

Semiconducting Materials

Semiconductors are materials whose electrical conductivity lies between that of good conductors and insulators. Elemental semiconductors include Antimony, Arsenic, Boron, Carbon, Germunium, Selenium, Sulfer and Tellurium. Silicon is the best known of these forming the basis of most integrated circuits. Common semiconductor compounds include Gallium Arsenide, Indium antimonide, and oxides of most metals. Of these Gallium arsenide (GaAs) is widely used in low noise, weak - signal amplifying devices.

At absolute zero, a semiconductor acts as an insulator. When temperature increases some of the valence electron are able to cross the small forbidden gap and reach to conduction band. Hence conductivity increases as temperature increases. The forbidden band gap does not exist in metal; it is narrow in semiconductor and wide in insulator.

Electron and Hole in Semiconductor

In semiconductor, at absolute zero the valence band is fully occupied and the conduction band is empty. For a valence electron to cross the band gap, the electron must gain energy at least equal to the band gap Eg. The newly entered electron to the conduction band can move freely throughout the solid is termed as free electron. The space left behind in the valence band by electron excited to the conduction band is called hole. So in intrinsic semiconductor both the conduction electron and hole are generated together called *electron hole pair*.

Electron hole pair can be generated not only by incident photons, but also by thermal energy. Thermal energy causes lattice vibration in crystal, which will periodically deform the covalent bond, and in certain region, the atoms may be moving in such a way that the bond is overstretched and rupturing of the over stretched bond takes place. As the bond ruptures, the electron corresponding to that bond becomes free and is excited to the conduction band.

When an external field is applied, the valence electrons move from the end at negative potential to the end at positive potential. The electron jump forward to the succeeding by creating a hole behind. Again the later electron jump succeeding hole by creating a hole behind. In this way there is a movement of electron from one hole to another. It seems as a movement of hole in the direction opposite to that of electron. Thus the current in semiconductor is due to both movement of electron in conduction band called electron current and movement of holes in valence band constitutes a current called hole current. Though the movement of electrons in conduction band and the movement of holes in valence band is in opposite direction, the electron current and hole current have same direction.

When an electron falls from the conduction band, fills the hole in valences band. This process is called *recombination*.

Electrical Conduction in Semiconductor

When an electric field is applied to the semiconductor valance holes drift in the direction of applied field where as conduction electron drift in the direction opposite to the applied field. The drifting of both electrons in conduction band and holes in valence band contribute to current in one direction. Therefore the total current density in semiconductor can be written as:

$$J = n e v_{de} + p e v_{dh}$$

Where, n = electrons concentration in conduction band

p = holes concentration in valence band

v_{de} = drift velocity of electron

v_{dh} = drift velocity of holes.

$$\text{Since, } \mu = \frac{v_d}{E} \Rightarrow v_d = \mu E$$

$$\therefore J = n e \mu_e E + p e \mu_h E$$

$$\sigma E = (n e \mu_e + p e \mu_h) E$$

$$\sigma = n e \mu_e + p e \mu_h$$

Where μ_e and μ_h are electrons and hole mobilities

$$\mu_e = \frac{e \tau_e}{m_e^*}, \mu_h = \frac{e \tau_h}{m_h^*}$$

In case of metal we used the mass of free electron to calculate mobility, but here we have to use effective mass of electron (m_e^*) in crystal to account for the internal forces that hinders its movement. For hole also we will use its effective mass (m_h^*) which is different from effective mass of electron in conduction band.

Electron Concentrations in Intrinsic Semiconductor

In an intrinsic semi conductor every electron transferred to the conduction band leaves behind a hole in the valence band. Therefore, the total number of electrons in the conduction band is equal to the total number of holes in the valence band.

If n is the density of electrons i.e. the number of electrons per unit volume in the conduction band, then in the equilibrium condition.

$$n = \int_{E_c}^{\infty} Z(E) F(E) dE \quad \dots(1)$$

Where, $Z(E) = \frac{4\pi}{h^3} (2m^*)^{3/2} (E)^{1/2} = \frac{4\pi}{h^3} (2m^*)^{3/2} (E - E_c)^{1/2}$... (2) is the density of states in conduction band. As the bottom of the conduction band has an energy E_c and not zero we substitute $(E - E_c)$ for E . The electrons in the conduction band are not completely free but interact with periodic potential of the semiconductor crystal we substitute effective mass m^* instead of m .

The Fermi-Dirac distribution function is given by

$$F(E) = \frac{1}{1 + e^{\frac{(E-E_F)}{KT}}} \approx e^{-\frac{(E-E_F)}{KT}} \text{ for } (E - E_F) \gg KT$$

$$\text{Therefore, } n = \frac{4\pi}{h^3} (2m^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-\frac{(E-E_F)}{KT}} dE$$

$$n = \frac{4\pi}{h^3} (2m^*)^{3/2} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{\frac{(E_F - E_c)}{KT}} \cdot e^{-\frac{(E-E_c)}{KT}} dE$$

$$n = \frac{4\pi}{h^3} (2m^*)^{3/2} e^{-\frac{(E_c - E_F)}{KT}} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-\frac{(E-E_c)}{KT}} dE$$

$$\text{Let, } \frac{E - E_c}{KT} = x \Rightarrow E - E_c = KTx$$

$$dE = (KT) dx$$

$$\text{When, } E = E_c, x = 0$$

$$E = \infty, x = \infty$$

$$\text{And, } (E - E_c)^{1/2} = (KT)^{1/2} x^{1/2}$$

$$\text{Now, } n = \frac{4\pi}{h^3} (2m^*)^{3/2} e^{-\frac{(E_c - E_F)}{KT}} \int_0^{\infty} (KT)^{1/2} x^{1/2} e^{-x} KT dx$$

$$n = \frac{4\pi}{h^3} (2m^* KT)^{3/2} e^{-\frac{(E_c - E_F)}{KT}} \int_0^{\infty} x^{1/2} e^{-x} dx$$

Using integration table

$$n = \frac{4\pi}{h^3} (2m^* KT)^{3/2} e^{-\frac{(E_c - E_F)}{KT}} \frac{\sqrt{\pi}}{2}$$

$$n = 2 \left(\frac{2\pi m^* KT}{h^2} \right)^{3/2} e^{-\frac{(E_c - E_F)}{KT}}$$

$$n = N_C e^{-\frac{(E_c - E_F)}{KT}}$$

... (3)

Where $N_c = 2 \left(\frac{2\pi m^* K T}{h^2} \right)^{3/2}$ is called *effective density of state at the conduction band edge*.

So by multiplying effective density of states at conduction band edge by Boltzmann's statistics we can calculate the electron concentration 'n' at E_c .

Hole Concentration in Intrinsic Semiconductor

A Hole is a state of energy in the valence band unoccupied by an electron. As $F(E)$ gives the probability of occupation for an energy states. The probability that it may be unoccupied by an electron is $[1 - F(E)]$ which is same thing as that it may be occupied by a hole.

