Chapter

5 Magnetic Materials

The materials which can be magnetised are called magnetic material.

Magnetising Field (H)

It can be defined as the magnetic flux passing through unit area at right angle to direction of lines of force. It is the field in vaccum or free space. It can be considered as pure applied field where no field due to induction is added. It depends purely upon external conduction current which is responsible to produce it.

Magnetic Field or Magnetic Field Intensity

[Magnetic Induction or Flux Density (B)]

The total magnetic flux per unit area at right angle to the flux due to magnetising field (H) and due to the field induced in the substance (M) is called flux density.

Magnetic Moment (P_m)

As similar to electric dipole moment it can be defined as the product of pole strength and distance between them.

It is observed that a coil carrying current 'I' when placed in a region of uniform magnetic flux density, will experience a torque whose magnitude depends upon area of the coil (A) and the current (I)

Thus magnetic moment (p_m) $= I \times A$

If there are N magnetic dipoles, total magnetic moment is $P_m = N\, p_m$

Magnetization [Intensity of Magnetization (M)]

It is the measure of the magnetization of a magnetized specimen. It is defined as magnetic moment per unit volume $M = \frac{Np_m}{V}$

Magnetic Susceptibility (χ)

I is the measure of how susceptible or strongly a material be magnetized by H. It is defined as the ratio of the intensity of magnetization induced to magenetising field.

$$\chi = \frac{M}{H} \Rightarrow M = \chi H$$

Permeability (μ)

The permeability measures the degree of penetration of magnetic field through the substance. In other words it measures the capacity of substance to take magnetization.

The ratio of permeability of a medium (μ) to that of free space (μ_o) is called relative permeability (μ_r).

$$\mu_r = \frac{\mu}{\mu_o} \Rightarrow \mu = \mu_o \mu_r$$

The permeability of the medium is also defined as the magnetic induction (B) per unit magnetizing field (H).

$$\mu = \frac{B}{H} \Rightarrow B = \mu H = \mu_o \mu_r H$$

For free space, $B_o = \mu_o H$, $\mu_o = 4\pi \times 10^{-7}$ Henry m^{-1}

Relation Between μ_r and χ

The magnetic flux density 'B' is the result of magnetizing field 'H' and field due to magnetization (M)

$$B = \mu_o (H + M)$$

$$\mu H = \mu_o (H + \chi H)$$

$$\mu H = \mu_o (1 + \chi) H$$

$$\frac{\mu}{\mu_o} = 1 + \chi$$

$$\mu_r = 1 + \chi$$

$$\chi = \mu_r - 1$$

Types of Magnetic Material

1. Diamagnetic Material

→ Diamagnetic substance have no magnetic moment in the absence of external field.

→ They are feebly magnetized in the direction opposite to the applied field so they are weakly repelled by magnet or magnetic field.

→ So they have small negative value of susceptibility and relative permeability ($\mu_r = \chi + 1$) is slightly less than unity. (The average value of $\chi = -10^{-6}$)

→ The diamagnetic properties of a substance is independent of temperature so they do not obey Curie's law.

Examples: Superconductor, Bismuth, Mercury, Silver, Diamond, Lead, Copper, Ethyl alcohol, Nitrogen, Gold, Antimony, Water, Hydrogen etc.

2. **Paramagnetic Materials**

- → Paramagnetic substance have no magnetic moment (or magnetization) in the absence of external field, this is because the atomic moments are randomly oriented in the absence of field.
- → When placed in magnetic field, they are feebly magnetized in the direction of applied field.
- → So they have small positive value of succentibility (E.g. $\chi = 1.2 \times 10^{-5}$ for metals)
- → The succeptibility decreases with rise in temperature. Because of higher temeprature, the molecular collision destroy the allignment of molecular magnetic moments with applied field. i.e. Curie law ($\chi = C/T$) is obeyed.

Examples: Aluminium, Chromium, Platinum, Oxygen, Tungsten, Magnesium, Solution of Ferro magnetic materials etc.

3. **Ferromagnetic Materials:**

- → Ferro magnetic material have large permanent magnetization even in the absence of field.
- → They are strongly magnetized in the direction of applied field.
- → So they have positive and very large value of susceptibility (χ)
- → The susceptibility decreases with rise in temperature at certain temperature T_c, Ferro magnetism is lost and the substance becomes paramagnetic. This value of temperature is called critical or curie temperature.

Examples: Iron, Cobalt, Nickel and their alloys such as AlNiCo.

4. **Antiferromagnetic Materials:**

- → Anti Ferromagnetic materials have permanent magnetic moments but have opposite alignment on alternating atoms.
- → So they have no net magnetization in the absence of external field.
- → They are feebly magnetized when subjected to a strong magnetic field.
- → So they have small positive value of susceptibility.
- → A temperature increases, antiferromagnetism is lost at certain temperature called Neel temperature above this they are paramagnetic.

Examples: Cr, Mn, MnS, MnO, MnO_4, NiO, CoO, FeO, $FeCl_3$ etc.

