**Chapter 4**

# Image Restoration

## Introduction

Restoration improves image in some predefined sense. It is an objective process. Restoration attempts to reconstruct an image that has been degraded by using a priori knowledge of the degradation phenomenon. These techniques are oriented toward modeling the degradation and then applying the inverse process in order to recover the original image. Restoration techniques are based on mathematical or probabilistic models of image processing. Enhancement, on the other hand is based on human subjective preferences regarding what constitutes a "good" enhancement result. Image Restoration refers to a class of methods that aim to remove or reduce the degradations that have occurred while the digital image was being obtained. All natural images when displayed have gone through some sort of degradation:

- During display mode
- Acquisition mode, or
- Processing mode
- ✓ Sensor noise
- ✓ Blur due to camera mis-focus
- ✓ Relative object-camera motion
- ✓ Random atmospheric turbulence

and others.

### 4.1 A model of the Degradation Model/Restoration Process

Degradation process operates on a degradation function that operates on an input image with an additive noise term. Input image is represented by using the notation f(x,y), noise term can be represented as η(x,y).These two terms when combined gives the result as g(x,y). If we are given g(x,y), some knowledge about the degradation function H or J and some knowledge about the additive noise teem η(x,y), the objective of restoration is to obtain an estimate f'(x,y) of the original image. We want the estimate to be as close as possible to the original image. The more we know about h and $\eta$ , the closer f(x,y) will be to f'(x,y). If it is a linear position invariant process, then degraded image is given in the spatial domain by

**$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$**

h(x,y) is spatial representation of degradation function and symbol * represents convolution. In frequency domain we may write this equation as

**$G(u,v)=F(u,v)H(u,v)+N(u,v)$ .**

The terms in the capital letters are the Fourier Transform of the corresponding terms in the spatial domain.

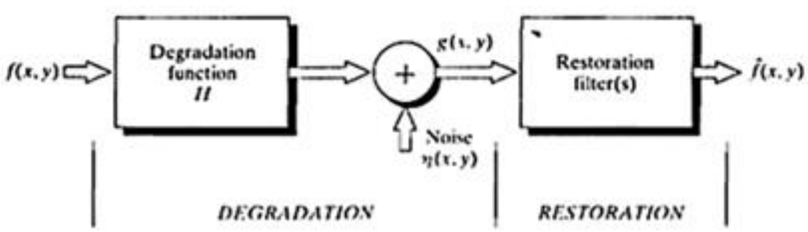



Figure 4.1: A model of the image Degradation / Restoration process

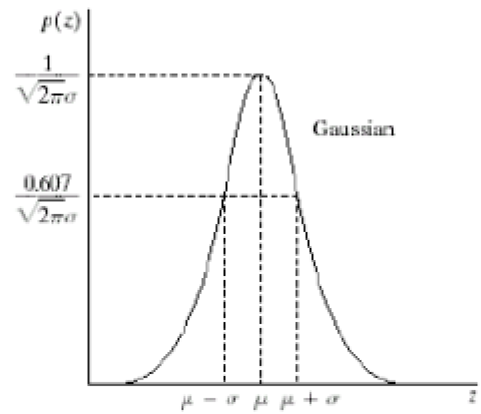
**4.2 Noise Models Restoration in the presence of Noise only- Spatial filtering:**

The principal source of noise in digital images arises during image acquisition and /or transmission. The performance of imaging sensors is affected by a variety of factors, such as environmental conditions during image acquisition and by the quality of the sensing elements themselves. Images are corrupted during transmission principally due to interference in the channels used for transmission. Since main sources of noise presented in digital images are resulted from atmospheric disturbance and image sensor circuitry, following assumptions can be made i.e. the noise model is spatial invariant (independent of spatial location). The noise model is uncorrelated with the object function.

**(a) Gaussian Noise**

Because of its mathematical tractability in both the spatial and frequency domains, Gaussian noise models are used frequently in practice. In fact this tractability is so convenient that it often results in Gaussian models being used in situations in which they are marginally applicable at best**.**

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2}$$ **Gaussian Noise**



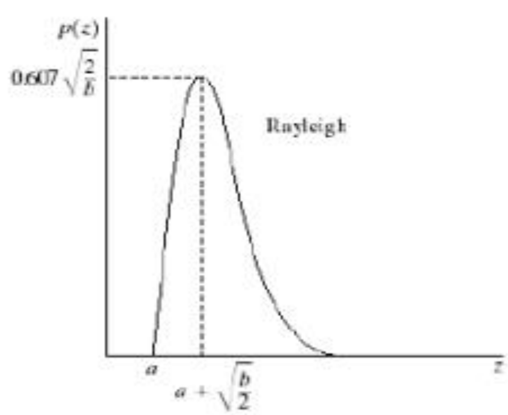

Where, z represents intensity, is the mean (average) value of z , and σ is its standard deviation σ 2 is variance of z.