Therefore the concentration of holes in valence band is given by

$$p = \int_{-\infty}^{E_V} Z(E) [1 - F(E)] dE \quad \dots(4)$$

$$\text{Here, } [1 - f(E)] = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{K T}\right)} = \frac{\exp\left(\frac{E - E_F}{K T}\right)}{1 + \exp\left(\frac{E - E_F}{K T}\right)}$$

As $E \ll E_F$ being in the valence band, so the term $\exp\left(\frac{E - E_F}{K T}\right)$ in the denominator can be neglected in comparison to 1.

$$\text{Therefore, } [1 - F(E)] = \exp\left(\frac{E - E_F}{K T}\right) \quad \dots(5)$$

From equation (5) we find that the value of $[1 - F(E)]$ i.e. the probability that the finding a hole decreases exponentially as we move from top to bottom of valence band. This implies that holes reside near the top of valence band. The value of $Z(E)$ near the top of valence band is given by,

$$Z(E) = \frac{4\pi}{h^3} (2m^*)^{3/2} (E_V - E)^{1/2}$$

Where m^* is the effective mass of hole.

$$\begin{aligned} \text{Therefore, } p &= \frac{4\pi}{h^3} (2m^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{\frac{(E - E_F)}{K T}} dE \\ &= \frac{4\pi}{h^3} (2m^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{\frac{(E_V - E_F)}{K T}} e^{\frac{-(E_V - E)}{K T}} dE \\ &= \frac{4\pi}{h^3} (2m^*)^{3/2} e^{\frac{-(E_F - E_V)}{K T}} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{\frac{-(E_V - E)}{K T}} dE \end{aligned}$$

$$\frac{E_v - E}{KT} = x \Rightarrow E_v - E = KT x$$

$$\Rightarrow dE = -KT dx$$

When, $E = -\infty, x = \infty$

When, $E = E_v, x = 0$

$$\text{and } (E_v - E)^{1/2} = (KT)^{1/2} x^{1/2}$$

$$\text{Therefore, } p = \frac{4\pi}{h^3} (2m^*)^{3/2} e^{-\frac{(E_F - E_v)}{KT}} \int_{-\infty}^0 (KT)^{1/2} x^{1/2} e^{-x} (-KT) dx$$

$$= \frac{4\pi}{h^3} (2m^* KT)^{3/2} e^{-\frac{(E_F - E_v)}{KT}} \int_0^\infty x^{1/2} e^{-x} dx$$

$$= \frac{4\pi}{h^3} (2m^* KT)^{3/2} e^{-\frac{(E_F - E_v)}{KT}} \cdot \frac{\sqrt{\pi}}{2}$$

$$p = 2 \left(\frac{2\pi m^* KT}{h^2} \right)^{3/2} e^{-\frac{(E_F - E_v)}{KT}}$$

$$p = N_v e^{-\frac{(E_F - E_v)}{KT}} \quad \dots(6)$$

Where $N_v = 2 \left(\frac{2\pi m^* KT}{h^2} \right)^{3/2}$ is called *effective density of states at the valence band edge*.

To show **Fermilevel in intrinsic semiconductor lies between conduction band and valence band.**

We have electron concentration in conduction band

$$n = N_c e^{-\frac{(E_c - E_F)}{KT}}$$

And the hole concentration in valence bond

$$p = N_v e^{-\frac{(E_F - E_v)}{KT}}$$

Multiplying,

$$n.p = N_c N_v e^{-\frac{(E_c - E_v)}{KT}}$$

This equation is commonly termed as "*Mass action law*"

$$n.p = N_c N_v e^{-\frac{E_g}{KT}}$$

Where, $E_g = E_c - E_v$

Therefore, the product of electron and hole concentration is constant for a given temperature.

For intrinsic semiconductors, the electrons and holes are generated simultaneously

$$\text{i.e. } n = p = n_i \text{ (say)}$$

From equation (7)

$$n_i^2 = N_c N_v e^{\frac{-E_g}{KT}}$$

$$n_i = (N_c N_v)^{1/2} e^{\frac{-E_g}{2KT}}$$

Since, $p = n_i$

$$N_v e^{\frac{-(E_F - E_V)}{KT}} = (N_c N_v)^{1/2} e^{\left(\frac{-E_g}{2KT}\right)}$$

For an intrinsic semiconductor, the Fermi energy is denoted by E_{Fi}

$$e^{\frac{-(E_{Fi} - E_V)}{KT}} = \left(\frac{N_c}{N_v}\right)^{1/2} e^{\left(\frac{-E_g}{2KT}\right)}$$

Taking \ln on both sides

$$-\left(\frac{E_{Fi} - E_v}{KT}\right) = \ln\left(\frac{N_c}{N_v}\right)^{1/2} - \frac{E_g}{2KT}$$

$$-E_{Fi} + E_v = \frac{KT}{2} \ln\left(\frac{N_c}{N_v}\right) - \frac{E_g}{2}$$

$$E_{Fi} = E_v + \frac{E_g}{2} - \frac{KT}{2} \ln\left(\frac{m_e^*}{m_h^*}\right)^{3/2}, \text{ Since } N_c = 2\left[\frac{2\pi m_e^* KT}{h^2}\right]^{3/2} \text{ and } N_v = 2\left[\frac{2\pi m_h^* KT}{h^2}\right]^{3/2}$$

$$E_{Fi} = E_v + \frac{E_g}{2} - \frac{3}{4} KT \cdot \ln\left(\frac{m_e^*}{m_h^*}\right)$$

If the elective mass of electron in conduction band and that of hole in valence band are equal, i.e $m_e^* = m_h^*$ (Or at absolute zero temperature i.e. $T = 0K$)

$$E_{Fi} = E_v + \frac{E_g}{2} \dots\dots(8)$$

$$E_{Fi} = E_v + \frac{E_c}{2} - \frac{E_v}{2} = \frac{E_c + E_v}{2} \Rightarrow E_{Fi} = \frac{E_c + E_v}{2} \dots\dots(9)$$

This shows that Fermi level lies in between conduction band and the valence band.

Again,

$$E_{Fi} = \frac{E_c + E_v}{2} = E_c - \frac{E_c}{2} + \frac{E_v}{2} = E_c - \left(\frac{E_c - E_v}{2}\right)$$

$$E_{Fi} = E_c - \frac{E_g}{2} \dots\dots(10)$$

In silicon (Si) the effective mass of hole is slightly greater than that of electron, so the intrinsic Fermi level is slightly above the mid gap.

Extrinsic Semiconductor

The conductivity of semiconductor rises when some elements belonging to either group III or group V in Mendeleev's periodic table are added on it. These elements (materials) are called

impurities. The intrinsic semiconductor doped with these impurities is called extrinsic semiconductor.

