Monte Carlo data association for multiple target tracking

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Abstract

The data association problem occurs for multiple target tracking applications. Since non-linear and non-Gaussian estimation problems are solved approximately in an optimal way using recursive Monte Carlo methods or particle filters, the association step will be crucial for the overall performance. We introduce a Bayesian data association method based on the particle filter idea and the joint probabilistic data association (JPDA) hypothesis calculations. A comparison with classical EKF based data association methods such as the nearest neighbor (NN) method and the JPDA method is made. The NN association method is also applied to the particle filter method. Multiple target tracking using particle filter will increase the computational burden, therefore a control structure for the number of samples needed is proposed. A radar target tracking application is used in a simulation study for evaluation.

1 Introduction

For multiple target tracking application the data association problem must be handled. Traditionally, the estimation problem is solved using linearized filters, such as the extended Kalman filter (EKF) [4], under a Gaussian noise assumption. The sufficient statistics from the linearized filter are used for data association. Several classical association methods have been proposed in the literature. When dealing with non-linear models in state equation and measurement relation and a non-Gaussian noise assumption, these estimation methods may lead to non-optimal solutions. The sequential Monte Carlo methods, or particle filters, provide general solutions to many problems where linearizations and Gaussian approximations are intractable or would yield too low performance. In this paper, we apply the classical particle filter *Bayesian bootstrap* [12], to a multiple target environment. In a simulation study we compare this approach to traditional methods. To handle the complexity problem we also propose a controller structure, to recursively chose the number of particles.

2 Sequential Monte Carlo methods

Monte Carlo techniques have been a growing research area lately due to improved computer performance. A rebirth of this type of algorithms came after the seminal paper of Gordon et al. [12], showing that Monte Carlo methods could be used in practice to solve the optimal estimation problem. In the recent article collection, [9], the theory and development in sequential Monte Carlo methods over the last years are summarized.

Consider the following non-linear discrete time system for a single target

$$\begin{aligned} x_{t+1} &= f(x_t) + v_t, \\ y_t &= h(x_t) + e_t. \end{aligned}$$

The sequential Monte Carlo methods, or particle filters, provide an approximative Bayesian solution to discrete time recursive problem by updating an approximative description of the posterior filtering density. Let $x_t \in \mathbb{R}^n$ denote the state of the observed system and $\mathbb{Y}_t = \{y_i\}_{i=0}^t$ be the set of observations until present time. Assume independent process noise v_t and measurement noise e_t with densities p_{v_t} respective p_{e_t}. The initial uncertainty is described by the density p_{x_0}. The particle filter approximates the probability density $p(x_t|\mathbb{Y}_t)$ by a large set of N particles $\{x_t^{(i)}\}_{i=1}^N$, where each particle has an assigned relative weight, $w_t^{(i)}$, such that all weights sum to unity. The location and weight of each particle reflect the value of the density in the region of the state space. The particle filter updates the particle location and the corresponding weights recursively with each new observation. The non-linear prediction density $p(x_t|\mathbb{Y}_{t-1})$ and filtering density $p(x_t|\mathbb{Y}_t)$ for the Bayesian interference are given by

$$p(x_t|\mathbb{Y}_{t-1}) = \int_{\mathbb{R}^n} p(x_t|x_{t-1})p(x_{t-1}|\mathbb{Y}_{t-1})dx_{t-1} \quad (1)$$

$$p(x_t|\mathbb{Y}_t) \propto p(y_t|x_t)p(x_t|\mathbb{Y}_{t-1}). \quad (2)$$

The main idea is to approximate $p(x_t|\mathbb{Y}_{t-1})$ with

$$p(x_t|\mathbb{Y}_{t-1}) \approx \frac{1}{N}\sum_{i=1}^{N} \delta(x_t - x_t^{(i)}), \quad (3)$$

where δ is the discrete Dirac function. Inserting (3) into (2) yields a density to sample from. This can be done by using the Bayesian bootstrap or Sampling Importance Resampling (SIR) algorithm from [12], given in Table 1. The estimate and uncertainty region for the

Bayesian bootstrap (SIR)

1. Set $t = 0$, generate N samples $\{x_0^{(i)}\}_{i=1}^N$ from the initial distribution $p(x_0)$.
2. Compute the weights $w_t^{(i)} = p(y_t|x_t^{(i)})$ and normalize, i.e, $\bar{w}_t^{(i)} = w_t^{(i)} / \sum_{j=1}^N w_t^{(j)}, i = 1, \ldots, N$.
3. Generate a new set $\{x_t^{(i\star)}\}_{i=1}^N$ by resampling with replacement N times from $\{x_t^{(i)}\}_{i=1}^N$, where $Pr(x_t^{(i\star)} = x_t^{(j)}) = \bar{w}_t^{(j)}$.
4. Predict (simulate) new particles, i.e, $x_{t+1}^{(i)} = f(x_t^{(i\star)}, v_t), i = 1, \ldots, N$ using different noise realizations for the particles.
5. Increase t and iterate to item 2.

