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TCS-409
Tutorial 4

Q1 $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$T(n) = aT\left(\frac{n}{2}\right) + f(n)$

$a \geq 1, \quad b > 1$

On comparing,
$a = 3, \quad b = 2, \quad f(n) = n^2$

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n^2)$

Q2 $T(n) = 4T(n/2) + n^2$

$a \geq 1, \quad b > 1$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

Q3 $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \Theta(2^n)$

Q4 $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$, $b = 2$; $f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$\therefore f(n) = n^c$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

Q5 $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Q6 $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Here, $n \log n > n$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(n \log n)$

Q7 $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$.

Since, $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Q8 $T(n) = 2T(n/4) + n^{0.5}$

$a = 2,\ b = 4,\ f(n) = n^{0.5}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Omega(n^{0.51})$

Q9 $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2$

Since, acc. to Master Theorem $a \geq 1$

but here, $a = 0.5$,

$\therefore$ Master Theorem cannot be applied

Q10 $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

Now, $n^c = n^2$

$n! > n^2$

$\therefore T(n) = \Theta(n!)$

Q11 $4T(n/2) + \log n$

$a = 4, b = 2, f(n) = \log n$

$c = \log_a b = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

$\log n < n^2$

$\therefore f(n) < n^c$

$T(n) = \Theta(n^c)$

$= \Theta(n^2)$

Q12 $T(n) = \text{opt}(n) T(n/2) + \log n$

$a = \sqrt{n}, b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2} \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log(n)$

$\therefore f(n) > n^c$

$\therefore T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

Q13 $T(n) = 3T(n/2) + n$

$a = 3, b = 2, f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$\therefore n^c = n^{1.5849}$

$\therefore n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$\therefore T(n) = \Theta(n^{1.5849})$

Q14 $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, b = 3$

$c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^{1/2} \quad n$

$\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n)$

Q15 $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$

$c = \log_a b = \log_2 4 = 2$

$\therefore n^c = n^2$

$\therefore cn < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

Q16 $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4, f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.792.$

$n^c = n^{0.792}$

$\therefore n^{0.792} < n\log n$

$\therefore T(n) = \Theta(\log(n))$

Q17 $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3$

$C = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^C = n^1 = n$

$n/2 < n$

$\therefore f(n) < n^C$

$\therefore T(n) = \Theta(n)$

Q18 $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3$

$C = \log_b a = \log_3 6 = 1.6309$

$n^C = n^{1.6309}$

$n^{1.6309} < n^2 \log n$

$\therefore T(n) = \Theta(n^2 \log n)$

Q19 $T(n) = 4T(n/2) + n \log n$

$a = 4, b = 2, f(n) = n \log n$

$C = \log_b a = \log_2 4 = 2$

$n^C = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \Theta(n^2)$

Q20 $T(n) = 64T(n/8) + n^2 \log n$

$a = 64, \quad b = 8$

$c = \log_a b = \log_8 64 = \log_8 (8)^2$

$c = 2$

$\therefore n^c = n^2$

$\therefore n^2 \log n > n^2$

$\therefore T(n) = \Theta(n^2 \log n)$

Q21 $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$\therefore T(n) = \Theta(n^2)$

Q22 $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2$

$c = \log_b a = \log_2 1 = 0$

$\therefore n^c = n^0 = n^1$

$\therefore n(2 - \cos n) > n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$