There are two types of extrinsic semiconductor.

i. n-type Semiconductor

When a pentavalent (Group V in periodic table) impurity like Arsenic (AS), Phosphorous (P), Antimony (Sb) is added to pure semiconductor a n-type semiconductor is formed. The four electrons out of five valence electron of these impurity make covalent bond with four valence electron of silicon (Si), so one electron remains free on every add of impurity. This extra electron continuously revolves around the impurity ion core as similar to the electron in hydrogen atom as shown in figure.

The energy required to free this extra electron from impurity site can be calculated in analogy to ionize hydrogen atom, which means to free an electron from the ground state of hydrogen.

The binding energy of the electron in hydrogen atom is

$$E_b = \frac{m_e e^4}{8 \epsilon_0^2 h^2} = 13.6 \text{ eV}$$

To free the extra electron from impurity ion core, the energy required is.

$$E_b^{si} = \frac{m_e^* e^4}{8 \epsilon_0^2 \epsilon_r h^2} = \frac{m_e^* e^4}{8 \epsilon_0^2 \epsilon_r h^2} = 0.032 \text{ eV}$$

Where, effective mass of electron $m_e^* = \frac{m_e}{3}$ and $\epsilon_r = 11.9$

This energy is comparable to average thermal energy of atomic vibration at room temperature $\sim \frac{3}{2} kT$ ($\sim 0.035 \text{ eV}$). Thus the fifth valence electron can be readily freed by thermal vibration of Si-lattice.

In this type of semiconductor the concentration of electron increases with every add of impurity. So it is called negative type (n-type) semiconductor. Since the pentavalent atom donate electron for conduction, so it is called donor. In n-type semiconductor electrons are majority charge carriers and holes are minority charge carriers.

If n and p represent extrinsic electron and hole concentration of semiconductor, N_d the donor concentration, then supposing all donor sites are ionized (i.e. all donor atoms contribute one electron to conduction band)

$$n = p + N_d$$

$$\text{but, } n \cdot p = n_i^2 \Rightarrow p = \frac{n_i^2}{n}$$

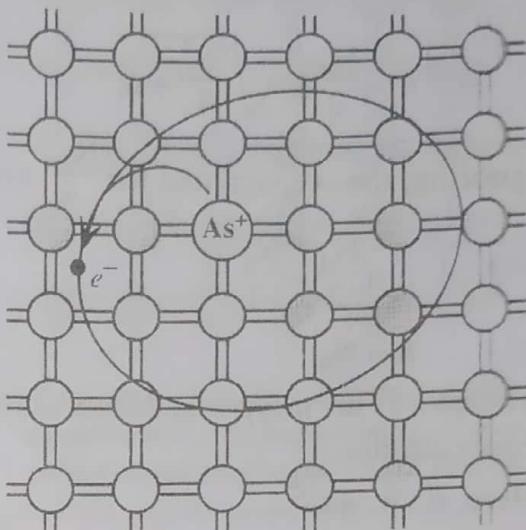


Figure: Arsenic doped Si crystal atom.

$$n = \frac{n_i^2}{n} + N_d = \frac{n_i^2 + n N_d}{n}$$

$$n^2 - n N_d - n_i^2 = 0$$

$$n = \frac{-(-N_d) \pm \sqrt{(-N_d)^2 - 4.1(-n_i^2)}}{2}$$

$$n = \frac{N_d}{2} \pm \sqrt{\frac{N_d^2}{4} + n_i^2}$$

Ignoring, the -ve sign and for $\frac{N_d^2}{4} \gg n_i^2$

$$n \approx \frac{N_d}{2} + \frac{N_d}{2}$$

$$n \approx N_d$$

Therefore in n-type semiconductor the electron concentration is nearly equal to donor concentration.

$$\text{Here, } n \cdot p = n_i^2$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d}$$

Therefore hole concentration in n type semiconductor $p \approx \frac{n_i^2}{N_d}$

The hole concentration in n-type semiconductor is less than the intrinsic hole concentration. This is because some of the electrons in conduction band recombine with holes in valence band so as to maintain $n p \approx n_i^2$. As we go an increasing concentration of donor atoms thereby increasing majority carrier concentration, the minority carrier concentration will be decreased simultaneously. This is called *minority carrier suppression*.

The conductivity of semiconductor is then

$$\begin{aligned}\sigma &= n e \mu_e + p e \mu_h \\ &= N_d e \mu_e + \left(\frac{n_i^2}{N_d} \right) e \mu_h\end{aligned}$$

$$\text{or, } \sigma \approx N_d e \mu_e$$

So the conductivity is mainly due to mobility of electrons in n-type semiconductor.

ii. p-type Semiconductor

When a trivalent (Group - III elements in periodic table) impurity like Boron (B), Aluminum (Al), Gallium (Ga) or Indium (In) is added to pure silicon a p - type semiconductor is formed.

The three valence electron of these impurities can form only three complete covalent bonds with silicon which has four valence electrons. There is deficiency of one electron to form forth bond. This means an electron vacancy is left in the fourth bond. This give rise to a hole.

A nearby electron can tunnel into this hole and displace the hole further away from the Boron atom. As the hole moves away, it gets attracted by the negative charge left behind on the Boron atom and therefore takes into orbit around the B^- ions as shown in figure. The binding energy of this hole to the B^- ion can be calculated using the hydrogenic atom analogy. This binding energy turns out to be small $\sim 0.05\text{eV}$, so at room temperature the thermal vibration of lattice can free the hole away from the B^- site.

In this type of semi conductor the concentration of holes increases with every add of impurity. So it is called positive type (p-type) semiconductor. Since the trivalent atom creates a hole and this hole has tendency to accept an electron, so trivalent impurity is called acceptor.

Let n and p are the electron and hole concentration in the semiconductor and N_a is the acceptor concentration. The doping of trivalent impurities will increase the hole concentration in valence band but does not increase the electron concentration in conduction band (or cannot push the valence electron to conduction band to create a hole in valence band) So,

$$p = N_a + n$$

$$\text{Since, } n \cdot p = n_i^2$$

$$n = \frac{n_i^2}{p}$$

$$\text{Therefore, } p = N_a + \frac{n_i^2}{p} = \frac{p N_a + n_i^2}{p}$$

$$p^2 - p N_a - n_i^2 = 0$$

$$p = \frac{-(-N_a) \pm \sqrt{(-N_a)^2 - 4 \cdot 1 \cdot (-n_i^2)}}{2}$$

$$p = \frac{N_a}{2} \pm \sqrt{\frac{N_a^2}{4} + n_i^2}$$

$$\text{Ignoring the negative sign and for } \frac{N_a^2}{4} \gg n_i^2$$

$$p \approx \frac{N_a}{2} + \frac{N_a}{2} = N_a$$

Therefore in p-type semiconductor hole concentration is nearly equal to acceptor concentration.
Here, $n \cdot p = n_i^2$

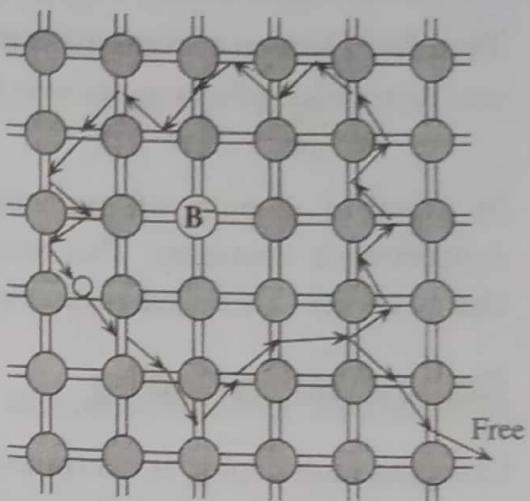


Figure: Boron-doped Si crystal. The hole orbits around the B^- site. Thermally vibrating Si atoms provide enough energy to free the hole from the B^- site into VB.

