Financial Engineering and Risk Management

Swaps

Martin Haugh **Garud Iyengar**

Columbia University
Industrial Engineering and Operations Research

Swaps

Definition. Swaps are contracts that transform one kind of cash flow into another.

Example.

- Plain vanilla swap: fixed interest rate vs floating interest rates
- Commodity swaps: exchange floating price for a fixed price. e.g. gold swaps, oil swaps.
- Currency swaps

Why swaps?

- Change the nature of cash flows
- Leverage strengths in different markets

Example of leveraging strengths

Two companies

Company	Fixed	Floating
A	4.0%	LIBOR + 0.3%
B	5.2%	LIBOR + 1.0%

Company A is "better" in both but relatively weaker in the floating rate market

Company A

- Borrows in fixed market at 4.0%
- Swap with B: pays LIBOR and receives 3.95%

Company B

- B borrow in the floating market at LIBOR + 1.0%
- Swap with A: pays 3.95% and receives LIBOR

Effective rates:

- A: $-4\% + 3.95\% - \text{LIBOR} = -(\text{LIBOR} + 0.05\%)$
- B: $-\text{LIBOR} - 1\% + \text{LIBOR} - 3.95\% = -4.95\%$

Both gain!

Role of financial intermediaries

Same two companies

Company	Fixed	Floating
A	4.0%	LIBOR + 0.3%
B	5.2%	LIBOR + 1.0%

Financial intermediary that constructs the swap.

Company A

- Borrows in fixed market at 4.0%
- Swap with Intermediary: pays LIBOR and receives 3.93%

Company B

- B borrow in the floating market at LIBOR + 1.0%
- Swap with Intermediary: pays 3.97% and receives LIBOR

Financial intermediary makes 0.04% or 4 basis points. Why?

- Compensation for taking on counterparty risk and providing a service

Pricing interest rate swaps

r_t = floating (unknown) interest rate at time t

Cash flows at time $t = 1, \ldots, T$

- Company A (long): receives Nr_{t-1} and pays NX X is the fixed interest rate
- Company B (short): receives NX and pays Nr_{t-1}

Value of swap to company A

- $N(r_0, \ldots, r_{T-1})$ = Cash flow of floating rate bond - Face value. Therefore, value of swap to company A

$$V_A = N\big(1 - d(0,T)\big) - NX \sum_{t=1}^{T} d(0,t) \quad ?$$

- Set X so that $V_A = 0$, i.e.

$$X = \frac{1 - d(0,T)}{\sum_{t=1}^{T} d(0,t)}$$