Table 1: Bayesian bootstrap (SIR) algorithm

particle filter can be calculated as

$$\hat{x}_t^{\text{MS}} = \sum_{i=1}^N w_t^{(i)} x_t^{(i)}, \tag{4}$$

$$P_t = \sum_{i=1}^N w_t^{(i)} (x_t^{(i)} - \hat{x}_t^{\text{MS}})(x_t^{(i)} - \hat{x}_t^{\text{MS}})'. \tag{5}$$

3 Particle number controller

The computational burden for the particle filter is dependent on the number of particles and on the resampling calculation. However, the resampling can be efficiently implemented using a classical algorithm for sampling N ordered independent identically distributed variables [5, 17]. For multiple target tracking applications the computational burden is increased. Therefore, it is essential to minimize the number of particles used in the estimation step. A novel approach is to apply a simple control structure according to Figure 1. The number of particles needed is determined by the controller using the residual $\epsilon_t = ||\mu_t^{(1)} - \mu_t^{(2)}||$, where $\mu_t^{(1)}$ and $\mu_t^{(2)}$ are some statistical property from the particle filters (PFs), using different number of particles. Possible choices are for instance some relevant statistics, such as the mean estimate from the particle filter or utilization of the probability density (pdf) or the cumulative density function (cdf). For instance the marginal distribution (density for each coordinate)

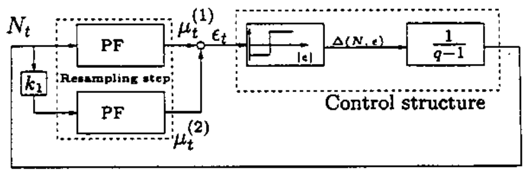

Figure 1: Controller of particles

could be used. The control structure used is a nonlinear block consisting of a relay and an integrator using

$$\Delta(N_t, \epsilon_t) = \begin{cases} \alpha_{\text{inc}}(N_t) & \text{, if } |\epsilon_t| > \Lambda \\ \alpha_{\text{dec}}(N_t) & \text{, if } |\epsilon_t| \leq \Lambda \end{cases},$$

For maneuvering targets in a tracking application the controller can reduce or increase the number of particles during the tracking envelope. However, performance may now depend on the parameters of the controller. Note that the controller is implemented in the resampling step (Table 1, step 3).

4 Data association

Data association is a problem of great importance for multiple target tracking applications. Several methods have been proposed in the literature and different methods are often discussed in estimation and tracking literature, [2, 3, 7, 8]. In general multi target tracking deals with state estimation of an unknown number of targets. Some methods are special cases which assume that the number of targets is constant or known. The observations are considered to originate from targets if detected or from clutter. The clutter is a special model for so-called false alarms, whose statistical properties are different from the targets. In some applications only one measurement is assumed from each target object, where in other applications several returns are available. This will of course reflect which data association method to use.

Several classical data association methods exist. The simplest is probably the *nearest neighbor* (NN). In [2], this is referred to as the *nearest neighbor standard filter* (NNSF) and uses only the closest observation to any given state to perform the measurement update step. The method can also be given as a global optimization, so the total observation to track statistical distance is minimized. Another multi target tracking association method is the *joint probability data association* (JPDA) which is an extension of the *probability data association* (PDA) algorithm to multi targets. It estimates the states by a sum over all the association hypothesis weighted by the probabilities from the likelihood. The most general method is a time-consuming algorithm called the *multi hypothesis tracking* (MHT), which calculates every possible update hypothesis. In [16], several algorithms for multiple target tracking are listed and categorized according to the underlying assumptions. A reference list to the different methods is also given. In [15], the so-called *probabilistic* MHT (PMHT) method is presented, using a maximum-likelihood method in combination with the *expectation maximization* (EM) method. A comparison between the JPDAF and the PMHT is also made. In [6], a Markov Chain Monte Carlo (MCMC) technique is used for data association of multiple measurements in an over the horizon radar application.