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a}$$

Therefore electron concentration in p-type semiconductor is

$$n \approx \frac{n_i^2}{N_a}$$

In trivalent doping of semiconductor, the hole concentration increases and electron concentration decreases. This means the minority charge carriers are suppressed by majority charge carrier. This phenomenon is called *minority carrier suppression*.

Compensation Doping

Compensation doping is a term used to describe the doping of semiconductor with both donors and acceptors to control the properties. The effect of donors compensates for the effect of acceptors and vice versa.

For example, a p-type semiconductor doped with N_a acceptor can be converted to an n-type semiconductor by simply adding donor until the concentration N_d exceed N_a . The holes due to acceptor doping will recombine with the electrons due to donor doping.

So the electron concentration is, $n = N_d - N_a$

From mass action law, $n p = n_i^2$

$$\text{Therefore, Hole concentration } p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a}$$

Similarly, a n-type semiconductor doped with N_d donor can be converted to a p-type semiconductor by simply adding acceptor until the concentration N_a exceed N_d . The electrons due to donor doping will recombine with the holes due to acceptor doping.

So the hole concentration is, $p = N_a - N_d$

From mass action law, $n \cdot p = n_i^2$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a - N_d}$$

Where n_i is the intrinsic concentration in semiconductor.

Temperature Dependence of Conductivity

To study the temperature dependence of conductivity we have to consider two factors. The temperature dependence of the carrier concentration and the drift mobility.

1. Temperature Dependence of Carrier Concentration

The temperature dependence of electron concentration has three regions.

a. Low - Temperature Range ($T < T_s$)

As the temperature is increased starting from very low temperature. Some of donors become ionized and donate the electrons to CB as shown in figure (1). The si-si bond breaking i.e.

thermal excitation from VB to CB is difficult because it takes too much energy. Since the donor ionization energy $\Delta E = E_c - E_d$ is very small (i.e. $\Delta E \ll E_g$), thermal generation involves exciting electrons from E_d to E_c .

The electron concentration at low temperature is given by an expression as similar to intrinsic case

$$n = (N_c N_v)^{1/2} e^{\frac{-E_g}{2KT}} \quad \dots(1)$$

In our case it is given by,

$$n = \left(\frac{1}{2} N_c N_d\right)^{1/2} e^{\frac{-\Delta E}{2KT}} \quad \dots(2)$$

The numerical factor $\frac{1}{2}$ in equation (2) arises because donor occupation statistics is different by this factor from the usual Fermi-Dirac function.

The donor ionization continues until we reach a temperature called saturation temperature (T_s) where all donor atoms have been ionized. This temperature range up to T_s is called *ionization range*.

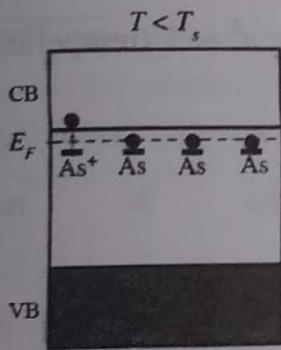


Figure (1)

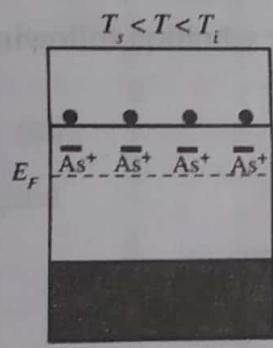


Figure (2)

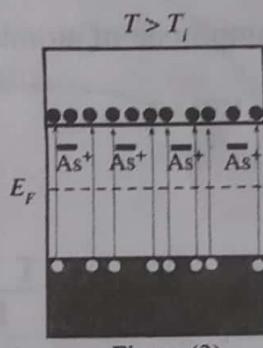


Figure (3)

b. Medium - Temperature Range ($T_s < T < T_i$)

Nearly all donors have been ionized in this range, $n_i = N_d$. This condition remain unchanged until $T = T_i$ as shown in figure (2). This is the temperature when $n_i = N_d$. It is that temperature range $T_s < T < T_i$ that utilizes the n - type doping in pn junction device applications. This temperature range is often referred to as the *extrinsic range*.

c. High Temperature Range ($T > T_i$)

At very high temperature, thermal vibrations of the atoms will be so strong that many si-si bonds will be broken. The electron concentration in the CB will then be mainly due to thermal excitation from VB to CB as shown in figure (3). So n_i is now much larger than N_d . But this process also generates an equal concentration of holes in the VB. Accordingly the semiconductor behaves as if it were intrinsic. The electron concentration at this temperature will therefore be equal to intrinsic concentration n_i as

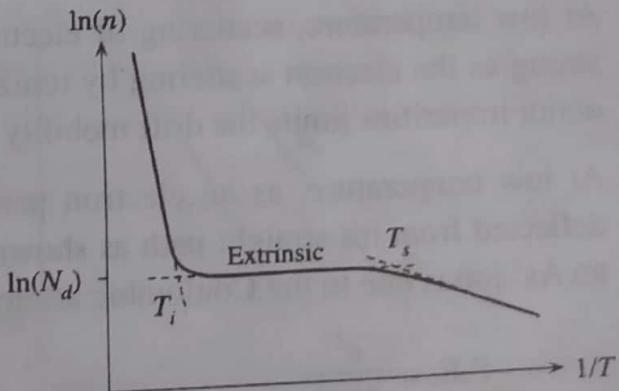


Figure (4): The temperature dependence of the electron concentration in an n-type Semiconductor.

given by equation (1). This temperature range is often referred to as *intrinsic range*.

Figure (4) shows the change in electron concentration with temperature in n-type semiconductor.

2. Drift mobility: Temperature and Impurity dependence.

For High Temperature Range

The electron drift mobility is given by.

$$\mu = \frac{e\tau}{m^*_e}, \tau = \text{mean free time between scattering.}$$

Where, $\tau = \frac{1}{A v_{th} N}$

A = Cross sectional area of scatter.

v_{th} = Mean speed of electron called thermal velocity

N = Number of scatters per unit volume.

