CSE 2002

Theory of Computation and Compiler Design

Digital Assignment - 2

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Q1 Construct PDA accepting strings by final state for the language $L = \{ a^n b^{n+m} c^m ; n \geq 0, m \geq 1 \}$

Ans. Let PDA be formally defined by $M = (S, \Sigma, \Gamma, \delta, S_0, z_0, A)$ where $S = \{ s_0, s_1, s_2 \}$ $\Sigma = \{a, b, c\}$ $A = \{s_2\}$ and $\Gamma = \{1, z_0\}$

The transition function δ:

1. Transition to accept null string

$$\delta(s_0, \Lambda, z_0) = \{(s_2, z_0)\}$$

2. Set of transitions to push '1' into stack, when reading 'a' ~~…~~ :

$$\delta(s_0, a, z_0) = \{(s_0, 1z_0)\}$$
$$\delta(s_0, a, 1) = \{(s_0, 11)\}$$

3. Set of transitions to pop '1' from stack when reading 'b' until top of the stack is 'z_0'.

$$\delta(s_0, b, 1) = \{(s_1, \Lambda)\}$$
$$\delta(s_1, b, 1) = \{(s_1, \Lambda)\}$$

4. Set of transitions to push '1' into stack when reading 'b'.

$$\delta(s_1, b, z_0) = \{(s_1, 1z_0)\}$$
$$\delta(s_1, b, 1) = \{(s_1, 11)\}$$

5. Set of transition to pop '1' when reading 'c'

$$\delta(s_1, c, 1) = \{(s_1, \Lambda)\}$$

~~δ~~

6. Finally, transition to make a recognize match:

$$\delta(s_1, \Lambda, z_0) = \{(s_2, z_0)\}$$

Q2 Construct a PDA accepting strings by final state for L; $n_a(w)$ represents number of a in w.

$$L = \{w \in \{a, b, c\}^+ : n_a(w) + n_b(w) = n_c(w)\}$$

Ans. Let PDA be formally defined by $M = (S, \Sigma, \Gamma, \delta, s_0, z_0, A)$ given by $S = \{s_0, s_1\}$, $\Sigma = \{a, b\}$, $A = \{s_1\}$ and $\Gamma = \{0, 1, z_0\}$

Transition function δ:

1. Transition to accept null string:

$$\delta(s_0, \Lambda, z_0) = \{(s_1, z_0)\}$$

2. Transitions to push '0' onto stack when reading symbol 'a' and 'b':

$$\delta(s_0, a, z_0) = \{(s_0, 0z_0)\}$$

$$\delta(s_0, a, 0) = \{(s_0, 00)\}$$

$$\delta(s_0, b, z_0) = \{(s_0, 0z_0)\}$$

$$\delta(s_0, b, 0) = \{(s_0, 00)\}$$

3. Transition to push symbol '1' onto stack when reading 'c'

$$\delta(s_0, c, z_0) = \{(s_0, 1z_0)\}$$

$$\delta(s_0, c, 1) = \{(s_0, 11)\}$$

4. Transition to pop a symbol when reading symbol 'c' with top of the stack 0.

$$\delta(s_0, c, 0) = \{(s_0, \Lambda)\}$$

5. Transition to pop a symbol, when reading 'a' or 'b' with

top of the stack 1

$$\delta(s_0, a, 1) = \{(s_0, \Lambda)\}$$
$$\delta(s_0, b, 1) = \{(s_0, \Lambda)\}$$

6. Finally, transition to make a recognize match:

$$\delta(s_0, \Lambda, z_0) = \{(s_1, z_0)\}$$

Q3 Construct PDA accepting strings by empty stack for language $L = \{0^n 1^m ;\ n \leq m < 3n\}$

Ans. Case 1: When $n = m$, $L = \{0^n 1^n ;\ n \geq 1\}$

Let PDA be formally defined by $M = (S, \Sigma, \Gamma, \delta, s_0, z_0, A)$ given by $S = \{s_0, s_1\}$, $\Sigma = \{0, 1\}$, $A = \{s_1\}$ and $\Gamma = \{0, 1, z_0\}$

Transition function δ:

1. Transition to push '0' when '0' is read

$$\delta(s_0, 0, z_0) = \{(s_0, 0z_0)\}$$
$$\delta(s_0, 0, 0) = \{(s_0, 00)\}$$

2. Transition to pop '0' when '1' is read

$$\delta(s_0, 1, 0) = \{(s_1, \Lambda)\}$$
$$\delta(s_1, 1, 0) = \{(s_1, \Lambda)\}$$

