Taxonomy of multiple target tracking methods

G.W. Pulford

Abstract: A concise summary of techniques for multiple target tracking is provided and their main characterics assessed qualitatively. The techniques have been catergorised into more than 35 different algorithmic types. A comparison chart is provided that lists each algorithm and categorises the processing scheme, data association mechanism, complexity scaling (with number of targets and with state dimension), overall complexity and a subjective performance figure. Although some recent filtering theory developments have been omitted, the survey should serve to demonstrate the large variety of 'classical' estimation theoretic algorithms already available for the design of multiple target tracking systems. A number of areas deserving of further study are identified in the concluding remarks.

1 Introduction

It has been over a quarter of a century since Bar-Shalom's survey of multiple target tracking (MTT) methods was published in *IEEE Transactions on Automatic Control* [1]. At that time there were about half a dozen MTT algorithms in evidence in the public domain. The lower visibility but more recent survey report by Dézert [2] also lists fewer than 10 algorithms. Since that time many new approaches to MTT and the inseparable problem of data association have been developed and it is expedient to take stock of what has transpired in the interim. This is especially true when we consider that quite recent developments such as finite set statistics and particle filtering, which, although not explicitly covered in this taxonomy, are poised to offer the next generation of advanced tracking methods. In view of this potential 'generational' change in MTT algorithms, we provide in this article a fairly complete survey of what we call 'classical' methods for the MTT problem. These methods are based largely on probability, stochastic processes and estimation theory, which, when combined with systems theory and combinatorial optimisation, lead to a plethora of approaches that can seem somewhat daunting to the uninitiated. Although the recent book by Blackman and Popoli [3] contains a detailed coverage of approaches for the MTT data association problem, the presentation is quite specialised and connections between the approaches may not always be apparent. The approach taken in this article should be easier to digest if only because of its brevity.

Automatic target tracking, which is founded on Kalman filtering [4], had its genesis in the early 1970s and was driven primarily by aerospace applications such as radar, sonar, guidance, navigation, and air traffic control (see chapter 8 of [5] and associated references). Most early systems used approximations to the Kalman filter called Alpha-Beta filters.

Today it is increasingly the case that MTT approaches are being applied in non-aerospace arenas. Examples of this trend include image processing [6–10], oceanography [11–13], autonomous vehicles and robotics [14–17], remote sensing [18, 19] and biomedical research [20–22]. Whereas in some of these areas the application of MTT techniques is already advanced, in others the full power of MTT methods is yet to be realised. In image processing and computer vision, the MTT problem goes by the name of motion correspondence. The 1993 article by Cox [23] reviews a number of MTT methods such as multi-hypothesis tracking (MHT) and joint probabilistic data association (JPDA) in this context. Cox also adds to the list of areas in which data association arises, viz. psychology, biological vision, molecular dynamics and particle physics. A popular account of MTT due to Uhlmann [24], including an efficient gating strategy, appeared in 1992 in connection with the Strategic Defense Initiative in the United States.

In order to maximise accessibility of the material we have adopted a somewhat informal, tutorial style of presentation in this article. In Section 2 we discuss the major aspects of the multiple target tracking problem. We then provide in Section 3 a brief overview of some of classical algorithms put forward as solutions to the MTT data association problem. We mention some more recent approaches (particle filering, finite set statistics) in Section 4 and clarify the scope of the article. In Section 5 we discuss the area of performance assessment for MTT algorithms. This is followed in Section 6 by a taxonomy of the more than 35 different MTT approaches that are now in existence. By way of justification for the lack of mathematical detail, we point out that an exposition of all these techniques would be ambitious in the confines of a journal article.

2 The multiple target tracking problem

The MTT problem as understood in this paper concerns the single-sensor, automatic tracking of multiple, independent point targets. This is distinct from the area of extended object tracking that arises, for instance, in image processing. A point target is modelled as having neither physical length nor resolvable features. It exists purely in a dynamical state-space usually consisting of position, velocity

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The author is with QinetiQ, Bldg A9, Cody Technology Park, Farnborough, Hampshire, GU14 0LX, UK

E-mail: gwpulford@qinetiq.com

and acceleration. The basic tracking problem concerns the estimation of the target state given noisy measurements from a sensor whose field of view includes the target of interest.

Multiple target tracking techniques are fundamentally different from single target tracking techniques. The difference lies in the state-space model used. In a single target tracking algorithm, the state of only one target is modelled; detections from other targets are assumed to be false alarms and problems result when tracking closely-spaced or crossing targets. Multiple target tracking algorithms, however, take the existence of more than one target into account simultaneously in their measurement association processes. In theory, MTT algorithms are capable of tracking closely-spaced and crossing targets. Usually, only one measurement is assumed to be produced by each target at a given time and the targets are assumed to have independent dynamics. While these assumptions can be relaxed, it comes at a high price in terms of computational requirements.

The target motion is modelled as a stochastic, or in some cases deterministic, process involving a vector of state variables $x(k)$ where k denotes the discrete time. For example, in a passive sonar system the state might consist of frequency, frequency-rate, bearing and bearing-rate for each target, whereas in a 3-D active radar system the state variables could include the Cartesian (x,y,z) position and velocity for each target. For a deterministic model, the motion is determined completely once the initial state is specified, whereas for a stochastic model one seeks to characterise the probability density function (PDF) of the target state. The sensor measurement model describes how the measurements $y(k)$ are coupled to the target state $x(k)$ and also models errors due to measurement noise. In many applications it is possible to write the system and measurement equations as a linear, discrete-time Gauss-Markov system:

$$\begin{aligned} x(k+1) &= F(k)\,x(k) + v(k) \\ y(k) &= H(k)\,x(k) + w(k) \end{aligned} \tag{1}$$

where $v(k)$ and $w(k)$ are *iid* zero-mean Gaussian random processes with known covariance matrices $Q(k)$ and $R(k)$ respectively. Ignoring data association for the time being, the estimation theoretic (filtering) problem is then to compute the PDF of the state given a sequence of noisy measurements $Y^k = \{y(1), \ldots, y(k)\}$. When the initial mean and covariance of the state vector are specified, the solution is given by the recursive discrete-time Kalman filter (KF) [25]. In practice it is necessary to use a combination of raw data and prior knowledge to establish the initial state and covariance – a process called *track* initiation [26]. If system nonlinearities are present, then extended Kalman filtering or one of its variants may be applied. Finite-dimensional nonlinear filters also exist for certain specific types of nonlinear systems [27].

In real applications spurious measurements called false alarms or clutter are also generally present so that there is ambiguity in the origin of the measurements; in other words, it is not clear *a priori* which measurement corresponds to the target of interest. This problem is aggravated by the presence of multiple targets and the non-unity detection probability of real sensors. All of these factors conspire to produce data association ambiguity, which is largely to blame for the complexity of the MTT problem (see [26, 28]).

When there is data association ambiguity, the model in (1) must be modified. Instead of a single measurement vector, a scan consisting of a variable number M_k of measurements $Y(k) = \{y_1(k), \ldots, y_{M_k}(k)\}$ is received at each time. The cumulative set of measurement data is denoted $Y^k = \{Y(1), \ldots, Y(k)\}$. It is now not possible to write down the posterior PDF of the state $\mathrm{p}(x(k)|Y^k)$ without defining a set of measurement-to-target data association hypotheses $\{\lambda_i(k)\}$ under each of which there applies a system of the form:

$$x(k+1) = F(k)\,x(k) + v(k) \tag{2}$$

$$y(k) = \begin{cases} H(k)\,x(k) + w(k) & \text{for target measurements} \\ \text{clutter} & \text{otherwise,} \end{cases}$$

where it should be understood that the measurement matrix $H(k)$ is modified to account for those targets that are actually detected under the association hypothesis. Note that the association hypotheses effectively partition the data set by defining disjoint events in which there is no uncertainty as to the origin of the measurements. Assuming prior Gaussianity, under each possible $\lambda_i(k)$ the posterior PDF of the joint target state is Gaussian. However the correct association hypothesis is unknown and must be accounted for in a probabilistic manner, resulting in a (Gaussian) mixture PDF of the form

$$\mathrm{p}(x(k)|Y^k) = \sum_{i=1}^{N_k} \beta_i(k)\,\mathrm{N}\{x(k); \hat{x}_i(k|k), P_i(k|k)\} \tag{3}$$

where N_k is the number of terms in the mixture, $\beta_i(k) = \Pr(\lambda_i(k)|Y^k)$ is the association probability under hypothesis i, and $\mathrm{N}\{x; m, P\}$ is a multivariate Gaussian PDF in the variable x with mean vector m and covariance matrix P. The terms $\hat{x}_i(k|k)$ and $P_i(k|k)$ are none other than the state estimate and error covariance from a Kalman filter for the system in (2). Unfortunately the hypothesis tree, representing all possible measurement-to-target data associations, undergoes continual branching, resulting in a number of association hypotheses N_k that increases exponentially with time. The optimal estimators for the MTT problem are therefore not implementable and many suboptimal approaches have been developed. We briefly discuss some of these in the following Section.

3 Classical solution approaches

Different tracking methods can be distinguished on the basis of the estimation criterion used. For instance, in a Bayesian approach, the posterior PDF of the state $\mathrm{p}(x(k)|Y^k)$ is sought, from which the conditional mean (CM) or minimum mean-square-error (MMSE) estimate of the state may be obtained by taking the expectation $\mathrm{E}(x(k)|Y^k)$. In a maximum *a posteriori* probability (MAP) approach, one wishes to find the state that maximises the posterior probability: argmax $\mathrm{p}(x(k)|Y^k)$. In problems where a deterministic target motion model has been used, it is often required to find the maximum likelihood (ML) estimate of the initial target state by maximising $\mathrm{p}(Y^k|x(0))$ with respect to $x(0)$.

MTT methods also differ in their processing scheme. Two major variants are recursive and batch methods. In a recursive estimator, the state at time k is estimated based on data at time k and the previous state estimate from time $k-1$ (and usually its error covariance). A batch method is one that uses a given sequence of data $Y^K = \{y(1), \ldots, y(K)\}$ from which a sequence of state estimates $\{\hat{x}(1|K), \hat{x}(2|K), \ldots, \hat{x}(k|K), \ldots, \hat{x}(K|K)\}$ is produced. Another variant is referred to as fixed-lag smoothing in which a look-ahead is set of, say, $L-1$ samples and the current state estimate $\hat{x}(k|k+L-1)$ is generated using the measurement set $\{y(1), \ldots, y(k+L-1)\}$. Broadly speaking, there is an inverse relationship between decision delay ($L-1$ in this case) and the estimation error variance. In commonsense terms this says that the longer we are prepared to wait, the better the estimates will be (on average). A further scheme is multi-pass batch in which the entire data set is reprocessed several times to refine the state estimates. Typically these approaches are based on the expectation-maximisation (EM) algorithm [29, 30] and a sequence of initial state estimates is required for initialisation.