Here, A = πa^2 , a = amplitude of atomic vibration following $a^2 \propto T$ (temperature)

Since $\frac{1}{2} m^*_e v_{th}^2 = \frac{3}{2} KT$

$$v_{th} \propto T^{1/2}$$

$$\tau = \frac{1}{\pi a^2 v_{th} N} \propto \frac{1}{T T^{1/2} N} \propto \frac{T^{-3/2}}{N}$$

Therefore, $\mu_L \propto \frac{T^{-3/2}}{N_s}$... (1)

Here μ_L is called "*lattice vibration scattering mobility*". Therefore at high temperature, the drift mobility is limited by scattering from lattice vibrations. As the magnitude of atomic vibration increases with temperature, the drift mobility decreases in the fashion $\mu \propto T^{-3/2}$.

For Low Temperature Range

At low temperature, scattering of electrons by thermal vibrations of the lattice will not be so strong as the electron scattering by ionized donor impurities. The electron scattering by ionized donor impurities limits the drift mobility at low temperature.

At low temperature, as an electron passes by an ionized donor As^+ , it is attracted and thus deflected from its straight path as shown in figure (5). The PE of electron at a distance 'r' from an As^+ ion is due to the Coulombic attraction, and its magnitude is given by.

$$P.E. = \frac{e^2}{4\pi\epsilon r}$$

When the electron is just scattered

$$P.E. = K.E.$$

$$\frac{e^2}{4\pi\epsilon r} = \frac{3}{2} K.T.$$

$$r = \frac{e^2}{6\pi\epsilon K T}$$

The scattering cross section.

$$S = \pi r^2 = \frac{\pi e^4}{(6\pi\epsilon K T)^2} \propto T^{-2}$$

$$\text{Also from, } \frac{1}{2} m v_{th}^2 = \frac{3}{2} K T$$

$$v_{th} \propto T^{1/2}$$

Therefore mean scattering time,

$$\tau = \frac{1}{A V_{th} N} \propto \frac{1}{T^{-2} T^{1/2} \cdot N_I}$$

$\tau \propto \frac{T^{3/2}}{N_I}$, Here N_I is the concentration of ionized impurities.

$$\text{Therefore, } \mu_I \propto \frac{T^{3/2}}{N_I}$$

Here, μ_I is called "ionized impurity scattering limited mobility".

Note also that as temperature increases N increases and μ_I decreases.

The over all temperature dependence of the drift mobility is then simply the reciprocal addition of μ_I and μ_L by virtue of Matthiessen's rule.

$$\text{i.e. } \frac{1}{\mu_e} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

The conductivity of semiconductor as given by $\sigma = n\mu_e + p\mu_h$ depends on carrier concentration and drift mobility. For n-type semiconductor (extrinsic range), it will depend on donor concentration and electron drift mobility where as in p-type semiconductor it will depend on acceptor concentration and hole drift mobility. In n-type semiconductor, at lowest temperatures in the ionization range the

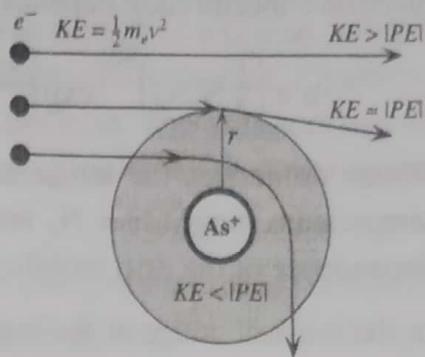


Figure (5): Scattering of electrons by an ionized impurity

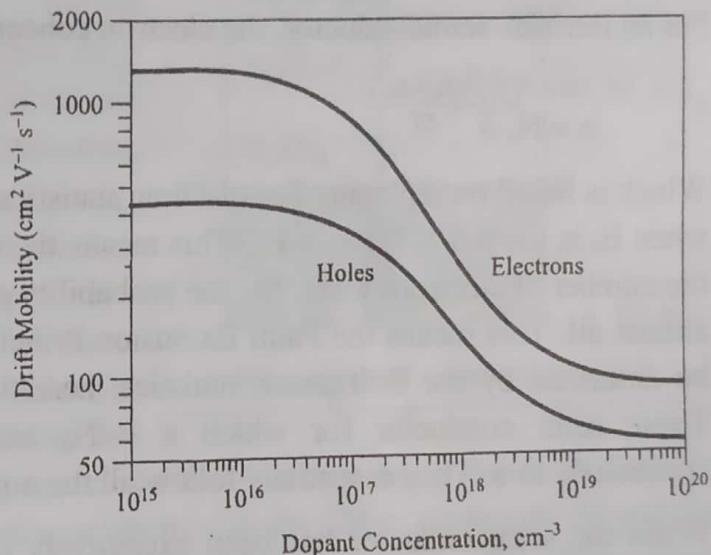


Figure (6): The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K.

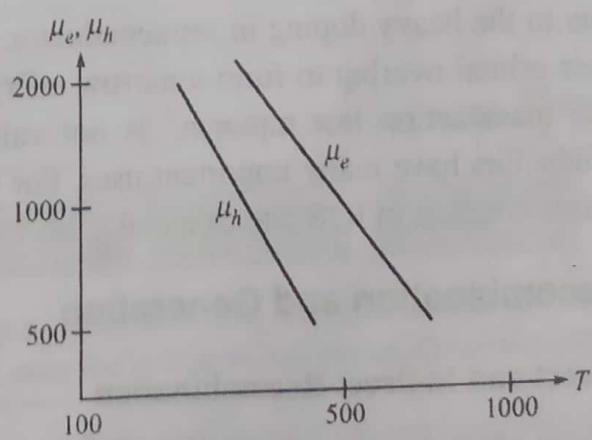


Figure (7): Temperature dependence of mobilities in extrinsic semiconductor.

electron concentration depends exponentially on the temperature.

$$n = \left(\frac{1}{2} N_c N_d\right)^{1/2} \exp\left[\frac{-(E_c - E_d)}{2KT}\right]$$

Which dominates the temperature dependence of conductivity. In the extrinsic range (higher temperature range) $n = N_d$ and is constant, so that the conductivity follows the temperature dependence of the drift mobility.

In the intrinsic range at the highest temperatures, the conductivity is dominated by temperature dependence of n_i . Since, $\sigma = n_i e (\mu_e + \mu_h)$ and n_i is exponential function of temperature in contrast to $\mu \propto T^{-1/2}$.

Degenerate and Non-degenerate Semiconductor

For an intrinsic semiconductor, the electron concentration is given as.

$$n = N_c e^{\frac{-(E_c - E_F)}{KT}} \quad \dots(1)$$

Which is based on replacing Fermi-Dirac statistics with Boltzmann statistics which is only valid when E_c is several KT above E_F . This means there are very large number of energy states than the number of electrons there. So, the probability of occupation of same state by two electrons is almost nil. This means the Pauli Exclusion Principle has no role and the electron statistics can be described by the Boltzmann statistics. here the equation (1) is valid only when $n \ll N_c$. These semi conductor for which $n \ll N_c$ and $p \ll N_v$ are termed as *non degenerate semiconductors*. They essentially follow all the normal semiconductor properties.