5. **Ferrimagnetism:**

- → Ferri magnetic materials have some magnetic moment even in the absence of external field. They exhibit magnetic behaviour similar to Ferromagnetic material. In such materials. there are permanent magnetic moments having opposite alignment of alternating atoms. However, the value of magnetic moment in one atom is higher than in its alternating atom. Therefore there is net magnetization even in the absence of field.

→ They have higher value of susceptibility than antiferromagnetic materials and lower than that of Ferromagnetic material.

→ Examples: $MgFe_3O_4$, $CuFe_3O_4$, $MnFe_3O_4$, $NiFe_3O_4$, $ZnFe_3O_4$ $ZnFe_3O_4$, $CdFe_3O_4$ [i.e. ferrites of Mg, Cu, Mn, Ni, Zn, Cd]

Note: These are non conducting.

Domain Structure and Domain Wall

The internal field in a ferromagnetic material tend to produce localized region of magnetic dipoles called domains. The boundary between two domains is called *domain wall or Bloch wall.*

If a single crystal of iron is magnetized to form a single domain the crystal will be in a high energy state. (very strong field is to be applied). The potential energy called *magnetostatic energy* of the magnet can be reduced by dividing the crystal into two regions (domains). In this process, a domain wall is formed between the domains in which the dipole gradually changes it's orientiention as shown in figure (1). Hence due to reversal of magnetism in domain wall two domain of opposite orientation are formed on either side of domain wall.

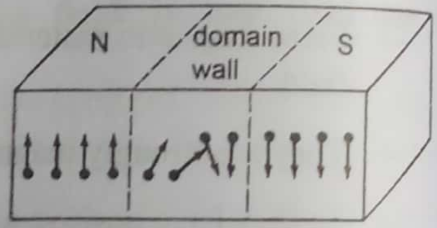

Figure 1: Reversal of magnetism across domain wall

The creation of magnetic domains continues until the potential energy reduction in creating an additional domain is same as the increase in addition of wall. The specimen then possesses minimum potential energy and is in equilibrium with no net magnetization.

In figure (2a) we have a single domain. In figure (2b) the magnetic energy is reduced by roughly one half by dividing the crystal into two domains magnetized in opposite direction. In (2c) with N domains (say) the magnetic energy is reduced to approximately 1/N of magnetic energy of (2a), because of the reduced spatial extension of the field. The magnetostatic energy associated with the field lines at the ends can be further reduced by closing the ends with side way domains with magnetization 90°. These sideway domains at 90° with the main domains core called *end domains* as shown in figure (2d) and figure (2e). Here the magnetic energy is zero.

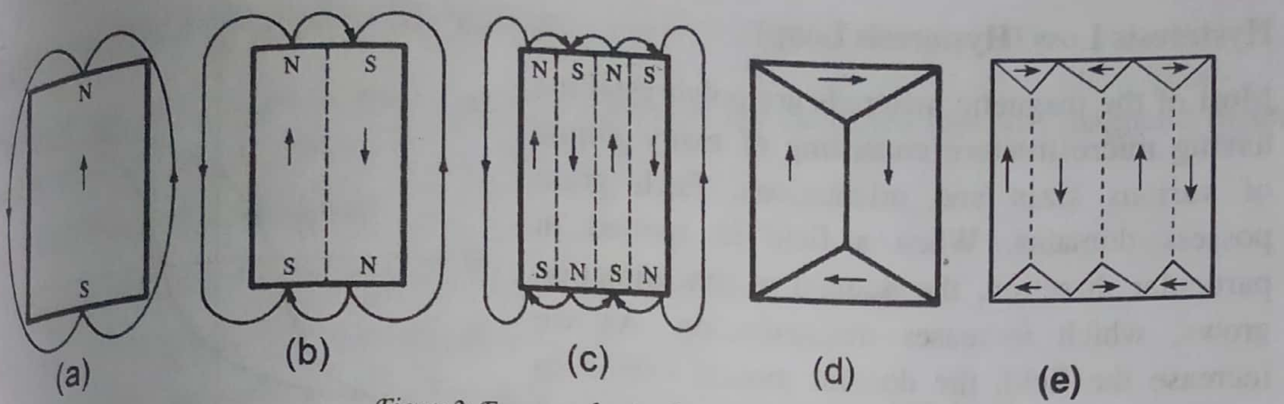

Figure 2: Energy reduction by formation of domains

When an external field is applied, the magnetization of the crystal along the applied field occurs by growth of the domain with magnetization along the applied field as shown in figure (3). Magnetization of iron crystals takes place most easily along {100} direction, moderately easily along {110} directions, and difficultly along {111} direction. The direction along which magnetization takes place easily is called *easy direction*. The magnetization process involves the motion of Bloch wall in the crystal.