Most of these methods rely upon that the mean and covariance is sufficient statistics for the problem. For linear and Gaussian problems the Kalman filter is the optimal estimator yielding sufficient information. For non-linear problems the EKF is often used as an approximation. To be able to fully use nonlinear and non-Gaussian estimation methods combined with data association to solve the joint data association and estimation problem there is a need to develop other methods. In [1], the solution to the assignment problem for data association is proposed to be within the Bayesian framework by simply incorporate it in the estimation equations. In [18], this idea is suggested for the particle filter, when the problem of maintaining a track on a target in the presence of intermittent spurious objects. In [11], a multiple target and multiple sensor estimation and association problem is solved using the Bayesian bootstrap filter. Samples are drawn from the overall target probability density. A special filter called *hybrid bootstrap filter* is constructed. The so-called joint-filter in [14], is a solution to the joint data association and estimation problem for particle filters. The estimation is done using a particle filter and a Gibbs sampler, [10], is used for the association. The case for unknown number of targets is handled by using a hypothesis test.

In this paper we focus on this idea for a multiple target problem in a cluttered environment, and compare the particle filter based estimation and association with classical association techniques.

5 Monte Carlo Probabilistic Data Association

In this paper we modify the classical SIR algorithm (Table 1) for estimation to handle multiple targets. The association principle proposed is based on a novel Monte Carlo approach for the JPDA algorithm. We have assumed time-invariant target models, which are the same for all targets. We use the same Bayesian approach as in [11], for the estimation. However, we extend the idea and introduce hypothesis calculations according to the JPDA method. The resampling is then executed over all target association hypotheses. The clutter or false alarm model is assumed uniformly distributed in the volume and the number of false alarms for a given time is assumed to be Poisson distributed.

Let x_t be the state at time t for the relative target locations, i.e, $x_t = \{x_t^1, \ldots, x_t^\tau\}$. The samples or particles in the SIR/MCJPDA method is defined as $\{x_t^{(i)}\}_{i=1}^{N_t} = \{x_0^{(i),1}, \ldots, x_0^{(i),\tau}\}_{i=1}^{N_t}$, where each initial target cloud is denoted $x_0^{(i),j}$ for targets $j = 1, \ldots, \tau$. The measurements for each time frame (scan) are denoted y_t^k, $k = 1, \ldots, M_t$. A special clutter model is used to handle false alarms, x_t^0 $(j = 0)$. The association likelihood (track j, measurement k) is given by $p_{jk} = p_{e_t}(y_t^k - h(x_t^j))$. A general expression for the probability in hypothesis H_n is:

$$P(H_n) = \delta_n P_D^{\tau - Z_n} (1 - P_D)^{Z_n} P_{FA}^{M_t - (\tau - Z_n)} l_n, \quad (6)$$

where Z_n is the number of false alarms (FA) in hypothesis n and l_n is the likelihood part. For more details, see hypothesis calculations in the example given in [8] (p. 354). We also have an extra option

$$\delta_n = \begin{cases} 1, \text{allow multiple measurement associations} \\ 0, \text{otherwise.} \end{cases}$$

For the particle filter each particle is associated with a weight:

$$w_t^{(i)} = \sum_{n=1}^{(M_t+1)^\tau} P(H_n^{(i)}).$$

Normalization yields the particle probability $\tilde{w}_t^{(i)}$. The joint particle filtering and association is summarized in Table 2. Similar ideas in the context of robot control appear in [19]. The optional particle number controller described in Section 3, is applied at step 3, in Table 2.

To simplify the algorithm some practical problems are discarded. The measurements within a scan is considered given at the same time instances and the number of targets (τ) is assumed constant during the simulation. If the number of targets is unknown or changing, the algorithm could be modified, for instance using a separate track start hypothesis. This could be done within the particle filter framework or possible to use some linearized method. To allow measurements with different time, the prediction step is modified with an increased computational load as a consequence, i.e, each track must be predicted to every measurement time, in the association step.