When the semiconductor has been excessively doped with donors then 'n' may be so large, typically $10^{19} - 10^{20} \text{ cm}^{-3}$, that it may be comparable to or greater than N_c . In that case the Pauli Exclusion Principle becomes important and we have to use the Fermi-Dirac statistics. Equation (1) for n is no longer valid. Such a semiconductor exhibits metal like properties than semiconductor like. For example resistivity follows $\rho \propto T$. Such semiconductors that have $n > N_c$ or $p > N_v$ are called *degenerate semiconductors*.

Due to the heavy doping in semiconductor, the donor atoms become so close to each other that their orbital overlap to form a narrow energy band and become a part of the conduction band. The mass action law $n.p = n_i^2$ is not valid for degenerate semiconductor. Degenerate semiconductors have many important uses. For example they are used in laser diodes, zener diodes, ohmic contacts in IC'S and as metal gates in many microelectronics MOS devices.

Recombination and Generation

Direct and Indirect Recombination

Above absolute zero of temperature, the thermal excitation of electrons from the VB to the CB continuously generates free electron hole pairs. It should be apparent that in equilibrium there

should be some annihilation mechanism that returns the electron from the CB down to an empty state (a hole) in the VB. When a free electron, wandering around in the CB of a crystal, meets a hole, it falls into this as shown in figure (1). This process is called recombination. Figure (1) shows a *direct recombination* mechanism for example it occurs in GaAs. The excess energy of an electron is lost as a photon of energy $hf = E_g$. In fact, it is this type of recombination that results in the emitted light from light emitting diodes (LEDs).

The recombination process between an electron and a hole, like every other process in nature, must obey the momentum conservation law. The wave function of an electron in the CB, $\psi_{cb}(k_{cb})$, will have a certain momentum $\hbar k_{cb}$ associated with the wave vector k_{cb} and, similarly, the electron wave function $\psi_{vb}(k_{vb})$ in the VB will have a momentum $\hbar k_{vb}$ associated with the wave vector k_{vb} . Conservation of linear momentum during recombination requires that when the electrons drop from the CB to the VB, its wave vector should remain the same, $k_{vb} = k_{cb}$. For the elemental semiconductors, Si and Ge, the electronic states $\psi_{vb}(k_{vb})$ with $k_{vb} = k_{cb}$ are right in the middle of the VB and are therefore fully occupied. Consequently there are no empty states in the VB that can satisfy $k_{vb} = k_{cb}$, and so direct recombination in Si and Ge is next to impossible. For some compound semiconductors, such as GaAs and InSb, for example, the states with $k_{vb} = k_{cb}$ are at the top of the valence band, so they are essentially empty (contain holes). Consequently an electron in the CB of GaAs can drop down to an empty electronic state at the top of the VB and maintain $k_{vb} = k_{cb}$. Thus direct recombination is highly probable in GaAs and this is the reason that makes GaAs and LED material.

In elemental semiconductor crystal, for example in Si and Ge, electrons and holes usually recombine through recombination centers. A recombination center increases the probability of recombination because it can 'take up' any momentum difference between a hole and electron. This process essentially involves a third body, which may be an impurity atom or a crystal defect. The electron is captured by the recombination center and thus becomes localized at this site. It is held at the center until some hole arrives and recombines with it as shown in figure (2). This type of recombination is called *indirect recombination*.

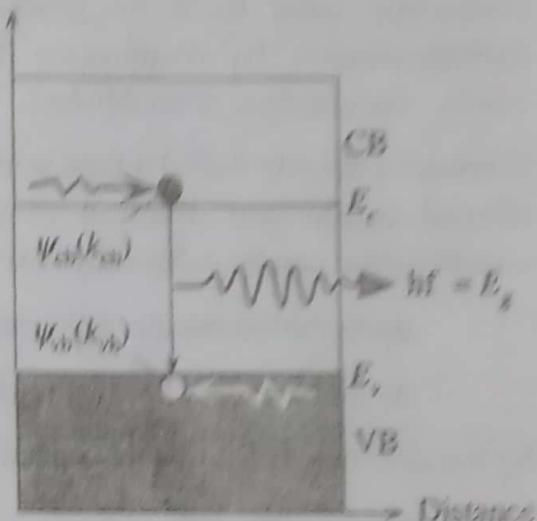


Figure 1: Direct recombination in GaAs

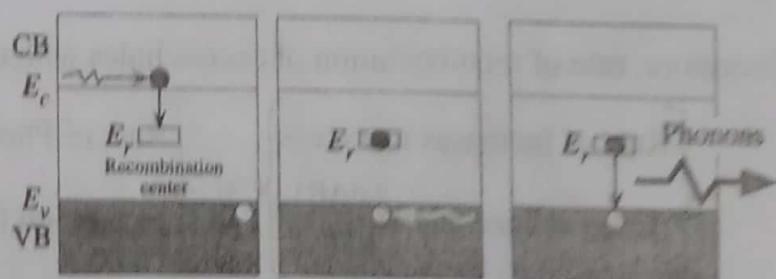


Figure 2: Recombination in Si via a recombination center that has a localized energy level at E_r in the band gap, usually near the middle.

In this recombination process, the energy of the electron is lost as lattice vibrations called phonons. A phonon is a quantum of energy associated with atomic vibrations in the crystal analogous to the photon in electro magnetic radiations. Typical recombination centers, other than the donor and acceptor impurities, might be metallic impurities and crystal defects such as dislocations, vacancies etc.

Photogeneration

When a photon of energy higher than band gap value is incident on semiconductor, it is absorbed by semiconductor. The energy given to the electron by the photon excites it into the conduction band there by generating electron hole pairs. This phenomenon is called photogeneration. By illuminating the surface of semiconductor, a non equilibrium state of carrier concentration is established.

Consider a heavily doped n-type semiconductor, n_0 and p_0 be electrons and hole concentration at thermal equilibrium. Because of photogeneration the departure from thermal equilibrium concentration can be defined by "excess concentration" as follows.

Δn is the excess electron (majority carrier) concentration.

Δp is the excess hole (minority carrier) concentration

So the total electron and hole concentrations are.

$$n = n_0 + \Delta n$$

$$p = p_0 + \Delta p$$

Photo excitation creates an equal number of electrons and holes, so $\Delta n = \Delta p$

When the light is switched off, excess holes recombine with electrons available and disappear. This however takes a time because the electrons and holes have to find each other. The minority carrier life time (or mean recombination time) τ_h is defined as the average time during which a hole is free before recombine with electron. The average probability per unit time that a hole will recombine with an electron is given by $\frac{1}{\tau_h}$.

