# Faraday Rotation of Light

**Saumya Prakash Sharma-1911151**

October 04, 2022

## Astract

The Faraday rotation in presence of a direct current has been measured, and so is the Verdat constant for the glass rod. We, now, use the AC magnetic field to demonstrate this phenomenon. As the alternating current produces low magnetic field as compared to DC, we use the Lock-in amplifier to detect the weak signals produced via the photodiode. The Lock-in amplifier has been fully understood before performing this experiment, and an external phase shifter has also been made for the reference to the lock-in.

## 1 Introduction and Theory

### 1.1 Phase shifter

The phase shift circuits maintain a steady gain while producing frequency-dependent phase shifts. All-pass or constant-delay filters are other names for these circuits. When a frequency is varied across a range of operating frequencies, the time delay between the input and the output remains constant.

Because a fixed gain is often maintained for all frequencies inside the operational range, this is known as an all-pass system. As the phase angles can lead or lag, there are two different types of circuits for performing these two operations.

#### 1.1.1 Phase-lag circuit:

An op-amp is used to build a phase lag circuit, and it is used in both inverting and non-inverting modes. In order to study the circuit's behaviour, it is assumed that input voltage v1 drives both a non-inverting amplifier with a low-pass filter and a straightforward inverting amplifier with an inverting input applied at the negative terminal of the OPAMP.

Additionally, it is assumed that non-inverting gain following the low-pass circuit is -1 and inverting gain is given by:

$$1 + \frac{R_f}{R_1} = 1 + 1 = 2 \quad \text{Since Rf = R1}$$

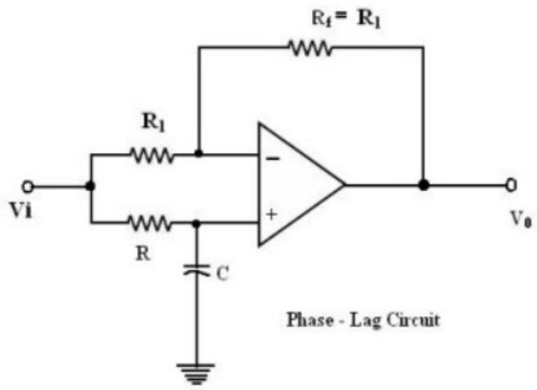


Figure 1: Lock-in amplifier

From the circuit, it can be written that:

$$V_O(jw) = -V_i(jw)\left(-1 + \frac{2}{1 + jwRC}\right)$$

and the relationship between output and input can be expressed by

$$\frac{V_O(jw)}{V_i(jw)} = \frac{(1 - jwRC)}{(1 + jwRC)}$$

### 1.1.2 Phase-lead circuit

$$\theta = -2\ \tan^{-1} RC\omega$$

$$\frac{V_O(jw)}{V_i(jw)} = -\frac{(1 - jwRC)}{(1 + jwRC)}$$

$$\theta = 180^\circ - 2\ \tan^{-1} RC\omega$$

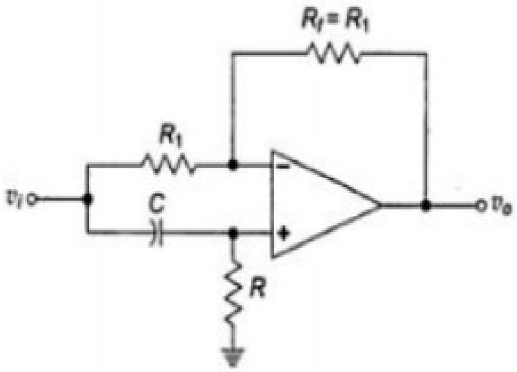


Figure 2: Phase-lead

## 1.2 LED and photodetector

To transmit data across a light channel, our device uses an LED and photodiode. Depending on how much we hide the system from external light sources like the room's lights, this channel is where noise can be added to the system. The beam then passes through the photodiode where it gets converted into a current signal that is fed into the Lock-in amplifier.
The aim here, is to detect the LED signal, among other noisy signals. More the noise(due to ambience lights), harder it is to detect the signal. But lock-in uses the reference and phase detection prnciple to spearate the deisred signal.