3. Transition to make recognize match

$$\delta(s_1, \Lambda, z_0) = \{(s_1, \Lambda)\}$$

Case 2: When $m = 2n$, $L = \{0^n 1^{2n}, n \geq 0\}$

Let PDA be formally defined by $M = (S, \Sigma, \Gamma, \delta, s_0, z_0, A)$ given by $S = \{s_0, s_1\}$, $\Sigma = \{0, 1\}$, $A = \{s_1\}$ and $\Gamma = \{a, z_0\}$

Transition function δ:

1. Transition to push symbol '0' when 0 is read:

$$\delta(s_0, 0, z_0) = \{(s_1, aaz_0)\}$$
$$\delta(s_1, 0, a) = \{(s_1, aaa)\}$$

2. Transition when reading 1s to pop 'a'

$$\cancel{\delta(s_0, 1, 0) = \{(s_1, 0)\}} \qquad \delta(s_1, 1, a) = \{(s_1, \Lambda)\}$$
$$\cancel{\delta(s_1, 1, 0) = \{(s_1, \Lambda)\}}$$
$$\cancel{\delta(s_1, 1, 0}$$

3. Transition to make recognize match

$$\delta(s_1, \Lambda, z_0) = \{(s_1, \Lambda)\}$$

<u>Case 3:</u> When $m = 3n$ $\quad L = \{0^n 1^{3n}, n \geq 0\}$

Let PDA be formally defined by $M = (S, \Sigma, \Gamma, \delta, s_0, z_0, A)$

where $S = \{s_0, s_1\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, z_0\}$, $A = \{s_1\}$

Transition function δ:

1. Transition to push 'a' when reading 0s

$$\delta(s_0, 0, z_0) = \{(s_1, aaaz_0)\}$$
$$\delta(s_1, 0, a) = \{(s_1, aaaa)\}$$

2. Transition to pop 'a' when reading 1s

$$\delta(s_1, 1, a) = \{(s_1, \Lambda)\}$$

3 Transition to make recognize match

$$\delta(s_1, \Lambda, z_0) = \{(s_1, \Lambda)\}$$

Q4: Determine whether the string $z = 1101001$ is in the language having productions $P = \{ q_0 \to 1q_0 \mid ABC \mid 0,\ A \to 10 \mid BB,\ B \to 0 \mid 1,\ C \to AB \mid 1 \}$. Use CYK Algorithm

Ans. $z = 1101001 = a_1 a_2 a_3 a_4 a_5 a_6 a_7$.

$n = 7$

$V_{11} = \{B, C\}$
$V_{22} = \{B, C\}$
$V_{33} = \{q_0, B\}$
$V_{44} = \{B, C\}$
$V_{55} = \{q_0, B\}$
$V_{66} = \{q_0, B\}$
$V_{77} = \{B, C\}$

$V_{12} = \{A \mid A \to BC,\ B \in V_{11} = \{B,C\},\ C \in V_{22} = \{B,C\}\} = \{A\}$
$V_{23} = \{A \mid A \to BC,\ B \in V_{22} = \{B,C\},\ C \in V_{33} = \{q_0, B\}\} = \{A\}$
$V_{34} = \{A \mid A \to BC,\ B \in V_{33} = \{q_0, B\},\ C \in V_{44} = \{B, C\}\} = \{A\}$
$V_{45} = \{A \mid A \to BC,\ B \in V_{44} = \{B, C\},\ C \in V_{55} = \{q_0, B\}\} = \{A\}$
$V_{56} = \{A \mid A \to BC,\ B \in V_{55} = \{q_0, B\},\ C \in V_{66} = \{q_0, B\}\} = \{A\}$
$V_{67} = \{A \mid A \to BC,\ B \in V_{66} = \{q_0, B\},\ C \in V_{77} = \{B, C\}\} = \{A\}$

$V_{13} = \{A \mid A \to BC,\ B \in V_{11} = \{B, C\},\ C \in V_{23} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{12} = \{A\},\ C \in V_{33} = \{q_0, B\}\} = \phi \cup \{C\} = \{C\}$

$V_{24} = \{A \mid A \to BC,\ B \in V_{22} = \{B, C\},\ C \in V_{34} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{23} = \{A\},\ C \in V_{44} = \{B, C\}\} = \phi \cup \{C\} = \{C\}$