The basic solution approaches for the MTT problem have already been adequately described elsewhere [23, 26]. The simplest of these is the nearest-neighbour Kalman filter (NNKF). This technique allows only the measurement closest in statistical distance to the predicted track to be used to update the target state estimate. A variant of this approach is the strongest neighbour filter in which the SNR is taken into account to resolve the association ambiguity. Such approaches ignore the other measurements in the data set although they can be surprisingly effective in practice [31].

A common suboptimal Bayesian approach is known as probabilistic data association (PDA) [32]. In this approach, all measurements that are close to the predicted track location are used in a Bayesian update of the form (3). The overall Gaussian mixture PDF is then condensed to a single Gaussian by retaining only the first and second moments (i.e. the mean and covariance). Both the NNKF and PDA are recursive single-target tracking algorithms.

The simplest 'true MTT' algorithm is the global nearest neighbour algorithm, also known as the 2-D assignment algorithm [33]. As the name suggests, an assignment problem is set up that accounts for the distances between all measurement and all tracks. The single best solution to this assignment problem allows each track to be updated with at most one measurement via Kalman filtering.

The preceding approaches consider data association decisions one scan at a time, summarising previous data by a single set of track estimates and covariances. More complex approaches are possible that consider data association across multiple scans or allow limited branching in the hypothesis tree. The multi-hypothesis tracking approach is of the latter type. MHT, which is essentially a MAP estimator, can be formulated in two principal ways referred to as measurement-oriented [34] and track-oriented [35], depending on the mechanism used to generate the association hypotheses. Implementations of MHT also differ in respect of the strategies used to compute hypothesis probabilities, cluster tracks, merge track histories and prune low probability branches from the hypothesis tree (see [3]). A recent article by Blackman [36] discusses these issues in a tutorial context and points to some areas for further study.

Some approaches to the MTT problem utilise the 'hard association' approach characterised by (3) in connection with the optimal Bayesian estimator. Other approaches seek to mitigate this complexity by applying a 'soft association' or non-enumerative approach. The latter approach ignores the constraint that a point target may only produce a single measurement from a single sensor at a given time. Alternatively, numerical approximation of the multi-target PDF (obtained via Bayes' rule) may be attempted [37]. In the latter approach, the complexity of the data association problem, which requires a mixture PDF with an ever-increasing number of terms, is replaced by an equally undesirable numerical approximation of a multi-dimensional PDF of arbitrary form, (current research in particle filtering may however be able to remedy this situation.)

Recognising that a track is basically a sequence of measurements, an alternative approach to solving the MTT problem is to map the measurement set onto a trellis and to seek the optimal measurement association sequence. These approaches may either be ML or MAP, depending on the assumed form of the cost function. Conceptually such approaches provide state estimates as a by-product of the data association process, although the state estimates are usually required to evaluate the cost function. Candidate techniques include the Viterbi algorithm [38–40], the EM algorithm [41, 42], multiple scan assignment (also known as matching or set partitioning) [43–48], and network theoretic algorithms [49]. The article by Pattipati *et al.* [50] provides an in-depth survey of assignment techniques for the MTT problem.

Although algorithms that seek to solve the data association problem directly are essentially taking a 'hard decision' approach, it is possible to hedge bets, so to speak, by retaining not only the optimal solution, but the N best solutions [50]. This kind of approach (called a list Viterbi algorithm) is also used in telecommunications where the N-best surviving paths in the trellis are retained [51] so as to minimise the risk of an incorrect decoding decision. For MTT the N-best assignment algorithm of [52] provides an efficient means to achieve this [6, 53, 54]. The N-best assignment formalism may also be generalised to higher-dimensional assignment problems.

Multi-scan assignment algorithms for MTT, which are equivalent to integer programming problems of the kind originally posed by Morefield [55], are known to be NP-hard problems in combinatorial optimisation. This means that the worst-case complexity of the problem is exponential in the size of the problem (i.e. the number of targets and measurements). The currently preferred solution is via Lagrangian relaxation [45, 56], wherein an S-dimensional assignment problem (with $S > 2$) is solved to desired accuracy by solving a sequence of lower-dimensional (preferably 2-D) assignment problems. In each stage of the process, a constraint is omitted or 'relaxed', being included as a term in the cost function of a dual problem via a Lagrange multiplier. The constraint is then reintroduced in another lower-dimensional problem (the primal problem). This iterative solution process works because the dual problem lower bounds the optimal cost while the primal problem upper bounds it, with the so-called duality gap decreasing at each iteration (see [43] for more detail).

An alternative approach in [57] starts with the integer programming formulation and, by relaxing the constraints for an integer solution, arrives at a standard linear programming (LP) problem. This makes the problem amenable to efficient polynomial time algorithms developed for linear programming, such as interior point methods [58]. Interestingly, since the interior point solution of an LP may sometimes be fractional, rather than integral, a probabilistic updating approach may be used. The fractional solutions can also be harnessed for track initiation. A linear programming approach to MHT has recently been presented in [59].

The problems of automatic detection and tracking are closely linked: the former is one of declaring the presence or absence of a target signal given noisy observations; the latter is that of estimating the parameters of the target (e.g. its location and speed) given noisy observations. When the signal-to-noise ratio is low or the false alarm rate high, the boundary between detection and tracking can become blurred. This is because it may not be reasonable to declare the presence of a target unless a number of detections from this target have been observed that correlate in space and time; but a number of detections from the same target also constitutes a track. Moreover the manner of correlating detections from one time to the next clearly depends on the location and motion of the target, which are generally unknown. Conventional tracking approaches implicitly assume that some form of thresholding has been applied to the raw sensor data in order to reduce the amount of measurement data for processing. Approaches that do not apply thresholding are referred to as track-before-detect (TBD). Such approaches tend to use discrete or quantised state and measurement spaces teamed with either hidden Markov models (HMMs) or multi-frame detectors (for 2-D image data), and their complexity/performance is therefore determined by the number of discrete states they require.

The HMM technique relies on a finite-state Markov chain model of the target dynamics accompanied by a (memory-less) probabilistic mapping that represents the observation process of the sensor. Under this formalism the problem is amenable to computation of the smoothed posterior state PDF $\mathrm{p}(x(k)|Y^K)$ via the Forward-Backward (FB) algorithm given a batch of (quantised) measurement data Y^K [60]. The sequence of discretised MAP state estimates

$$\tilde{x}(k|K) = \arg\max \mathrm{p}(x(k)|Y^K), k = 1, \ldots, K \tag{4}$$

can then be computed. The sequential MAP state estimator

$$\{\hat{x}(1|K), \ldots, \hat{x}(K|K)\} = \mathrm{argmax}\, \mathrm{p}(x(1), \ldots, x(K)|Y^K) \tag{5}$$

may also be computed by applying the Viterbi algorithm (VA) [61]. Note that the sequence of individually most probable states in (4) is not the same as the single most probable state sequence in (5). Although mathematically feasible, the HMM method is not practicable in the full MTT case except in low dimensional state-spaces (e.g. passive sonar).

4 Recent developments

As alluded to previously, some recent theoretical developments have not been covered in our survey. The reader should however be aware of these areas at least in name and we suggest some starting references. The areas in question are random set theory/finite set statistics (FISST) [62], unscented Kalman filters (UKFs) [63, 64], statistical bootstrapping [65] and particle filters (PFs) [66–69]. The last area in particular is experiencing very rapid growth at present, as evidenced by two recent books on the subject [70, 71]. It should be pointed out that the UKF and PF methods arose in the context of estimation for nonlinear systems and are thus not explicitly MTT methods, although they are highly relevant since they can be used to replace the traditional extended Kalman filter, which is widely used in MTT approaches.

Particle filtering is a random sampling approach, that is, it is non-parametric. As such, a set of sample vectors called particles is taken to represent the PDF. Estimation statistics may then be generated on the basis of these samples. The attraction of the approach is that the actual filtering procedure is conceptually much simpler than in conventional MTT algorithms and is unhampered by system nonlinearities. On the other hand, it should be stressed that there are many unanswered questions of practical importance concerning PFs, such as the number of particles required for a given level of accuracy and the dependence of the approximation error on the dimension of the state vector [72]. So-called Rao-Blackwellized particle filters have recently been applied directly to the MTT problem by representing some components of the posterior state PDF, for instance the means and covariances of the terms in the mixture PDF, in a parametric form [73]. This reduces the number of particles required to approximate the PDF in addition to reducing the estimation variance.

The FISST approach is primarily for multi-sensor multi-target estimation problems, in which the number of targets is also a random variable. Mahler, in a formidable body of work, presents a new differential and integral calculus as a foundation for this class of problems. In FISST, random variables are set-valued, that is, they are random sets. Conventional probability-mass functions are replaced by belief-mass functions, integrals are replaced by set-integrals, and so on. The methodology also encompasses Dempster-Schafer evidential reasoning. The interested reader should consult [74, 75] and references therein. At the time of writing FISST cannot be viewed as a practical alternative for Bayesian MTT owing to difficulties with its implementation, which, even in the approximate 'probability hypothesis density' (PHD) case, may require techniques such as particle filtering.

Finally we mention a number of other omissions from our review. Image processing approaches, e.g. Hough Transforms [76–78]) and 3-D matched filters for multi-frame detection [79–83], have not been included in the tables since they are designed for sequences of 2-D images and are not easily generalised to multidimensional state/measurement data. We have deliberately left out artificial neural networks: although some work relevant to MTT has been performed in this area, it tends to replicate known structures like JPDA (see, e.g. [84]) or replace parts of known adaptive filters [85]. An important area that is beyond the purview of this article is that of manoeuvring target tracking where the system and/or noise covariance matrices in (1) can change with time according to an unobservable stochastic process. A large class of algorithms has been developed for this problem including multiple model filters and filter banks [26] (chapter 4), interacting multiple model (IMM) [86] and generalised pseudo-Bayesian (GPB) algorithms [87]. Surveys of techniques for the manoeuvring target tracking problem may be found in [26, 88, 89].

