Two Firm-Level Labour-Market Tightness Metrics

Objective. Produce a single firm–level variable that captures labour–market tightness while recognising two facts:

- 1. Tightness differs across metros.
- 2. Occupations contribute unequally to a firm's head-count.

Below we document two weighting schemes that address (1)–(2).

- tight_wavg: all metros, weighted by where each occupation is employed and by its size in the firm's overall workforce.
- tight_hq: the *single* headquarters metro, weighted by the occupations actually staffed at HQ.

The metrics below relying the same underlying data from OEWS. But since we can either try to account for all locations or just look at the HQ (which is the most popular MSA in this case), we'll create two versions of the tightness metric.

1 OEWS building block (occupation-metro tightness)

OEWS gives raw employment counts

 $\begin{array}{lll} \operatorname{EMP}_{o,m} & - & \operatorname{employees} \text{ in occupation } o \text{ and metro } m \\ \operatorname{EMP}_m & - & \operatorname{total employment in metro } m = \sum_k \operatorname{EMP}_{k,m} \\ \operatorname{EMP}_o^{US} & - & \operatorname{national employment in occupation } o \\ \operatorname{EMP}^{US} & - & \operatorname{total US employment} = \sum_k \operatorname{EMP}_k^{US}. \end{array}$

Step 1: local versus national shares

$$\operatorname{share}_{o,m} \equiv \frac{\operatorname{EMP}_{o,m}}{\operatorname{EMP}_m} \qquad \text{(occupation's share in metro)}$$

$$\operatorname{share}_o^{US} \equiv \frac{\operatorname{EMP}_o^{US}}{\operatorname{EMP}^{US}} \qquad \text{(occupation's share nationally)}.$$

Step 2: location quotient

$$LQ_{o,m} = \frac{\text{share}_{o,m}}{\text{share}_{o}^{US}}.$$

When $LQ_{o,m} < 1$, the occupation is under-represented locally compared with its national prevalence.

Step 3: tightness index Taking the inverse turns scarcity into a measure that rises with hiring difficulty:

$$T_{o,m} = \frac{1}{\text{LQ}_{o,m}} = \frac{\text{share}_o^{US}}{\text{share}_{o,m}}.$$

Thus $T_{o,m} > 1$ signals thinner local supply (tighter market), while $T_{o,m} < 1$ indicates a looser labour market for that occupation.

Put differently, the index simply contrasts the occupation's *local* employment share with its *national* share—values above one mean the occupation is less prevalent (scarcer) in that metro than in the country as a whole.

The next sections combine these OEWS cell values into firm-level indices.

2 Metric tight_wavg: all metros, firm-wide mix

Goal. Deliver one scalar that reflects how tight labour markets were for the firm's 2019 workforce across all its sites and occupations. A useful metric must account simultaneously for

- where each occupation is based (otherwise we blur San Francisco with Omaha), and
- how many employees the firm has in that occupation (otherwise a lone intern could dominate the average).

The two-stage weighting scheme below meets both needs.

Step 2.1 Count heads by metro

For every firm f, occupation o and metro m we first count how many employees fall into that cell during 2019–H2:

$$h_{f,o,m} = \text{LinkedIn heads}(f, o, m; 2019-H2).$$

In words: "How many software engineers (for example) does the firm employ in San Francisco, how many in Chicago, and so on?"

Step 2.2 Firm-occupation tightness

The same occupation is often present in several cities. To roll the metro–level tightness figures into a single number we ask *where* the people in that occupation actually sit—the more heads a metro hosts, the larger its influence on the average. We formalise this by turning head-counts into shares:

$$\alpha_{f,o,m} = \frac{h_{f,o,m}}{\sum_k h_{f,o,k}}, \qquad \sum_m \alpha_{f,o,m} = 1.$$

