Two Firm-Level Labour-Market Tightness Metrics

Objective. Produce a single firm–level variable that captures labour–market tightness while recognising two facts:

- 1. Tightness differs across metros.
- 2. Occupations contribute unequally to a firm's head-count.

Below we document two weighting schemes that address (1)–(2).

- tight_wavg: all metros, weighted by where each occupation is employed and by its size in the firm's overall workforce.
- tight_hq: the *single* headquarters metro, weighted by the occupations actually staffed at HQ.

The metrics below relying the same underlying data from OEWS. But since we can either try to account for all locations or just look at the HQ (which is the most popular MSA in this case), we'll create two versions of the tightness metric.

1 OEWS building block (occupation-metro tightness)

OEWS gives raw employment counts

 $\begin{array}{lll} \operatorname{EMP}_{o,m} & - & \operatorname{employees} \text{ in occupation } o \text{ and metro } m \\ \operatorname{EMP}_m & - & \operatorname{total employment in metro } m = \sum_k \operatorname{EMP}_{k,m} \\ \operatorname{EMP}_o^{US} & - & \operatorname{national employment in occupation } o \\ \operatorname{EMP}^{US} & - & \operatorname{total US employment} = \sum_k \operatorname{EMP}_k^{US}. \end{array}$

Step 1: local versus national shares

$$\operatorname{share}_{o,m} \equiv \frac{\operatorname{EMP}_{o,m}}{\operatorname{EMP}_m} \qquad \text{(occupation's share in metro)}$$

$$\operatorname{share}_o^{US} \equiv \frac{\operatorname{EMP}_o^{US}}{\operatorname{EMP}^{US}} \qquad \text{(occupation's share nationally)}.$$

Step 2: location quotient

$$LQ_{o,m} = \frac{\text{share}_{o,m}}{\text{share}_{o}^{US}}.$$

When $LQ_{o,m} < 1$, the occupation is under-represented locally compared with its national prevalence.

Step 3: tightness index Taking the inverse turns scarcity into a measure that rises with hiring difficulty:

$$T_{o,m} = \frac{1}{\text{LQ}_{o,m}} = \frac{\text{share}_o^{US}}{\text{share}_{o,m}}.$$

Thus $T_{o,m} > 1$ signals thinner local supply (tighter market), while $T_{o,m} < 1$ indicates a looser labour market for that occupation.

Put differently, the index simply contrasts the occupation's *local* employment share with its *national* share—values above one mean the occupation is less prevalent (scarcer) in that metro than in the country as a whole.

The next sections combine these OEWS cell values into firm-level indices.

2 Metric tight_wavg: all metros, firm-wide mix

Goal. Deliver one scalar that reflects how tight labour markets were for the firm's 2019 workforce across all its sites and occupations. A useful metric must account simultaneously for

- where each occupation is based (otherwise we blur San Francisco with Omaha), and
- how many employees the firm has in that occupation (otherwise a lone intern could dominate the average).

The two-stage weighting scheme below meets both needs.

In practice we perform two passes:

- 1. Across metros within each occupation. We pool the tightness numbers of every city where the occupation is present, weighting by how many of the firm's employees in that occupation sit in each place.
- 2. Across occupations within the firm. We then pool those occupation averages into one firm number, weighting by the size of each occupation in the overall head-count.

The sub-steps below implement these two passes.

Step 2.1 Count heads by metro

For every firm f, occupation o and metro m we first count how many employees fall into that cell during 2019–H2:

$$h_{f,o,m} = \text{LinkedIn heads}(f, o, m; 2019-H2).$$

In words: "How many software engineers (for example) does the firm employ in San Francisco, how many in Chicago, and so on?"

Step 2.2 Firm-occupation tightness

The same occupation is often present in several cities. To roll the metro–level tightness figures into a single number we ask *where* the people in that occupation actually sit—the more heads a metro hosts, the larger its influence on the average. We formalise this by turning head-counts into shares:

$$\alpha_{f,o,m} = \frac{h_{f,o,m}}{\sum_k h_{f,o,k}}, \qquad \sum_m \alpha_{f,o,m} = 1.$$

Each $\alpha_{f,o,m}$ is the share of the firm's occupation o staff that works in metro m. We simply take a weighted average of the metro-level tightness numbers using these shares: if everyone is in one city the result is that city's $T_{o,m}$; if staff are spread across several metros the figure lands somewhere in between.

$$\widehat{T}_{f,o} = \sum_{m} \alpha_{f,o,m} T_{o,m}. \tag{A1}$$

Step 2.3 Collapse to firm level

We now pool together all metros and simply ask: "How many employees does the firm have in each occupation, regardless of location?" Call this total $H_{f,o}$ for occupation o.

The firm still employs many different occupations and some are far larger than others. To reflect this we convert the raw counts $H_{f,o}$ into shares of the total workforce—larger job families should carry more weight when we collapse to a single firm number. We obtain these shares by dividing each occupation's head-count by the firm's overall head-count; these are the occupation weights

$$\beta_{f,o} = \frac{H_{f,o}}{\sum_k H_{f,k}}, \qquad \sum_o \beta_{f,o} = 1.$$

The final static metric is then

$$tight_wavg_f = \sum_o \beta_{f,o} \widehat{T}_{f,o}.$$
(A2)

In plain terms, tight_wavg answers a simple question: "If the firm had to fill every single role it had in 2019, drawing workers from the same cities, how difficult would that be given the local supply of each occupation?"

Interpretation. Equations (A1)–(A2) give a direct answer: "How tight were the labour markets that supported the firm's full 2019 workforce, considering every metro where those employees were based?"

3 Metric tight_hq: headquarters metro, HQ mix

Goal. Gauge hiring difficulty in the headquarters city. We use a *single* metro (HQ(f)) and average OEWS tightness across occupations with weights given by their head-counts at HQ.

Step 3.1 Pick one metro

For each firm, the Stata routine assigns a head-quarters metro

$$HQ(f) = \underset{m}{\operatorname{arg\,max}} LinkedIn \text{ spells in } m.$$

Step 3.2 Heads located in HQ

$$h_{f,o,HO} = \text{LinkedIn heads}(f, o, HQ(f); 2019-H2).$$

That is, employees who both belong to occupation o and sit in the headquarters metro.

Step 3.3 Turn counts into weights

From these HQ head-counts we form occupation shares so the numbers sum to one and can be used as averaging weights:

$$w_{f,o} = \frac{h_{f,o,HQ}}{\sum_{k} h_{f,k,HQ}}, \qquad \sum_{o} w_{f,o} = 1.$$

Each $w_{f,o}$ is therefore the fraction of the HQ workforce in occupation o. Using these shares—as opposed to equal weights—prevents an occupation staffed by only a handful of employees from overtaking the signal coming from much larger functions.

Step 3.4 Combine OEWS values with the HQ mix

Finally, we take the metro-specific tightness numbers for the head quarters city, $T_{o,\mathrm{HQ}(f)}$, and average them using the weights from the previous step:

$$tight_hq_f = \sum_o w_{f,o} T_{o,HQ(f)}.$$
(B1)

Interpretation. Equation (B1) tells us: "Given the occupations the firm actually employs in its headquarters city, how tight is that single local labour market?"

4 Summary of differences

Metric	Geography entering average	Weights come from
tight_wavg	All metros with firm heads (2019–H2)	Entire 2019 firm workforce
tight_hq	Single metro: HQ(f)	Heads located in HQ(f)

Both variables are **time-invariant**: the occupational and metro weights are frozen at 2019–H2.