Tracking & association: SIR/MCJPDA

1. Set $t = 0$, generate N_t samples from each target $j = 1, \dots, \tau$, i.e, $x_0 = \{x_0^{(i)}\}_{i=1}^{N_t} = \{x_0^{(i),1}, \dots, x_0^{(i),\tau}\}_{i=1}^{N_t}$, where $x_0^{(i),j}$ from $p(x_0^j)$.
2. For each particle compute the weights for all measurement to track association $w_t^{(i)} = \sum_{n=1}^{(M_t+1)^\tau} P(H_n^{(i)})$ and normalize for each measurement, i.e, $\tilde{w}_t^{(i)} = w_t^{(i)} / \sum_{i=1}^{N_t} w_t^{(i)}$, where $P(H_n^{(i)})$ is the probability for hypothesis n using particle i according to equation (6).
3. Generate a new set $\{x_t^{(i*)}\}_{i=1}^{N_t}$ by resampling with replacement N_t times from $\{x_t^{(i)}\}_{i=1}^{N_t}$, where $Pr(x_t^{(i*)} = x_t^{(l)}) = \tilde{w}_t^{(l)}$.
4. Predict (simulate) new particles, i.e, $x_{t+1}^{(i),j} = f(x_t^{(i*),j}, v_t^{(i),j}), i = 1, \dots, N_t$, using different noise realizations for the particles, for each target $j = 1, \dots, \tau$.
5. Increase t and iterate to item 2.

Table 2: SIR/MCJPDA estimation and association

6 Simulations

In a simulation study, the proposed SIR/MCJPDA method is implemented for a multi target environment problem. The application at hand is a missile to air scenario. To simplify the simulations we assume that it is always possible to resolve the targets. In Cartesian coordinates the relative state vector is defined as $x_t = \bar{x}_t - \bar{x}_t^{own}$, such that $x_t = \begin{pmatrix} X(t) & Y(t) & Z(t) & V_x(t) & V_y(t) & V_z(t) \end{pmatrix}'$, where X, Y and Z are the Cartesian position coordinates and V_x, V_y and V_z the velocity components. The following discrete time system is used

$$x_{t+1} = \begin{pmatrix} I_{3x3} & TI_{3x3} \\ \hline O_{3x3} & I_{3x3} \end{pmatrix} x_t + \begin{pmatrix} \frac{T^2}{2} I_{3x3} \\ \hline TI_{3x3} \end{pmatrix} v_t,$$

$$y_t = h(x_t) = \begin{pmatrix} \sqrt{X_t^2 + Y_t^2 + Z_t^2} \\ \arctan(\frac{Y_t}{X_t}) \\ \arctan(\frac{-Z_t}{\sqrt{X_t^2+Y_t^2}}) \end{pmatrix} + e_t,$$

where the process noise v_t is assumed Gaussian, $v_t \in \mathcal{N}(0, Q)$. The three-by-three null matrix and unity matrix is denoted O_{3x3} and I_{3x3} respectively. The measurement noise is assumed Gaussian $e_t \in \mathcal{N}(0, R)$. The parametric models for false alarms are assumed $N_{\text{FA}} \in Po(\lambda V)$, with average number of false alarms per unit volume λ and the validation region volume V. In the simulations $E\{N_{FA}\} = \lambda V = 0.5$ is used. The detection probability is assumed $P_D = 0.9$. Assume the number of targets $\tau = 2$ and a sample time of $T = 1[s]$. The initial inertial target state vectors $\bar{x}_0^i$, initial own platform $\bar{x}_0^{own}$, measurement noise matrix R, process noise Q and initial state error matrix P_0 are

$$\bar{x}_0^1 = \begin{pmatrix} 6500 \\ -1000 \\ 2000 \\ -50 \\ 100 \\ 0 \end{pmatrix}, \bar{x}_0^2 = \begin{pmatrix} 5050 \\ -450 \\ 2000 \\ 100 \\ 50 \\ 0 \end{pmatrix}, \bar{x}_0^{own} = \begin{pmatrix} 0 \\ 0 \\ 3000 \\ 200 \\ -50 \\ 0 \end{pmatrix},$$

$$P_0 = \text{diag}\begin{pmatrix} 100^2 & 100^2 & 100^2 & 50^2 & 50^2 & 50^2 \end{pmatrix},$$

$$Q = \begin{pmatrix} 10^2 & 0 & 0 \\ 0 & 10^2 & 0 \\ 0 & 0 & 10^2 \end{pmatrix}, R = \begin{pmatrix} 50^2 & 0 & 0 \\ 0 & 0.01^2 & 0 \\ 0 & 0 & 0.01^2 \end{pmatrix}.$$

