RECURRENCE RELATION

$T(n) = 2T(n/2) + n$ A

Solution:

Iteration 1:

Put $n=n/2$, then

$T(n/2) = 2T(n/4) + n/2$ B

Put the value of $T(n/2)$ from B to A, then

$T(n) = 2\{2T(n/2^2) + n/2\} + n$

$T(n) = 2^2T(n/2^2) + n + n$ C

Iteration 2:

Put $n= n/2^2$ in A, then

$T(n/2^2) = 2T(n/2^3) + n/2^2$ D

Put the value of $T(n/2^2)$ from D to C, then

$T(n) = 2^2\{2T(n/2^3) + n/2^2\} + n + n$

$T(n) = 2^3T(n/2^3) + n + n + n$ E

Iteration 3:

Put $n= n/2^3$ in A, then

$T(n/2^3) = 2T(n/2^4) + n/2^3$ F

Put the value of $T(n/2^3)$ from F to E, then

$T(n) = 2^3\{2T(n/2^4) + n/2^3\} + n + n + n$

$T(n) = 2^4T(n/2^4) + n + n + n + n$ G

RECURRENCE RELATION

Iteration 4:

Put n= $n/2^4$ in A, then

$T(n/2^4) = 2T(n/2^5) + n/2^4$ H

Put the value of $T(n/2^4)$ from H to G, then

$T(n) = 2^4\{2T(n/2^5) + n/2^4\} + n + n + n + n$

$T(n) = 2^5T(n/2^5) + n + n + n + n + n$ I

Now, for “k” terms, it will be

$T(n) = 2^kT(n/2^k) + n + n + n + n + n + . . . +$ ‘k’ times

$T(n) = 2^kT(n/2^k) + n*k$

Assume:

$\frac{n}{2^k} = 1 => \boldsymbol{n = 2^k}$ and **T(1) = 0**

Taking $\log_2$ both sides

$=> \log_2 n = \log_2 2^k => \log_2 n = k * \log_2 2$

$=> \boldsymbol{\log_2 n = k}$ $\quad for\ base\ 2 \log$

$\frac{loga}{logb} = 1 \ \ if\ a = b$

Then, T(n) = n*0 + n*logn for n >>>>>1

Answer: T(n) = ϴ (n*log n)

Homework:

1.T(n) = 2T(n/2) + 1 **2.T(n) = T(n/2) + n**

3.T(n) = 2T(n/2) + n^2 **4.T(n) = T(n/2) + n^2**