**(b) Rayleigh Noise**: Unlike Gaussian distribution, the Rayleigh distribution is no symmetric. It is given by the formula.

$$p_z(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

The mean and variance of this density is

$$m = a + \sqrt{\pi b/4}, \sigma^2 = \frac{b(4-\pi)}{4}$$

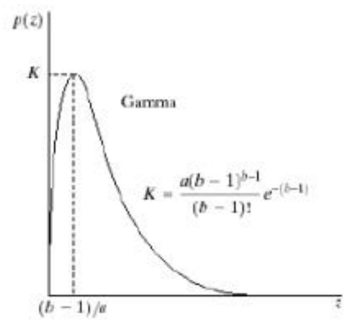


**(c ) Gamma Noise:** The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az}, \text{ for } z \geq 0 \\ 0, \text{ for } z < 0 \end{cases}$$

The mean and variance of this density are given by

$$\text{mean} : \mu = \frac{b}{a} \quad \text{variance} : \sigma^2 = \frac{b}{a^2}$$




Its shape is similar to Rayleigh disruption. This equation is referred to as gamma density it is correct only when the denominator is the gamma function.

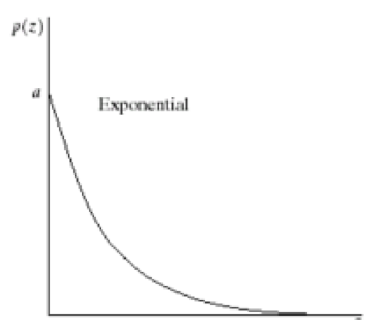
**(d) Exponential Noise**

Exponential distribution has an exponential shape. The PDF of exponential noise is given as

$$p_z(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

Where a>0. The mean and variance of this density are given by

$$m = \frac{1}{a}, \quad \sigma^2 = \frac{1}{a^2}$$



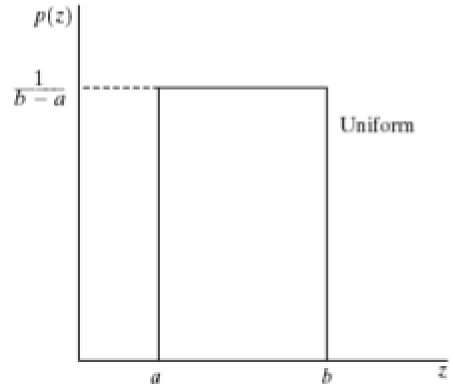
**(e) Uniform Noise:**

The PDF of uniform noise is given by

$$p_z(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this noise is

$$m = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}$$




**(f) Impulse (salt & pepper) Noise:**

In this case, the noise is signal dependent, and is multiplied to the image.

The PDF of bipolar (impulse) noise is given by

$$p_z(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

$$b > a$$

If b>a, gray level b will appear as a light dot in image. Level a will appear like a dark dot.

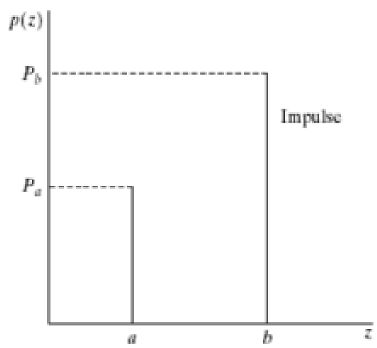



Figure 4.2 Different Noise Models



## Restoration in the presence of Noise only- Spatial filtering:

When the only degradation present in an image is noise, i.e.

**g(x,y)=f(x,y)+η(x,y)**

**or**

**G(u,v)= F(u,v)+ N(u,v)**

The noise terms are unknown so subtracting them from g(x,y) or G(u,v) is not a realistic approach. In the case of periodic noise it is possible to estimate N(u,v) from the spectrum G(u,v). So N(u,v) can be subtracted from G(u,v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present. The following techniques can be used to reduce the noise effect.

