Cs by nuclear accidents such as the one from Chernobyl. [4]

Markov processes for data analysis

Markov process is defined as a stochastic extension of a finite state automaton. In a Markov process, state transitions are probabilistic, and there is no input to the system, also the system is only in one state at each time step.[5] Formally, it can be defined as follows:

$$P(X_n = x \mid X_{n-1} = x_{n-1}, ..., X_1 = x_1)$$

$$= P(X_n = x \mid X_{n-1} = X_{n-1}) \tag{2}$$

with $P(A \mid B)$ as the probability of A given B.

We define then the state space S constituted by all the possible values X_i that can exist in the chain. Therefore, if the state space is finite and the transition probabilities are constant in time, the transition probability distribution can be represented by the transition matrix defined as:

$$M = (m_{ij})_{i,j \in S} \tag{3}$$

[1] The radiation of ^{137}Cs is $3.2x10^{12}$Bq/g

[2] It's estimated that, since 1986 until 2010, 60000 studies of the Cesium contamination have been produced.[1]

where m_{ij} are the elements of the matrix.

Now, the last step is to define a vector whose $i-$th component describe the probability of the system to be in state i at time state n:

$$x^n = M(X_n = i) \tag{4}$$

So, the transition probabilities are just the power of the transition matrix:

$$x^{n+1} = M.x^n$$

$$x^{n+2} = M.x^{n+1} = M^2 x^n$$

$$x^n = M^n.x^0 \tag{5}$$

Finally, an important remark for this method is that, after a certain time, the probability distribution converges towards a stationary distribution:

$$x^* = \lim_{n\to\infty} x^n \tag{6}$$

Absorption states for a Markov chain

We can define a transition matrix P for a time-homogeneous absorbing Markov with t transition states in the form:

$$M = \begin{bmatrix} Q & R \\ 0 & I_s \end{bmatrix} \tag{7}$$

Where Q is a $t \times t$ matrix, R is a $t \times s$ matrix and I_s the $s \times t$ identity matrix.

Many calculations can be performed by decomposing this matrix. In particular, we can define another matrix called N that contains the information about how many times a column j is visited in the process. This matrix is formed from the elements of this matrix and it's defined as:

$$N = (I - Q)^{-1} = \sum_{k=0}^{\infty} Q^k$$

$$N_{ij} = Prob(X_0 = j|X_0 = i) + Prob(X_1 = j|X_0 = i)$$

$$+ Prob(X_2 = j|X_0 = i) + ... \tag{8}$$

Additional to this, the probability of being absorbed by state j after starting in state i is given by the (i, j) entry of the $t \times s$ matrix $P = NR$. [6]

Markov Chain Simulation

According to the given data, the coefficients at which the ^{137}Cs goes into the atmosphere, ground, humans, cows and vegetables and the routes that the isotope takes to reach each one of them are represented in the variables shown in the table 1.

	Atmosphere	Ground	Cows	Humans	Vegetables
Atmosphere		g	x	x	v
Ground					a
Cows				y	
Vegetables			c	z	

Table 1: Constant rates at which ^{137}Cs isotopes go from row to column states

Taken into account the values for these variables[3] , the rate at which ^{137}Cs decays (Eq. 1) and the fact that the row sum of the M matrix must be equal to 1 gives:

$$M = \begin{bmatrix} 1 & 0.0099 & 0.0001 & 0.0001 & 0.0001 & 0.0001 \\ 0 & 0 & 0.7460 & 0 & 0 & 0 \\ 0 & 0 & 0.0020 & 0.7999 & 0.0667 & 0 \\ 0 & 0.999 & 0.0020 & 0.20 & 0.083 & 0 \\ 0 & 0 & 0.050 & 0 & 0.8499 & 0.033 \\ 0 & 0 & 0.2000 & 0 & 0 & 0.966 \end{bmatrix}$$

With states in the following order: Stable, Humans, Atmosphere, Cows, Vegetables, Ground.

It is important to notice that ^{137}Cs has a biological half-life of 70 days, which is considerably different than its standard half-life. Biological half-life is defined as the time a substance takes to reduce its concentration by half in human blood plasma.

Now we are interested in to see how much ^{137}Cs humans will absorb through time. Before doing this, we can analyze the structure and evolution of the Markov chain to see the routes of each state. According to the figure 1 we can see a graphic representation of the Markov chain.

