DS203: Programming for Data Sciences

Assignment 1

Exercise 1 (8 marks)**.** *A particular webserver may be working or not working. If the webserver is not working, any attempt to access it fails. Even if the webserver is working, an attempt to access it can fail due to network congestion beyond the control of the webserver. Suppose that the a priori probability that the server is working is* 0.8*. Suppose that if the server is working, then each access attempt is successful with probability* 0.9*, independently of other access attempts. Find the following quantities.*

1. *P(first access attempt fails)*
2. *P(server is working | first access attempt fails)*
3. *P(second access attempt fails | first access attempt fails)*
4. *P(server is working | first and second access attempts fail).*

Exercise 2 (5 marks)**.** *Let X represent the lifetime, rounded up to an integer number of years, of a certain car battery. Suppose that the pmf of X is given by $p_X(k) = 0.2$ if $3 \le k \le 7$ and $p_X(k) = 0$ otherwise.*
(i) Find the probability, $P\{X > 3\}$, that a three year old battery is still working.
(ii) Given that the battery is still working after five years, what is the conditional probability that the battery will still be working three years later? (i.e. what is $P(X > 8|X > 5)$)?

Exercise 3 (5 marks)**.** *A certain Illini basketball player shoots the ball repeatedly from half court during practice. Each shot is a success with probability p and a miss with probability $1-p$, independently of the outcomes of previous shots. Let Y denote the number of shots required for the first success.*
(i) Express the probability that she needs more than three shots for a success, $P\{Y > 3\}$, in terms of p.
(ii) Given that she already missed the first five shots, what is the conditional probability that she will need more than three additional shots for a success? (i.e. what is $P\,(Y > 8|Y > 5)$)?
(iii) What type of probability distribution does Y have?

Exercise 4 (5 marks)**.** *(a) Suppose that an event E is independent of itself. Show that either $P(E) = 0$ or $P(E) = 1$.*
(b) Events A and B have probabilities $P(A) = 0.3$ and $P(B) = 0.4$. What is $P(A \cup B)$ if A and B are independent? What is $P(A \cup B)$ if A and B are mutually exclusive?
(c) Now suppose that $P(A) = 0.6$ and $P(B) = 0.8$. In this case, could the events A and B be independent? Could they be mutually exclusive?

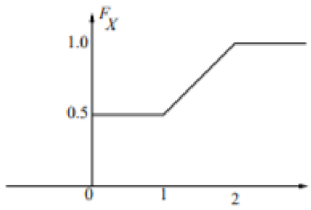
Exercise 5 (5 marks)**.** *Which of the following are valid CDF's? For each that is not valid, state at least one reason why. For each that is valid, find $P(X^2 > 5)$.*

1.
$$F(x) = \begin{cases} e^{-x^2}/4 & \text{if} \quad x < 0 \\ 1 - e^{-x^2}/4 & \text{if} \quad x \ge 0 \end{cases}$$

2.
$$F(x) = \begin{cases} 0 \quad \textit{if} \quad x < 0 \\ 0.5 + e^{-x} \quad \textit{if} \quad 0 \leq x < 3 \\ 1 \quad \textit{if} \quad x \geq 3 \end{cases}$$

3.
$$F(x) = \begin{cases} 0 \quad \textit{if} \quad x < 0 \\ 0.5 + x/20 \quad \textit{if} \quad 0 \leq x \leq 10 \\ 1 \quad \textit{if} \quad x \geq 10 \end{cases}$$

Exercise 6 (6 marks)**.** *Let X have the CDF shown.*



1. *Find $P(X \leq 0.8)$.*
2. *Find $E(X)$.*
3. *Find $Var(x)$.*

Exercise 7 (5 marks)**.** *If the density function of X equals*

$$f(x) = \begin{cases} ce^{-2x} & \textit{if}\, 0 \leq x < \infty \\ 0 & \textit{if}\, x < 0 \end{cases}$$

find c. What is the value of $P\{X > 2\}$?

Exercise 8 (5 marks)**.** *Suppose a coin having probability 0.7 of coming up heads is tossed three times. Let X denote the number of heads that appear in the three tosses. Determine the probability mass function of X.*

Exercise 9 (5 marks)**.** *Let X is a random variable with probability density function*

$$f_X(x) = \begin{cases} 2x & \textit{if}\, 0 \leq x \leq 1, \\ 0 & \textit{otherwise}. \end{cases}$$

Find $P(X \geq 0.4 | X \leq 0.8)$.

Exercise 10 (3 marks)**.** *Let X is an exponentially distributed random variable with parameter λ. For any $a, b > 0$, find $P(X > a + b | X > a)$.*

Exercise 11 (2 marks)**.** *Suppose five fair coins are tossed. Let E be the event that all coins land heads. Define a random variable I_E*

$$I_E = \begin{cases} 1 & \textit{if } E \textit{ occurs} \\ 0 & \textit{if } E^c \textit{ occurs} \end{cases}$$

For what outcomes in the original sample space does I_E equals 1 ? what is $P\{I_E = 1\}$

Exercise 12 (2 marks)**.** *Suppose the distribution function of X is given by*

$$F(b) = \begin{cases} 0, & b < 0 \\ \frac{1}{2}, & 0 \leq b < 1 \\ 1, & 1 \leq b < \infty \end{cases}$$

What is the probability mass function of X?

Exercise 13 (5 marks)**.** *A ball is drawn from an urn containing three white and three black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes on indefinitely. What is the probability that of the first four balls drawn, exactly two are white?*

Exercise 14 (5 marks)**.** *A coin having probability p of coming up heads is successively flipped until the rth head appears. Argue that X, the number of flips required, will be n, $n \geq r$, with probability*

$$P(X = n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}, n \geq r$$