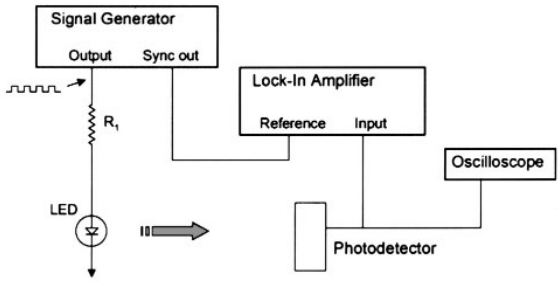


Figure 3: The experiment to show that a little signal buried in noise can be recovered via lock-in detection. The signal should be strong and visible on the oscilloscope when the LED is close to the photodetector. It may not be visible on the oscilloscope as the LED gets further away, but that's where the lock-in detection comes into picture that should make it simple to find the signal.

Here, we wish to modulate an LED with a simple square wave, for example, 500 Hz, and then observe the light output using a photodiode. When the LED is close to the photodiode, an oscilloscope can clearly display the signal, enabling direct measurement. At greater separations, the photodiode signal gets so weak that it is virtually undetectable on the oscilloscope trace.

However, even when the photodiode signal is buried in the background noise, the signal is still strong at the lock-in output when using the lock-in detector. .

## 1.3 Lock-in amplifier

For the Faraday rotation using AC magnetic fields, we employ the Lock-in Amplifier to detect the signal from the photodiode.

$$\begin{aligned} V_{\text{signal}} &= V_0 \sin(\omega t + \phi) \\ V_{\text{ref}} &= V_0 \sin(\omega t) \end{aligned}$$

Output of the amplifier will be,

$$V_{\text{out}} = \mu V_{\text{signal}}$$

The output from the Lock-in would be in the form of a full-wave rectified output at the right phase settings varied with the help of the phase shifter. This gives maximum DC Voltage. We can pass this signal to a low pass filter to average this and DC output.

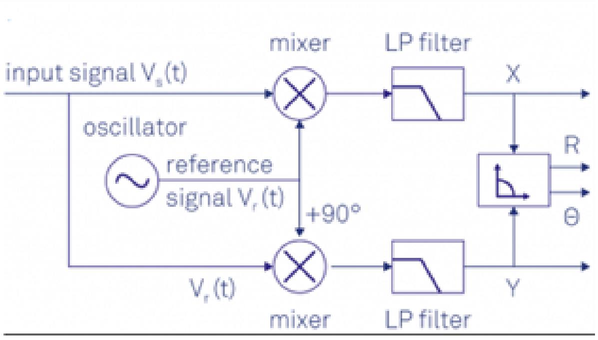


Figure 4: Lock-in amplifier

### 1.4 Detection and determination of Faraday rotation angle using Lock-in amplifier and Verdat constant from it

The light falling on the sample is aready polarised, let's say, in along x-axis and is propagating along z-direction, then the electric field of the light beam can be given as,

$$E_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_0 \exp(-i\omega t + ikz),$$

where $A_0$ is the amplitude of the electric field of the beam. WHen the beam passes through the sample, the polarization is rotated by a small angle, $\theta$, so,

$$E = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} A_0 \exp(-i\omega t + ikz).$$

After passing through the analyzer, which is at $\phi$ angle from the polarizer. Then, the electric field,

$$E = \begin{pmatrix} \cos(\phi-\theta)\cos\phi \\ \cos(\phi-\theta)\sin\phi \end{pmatrix} A_0 \exp(-i\omega t + ikz)$$

and the light intensity measured by the detector is given by

$$I = \cos^2(\phi-\theta)A_0^2.$$

For maximum modulation of the intensity, we have,

$$\frac{\partial I}{\partial \theta} = \sin 2(\phi-\theta)A_0^2.$$

As the angle of rotation is very small, it can be ignored. So, the analyzer should be at an angle of 45° with respect to the polarizer for maximum intensity.

$$I = \frac{1}{2}(1+2\theta)A_0^2.$$

Now, writing I in terms of angle of rotation is, Here, the ac magnetic field is sinusoidal $B = B_0\sin(\Omega t)$ and $\theta$ is proportional to $B$, the rotation angle can be written in the form of $\theta = \theta_0 \sin(\Omega t)$. Then, writing I in terms of the angle of rotation is,

$$I = \frac{1}{2}\left(1 + 2\theta_0\sin(\Omega t)\right)A_0^2 = I_0 + \Delta I \sin(\Omega t).$$

From the relative change in the light intensity $\Delta I/I_0$, the amplitude of the angle of rotation is found to be,

$$\theta_0 = \frac{1}{2}\frac{2\theta_0 A_0^2}{A_0^2} = \frac{1}{2}\frac{\Delta I}{I_0}.$$

The Faraday rotation angle $\theta$ is,

$$\theta_0 = \frac{V_{ac}}{2V_{dc}} = \frac{V_{ac(RMS)}}{\sqrt{2}V_{dc}}$$

$V_{dc}$ is the dc component of the output from the photodiode and $V_{ac}$ is the ac component which we get from the Lockin Amplifier.