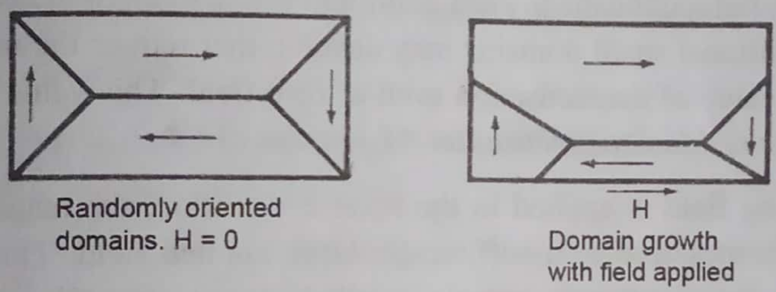

Figure 3: Domain growth in a ferromagnetic material

The domain wall is not simply one atomic spacing but has a finite thickness, which for iron is of the order of 0.1 micron. The equilibrium wall thickness is that which minimizes the total potential energy which is the sum of exchange energy (during formation of domains) and anisotropy energy. The magnetic moments that are oriented away from easy direction possesses excess of energy called *anisotropy energy*.

Losses in Magnetic Materials

When magnetic materials are subjected to changing magnetic field (flux) two types of losses occur which are (1) Eddy current loss (2) Hysteresis loss.

1. **Eddy Current Loss**

   When applied magnetic field is changed current is induced in magnetic materials. This current is called *eddy current*. This eddy current produce loss in power resulting in heating of materials called eddy current loss. To reduce this loss, resistance is made large. Very high resistance of material allow only very small current and consequently very small eddy current loss.

2. Hysteresis Loss (Hysteresis Loop)

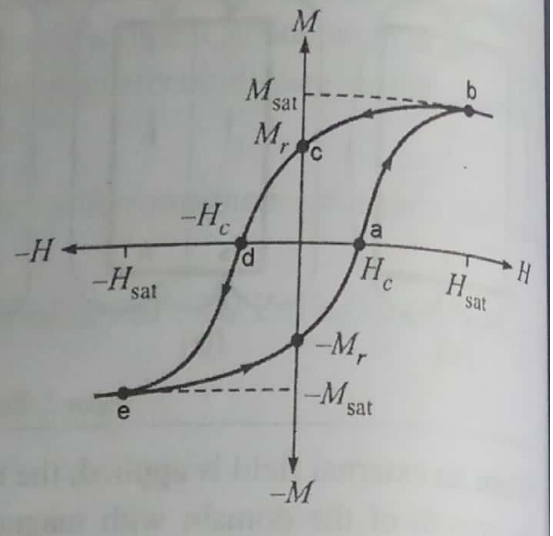

Figure : Hysteresis loop

Most of the magnetic materials are polycrystalline having microstructure consisting of many *grains* of various sizes and orientations. Each grain possess domains. When a field is applied in particular direction, the domain in that direction grows, which increases magnetization. As we increase the field, the domain motion extend to large distance until the domain wall stuck at an imperfection (due to crystal defect, impurities). Domain wall cannot move until the field applied is sufficient to break it. This sudden jerks in wall motion leads to a small jump in magnetization of specimen. This phaenomenon is called *Barkhausen effect*. As we continue to increase the field to H_{sat}, Magnetization reaches to saturation value M_{sat} [point b].

If we decrease the field, domains in each grain will rotate to align in nearest easy direction. In some grain additional small domains may develop that reduce the magnetization. There exist some finite value of magnetisation even at zero field. This value of magnetization is called *remanent or residual magnetization* (M_r) [point c]

If now magnetizing field is applied in the reverse direction, the magnetization decreases and gradually becomes zero at a sufficiently large applied field. This magnetizing field required to totally demagnetize the sample is called *coercive field* H_c. [point d]

If the field is further increased in reverse direction, the similar loop is obtained in reverse direction. The complete 'M' versus 'H' curve obtained by changing magnetizing field in both positive and negative direction is a closed loop called Hysteresis loop.

The area enclosed with in B versus H curve is the measure of energy dissipated per unit volume percycle of applied field variation. It is given by

$$E = \oint H dB$$

Steinmetz developed an empirical relation for hysteresis power loss as,

$$P_h = K_h f B_m^k$$

Where, P_h = Hysteresis power loss in watt/m^3

K_h = Hysteresis coefficient (Joules /m^3)

f = Frequency of magnetization (Hz)

B_m = Maximum flux density in Wb/m^2 (or T)

k = Steinmetz coefficient

Soft and Hard Magnetic Materials

On the basis of Hysteriesis loop magnetic materials are classified into soft magnetic materials and hard magnetic materials.

1. **Soft Magnetic Materials**
   → These are the materials having small hysteresis loop area.
   → Power loss/ volume is small
   → They are easy to magnetize and demagnetize so require low field.
   → They have high susceptibility and high permeability.
   → low residual magnetization.
   → low coercivity i.e. having small value of H_c.
   → These are suitable for the application where cycles of magnetization and demagnetization involved. Such as in Electric motors, transformers, inductors etc.
   → They are composed of very large grain and the anisotropy energy is small. So small field is sufficient to magnetize them to saturation as a single domain.

2. **Hard Magnetic Materials**
   → These are the materials having large hysteresis loop area.
   → So power loss per unit volume is large.
   → They are hard to magnetize and demagnetize so require large field.
   → The have low value of susceptibility and permeability.
   → They have large residual magnetization.
   → They have large coercivity, coercive field (H_c) for these material is millions of times greater than that of soft magnetic material.
   → They are suitable to make permanent magnet, suitable for magnetic storage of data.
   → They are composed of very small grain and the anisotropy energy is high. So sufficiently large field is required to magnetize to saturation as a single domain.

