CONTENTS

CHAPTER 01

UNITS AND MEASUREMENTS

PHYSICAL QUANTITY:

- **Fundamental quantities** are basic and do not depend on other quantities (e.g., mass, length, time).
- **Derived quantities** are expressed using fundamental quantities (e.g., velocity, force).

SYSTEM OF UNITS

- **Common Systems:**
- **FPS:** Foot (length), Pound (mass), Second (time)
- **CGS:** Centimetre (length), Gram (mass), Second (time)
- **MKS:** Metre (length), Kilogram (mass), Second (time)
- **International System (SI):** This is a modified version of the MKS system. It has seven fundamental units (metre, kilogram, second, kelvin, ampere, candela, mole) and two supplementary units (radian for plane angle, steradian for solid angle).

DIMENSIONS OF PHYSICAL QUANTITIES

- Dimensions show how a physical quantity is related to the fundamental quantities.
- For example, the dimension of force is $[MLT^{-2}]$. Time
  Mass length

PRINCIPLE OF HOMOGENEITY

- Only quantities with the same dimensions can be added or subtracted. M + M = M
- In any correct equation, the dimensions on both sides must be the same.

DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

- **Checking Consistency:** Use the principle of homogeneity to check if an equation is dimensionally correct.
- **Deducing Relations:** Find the relationship between physical quantities if you know the factors they depend on.
- **Unit Conversion:** Convert a quantity from one system of units to another using the formula $n_1u_1 = n_2u_2$.

LIMITATIONS OF DIMENSIONAL ANALYSIS

- It cannot determine numerical constants (like 1/2 or π) in formulas.
- It cannot be used for equations involving trigonometric, logarithmic, or exponential functions.

SIGNIFICANT FIGURE OR DIGITS

- Significant figures are the reliable digits in a measurement plus the first uncertain digit.
- There are specific rules for counting significant figures and for performing calculations (addition, subtraction, multiplication, division) with them.

REPRESENTATION OF ERRORS

- **Mean absolute error** is the average of the absolute values of the errors in individual measurements.
- **Relative error** is the ratio of the mean absolute error to the mean value of the quantity.
- **Percentage error** is the relative error expressed as a percentage $\left(\text{Percentage Error} = \frac{\Delta a_m}{a_m} \times 100\%\right)$.

COMBINATION OF ERRORS

- When adding or subtracting quantities, their absolute errors add up: $\Delta Z = \Delta A + \Delta B$.
- When multiplying or dividing quantities, their fractional errors add up: $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$.
- For a quantity raised to a power, like $Z = A^n$, the fractional error is $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$.

ROUNDING OFF

- This is done to keep the result of a calculation consistent with the precision of the measurements used.
- **Rules:** If the digit to be dropped is > 5, the preceding digit is increased by 1. If it's < 5, the preceding digit is unchanged. If it's exactly 5, the preceding digit is made even.

MEASURING INSTRUMENTS

- **Least Count:** The smallest value that can be accurately measured by an instrument.
- **Accuracy vs. Precision:** Accuracy is how close a measurement is to the true value. Precision is how close multiple measurements are to each other.

VERNIER CALLIPERS

- A **Vernier Calliper** is used for precise length measurements.
- In case nVSD = (n – 1)MSD then, its least count is defined as the difference between one main scale division (MSD) and one vernier scale division (VSD).

SCREW GAUGE

- This instrument measures very small lengths, like the diameter of a wire, based on the principle of a micro-meter screw.
- Its least count is calculated by dividing the pitch by the total number of divisions on the circular scale

Single Correct Type Questions

1. The distance travelled by an object is given by $x = at + \frac{bt^2}{c+a}$ where t is time and a, b, c are constants. The dimensions of b and c respectively are
(1) $[LT^{-2}],[LT^{-1}]$ (2) $[L^2T^{-3}],[LT^{-1}]$
(3) $[LT^{-1}],[L^2T^{-1}]$ (4) $[LT^{-1}],[LT^{-1}]$

2. The effective length of a simple pendulum is the sum of the following three: Length of string, radius of bob, and length of hook In a simple pendulum experiment, the length of the string, as measured by a metre scale, is 92.0 cm. The radius of the bob combined with the length of the hook, as measured by a vernier callipers, is 2.17 cm. The effective length of the pendulum is
(1) 94.1 cm (2) 94.2 cm
(3) 94.17 cm (4) 94 cm

3. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
(1) A meter scale.
(2) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(3) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(4) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

4. If n is the number of particles crossing a unit area perpendicular to the x – axis in a unit time is given by $n = -D\left(\frac{N_2 - N_1}{x_2 - x_1}\right)$, where N_1 and N_2 are the number of particles per unit volume at positions $x = x_1$ and $x = x_2$, respectively, and D is the diffusion constant. The dimensions of D are
(1) $[M^0LT^{-2}]$ (2) $[M^0L^2T^{-4}]$
(3) $[M^0L^2T^{-2}]$ (4) $[M^0L^2T^{-1}]$

5. Linear momentum of a particle moving along a straight line as a function of time is given as $p = p_0e^{-\alpha t^3}$; where p_0 and α are constants. Time is measured with a stop watch of least count 10^{-2} s and value of α is 1 s^{-3}. The percentage error in the measurement of p at t = 1 s is
(1) 2.5% (2) 3.0%
(3) 1.5% (4) 1%

6. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is closest to:
(1) 2% (2) 3%
(3) 1% (4) 5%

7. In a special vernier calipers 5 vernier scale divisions are equal to 4 main scale divisions. One main scale division is equal to 5 mm. A screw gauge with pitch equal to least count of above vernier calipers is designed. Number of divisions on circular scale of this screw gauge is equal to 100. Least count of this screw gauge is:
(1) 0.1 mm (2) 0.01 mm
(3) 0.001 mm (4) 0.0001 mm

8. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:
(1) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$
(2) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$
(3) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$
(4) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A]$

9. Percentage error in the calculation of volume of sphere is 4.8 %, percentage error in the measurement of radius of sphere is:
(1) 4.8% (2) 2.4%
(3) 1.6% (4) 1.2%

10. The dimensional formula of dielectric strength is
(1) $M^0L^0T^0I^0$
(2) $M^1L^1I^{-1}T^{-3}$
(3) $M^1L^2T^{-3}I^{-1}$
(4) $M^{-1}L^{-3}T^{-4}I^2$

11. If momentum (P), area (A) and time (T) are taken to be the fundamental quantities then the dimensional formula for energy is

(1) $\left[P^{\frac{1}{2}}AT^{-1}\right]$ (2) $[P^2AT^{-1}]$

(3) $\left[PA^{\frac{1}{2}}T^{-1}\right]$ (4) $[PA^{-1}T^{-2}]$

12. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure $5\mu m$ diameter of a wire is

(1) 200 (2) 50

(3) 100 (4) 500

13. A watt meter reads 21.23 W. The absolute error in the measurement is –0.11W. The true value of power is

(1) 21.34 W (2) 21.12 W

(3) 21.23 W (4) 21.00 W

14. In a theoretical exercise if acceleration (A), Mass (M) and Energy (E) are chosen as the base quantities, the dimensions of the time will be :

(1) $[AM^{-2}E]$ (2) $[A^{-1}M^{-1/2}E^{1/2}]$

(3) $[A^{-1/2}ME^{1/2}]$ (4) $[A^{1/2}M^{-1/2}E^{1/2}]$

15. Two resistance are given as $R_1 = (20 \pm 0.2)\Omega$ and $R_2 = (10 \pm 0.1)\Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is

(1) 1 (2) 2

(3) 3 (4) 4

16. Identify the pair of physical quantities that have same dimensions:

(1) Velocity gradient and decay constant

(2) Wien's constant and Stefan constant

(3) Angular frequency and angular momentum

(4) Wave number and Avogadro number

17. N divisions on the main scale of a vernier callipers coincide with $N+1$ divisions on the vernier scale. If each division on the main scale is of a units, determine the least count of the instrument (in the same system of units).

(1) $a(N+1)$ (2) Na

(3) $\frac{Na}{N+1}$ (4) $\frac{a}{N+1}$

18. The values of kinetic energy (K) and potential energy (U) are measured as follows $K = 300.0 \pm 7.0$ J, $U = 500.0 \pm 9.0$ J. The percentage error in measurement of mechanical energy is

(1) 1% (2) 2%

(3) 3% (4) 4%

19. The Current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K. The error in the value of current in mA is n then $\frac{n}{0.2}$ is:

(1) 1 (2) 2

(3) 3 (4) 4

20. If dimensional formula for coefficient of viscosity is $[M^xL^yT^z]$. Then $|x+y+z|$ is______

(1) 1 (2) 2

(3) 3 (4) 4

Integer Type Questions

21. An experiment is performed to obtain the value of acceleration due to gravity 'g' by using a simple pendulum of length L. In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90 seconds. The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error (in %) in the determination of g is equal to $\frac{49}{y}\%$.

Value of y is equal to _____.

