

**ML Assignment 4**  
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**Section A**

Q1: (a) Forward pass of a CNN

input dimensions of image =  $M \times N$  ( $\min(M, N) \geq 1$ )  
channels =  $P$  ( $P \geq 1$ )  
kernel size =  $K \times K$  ( $1 \leq K \leq \min(M, N)$ )

(a) stride = 1

No padding

Dimensions of feature map =

$$M_{\text{output}} = M - K + 1 \quad N_{\text{output}} = N - K + 1$$

$$\Rightarrow (M_{\text{output}}, N_{\text{output}}) = (M - K + 1, N - K + 1)$$

final dimensions  $\Rightarrow (M - K + 1) \times (N - K + 1)$

(b) No. of multiplications =  ~~$K \times K \times P$~~

No. of additions =  $K \times K \times P - 1$

$$\Rightarrow \text{Total operations} = K^2 P + (K^2 P - 1)$$
$$= 2K^2 P - 1$$

(c)  $Q$  kernels ( $Q \geq 1$ ) of size  $K \times K$

feature map dimension =  $(M - K + 1) \times (N - K + 1)$

Total operations per pixel per kernel =  $K^2 P$

$$\Rightarrow \text{complexity} = O(Q \cdot (M - K + 1) \cdot (N - K + 1) \cdot K^2 \cdot P)$$

if  $(M, N) \gg K$  then  $M - K + 1 \approx M$   
then  $N - K + 1 \approx N$

$$\Rightarrow O(Q \cdot M \cdot N \cdot K^2 \cdot P)$$

(d) Assignment Step:

- For each data pt in dataset  $X$ , we calculate the distance of data pt to each cluster's centroid.
- We assign the data pt to that cluster which gives the minimum euclidean distance.

Update Step:

- After we are done with the assignment step, we recalculate the cluster centroids.
- New centroid is calculated by taking the mean of all the data pts in that particular cluster.

The method which helps to determine optimal number of clusters is Elbow Method. It's algorithm is as follows:

- We run the code for  $\text{max\_k}$  clusters, For starting from 1 to  $\text{max\_k}$ . Let us suppose the clusters at  $i^{\text{th}}$  iteration is  $k$ .
- We randomly select  $k$  pts from our datapts and assume that they are our centroids. Then we cluster them according to those centroids.
- We know calculate within cluster sum of squares using following formula:



$$WSS = \sum_{i=1}^k \sum_{x \in C_i} \|x - \mu_i\|^2$$

$C_i$  are the pts in cluster  $i$  &  $\mu_i$  is the centroid of that cluster.

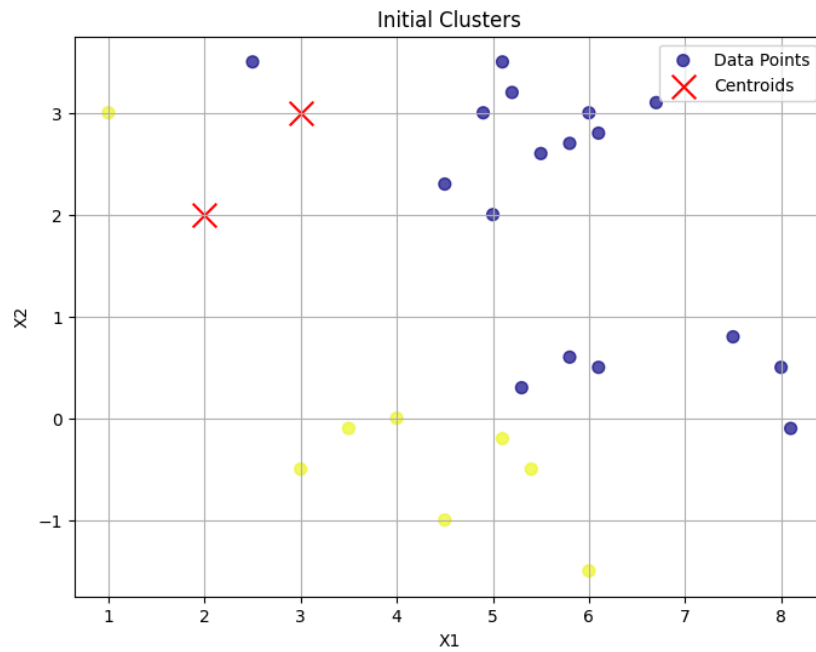
→ Then we plot WSS wss v/s  $k$  plot and find the elbow pt. It is the pt on the graph where rate decreases in wss significantly.

No, we cannot assign clusters centroids & arrive at global minima because it may lead to slow ~~conv~~ convergence. K-means may converge to a local minimum based on the initial random assignment of centroids.