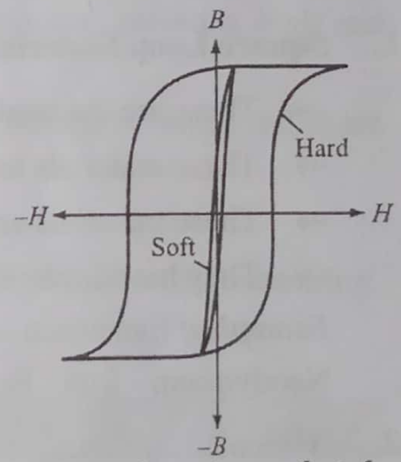

Figure : Hysteresis curve for Soft and Hard magnetic material

Some Soft Magnetic Materials

1. **Fe-Si Alloys (Silicon Steel)**

Pure iron despite being soft magnetic material causes more eddy current losses due to its higher electrical conductivity. By adding (3-4)% of Silicon to Iron produces iron silicon alloys with improved characteristics. Silicon increases the electrical resistivity thus reduces eddy *current losses*. It reduces magnetostriction and therefore reduces transformer noise. It is used widely in *power transformers*.

2. **Fe-Ni Alloys**

Based on content of Ni, these alloys are devided into three groups. 36% Ni alloys have high resistivity and low permeability and are used for high frequency devices such as *speed relays, wide band transformers and inductors*. 50% Ni alloys are used where low loss and small size are required. They have moderate permeability and high saturation induction. They are used in small *motors, synchores* etc. The 79% Ni alloys have high permeability but lower saturation induction and are used in *recording heads, pulse transformers, sensitive relays* etc.

3. **Ferrites (Soft Ferrites)**

Ferrites are an important class of Ferromagnets, which are typically oxides of mixed transition metals. They have large magnetization and high resistivity. They are therefore more suitable than Ferromagnets *for use at high frequencies* where eddy current losses are a serious problem. They are used in memory storage in computer, audio visual recording, modulators, couplers, phase shifter etc.

Manganese Zinc Ferites are useful up to frequency of 1MHz, Nickel Zinc Ferrites can be used up to frequency of 200 MHz. The Garnets such as Yttrium Iron Garnet ($Y_3\ Fe_5O_{12}$) can be used at highest frequency covering the microwave range (1-300 GHZ).

Some Hard Magnetic Materials

1. **Square Loop Materials**

→ These are the hard magnetic materials having square shape hysteresis loop.

→ These materials have very high coercivities.

→ These materials are most suited to store digital information.

→ They have lower Curie temperature

Examples: Samarium - cobalt [S_mCo_5]

Neodymium - Iron - Boron [Nd FeB]

2. **Alnico**

→ It is widely used as permanent magnetic material

→ High coercivity

→ High curie temperature (700°C)

→ It is the alloy of Al, Ni, Co and Iron

→ It is manufactured by appropriate heat treatment to allow Fe-Co particles to precipitate out from a solid solution of alloy.

→ Fe-Co particles possess strong magnetic field, where as Al-Ni particles posses weak magnetic field.

→ When heat treatment is carried out in the presence of strong magnetic field, Fe-Co particles have their magnetization along elongated side.

→ They are mechanically hard and brittle.

3. **Barium Femite [Be $Fe_{12}O_9$]**

→ It is very popular as permanent magnet.

→ It has hexagonal crystal structure with large magnetocrystalline anisotropy energy giving high coercivity.

→ These magnets are formed by wet pressing Ferrite powder in the presence of strong magnetic field to allow the single domain fine particles to be aligned in one of easy direction and then drying and carefully sintering the ceramic.

→ These ceramic magnets are used to manufacture permanent magnets in low cost applications.

4. **Carbon Steel**

→ These steels are iron in which main alloying constituent is carbon in the range of 0.12-2%.

→ The other elements such as Chromium, Cobalt, Nickel, Tungsten, Vanadium etc are add to obtain a desired alloying effect.

→ It is not stainless steel

→ Depending on carbon percentage content they are classified as low, medium, high and ultra high carbon steel.

→ As the carbon percentage rises, steel has ability to become harder, stronger and more brittle.

→ The higher carbon content lower the melting point.

→ Heat treatment is carried out to change mechanical properties of steel such as ductility, hardness, conductivity.