22. While measuring the acceleration due to gravity by a simple pendulum, a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of time period. Her maximum percentage error in the calculation of g by the relation $g = 4\pi^2\left(l/T^2\right)$ will be

23. In a standard Vernier calipers the Vernier constant of the calipers is 0.1 mm. With time due to certain particle impurities deposited on the jaws its zero error becomes +0.3 mm. Using this Vernier calipers to measure the width of an object, the main scale reads 2.3 cm and the matching Vernier division is 4. The width of the object will be ______ $\times 10^{-1}$ mm.

24. The frequency of vibration (f) of a mass (m) suspended from a spring of spring constant (K) is given by a relation of this type $f = Cm^x K^y$; where c is a dimensionless quantity. The value of $x + y$ [where unit of spring constant is N/m]

25. If the unit of velocity is run, the unit of time is second and the unit of force is strength in hypothetical system of unit. In this system of unit the unt of mass is (strength)^x (second)^y (run)^z. The value of $\frac{y}{x}$ is ....

ANSWER KEY

1.	**(2)**	**6.**	**(2)**	**11.**	**(3)**	**16.**	**(1)**	**21.**	**(18)**
2.	**(2)**	**7.**	**(2)**	**12.**	**(1)**	**17.**	**(4)**	**22.**	**(7)**
3.	**(2)**	**8.**	**(2)**	**13.**	**(1)**	**18.**	**(2)**	**23.**	**(231)**
4.	**(4)**	**9.**	**(3)**	**14.**	**(2)**	**19.**	**(1)**	**24.**	**(0)**
5.	**(2)**	**10.**	**(2)**	**15.**	**(1)**	**20.**	**(1)**	**25.**	**(1)**

HINT & SOLUTIONS

1. **(2)**
Dimension of a and c will be of velocity.

2. **(2)**
(92.0+2.17) cm = 94.15 cm. Rounding off to first decimal place, we get 94.2 cm

3. **(2)**
If student measure 3.50 cm it means that there in an uncertainly of order 0.01 cm
L.C of V.C = 1 MSD – 1VSD

$$=\frac{1}{10}\left[1-\frac{9}{10}\right]=\frac{1}{100}\text{ cm}$$

4. **(4)**

$$n=-\frac{D(n_2-n_1)}{x_2-x_1}\Rightarrow T^{-1}L^{-2}=\frac{D\left(L^{-3}\right)}{L}$$

$$\Rightarrow D=\frac{T^{-1}L^{-2}\times L}{L^{-3}}\Rightarrow D=\left[M^0L^2T^{-1}\right]$$

5. **(2)**

$$p=p_0e^{-\alpha t^3}$$

$$\frac{dp}{dt}=p_0e^{-\alpha t^3}(-3\alpha t^2)$$

$$\frac{dp}{p}=-3\alpha t^2dt=-3\times1\times1\times10^{-2}=-0.03$$

% error = 3%

6. **(2)**

Given $\frac{\Delta L}{L}=\frac{0.1}{20}$

and $\frac{\Delta t}{t}=\frac{1}{90}$

$$g=\left(\frac{1}{4\pi^2}\right)\frac{L}{T^2}=\left(\frac{1}{4\pi^2}\right)\frac{L}{\left(\frac{t}{100}\right)^2}$$

$$\Rightarrow\frac{\Delta g}{g}\times100\%=\frac{\Delta L}{L}\times100+\frac{2\Delta t}{t}\times100$$

$$\frac{\Delta g}{g}\times100\%=\left(\frac{0.1}{20}\right)100+2\left(\frac{1}{90}\right)100=2.72\%$$

So closest option is 3%.

7. **(2)**

$5\ VSD = 4\ MSD$

$1VSD=\frac{4}{5}MSD$

$LC = 1\ MSD - 1\ VSD = 1$ mm

For Screw Gauge $LC=\frac{\text{pitch}}{100}=0.01$ mm

8. **(2)**

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2}$$

$$\varepsilon_0=\frac{q_1q_2}{4\pi FR^2}$$

Hence $\varepsilon_0=\frac{C^2}{N.m^2}=\frac{[AT]^2}{MLT^{-2}\,.\,L^2}$

$=[M^{-1}L^{-3}T^4A^2]$

9. **(3)**

The volume of a sphere is $V=\frac{4}{3}\pi r^3$

The error in volume is related to the error in radius by

$\frac{\Delta V}{V} = 3\frac{\Delta r}{r}$

$\frac{4.8}{100} = 3\frac{\Delta r}{r}$

$\frac{\Delta r}{r} = \frac{4.8}{300}$ $\quad \frac{\Delta r}{r}\times 100 = \frac{4.8}{300}\times 100$

$\frac{\Delta r}{r}\times 100 = \frac{4.8}{3}$ $\quad \frac{\Delta r}{r}\times 100 = 1.6$

10. (2)

Electric field $[E] = [MLT^{-3}I^{-1}]$

11. (3)

Dimension of energy (E) ML^2T^{-2} ...(i)

Dimension of momentum (P) = MLT^{-1} ...(ii)

Dimension of area (A) = L^2or $A^{1/2}$ ...(iii)

Substituting (ii) and (iii) in equation (i) we get

Dimensional formula for energy (e) $P^1T^{-1}A^{1/2}$

12. (1)

$$\text{L.C.} = \frac{\text{pich}}{\text{No.of division on circulation scale}}$$

$\Rightarrow 5\times 10^{-6} = \frac{10^{-3}}{N}$

$\Rightarrow N = 200$

13. (1)

Measured value = 21.23 W

Absolute error = –0.11W

Absolute error = Measured value – True value

–0.11 = 21.23 – true value

True value = 21.23 + 0.11

= 21.34 W.

14. (2)

$\Rightarrow \quad E = MAL$

$\Rightarrow \quad L = \frac{E}{MA}$

$\Rightarrow \quad A = LT^{-2}$

$\Rightarrow \quad T^2 = LA^{-1}$

$\Rightarrow \quad T^2 = EM^{-1}A^{-2}$

$\Rightarrow \quad T = A^{-1}M^{-1/2}E^{1/2}$

15. (1)

When R_1 and R_2 are connected in parallel

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\left[R = \frac{R_1R_2}{R_1 + R_2} = \frac{10\times 20}{10+20} = \frac{20}{3}\right]$

Differentiating (i) both sides

$\frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$

$\Rightarrow \frac{\Delta R}{R} = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right)R$

$\left(\frac{0.2}{400} + \frac{0.1}{100}\right)\frac{20}{3} = \frac{\Delta R}{R}$

$\frac{\Delta R}{R}\times 100 = 1\%$

16. (1)

We know that,

Velocity gradient = $\frac{dv}{dx}$

Decay constant (λ) = $\frac{0.693}{T_{1/2}} = [T^{-1}]$

$\therefore$ Dimension of velocity gradient $\left(\frac{dv}{dx}\right) = \frac{LT^{-1}}{L} = [T^{-1}]$

17. (4)

$(N+1)$ divisions on the vernier scale = N divisions on main scale

$\therefore$ 1 division on vernier scale $= \frac{N}{N+1}$ divisions on main scale

Each division on the main scale is of a units.

So, 1 division on vernier scale $= \left(\frac{N}{N+1}\right)$ a unit

= a' (say)

Least count = 1 MSD – 1 VSD

$\Rightarrow$ Least count $= a - a' = a - \left(\frac{N}{N+1}\right)a = \frac{a}{N+1}$

18. (2)

$E = k + U$

$E = 800 \pm 16$ J

$\frac{\Delta E}{E}\times 100 = \frac{16}{800}\times 100 = 2\%$

19. (1)

$I = e^{\frac{1000V}{T}} - 1$

$I + 1 = e^{\frac{1000V}{T}}$

$$\log(I+1) = \frac{1000}{T} dV$$

$$\frac{d(I+1)}{I+1} = \frac{1000}{T} dV$$

$$\frac{dI}{I+1} = \frac{1000}{T} dV$$

$$\frac{dI}{(5+1)mA} = \frac{1000}{300}(0.01)$$

$dI = 0.2$ mA

So $n = 0.2$ and $\frac{n}{0.2} = 1$.

20. **(1)**

$[\text{Viscosity}] = [M^1L^{-1}T^{-1}]$

21. **(18)**

$$T = 2\pi\sqrt{\frac{L}{g}}$$

or $T^2 = 4\pi\left(\frac{L}{g}\right)$

$$\Rightarrow g = \frac{4\pi^2 L}{T^2} \Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

% error in $g = \frac{\Delta g}{g} \times 100$

$$\left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right) \times 100$$

$$= \left(\frac{\frac{1}{10}}{20} + 2 \times \frac{1}{90}\right) \times 100 = 2.72\%$$

22. **(7)**

Given that $\frac{\Delta l}{l} \times 100 = +1\%$

and $\frac{\Delta T}{T} \times 100 = -3\%$

Percentage error in the measurement of g is

$$\left[\frac{4\pi^2 l}{T^2}\right] = 100 \times \frac{\Delta l}{l} - 2 \times \frac{\Delta T}{T} \times 100$$

$$= 1\% - 2[-3\%] = +7\%$$

23. **(231)**

Vernier scale reading $= 4 \times 0.1 = 0.4$ mm

Diameter of wire = Main scale reading + vernier scale reading + zero correction

$= 2.3 + 0.04 - 0.03 = 2.31$ cm $= 231 \times 10^{-1}$ mm

24. **(0)**

By putting the dimensions of each quantity both the sides we get $\left[T^{-1}\right] = [M]^x \left[MT^{-2}\right]^y$

Now comparing the dimensions of quantities in both sides we get $x + y = 0$ and $2y = 1$ $\therefore x = -\frac{1}{2}, y = \frac{1}{2}$

25. **(1)**

$\because F = ma$

$$\Rightarrow m = \frac{F}{a} = (\text{Force}) \div \frac{\text{Change in velocity}}{\text{Time}}$$

$$= \frac{\text{Force} \times \text{Time}}{\text{Change in velocity}}$$

$= (\text{strength})(\text{second})(\text{run})^{-1}$

Thus, $x = 1, y = 1$ and $z = -1$

$$\therefore \frac{y}{x} = 1$$

CHAPTER

02

MOTION IN A STRAIGHT LINE

DISTANCE

- Distance is the total length of the path covered by a particle. It is a scalar quantity.

