## Chapter -6 Introduction to Morphological Image Processing

## Introduction

**Morphology**: a branch of biology that deals with the form and structure of animals and plants. Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull. Morphological operations rely only on the relative ordering of pixel values, not on their numerical values, and therefore are especially suited to the processing of binary images.

Morphological techniques probe an image with a small shape or template called a **structuring element**. The structuring element is positioned at all possible locations in the image and it is compared with the corresponding neighbourhood of pixels. Some operations test whether the element "fits" within the neighbourhood, while others test whether it "hits" or intersects the neighbourhood:

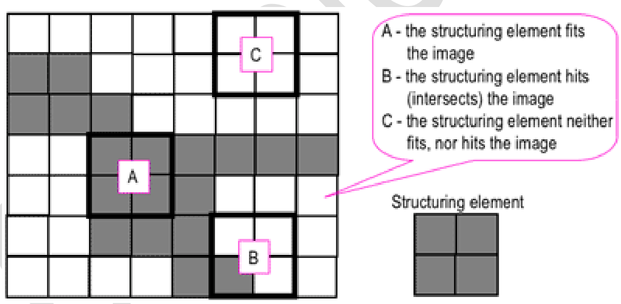



Figure 6.1 Probing of an image with a structuring element
(white and grey pixels have zero and non-zero values, respectively).

A morphological operation on a binary image creates a new binary image in which the pixel has a non-zero value only if the test is successful at that location in the input image.

The **structuring element** is a small binary image, i.e. a small matrix of pixels, each with a value of zero or one:

- The matrix dimensions specify the *size* of the structuring element.
- The pattern of ones and zeros specifies the *shape* of the structuring element.
- An *origin* of the structuring element is usually one of its pixels, although generally the origin can be outside the structuring element.

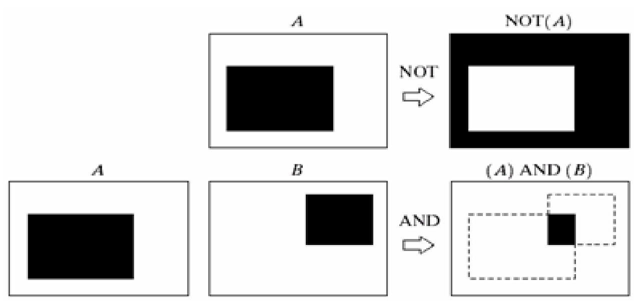
Set in mathematic morphology represent objects in an image. Binary image (0 = white, 1 = black) : the element of the set is the coordinates (x,y) of pixel belong to the object ⇨ $Z^2$.Gray-scaled image : the element of the set is the coordinates (x,y) of pixel belong to the object and the gray levels ⇨ $Z^3$

### 6.1 Logical Operations involving Binary Images

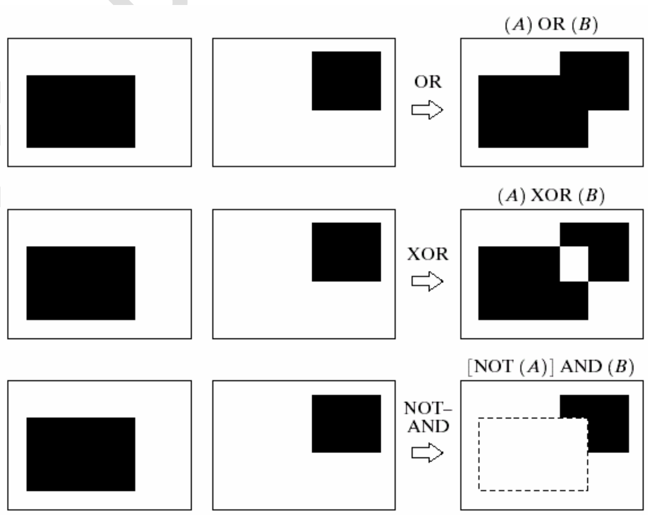
- The principal logic operations used in image processing are: AND, OR, NOT (COMPLEMENT).These operations are *functionally complete*.
- Logic operations are performed on a pixel by pixel basis between corresponding pixels (bitwise).Other important logic operations : XOR (exclusive OR), NAND (NOT-AND) .
- Logic operations are just a private case for a binary set operations, such: AND – Intersection, OR – Union, NOT-Complement.