5 Performance assessment

Whatever the approach used to solve the MTT problem, it is necessary to devise measures of estimation performance so that different algorithms may be compared or benchmarked. For problems involving only parameter estimation (maximum likelihood techniques) an absolute limit exists called the Cramér-Rao lower bound. This is a lower bound on the estimation variance for any unbiased estimator and it requires knowledge of the true target state. Extensions of this result are available for MAP estimators (see [90] or [42], chapter 4), although the problem of missed detections and data association can greatly complicate the analysis [91]. The evaluation of CRLBs, when possible, is often laborious.

Tracker performance analysis is still an open research area: very few analytical performance results are available for even basic algorithms like the nearest neighbour Kalman filter. It is therefore commonplace to perform Monte Carlo trials in order to accumulate track error data from which performance metrics such as average errors and false track probabilities can be computed. A promising method is the tracker operating characteristic (TOC) [92–94], which is the tracking analogy of the receiver operating characteristic in detection theory [95]. The TOC plots the true and false track probabilities for given input data statistics (detection and false alarm probabilities), and can be visualised as a set of surfaces.

Owing to the many different types of possible tracking errors, one must be careful to define what is understood by 'tracker performance' [31, 96]. In MTT studies, comparative performance is typically gauged on the basis of state estimation errors. For instance the average position and velocity errors or the normalised-squared innovations [26] may be computed for a given scenario. This interpretation of performance is due in part to the classical control-theoretic viewpoint that is the legacy of the Kalman filtering era. Unfortunately such an approach fails to characterise adequately an algorithm's performance in solving the data association problem in an MTT environment. Thus such quantities as track initiation delay, overshoot, label swaps, misassociation errors, track loss and coalescence, etc., are generally not taken into account. It is also rare to see algorithm robustness characterised, that is, the sensitivity of an algorithm to modelling errors and mistuning. These considerations, while more difficult to analyse in a theoretical context, are often paramount in the design of a multi-target tracking system for a real application.

In practice algorithm performance against an agreed set of criteria for a given application can usually only be assessed after extensive testing on both simulated and real data. A comprehensive set of simulation scenarios might include: crossing tracks, parallel tracks and manoeuvres in light/heavy clutter and at high/low probability of detection. Factors such as finite sensor resolution, field of view and target detection probability modelling should also be taken into account at least in the data generation stage. Furthermore the dimension of the measurement data can have a dramatic impact on tracker performance for the same algorithm. It is easy to be fooled by simulations on multidimensional measurement data in which the effective volumetric clutter density is very low.

Apart from performance, other important criteria used in the evaluation of MTT algorithms include the computational complexity (as a function of the number of targets, measurements, and state dimension), implementational complexity (data structures, hypothesis management), whether automatic track initiation and maintenance are a part of the algorithm, and the decision delay (also referred to as processing lag). The decision delay is important since the theoretically optimal approach is to base the state estimation on as much data as possible, delaying the actual production of tracks. This must be weighed against the practical or operational consequences of not displaying the filtered track data in a timely manner. Some algorithms like the VA and MHT may make retrospective changes to the track history in view of new data. We mention some further outstanding issues to do with performance comparisons in the Conclusion.

6 Taxonomy of MTT methods

In the following tables the open literature algorithms for the single-sensor, multiple target tracking problem have been summarised (although many of the approaches are also applicable to the multi-sensor case). This list covers the major journals, in addition to a number of conference proceedings and reports. Our categorisation of MTT algorithms is motivated by a book review by Daum [97], which deals with numerous MTT approaches and focuses on finite sensor resolution. We confess that we have sided with the majority of researchers in not covering finite resolution in this literature review, despite its obvious practical importance. The interested reader should consult [3] (chapter 6).

All algorithms we include herein relate to the multi-target tracking problem with unassociated or unlabelled data, i.e. measurements are not tagged in terms of their origin. This excludes multiple target localisation and tracking methods for which the data association problem does not arise since the individual targets signals are identified by their angles of arrival at a receiver array [98–102]. We now go on to describe the scheme we have adopted to organise and tabulate the various approaches.

First the processing scheme is stated. This may be either recursive (sequential), single- or multi-pass batch. In a similar manner to the fast Fourier transform, most batch algorithms can be implemented in a sliding window or fixed-lag format that may or may not entail overlap of data segments.

Next the data association rule for the algorithm is described. This is a very important factor in determining the performance/complexity trade-off of the algorithm. The methods can be loosely categorised as either enumerative or non-enumerative. Much of the complexity of the data association problem stems from the enumeration of measurement-to-target association hypotheses. Enumerative methods include the NNKF (nearest neighbour); PDA (all-neighbours); 1:1 single and multiple scan assignment; 1:1 association along each branch of a hypothesis tree; 1:1 association along each path of a trellis. Non-enumerative (soft-association) algorithms, such as the SME [103], PMHT [104], event-averaged mean field [105] and Markov random field [106] approaches, do not require explicit enumeration of data association hypotheses; instead, the computations occur in a higher dimensional state space (avoiding data association) or some kind of average is used to take all associations into account in a statistical sense (this can be described as many-to-one association).

The approximate scaling of computational complexity with the number of targets (T) and with the state dimension (n_x) is subsequently tabulated. This is described in order of increasing complexity as linear, polynomial, combinatorial and exponential. Some licence has been used since actual algorithmic complexity depends strongly on the implementation (e.g. gating and hypothesis management strategies). The Kalman filter has a generic complexity of $O(n_x^3)$. The scaling for a single KF is therefore quadratic in the state dimension. Many algorithms employ multiple KFs and so this scaling will be multiplied by the number of filters required, which is a function of the number of targets and measurements in general. The (average) number of measurements in a single scan is denoted by M. The batch length or history is denoted by K. Some recursive algorithms like PDA have a history of 0; others like GPB consider association hypotheses over several scans of data.

Following this a measure of overall computational complexity has been given as reflected in Table 1. The reader should be mindful that this simplistic linear characterisation of complexity spans many orders of magnitude. Simple algorithms such as the NNKF are classed as 'low' complexity whereas algorithms such as

Table 1: Scale used for both algorithmic complexity and performance

low	low/medium	medium	medium/high	high

the optimal Bayesian filter are classed as 'high' complexity, although the latter cannot be implemented since its computational requirements increase with time. Even for the same algorithm, complexity may vary widely depending on the implementation details.

Memory requirements are not addressed since these are highly implementation dependent and are not usually detailed in the literature. Clearly, however, the higher complexity algorithms also tend to have higher memory requirements, as do multiple scan and batch algorithms. Some algorithms, such as hidden Markov models, use a discretised state space (with N states) and their complexity cannot be directly compared with the majority of approaches (which are based on continuous state spaces).

Table 2: Single-target Bayesian tracking algorithms: nearest neighbour (no branching); all-neighbour (single scan branching and merging); optimal (branching with no merging)

Algorithm	Nearest neighbour Kalman filter	Probabilistic data association	Optimal Bayesian filter
Processing	recursive	recursive	recursive
History (scans)/batch length	0	0	unlimited
Association rule	nearest neighbour	all neighbour	all paths/all neighbours
Scaling for number of targets	–	–	–
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	low	low	high
Performance	low	low	NA
Comments	single target	single target	single target optimal MMSE estimator; infeasible
References	[26, 109]	[32, 26, 43]	[26, 1, 110, 111]

Table 3: Single target tracking algorithms (continued): the GPB filter approximates the OBF by allowing limited branching followed by merging. The TSF is a non-Bayesian, multi-target branching filter with likelihood-based pruning

Algorithm	Generalised pseudo-Bayesian filter	Track splitting filter (optimal)	Track splitting filter (approximate)
Processing	recursive (fixed lag)	recursive	recursive
History (scans)/batch length	$K > 0$	unlimited	limited
Association type	all paths/all neighbours	1:1 per branch	1:1 per branch
Scaling for number of targets	–	exponential	combinatorial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	medium	high	medium
Performance	medium	NA	medium
Comments	single target, K scan back; also called multiple frame data association.	single target assumed in likelihood calculations; infeasible	single target likelihood with pruning
References	[111]	[26, 1]	[112, 113]

Table 4: Also known as 'global nearest neighbour', 2-DA is the simplest 'true MTT' algorithm. Its extension to multiple scans is an NP-hard integer programming problem

Algorithm	Nearest neighbour 2-D assignment	Multiple scan assignment	Integer programming
Processing	recursive	batch	batch
History (scans)/batch length	0	$K > 0$	$K > 0$
Association type	1:1	1:1 per scan	1:1 per scan
Scaling for number of targets	polynomial	polynomial	polynomial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	low/medium	medium/high	medium/high
Performance	low/medium	medium /high	medium/high
Comments	Complexity of 2-D assignment is $O(N^3)$, $N = \min\{M, T\}$, T targets, M measurements	3-D assignment solved as sequence of 2-D problems; NP-hard for $K > 0$.	MTT problem reformulated as constrained integer programming problem with likelihood cost function
References	[33, 3, 114]	[43–45, 47, 48, 20, 50, 57, 115–117]	[1, 43, 3, 55]

The complexity of HMM-based approaches is usually written in terms of the number of discrete states.

