

Pi function

$$A'(x, y, z) = A((x + 3y) \bmod 5, x, z)$$

Now to perform the pre-image on this equation,

$$x' = (x + 3y) \bmod 5 \quad - (1)$$

$$y' = x \quad - (2)$$

$$z' = z \quad - (3)$$

Now,

$$3y = (x' - x) \bmod 5$$

$$y = \frac{x' - x}{3} \bmod 5$$

$$y = \frac{x' - y'}{3} \bmod 5$$

$$y = 3^{-1}(x' - y') \bmod 5$$

$$y = 2 * (x' - y') \bmod 5$$

So the pre-image equation will be

$$A[x, y, z] = A'[y', 2(x' - y') \bmod 5, z]$$

While performing the pre-image, one thing to notice is that $(x' - y')$ will produce negative result sometimes. To make that positive we will add 5 to the result to counteract the negative result.

So the final equation will be

$$A[x, y, z] = A'[y', 2(x' - y') \bmod 5, z]$$

We have done the verification using the code.