$V_{35} = \{A \mid A \to BC,\ B \in V_{33} = \{q_0, B\},\ C \in V_{45} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{34} = \{A\},\ C \in V_{55} = \{q_0, B\}\} = \phi \cup \{C\} = \{C\}$

Scanned by CamScanner

$$V_{46} = \{A \mid A \to BC,\ B \in V_{44} = \{B,C\},\ C \in V_{56} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{45} = \{A\},\ C \in V_{66} = \{Q_0, B\}\} = \phi \cup \{C\} = \{C\}$$

$$V_{57} = \{A \mid A \to BC,\ B \in V_{55} = \{Q_0, B\},\ C \in V_{67} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{56} = \{A\},\ C \in V_{77} = \{B,C\}\} = \phi \cup \{C\} = \{C\}$$

$$V_{14} = \{A \mid A \to BC,\ B \in V_{11} = \{B,C\},\ C \in V_{24} = \{C\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{12} = \{A\},\ C \in V_{34} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{13} = \{C\},\ C \in V_{44} = \{B,C\}\} = \phi \cup \phi \cup \phi = \phi$$

$$V_{25} = \{A \mid A \to BC,\ B \in V_{22} = \{B,C\},\ C \in V_{35} = \{C\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{23} = \{A\},\ C \in V_{45} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{24} = \{C\},\ C \in V_{55} = \{Q_0, B\}\} = \phi \cup \phi \cup \phi = \phi$$

$$V_{36} = \{A \mid A \to BC,\ B \in V_{33} = \{Q_0, B\},\ C \in V_{46} = \{C\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{34} = \{A\},\ C \in V_{56} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{35} = \{C\},\ C \in V_{66} = \{Q_0, B\}\} = \phi \cup \phi \cup \phi = \phi$$

$$V_{47} = \{A \mid A \to BC,\ B \in V_{44} = \{B,C\},\ C \in V_{57} = \{C\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{45} = \{A\},\ C \in V_{57} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{46} = \{C\},\ C \in V_{77} = \{B,C\}\} = \phi \cup \phi \cup \phi = \phi$$

$$V_{15} = \{A \mid A \to BC,\ B \in V_{11} = \{B,C\},\ C \in V_{25} = \phi\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{12} = \{A\},\ C \in V_{35} = \{C\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{13} = \{C\},\ C \in V_{45} = \{A\}\} \cup$$
$$\{A \mid A \to BC,\ B \in V_{14} = \phi,\ C \in V_{55} = \{Q_0, B\}\} = \phi \cup \phi \cup \phi \cup \phi = \phi$$

$V_{26} = \{A \mid A \to BC,\ B \in V_{22} = \{B,C\},\ C \in V_{36} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{23} = \{A\},\ C \in V_{46} = \{C\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{24} = \{C\},\ C \in V_{56} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{25} = \phi,\ C \in V_{66} = \{S_0, B\}\} = \phi \cup \phi \cup \phi \cup \phi = \phi$

$V_{37} = \{A \mid A \to BC,\ B \in V_{33} = \{S_0, B\},\ C \in V_{47} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{34} = \{A\},\ C \in V_{57} = \{C\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{35} = \{C\},\ C \in V_{67} = \{B\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{36} = \phi,\ C \in V_{77} = \{B, C\} = \phi \cup \phi \cup \phi \cup \phi = \phi$

$V_{16} = \{A \mid A \to BC,\ B \in V_{11} = \{B, C\},\ C \in V_{26} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{12} = \{A\},\ C \in V_{36} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{13} = \{C\},\ C \in V_{46} = \{C\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{14} = \phi,\ C \in V_{56} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{15} = \phi,\ C \in V_{66} = \{S_0, B\}\} = \phi \cup \phi \cup \phi \cup \phi \cup \phi = \phi$

$V_{27} = \{A \mid A \to BC,\ B \in V_{22} = \{B, C\},\ C \in V_{37} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{23} = \{A\},\ C \in V_{47} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{24} = \{C\},\ C \in V_{57} = \{C\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{25} = \phi,\ C \in V_{67} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{26} = \phi,\ C \in V_{77} = \{B, C\}\} = \phi \cup \phi \cup \phi \cup \phi \cup \phi = \phi$