DISPLACEMENT

- Displacement is the shortest distance from the initial to the final position. It is a vector quantity.

AVERAGE SPEED AND INSTANTANEOUS SPEED

- **Average speed** is the total distance covered divided by the total time taken. → $\frac{D}{T}$
- **Instantaneous speed** is the speed of a particle at a specific moment in time, given by $\boxed{v = \frac{ds}{dt}}$ → Kisi inst. pe

AVERAGE VELOCITY

- Average velocity is the total displacement divided by the total time interval, $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{\text{disp.}}{\text{T. Time}}$

INSTANTANEOUS VELOCITY

- Instantaneous velocity is the velocity at a particular instant, given by $v = \frac{dx}{dt}$. → Kisi inst. pe vel.
- Its magnitude is always equal to the instantaneous speed.

ACCELERATION

- Acceleration is the rate of change of velocity with time, given by $a = \frac{dv}{dt}$.

$a = 0$

MOTION WITH CONSTANT ACCELERATION: EQUATIONS OF MOTION

- In vector form, the main equations are $\vec{v} = \vec{u} + \vec{a}t$ and $\Delta\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$.
- In scalar form for 1D motion, the key equations are $\boxed{v = u + at,\ s = ut + \frac{1}{2}at^2, \text{ and } v^2 = u^2 + 2as}$

DISPLACEMENT-TIME GRAPHS AND THEIR CHARACTERISTICS (1-7)

- The slope of a displacement-time graph at any point gives the instantaneous velocity.

$\text{Slope}_{(\text{disp-time})}$ = Inst. Vel.

- A horizontal line indicates that the object is at rest (velocity is zero). $V = 0$ if ——
- A straight line with a constant slope indicates motion with constant velocity. Cost Slope = Const. Vel.

VELOCITY-TIME GRAPHS AND THEIR CHARACTERISTICS

- The slope of a velocity-time graph gives the instantaneous acceleration. $(V-t)_{\text{Slope}}$ = Inst. Acc.
- The area under a velocity-time graph represents the displacement. $(V-t)_{\text{Area}}$ = Displacement.
- A horizontal line indicates motion with constant velocity (zero acceleration). (——) ⇒ $a = 0$

MOTION UNDER GRAVITY (NO AIR RESISTANCE)

- For a freely falling object (taking downward as positive), the equations of motion use acceleration g. ↓ (+ve)
- Key formulas are $v = u + gt$, $h = ut + \frac{1}{2}gt^2$, and $v^2 = u^2 + 2gh$. In terms of g.
- For a body thrown vertically up, maximum height is $\boxed{H = \frac{u^2}{2g}}$ and total time of flight is $\boxed{T = \frac{2u}{g}}$.

MOTION WITH VARIABLE ACCELERATION

- When acceleration is not constant, we must use calculus (integration) to find velocity and position.
- If acceleration is a function of time, $a(t)$, then velocity is $v(t) = u + \int_0^t a(t)\,dt$.
- If acceleration is a function of position, $a(x)$, we use the relation $a = v\frac{dv}{dx}$ to solve for velocity.

RELATIVE MOTION

- The relative position of object B with respect to A is $\boxed{x_{BA} = x_{BO} - x_{AO}}$.
The relative velocity of B with respect to A is found by differentiating the position equation: $\boxed{V_{BA} = V_{BO} - V_{AO}}$.

PRACTICE QUESTIONS

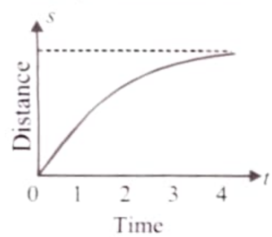
Single Correct Type Questions

1. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:

(1) $a+\frac{b^2}{4c}$ (2) $a+\frac{b^2}{c}$

(3) $a+\frac{b^2}{2c}$ (4) $a+\frac{b^2}{3c}$

2. The displacement of a particle as a function of time is shown in fig. The fig. indicates that



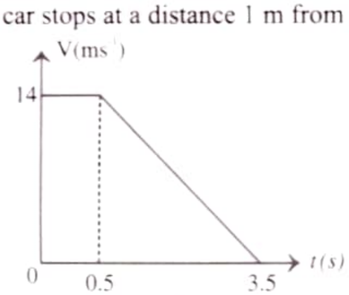
(1) the particle starts with a certain velocity, but the motion is retarded and finally the particle stops

(2) the velocity of particle is constant through

(3) the acceleration of the particle is constant throughout

(4) the particle starts with a constant velocity, the motion is accelerated and finally the particle moves with another constant velocity.

3. A car is moving uniformly on a road and a dog suddenly appears at $t = 0$ in front of the car at a distance d m. When the driver sees the dog, he apply brakes to the car immediately. The following is the velocity-time graph of car. The car stops at a distance 1 m from the dog. Find d.



(1) 22 m (2) 15 m

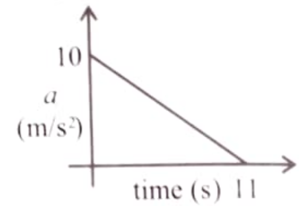
(3) 29 m (4) 28 m

4. Two balls are dropped from different heights. One ball is dropped 2 sec after the other but they both strike the ground at the same time, 3 sec after the first is dropped. The difference in the heights at which they were dropped is ($g = 9.8$ m/s^2)

(1) 7.8 m (2) 78 m

(3) 15.6 m (4) 39.2 m

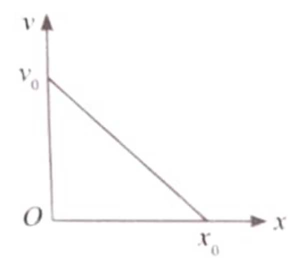
5. A particle is initially at rest, It is subjected to a linea acceleration a, as shown in the figure. The maximum speed attained by the particle is



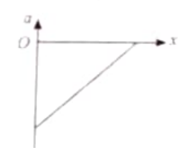
(1) 605 m/s (2) 110 m/s

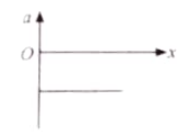
(3) 55 m/s (4) 550 m/s

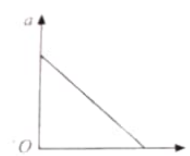
6. The velocity - displacement graph of a particle is show in the figure.

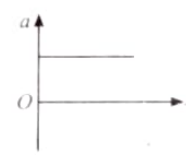


The acceleration - displacement graph of the sa particle is represented by:

(1) 

(2) 

(3) 

(4) 

7. A stone is dropped from rising balloon at a height of m above the ground and reaches the ground in 6 s. Wh was the magnitude of velocity of the balloon when t stone was dropped? Take $g = 10\,\text{ms}^{-2}$

(1) 52/3 ms^{-1} (2) 27/5 ms^{-1}

(3) 13/4 ms^{-1} (4) 10 ms^{-1}

8. A motor car covers $\frac{1}{3}$rd part of total distance with v 10 km/hr. second $\frac{1}{3}$rd part with $v_2 = 20$ km/hr and rest $\frac{1}{3}$rd part with $v_3 = 60$ km/hr. What is the average speed of the car?

(1) 18 km/hr (2) 45km/hr

(3) 6 km/hr (4) 22.5 km/hr

9. A particle is moving so that its displacement is given as $s = t^3 - 6t^2 + 3t + 4$ meter. Its velocity at the instant when its acceleration is zero will be

(1) 3 m/s (2) –12 m/s
(3) 42 m/s (4) –9 m/s

10. **STATEMENT-1** : A particle having negative acceleration will slow down.

STATEMENT-2: Direction of the acceleration is not dependent upon direction of the velocity.