Additionally, the heatmap of the transition probabilities, show the probabilities for going from a column state

[3] They correspond to $x = \frac{1}{500}$, $v = \frac{1}{20}$, $g = \frac{1}{5}$, $a = \frac{1}{30}$, $c = \frac{1}{15}$, $z = \frac{1}{12}$ and $y = \frac{1}{5}$.

to the row state, using matrix M, as shown in the figure 2. The map shows that the state with the bigger probability to reach humans is the cows state.

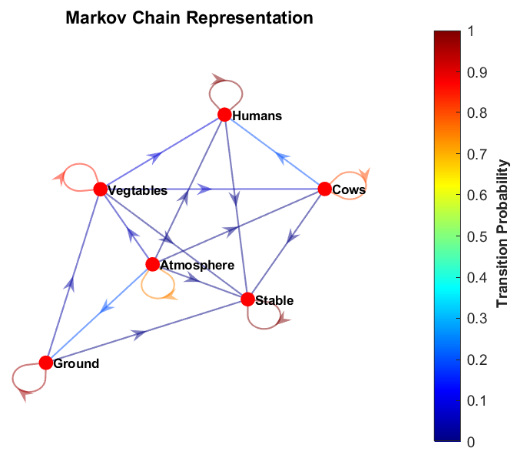

Figure 1: Graphical representation of the Markov chain analyzed.

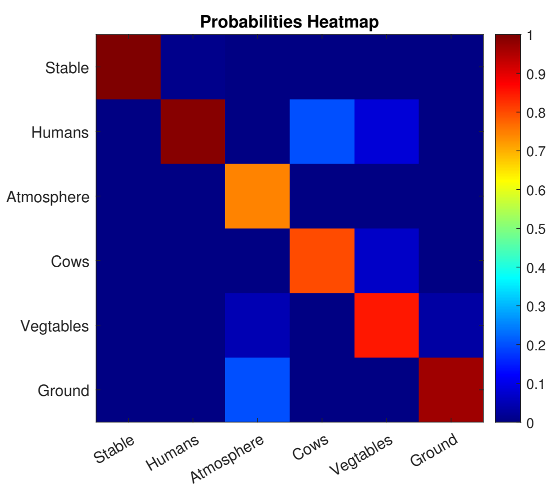

Figure 2: Transition probabilities from column to row states. M.

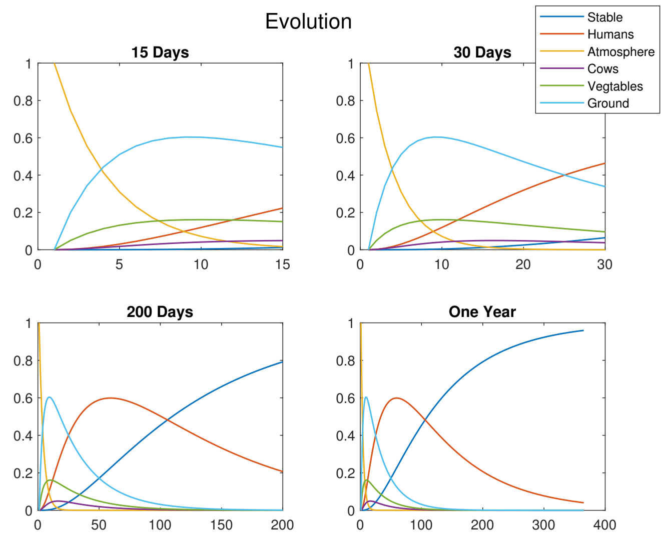

Figure 3: Different time scale evolution of the Markov chain

The evolution obtained following the next code is shown in different time scales to see the propagation of the ^{137}Cs proportion in each state (Fig. 3). [4]:

```
NumDays = 365;
Evolution = zeros(6,NumDays);
Evolution(3,1) = 1;
Humans = zeros(3,NumDays);
for t = 2:NumDays
Humans(:,t-1) = Evolution(3:5,t-1) .* [x y z]';
Evolution(:,t) = M * Evolution(:,t-1);
end
```

Listing 1: For-cycle for the Markov process

The evolution starts with all isotopes being released to the atmosphere during the nuclear meltdown and subsequent explosions and fires that occurred in the Chernobyl disaster. During the first 15 days ^{137}Cs is rapidly depleted from the atmosphere and is mainly deposited in the ground. After the first 20 days, human contamination becomes predominant. After 100 days the proportion of stable nucleus surpasses the radioactive ones. At one year most of the ^{137}Cs has become stable and left the humans.