The Verdet constant of the material is then determined using the relation $\theta_0 = VB_0L$, where $\theta_0$, $L$, and $B_0$ are experimentally determined.

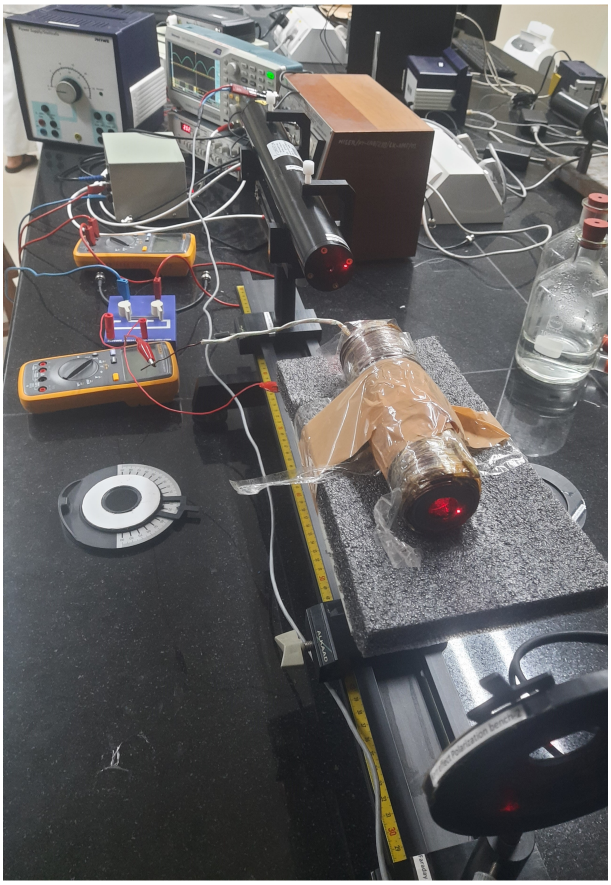

Figure 5: The experimental setup

# 2 Observations and Data Analysis

## 2.1 Phase shifter circuit

The circuit for the phase shifter oscillator is:

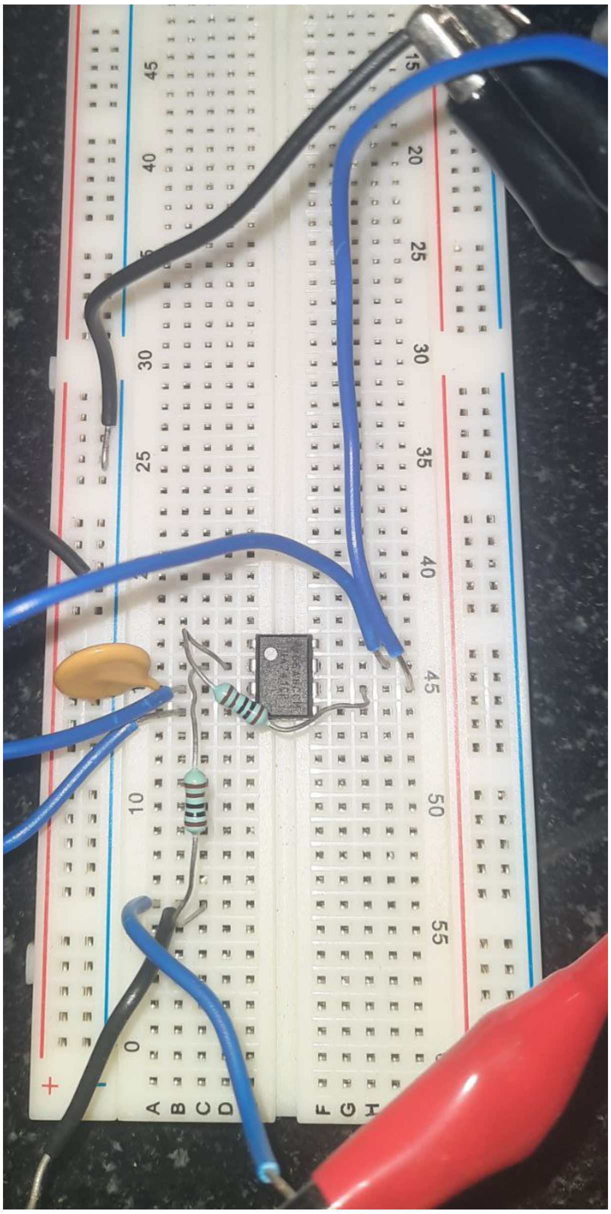

Figure 6: The phase shifter circuit

The components used here are OPAMP 747, two resistors of $10\Omega$, a capacitor of $10nF$ and a potentiometer of range $100k\Omega$.

The output from the circuit is connected to the oscilloscope to compare with the input signal. This is how it looks like:

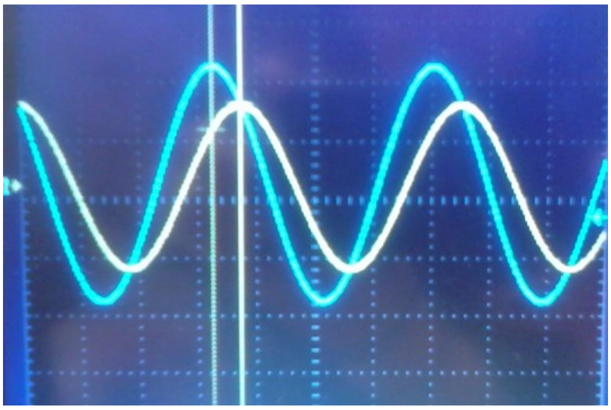

Figure 7: Input and the phase shifted output in the oscilloscope

It has been found that the phase of the output signal changes when the value of the potentiometer is changed. This circuit is capable of changing the phase from $0°$ to $180°$. The same has been verified via the output of the oscilloscope.
Both the input and output waves are in same phase when the knob of the potentiometer is at one end, i.e., when the external resistance, R is zero. Similarly, the output wave becomes out of phase when the knob of the potentiometer is fully rotated, depicting that this phase shifter works and any phase difference can be achieved using it.

## 2.2 LED as a lamp and the photodetector

The LED was first supplied the square pulse and then the sine pulse for a better study of the phase angle.

Here the distance between the LED and the photodiode is kept to be 35cm, to study the output of the photodiode and thereby the lock-in detection.
Here is the output of the signal from the Lock-in amplifier:

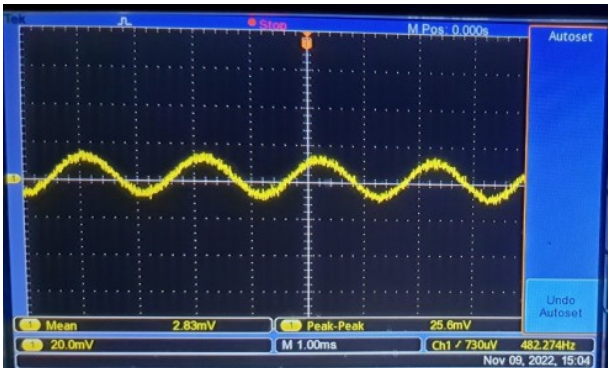


Figure 8: Output from the photodiode in dark condition

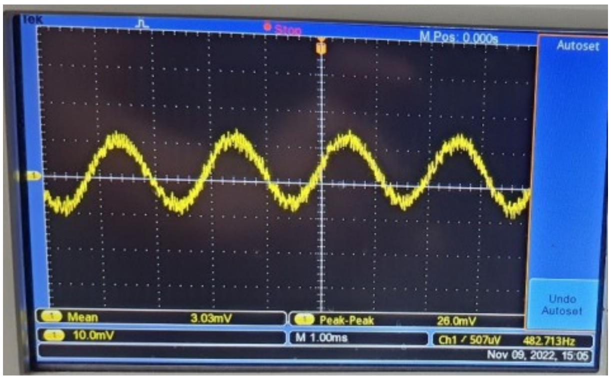


Figure 9: Output from the photodiode in ambient light condition


Figure 10: The output of the Lock-in in absence of any ambience light


Figure 11: The output of the photodiode in presence of ambience lights

When the ambience lights are turned on, the output from the photodiode is buried in noise, but can be detected from the lock-in. The irregularities in the shape of the signal are due to the dc-offset caught by the photodiode due to the noises.
This experiment proves the fact that photodiode is properly functioning and the Lock-in Amplifier is able to detect signal with and without the ambient light.
So, there is no problem with the photo diode or the Lock-in processing and they can be used further for the experiment.