Penultimately a performance figure is given for each algorithm. In assessing performance, rated on a scale of 'low' to 'high' as in Table 1, a rather subjective assessment has been made based on (i) the available public domain simulation results; (ii) theoretical considerations such as MAP or ML 'optimality'; and (iii) the author's own experience. In many cases, however, no comparative performance results are available. Even though a benchmark problem has been proposed for manoeuvring target tracking [107, 108], there is no agreed MTT benchmark problem, which makes claims of algorithm superiority at times debatable. To complicate matters, algorithm performance is strongly application- and data-dependent, although there seems to be consensus in the tracking community that

Table 5: JPDA is the MTT generalisation of PDA. Its variants use simplified formulae for the data association probabilities

Algorithm	Joint PDA	Cheap JPDA	Suboptimal/fast JPDA
Processing	recursive	recursive	recursive
History (scans)/batch length	0	0	0
Association type	all neighbour	all neighbour	all neighbour
Scaling for number of targets	combinatorial	linear	polynomial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	medium	low/medium	low/medium
Performance	medium	low/medium	low/medium
Comments	Complicated by association probability calculations	Greatly simplified association probability calculations	Simplified association probability calculations
References	[26, 43, 118–122]	[123]	[124, 125]

Table 6: Further variants of JPDA use, e.g. assignment algorithms, to obtain the single best or N-best joint association hypotheses. The SME filter avoids data association by nonlinear filtering in a higher dimensional space

Algorithm	Nearest neighbour JPDA	N-best JPDA	Symmetric measurement equation (SME) filter
Processing	recursive	recursive	recursive
History (scans)/batch length	0	0	0
Association type	1:1	all neighbour	non-enumerative
Scaling for number of targets	linear	polynomial	linear
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	low/medium	medium	medium
Performance	low/medium	low/medium	Medium
Comments	Better for M and T approximately equal; single joint association event selected.	Sequence of 2-D assignment problems to solve.	EKF on symmetric combinations of position-only measurements.
References	[123, 31]	[126, 127, 31]	[128, 103]

Table 7: JPDA hypotheses can be extended across multiple scans. Smoothing (retrodiction) of state estimates may also be introduced

Algorithm	JPDA smoothing	Multi-scan JPDA	Bayesian MHT (GCDA_SMRN)
Processing	recursive	recursive (fixed lag)	recursive (fixed lag)
History (scans)/batch length	$K>0$	$K>0$	$m+n+1$
Association type	multiple scan, all neighbours	multiple scan, all neighbours	1:1 per branch with PDA combining
Scaling for number of targets	combinatorial	combinatorial	combinatorial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	medium/high	medium/high	medium/high
Performance	medium	medium	medium
Comments	State vector concatenation and Kalman smoothing	Association probabilities updated with K scans of data: multiple observation combining	Global combining data association suspended m frames and retrodicted n frames: multiple tracks are combined.
References	[129]	[3, 57, 130]	[3, 131, 132]

Table 8: Mixture reduction is a structured approach to reducing the number of components in a mixture PDF for single or multiple target states. ML-PDA uses a PDA likelihood to set up an optimisation problem for batch estimation

Algorithm	Clustering algorithm filter	Multi-target mixture reduction	ML-PDA
Processing	recursive	recursive	multi-pass batch
History (scans)/batch length	0	0	$K>0$
Association type	all neighbour	all neighbour	all neighbour (in cost function)
Scaling for number of targets	--	combinatorial	combinatorial
Scaling for state dimension	polynomial	polynomial	exponential
Computational complexity	medium	medium/high	medium/high
Performance	medium	NA	NA
Comments	single-target Gaussian mixture reduction algorithm. Successive combining until only B branches	Extension of Gaussian mixture reduction algorithm to multi-target case	Requires solution of non-linear multi-dimensional optimisation problem (e.g. quasi-Newton)
References	[3, 133, 134]	[3, 135]	[136, 137]

Table 9: MHT is a Bayesian MAP estimator with unbounded computational requirements. Various types of clustering, hypothesis selection, pruning and merging of state estimates may be applied in an implementation

Algorithm	MHT (optimal)	MHT (approximate)	N-best MHT
Processing	recursive	recursive (fixed lag)	recursive
History (scans)/batch length	unlimited	$K>0$	$K>0$
Association type	1:1 per branch	1:1 per branch	1:1 per branch
Scaling for number of targets	combinatorial	combinatorial	polynomial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	high	medium/high	medium
Performance	NA	medium/high	medium/high
Comments	Optimal branching MAP estimator is infeasible; Track initiation and termination included	Clustering, history combining, pruning to reduce complexity	Sequence of 2-D assignment problems to solve at each step
References	[34]	[33, 138–144, 35]	[53, 6, 145–147, 54, 52]

Table 10: PMHT takes a soft association approach in the EM algorithm to produce smoothed state estimates. HMM approaches use a discretised state space coupled with the Viterbi or FB algorithm to produce state estimates across a batch

Algorithm	PMHT	Single-target HMM	HMM (decoupled)
Processing	multi-pass batch	batch	batch
History (scans)/batch length	$K>0$	$K>0$	$K>0$
Association type	non-enumerative many:1	NN or PDA	NN or PDA
Scaling for number of targets	linear	–	--
Scaling for state dimension	polynomial	exponential, base N	exponential, base n
Computational complexity	medium	$O(N^2K) \sim O(n^{2D}K)$	$O(n^{D+2}K)$
Performance	low/medium	medium/high	medium
Comments	EM algorithm with Kalman smoother; approximate sum over all association sequences	Single target, N discrete states; forward backward or Viterbi algorithm, both $O(N^2K)$	Single target, n discrete states per dimension; D-dimensional state; Component transitions separable
References	[148–153, 104]	[154–164]	[165–167]

the MHT and multi-scan assignment approaches provide the best currently available performance (note however that there are a large number of variants of both of these methods).

The reader should bear in mind that some of the algorithms are only for single target tracking but can be applied to the MTT problem by performing computations independently for each target; thus their complexity scaling

Table 11: HMM approaches may be applied to unthresholded data for low SNR single targets. The Bayes-Markov approach is a discretised, recursive MAP estimator. [172] applies to 2-D image data only

Algorithm	HMM-TBD	Single target Bayes-Markov TBD	Multi-target Bayes-Markov TBD
Processing	batch	recursive	recursive
History (scans)/batch length	$K > 0$	0	0
Association type	observation PDF	observation PDF	observation PDF
Scaling for number of targets	–	–	linear
Scaling for state dimension	exponential	exponential	exponential
Computational complexity	$O(N^2K)$ for N states	$O(N)$ for N states	$O(NT)$ for N states, T targets
Performance	NA	NA	NA
Comments	unthresholded data; designed for very low SNR tracking; Viterbi algorithm	unthresholded data; discrete state. forward recursion of FB algorithm	unthresholded data discrete state; non-crossing tracks assumption
References	[168–170]	[160, 171, 172]	[173, 174, 172]

Table 12: Multi-target HMM requires a Cartesian product state space that is computationally prohibitive. Simplifications use a 'mixed state' representation that ignores target identity or a mutual exclusivity assumption

Algorithm	Multi-target HMM (full state)	Multi-target HMM (mixed state)	Multi-target HMM-FB
Processing	batch	batch	batch
History (scans)/batch length	$K > 0$	$K > 0$	$K > 0$
Association type	NN or PDA	NN or PDA	all neighbour (PDA)
Scaling for number of targets	exponential	combinatorial	linear
Scaling for state dimension	exponential	exponential	exponential
Computational complexity	$O(N^{2T}K)$ for N states, T targets	$O\left(\left(\frac{N^T}{T!}\right)^2 K\right)$	$O(N^2K)$ for N states
Performance	NA	NA	NA
Comments	discrete state; infeasible for most applications	discrete permutation state; requires a hypothesis testing stage to separate mixed track	discrete state; forward backward algorithm; assumes separability of tracks in state space
References	[175, 176]	[177–179]	[180, 181]

Table 13: VDA harnesses the Viterbi algorithm to perform data association on a trellis. The EM algorithm may also be applied to estimate either the association sequence or the target state (parameters)

Algorithm	Viterbi data association	Expectation maximisation data association	Maximum likelihood data association
Processing	recursive (fixed lag)	multi-pass batch	multi-pass batch
History (scans) / batch length	$K > 0$	$K > 0$	$K > 0$
Association type	1:1 per path	1:1 per path	all neighbour
Scaling for number of targets	–	–	combinatorial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	low/medium	medium	medium
Performance	medium	NA	medium /high
Comments	single target; Viterbi algorithm on M-state trellis	Single target MAP estimator for association sequence; VA + Kalman smoother per pass	Maximum likelihood deterministic estimator for target parameters; Based on EM algorithm
References	[38, 39]	[41, 42]	[182]

is linear in the number of targets. For single target algorithms, the performance figure relates only to single target tracking: these algorithms are not intended for closely-spaced multiple targets.

An additional space has been provided for general comments as to the particulars of the algorithm in question. Note that a dash in the table indicates that a given term does not apply to a given algorithm. When there is insufficient

Table 14: A number of operations research algorithms have been applied to solve the MTT problem on a trellis

Algorithm	Multi-target Viterbi	Multi-target Viterbi network flow	Viterbi MHT
Processing	batch	batch	recursive (fixed lag)
History (scans) / batch length	$K>0$	$K>0$	$K>0$
Association type	1:1 per path	1:1 per path	1:1 per path
Scaling for number of targets	combinatorial	polynomial	combinatorial
Scaling for state dimension	polynomial	polynomial	polynomial
Computational complexity	medium/high	medium	medium /high
Performance	NA	NA	NA
Comments	non-intersecting tracks; Markov assumption; VA + assignment algorithm	non-intersecting tracks; Markov assumption; relaxation algorithm	list VA on trellis with permutation states
References	[40]	[49]	[183]

Table 15: Ideas from statistical thermodynamics have been applied to approximate the data association process. These include mean field theory and MRFs. Direct updating of the multi-target state PDF via Bayes' rule may be attempted using numerical methods

Algorithm	Mean field theory (event averaged MLE)	Markov random field	Numerical Bayes
Processing	recursive	multi-pass batch	recursive
History (scans) / batch length	0	$K>0$	0
Association type	non-enumerative many:1	non-enumerative many:1	non-enumerative (joint state space)
Scaling for number of targets	polynomial	linear	exponential
Scaling for state dimension	polynomial	polynomial	exponential
Computational complexity	medium	medium	medium /high
Performance	NA	NA	NA
Comments	thermodynamic approximation; 2nd order numerical optimisation required at each step	MRF model for association probabilities; EM algorithm used	Grid-based method; Requires numerical integration for Markov prediction and Bayes update
References	[105, 184]	[106]	[37, 185]

Table 16: Abbreviations used in the paper

2-DA	2-dimensional assignment	MRF	Markov random field
CRLB	Cramér-Rao lower bound	MTT	Multiple target tracking
FB	Forward Backward (algorithm)	NA	Not assessed
EKF	Extended Kalman filter	NN	Nearest neighbour
EM	Expectation-maximisation	OBF	Optimal Bayesian filter
HMM	Hidden Markov model	PDA	Probabilistic data association
IMM	Interacting multiple model	PDF	Probability density function
FISST	Finite set statistics	PMHT	Probabilistic multi-hypothesis tracking
JPDA	Joint probabilistic data association	SME	Symmetric measurement equation
ML	Maximum likelihood	SNR	Signal-to-noise ratio
MAP	Maximum *a posteriori*	TBD	Track-before-detect
MFT	Mean field theory	TOC	Tracker operating characteristic
MHT	Multiple hypothesis track{er/ing}	TSF	Track-splitting filter
MLE	Maximum likelihood estimat{ion/or}	UKF	Unscented Kalman filter
MMSE	Minimum mean-square error	VA	Viterbi algorithm

information in the literature on which to assess performance, the abbreviation 'NA' (not assessed) has been used Table 2–15. Further abbreviations have been listed in Table 16 at the end of the paper.