The three basic logical operations.

| $p$ | $q$ | $p$ AND $q$ (also $p \cdot q$) | $p$ OR $q$ (also $p + q$) | NOT ($p$) (also $\bar{p}$) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 |




Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.



Along with logical operations image processing supports basic set operations .

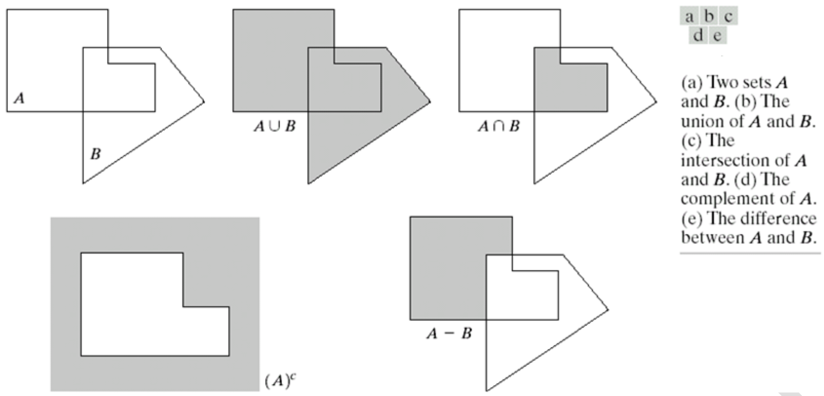



**Figure 6.2 Logical and Set Operationss**

**Reflection and Translation**

**Reflection**

- Reflection is flipping an object across a line without changing its size or shape.
- Reflection of a set B is defined as

$$\hat{B} = \{w \mid w \in -b, \quad for\ b \in B\}$$

**Translation**

- Translation is sliding a figure in any direction without changing its size, shape or orientation.
- A translation always moves an object but it does not turn it, flip it, or change its size
- Translation of a set A by a point z = ( z1 , z2 )is defined as:

$$(A)_z = \{c \mid c \in a + z, \quad for\ a \in A\}$$

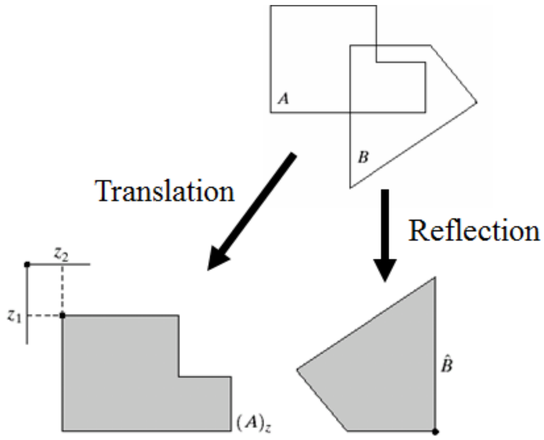



Figure 6.3 Translation and Reflection

### 6.2 Dilation and Erosion

**Dilation** operation makes an object to grow by size. The extent to which it grows depends upon nature and shape of Structuring Elements

- With *A* and *B* as sets in $Z^2$, the dilation of *A* by *B* is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\}$$

Where $\hat{B}$ the reflection of *B* about its origin and shifting this reflection by *z*

- The dilation of *A* by *B* is the set of all displacements, *z*, such that $\hat{B}$ and *A* overlap by at least one element. Thus,

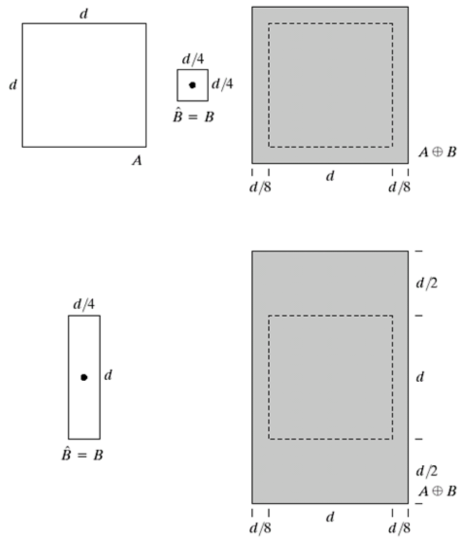
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

- Set B is referred to as the structuring element in dilation.