### 2.3 Faraday rotation with DC magnetic field

We have used a different coil and a different glass rod to observe the Faraday rotation and determine the Verdat constant. As we know the relation between Verdat constant, magnetic field and the length of the rod is:

$$\Delta\theta = vdB$$

where v is the Verdat constant, d is the length, B is the magnetic field.

| Current (A) | mag field (mT) | $2\theta$ (°) | $\theta$ (°) | $\theta$(rad) |
|---|---|---|---|---|
| 1 | 12.1 | 0.5 | 0.25 | 0.00436332313 |
| 1.5 | 17 | 1 | 0.5 | 0.00872664626 |
| 2 | 23 | 1.5 | 0.75 | 0.01308996939 |
| 2.5 | 28.5 | 2 | 1 | 0.01745329252 |
| 3 | 31.7 | 2.5 | 1.25 | 0.02181661565 |
| 3.5 | 37.5 | 3 | 1.5 | 0.02617993878 |

Table 1: Data for coil 2

| current (A) | Magnetic field (mT) | current (A) | Magnetic field (mT) |
|---|---|---|---|
| 0.22 | 12.60 | 1.2 | 74.00 |
| 0.31 | 18.10 | 1.3 | 79.40 |
| 0.4 | 23.90 | 1.4 | 87.00 |
| 0.5 | 30.50 | 1.5 | 92.50 |
| 0.6 | 36.00 | 1.6 | 96.80 |
| 0.7 | 42.10 | 1.7 | 104.00 |
| 0.8 | 50.30 | 1.8 | 110.20 |
| 0.9 | 53.80 | 1.9 | 117.10 |
| 1 | 61.50 | 2 | 121.10 |

Table 2: Calibration for coil 1

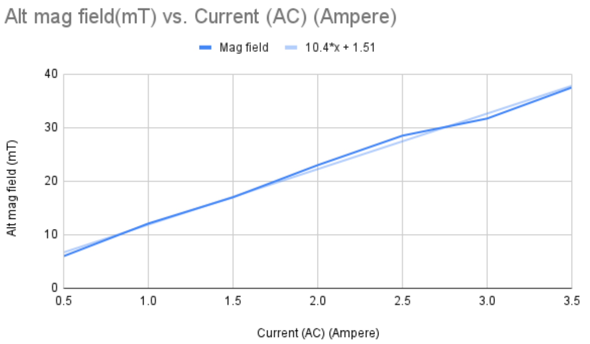


Figure 12: Calibration curve for DC field for coil 2

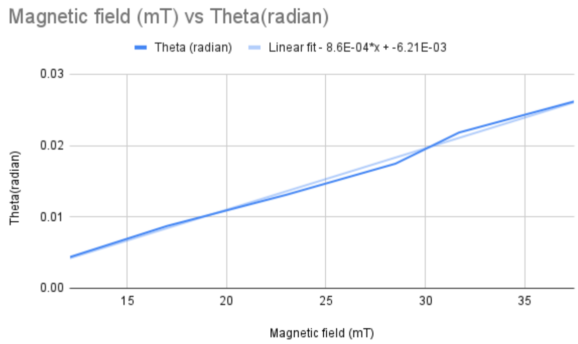


Figure 13: Variation of polarization angle rotation with Magnetic field.

From above tables, we can observe that for a particular value of DC current, the magnetic field in coil 2 is much lesser than coil 1. So, the Faraday rotation can be better observed in coil 1.

From the plot in Figure 14, it can be seen that the Faraday rotation is directly proportional to the Magnetic field. Also,the slope of this curve is used in calculating the Verdat constant of the sample.

#### 2.3.1 Calculations

The slope of the curve of Mag. field v/s Theta = 0.86 rad/Tesla
As, $\theta = VBl$,
$\implies V = (\theta/B) * 1/l$
Here, length of rod, l = 3 cm
so, Verdet constant,

$$V = 0.86/0.03 = 28.66\frac{radian}{Tesla.meter}$$

**Error in Verdat Constant:**
Error in the slope; $\Delta m = 0.031$

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}$$

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{0.031}{0.537}\right)^2 + \left(\frac{0.001}{1}\right)^2} = 0.048$$

$$\Delta v = 0.048 \times 28.67 = 1.38$$

Thus, Verdet Constant for the given SF glass is **28.67**$\pm$ 1.38 rad $\mathrm{T}^{-1}$ $\mathrm{m}^{-1}$.

