

RECURRENCE RELATION

$$T(n) = T(n-1) + n^2 \quad A$$

Solution:

Iteration 1:

Put $n=n-1$, then

$$T(n-1) = T(n-2) + (n-1)^2 \quad B$$

Put the value of $T(n-1)$ from B to A, then

$$T(n) = T(n-2) + (n-1)^2 + n^2 \quad C$$

Iteration 2:

Put $n=n-2$ in A, then

$$T(n-2) = T(n-3) + (n-2)^2 \quad D$$

Put the value of $T(n-2)$ from D to C, then

$$T(n) = T(n-3) + (n-2)^2 + (n-1)^2 + n^2 \quad E$$

Iteration 3:

Put $n=n-3$ in A, then

$$T(n-3) = T(n-4) + (n-3)^2 \quad F$$

Put the value of $T(n-3)$ from F to E, then

$$T(n) = T(n-4) + (n-3)^2 + (n-2)^2 + (n-1)^2 + n^2 \quad G$$

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Iteration 4:

Put $n=n-4$ in A, then

$$T(n-4) = T(n-5) + (n-4)^2 \quad H$$

Put the value of $T(n-4)$ from H to G, then

$$T(n) = T(n-5) + (n-4)^2 + (n-3)^2 + (n-2)^2 + (n-1)^2 + n^2 \quad I$$

Now, for “k” terms, it will be

$$T(n) = T(n-k) + n^2 + (n-1)^2 + (n-2)^2 + (n-3)^2 + (n-(k-1))^2$$

$$T(n) = T(n-k) + n^2 + (n-1)^2 + (n-2)^2 + (n-3)^2 + (n-k+1)^2$$

$$T(n) = T(n-k) + (n^2 + n^2 + n^2 + n^2 + \dots + n^2 \text{ for } k \text{ times}) - \text{discarded values}$$

Assume;

$n-k = 0$ (zero), $n=k$ and $T(0) = 0$ (zero), then

$$T(n) = n^2 * k$$

$$T(n) = n^2 * n \quad \text{as } n=k \text{ assumed}$$

$$T(n) = n^3$$

Answer: $T(n) = O(n^3)$

Homework: Solve $T(n) = 2T(n-1) + 1$