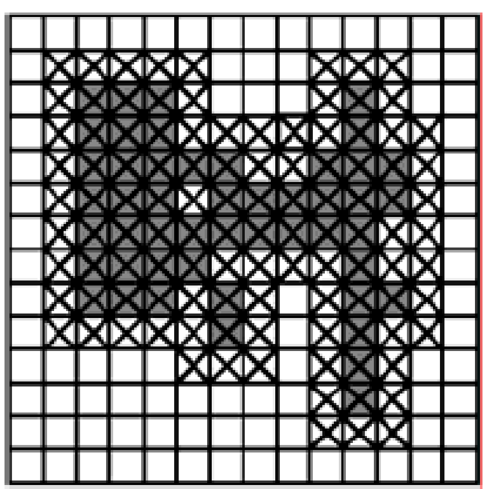
- It is basically used to fill the holes(missing pixels) in a continuous object

a b c
d e

(a) Set $A$.
(b) Square structuring element (dot is the center).
(c) Dilation of $A$ by $B$, shown shaded.
(d) Elongated structuring element.
(e) Dilation of $A$ using this element.




- Does the structuring element **hit the set?**
- dilation of a set A by structuring element B: all z in A such that B hits A when origin of B=z $A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$
- **grow the object**



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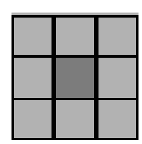
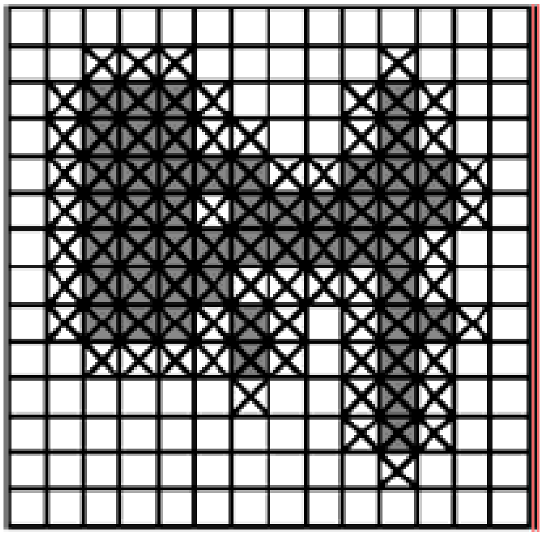
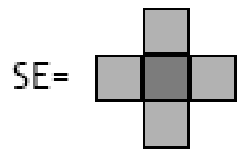
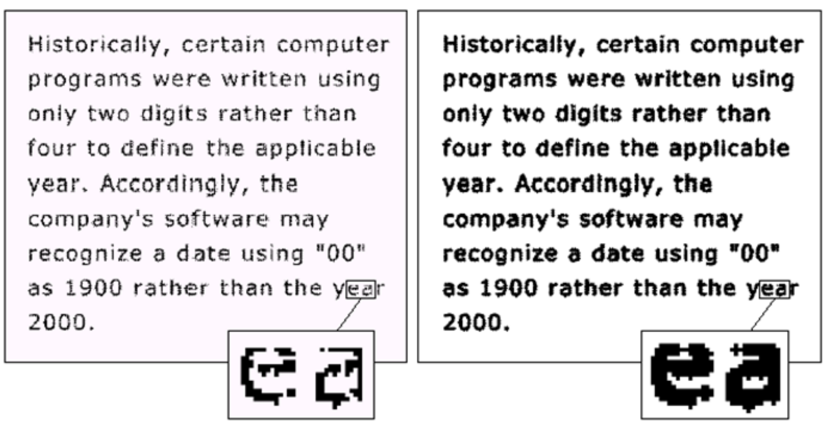