(1) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1

(2) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for STATEMENT-1

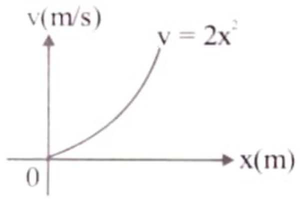
(3) STATEMENT-1 is true, STATEMENT-2 is false

(4) STATEMENT-1 is false, STATEMENT-2 is true

11. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by: $\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be:

(1) 1 s (2) 2 s
(3) 4 s (4) 8 s

2. The velocity-position graph of a particle moving in a straight line along x-axis is given below. Acceleration of particle at x = 2 m is



(1) 8 m/s² (2) 16 m/s²
(3) 64 m/s² (4) 32 m/s²

13. The velocity of a particle moving along y-axis is given by $v = \frac{2}{3}t^{3/2}\,ms^{-1}$ where 't' is in seconds. Then acceleration of the particle is

(1) $\left(\frac{3}{2}v\right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3}v\right)^{\frac{1}{3}}$

(3) $\left(\frac{3}{2}v\right)^{\frac{2}{3}}$ (4) $\left(\frac{2}{3}v\right)^{\frac{2}{3}}$

4. Front end of a locomotive, accelerating at a constant rate, crosses a signal post at time $t = 0$ with an initial velocity u. The rear end of last carriage crosses the same signal post with a final velocity v. After $t = 0$ what is the ratio of the time interval taken by the midpoint of the locomotive to the time interval taken by the rear carriage to cross the signal post after $t = 0$?

(1) $\frac{u+v}{2(v-u)}$ (2) $\frac{\sqrt{\frac{v^2+u^2}{2}}-u}{v-u}$

(3) $\frac{v-u}{2(v+u)}$ (4) $\sqrt{\frac{v^2-u^2}{2(v^2+u^2)}}$

15. A man standing on the edge of a cliff throws a stone straight up with initial speed u and then throws another stone straight down with same initial speed u from the same position. Find the ratio of speeds, the stones would have attained when they hit the ground at the base of the cliff?

(1) 2 : 1 (2) 1 : 2
(3) 1 : 1 (4) 3 : 1

16. A body is thrown upward and reaches its maximum height. At that position

(1) Its velocity is zero and its acceleration is also zero

(2) Its velocity is zero but its acceleration is maximum

(3) Its acceleration is minimum

(4) Its velocity is zero and its acceleration is the acceleration due to gravity

17. The displacement-time graph for the two particles A and B are straight lines inclined at angles 30° and 60° with the time axis. The ratio of the velocities of A to B will be

(1) 1 : 2 (2) $1:\sqrt{3}$
(3) $\sqrt{3}:1$ (4) 1 : 3

18. A circus performer is juggling balls such that by the time a ball reaches maximum height next ball leaves the hand of juggler. He throws each ball vertically upward with the same initial velocity. If he throws n balls per second, and the time between throwing each ball is the same, what is the initial velocity of each ball?

(Assume little movement of juggler's hand)

(1) $2g/n$ (2) $g/2n$
(3) g/n (4) $3g/2n$

19. A particle is thrown upwards from grou- experiences a constant air resistance which can- retardation of 2 m/s² opposite to the directi- -city of particle. The ratio of time of ascent t- -t descent is:
[g = 10 m/s²]

$\sqrt{\frac{2}{3}}$

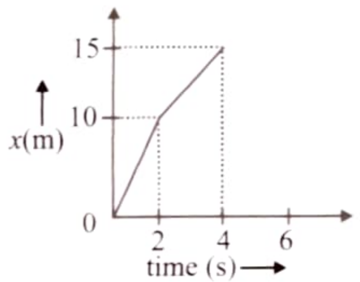
(1) 1 : 1 (4) $\sqrt{\frac{3}{2}}$

(3) 2

20. A thief is running away on a straight road in a car moving with a constant speed of $9\ ms^{-1}$. A policeman chases him on a motor bike moving at a constant speed of $20\ ms^{-1}$. If the instantaneous separation of the car from the motor bike is 99 m, how long (in seconds) will it take for the policeman to catch the thief?
(1) 4 (2) 9
(3) 25 (4) 6

Integer Type Questions

21. The position-time graph for a bus moving on a straight path is shown in the following figure. If the velocity of bus for time interval between 0 to 2 second is v_1 and for time interval between 2 seconds to 4 seconds is v_2. Find $\frac{v_1}{v_2}$

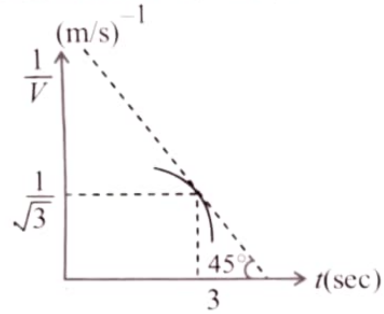


22. Two cars A and B are at rest at same point initially at $t = 0$ sec. If A starts with uniform velocity of 40 m/sec and B starts from rest in the same direction with constant acceleration of $4\ m/s^2$, then B will catch A after how much time (in sec)?

23. A locomotive traveling with an initial velocity of 50 m/s must engage its braking system 600 m from the station to achieve a complete stop precisely at the station. If the braking were initiated at one-third of this distance from the station, the locomotive would exit the station with a velocity of $10\sqrt{x}$ m/s. Determine the value of $0.3x$, assuming the retardation produced by the braking system remains constant.

24. A man standing in a lift throws a ball with a speed of 20 m/s in upward direction relative to lift and lift is moving upwards with an acceleration $a = 2\ m/s^2$. Find the time (in seconds) when ball comes back to man in seconds. {Take $g = 10\ m/s^2$}. {Assume the ball does not hit the roof of lift}.

25. The graph shows the variation of $\frac{1}{V}$ (where V is the velocity of the particle) with respect to time. Then find the value of acceleration (in m/s^2) at $t = 3$ sec.



ANSWER KEY

1.	(4)	6.	(1)	11.	(2)	16.	(4)	21.	(2)
2.	(1)	7.	(1)	12.	(3)	17.	(4)	22.	(20)
3.	(3)	8.	(1)	13.	(1)	18.	(3)	23.	(5)
4.	(4)	9.	(4)	14.	(2)	19.	(2)	24.	(4)
5.	(3)	10.	(4)	15.	(3)	20.	(2)	25.	(03.00)

HINT & SOLUTIONS

1. (4)
x = at + bt2 – ct3

$$v = \frac{dx}{dt} = a + 2bt - 3ct^2$$

$$a = \frac{dv}{dt} = 2b - 6ct = 0 \Rightarrow t = \frac{b}{3c}$$

$$v_{\left(at\, t = \frac{b}{3c}\right)} = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = a + \frac{b^2}{3c}$$

2. (1)
As the slope of tangent decreases, velocity also decreases with time.

after time distance becomes constant i.e particle stops.

3. (3)

$$d = 1 + 14 \times 0.5 + \frac{1}{2} \times 14 \times 3$$

$$= (1 + 7 + 21)\ m$$

$$d = 29\,m$$

4. (4)

$$h_1 = \frac{1}{2} g\,(3)^2$$

$$h_2 = \frac{1}{2} g (3-2)^2$$

$$h_1 - h_2 = \frac{1}{2} g (9-1) = 4 \times 9.8 = 39.2 \text{ m}$$

5. **(3)**

Area under acceleration-time graph gives the change in velocity.

Hence,

$$v_{max} = \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$$

6. **(1)**

The slope of the given v versus x graph is

$m = -\frac{v_0}{x_0}$ and intercept is $c = +v_0$.

Hence, varies with x as

$$v = -\left(\frac{v_0}{x_0}\right)x + v_0 \qquad \ldots(i)$$

Using equation (i) in equation (ii), we get

$$a = v\frac{dv}{dx} = -\left(\frac{v_0}{x_0}\right)\left(-\frac{v_0}{x_0}x + v_0\right)$$

$$\Rightarrow \quad a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

Thus the graph of a versus x is a straight line having a positive slope $= \left(\frac{v_0}{x_0}\right)^2$ and negative intercept $= \frac{v_0^2}{x_0}$

7. **(1)**

$$S = ut + \frac{1}{2}at^2$$

$$\Rightarrow -76 = u \times 6 - \frac{1}{2} \times 10 \times (6)^2 \Rightarrow u = \frac{52}{3}\text{ms}^{-1}$$

8. **(1)**

$$v_{avg} = \frac{3}{\frac{1}{10}+\frac{1}{20}+\frac{1}{60}} = \frac{3}{\frac{10}{60}} = 18 \text{ km/h}$$

9. **(4)**

$$s = t^3 - 6t^2 + 3t + 4$$

$$v = \frac{ds}{dt} = 3t^2 - 12t + 3$$

$$a = \frac{dv}{dt} = 6t - 12 = 0 \therefore t = 2 \text{ sec}$$

$$v = 3(2)^2 - 12 \times 2 + 3$$
$$= 12 - 24 + 3 = -9 \text{m/s}$$

10. **(4)**

Conceptual

11. **(2)**

$$\int_{6.25}^{0} \frac{dv}{\sqrt{v}} = -2.5\int_0^t dt$$

$$\left|2\sqrt{v}\right|_{6.25}^{0} = -2.5 \text{ t}$$

$$2.\sqrt{6.25} = 2.5 \text{ t} \qquad \Rightarrow \quad t = 2 \text{ sec.}$$

12. **(3)**

$$a = v\frac{dv}{dx}, \quad v = 2x^2 \Rightarrow a = (2x^2)\frac{d(2x^2)}{dx} = (2x^2)(4x) = 8x^3$$

At $x = 2$ m:

$$a = 8(2)^3 = \boxed{64 \text{ m/s}^2}$$

13. **(1)**

$$v_{(t)} = \frac{2}{3}t^{3/2} \Rightarrow a_{(t)} = \frac{dv}{dt}(t) = t^{1/2} = \left(\frac{3}{2}v\right)^{\frac{1}{3}}$$

14. **(2)**

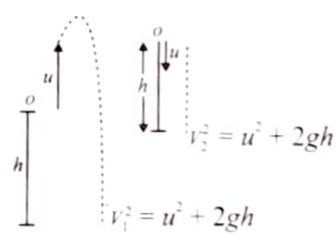
Let the length of train be l and its acceleration be a. From kinematic equation.

$$\Rightarrow \quad al = \frac{v^2 - u^2}{2}$$

Velocity when middle point crosses the post,

$$V_m = \sqrt{u^2 + 2a\frac{l}{2}} = \sqrt{u^2 + \frac{v^2-u^2}{2}} = \sqrt{\frac{u^2+v^2}{2}}$$

15. **(3)**



$V_1 = V_2$ $\qquad$ so $\frac{V_1}{V_2} = 1 : 1$

16. **(4)**

At maximum height velocity is zero and acceleration is g.