(i) **Mean Filter**

**ii) (a) Arithmetic Mean filter:** It is the simplest mean filter. Let Sxy represents the set of coordinates in the sub image of size m*n centered at point (x,y). The arithmetic mean filter computes the average value of the corrupted image g(x,y) in the area defined by Sxy. The value of the restored image f at any point (x,y) is the arithmetic mean computed using the pixels in the region defined by Sxy.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in Sxy} g(s,t)$$

This operation can be using a convolution mask in which all coefficients have value 1/mn. A mean filter smoothes local variations in image Noise is reduced as a result of blurring. For every pixel in the image, the pixel value is replaced by the mean value of its neighboring pixels with a weight .This will resulted in a smoothing effect in the image.

**(b)Geometric Mean filter:**

An image restored using a geometric mean filter is given by the expression.

$$\hat{f}(x,y) = \left( \prod_{(s,t)\in Sxy} g(s,t) \right)^{1/mn}$$

Here, each restored pixel is given by the product of the pixel in the sub image window, raised to the power 1/mn. A geometric mean filters but it to loose image details in the process.

(c)**Harmonic Mean filters:**

The harmonic mean filtering operation is given by the expression:

$$\hat{f}(x,y) = \sum_{(s,t)\in Sxy} g(s,t)^{Q+1} \Big/ \sum_{(s,t)\in Sxy} g(s,t)^{Q}$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It does well with Gaussian noise also.

**(d)Order statistics filter:** Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter. The response of the filter at any point is determined by the ranking result.

**(e)Median filter:** It is the best order statistic filter; it replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x,y) = \underset{(s,t)\in Sxy}{\text{median}}\{g(s,t)\}$$

The original of the pixel is included in the computation of the median of the filter are quite possible because for certain types of random noise, the provide excellent noise reduction capabilities with considerably less blurring then smoothing filters of similar size. These are effective for bipolar and unipolor impulse noise.

**(e)Max and Min filter:** Using the l00th percentile of ranked set of numbers is called the max filter and is given by the equation:

$$\hat{f}(x,y) = \max_{(s,t)\in Sxy}\{g(s,t)\}$$

It is used for finding the brightest point in an image. Pepper noise in the image has very low values, it is reduced by max filter using the max selection process in the sublimated area sky. The 0th percentile filter is min filter.

$$\hat{f}(x,y) = \min_{(s,t)\in Sxy}\{g(s,t)\}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noise of the min operation.

**(f)Midpoint filter:** The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by:

$$\hat{f}(x,y) = \left( \max_{(s,t)\in Sxy} \{g(s,t)\} + \min_{(s,t)\in Sxy} \{g(s,t)\} \right) \Big/ 2$$

It comeliness the order statistics and averaging .This filter works best for randomly distributed noise like Gaussian or uniform noise

**4.3 Periodic Noise Reduction by Frequency Domain Filtering**

Typically arises due to electrical or electromagnetic interference which gives rise to regular noise patterns in an image. Frequency domain techniques in the Fourier domain are most effective at removing periodic noise.

1. **Band Reject Filters**

Removing periodic noise form an image involves removing a particular range of frequencies from that image. *Band reject* filters can be used for this purpose. An ideal band reject filter is given as follows:

**(a) Ideal Band Reject Filter:** An ideal band reject filter is given by the expression:

$$H(u,v) = \begin{cases} 1 & if\ D(u,v) < D_0 - \frac{W}{2} \\ 0 & if\ D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & if\ D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

D(u,v)- the distance from the origin of the centered frequency rectangle. W- the width of the band Do- the radial center of the frequency rectangle.

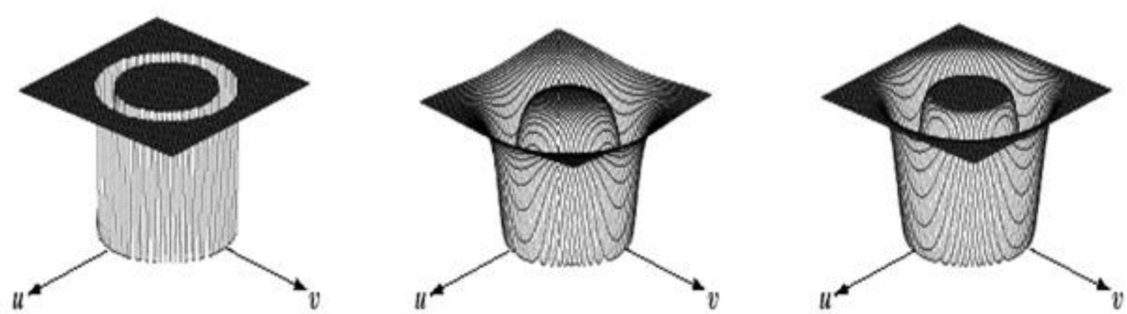
**(b) Butterworth Band Reject Filter:**

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^{\,2}}\right]^{2n}}$$

**(c) Gaussian Band reject Filter:**

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^{\,2}}{D(u,v)W}\right]^2}$$