**Dilation: Bridging the Gaps**




a c
b

(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

**Erosion**

- For sets $A$ and $B$ in $Z^2$, the erosion of $A$ by $B$ is defined as

$$A \Theta B = \left\{ z \mid (\hat{B})_z \subseteq A \right\}$$

shrink the object

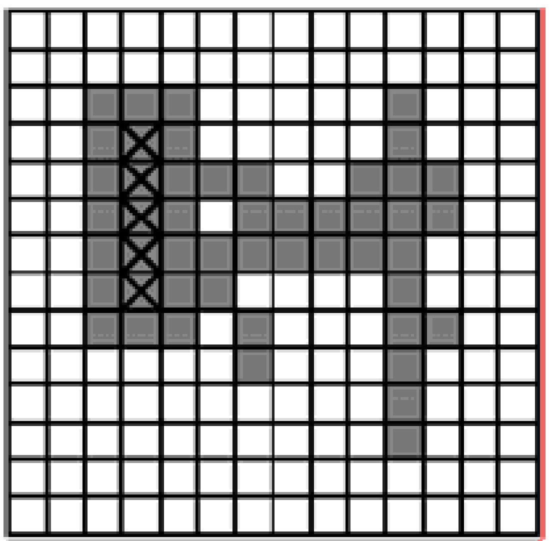
Where, the reflection of $B$ about its origin and shifting this reflection by $z$

- The erosion of *A* by *B* is the set of all points *z*, such that *B*, translated by *z* is contained in *A*.
- $$(A\Theta B)^c = A^c \oplus \hat{B}$$

$$(A\Theta B)^c = \{z \mid (B)_z \subseteq A\}^c$$
$$= \{z \mid (B)_z \subseteq A^c = \phi\}^c$$
$$= \{z \mid (B)_z \subseteq A^c \neq \phi\} \quad = A^c \oplus \hat{B}$$

- Does the structuring element fit the set?

Erosion of a set A by structuring element B: all z in A such that B is in A when origin of B=z