17. **(4)**

$V_A = \tan 30°$

$V_B = \tan 60°$

$$\frac{V_A}{V_B} = \frac{\tan 30°}{\tan 60°} = \frac{1}{\sqrt{3}.\sqrt{3}} = 1 : 3$$

18. **(3)**

Time to reach highest point $= \left(\frac{1}{n}\right)$

$$\frac{u}{g} = \frac{1}{n}$$

$$u = \left(\frac{g}{n}\right)$$

19. **(2)**

Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and time of descent respectively. If the particle rises upto a height 'h'.

Then $h = \frac{1}{2}(g + a)\, t_a^2$ and $h = \frac{1}{2}(g - a)\, t_d^2$

$$\therefore \quad \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$$

20. **(2)**

Relative velocity of policeman w.r.t. the thief is 11. Since the relative separation between them is 99 m, the time taken will be = relative separation relative velocity =99/11 = 9 s

21. **(2)**

$$(\text{Slope})_{0 \text{ to } 2} = \frac{10-0}{2-0} \Rightarrow 5 \text{ m/s}$$

$$(\text{Slope})_{2 \text{ to } 4} = \frac{15-10}{4-2} \Rightarrow \frac{5}{2} \text{ m/s}$$

$$\frac{v_1}{v_2} = 2$$

22. **(20)**

Let A and B will meet after time t sec. it means the distance travelled by both will be equal.

$$S_A = ut = 40t \text{ and } S_B = \frac{1}{2}at^2 = \frac{1}{2} \times 4 \times t^2$$

$$S_A = S_B \quad \Rightarrow 40t = \frac{1}{2}4t^2 \Rightarrow t = 20 \text{ sec}$$

23. **(5)**

$u = 50$ m/s, $S_1 = 600$ m, $v = 0$

Using third equation of motion

$0 = (50)^2 - 2a\,(600)$

$$\Rightarrow \quad a = \frac{25}{12} \text{ m/s}^2$$

After brakes are applied

$u = 50$m/s, $S_2 = 200\ m$, $v = ?$

$v^2 = (50)^2 - 2a.200$

$$v = 10\sqrt{\frac{50}{3}} \text{ m/s}$$

$$\therefore \quad x = \frac{50}{3}$$

$$0.3x = 5$$

24. **(4)**

$V_{rel} = 24$ m/s

$a_{rel} = (10 + 2) = 12$ m/s^2

$$t = \left(\frac{2v_{rel}}{a_{rel}}\right) = 2 \times 2 = 4 \text{sec}$$

25. **(03.00)**

At $t = 3$

$$\text{Slope} = \frac{d\left(\frac{1}{V}\right)}{dt} = -1$$

$$\Rightarrow -\frac{1}{V^2}\frac{dV}{dt} = -1$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2 \frac{dV}{dt} = 1$$

$$\Rightarrow a = \frac{dV}{dt} = 3 \text{ m / s}^2$$

CHAPTER

03 MOTION IN A PLANE

ALAR AND VECTOR

A **scalar** quantity has only magnitude (e.g., mass, temperature).

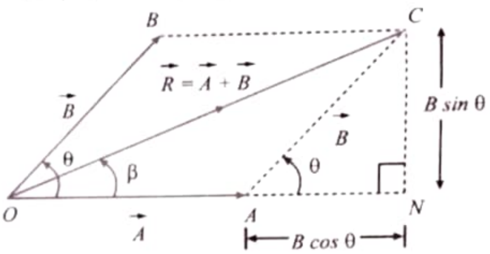
A **vector** quantity has both magnitude and direction (e.g., velocity, force).

ARALLELOGRAM LAW OF VECTOR ADDITION

If two vectors are represented by the adjacent sides of a parallelogram, their resultant is the diagonal starting from the common point.

The magnitude of the resultant is

$R = \sqrt{A^2 + B^2 + 2ABcos\theta}$.



RIANGLE LAW OF VECTOR ADDITION OF TWO ECTORS

To add two vectors, place the tail of the second vector at the tip of the first. The resultant vector is drawn from the tail of the first vector to the tip of the second.

The magnitude of the resultant vector $\vec{R} = \vec{A} + \vec{B}$ is $R = \sqrt{A^2 + B^2 + 2ABcos\theta}$, where θ is the angle between $\vec{A}$ and $\vec{B}$.

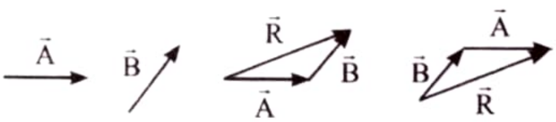


Fig. (i) Fig. (ii) Fig. (iii)

ALAR PRODUCT (DOT PRODUCT)

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity defined as $\vec{A}\cdot\vec{B} = ABcos\theta$.

An example is work done: $W = \vec{F}\cdot\vec{S}$.

ROSS PRODUCT (VECTOR PRODUCT)

The cross product $\vec{A} \times \vec{B}$ is a vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

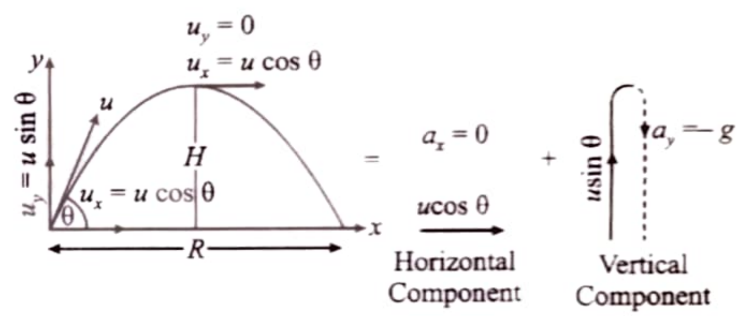
Its magnitude is $ABsin\theta$, and its direction is given by the right-hand thumb rule.

RECTANGULAR COMPONENTS IN THREE DIMENSION

- A vector $\vec{A}$ can be broken down into components along the X, Y, and Z axes: $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.
- The magnitude is given by $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

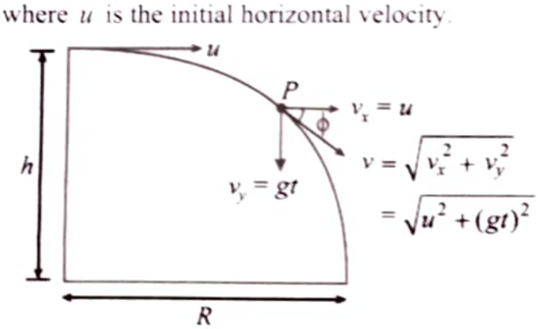
PROJECTILE THROWN FROM THE GROUND LEVEL

- For a projectile launched with initial velocity u at an angle θ with the horizontal:
- **Time of Flight:** $T = \dfrac{2usin\theta}{g}$
- **Maximum Height:** $H = \dfrac{u^2sin^2\theta}{2g}$
- **Horizontal Range:** $R = \dfrac{u^2sin(2\theta)}{g}$



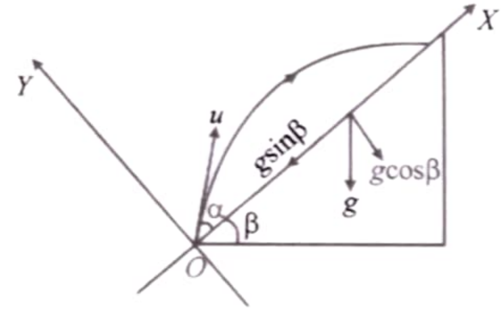
HORIZONTAL PROJECTION

- This is when an object is projected horizontally from a certain height, h.
- The time taken to reach the ground depends only on the height: $t = \sqrt{\dfrac{2h}{g}}$.
- The horizontal distance covered (Range) is $R = u \times t$, where u is the initial horizontal velocity.



PROJECTED FROM SOME HEIGHT AT SOME ANGLE

- When a projectile is launched from a height, its time of flight is calculated using the vertical motion equation $s_y = u_y t + \frac{1}{2}a_y t^2$.
- If projected upwards, the initial vertical velocity u_y is positive. If projected downwards, it is negative.