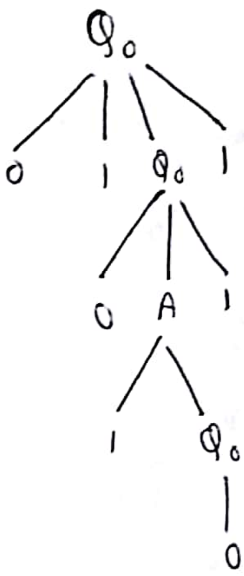
$V_{17} = \{A \mid A \to BC,\ B \in V_{11} = \{B, C\},\ C \in V_{27} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{12} = \{A\},\ C \in V_{37} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{13} = \{C\},\ C \in V_{47} = \phi\} \cup$
$\{A \mid A \to BC,\ B \in V_{14} = \phi,\ C \in V_{57} = \{C\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{15} = \phi,\ C \in V_{67} = \{A\}\} \cup$
$\{A \mid A \to BC,\ B \in V_{16} = \phi,\ C \in V_{77} = \{B, C\}\} = \phi \cup \phi \cup \phi \cup \phi \cup \phi \cup \phi = \phi$

So we see that $q_0 \notin V_{17}$ and hence $z = 1101001$ is not in the given language.

Q5 $q_0 \to 0 \mid 01q_01 \mid 0A1 \qquad A \to 1q_0 \mid 0AA1$

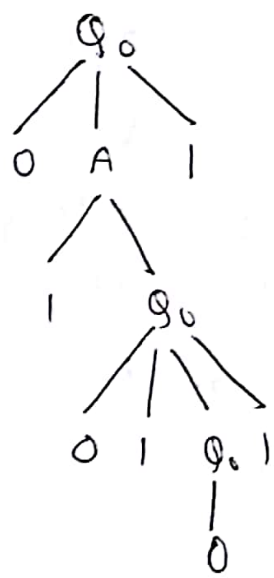
Find whether the grammar is ambiguous or not.

Ans. <u>Leftmost derivation 1</u>



$q_0 \to 01q_01$
$q_0 \to 010A11 \quad (q_0 \to 0A1)$
$q_0 \to 0101q_011 \quad (A \to 1q_0)$
$q_0 \to 0101011 \quad (q_0 \to 0)$

<u>Leftmost derivation 2</u>



$q_0 \to 0A1$
$q_0 \to 01q_01 \quad (A \to 1q_0)$
$q_0 \to 0101q_011 \quad (q_0 \to 01q_01)$
$q_0 \to 0101011 \quad (q_0 \to 0)$

$\because$ strings generated by 2 leftmost derivations are the same
$\therefore$ The grammar is ambiguous.

Q6 Construct a grammar equivalent to the grammar having no unit production where $P = \{ Q_0 \to aQ_0 \mid AB \mid aB \mid C,$
$A \to b \mid \Lambda$, $B \to a$, $C \to \Lambda \}$

Ans Eliminating null production:

Let grammar be $G = \{ V_N, T, P, Q_0 \}$

where $V_N = \{ Q_0, A, B, C \}$ $T = \{ a, b \}$

$P = \{ Q_0 \to aQ_0 \mid AB \mid aB \mid C,$
$A \to b \mid \Lambda,$
$B \to a$
$C \to \Lambda \}$

We must construct $G_1 = (V_N', T, P', Q_0)$ that has no null production:

1. Construction of the set of nullable variables W

$W_1 = \{ Q \in V_N \mid Q \to \Lambda$ is a production in $P \}$
$= \{ A, C \}$

$W_2 = W_1 \cup \{ Q \in V_N \mid Q \to \alpha$ with $\alpha \in W_1^* \}$
$= \{ A, \Lambda \} \cup \{ Q_0 \}$
$= \{ A, C, Q_0 \}$

$W_3 = W_2 \cup \{ Q \in V_N \mid Q \to \alpha$ with $\alpha \in W_2^* \}$
$= W_2 \cup \phi = W_2$

$\therefore W = W_2 = \{ Q_0, A, C \}$

2. Construction of P'

(a) The productions $A \to b$, $B \to a$ are included in P'.

(b) The production $Q_0 \to aB$ is included in P'

(c) (i) The production $Q_0 \to aQ_0$ leads to $Q_0 \to aQ_0$ and $Q_0 \to a$

(ii) The production $Q_0 \to AB$ leads to $Q_0 \to AB$ and $Q_0 \to B$

(iii) The production $Q_0 \to C$ does not give $Q_0 \to C$ as it leads no where

$\therefore G_1 = (V_N', T, P', S_0)$

$V_N' = \{S_0, A, B\}$

$T = \{a, b\}$

$P' = \{S_0 \to aB \mid aS_0 \mid a \mid AB \mid B \; , A \to b \; , B \to a\}$

Eliminating unit production :