The sources of human contamination were analyzed to determine which one is contaminating the most to the human body. Knowing the evolution of the matrix M and the rate factor for the cows, vegetables and atmosphere it is possible to determine the contribution of each state (Fig. 4). Atmosphere direct contribution to human contamination is negligible compared to the other sources. Vegetable and cow contamination are comparable, however vegetable is found to be the main source of contamination (Fig 5). This is expected, as noted before most of the initial population goes rapidly into the ground which is eventually absorbed into vegetables.

Alternatively this results can be inferred by the transient probabilities matrix matrix in order to obtain the transition probabilities for the states involved in the human contamination using the following code:

```
P = flip(flip(M',2));
Q = P(1:5,1:5);
```

[4]To see the full code of MATLAB implemented in this analysis please see the *"Appendix"* section

```
R = P(1:5,6);
N = inv(eye(5) - Q);
M2 = (eye(5) - Q) \ R;
H = (N - eye(5)) / ((eye(5) .* diag(N)));
```

Listing 2: Absorption matrix and transition probabilities for *"Humans"* state

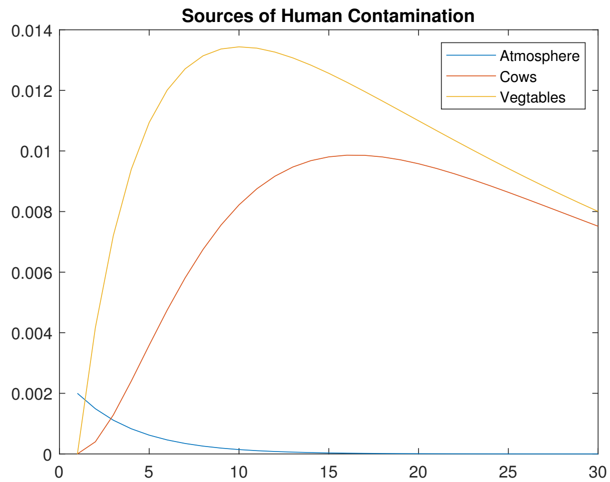

Figure 4: Contamination from the cows, atmosphere and vegetables for humans.

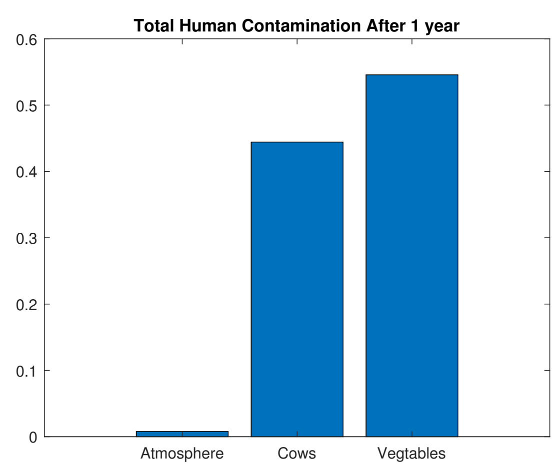

Figure 5: Total contamination in humans after 1 year.

In which we can find that the probabilities of reaching the humans state starting at each of the three other states are:

$$T = (P_a, P_c, P_v)$$

$$= (0.9994, 0.9997, 0.9977)$$

Hence, we can confirm that these values match with the information presented above.

Conclusions

It was determined that, due to the consumption of contaminated foods such as beef and vegetables, the human body absorbs a large amount of cesium, with respect to the presence of cesium in the atmosphere. Mainly due to its rapid deposit to the Earth's ground, while in the case of the ground itself it decreases at an high speed rate due to the absorption by vegetables.

Expulsion of ^{137}Cs from the human body is obtained after a year of the initial release of the radioactive material. However, multiple experiments conducted in mice and other test subjects showed that a high radiation exposure (around 200 MBq/kg) results in dead within 30 days.

The simulation conducted takes into account several simplification that a more in-depth analysis could further explore. To begin with, the rates are given as time and concentration independent. It is also worth noting that in this analysis a radioactive isotope is considered to be stable as it removed from the human body. Which in practice is false. The biological half-life is only associated with the removal from the human body but it by no means signify that the ^{137}Cs is no longer radioactive. A more complex chain should be implemented to account for contamination of other structures such as water deposits from ^{137}Cs released from humans. With a possible re contamination due to the consumption of marine life.