7 Concluding remarks

Motivated by Daum's 1996 book review [97], we have provided a comprehensive survey of single and multiple target tracking methods in an estimation theoretic framework. This survey updates earlier reviews by Bar-Shalom and Dézert. A taxonomy of the methods was presented that comprised over 35 different algorithms, classified according to (i) the central algorithmic idea; (ii) the processing strategy; (iii) the history or batch length; (iv) the scaling as a function of the number of targets; (v) the scaling as a function of the state dimension; and (vi) the computational complexity. A subjective performance measure was also provided. In some cases the theoretically optimal version of an algorithm was distinguished from its practical implementation (as done in [97]). We reiterate that no coverage was given of finite sensor resolution, target manoeuvres or multi-sensor tracking.

Even without considering new approaches like finite-set statistics and particle filters, it is clear that the current range of MTT algorithms is indeed large. It is also apparent that more needs to be done to establish practically meaningful performance benchmarks, using freely available measurement and truth data, so that the relative merits of the individual algorithms can be better understood.

In comparing MTT algorithms regard should be paid to the computational complexity of the approach (which is defined differently for continuous and discrete state spaces) and to whether the approach is a recursive filter as opposed to a smoother/batch algorithm. There are many cases in which smoothers have been, in the author's opinion, unfairly compared with filters, when a more appropriate yardstick might have been a smoother with associated (labelled) data. Another potentially misleading statistic that arises in the tracking literature is that of performance metrics, such as true and false track probabilities: these depend strongly on track initiation and maintenance logic, which is often defined differently by different authors. A further example is that of track loss where algorithms, such as PDA, that provide good track continuity for true tracks also tend to have high false track rates. Given the conflicting nature of many tracking performance metrics, end-users stand to benefit from studies that highlight the numerous trade-offs that are involved.

Whilst considerable effort has been expended by the research community on approaches like JPDA, MHT and HMMs for state estimation, relatively little attention has been paid to approaches that concentrate on the data association problem. Such techniques as Viterbi and EM algorithms show potential that merits further study in this context. In contrast to the large number of papers on multiple-scan assignment algorithms, other techniques from operations research and combinatorial optimisation (e.g. network theory, integer and linear programming) are somewhat underrepresented in the MTT literature. Fundamental questions remain as to the effectiveness of so-called non-enumerative or 'soft association' approaches (as used in PMHT, MFT and MRF models) when compared with conventional 'hard association' methods that use a single measurement per track assumption.

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9 References

1 Bar-Shalom, Y.: 'Tracking methods in a multitarget environment', *IEEE Trans. Autom. Control*, 1978, **AC-23**, (4), pp. 618–626
2 Dézert, J.: 'Poursuite multi-cibles mono-senseur: Analyse des principales approches développées dans le domaine' (ONERA, Châtillon, France, 1988
3 Blackman, S., and Popoli, R.: 'Design and analysis of modern tracking systems' (Artech House, 1999)
4 Gelb, A. (Ed.): 'Applied optimal estimation' (MIT Press, 1974)
5 Skolnik, M.L. (Ed.): 'Radar handbook' (McGraw-Hill, 1990, 2nd edn.)
6 Cox, I.J., and Hingorani, S.L.: 'An efficient implementation of Reid's multiple hypothesis tracking algorithm and its evaluation for the purpose of visual tracking', *IEEE Trans. Pattern Anal. Mach. Intell*, 1996, **18**, (2), pp. 138–150
7 Cox, I.J., Rehg, J.M., and Hingorani, S.L.: 'A Bayesian multiple hypothesis approach to contour segmentation', *Int. J. Comput. Vis.*, 1993, **11**, (1), pp. 5–24
8 Lane, D.M., Chantler, M.J., and Dai, D.: 'Robust tracking of multiple objects in sector-scan sonar image sequences using optical flow motion estimation', *IEEE J. Ocean. Eng.*, 1998, **23**, (1), pp. 31–46
9 MacCormick, J., and Blake, A.: 'A probabilistic exclusion principle for tracking multiple Objects'. Proc. 7th Int. Conf. Computer Vision, 1999, pp. 572–587
10 Rasmussen, C., and Hager, G.D.: 'Probabilistic data association methods for tracking complex visual objects', *IEEE Trans. Pattern Anal. Mach. Intell.*, 2001, **23**, (6), pp. 560–576
11 Moran, B.A., Leonard, J.J., and Chryssostomidis, C.: 'Curved shape reconstruction using multiple hypothesis tracking', *IEEE J. Ocean. Eng.*, 1997, **22**, (4), pp. 625–638
12 Kocak, D.M., da Vitoria Lobo, N., and Widder, E.A.: 'Computer vision techniques for quantifying, tracking, and identifying bioluminescent plankton', *IEEE J. Ocean. Eng.*, 1999, **24**, (1), pp. 81–95
13 Xie, Y.: 'A range-dependent echo-association algorithm and its application in split-beam sonar tracking of migratory salmon in the Fraser River watershed', *IEEE J. Ocean. Eng.*, 2000, **25**, (3), pp. 387–398
14 Cox, I.J., and Leonard, J.J.: 'Probabilistic data association for dynamic world modeling: A multiple hypothesis approach'. Int. Conf. Advanced Robotics, June 1991, vol. 2, pp. 1287–1294
15 Jensfelt, P., and Kristensen, S.: 'Active global localization for mobile robot using multiple hypothesis tracking', *IEEE Trans. Robot. Autom.*, 2001, **17**, (5), pp. 748–760
16 Leonard, J.J., and Durrant-Whyte, H.F.: 'Application of multi-target tracking to sonar based mobile robot navigation'. Proc. 29th Conf. Dec. Control., Hawaii, Dec. 1990, pp. 3118–3123
17 Leonard, J.J., Moran, B.A., Cox, I.J., and Miller, M.L.: 'Underwater sonar data fusion using an efficient multiple hypothesis algorithm'. Proc. Int. Conf. Robotics and Automation, 1995, vol. 3, pp. 2995–3002
18 Nicoli, M., Rampa, V., and Spagnolini, U.: 'Hidden Markov model for multidimensional wavefront tracking', *IEEE Trans. Geosci. Remote Sens.*, 2002, **40**, (3), pp. 651–662
19 Spagnolini, U., and Rampa, V.: 'Multitarget detection/tracking for monostatic ground penetrating radar: application to pavement profiling', *IEEE Trans. Geosci. Remote Sens.*, 1999, **37**, (1), pp. 383–394
20 Kirubarajan, T., Bar-Shalom, Y., and Pattipati, K.R.: 'Multiassignment for tracking a large number of overlapping objects', *IEEE Trans. Aerosp. Electron. Syst.*, 2001, **37**, (1), pp. 2–21
21 Goobic, A.P., Welser, M.E., Acton, S.T., and Ley, K.: 'Biomedical application of target tracking in clutter' Dept. EE, Virginia Univ. Charlottesville, 2001
22 Hammarberg, B., Forster, C., and Torebjörk, E.: 'Parameter estimation of human nerve C-fibers using matched filtering and multiple hypothesis tracking', *IEEE Trans. Biomed. Eng.*, 2002, **49**, (4), pp. 329–336
23 Cox, I.J.: 'A review of statistical data association techniques for motion correspondence', *Int. J. Comput. Vis.*, 1993, **10**, (1), pp. 53–66
24 Uhlmann, J.K.: 'Algorithms for multiple target tracking', *Am. Sci*, 1992, **80**, pp. 128–141
25 Anderson, B.D.O., and Moore, J.B.: 'Optimal filtering' (Prentice-Hall, NJ, 1979)
26 Bar-Shalom, Y., and Fortmann, T.E.: 'Tracking and data Association' (Academic Press, 1988)
27 Daum, F.E.: 'Exact finite-dimensional nonlinear filters', *IEEE Trans. Autom. Control.*, 1986, **31**, (7), pp. 616–622
28 Sittler, R.: 'An optimal data association problem in surveillance theory', *IEEE Trans. Mil. Electron*, 1964, **MIL-8**, pp. 125–139
29 Baum, L.E., Petrie, T., Soules, G., and Weiss, N.: 'A maximization technique occurring in the statistical analysis probabilistic functions of Markov chains', *Ann. Math. Stat.*, 1970, **41**, pp. 164–171
30 Titterington, D.M., Smith, A.F.M., and Makov, U.E.: 'Statistical analysis of finite mixture distributions' (John Wiley, NY, 1985)