Therefore, rate of recombination of excess holes with electrons = $\frac{\Delta p}{\tau_h}$

$$\left\{ \begin{array}{l} \text{Rate of increases in excess} \\ \text{Hole concentration } \left(\frac{d\Delta P}{dt} \right) \end{array} \right\} = \left\{ \begin{array}{l} \text{Rate of Photo} \\ \text{generation } (G_{ph}) \end{array} \right\} - \left\{ \begin{array}{l} \text{Rate of recombination} \\ \text{of excess holes } \frac{\Delta p}{\tau_h} \end{array} \right\}$$

$$\frac{d\Delta p}{dt} = G_{ph} - \frac{\Delta p}{\tau_h} \quad \dots(1)$$

$$\text{or, } \frac{d\Delta p}{dt} = \frac{G_{ph} \tau_h - \Delta p}{\tau_h}$$

$$\frac{d\Delta p}{G_{ph}\tau_h - \Delta p} = \frac{dt}{\tau_h}$$

Integrating, $\int \frac{d(\nabla p)}{(G_{ph}\tau_h - \nabla p)} = \frac{1}{\tau_h} \int dt$

$$\int \frac{-d(\nabla p)}{(G_{ph}\tau_h - \nabla p)} = \frac{-1}{\tau_h} \int dt$$

$$\ln(G_{ph}\tau_h - \Delta p) = -\frac{t}{\tau_h} + C$$

at $t = 0, \Delta p = 0$, so that, $C = \ln(G_{ph}\tau_h)$

Hence,

$$\ln(G_{ph}\tau_h - \Delta p) = \frac{-t}{\tau_h} + \ln(G_{ph}\tau_h)$$

$$\ln\left(1 - \frac{\Delta p}{G_{ph}\tau_h}\right) = \frac{-t}{\tau_h}$$

$$1 - \frac{\Delta p}{G_{ph}\tau_h} = \exp\left(-\frac{t}{\tau_h}\right)$$

$$\frac{\Delta p}{G_{ph}\tau_h} = 1 - \exp\left(-\frac{t}{\tau_h}\right)$$

$$\Delta p = G_{ph}\tau_h \left(1 - e^{-\frac{t}{\tau_h}}\right) \quad \dots(2)$$

For large time ($t \rightarrow \infty$) $\Delta p = G_{ph}\tau_h$. From equation (2) it is seen that minority carrier concentration (Δp) rises exponentially towards steady state value $\Delta p(\infty) = G_{ph}\tau_h$.

When the light is switched off (i.e. at $t = t_{off}$), the photogeneration will be stopped, so, $G_{ph} = 0$

From (1) $\frac{d(\Delta p)}{dt} = \frac{-\Delta p}{\tau_h}$

$$\frac{d(\Delta p)}{\Delta p} = -\frac{dt}{\tau_h}$$

Integrating, $\int \frac{d(\Delta p)}{\Delta p} = -\frac{1}{\tau_h} \int dt$

$$\ln(\Delta p) = -\frac{t}{\tau_h} + C$$

In this case, when the light source is switched off at $t = 0$, the minority carrier concentration start decaying from its steady state value $\Delta p = G_{ph}\tau_h$.

So for this case, at $t = 0, \Delta p = G_{ph}\tau_h$

Therefore, $\ln(G_{ph}\tau_h) = C$

$$\text{Hence, } \ln(\Delta p) = -\frac{t}{\tau_h} + \ln(G_{ph} \tau_h)$$

$$\ln\left(\frac{\Delta p}{G_{ph} \tau_h}\right) = \frac{-t}{\tau_h}$$

$$\Delta p = G_{ph} \tau_h e^{-\frac{t}{\tau_h}}$$

....(3)

Hence, from equation (3) it is seen that the minority carrier concentration decays exponentially because of recombination between holes and electrons from the instant the light is switched off. The rate of recombination is obtained by differentiating equation (3) with respect to time.

$$\text{i.e. } \frac{d(\Delta p)}{dt} = -\frac{1}{\tau_h} \cdot G_{ph} \tau_h e^{-\frac{t}{\tau_h}}$$

$$= -G_{ph} e^{-\frac{t}{\tau_h}}$$

$$\frac{d(\Delta p)}{dt} = -\frac{\Delta p}{\tau_h} \quad [\text{Using equation (3)}]$$

Some applications require a short τ_h , as in fast switching of p-n junctions. Where as other require a long τ_h for example, persistent luminescence.

Continuity Equation for Carriers.

1. Continuity Equation for Holes

Consider small portion of a volume of n-type semiconductor. The hole current density $J(x + \Delta x)$ leaving volume can be larger or smaller than the current density $J(x)$ entering depending upon the generation and recombination in the sample.

The net increase in the hole concentration per unit time

$\left(\frac{\partial p}{\partial t}\right)$ is the difference between the hole flux per unit

volume entering and leaving minus the recombination

rate. The hole current density can be converted to hole particle flux density by dividing J by 'e'.

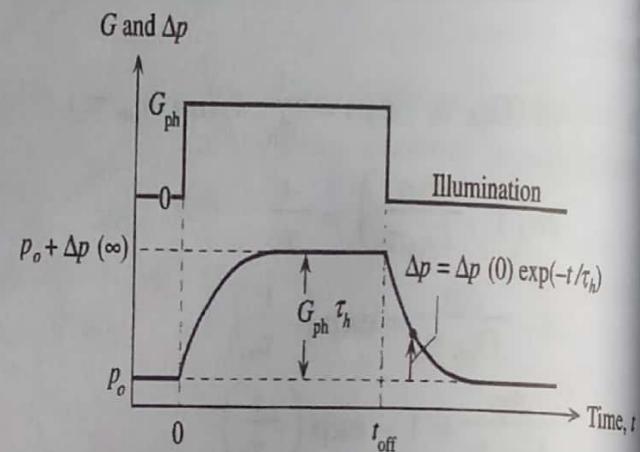
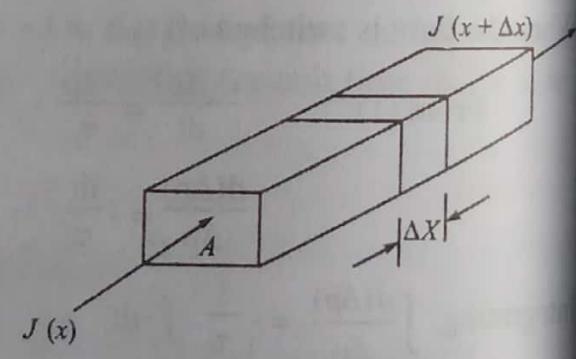


Figure: Hole concentration profile when illumination is switched on at $t = 0$ and switched off at $t = t_{off}$. Excess minority carrier concentration rises and decays exponentially.