### 2.4 Faraday rotation with AC magnetic field


Figure 14: Lock-in output with AC magnetic field

| AC current mA | Magnetic field mT | $V_{ac(RMS)}$ mV | $V_{dc}$ (mV) mV | $\theta$ (radian) | verdet constant (rad $T^{-1}$ $m^{-1}$) |
|---|---|---|---|---|---|
| 950 | 11.2 | 1900 | 289 | 0.00929900207 | 27.6756014 |
| 800 | 9.4 | 1790 | 289 | 0.008760638793 | 31.06609501 |
| 600 | 7.2 | 1280 | 289 | 0.006264590868 | 29.0027355 |
| 400 | 4.8 | 850 | 289 | 0.004160079874 | 28.88944357 |
| 200 | 2.4 | 420 | 289 | 0.002055568879 | 28.54956776 |
| 500 | 5.9 | 1100 | 289 | 0.005383632778 | 30.41600439 |
| 100 | 1.2 | 200 | 289 | 0.0009788423232 | 27.19006453 |

Table 3: Angle of Rotation for AC Faraday Effect

| Current (AC)(Ampere) Ampere | Alt mag field (mT) milli Tesla |
|---|---|
| 0.2 | 2.4 |
| 0.4 | 4.8 |
| 0.5 | 5.9 |
| 0.6 | 7.2 |
| 0.8 | 9.4 |
| 0.95 | 11.2 |

Table 4: Calibration for AC magnetic field

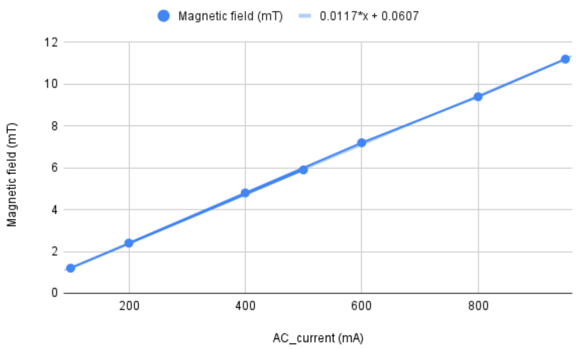


Figure 15: Calibration curve for AC magnetic field

Here, for the calculation of the angle of rotation, we use the relation,

$$\theta = \frac{1}{\sqrt{2}} \frac{\Delta V_{ac(RMS)}}{V_{dc}}$$

Here, $V_{ac(RMS)}$ is the output from the Lock-in Amplifier, and
$V_{dc}$ is DC output from the photodiode.

### 2.4.1 Error:

| Magnetic Field (T) | $\theta$ rad | Verdet Constant rad $T^{-1}m^{-1}$ | $\Delta\theta$ $\times10^{-5}$ | $\Delta$ v |
|---|---|---|---|---|
| 11.2 | 0.0093 | 27.676 | 3.255 | 1.005 |
| 9.4 | 0.0088 | 31.066 | 3.071 | 1.019 |
| 7.2 | 0.0063 | 29.003 | 2.222 | 1.051 |
| 5.9 | 0.0054 | 30.416 | 1.932 | 1.088 |
| 4.8 | 0.0042 | 28.889 | 1.520 | 1.144 |
| 2.4 | 0.0021 | 28.550 | 86.34 | 1.551 |
| 1.2 | 0.0010 | 27.190 | 59.52 | 2.606 |

Table 5:

Error in slope, $\delta m = 0.063$
Relative error in $\theta$ is given by,

$$\frac{\Delta\theta}{\theta} = \sqrt{\left(\frac{\Delta V_{ac(RMS)}}{V_{ac(RMS)}}\right)^2 + \left(\frac{\Delta V_{dc}}{V_{dc}}\right)^2}$$

Relative error in Verdet Constant would be given by,

$$\frac{\Delta v}{v} = \sqrt{\left(\frac{\Delta\theta}{\theta}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$$

Final error, $\Delta$v = 0.55
So, the **verdet constant is** 28.97 $\pm$ 0.55rad$T^{-1}$ $m^{-1}$

## 2.5 A general variation of $V_{ac(RMS)}$ and $V_{dc}$

An extension to the measurements of previous sections has been made to observe the general variation of $V_{ac)(rms)}$ with $V_{dc}$ at constant magnetic field. A constant magnetic field is attained by maintaining a fixed alternating current.