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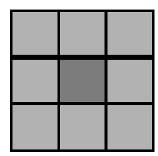
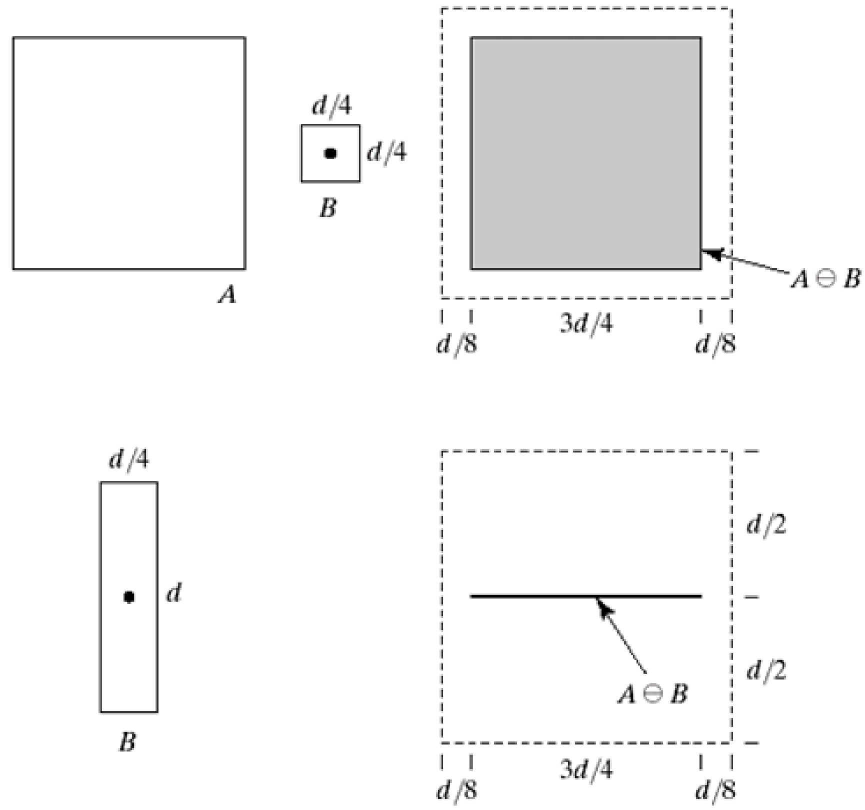
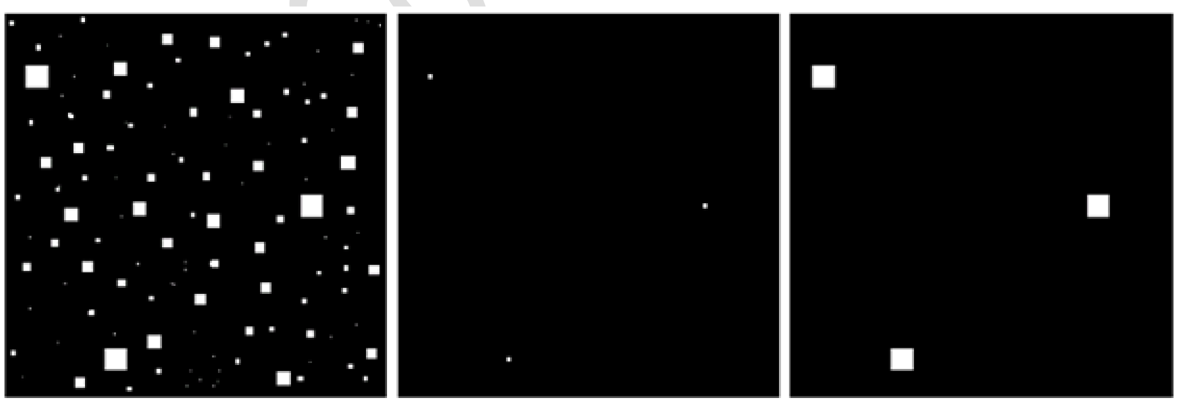




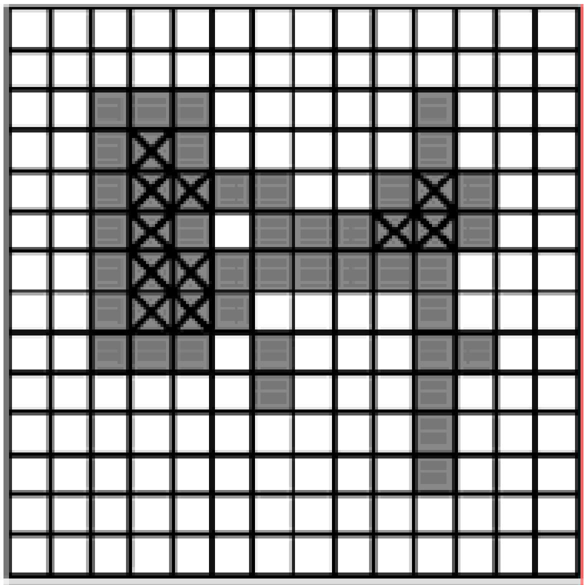

a b c
d e

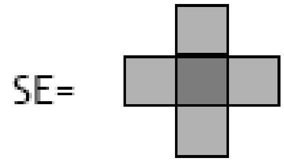
(a) Set A. (b) Square structuring element. (c) Erosion of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Erosion of $A$ using this element.



a b c

(a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.






**Differences between Dilation and Erosin**

| Dilation | Erosion |
|---|---|
| Dilation adds pixels to the boundaries of objects in an image | Erosion removes pixels on object boundaries |
| It increases the size of the objects. | It decreases the size of the objects. |
| It fills the holes and broken areas. | It removes the small anomalies |
| It increases the brightness of the objects | It reduces the brightness of the bright objects. |
| It is used prior in Closing operation. | It is used later in Closing operation. |
| It preserves pixel detail | It eliminats irrelevant detail |

### 6.3 Opening and Closing

**Opening:** Smooth the contour of an object, break narrow isthmuses, and eliminate thin protrusions. The opening *A* by *B* is the erosion of *A* by *B*, followed by a dilation of the result by *B*

$$A \circ B = (A \ominus B) \oplus B$$
$$A \circ B = \cup \{(B)_z \mid (B)_z \subseteq A\}$$

**Closing:** Smooth sections of contours but it generally fuses narrow breaks and long thing gulfs, eliminates small holes, and fills gaps in the contour.