These filters are mostly used when the location of noise component in the frequency domain is known. Sinusoidal noise can be easily removed by using these kinds of filters because it shows two impulses that are mirror images of each other about the origin of the frequency transform.




a b c

From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

4.3 Different Band Reject Filters

**2. Band Pass Filter:**

The function of a band pass filter is opposite to that of a band reject filter.It allows a specific frequency band of the image to be passed and blocks the rest of frequencies. The transfer function of a band pass filter can be obtained from a corresponding band reject filter with transfer function Hbr(u,v) by using the equation .

$$H_{bp}(u,v)=1-H_{br}(u,v)$$

These filters cannot be applied directly on an image because it may remove too much details of an image but these are effective in isolating the effect of an image of selected frequency bands.

### 3. Notch Filters

A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency .Due to the symmetry of the Fourier transform notch filters must appear in symmetric pairs about the origin. The transfer function of an ideal notch reject filter of radius $D_0$ with centers a ($u_0$ , $v_0$) and by symmetry at ($-u_0$ , $v_0$) is

$$D_1(u,v)=\sqrt{(u-M/2-u_0)^2+(v-N/2-v_0)^2}$$
$$D_2(u,v)=\sqrt{(u-M/2+u_0)^2+(v-N/2+v_0)^2}$$

**Ideal:**

$$H(u,v)=\begin{cases}0 & if\ \ D_1(u,v)\le D_0\ or\ D_2(u,v)\le D_0\\ 1 & \quad otherwise\end{cases}$$

$$D_1(u,v)=\left[\left(u-\frac{M}{2}-u_0\right)^2+\left(v-\frac{N}{2}-v_0\right)^2\right]^{\frac{1}{2}}$$

$$D_1(u,v)=\left[\left(u-\frac{M}{2}+u_0\right)^2+\left(v-\frac{N}{2}+v_0\right)^2\right]^{\frac{1}{2}}$$

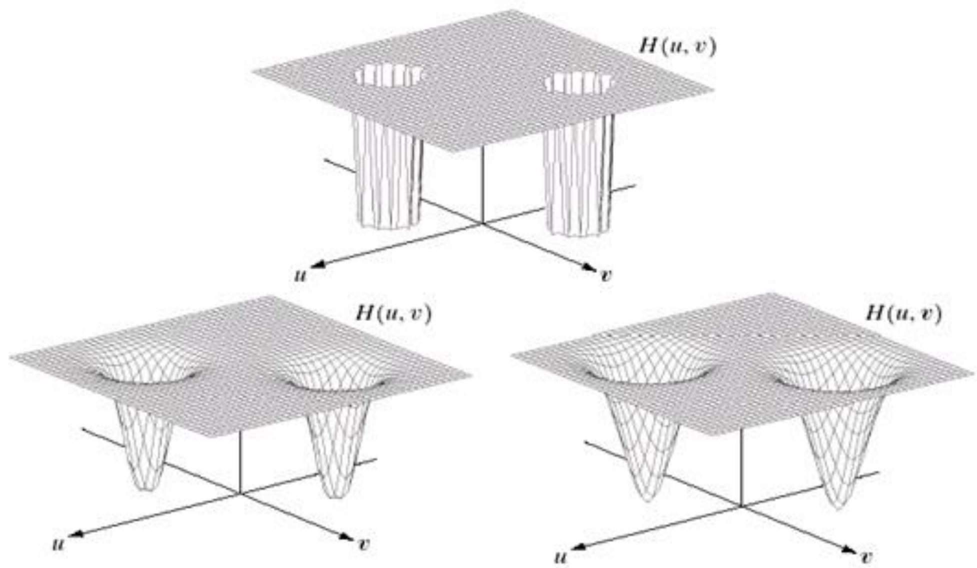
The center of the frequency rectangle has been shifted to the point (M/2,N/2)

**Butterworth:**

$$H(u,v)=\frac{1}{1+\left[\dfrac{{D_0}^2}{D_1(u,v)D_2(u,v)}\right]^n}$$

**Gaussian :**

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0{}^2}\right]}$$




a
b c

Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Figure 4.4 Different Notch Filters