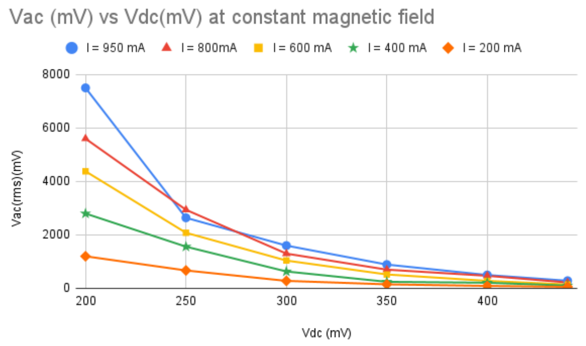


Figure 16: $V_{ac(RMS)}$ vs $V_{dc}$ at constant Magnetic Fields

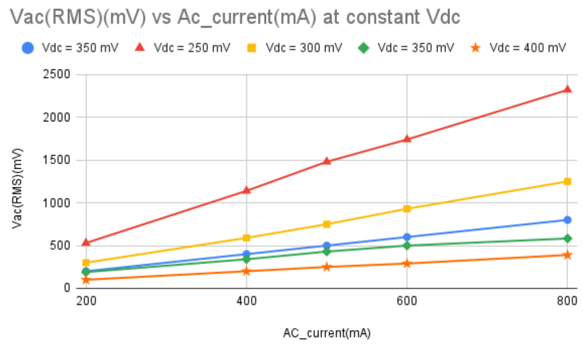


Figure 17: Vac(RMS) vs Magnetic FIeld at constant Vdc

# 3 Discussion

- We started with the **Demonstration of Faraday rotation using DC field.** We have performed this for 3 different coils (2 cylindrical coils, and the PhyWe coil). We have found that all the coils have different impedances and the 2nd cylindrical coil is best for performing the experiment as it produces a larger magnetic field ( 100mT).

- **Working of the Lock-in amplifer** has been understood fully and all the components have been tested under multiple experimental parameters. The phase shifter has been found to be not working and hence an external circuit has been used.

- Before the Faraday detection, the working of the lock-in has been verified by the LED experiment.

- The faraday rotation angle and Verdat constant has been found under **different AC magnetic fields**, and they lie in the range of 27.19 - 31.06$rad\ T^{-1}m^{-1}$. This proves that the Verdat constant is a property of the sample, and it is invariant of the experimental parameters.

- **The Use of a polarized laser** is suggested. We have performed the experiment using two different lasers. The first one was unpolarised and of 5mWatts power. The signal produced by the beam of this laser was too weak to be undetected. The other laser is p-polarized and has a maximum power of 10mW. The signal produced then, was detected successfully.

- The results observed in this experiment by above mentioned experimental setup is robust enough to be reproduced under various parameters, such as input currents, gain of the amplifier, etc.

# 4 Results and Conclusion

- The Verdet constant of the SF glass rod has been found to be 28.6$\pm$2.25 rad/T/m, from the analysis with magnetic due to the Direct Current.

- The Verdet constant from the AC analysis with different Alternating current varies between 27.019 to 31.006 rad/T/m.

- The external phase shifter has been found to be working correctly.

- The principle of Lock-in detection has also been verified from the experiment of detecting the LED signals via the photodiode.

- The principles of polarization, lock-in detection and DC faraday rotation has been observed and studied.


# 5 Acknowledgement

We acknowledge and are grateful for the immense guidance provided by our instructor **Dr G. Santosh Babu** throughout the semester. The support from **Dr Ritwick Das** is also appreciated. We also appreciate the constant support from **Mr. Prabhakar** and **Mr. Shakthivel**. Without their help and inputs for the instruments, it would not have been possible to reach this far.

# 6 References


1. https://www.physics.uci.edu/ advanlab/ faraday ·pdf
2. Malak et al. Faraday rotation measurement using a lock-in amplifier.Department of Physics, State University of New York at Binghamton. 2011
3. Faraday Rotation manual
4. Teachspin Lockin Amplifier Manual
5. Jain A et al. A simple experiment for determining Verdet constants using alternating current magnetic fields
6. Valev V K et al. Precise measurements of Faraday rotation using ac magnetic fields