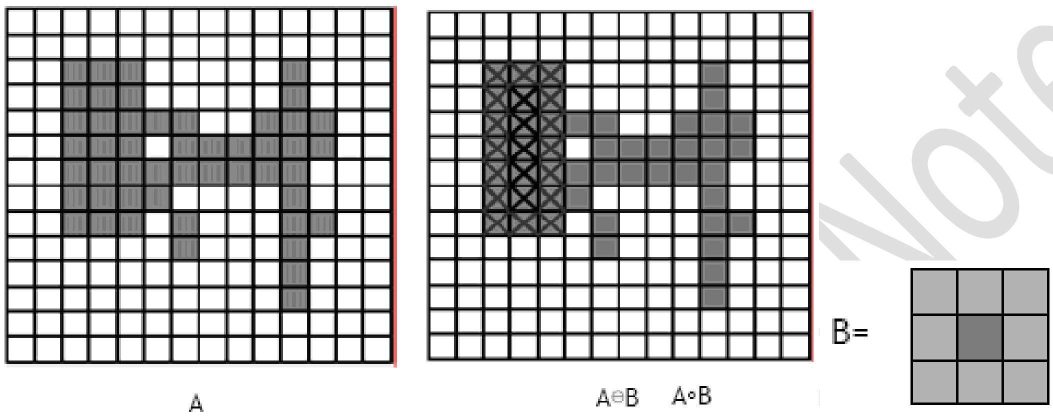
$$A \bullet B = (A \oplus B) \ominus B$$

