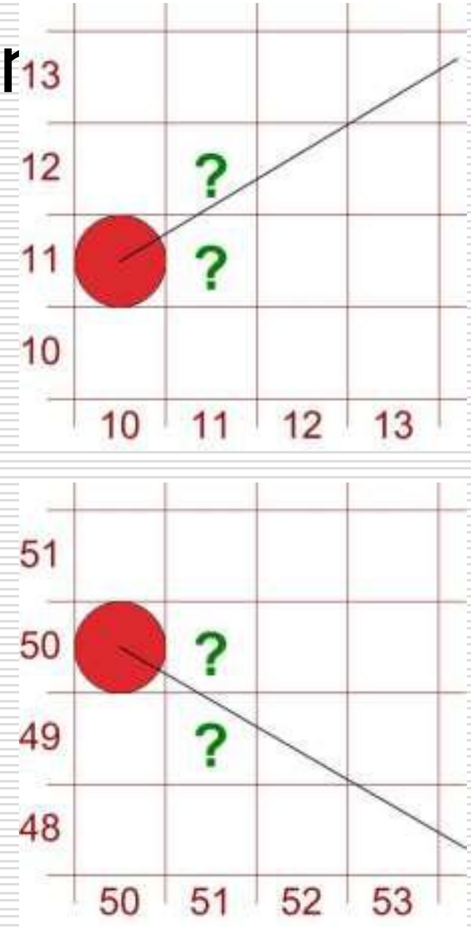
Computer Graphics (L04)
EG678EX

2-D Algorithms

Bresenham's Line Algorithm

- Uses only incremental integer calculations
- Which pixel to draw ?
  - (11,11) or (11,12) ?
  - (51,50) or (51,49) ?
  - Answered by Bresenham



- For $|m|<1$
  - Start from left end point (x_0,y_0) step to each successive column (x samples) and plot the pixel whose scan line y value is closest to the line path.
  - After (x_k,y_k) the choice could be (x_k+1,y_k) or (x_k+1,y_k+1)

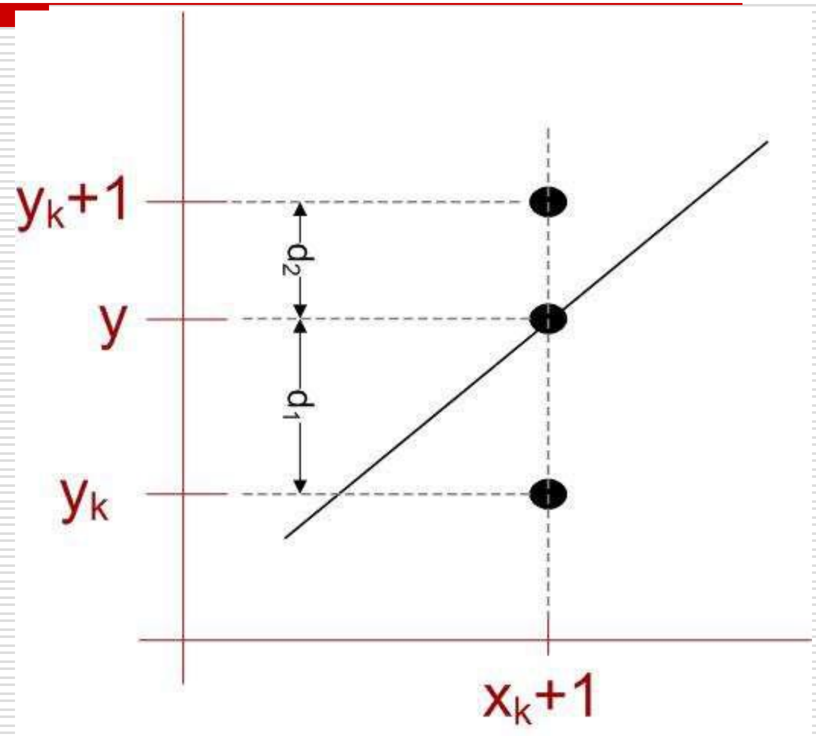
$$y = m(x_k + 1) + b$$

Then

$$d_1 = y - y_k$$
$$= m(x_k + 1) + b - y_k$$

And

$$d_2 = (y_k + 1) - y$$
$$= y_k + 1 - m(x_k + 1) - b$$



Difference between separations

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

Defining decision parameter

$$p_k = \Delta x(d_1 - d_2) \quad [1]$$
$$= 2\Delta y.x_k - 2\Delta x.y_k + c$$

Constant=2Δy + Δx(2b-1) Which is independent of pixel position

Sign of p_k is same as that of d_1-d_2 for Δx>0 (left to right sampling)

$$p_{k+1} = 2\Delta y.x_{k+1} - 2\Delta x.y_{k+1} + c$$

c eliminated here

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

because $x_{k+1} = x_k + 1$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

y_{k+1}-y_k = 0 if $p_k < 0$
y_{k+1}-y_k = 1 if $p_k \geq 0$

For Recursive calculation, initially

$$p_0 = 2\Delta y - \Delta x$$

Substitute $b = y_0 - m.x_0$ and $m = \Delta y/\Delta x$ in [1]

Algorithm Steps ($|m|<1$)

1. Input the two line endpoints and store the left endpoint in (x_0,y_0)
2. Plot first point (x_0,y_0)
3. Calculate constants Δx, Δy, $2\Delta y$ and $2\,\Delta y-2\Delta x$, and obtain $p_0 = 2\Delta y - \Delta x$
4. At each x_k along the line, starting at $k=0$, perform the following test:

   If $p_k<0$, the next point plot is (x_k+1,y_k) and

   $$P_{k+1} = p_k + 2\Delta y$$

   Otherwise, the next point to plot is $(x_k + 1, y_k+1)$ and

   $$P_{k+1} = p_k + 2\Delta y - 2\Delta x$$
5. Repeat step 4 Δx times

What's the advantage?

- Answer: involves only the calculation of constants Δx, Δy, $2\Delta y$ and $2\Delta y - 2\Delta x$ once and integer addition and subtraction in each steps

Example

Endpoints (20,10) and (30,18)

Slope m = 0.8

$\Delta x = 10$, $\Delta y = 8$

$P_0 = 2\Delta y - \Delta x = 6$

$2\Delta y = 16$, $2\Delta y - 2\Delta x = -4$

Plot ?

Plot $(x_0, y_0) = (20,10)$

k	p_k	(x_{k+1}, y_{k+1})	k	p_k	(x_{k+1}, y_{k+1})
0	6	(21,11)	5	6	(26,15)
1	2	(22,12)	6	2	(27,16)
2	-2	(23,12)	7	-2	(28,16)
3	14	(24,13)	8	14	(29,17)
4	10	(25,14)	9	10	(30,18)