PROJECTION ON AN INCLINED PLANE

- For projectile motion on an incline, the acceleration due to gravity (g) is resolved into components parallel and perpendicular to the plane.
- For projection **up an incline** with angle β, the time of flight is $T = \frac{2u sin\alpha}{g cos\beta}$, where α is the projection angle relative to the incline.

RELATIVE VELOCITY OF RAIN W.R.T. THE MOVING MAN

- The relative velocity of rain with respect to a person ($\vec{v}_{rm}$) is the vector difference between the velocity of rain ($\vec{v}_r$) and the velocity of the person ($\vec{v}_m$).
- Formula: $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$.

SWIMMING INTO THE RIVER

- The velocity of a swimmer relative to the ground ($\vec{v}_m$) is the vector sum of their velocity in still water ($\vec{v}$) and the velocity of the river ($\vec{v}_R$).
- Formula: $\vec{v}_m = \vec{v} + \vec{v}_R$.

CIRCULAR MOTION

- **Angular velocity** (ω) is the rate of change of angular displacement, related to linear velocity by $v = r\omega$.
- **Centripetal acceleration** (a_C) is directed towards the center and is responsible for changing the direction of velocity. Its magnitude is $a_C = \frac{v^2}{r} = \omega^2 r$.

RADIUS OF CURVATURE

- Any curved path can be thought of as being made up of tiny circular arcs.
- The radius of curvature (R) at any point on a path is the radius of the circle that best fits the curve at that point and is calculated as $R = \frac{v^2}{a_n}$, where a_n is the normal component of acceleration.

PRACTICE QUESTIONS

Single Correct Type Questions

1. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by:

(1) $\sqrt{\alpha^2 + \beta^2}$ (2) $3t^2\sqrt{\alpha^2 + \beta^2}$

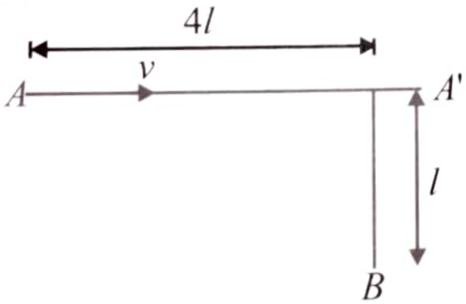
(3) $t^2\sqrt{\alpha^2 + \beta^2}$ (4) $3t\sqrt{\alpha^2 + \beta^2}$

2. Two particles 1 and 2 are moving with velocities $\vec{v}_1 = 4\hat{i} - 3\hat{j}$ m/s and $\vec{v}_2 = b\hat{i} - \hat{j}$ m/s respectively. The position vectors of the particles at time t = 0 are $\vec{r}_1 = 5\hat{i} + 2\hat{j}$ m/s and $\vec{r}_2 = -4\hat{i} - 4\hat{j}$ m. If they collide at t = 3 s, the value of b is

(1) $\frac{10}{3}$ (2) 5

(3) –1 (4) 7

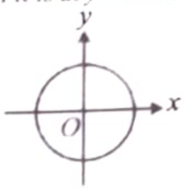
3. A man, running, on the road AA', with speed v, wants to reach his friend B trapped in the mudfield as early as possible. The speed of man in mudfield is one third of running speed along AA'. The minimum time taken (from A to B)



(1) $\frac{l}{v}\left[4 + \sqrt{2}\right]$ (2) $\frac{l}{v}\left[4 + \sqrt{8}\right]$

(3) $\frac{l}{v}\left[4 - \sqrt{2}\right]$ (4) $\frac{l}{v}\left[2 + \sqrt{8}\right]$

An object moves at constant speed along a circular path in a horizontal xy plane, with the center as origin. When the object is at $x = -2$ m, its velocity is $-4\hat{j}$ m/s. Its acceleration when it is at $y = 2$ m is



(1) $-8\hat{i}$ m/s^2 (2) $-2\hat{j}$ m/s^2

(3) $-8\hat{j}$ m/s^2 (4) $2\hat{i}$ m/s^2

A boatman moves his boat with a velocity 'v' (relative to water) in river and finds that velocity of river 'u' (with respect to ground) is more than 'v'. He has to reach a point directly opposite to the starting point on another bank by travelling minimum possible distance. Then

(1) He must steer the boat (with velocity v) at certain angle with river flow so that he can reach the opposite point on other bank directly.

(2) His velocity 'v' must be towards directly opposite point, So, that he can travel rest of distance by walking on other bank to reach the directly opposite point.

(3) Boatman should maintain velocity v of boat at certain angle greater than 90° with direction of river flow to minimize drifting and then walk rest of distance on other bank.

(4) Boat velocity 'v' should be at an angle less than 90° with direction of river flow to minimize the drift and then walk to the point.

A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R form it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is:

(1) R/g (2) $R/4g$

(3) $2R/g$ (4) $R/2g$

The position co-ordinates of a particle moving in a space varries with time t as $x = a\cos\omega t$, $y = a\sin\omega t$ and $z = a\omega t$. The speed of particle is equal to

(1) $2a\omega$ (2) $\sqrt{2}a\omega$

(3) $\sqrt{3}a\omega$ (4) insufficient data

u_A and u_B represent the velocities of two balls A and B thrown from same point over a horizontal ground at angle of projections 37° and 45° respectively with horizontal such that both the balls land at same point on ground. Value of $\frac{u_A}{u_B}$ is equal to

(1) $\frac{24\sqrt{6}}{5}$ (2) $\frac{5\sqrt{6}}{12}$

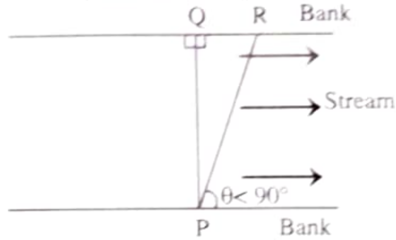
(3) $\frac{12}{5\sqrt{6}}$ (4) $\frac{5\sqrt{6}}{24}$

9. A particle has initial velocity, $\vec{v} = (9\hat{i} - 6\hat{j})$m/s and a constant acceleration $\vec{a} = (2\hat{i} + 3\hat{j})$ m/s^2 acts on the particle. The path of particle will be

(1) Straight line (2) Parabolic

(3) Circular (4) Elliptical

10. In a lake, stream direction is as shown in the figure. A man starting from the point P on the bank 1 wants to move to the bank 2 in shortest time. He should swim (relative to the river)



(1) along PQ

(2) along PR

(3) In a direction in-between PQ and PR

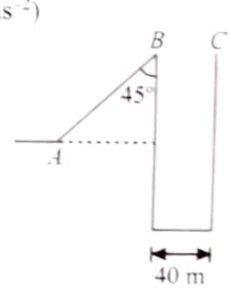
(4) in all cases he would reach at the same time

11. A ball-1 of mass m is thrown vertically upward. Another ball-2 of mass $2m$ is thrown at angle θ with the vertical such that by the time ball-1 comes to the ground ball-2 is at its highest point. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is __________.

(1) 8 (2) 2

(3) 16 (4) 4

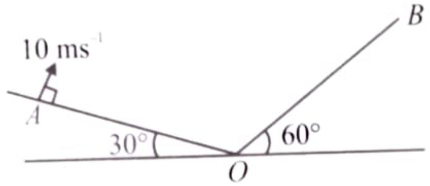
12. The diagram shown depicts a well of diameter 40 m with point B and C being the two diametrically opposite points on the top periphery at the mouth of the well. Point B is connected to point A with an inclined plane of angle of inclination 45° as shown. A particle projected at point A along incline AB of length $20\sqrt{2}$ m, with speed v_0, just manages to reach point C. Value of v_0 will be equal to (g = 10 ms^{-2})



(1) 20 ms^{-1} (2) $20\sqrt{2}$ ms^{-1}

(3) 40 ms^{-1} (4) $40\sqrt{2}$ ms^{-1}

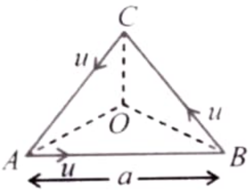
13. An object is thrown from point A, with 10 ms^{-1} at an angle of 60° with the horizontal, from point A of an inclined plane OA. During the journey, it hits another inclined plane OB, perpendicularly. Its speed at this instant is



(1) $2.5\ ms^{-1}$ (2) $5\ ms^{-1}$

(3) $\frac{10}{\sqrt{3}}\ ms^{-1}$ (4) $\frac{5}{\sqrt{3}}\ ms^{-1}$

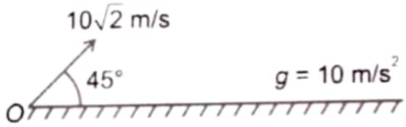
14. Three particles A, B and C situated at vertices of an equilateral triangle, all moving with same constant speed such that A always move towards B, B always towards C and C always towards A. Initial separation between each of the particle is a. O is the centroid of the triangle. Distance covered by particle A when it completes one revolution around O is



(1) $2a\left(1-e^{-2\sqrt{3}\pi}\right)$ (2) $\frac{2a}{3}\left(1-e^{-2\sqrt{3}\pi}\right)$

(3) $a\left(1+e^{-2\sqrt{3}\pi}\right)$ (4) $\frac{2a}{3}\left(1-e^{-\sqrt{3}\pi}\right)$