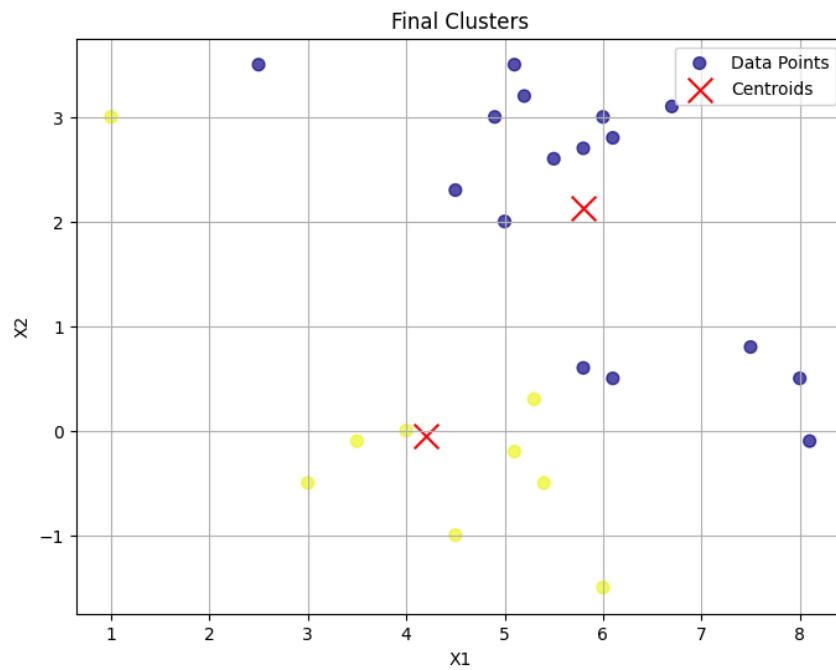
## Section B

(b).

Initial Clusters:



Final Clusters:



(c).

### Final Centroids:

Converged after 3 iterations

```
array([[ 5.8      ,  2.125     ],  
       [ 4.2      , -0.05555556]])
```

### Random Centroids:

Converged after 7 iterations

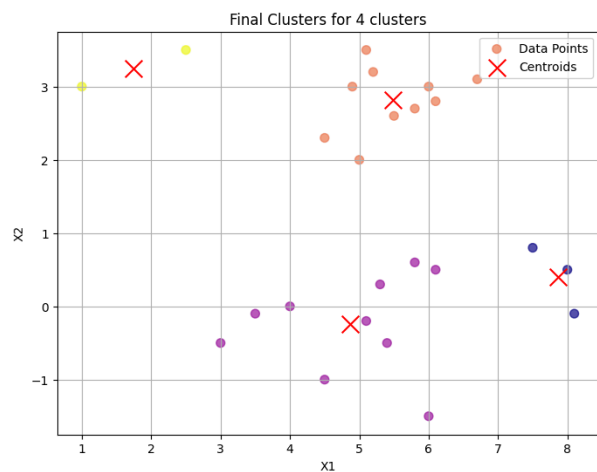
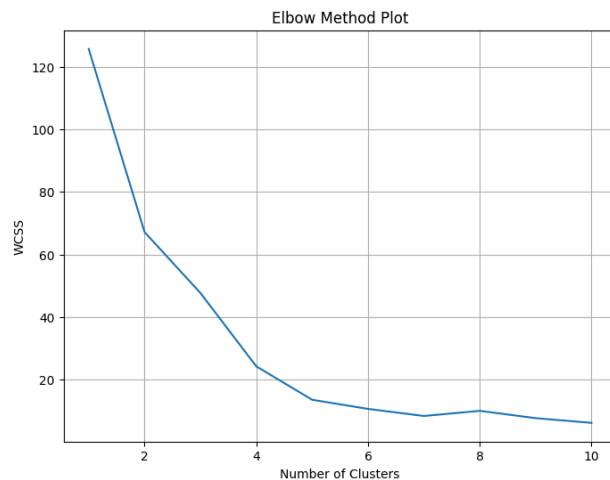
```
array([[ 4.85833333,  2.89166667],  
       [ 5.56153846, -0.09230769]])
```

The algorithm is able to find the clusters but it is taking a lot more iterations. Therefore random centroid assignment takes a longer time to converge. Convergence may take a longer time if the dataset is huge.

(d).

Maximum number of clusters takes = 10

Optimal number of clusters,  $M = 4$  (In the plot after running the algorithm again and again, I noticed that the value of  $M$  (optimal number of clusters) is 4)



## Section C

2.

Class: plane



Class: car



Class: bird

### References:

- [https://pytorch.org/tutorials/beginner/basics/data\\_tutorial.html](https://pytorch.org/tutorials/beginner/basics/data_tutorial.html)
- [https://pytorch.org/tutorials/beginner/blitz/cifar10\\_tutorial.htm](https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.htm)