31 Pulford, G.W.: 'Relative performance of single-scan algorithms for passive sonar tracking'. Proc. Aust. Acoustical Soc. Ann. Conf, Adelaide, Nov. 2002, pp. 212–221
32 Bar-Shalom, Y., and Tse, E.: 'Tracking in a cluttered environment with probabilistic data Association', *Automatica*, 1975, **11**, pp. 451–460
33 Blackman, S.: 'Multiple target tracking with radar applications' (Artech House, MA, 1986)
34 Reid, D.B.: 'An algorithm for tracking multiple targets', *IEEE Trans. Autom. Control*, 1979, pp. 843–854
35 Kurien, T.: 'Issues in the design of practical multitarget tracking Algorithms', in Bar-Shalom, Y. (Ed.): 'Multitarget-multisensor tracking: advanced applications' (Artech House, 1990), pp. 43–83
36 Blackman, S.: 'Multiple hypothesis tracking for multiple target tracking', *IEEE Trans. Aerosp. Electron. Syst. Mag. Tutor*, 2004, **19**, (1), pp. 5–18
37 Stone, L.D.: 'A Bayesian approach to multiple target tracking', in Hall, D.L., and Llinas, J. (Eds.): 'Handbook of multisensor data fusion' (CRC Press, 2001), pp. 101–1031
38 La Scala, B., and Pulford, G.W.: 'A Viterbi algorithm for data association'. Proc. Int. Radar Symp Munich, Sept. 1998, vol. 3, pp. 1155–1164
39 Quach, T., and Farooq, M.: 'Maximum likelihood track formation with the Viterbi algorithm'. Proc. 33rd IEEE Conf. on Decision and Control, Orlando, FL, Dec. 1994, pp. 271–276
40 Wolf, J.K., Viterbi, A.J., and Dixon, G.S.: 'Finding the best set of K paths through a trellis with application to multitarget tracking', *IEEE Trans. Aerosp. Electron. Syst.*, 1989, pp. 287–296
41 Pulford, G.W., and Logothetis, A.: 'An expectation-maximisation tracker for multiple observations of a single target in clutter'. Proc. 36th Conf. Decision and Control, San Diego, 1997, pp. 4997–5003
42 Bergman, N.: 'Linköping Studies in Science & Technology Dissertations No. 579'. 1999
43 Bar-Shalom, Y., and Li, X.-R.: 'Multitarget-multisensor tracking: principles and techniques' (YBS Publishing, 1995)
44 Kirubarajan, T., Bar-Shalom, Y., and Pattipati, K.R.: 'Multiassignment for tracking a large number of overlapping objects', in Bar-Shalom, Y., and Blair, W.D. (Eds.): 'Multitarget-multisensor tracking: applications and advances, volume 3' (Artech House, 2000), pp. 199–231
45 Pattipati, K.R., Deb, S., Bar-Shalom, Y., and Washburn, R.B.: 'A new relaxation algorithm and passive sensor data association', *IEEE Trans. Autom. Control.*, 1992, **37**, (2), pp. 198–213
46 Pattipati, K.R., Deb, S., Bar-Shalom, Y., and Washburn, R.B.: 'Passive multisensor data association using a new relaxation algorithm', in Bar-Shalom, Y. (Eds.): 'Multitarget-multisensor tracking: advanced applications' (Artech House, 1990)
47 Poore, A.B.: 'Multidimensional assignment formulation of data association problems arising from multitarget and multisensor tracking', *Comput. Optim. Appl.*, 1994, **3**, pp. 27–57
48 Deb, S., Yeddanapudi, M., Pattipati, K.R., and Bar-Shalom, Y.: 'A generalized S-D assignment algorithm for multisensor-multitarget state estimation', *IEEE Trans. Aerosp. Electron. Syst.*, 1997, **33**, (2), pp. 523–538
49 Castanon, D.A.: 'Efficient algorithms for finding the K best paths through a trellis', *IEEE Trans. Aerosp. Electron. Syst.*, 1990, **26** pp. 405–409
50 Pattipati, K.R., Popp, R.L., and Kirubarajan, T.: 'Survey of assignment techniques for multitarget tracking', in Bar- Shalom, Y., and Blair, W.D. (Eds.): 'Multitarget-multisensor tracking: applications and advances' (Artech House, 2000) vol. 3, pp. 77–159
51 Seshadri, N., and Sundberg, C.E.W.: 'List Viterbi decoding algorithms with applications', *IEEE Trans. Commun.*, 1994, **42**, pp. 313–323
52 Murty, K.G.: 'An algorithm for ranking all the assignments in order of increasing cost', *Oper. Res.*, 1968, **16**, pp. 682–687
53 Brogan, W.: 'Algorithm for ranked assignments with applications to multiobject tracking', *J. Guid.*, 1989, **12**, (3), pp. 357–364
54 Miller, M.L., Stone, H.S., and Cox, I.J.: 'Optimizing Murty's ranked assignment method', *IEEE Trans. Aerosp. Electron. Syst.*, 1997, **33**, (3), pp. 851–861
55 Morefield, C.L.: 'Application of 0-1 integer programming for multitarget tracking problems', *IEEE Trans. Autom. Control*, 1977, **22**, pp. 302–312
56 Poore, A.B., and Rijavec, N.: 'A Lagrangian relaxation algorithm for multidimensional assignment problems arising from multitarget tracking', *SIAM J. Optim.*, 1993, **3**, (3), pp. 544–563
57 Li, X., Luo, Z.Q., Wong, K.M., and Bosse, E.: 'An interior point linear programming approach to two-scan data association', *IEEE Trans. Aerosp. Electron. Syst.*, 1999, **35**, (2), pp. 474–490
58 Karmarkar, N.K.: 'A new polynomial time algorithm for linear programming', *Combinatorica*, 1984, **4**, pp. 163–174
59 Coraluppi, S., and Carthel, C.: 'Multi-hypothesis sonar tracking'. Proc. 7th Int. Conf. Info. Fusion, June 2004, pp. 33–40
60 Rabiner, L.R., and Juang, B.H.: 'An introduction to hidden Markov models', *IEEE ASSP Mag.*, 1986, **3**, pp. 4–16
61 Forney, Jr, G.D.: 'The Viterbi algorithm', *Proc. IEEE.*, 1973, **61**, (3), pp. 268–278
62 Mahler, R.: 'Random set theory for target tracking and identification', in Hall, D.L., and Llinas, J., (Eds.): 'Handbook of Multisensor data Fusion' (CRC Press, 2001), pp. 14-1–14-33
63 Julier, S., and Uhlmann, J.K.: 'Data Fusion in Nonlinear Systems', in Hall, D.L., and Llinas, J. (Eds.): 'Handbook of Multisensor data fusion' (CRC Press, 2001), pp. 13-1–13-22
64 Julier, S.J., and Uhlmann, J.K.: 'Unscented filtering and nonlinear estimation', *Proc. IEEE*, 2004, **92**, (3), pp. 401–422
65 Gordon, N.J., Salmond, D.J., and Smith, A.F.M.: 'Novel approach to nonlinear/non-Gaussian Bayesian state estimation', *IEE Proc. F, Radar Signal Process.*, 1993, **140**, (2), pp. 107–113
66 Avitzour, D.: 'Stochastic simulation Bayesian approach to multitarget tracking', *IEE Proc., Radar Sonar Navig.*, 1995, **142**, (2), pp. 41–44
67 Arulampalam, S., Maskell, S., Gordon, N., and Clapp, T.: 'A tutorial on particle filters for online non-linear/non-Gaussian Bayesian tracking', *IEEE Trans. Signal Process*, 2002, **50**, (2), pp. 174–188
68 Hue, C., Le Cadre, J.P., and Perez, P.: 'Sequential Monte Carlo methods for multiple target tracking and data fusion', *IEEE Trans. Signal Process*, 2002, **50**, (2), pp. 309–325
69 Musick, S., Greenwald, J., Kreucher, C., and Kastella, K.: 'Performance comparison of particle method and finite difference nonlinear filters for low SNR target tracking'. Proc. 4th Int. Conf. on Information Fusion, Montreal, 2001
70 Doucet, A., de Freitas, N., and Gordon, N. (Eds.): 'Sequential Monte Carlo methods in practice' (Springer-Verlag, 2001)
71 Ristic, B., Arulampalam, S., and Gordon, N.: 'Beyond the Kalman filter: particle filters for tracking applications' (Artech House, Boston, 2004)
72 Daum, F.E., and Huang, J.: 'Curse of dimensionality and particle filters'. Proc. Aerospace Conf., Mar. 2003, **4**, pp. 4-1979–4-1993
73 Särkkä, S., Vehtari, A., and Lampinen, J.: 'Rao-Blackwellized Monte Carlo data association for multiple target tracking'. Proc. 7th Int. Conf. Info. Fusion, June 2004, pp. 583–590
74 Mahler, R.P.S.: 'Multitarget Bayes filtering via first-order multitarget moments', *IEEE Trans. Aerosp. Electron. Syst.*, 2003, **39**, (4), pp. 1152–1178
75 Mahler, R.P.S.: 'Statistics 101 for multisensor, multitarget data fusion', *IEEE Trans. Aerosp. Electron. Syst. Mag. Tutor.*, 2004, **19**, (1), pp. 53–64
76 Carlson, B.D., Evans, E.D., and Wilson, S.L.: 'Search radar detection and track with the Hough transform Part I: system concept', *IEEE Trans. Aerosp. Electron. Syst.*, 1994, **30**, (1), pp. 102–108
77 Hu, Z., Leung, H., and Blanchette, M.: 'Statistical performance analysis of track initiation techniques', *IEEE Trans. Signal Process.*, 1997, **45**, (2), pp. 445–456
78 Liou, R.-J., and Azimi-Sadjadi, M.R.: 'Multiple target detection using modified high order correlations', *IEEE Trans. Aerosp. Electron. Syst.*, 1998, **34**, (2), pp. 553–568
79 Im, H., and Kim, T.: 'Optimization of multiframe target detection schemes', *IEEE Trans. Aerosp. Electron. Syst.*, 1999, **35**, (1), pp. 176–187
80 Porat, B., and Friedlander, B.: 'A frequency domain algorithm for multiframe detection and estimation of dim targets', *IEEE Trans. Pattern Anal. Mach. Intell.*, 1990, **12**, (4), pp. 398–401
81 Reed, I.S., Gagliardi, R.M., and Stotts, L.B.: 'Optical moving target detection with 3-D matched filtering', *IEEE Trans. Aerosp. Electron. Syst.*, 1988, **24**, (4), pp. 327–336
82 Reed, I., Gagliardi, R., and Stotts, L.: 'A recursive moving-target-indication algorithm for optical image sequences', *IEEE Trans. Aerosp. Electron. Syst.*, 1990, **26**, pp. 434–440
83 Reed, I., Gagliardi, R., and Shao, H.M.: 'Application of three-dimensional filtering to moving target detection', *IEEE Trans. Aerosp. Electron. Syst.*, 1983, **19**, pp. 898–905
84 Silven, S.: 'Neural approach to the assignment algorithm for multiple-target tracking', *IEEE J. Ocean. Eng.*, 1992, **17**, (4), pp. 326–332
85 Chin, L.: 'Application of neural networks in target tracking data fusion', *IEEE Trans. Aerosp. Electron. Syst.*, 1994, **30**, (1), pp. 281–287
86 Blom, H.P., and Bar-Shalom, Y.: 'The interacting multiple model algorithm for systems with Markovian switching coefficients', *IEEE Trans. Autom. Control.*, 1988, **33**, (8), pp. 780–783
87 Bar-Shalom, Y., and Li, X.-R.: 'Estimation and tracking: principles, techniques, and software' (Artech House, MA, 1993)
88 Mazor, E., Averbuch, A., Bar-Shalom, Y., and Dayan, J.: 'Interacting multiple model methods in target tracking: a survey', *IEEE Trans. Aerosp. Electron. Syst.*, 1998, **34**, (1), pp. 103–123
89 Pulford, G.W., and La Scala, B.F.: 'A survey of manoeuvring target tracking methods and their applicability to over-the-horizon radar', CSSIP Report No. 14/96 to High Frequency Radar Division', July 1996, http://www.ee.mu.oz.au/research/cssip/publications/9798/manoeuvre_survey.pdf
90 Kerr, T.H.: 'Status of CR-like lower bounds for nonlinear filtering', *IEEE Trans. Aerosp. Electron. Syst.*, 1989, **25**, (5), pp. 590–601
91 Farina, A., Ristic, B., and Timmoneri, L.: 'Cramer-Rao bound for nonlinear filtering with $Pd < 1$ and its application to target tracking', *IEEE Trans. Signal Process*, 2002, **50**, (8), pp. 1916–1924
92 Fortmann, T.E., Bar-Shalom, Y., Scheffe, M., and Gelfand, S.: 'Detection thresholds for tracking in clutter – a connection between estimation and signal processing', *IEEE Trans. Autom. Control.*, 1985, **AC-30**, (3), pp. 221–229
93 Bar-Shalom, Y., Campo, L.J., and Luh, P.B.: 'Receiver operating characteristic to system operating characteristic: evaluation of a track formation system', *IEEE Trans. Autom. Control*, 1990, **35**, (2), pp. 172–179
94 Pulford, G.W.: 'Markov chain analysis of the sequential probability ratio test for automatic track maintenance'. Proc. 6th Conf. Info. Fusion, Cairns, July 2003, pp. 1258–1265
95 Van Trees, H.L.: 'Detection, estimation, and modulation theory. Part 1' (John Wiley, New York, 1968)