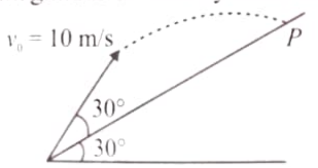
15. A particle is projected with speed $10\sqrt{2}$ m/s in vertical plane as shown. Time after which its velocity will be perpendicular to initial velocity, is



(1) $\frac{3}{2}$ s (2) $\frac{3}{4}$ s

(3) 1 s (4) 2 s

16. A particle is projected up an inclined plane of inclination 30° with the horizontal as shown in the figure. It strikes the inclined plane at point P. If magnitude of the initial velocity is 10 m/s and the angle of projection with the inclined plane is 30°. The magnitude of velocity at the striking point P is



(1) $5\ ms^{-1}$ (2) $5\sqrt{3}\ ms^{-1}$

(3) $\frac{10}{\sqrt{3}}\ ms^{-1}$ (4) $\frac{10}{3}\ ms^{-1}$

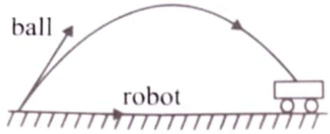
17. A particle is thrown with speed u, at an angle α w... horizontal, from the bottom of an inclined plane, hav... an angle of inclination β with horizontal. The part... strikes the inclined plane horizontally. Then

(1) $\frac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta}=\frac{1}{2}$ (2) $\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}=\frac{1}{2}$

(3) $\frac{\sin(\alpha-\beta)}{\sin\alpha\cos\beta}=\frac{1}{2}$ (4) $\frac{\sin(\alpha-\beta)}{\cos\alpha\sin\beta}=\frac{1}{2}$

18. At $t = 0$, a golf ball is projected with some initial spee... an angle θ above the horizontal. Suppose that at instant the ball is launched, a retriever robot on whee... sent off from the launch point. The robot starts from... and moves with constant acceleration in a straight... along the ground. What must be the magnitude a of... robot's acceleration such that the ball comes down... on the robot?

Given $\tan\theta = 5/4$ and horizontal level of projection... that of retriever robot same. (Take $g = 10\ m/s^2$)



(1) $2\ m/s^2$ (2) $4\ m/s^2$

(3) $6\ m/s^2$ **(4) $8\ m/s^2$**

19. A particle moves such that its position ve... $\vec{r}(t)=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ where ω is a constant and... time. Then which of the following statements is tru... the velocity $\vec{v}(t)$ and acceleration a $\vec{a}(t)$ of the parti...

(1) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed tow... the origin.

(2) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$.

(3) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed a... from the origin.

(4) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$.

20. In a circular motion of a particle the tange... acceleration of the particle is given by $a_t = 2t\ m/s^2$... radius of the circle described is 4m. The partic... initially at rest. Time after which total acceleration o... particle makes 45° with radial acceleration is

(1) 1 sec (2) 2 sec

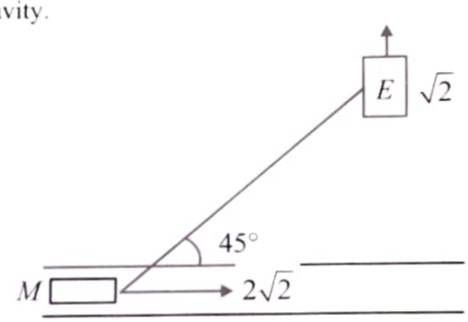
(3) 3 sec (4) 4 sec

Integer Type Questions

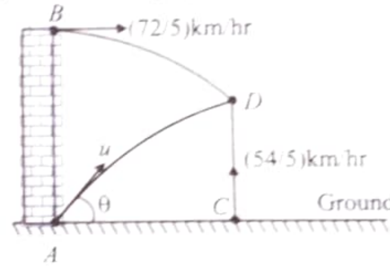
21. Two similar objects A and B are projected with s... speeds u but at different angles with horizontal... maximum possible horizontal range of A with the g... speed is twice the horizontal range of B. If the proje... angle of B is X degrees, then find X

2. A swimmer tries to cross the river normally to the flow of river of width 1.5 km. His speed in still water is 5 km/h. He drifted 500 m along the direction of river water flow, by the time he reaches opposite bank. The speed of river water is $\frac{x}{y}$ km/h. Find $x + y$ [x and y are nearest positive integer]

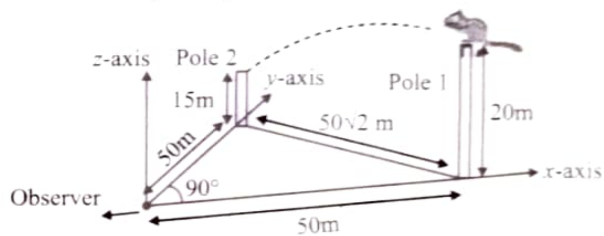
. A bow man is riding on a horse moving with speed of $2\sqrt{2}$ ms^{-1} along a straight road. He aims at his enemy moving perpendicularly to the road at speed of $\sqrt{2}$ ms^{-1}. At the instant when he fires the arrow, the line joining man and his enemy makes an angle of 45° with the road. Find the angle (in degree) with the road at which he should aim to hit his enemy? Muzzle velocity of arrow is 5 ms^{-1}. (given that sin37° = 3/5). Neglect effect of gravity.



In the given figure points A and C are on the horizontal ground & A and B are in same vertical plane. Simultaneously bullets are fired from A, B and C and they collide at D. The bullet at B is fired horizontally with speed of $\frac{72}{5}$ km/hr and the bullet at C is projected vertically upward at velocity of $\frac{54}{5}$ km/hr. Find velocity of the bullet projected from A in m/s.



25. A small squirrel jumps from pole 1 to pole 2 in horizontal direction. Squirrel is observed by a very small observer at origin. What is average velocity vector of squirrel? If average velocity vector is expressed as $v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, express your answer as sum of magnitudes of its components $|v_x| + |v_y| + |v_z|$ in unit m/s.



ANSWER KEY

	(2)	6.	(3)	11.	(4)	16.	(3)	21.	(15)
	(4)	7.	(2)	12.	(2)	17.	(3)	22.	(8)
	(2)	8.	(2)	13.	(3)	18.	(4)	23.	(82)
	(3)	9.	(2)	14.	(2)	19.	(1)	24.	(5)
	(3)	10.	(1)	15.	(4)	20.	(2)	25.	(105)

HINT & SOLUTIONS

(2)

$x = \alpha t^3, y = \beta t^3$

$v_x = \frac{dx}{dt} = 3\alpha t^2$

$v_y = \frac{dy}{dt} = 3\beta t^2$

Resultant velocity

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$

$= 3t^2\sqrt{\alpha^2 + \beta^2}$

2. **(4)**

$\vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$

$(5\hat{i} + 2\hat{j}) + (4\hat{i} - 3\hat{j}) = (-4\hat{i} - 4\hat{j}) + (b\hat{i} - \hat{j})(3)$

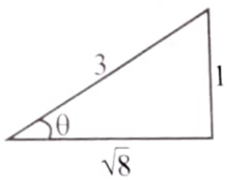
$17\hat{i} - 7\hat{j} = (-4 + 3b)\hat{i} - 7\hat{j}$

So, (3b – 4) 17

$\Rightarrow b = 7$

3. **(2)**

$\Rightarrow \quad \sin\theta = \frac{v_2}{v_1} = \frac{1}{3}$



$$t = \frac{AC}{v_1} + \frac{CB}{v_2} = \frac{AC}{v_1} + \frac{BD}{\cos\theta \times v_2}$$

$$= \frac{AD - CD}{v_1} + \frac{BD}{\cos\theta \times v_2}$$

$$= \frac{4l - l\tan\theta}{v_1} + \frac{l}{v_2\cos\theta}$$

$$= \frac{l}{v}\left[4 + \sqrt{8}\right]$$

4. **(3)**

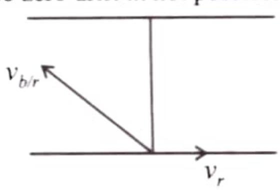
Acceleration $= \frac{v^2}{r}$ towards centre

$\therefore |\vec{a}| = \frac{4^2}{2} = \frac{16}{2} = 8$ towards centre

i.e., $\vec{a} = 8(-\hat{j})$

5. **(3)**

$v_r > v_{b/r}$, so zero drift in not possible.



(C) boatman should be maintain velocity v of boat at certain angle greater than 90° with direction of river flow to minimize drifting & than walk rest of distance on other bank.

6. **(3)**

a $t_1 = \frac{2u\sin\theta}{g}, t_2 = \frac{2u\sin(90° - \theta)}{g}$

and $R = \frac{u^2\sin 2\theta}{g}$

$$t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$$

$$= \frac{2}{g}\left[\frac{2u^2\sin\theta\cos\theta}{g}\right] = \frac{2R}{g}$$

7. **(2)**

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$V_x = -a\omega\sin\omega t$

$V_y = a\omega\cos\omega t$

$V_2 = a\omega$

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{2}a\omega$$

8. **(2)**

For ball A

$R = \frac{u_A^2\sin 2\theta}{g}$ | Given $\theta = 37°$

$$R = \frac{u_A^2 \times 2\sin 37°\cos 37°}{g}$$

$$R = \frac{24}{25}\frac{u_A^2}{g}$$

For ball B

$R = \frac{u_B^2\sin 2\theta}{g}$ | $\theta = 45°$

$$= \frac{u_B^2\sin 90°}{g}$$

$$R = \frac{u_B^2}{g}$$

(1) ÷ (2)

$$1 = \frac{24u_A^2}{25u_B^2} \Rightarrow \frac{u_A}{u_B} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

9. **(2)**

Parabolic

10. **(1)**

The man must swim along the direction of shortest distance.