Erosion followed by dilation, denoted ∘

- eliminates protrusions
- breaks necks
- smoothes contour

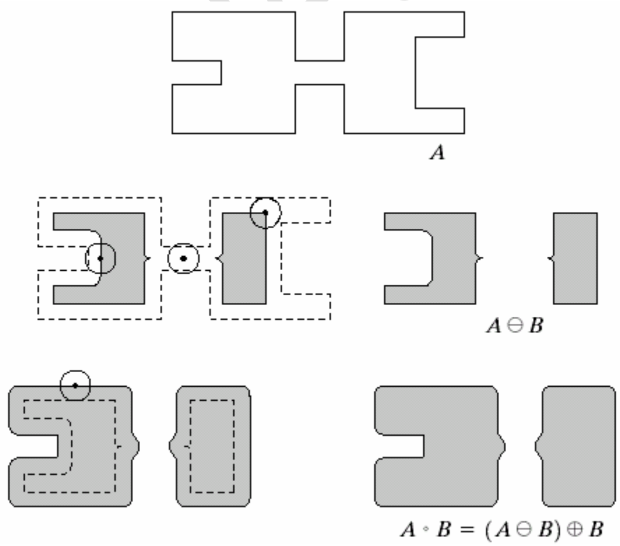
$$A \circ B = (A - B) \oplus B$$

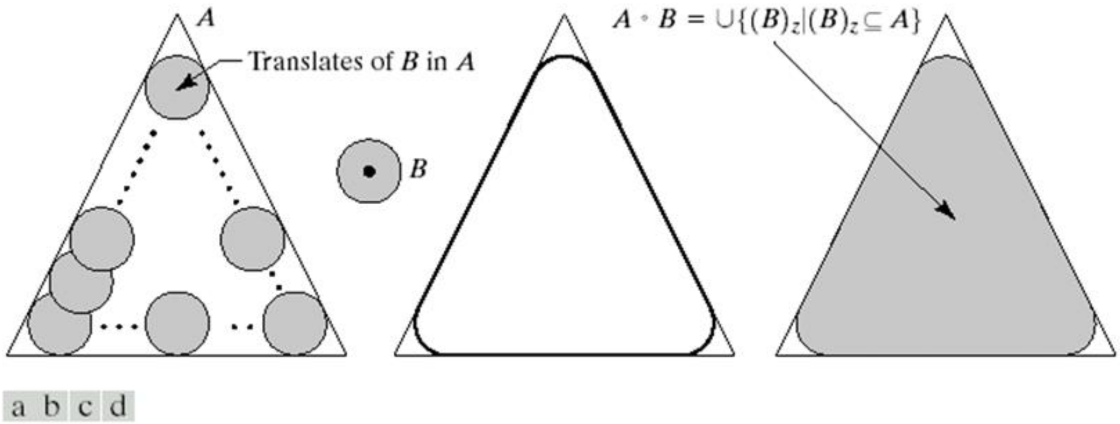
**Opening**




a
b c
d e
f g
h i

Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.





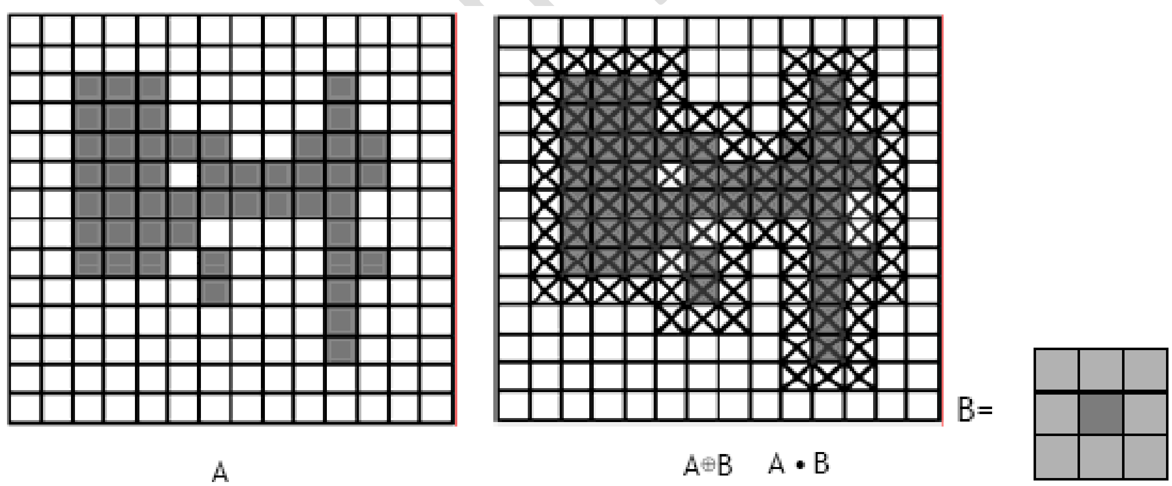

(a) Structuring element $B$ "rolling" along the inner boundary of $A$ (the dot indicates the origin of $B$). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