96 Manson, K., and O'Kane, P.A.: 'Taxonomic performance evaluation for multitarget tracking systems', *IEEE Trans. Aerosp. Electron. Syst.*, 1992, **28**, (3), pp. 775–787
97 Daum, F.E.: 'Review of "Multitarget-multisensor tracking: principles and techniques"', *IEEE Aerosp. Electron. Syst. Syst. Mag.*, 1996, pp. 41–44
98 Frenkel, L., and Feder, M.: 'Recursive expectation-maximization (EM) algorithms for time-varying parameters with applications to multiple target tracking', *IEEE Trans. Signal Process*, 1999, **47**, (2), pp. 306–320
99 Affes, S., Gazor, S., and Grenier, Y.: 'An algorithm for multisource beamforming and multitarget tracking', *IEEE Trans. Signal Process*, 1996, **44**, (6), pp. 1512–1522
100 Satish, A., and Kashyap, R.L.: 'Multiple target tracking using maximum likelihood principle', *IEEE Trans. Signal Process*, 1995, **43**, (7) pp. 1677–1695
101 Rao, C.R., and Zhou, B.: 'Tracking the direction of arrival of multiple moving targets', *IEEE Trans. Signal Process*, 1994, **42**, (5), pp. 1133–1144
102 Sword, C.K., Simaan, M., and Kamen, E.W.: 'Multiple target angle tracking using sensor array outputs', *IEEE Trans. Aerosp. Electron. Syst.*, 1990, **26**, (2), pp. 367–372
103 Kamen, E.W.: 'Multiple target tracking based on symmetric measurement equations', *IEEE Trans. Autom. Control*, 1992, **37**, (3), pp. 371–374
104 Streit, R.L., and Luginbuhl, T.E.: 'Probabilistic multi-hypothesis tracking', NUWC-NPT Technical Report 10,428, Naval Undersea Warfare Center, Newport, Rhode Island, Feb. 1995
105 Kastella, K.: 'Event-averaged maximum likelihood estimation and mean-field theory in multitarget tracking', *IEEE Trans. Autom. Control*, 1995, **40**, (6), pp. 1070–1074
106 Molnar, K.J., and Modestino, J.W.: 'Application of the EM algorithm for the multitarget/multisensor tracking problem', *IEEE Trans. Signal Process.*, 1998, **46**, (1), pp. 115–128
107 Blair, W.D., Watson, G.A., and Hoffman, S.A.: 'Benchmark problem for beam pointing control of phased array radar against maneuvering targets'. Proc. Am. Control Conf., Baltimore, MD, 1994, vol. 2, pp. 2071–2075
108 Watson, G.A., and Blair, W.D.: 'Solution to second benchmark problem for tracking manoeuvring targets in the presence of false alarms and ECM', *Proc. SPIE*, 1995, **2561**, pp. 263–274
109 Singer, R.A., Sea, R.G., and Housewright, K.B.: 'New results in optimizing surveillance system tracking and data correlation performance in dense multitarget environments', *IEEE Trans. Autom. Control.*, 1973, **AC-18**, (6), pp. 571–582
110 Emre, E., and Seo, J.: 'A unifying approach to multitarget tracking', *IEEE Trans. Aerosp. Electron. Syst.*, 1989, **25**, (4), pp. 520–528
111 Singer, R.A., Sea, R.G., and Housewright, K.B.: 'Derivation and evaluation of improved tracking filters for use in dense multitarget environments', *IEEE Trans. Inf. Theory*, 1974, **IT-20**, (4), pp. 423–432
112 Fraser, E.C., and Meier, L.: 'Mathematical models and optimum computation for computer-aided active sonar systems', U.S. Navy Electronic Lab., SRI Final Report, San Diego, Mar. 1967
113 Smith, P.L., and Buechler, G.: 'A branching algorithm for discriminating and tracking multiple objects', *IEEE Trans. Autom. Control*, 1975, **AC-20**, pp. 101–104
114 Drummond, O.E.: 'Comparison of 2-D assignment algorithms for sparse, rectangular, floating point, cost matrices', *J. SDI Panels on Tracking*, 1990, pp. 481–497, Iss. 4/1990
115 Chummun, M.R., Kirubarajan, T., Pattipati, K.R., and Bar-Shlom, Y.: 'Fast data association using multidimensional assignment with clustering', *IEEE Trans. Aerosp. Electron. Syst.*, 2001, **37**, (3), pp. 898–913
116 Kirubarajan, T., Wang, H., Bar-Shalom, Y., and Pattipati, K.R.: 'Efficient multisensor fusion using multidimensional data association', *IEEE Trans. Aerosp. Electron. Syst.*, 2001, **37**, (2), pp. 386–400
117 Popp, R.L., Pattipati, K.R., and Bar-Shalom, Y.: 'Dynamically adaptable M-best 2-D assignment algorithm and multilevel parallelization', *IEEE Trans. Aerosp. Electron. Syst.*, 1999, **35**, (4), pp. 1145–1160
118 Chang, K., and Bar-Shalom, Y.: 'Joint probabilistic data association for multitarget tracking with possibly unresolved measurements and maneuvers', *IEEE Trans. Autom. Control*, 1984, **29**, (7), pp. 585–594
119 Colegrove, S.B.: 'Multi-target tracking in a cluttered environment'. Proc. ISSPA'87, Adelaide, Aug. 1987, pp. 307–314
120 Dézert, J., and Bar-Shalom, Y.: 'Joint probabilistic data association for autonomous navigation', *IEEE Trans. Aerosp. Electron. Syst.*, 1993, **29**, (4), pp. 1275–1285
121 Fortmann, T.E., Bar-Shalom, Y., and Scheffe, M.: 'Sonar tracking of multiple targets using joint probabilistic data association', *IEEE J. Ocean. Eng.*, 1983, **OE-8**, (3), pp. 173–184
122 Zhou, B., and Bose, N.K.: 'An efficient algorithm for data association in multitarget tracking', *IEEE Trans. Aerosp. Electron. Syst.*, 1995, **31**, (1), pp. 458–468
123 Fitzgerald, R.J.: 'Development of practical PDA logic for multitarget tracking by microprocessor', in Bar-Shalom, Y. (Ed.): 'Multitarget-multisensor tracking: advanced applications' (Artech House, 1990), pp. 1–23
124 Roecker, J.A., and Phillis, G.L.: 'Suboptimal joint probabilistic data association', *IEEE Trans. Aerosp. Electron. Syst.*, 1993, **29**, (2), pp. 510–517
125 Fisher, J., and Casasent, D.: 'Fast JPDA multitarget tracking algorithm', *Appl. Opt.*, 1989, **28**, pp. 371–376
126 Roecker, J.A.: 'A class of near optimal JPDA algorithms', *IEEE Trans. Aerosp. Electron. Syst.*, 1994, **30**, (2), pp. 504–510
127 Zhou, B., and Bose, N.K.: 'Multitarget tracking in clutter: fast algorithms for data association', *IEEE Trans. Aerosp. Electron. Syst.*, 1993, **29**, (2), pp. 352–363
128 Kamen, E.W., and Sastry, C.R.: 'Multiple target tracking using products of position measurements', *IEEE Trans. Aerosp. Electron. Syst.*, 1993, **29**, (2), pp. 476–493
129 Mahalanabis, A.K., Zhou, B., and Bose, N.K.: 'Improved multi-target tracking in clutter by PDA smoothing', *IEEE Trans. Aerosp. Electron. Syst.*, 1990, **26**, (1), pp. 113–121
130 Roecker, J.A.: 'Multiple scan joint probabilistic data association', *IEEE Trans. Aerosp. Electron. Syst.*, 1995, **31**, (3), pp. 1204–1210
131 Drummond, O.E.: 'Multiple target tracking with multiple frame, probabilistic data association', *Proc. SPIE*, 1993, **1954**, pp. 394-408
132 Koch, W.: 'Experimental results on Bayesian MHT for maneuvering closely-spaced objects in a densely cluttered environment'. Proc. Radar 97, Oct. 1997, pp. 729–733
133 Salmond, D.J.: 'Mixture reduction algorithms for target tracking in clutter', *Proc. SPIE*, 1990, **1305**, pp. 434–445
134 Williams, J.L., and Maybek, P.S.: 'Cost-function-based Gaussian mixture reduction for target tracking'. Proc. 6th Conf. Info. Fusion, Cairns, 2003, pp. 1047–1054
135 Pao, L.Y.: 'Multisensor multitarget mixture reduction algorithms for tracking', *J. Guid. Control Dyn.*, 1994, **17**, (6), pp. 1205–1211
136 Kirubarajan, T., and Bar-Shalom, Y.: 'Target tracking using probabilistic data association-based techniques with applications to sonar, radar, and EO sensors', in Hall, D.L., and Llinas, J. (Eds.): 'Handbook of Multisensor Data Fusion' (CRC Press, 2001), pp. 81–838
137 Shertukde, H.M., and Bar-Shalom, Y.: 'Detection and estimation for multiple targets with two omnidirectional sensors in the presence of false alarms', *IEEE Trans. Acoust. Speech Signal Process.*, 1990, **38**, (5), pp. 749–763
138 Blackman, S., Dempster, R., and Broida, T.: 'Multiple hypothesis track confirmation for infrared surveillance systems', *IEEE Trans Aerosp. Electron. Syst.*, 1993, **29**, (3), pp. 810–823
139 Blostein, S.D., and Richardson, H.S.: 'A sequential detection approach to target tracking', *IEEE Trans. Aerosp. Electron. Syst.*, 1994, **30**, (1), pp. 197–211
140 Chang, K.-C., Mori, S., and Chong, C.-Y.: 'Performance evaluation of a multiple-hypothesis multi-target tracking algorithm'. IEEE Conf. Dec. and Control, Hawaii, 1990, vol. TP-9, pp. 2258–2263
141 Koch, W., and Van Keuk, G.: 'Multiple hypothesis track maintenance with possibly unresolved measurements', *IEEE Trans. Aerosp. Electron. Syst.*, 1997, **33**, (3), pp. 883–892
142 Quach, T., and Farooq, M.: 'Application of the MHT to track formation of a single maneuvering acoustic source in clutter'. Proc. 36th Midwestern Symp. on Circuits and Syst., 1993, vol. 1, pp. 684–687
143 Torelli, R., Graziano, A., and Farina, A.: 'IM3HT algorithm: a joint formulation of IMM and MHT for multitarget tracking', *Eur. J. Control*, 1999, **5**, (1), pp. 46–53
144 Van Keuk, G.: 'Multiple hypothesis tracking with electronically scanned radar', *IEEE Trans. Aerosp. Electron. Syst.*, 1995, **31**, (3), pp. 916–927
145 Cox, I.J., and Miller, M.L.: 'On finding ranked assignments with application to multitarget tracking and motion correspondence', *IEEE Trans. Aerosp. Electron. Syst.*, 1995, **31**, (1), pp. 486–489
146 Cox, I.J., Miller, M.L., Danchick, R., and Newnam, G.E.: 'A comparison of two algorithms for determining ranked assignments with application to multitarget tracking and motion correspondence', *IEEE Trans. Aerosp. Electron. Syst.*, 1997, **33**, (1), pp. 295–301
147 Danchick, R., and Newnam, G.E.: 'A fast method for finding the exact N-best hypotheses for multitarget tracking', *IEEE Trans. Aerosp. Electron. Syst.*, 1993, **29**, (2), pp. 555–560
148 Gauvrit, H., Le Cadre, J.P., and Jauffret, C.: 'A formulaton of multitarget tracking as an incomplete data problem', *IEEE Trans. Aerosp. Electron. Syst.*, 1997, **33**, (4), pp. 1242–1257
149 Malyutov, M., Nikiforov, A., and Protassov, R.: 'Multitrajectory estimation in noise and clutter'. Proc. 4th Conf. Info. Fusion, Montréal, Aug. 2001, pp. 17–25
150 Pulford, G.W.: 'Is the probabilistic mulithypothesis tracking algorithm optimal?', University of Melbourne, Dept. Electrical & Electronic Eng. Technical Note, March 1998
151 Streit, R.L., and Luginbuhl, T.E.: 'Maximum likelihood method for probabilistic multi-hypothesis tracking', *Proc. SPIE*, 1994, **2335**, pp. 394–405
152 Rago, C., Willett, P., and Streit, R.L.: 'A comparison of the JPDAF and PMHT tracking algorithms'. Proc. ICASSP, Detroit, May 1995, vol. 5, pp. 3571–3574
153 Willett, P., Ruan, Y., and Streit, R.: 'PMHT: problems and some solutions', *IEEE Trans. Aerosp. Electron. Syst.*, 2002, **38**, (3), pp. 738–754
154 Arnold, J., Shaw, S., and Pasternack, H.: 'Efficient target tracking using dynamic programming', *IEEE Trans. Aerosp. Electron. Syst.*, 1993, **29**, pp. 44–56
155 Barrett, R.F., and Holdsworth, D.A.: 'Frequency tracking using hidden Markov models with amplitude and phase information', *IEEE Trans. Signal Process.*, 1993, **41**, (10), pp. 2965–2976

