1 Perfect Detection Data

1.1 Assumptions

1. Constant and known # of Targets (P)
2. No missed Detections
3. No False Alarms

1.2 Original Formulation

$$
\begin{aligned}
\underset{y_{itj},\alpha_{it},\beta_{it}}{\text{minimize:}} \quad & \sum_{i=1}^{P}\sum_{t=1}^{T} |x_{it} - \alpha_{it} - \beta_{it} * t| \\
\text{subject to:} \quad & \sum_{j=1}^{P} y_{itj} = 1 \qquad \forall i, t \\
& \sum_{i=1}^{P} y_{itj} = 1 \qquad \forall j, t \\
& M * (1 - y_{itj}) \geq |\alpha_{it} - \mu_j| \qquad \forall i, t, j \\
& M * (1 - y_{itj}) \geq |\beta_{it} - \lambda_j| \qquad \forall i, t, j \\
& y_{itj} \in \{0, 1\} \quad \forall i, t, j \\
& \alpha_{it} \in \mathbb{R}^n, \quad \beta_{it} \in \mathbb{R}^n \quad \forall i, t \\
& \mu_j \in \mathbb{R}^n, \quad \lambda_j \in \mathbb{R}^n \quad \forall j
\end{aligned}
$$

1.3 Alternate Formulation

$$
\begin{aligned}
\underset{y_{itj},\alpha_j,\beta_j,z_{jt}}{\text{minimize:}} \quad & \sum_{j=1}^{P}\sum_{t=1}^{T} |z_{jt} - \alpha_j - \beta_j * t| \\
\text{subject to:} \quad & \sum_{j=1}^{P} y_{itj} = 1 \qquad \forall i, t \\
& \sum_{i=1}^{P} y_{itj} = 1 \qquad \forall j, t \\
& x_{it}y_{itj} + M(1 - y_{itj}) \geq z_{jt} \qquad \forall i, t, j \\
& x_{it}y_{itj} - M(1 - y_{itj}) \leq z_{jt} \qquad \forall i, t, j \\
& y_{itj} \in \{0, 1\} \quad \forall i, t, j \\
& \alpha_j \in \mathbb{R}^n, \quad \beta_j \in \mathbb{R}^n \quad \forall j \\
& z_{jt} \in \mathbb{R}^n, \quad \forall j, t
\end{aligned}
$$

1.4 Missed Detections Only

$$
\begin{aligned}
\underset{y_{itj},\alpha_j,\beta_j,z_{jt}}{\text{minimize:}} \quad & \sum_{j=1}^{P}\sum_{t=1}^{T} |z_{jt} - \alpha_j - \beta_j * t| \\
\text{subject to:} \quad & \sum_{j=1}^{P} y_{itj} \leq 1 \qquad \forall i, t \\
& \sum_{i=1}^{P} y_{itj} = 1 \qquad \forall j, t \\
& x_{it}y_{itj} + M(1 - y_{itj}) \geq z_{jt} \qquad \forall i, t, j \\
& x_{it}y_{itj} - M(1 - y_{itj}) \leq z_{jt} \qquad \forall i, t, j \\
& y_{itj} \in \{0, 1\} \quad \forall i, t, j \\
& \alpha_j \in \mathbb{R}^n, \quad \beta_j \in \mathbb{R}^n \quad \forall j \\
& z_{jt} \in \mathbb{R}^n, \quad \forall j, t
\end{aligned}
$$

2 Imperfect Detection Data

2.1 Assumptions

1. Constant and unknown # of Targets (P)
2. Missed Detections exist with probability P_d
3. False Alarms exist at rate λ_{FA}

2.2 Formulation 1- Equal Penalties (θ_0) for TF and TM

$$
\begin{aligned}
\underset{y_{itj},\alpha_j,\beta_j,z_{jt},S_t}{\text{minimize:}} \quad & \sum_{j=1}^{N_1}\sum_{t=1}^{T} |z_{jt} - \alpha_j - \beta_j * t| + \theta_o \sum_{t=1}^{T} S_t \\
\text{subject to:} \quad & \sum_{j=1}^{N_1} y_{itj} \leq 1 \qquad \forall i,t \\
& \sum_{i=1}^{n_t} y_{itj} \leq 1 \qquad \forall j,t \\
& x_{it}y_{itj} + M(1-y_{itj}) \geq z_{jt} \qquad \forall i,t,j \\
& x_{it}y_{itj} - M(1-y_{itj}) \leq z_{jt} \qquad \forall i,t,j \\
& N_0 \leq \sum_{i,j} y_{itj} \qquad \forall t \\
& \sum_{i,j} y_{itj} + S_t = N_1 \qquad \forall t \\
& y_{itj} \in \{0,1\} \quad \forall i,t,j \\
& \alpha_j \in \mathbb{R}^n, \quad \beta_j \in \mathbb{R}^n \quad \forall j \\
& z_{jt} \in \mathbb{R}^n, \quad \forall j,t \\
& S_t \in \mathbb{Z} \quad \forall t
\end{aligned}
$$

2.3 Formulation 2 - Different Penalties $\theta_0(\phi_0)$ for TF (TM) respectively

$$
\begin{aligned}
\underset{y_{itj},\alpha_j,\beta_j,z_{jt},TF,TM}{\text{minimize:}} & \sum_{j=1}^{N_1}\sum_{t=1}^{T}|z_{jt}-\alpha_j-\beta_j*t|+\theta_o TF+\phi_0 TM \\
\text{subject to: } & \sum_{j=1}^{N_1} y_{itj}+F_{it}=1 \qquad \forall i,t \\
& \sum_{i=1}^{n_t} y_{itj}+M_{jt}=w_j \qquad \forall j,t \\
& \sum_{i=1}^{n_t}\sum_{t=1}^{T} F_{it}=TF \\
& \sum_{j=1}^{N_1}\sum_{t=1}^{T} M_{jt}=TM \\
& N_0 \le \sum_{j=1}^{N_1} w_j \le N_1 \\
& |\alpha_j|+|\beta_j| \le M_0 w_j \qquad \forall j,t \\
& x_{it}y_{itj}+M_1(1-y_{itj}) \ge z_{jt} \qquad \forall i,t,j \\
& x_{it}y_{itj}-M_1(1-y_{itj}) \le z_{jt} \qquad \forall i,t,j \\
& y_{itj}\in\{0,1\} \quad \forall i,t,j \\
& \alpha_j\in\mathbb{R}^n, \quad \beta_j\in\mathbb{R}^n \quad \forall j \\
& z_{jt}\in\mathbb{R}^n, \quad \forall j,t \\
& TF\in\mathbb{Z}, \quad TM\in\mathbb{Z}
\end{aligned}
$$

3 Notation

n_t = number of detections at time t

$$N_1 = \max_t n_t$$

$$N_0 = \min_t n_t$$

$$y_{itj} \begin{cases} 1 & \text{if detection } i \text{ at time } t \text{ is assigned to trajectory } j \\ 0 & \text{otherwise} \end{cases}$$

$$F_{it} \begin{cases} 1 & \text{if detection } i \text{ at time } t \text{ is a False Alarm} \\ 0 & \text{otherwise} \end{cases}$$

$$M_{jt} \begin{cases} 1 & \text{if detection for trajectory } j \text{ at time } t \text{ is a Missed Detection} \\ 0 & \text{otherwise} \end{cases}$$

$$w_j \begin{cases} 1 & \text{if trajectory } j \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$