The implemented EKF is according to the *discretized linearization* technique [13], i.e, first linearize the underlying continuous time system and then discretize. Initial values for the tracks is draw from the initial uncertainty region P_0 around the true value. We compare the SIR/MCJPDA method with an NN data association where the estimation is done by the particle filter and where the covariance matrix needed for the association is similar to equation (5). A comparison is also made to an EKF using the NN or JPDA association in a similar way. In Figure 2, a data association and estimation using the SIR/MCJPDA filter is presented. To evaluate the performance a *root mean square error*

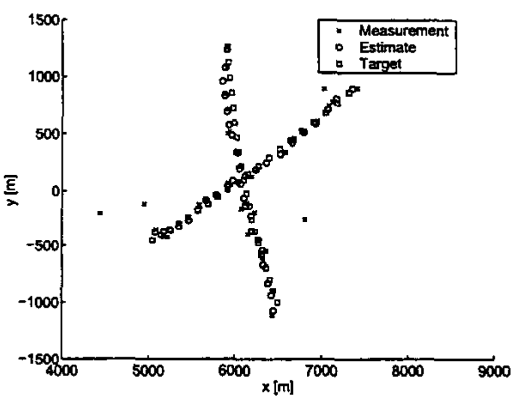

Figure 2: Data association & tracking

(RMSE) analysis is performed over $N_{mc} = 60$ simulations and time samples. In Table 3, the results for the different methods are summarized, using RMSE for the two targets when $t \geq 3$, ignoring initial transients. The particle filter used $N = 25000$ samples. In Figure 3, the RMSE values for different times are presented for the methods described in Table 3 (target 1). In Figure 4, the particle number controller (Section 3), for SIR/MCJPDA is used with $k_1 = \frac{1}{2}, k_2 = 0.1, \Lambda = 9.5$ and $\alpha_{\text{inc}}(N_t) = 0.2N_t, \alpha_{\text{dec}}(N_t) = -0.1N_t$, for the marginal case, for 20 Monte Carlo simulations.

Estimation	Association	RMSE #1	RMSE #2
SIR	MCJPDA	51.6988	51.3957
SIR	NN	55.8878	55.4883
EKF	JPDA	52.1159	51.5462
EKF	NN	52.6854	54.0163

Table 3: Association & estimation – RMSE analysis

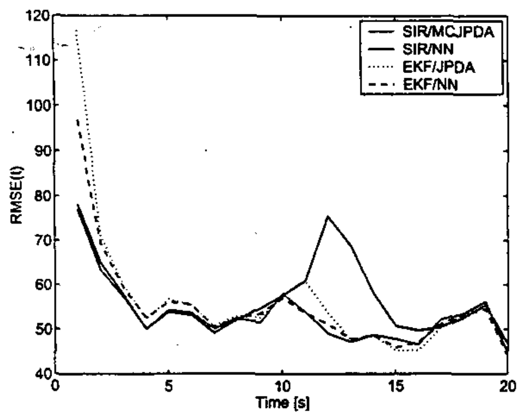

Figure 3: RMSE(t) for different methods

7 Conclusions

In this paper a novel Monte Carlo data association method for jointly estimation and association in a probabilistic data association framework is presented. This method (SIR/MCJPDA) is compared to EKF based classical association methods such as NN and JPDA. The NN association is also applied to the SIR method, where the covariance is calculated from the particle filter cloud. A novel approach to determine the number of particles for each target is also developed, using a relay and an integrator in a feedback system. In the simulation study in Section 6, the methods are compared and the RMSE is used to describe the performance. For more non-linear problems and problems where the noise distribution is highly non-Gaussian, the proposed simulation based algorithms may increase the overall tracking performance.

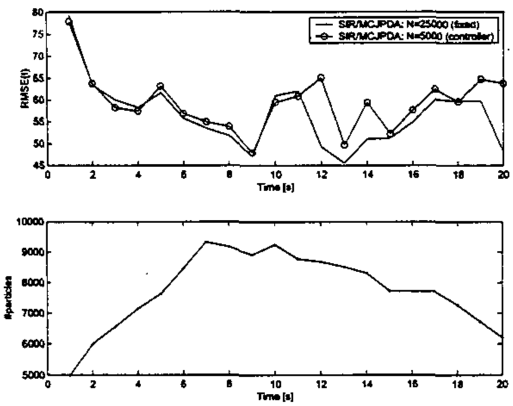

Figure 4: The particle number controller

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