A lot of factors could be considered in the process and this is a main factor on the complexity of an estimation of the real impact of such a nuclear disaster. A first hand approach showed the channels of contamination for humans and shines some light on the possible impact to the health of someone exposed to large amounts of ^{137}Cs
.

Although the half-life of cesium-137 is only 30 years, its absorption continues to occur even after that time, so it is recommended to raise awareness about the effects of nuclear contamination that may occur on the planet due to an environmental catastrophe such as the one that occurred in Chernobyl.

References

[1] *Chernobyl Caesium-137* Obtained from https://www.radioactivity.eu.com/site/pages/Chernobyl_Caesium.htm.

[2] Wessells, Colin. (2012). *Cesium-137: A Deadly Hazard* Obtained from http://large.stanford.edu/courses/2012/ph241/wessells1/

[3] Wai, KM., Krstic, D., Nikezic, D.. (2020). *xternal Cesium-137 doses to humans from soil influenced by the Fukushima and Chernobyl nuclear power plants accidents: a comparative study.* Sci Rep **10** Obtained from https://doi.org/10.1038/s41598-020-64812-9.

[4] National Center for Environmental Health (NCEH). (2018, 4th April). *CDC Radiation Emergencies — Radioisotope Brief: Cesium-137 (Cs-137)* Obtained from https://www.cdc.gov/nceh/radiation/emergencies/isotopes/cesium.htm.

[5] *Markov Process* Obtained from https://www.ifi.uzh.ch/dam/jcr:00000000-2826-155d-ffff-ffff86200612/f-chapter4.pdf.

[6] *Absorbing Markov Chains. Brilliant.org.* Obtained from https://brilliant.org/wiki/absorbing-markov-chains/.

Appendix

Code used in the present paper.

```
%% Chernobyl
clc
clear
close all
%Defining Variables
s = log(2) / 30.15 / 365;
sBio = log(2) / 70;
x = 1/500;
v = 1/20;
g = 1/5;
a = 1/30;
c = 1/15;
z = 1/12;
y = 1/5;
dt = 5/100;
%Building Transition Matrix
Names = ["Stable", "Humans","Atmosphere", "Cows",
    "Vegtables", "Ground"];
M = [1 sBio s s s s; 0 1-sBio x y z 0; 0 0
    1-s-2*x-v-g 0 0 0; 0 0 x 1-s-y c 0; 0 0 v 0
    1-s-c-z a; 0 0 g 0 0 1-s-a];
mc = dtmc(M','StateNames',Names);
%% Plot graph
figure(1);
graphplot(mc, 'ColorEdges', true);
title('Markov Chain Representation')
%% Plot Heatmap
figure(2);
imagesc(M);
colormap(jet);
colorbar;
axis square
xticklabels(Names);
yticklabels(Names);
title('Probabilities Heatmap')
%% Evolution
NumDays = 365;
Evolution = zeros(6,NumDays);
Evolution(3,1) = 1;
Humans = zeros(3,NumDays);
for t = 2:NumDays
    Humans(:,t-1) = Evolution(3:5,t-1) .* [x y z]';
    Evolution(:,t) = M * Evolution(:,t-1);
end
%% Plot Evolution
figure(3)
subplot(2,2,1)
plot(Evolution(:,1:15)',LineWidth=1);
title('15 Days')
subplot(2,2,2)
plot(Evolution(:,1:30)',LineWidth=1);
title('30 Days')
subplot(2,2,3)
plot(Evolution(:,1:200)',LineWidth=1);
title('200 Days')
subplot(2,2,4)
plot(Evolution(:,1:365)',LineWidth=1);
title('One Year')
```

```
legend(Names);
sgtitle('Evolution')
%% Plot Human Contamination
figure(4)
plot(Humans(:,1:30)',LineWidth=1);
legend(Names(3:5));
title('Sources of Human Contamination')
%% Plot Total Human Contamination
figure(5)
bar(sum(Humans,2));
xticklabels(Names(3:5));
title('Total Human Contamination After 1 year')
%% Absorbing Matrix
%Treat humans as an absorbing state
P = flip(flip(M',2));
Q = P(1:5,1:5);
R = P(1:5,6);
N = inv(eye(5) - Q);
A = (eye(5) - Q) \ R; %Absorbing Matrix
H = (N - eye(5)) / ((eye(5) .* diag(N)));
    %Transient Probabilities
```