Each $\alpha_{f,o,m}$ is therefore the fraction of the firm's occupation o workforce that sits in metro m. Using these shares as weights we average the metro–specific tightness values:

$$\alpha_{f,o,m} = \frac{h_{f,o,m}}{\sum_{k} h_{f,o,k}}, \qquad \sum_{m} \alpha_{f,o,m} = 1.$$

Each $\alpha_{f,o,m}$ is the share of the firm's occupation o staff that works in metro m (e.g. $\alpha=0.7$ means 70 % of those workers sit in San Francisco). We simply take a weighted average of the metro-level tightness numbers using these shares: if everyone is in one city the result is that city's $T_{o,m}$; if people are spread across several metros the figure lands somewhere in between, reflecting how the workforce is split.

$$\widehat{T}_{f,o} = \sum_{m} \alpha_{f,o,m} T_{o,m}. \tag{A1}$$

Step 2.3 Collapse to firm level

Next we turn to the composition of the workforce across occupations. Define $H_{f,o} = \sum_m h_{f,o,m}$, the total number of employees the firm has in occupation o after pooling across all metros.

The firm still employs many different occupations and some are far larger than others. To reflect this we convert the raw counts $H_{f,o}$ into shares of the total workforce—larger job families should carry more weight when we collapse to a single firm number. We obtain these shares by dividing each occupation's head-count by the firm's overall head-count; these are the occupation weights

$$\beta_{f,o} = \frac{H_{f,o}}{\sum_k H_{f,k}}, \qquad \sum_o \beta_{f,o} = 1. \label{eq:betaform}$$

The final static metric is then

$$tight_-wavg_f = \sum_o \beta_{f,o} \, \widehat{T}_{f,o}.$$
(A2)

In plain terms, tight_wavg answers a simple question: "If the firm had to fill every single role it had in 2019, drawing workers from the same cities, how difficult would that be given the local supply of each occupation?"

Interpretation. Equations (A1)–(A2) give a direct answer: "How tight were the labour markets that supported the firm's full 2019 workforce, considering every metro where those employees were based?"

3 Metric tight_hq: headquarters metro, HQ mix

We can also just look at the labour market around the corporate headquarters. For that purpose we build a second metric, tight_hq, that looks at *one* city only —the firm's main office—and asks how hard it is to fill the jobs that are actually based there.

Step 3.1 Pick one metro

First we pick a headquarters metro for each firm: the city where it has the largest number of LinkedIn spells in 2019–H2.

$$HQ(f) = \underset{m}{\operatorname{arg\,max}} \text{ LinkedIn spells in } m.$$

Step 3.2 Count heads in HQ

Let

$$h_{f,o,HQ} = \text{LinkedIn heads}(f, o, HQ(f); 2019-H2)$$

be the number of employees who are in occupation o and located in the headquarters city.

Step 3.3 Turn counts into weights

We again turn the counts into shares so that larger occupations matter more when we average:

$$w_{f,o} = \frac{h_{f,o,HQ}}{\sum_k h_{f,k,HQ}}, \qquad \sum_o w_{f,o} = 1.$$

Each $w_{f,o}$ is the fraction of the HQ workforce in occupation o; this prevents a tiny job family from outweighing a much larger one.

Step 3.4 Combine OEWS values with the HQ mix

Finally, we grab the OEWS tightness values for that single city and average them using the occupation shares:

$$tight_hq_f = \sum_{o} w_{f,o} T_{o,HQ(f)}.$$
(B1)

Interpretation. Equation (B1) tells us: "Given the occupations the firm actually employs in its headquarters city, how tight is that single local labour market?" Put differently, tight_hq answers: "If the firm had to re-hire everyone who sits in HQ, how tough would that be given the local supply of each occupation?"

4 Summary of differences

Metric	Geography entering average	Weights come from
tight_wavg	All metros with firm heads (2019–H2)	Entire 2019 firm workforce
tight_hq	Single metro: HQ(f)	Heads located in HQ(f)

Both variables are **time-invariant**: the occupational and metro weights are frozen at 2019–H2, so the numbers summarise structural hiring difficulty rather than short-run fluctuations.