Financial Engineering and Risk Management

Forwards contracts

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Forward contract

Definition. A forward contract gives the buyer the right, and also the obligation, to purchase

- a specified amount of an asset
- at a specified time T
- at a specified price F (called the forward price) set at time $t = 0$

Example.

- Forward contract for delivery of a stock with maturity 6 months
- Forward contract for sale of gold with maturity 1 year
- Forward contract to buy 10m $ worth of Euros with maturity 3 months
- Forward contract for delivery of 9-month T-Bill with maturity 3 months.

Setting the forward price F

Goal: Set the forward price F for a forward contract at time $t = 0$ for 1 unit of an asset with

- asset price S_t at time t
- and maturity T

f_t = value/price at time t of a long position in the forward contract

Value at time T: $f_T = (S_T - F)$

- long position in forward: must purchase the asset at price F
- spot price of asset: S_T

Forward price F is set so that time $t = 0$ value/price f_0 is 0

Use no-arbitrage principle to set F

Short selling an asset

Short selling is the selling of shares in a stock that the seller doesn't own

- The seller borrows the shares from the broker
- The shares comes from the brokerage's own inventory
- The shares are sold and the proceeds are credited to the seller's account

However ... sooner or later

- the seller must "close" the short by buying back the shares (called covering)

Profit/loss associated with a short sale

- Results in a profit when the price drops
- Results in a loss when the price increases

Short positions can be very risky

- Price can only drop to zero ... potential profit is bounded
- Price can increase to arbitrarily large values ... potential loss is unbounded

No-arbitrage argument to set F

Assume asset has no intermediate cash flows, e.g. dividends, or storage costs.

Portfolio: Buy contract, **short** sell the underlying and lend S_0 up to time T

Cash flow	$t = 0$	$t = T$
Buy contract	$f_0 = 0$	$f_T = S_T - F$
Short sell asset and buy back at time T	$+S_0$	$-S_T$
Lend S_0 up to T	$-S_0$	$S_0/d(0,T)$
Net cash flow	0	$S_0/d(0,T) - F$

The portfolio has a deterministic cash flow at time T and the cost $= 0$.
Therefore,

$$0 = \left(\frac{S_0}{d(0,T)} - F\right) d(0,T) \quad \Rightarrow \quad F = \frac{S_0}{d(0,T)}$$

Why is F strictly greater than the spot price S_0?

- Cost of carry

Examples of forward contracts

Example. Forward contract on a non-dividend paying stock that matures in 6 months. The current stock price is \$50 and the 6-month interest rate is 4% per annum.

Solution. Assuming semi-annual compounding, the discount factor

$$d(0,.5) = \frac{1}{1+\frac{0.04}{2}} = 0.9804.$$

Therefore,

$$F = 50/0.9804 = 51.0$$

Forward value f_t for $t > 0$

Recall the value of a long forward position

- at time 0: $f_0 = 0$
- at time T: $f_T = S_T - F$

- F_0: Forward price at time 0 for delivery at time T
- F_t: Forward price at time t for delivery at time T

Pricing via the no-arbitrage arguments

Cash flow	$t = t$	$t = T$
Short F_t contract	0	$F_t - S_T$
Long F_0 contract	$-f_t$	$S_T - F_0$
Net cash flow	$-f_t$	$F_t - F_0$

The portfolio has a deterministic cash flow. Therefore,

$$f_t = (F_t - F_0)\, d(t, T)$$