Solved Examples

1. **Calculate the permeability and susceptibility of an iron bar of cross sectional area 0.2 cm^2 when a magnetizing field of 1200 Am^{-1} produces magnetic field of 24 micro-Weber.**

Solution:

Here, $\phi = 24 \times 10^{-6}$ Wb $= 2.4 \times 10^{-5}$ Wb

$A = 0.2\ cm^2 = 0.2 \times 10^{-4}\ m^2$

$H = 1200\ Am^{-1}$

Using the relation, $\phi = B.A$

$$\Rightarrow B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2\ T$$

The permeability is given by, $B = \mu H$

$$\Rightarrow \mu = \frac{B}{H} = \frac{1.2}{1200} = 1 \times 10^{-3}\ H/m$$

The susceptibility is given by,

$$\chi = \mu_r - 1 = \frac{\mu}{\mu_o} - 1 = \left(\frac{1 \times 10^{-3}}{4\pi \times 10^{-7}} - 1\right) = 795.2$$

2. **Calculate the magnetization and flux density in a diamagnetic sample having susceptibility -0.3 × 10^{-5} and magnetic field strength 1000 A/m.**

Solution:

Here, $\chi = -0.3 \times 10^{-5}$, $H = 1000$ A/m

We have, $M = \chi H = -0.3 \times 10^{-5} \times 1000 = -3 \times 10^{-3}$ A/m

Now, the magnetic flux density is given by

$$B = \mu_o (H + M)$$
$$= 4\pi \times 10^{-7} (1000 - 3 \times 10^{-3})$$
$$= 1.256 \times 10^{-3}\ T$$

3. **Determine the permeability and relative permeability of a diamagnetic sample having susceptibility - 9.5 × 10^{-9}**

Solution:

Here, $\chi = -9.5 \times 10^{-9}$

$$\mu_r - 1 = -9.5 \times 10^{-9}$$
$$\mu_r = 1 - (9.5 \times 10^{-9}) = 0.999$$
$$\Rightarrow \frac{\mu}{\mu_o} = 0.999 \Rightarrow \mu = \mu_o \times 0.999 = 1.256 \times 10^{-6}$$

4. **Calculate the relative permeability of a paramagnetic material at -73°C and 227°C if the susceptibility of the paramagnetic material at 27°C is 3.7 × 10^{-3}.**

Solution:

Here, at T = 27°C = 300K, $\chi = 3.7 \times 10^{-3}$

From curie law, $\chi = \frac{C}{T} \Rightarrow C = \chi T$

$C = 3.7 \times 10^{-3} \times 300 = 1.11$

a. Now at, T = -73°C = (-73 + 273) = 200 K

$$\chi = \frac{C}{T} = \frac{1.11}{200} = 5.55 \times 10^{-3}$$

$$\mu_r - 1 = 5.55 \times 10^{-3}$$

$$\mu_r = 1.006$$

b. Again, at, $T = 227^{\circ}C = (227 + 273)K = 500K$

$$\chi = \frac{C}{T} = \frac{1.11}{500} = 2.22 \times 10^{-3}$$

$$\Rightarrow \mu_r - 1 = 2.22 \times 10^{-3}$$

$$\mu_r = 1.002$$

5. **The magnetic field strength in a piece of Fe_2O_3 is 10^6 Am^{-2}. Given that the susceptibility at room temperature is 1.4×10^{-3}, find the flux density and magnetization in the material.**

Solution:

Here, $H = 10^6$ Am^{-2}, $\chi = 1.4 \times 10^{-3}$, $B = ?$, $M = ?$

We have, $\chi = \frac{M}{H} \Rightarrow M = \chi H$

$$M = 1.4 \times 10^{-3} \times 10^6 = 1.4 \times 10^3 \text{ Am}^{-1}$$

Again $B = \mu_o (H + M)$

$$B = 4\pi \times 10^{-7} (10^6 + 1.4 \times 10^3)$$

$$B = 1.257 \text{ T}$$

6. **Calculate the magnetic moment of a bar of iron having length 0.5 m and cross sectional area 2mm² when placed in a long solenoid of 25 turns/cm carrying current of 2A. Take the relative permeability of the material 400.**

Solution:

Here, $l = 0.5$ m, $A = 2mm^2 = 2 \times 10^{-6}$ m^2, $I = 2A$ and $n = 25$ turns/cm $= \frac{25}{10^{-2}}$ turns/m

$= 2500$ tuns/m

We have,

$$B = \mu In = \mu_o \mu_r In$$

$$= 4\pi \times 10^{-7} \times 400 \times 2 \times 2500 = 2.512 \text{ T}$$

We have, $B = \mu_o (H + M)$, Here, applied field, $H = 0$

So, $B = \mu_o (H+M) = \mu_o M$

$$\Rightarrow M = \frac{B}{\mu_o}$$

$$\therefore M = \frac{2.512}{4\pi \times 10^{-7}} = 2 \times 10^{6}\ Am^{-1}$$

Hence, magnetic moment $= MV = MAl$

$$= 2 \times 10^{6} \times 2 \times 10^{-6} \times 0.5$$

$$= 2\ Am^{2}$$

7. **Calculate the field intensity of a magnetic field and the intensity of magnetization if 0.2A current is passed through a winding of 20 turns/cm over an iron anchor ring having magnetic field 1.26 T.**

Solution:

Here, I = 0.2 A, n = 20 turns/cm = 2000 turns/m, B = 1.26 T

We have, $B = \mu I n \Rightarrow \mu = \frac{B}{I.n} = \frac{1.26}{0.2 \times 2000}$