(a) construction of $W(S)$ for all $S \in V_N$

$W_0(S_0) = \{S_0\}$

$W_1(S_0) = W_0(S_0) \cup \{A \in V_N \mid B \to A \text{ is in } P \text{ with } B \in W_0(S)\}$

$= W_0(S_0) \cup \{B\}$

$= \{S_0, B\}$

$W_2(S_0) = W_1(S_0) \cup \phi$

$= W_1(S_0)$

$\therefore W(S_0) = W_1(S_0) = \{S_0, B\}$

$W_0(A) = \{A\}$

$W_1(A) = W_0(A) \cup \phi = W_0(A)$

$W(A) = W_0(A) = \{A\}$

$W_0(B) = \{B\}$

$W_1(B) = W_0(B) \cup \phi = W_0(B)$

$W(B) = W_0(B) = \{B\}$

(b) Construction of productions in G_2

$S_0 \to aS_0$, $S_0 \to aB$, $S_0 \to a$, $S_0 \to AB$, $A \to b$, $B \to a$ are included

We finally get $G_2 = (V_N', T, P', S_0)$

$V_N' = \{S_0, A, B\}$

$T = \{a, b\}$

$P' = \{S_0 \to aS_0 \mid a \mid aB \mid AB, \; A \to B, \; B \to a\}$

Q7 Reduce the grammar having productions $P = \{ Q_0 \rightarrow 1ABQ_0 | A0B1C$
$| 0, \; A \rightarrow 10 | 11BB, \; B \rightarrow 0 | 1A, \; C \rightarrow 0 | 1 \}$ into CNF.

Ans. The given grammar is $G = (V_N, T, P, Q_0)$ where

$V_N = \{Q_0, A, B, C\}$

$T = \{0, 1\}$

$P = \{ Q_0 \rightarrow 1ABQ_0 | A0B1C | 0,$
$A \rightarrow 10 | 11BB,$
$B \rightarrow 0 | 1A,$
$C \rightarrow 0 | 1 \}$

Step 1: Eliminate null and unit productions. There are none, so we continue

Step 2: Elimination of terminals from RHS

We aim to get $G_1 = (V_N', T', P', Q_0)$

$Q_0 \rightarrow 0, \; B \rightarrow 0, \; C \rightarrow 0, \; C \rightarrow 1$ are included in P'.

Introduce $D \rightarrow 1$ and $E \rightarrow 0$ so that

$Q_0 \rightarrow 1ABQ_0$ becomes $Q_0 \rightarrow DABQ_0$

$Q_0 \rightarrow A0B1C$ becomes $Q_0 \rightarrow AEBDC$

$A \rightarrow 10$ becomes $A \rightarrow DE$

$A \rightarrow 11BB$ becomes $A \rightarrow DDBB$

$B \rightarrow 1A$ becomes $B \rightarrow DA$

The new productions are added to P'

$P' = \{ Q_0 \rightarrow DABQ_0 | AEBDC | 0, \; A \rightarrow DE | DDBB$
$B \rightarrow 0 | DA, \; C \rightarrow 0 | 1 \}$

$V_N' = \{Q_0, A, B, C, D, E\} \quad T = \{0, 1\}$

Step 3: Restricting the number of variables on the RHS

We aim to get $G' = (V_N'', T', P'', Q_0)$

(a) $Q_0 \to 0$, $A \to DE$, $B \to 0$, $B \to DA$, $C \to 0$, $C \to 1$ are included in P''. V_N' variables are included in V_N''.

(b) $Q_0 \to DABQ_0$ becomes $Q_0 \to DD_1$, $D_1 \to AD_2$,

$D_2 \to BQ_0$.

$Q_0 \to AEBDC$ becomes $Q_0 \to AD_3$; $D_3 \to ED_4$,

$D_4 \to BD_5$, $D_5 \to DC$

$\therefore P'' = \{ Q_0 \to DD_1 \mid AD_3 \mid 0,$

$A \to DE \mid DD_6,$

$B \to 0 \mid DA,$

$C \to 0 \mid 1,$

$D_1 \to AD_2,\ D_2 \to BQ_0,$

$D_3 \to ED_4,\ D_4 \to BD_5,\ D_5 \to DC,$

$D_6 \to DD_7,\ D_7 \to BB \}$

$V_N'' = \{Q_0, A, B, C, D, E, D_1, D_2, D_3, D_4, D_5, D_6, D_7\}$

$T' = \{0, 1\}$

and $G_2 = (V_N'', T', P'', Q_0)$ is the grammar G in CNF.