156 Demirbaş, K.: 'Manoeuvring-target tracking with the Viterbi algorithm in the presence of interference', *IEE Proc. F, Radar Signal Process.*, 1989, **136**, pp. 262–268
157 Martin, T., and Jauffret, C.: 'Poursuite de cible manoeuvrante en présence de fausses alarmes pour un système sonar actif'. Proc. 16th GRETSI, Grenoble, France, 1997, pp. 801–804
158 Martinerie, F., and Forster, P.: 'Data association and tracking using hidden markov models and dynamic programming'. IEEE Int. Conf. Acoust. Speech Signal Process, 1992, **II**, pp. 449–452
159 Martinerie, F.: 'New data fusion and tracking approaches in multiple targets / distributed sensors network contexts'. IEEE Int. Conf. Acoust. Speech Signal Process, 1993, **I**, pp. 249–252
160 Paris, S., Jauffret, C., and Goullet, G.: 'Frequency line tracking in passive sonar system'. Proc. GDR-PCR ISIS, Nov. 1998
161 Paris, S., and Jauffret, C.: 'Frequency line tracking using HMM-based schemes', *IEEE Trans. Aerosp. Electron. Syst.*, 2003, **39**, (2), pp. 439–449
162 Sitbon, S.: 'Comparative study between a new HMM tracker and conventional passive tracking algorithms'. Proc. Undersea Defence Technology, 1994, pp. 441–445
163 Sitbon, S., and Passerieux, J.M.: 'New efficient target tracking based upon hidden markov model and probabilistic data association'. Proc. 29th Asilomar Conf, Pacific Grove, USA, Nov. 1995, vol. 2, pp. 849–854
164 Streit, R.L., and Barrett, R.F.: 'Frequency line tracking using hidden Markov models', *IEEE Trans. Acoust. Speech Signal Process*, 1990, **38**, pp. 586–598
165 Logothetis, A., Evans, R.J., and Sciacca, L.J.: 'Bearings-only tracking using hidden Markov models'. Proc. 33rd Conf. Decision & Control, Orlando, FL, Dec. 1994, vol. 4, pp. 3301–3302
166 Van Cappel, D., and Alinat, P.: 'Frequency line extractor using multiple hidden Markov models'. Proc. IEEE Ocean. Eng. Soc. OCEANS, Sept. 1998, vol. 3, pp. 1481–1485
167 Van Cappel, D., and Alinat, P.: 'HMM-based frequency line extracting in lofargrams'. Proc. IMDEX ASIA, 1999, pp. 81–86
168 Barniv, Y.: 'Dynamic programming solution for detecting dim moving targets', *IEEE Trans. Aerosp. Electron. Syst. Aerosp. Electron. Syst.*, 1985, **AES-21**, (1), pp. 144–156
169 Barniv, Y., and Kella, O.: 'Dynamic programming solution for detecting dim moving targets part II: analysis', *IEEE Trans. Aerosp. Electron. Syst.*, 1987, **23**, pp. 776–788
170 Barniv, Y.: 'Dynamic programming solution for detecting dim moving targets', 'Multitarget-multisensor tracking: advanced applications' (Artech House, MA, 1990), pp. 85–154
171 Bethel, R.E., and Rahikka, R.G.: 'An optimum first-order time delay tracker', *IEEE Trans Aerosp. Electron. Syst.*, 1987, **23**, (6), pp. 718–725
172 Bruno, M.G.S., and Moura, J.M.F.: 'Multiframe detector/tracker: optimal performance', *IEEE Trans. Aerosp. Electron. Syst.*, 2001, **37**, (3), pp. 925–945
173 Bethel, R.E., and Rahikka, R.G.: 'Multisignal time delay detection and tracking', *IEEE Trans Aerosp. Electron. Syst.*, 1992, **28**, (3), pp. 675–696
174 Bethel, R.E., and Paras, G.J.: 'A PDF multitarget tracker', *IEEE Trans Aerosp. Electron. Syst.*, 1994, **30**, (2), pp. 386–403
175 Pulford, G.W., and Evans, R.J.: 'A survey of hidden Markov model tracking with emphasis on OTHR - first report to High Frequency Radar Division', CSSIP Report No. 7/95, Adelaide, May 1995, http://www.ee.mu.oz.au/research/cssip/publications/9798/hmm_survey.pdf
176 Xie, X., and Evans, R.J.: 'Frequency wavenumber tracking using hidden Markov models', *IEEE Trans. Signal Proc.*, 1993, **41**, (3), pp. 1391–1394
177 Chen, B., and Willett, P.: 'Superimposed HMM transient detection via target tracking ideas', *IEEE Trans. Aerosp. Electron. Syst.*, 2001, **37**, (3), pp. 946–956
178 Xie, X., and Evans, R.J.: 'Multiple target tracking and multiple frequency line tracking using hidden Markov models', *IEEE Trans. Signal Procss.*, 1991, **39**, (12), pp. 2659–2676
179 Xie, X., and Evans, R.J.: 'Multiple frequency line tracking with hidden Markov models – further results', *IEEE Trans. Signal Process.*, 1993, **41**, (1), pp. 334–343
180 Paris, S., Jauffret, C., and Goullet, G.: 'Extraction et détection automatique de pistes fréquentielles en sonar passif'. Proc. 17th GRETSI, Vannes, France, Sept. 1999, pp. 535–538
181 Paris, S., and Jauffret, C.: 'A new tracker for multiple frequency lines'. Proc. IEEE Aerospace Conf., Mar. 2001, vol. 4, pp. 4/1771–4/1782
182 Avitzour, D.: 'A maximum likelihood approach to data association', *IEEE Trans. Aerosp. Electron. Syst.*, 1992, **28**, (2), pp. 560–565
183 Perry, R., Vaddiraju, A., and Buckley, K.: 'Multitarget list Viterbi tracking algorithm'. Proc. 32nd Asilomar Conf. on Signals, Systems & Computers, Nov. 1998, vol. 1, pp. 436–440
184 Kastella, K.: 'A Maximum likelihood estimator for report-to-track association', *Proc. SPIE*, 1993, **1954**, pp. 386–393
185 Towers, J.J., and Chan, Y.T.: 'Tracking with Doppler and bearing measurements using conditional PDF', in Moura, and Lourtie (Eds.): 'Acoustic Signal Processing for Ocean Exploration' (Kluwer, Boston, 1993), NATO ASI Series C: Math. and Phys. Sciences, pp. 301–308