$\mu = 3.15 \times 10^{-3}$ H/m

Again $B = \mu H \Rightarrow H = \frac{B}{\mu} = \frac{1.26}{(3.15 \times 10^{-3})}$

$H = 400\ Am^{-1}$

Again, $B = \mu_o (H + M) \Rightarrow (M + H) = \frac{B}{\mu_o}$

$M = \frac{B}{\mu_o} - H$

$M = \frac{1.26}{(4\pi \times 10^{-7})} - 400$

$M = 1 \times 10^{6}\ Am^{-1}$

Exercise

1. What is permeability and susceptibility? Differentiate between paramagnetism and diamagnetism.
2. Define relative permeability and susceptibility. Show that $M = H(\mu_r - 1)$, where symbols have their usual meaning.
3. Based on magnetization vector, explain the diamagnetism, ferromagentism and ferrimagnetism.
4. Classify magnetic material based on their magnetic susceptibilities. What is the basic difference between frerromagentic and ferrimagnetic material?
5. Based on magnetization, differentiate between ferromagnetism, ferrimagnetism, anti-ferromagnetism and paramagnetism.

Superconductivity

Superconductor

Kammerlingh Onnes discovered that the resistivity of mercury(Hg) becomes zero below the temperature of 4K. This property of material being of zero resistivity below certain temperature is called *superconductivity.* Such materials are called *superconductor* and this temperature below which the material is in superconductive state above which the material is in normal state is called *critical temperature* (T_c).

In superconductor, current once created persist for several years without diminution because there is no thermal energy loss.

The superconductive state having existence at highest temperature found yet is Hg Ba Ca Cu O at 164K under high pressure.

BCS Theory

The microscopic theory put forward by Bardeen, Cooper and Schrieffer (BCS) provides the better quantum explanation of superconductivity. It accounts very well for all the properties exhibited by the superconductor. This theory is called BCS theory.

According to this theory the superconductivity occur when a strong interaction between two electrons takes place to distort the lattice. This interaction is strongest when the two electrons have equal and opposite moment and spin. Such an interaction occurs by means of phonon exchange. The superconductivity occurs when this attractive interaction over come the usual repulsive Columb interaction. Two such electrons which interact attractively in the phonon field are called *cooper pair.*

The paired electrons (cooper pair) can maintain their coupled motion up to a certain distance called *coherence length.* The coherence length is given by

$$L_g = \frac{\hbar V_F}{B_E}$$

Where, V_F is called Fermi velocity

B_E is binding energy of the pairs.

Properties of Superconductor

Meissner Effect

Messiner in 1935, found that if a superconductor is cooled in a magnetic field down to critical temperature, the lines of induction of magnetic field (B) are pushed out. This effect is called Meissner effect.

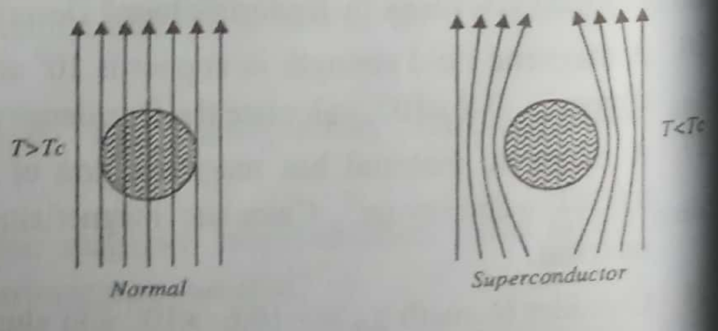

Figure: Meissner effect

This means superconductor shows perfect diamagnetic effect i.e. magnetic field inside the superconductor is zero i.e. $B = 0$.

We have, $B = \mu_o (H + M)$

$$0 = \mu_o (H + M)$$

$$H = -M$$

Therefore, susceptibility, $\chi = \frac{M}{H} = -1$

Which is true for perfect diamagnet.

We thus conclude that the conditions defining superconducting state are.

$E = 0$ (From the absence of resistivity)

$B = 0$ (From Meissner effect)

Critical Magnetic Field

A sufficiently strong magnetic field can destroy superconductivity. The critical value of applied magnetic field which can destroy superconductivity is called *critical magnetic field H_C (T).*

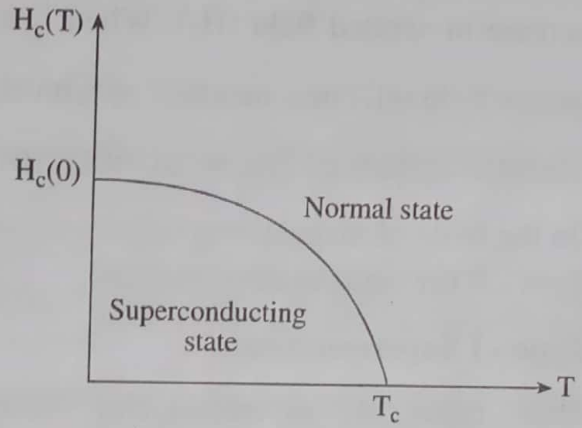

Figure : Variation of critical field with temperature

Figure shows the variation of critical magnetic field with temperature. At critical temperature, the critical field is zero. The nature of the curve is approximately parabolic and satisfies the parabolic relation.

$$H_c (T) = H_c (0) \left(1 - \frac{T^2}{T_c^2}\right).$$

Where H_c (0) is critical field at 0K.