11. **(4)**

$T_2 = 2T_1$

$u_2 = 2u_1$

$H \propto u^2$

$\Rightarrow \quad \frac{H_1}{H_2} = \frac{1}{4}$

12. **(2)**

Let v be the velocity acquired by the body at B which be moving making an angle 45° with the horiz direction. As the body just crosses the well so, $\frac{v^2}{g} =$

or $v^2 = 40g = 40 \times 10 = 400$

or $v = 20\,ms^{-1}$

Taking motion of the body from A to B along the incl plane we have

$$u = v_0, a = -g\sin 45° = -\frac{10}{\sqrt{2}}ms^{-2}$$

$s = 20\,m, v = 20\,ms^{-1}$

As $v^2 = u^2 + 2as$

$$\therefore 400 = v_0^2 + 2\left(-\frac{10}{\sqrt{2}}\right)\times 20\sqrt{2}$$

$$v_0^2 = 400 + 400 = 800 \text{ or } v = 20\sqrt{2}\text{ ms}^{-1}$$

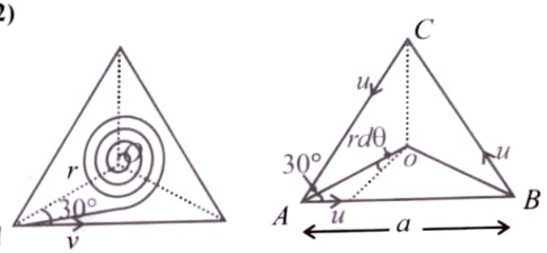
13. **(3)**

The object hits plane OB perpendicular, so

$$t = \frac{v}{g\sin 60^\circ} = \frac{10\times 2}{10\times\sqrt{3}} = \frac{2}{\sqrt{3}}\text{sec}$$

$$v = 10\times\cot 60^\circ = \frac{10}{\sqrt{3}}\text{ m/s}$$

14. **(2)**



$$\frac{dr}{dt} = -v\cos 30^\circ = -\frac{\sqrt{3}}{2}v$$

$$r\frac{d\theta}{dt} = v\sin 30^\circ = v/2$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\sqrt{3}$$

$$\int_{r_0}^{r}\frac{dr}{r} = -\sqrt{3}\int_0^\theta d\theta \Rightarrow r = r_0 e^{-\sqrt{3}\theta}$$

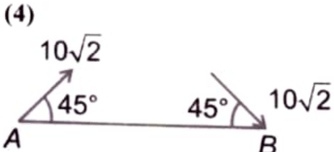
When A completes one revolution $\theta = 2\pi$

Time taken $t = \dfrac{r_0\left(1-e^{-2\sqrt{3}\pi}\right)}{\sqrt{3}v/2}$

Distance travelled $D = vt = \dfrac{2r_0}{\sqrt{3}}\left(1-e^{-2\sqrt{3}\pi}\right)$

$$D = \frac{2a}{3}\left(1-e^{-2\sqrt{3}\pi}\right)$$

15. **(4)**



$t = T$

$= \dfrac{(2)10}{10}$

$= 2$ s

16. **(3)**

Let the particle strikes the inclined plane at point P at an angle θ from the inclined plane

$v\sin\theta = v_0\sin 30^\circ$ ....(i)

$v\cos\theta = v_0\cos 30^\circ - g\sin 30^\circ\left(\dfrac{2v_0\sin 30^\circ}{g\cos 30^\circ}\right)$ ....(ii)

Solving equation (i) and (ii)

$\theta = 60^\circ$ and $v = \dfrac{10}{\sqrt{3}}ms^{-1}$

17. **(3)**

Time of flight upon inclined plane

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

As $v_\perp$ at inclined plane $= 0$

Also, $T = \dfrac{u\sin\alpha}{g}$

$$\Rightarrow \frac{2u(\sin\alpha-\beta)}{g\cos\beta} = \frac{u\sin\alpha}{g}$$

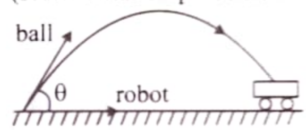
$$\Rightarrow \frac{\sin(\alpha-\beta)}{\sin\alpha\cos\beta} = \frac{1}{2}$$

18. **(4)**

When the ball reaches the robot,

$$u\cos\theta T = \frac{1}{2}aT^2$$

(Horizontal displacement is same)



$$\Rightarrow u\cos\theta = \frac{a}{2}T$$

where T is time of flight of ball $= \dfrac{2u\sin\theta}{g}$

$$\therefore u\cos\theta = \frac{a}{2}\left[\frac{2u\sin\theta}{g}\right]$$

$$\therefore a = g\cot\theta = 8\text{ m/s}^2$$

19. **(1)**

$$\vec{r} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\,\hat{i} + \cos\omega t\,\hat{j})$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$$

$$\vec{a} = -\omega^2\vec{r}$$

$\therefore \vec{a}$ is antiparallel to $\vec{r}$

$\vec{v}\cdot\vec{r} = \omega(-\sin\omega t\cos\omega t + \cos\omega t\sin\omega t = 0)$

so $\vec{v}\perp\vec{r}$

20. **(2)**

When total acceleration vector makes 45° with radial acceleration, then

$a_c = a_t = 2t$ ....(i)

$$a_t = \frac{dv}{dt} = 2t \Rightarrow v = t^2$$

and $a_c = \frac{v^2}{R} = \frac{t^4}{R}$ ....(ii)

From eqn. (i) and (ii),

$$2t = \frac{t^4}{R} \Rightarrow t^3 = 2R = 8$$

$$\Rightarrow t = 2\,\text{sec}$$

21. **(15)**

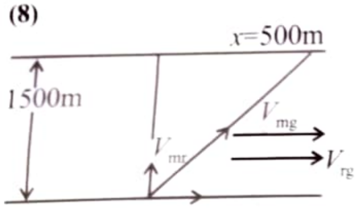
$$\left(\frac{u^2}{g}\right)_A = 2\left(\frac{u^2 \sin 2\theta}{g}\right)_B$$

$$\frac{1}{2} = \sin 2\theta$$

$$2\theta = 30^\circ$$

$$\theta = 15^\circ$$

22. **(8)**



time to cross the river of width 1500 m is $\frac{1.5\text{ km}}{5\text{km/h}}$

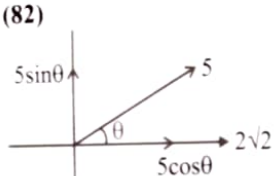
Drift $x = V_{rg} \times t$

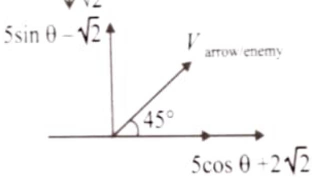
Where V_{rg} is velocity of river w.r. t. to ground.

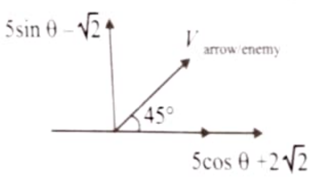
$$x = V_{rd} \times \frac{1.5}{5}$$

$$V_{rg} = \frac{5}{3}\text{ km/hr}$$

23. **(82)**







w.r.t. Enemy

$$5\cos\theta + 2\sqrt{2} = 5\sin\theta - \sqrt{2}$$

$$5\sin\theta - 5\cos\theta = 3\sqrt{2}$$

$$\frac{1}{\sqrt{2}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta = \frac{3}{5}$$

$$\sin(\theta - 45^\circ) = \sin 37^\circ$$

$$\theta = 45^\circ = 37^\circ$$

$$\theta = 82^\circ$$

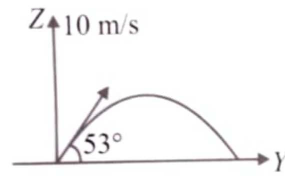
24. **(5)**

For collision

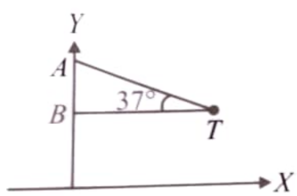
$$u = \sqrt{u_B^2 + u_C^2} = \sqrt{\left(\frac{72}{5} \times \frac{5}{18}\right)^2 + \left(\frac{54}{5} \times \frac{5}{18}\right)^2}$$

$$= \sqrt{4^2 + 3^2} = 5\text{ m/s}$$

25. **(105)**



YZ plane is vertical plane



XY plane is horizontal plane

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1\text{ s}$$

$$v_x = \frac{x}{t} = \frac{50}{1} = 50\text{ m/s}$$

$$v_y = \frac{y}{t} = \frac{50}{1} = 50\text{ m/s}$$

$$v_z = \frac{z}{t} = \frac{5}{1} = 5\text{ m/s}$$