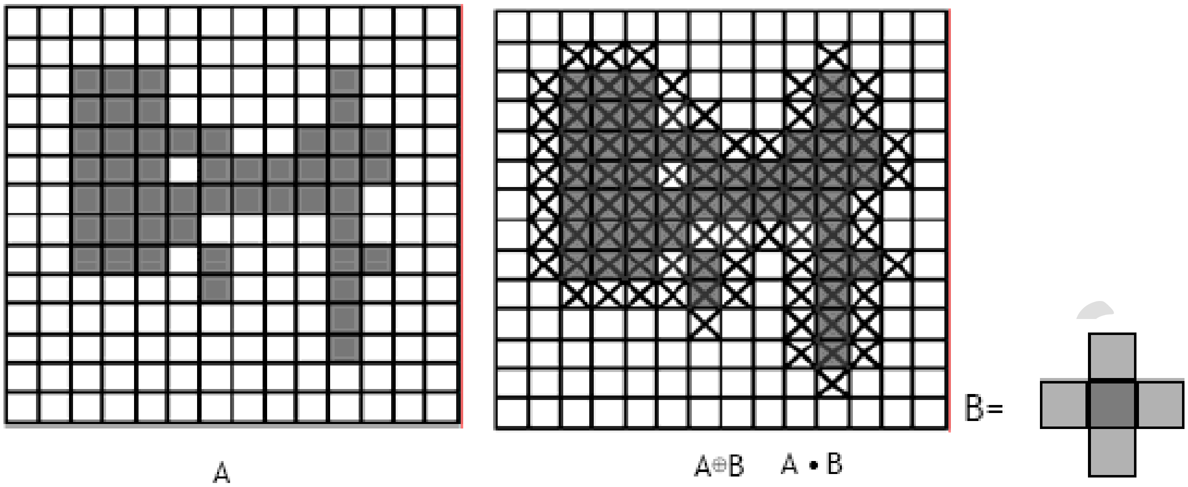
**Closing**

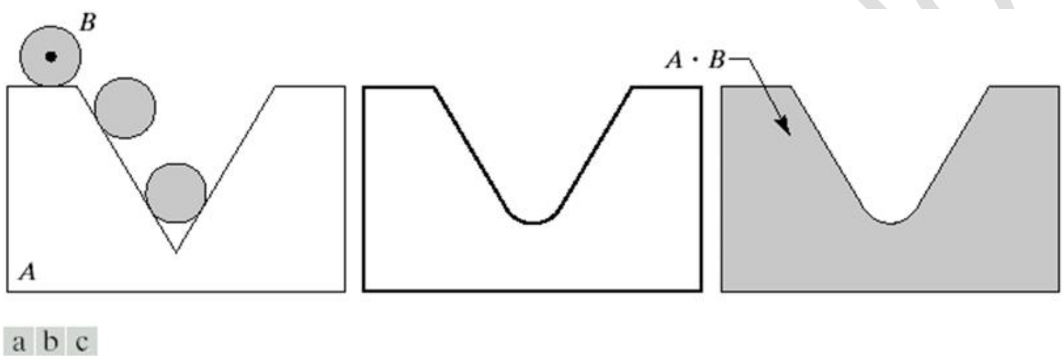
Dilation followed by erosion, denoted •

- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour

$$A \bullet B = (A \oplus B) - B$$





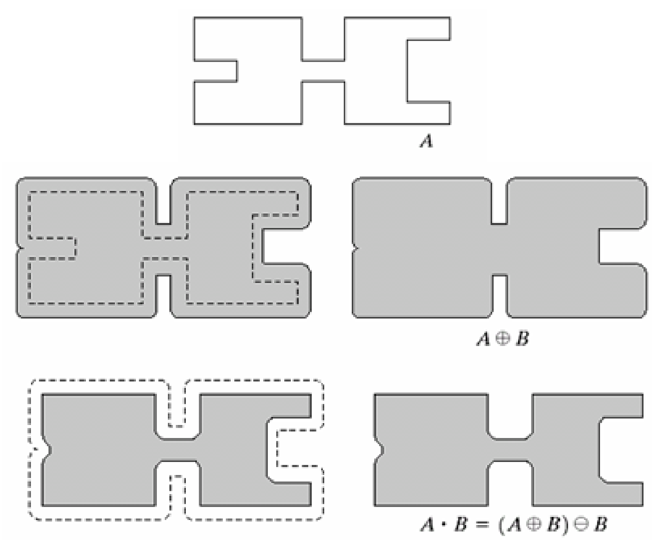




(a) Structuring element $B$ "rolling" on the outer boundary of set $A$. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

**Closing Example**

a
b c
d e
f g
h i

Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



### Opening and Closing Properties

The opening operation satisfies the following properties:

– is a subset (subimage) of $A$.

– If $C$ is a subset of $D$, then $C \circ B$ is a subset of $D \circ B$

$$(A \circ B) \circ B = A \circ B$$

- The closing operation satisfies the following properties:

– $A$ is a subset (subimage) of $A \bullet B$

– If $C$ is a subset of $D$, then $C \bullet B$ is a subset of $D \bullet B$.

$$(A \bullet B) \bullet B = A \bullet B$$



### Useful: Open & Close