Critical Current

A superconductor cannot support an electric current density greater than a certain value called *critical current density* J_C. The magentic field which destroy the superconductivity is not necessarily an external applied field, it may also be the magnetic field due to the current flowing through superconductor. The critical current is the critical value of current in a superconductive material that can persist through superconductor. For the current greater than critical value material becomes normal and for smaller value than critical current the material becomes super conducting.

If a super conducting wire of radius 'r' carries current 'I' then magnetizing field due to current is,

$$H_I = \frac{I}{2\pi r} \qquad ...(1)$$

In addition to this current, if transverse field H_T is applied the value of critical current decrease. Then the condition for transition to normal is

$$H_C = H_I + 2\,H_T$$

$$H_I = H_C - 2\,H_T$$

$$\frac{I}{2\pi r} = H_C - 2H_T$$

$$I = 2\pi r\,(H_C - 2\,H_T)$$

At critical value, $I = I_C$

Therefore, $I_C = 2\pi r\,(H_c - 2H_T)$ ...(2)

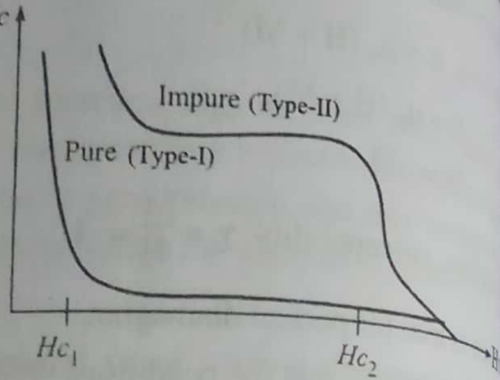

Figure: Variation of critical current of pure and impure materials with transverse applied field

Which is Silsbee's rule. From equation (2) it is seen that critical current (I_c) decreases with increase in applied field (H_T). When $I_c = 0$ from (2), $H_T = \frac{H_c}{2}$. Therefore when the transverse applied field (H_T) becomes half of critical field, the critical current becomes zero.

Classification of Superconductors

On the basis of magnetizing behaviour, superconductors can be classified as type I (or soft) and Type - II (or hard) superconductors.

Type - I Superconductor

This type of superconductor obeys complete Meissner's effect up to critical field. They are completely diamagnetic. The magnetization curve for Type - I materials is as shown in figure. At the critical magnetizing field, the magnetization decreases abruptly and the material becomes normal.

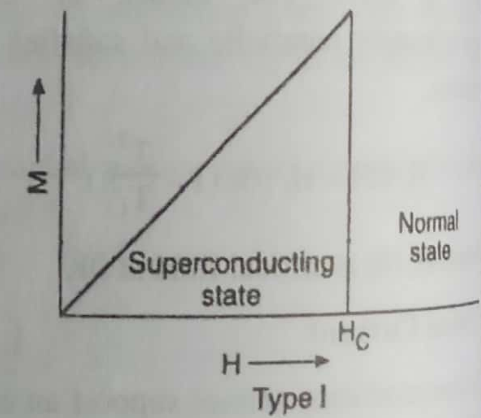

Figure : Magnetization curve of type-I Superconductor

These materials give away their superconductivity at lower field strength so these are referred to as soft superconductors.

Type - II Superconductor

This type of superconductor losses magnetization gradually as shown in figure. For applied field below H_{C1}, the material is diamagnetic and hence the field is completely excluded.

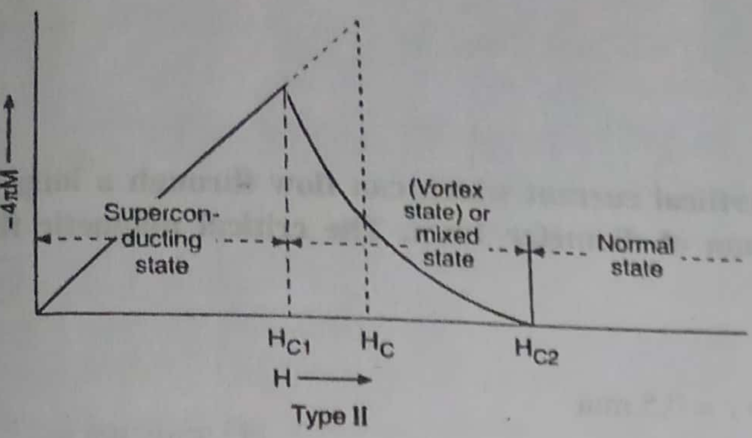

Figure : Magnetization curve for type-II super conductor

Here H_{C1} is called lower critical field. Above H_{C1}, the field starts penetrating into the material until the upper critical field H_{C2} is reached. Between the two critical magnetic fields H_{C2} and H_{C1}, the material is said to be in *mixed or vortex state*. Above the magnetic field H_{C2}, the material becomes normal conductor.

This type of superconductors are obtained by adding some impurities on type- I superconductor so these are also called impure superconductor. Since high value of critical field is required to destroy superconductivity so they are referred as hard superconductor.

Solved Examples

1. **Determine the temperature at which the critical field becomes half of its value at 0K, if the critical temperature of a superconductor at zero magnetic field is Tc.**

Solution:

According to question. $H_c(T) = \frac{H_c(0)}{2}$

We have, $H_c(T) = H_c(0)\left[1 - \frac{T^2}{T_c^2}\right]$

$$\frac{H_c(0)}{2} = H_c(0)\left[1 - \frac{T^2}{T_c^2}\right]$$

$$\frac{1}{2} = \frac{T_c^2 - T^2}{T_c^2}$$

$$T_c^2 = 2T_c^2 - 2T^2$$

$$2T^2 = T_c^2$$

$$T^2 = \frac{T_c^2}{2}$$

$$T = \frac{T_c}{\sqrt{2}}$$

$$T = 0.707\ T_c$$

2. **Determine the critical current which can flow through a long thin superconducting wire of Aluminum of diameter 1mm. The critical magnetic field for Aluminum is $8 \times 10^3\ Am^{-1}$**

Solution:

Here, $d = 1mm \Rightarrow r = 0.5\ mm$

$$H_c = \frac{I}{2\pi r}$$

$$\begin{aligned} I &= H_c .\ 2\pi r \\ &= 8\times10^3 \times 2 \times 3.14 \times 0.5 \times 10^{-3} \\ &= 25.12\ A \end{aligned}$$

3. **Calculate the coharence length of an aluminium if the size of energy gap is 3.4×10^{-4} eV and Fermi velocity is $2.02 \times 10^6\ ms^{-1}$.**

Solution:

Here, $B_E = 3.4 \times 10^{-4}$ eV, $V_F = 2.02 \times 10^6\ ms^{-1}$

We have, coherence length, $L_g = \frac{\hbar V_F}{B_E}$

$$L_g = \frac{1.055 \times 10^{-34} \times 2.02 \times 10^6}{3.4 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

$$= 3.9 \times 10^{-6} m$$

4. **For a superconducting specimen, the critical fields are 1.4×10^5 and $4.2\times10^5\ Am^{-1}$ for 14 K and 13K. Estimate the transistion temperature and critical fields at 0K and 4.2 K.**

Solution:

Here $H_c\ (T) = 1.4 \times 10^5\ Am^{-1}$ for $T = 14K$ and $H_c\ (T) = 4.2 \times 10^5\ Am^{-1}$ for $T = 13$ K

We have, $H_c(T) = H_c\ (0)\left[1 - \frac{T^2}{T_c^2}\right]$

$$H_c(T) = H_c(0)\left(\frac{T_c^2 - T^2}{T_c^2}\right)$$

For first case, $1.4 \times 10^5 = H_c(0)\left[\frac{T_c^2 - 14^2}{T_c^2}\right]$ ...(1)

For second case, $4.2 \times 10^5 = H_c\ (0)\left[\frac{T_c^2 - 13^2}{T_c^2}\right]$ ...(2)

Dividing (2) by (1)

$$3 = \frac{T_c^2 - 169}{T_c^2 - 196}$$

$$3T_c^2 - 588 = T_c^2 - 169$$

$$2T_c^2 = 588 - 169$$

$$T_c = \sqrt{\frac{588-169}{2}}$$

$$T_c = 14.47 \text{ K}$$

2. Using value of T_c in equation (1)

$$1.4 \times 10^5 = H_c(0)\left[\frac{14.47^2 - 14^2}{14.47^2}\right]$$

$$1.4 \times 10^5 = 0.064\ H_c(0)$$

$$H_c(0) = 2.2 \times 10^6 \text{ Am}^{-1}$$

3. Now critical field at T = 4.2 K is given by

$$H_c(T) = H_c(0)\left[1 - \frac{4.2^2}{T_c^2}\right]$$

$$H_c(4.2\text{ K}) = 2.2 \times 10^6\left[1 - \frac{4.2^2}{14.47^2}\right]$$

$$= 2.01 \times 10^6 \text{ Am}^{-1}.$$

Exercise

1. What is Meissner effect? Explain in brief about type - I and type - II super conductor.
2. What is Meissner effect in superconducting state? How Meissner effect helps to differentiate superconductors as Type I and Type II? Explain in brief.
3. What is Meissner effect? Explain how Meissner's effect is complete for type I and incomplete for type II superconductors.
4. What is superconductor? Explain about the electrical and magnetic properties.
5. Define superconductor. Show that the superconducting sate has mutually independent properties, zero resistivity and perfect diamagnetism.
6. Explain how strong magnetic field effects superconductor. Derive the relation of critical current in superconductor with necessary diagram.
7. Define critical magnetic field and critical current in a super conductor with mathematical relation involved.
8. Define critical magnetic field. Describe the characteristic properties of superconductor. Write some applications of superconductor.