$$\left\{ \begin{array}{l} \frac{\partial p}{\partial t} \\ \text{Rate of} \\ \text{hole build up} \end{array} \right\} = \left\{ \begin{array}{l} \frac{1}{e} \left[\frac{J(x) - J(x + \Delta x)}{\Delta x} \right] \\ \text{Increase of hole concentration} \\ \text{in volume } A\Delta x \text{ per unit time} \end{array} \right\} - \left\{ \begin{array}{l} \frac{\Delta p}{\tau} \\ \text{Recombination} \\ \text{rate} \end{array} \right\} \dots(1)$$

Using Taylor series expansion

$$J(x + \Delta x) = J(x) + \frac{\partial J(x)}{\partial x} \Delta x$$

$$\left[\text{Note: } \frac{J}{ex} = \frac{ven}{ex} = \frac{vn}{x} = \frac{xn}{tx} = \frac{n}{t} \right]$$

Again $\Delta p = p - p_0$

$$\frac{\partial(\Delta p)}{\partial t} = \frac{\partial p}{\partial t}$$

Therefore equation (1) can be written as,

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{1}{e} \frac{\partial J}{\partial x} - \frac{\Delta p}{\tau} \quad \dots(2)$$

Equation (2) is called *continuity equation for holes*. The total current density is due to the diffusion and drifting.

$$\text{i.e. } J = J_{\text{drift}} + J_{\text{diffusion}} = pe \mu E - e D \frac{\partial p}{\partial x}$$

Now substituting J in equation (2)

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau} - \frac{1}{e} \frac{\partial}{\partial x} (pe \mu E - e D \frac{\partial p}{\partial x})$$

For uniformly doped semiconductor \bar{E} is small, so neglecting the term containing \bar{E} .

$$\frac{\partial(\Delta p)}{\partial t} = -\frac{\Delta p}{\tau_h} + D \frac{\partial^2 p}{\partial x^2} \quad \dots(3)$$

Which is called *Fick's second law (or diffusion equation for holes)*.

Under steady state condition $\frac{\partial(\Delta p)}{\partial t} = 0$

$$\text{From (3)} \quad D \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau} = 0$$

$$\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{D\tau} = 0$$

$$\frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{L^2} = 0 \quad \dots(4)$$

Where $L = \sqrt{D\tau}$ is called diffusion length for the holes in n-type semiconductor. It is defined as the length; a hole diffuses in average before it recombines with an electron in n-type semiconductor. 'L' is also called *minority carrier diffusion length*.

The solution of equation (4) is

$$\Delta p = A e^{\frac{x}{L}} + B e^{-\frac{x}{L}} \quad \dots(5)$$

Where 'A' and 'B' are constant those have to be found from the boundary conditions.

For an infinite bar, at $x = \infty$, $\Delta p(\infty) = 0$

From (5), $0 = A + 0 \Rightarrow A = 0$

Now, equation (5) becomes,

$$\Delta p(x) = B e^{\frac{-x}{L}} \quad \dots(6)$$

At $x = 0$, $\Delta p(x) = \Delta p(0)$

From (6) $\Delta p(0) = B$

Thus the excess hole concentration at position x is

$$\Delta p(x) = \Delta p(0) e^{\frac{-x}{L}} \quad \dots(7)$$

2. Continuity Equation for Electrons

Consider small portion of a volume of p-type semiconductor. The electron current density $J(x + \Delta x)$ leaving the portion can be larger or smaller than the current density $J(x)$ entering, depends upon the generation and recombination in the sample. The net increase in electron concentration $\left(\frac{\partial n}{\partial t}\right)$ is the difference between the electron concentration entering and leaving minus the recombination rate.

$$\frac{\partial n}{\partial t} = \frac{1}{(-e)} \left[\frac{J(x) - J(x + \Delta x)}{\Delta x} \right] - \frac{\Delta n}{\tau} \quad \dots(1)$$

Using Taylor series expansion. $J(x + \Delta x) = J(x) + \frac{\partial J(x)}{\partial x} \Delta x$

$$\text{Again, } \Delta n = n - n_0 \Rightarrow \frac{\partial(\Delta n)}{\partial t} = \frac{\partial n}{\partial t}$$

Therefore, equation (1) can be written as,

$$\begin{aligned} \frac{\partial(\Delta n)}{\partial t} &= \frac{1}{(-e)} \left[\frac{J(x) - J(x) - \frac{\partial J(x)}{\partial x} \Delta x}{\Delta x} \right] - \frac{\Delta n}{\tau} \\ \frac{\partial(\Delta n)}{\partial t} &= \frac{1}{e} \frac{\partial J(x)}{\partial x} - \frac{\Delta n}{\tau} \end{aligned} \quad \dots(2)$$

Equation (2) is called *continuity equation for electrons*.

The total current density is due to both diffusion and drifting of electrons.

$$\text{i.e. } J = J_{\text{drift}} + J_{\text{diffusion}} = ne\mu E + e D \frac{\partial n}{\partial x}$$

Now substituting 'J' in equation (2)

$$\frac{\partial(\Delta n)}{\partial t} = \frac{1}{e} \frac{\partial}{\partial x} \left[ne\mu E + eD \frac{\partial n}{\partial x} \right] - \frac{\Delta n}{\tau}$$

For uniformly doped semiconductor \vec{E} is small, so neglecting the term containing \vec{E} .

$$\frac{\partial(\Delta n)}{\partial t} = -\frac{\Delta n}{\tau} + D \frac{\partial^2 n}{\partial x^2} \quad \dots(3)$$

Which is the *diffusion equation for electrons*.

Under steady state condition $\frac{\partial(\Delta n)}{\partial t} = 0$

$$\text{From (3)} D \frac{\partial^2(\Delta n)}{\partial x^2} = \frac{\Delta n}{\tau} \Rightarrow \frac{\partial^2(\Delta n)}{\partial x^2} = \frac{\Delta n}{D\tau}$$

$$\frac{\partial^2(\Delta n)}{\partial x^2} - \frac{\Delta n}{D\tau} = 0$$

$$\frac{\partial^2(\Delta n)}{\partial x^2} - \frac{\Delta n}{L^2} = 0 \quad \dots(4)$$

Where $L = \sqrt{D\tau}$ is called *diffusion length for the electrons*. The solution of equation (4) is

$$\Delta n = A e^{\frac{x}{L}} + B e^{-\frac{x}{L}} \quad \dots(5)$$

Here, L is defined as the length; an electron diffuses in average before it recombines with holes in p-type semiconductor. It is also called *minority carrier diffusion length*.

Ideal pn Junction

1. No Applied Bias: Open Circuit

When a piece of p-type material is brought in contact with n-type material then there occurs abrupt discontinuity at the interface between the two regions. The *abrupt pn junction* is called *metallurgical junction*.

The transition region can be considered to be essentially devoid of (out of) mobile charge carriers, but it is important to remember that there are some mobile carriers in this region. This approximation is more accurate toward the middle of the transition region than near the boundaries. It is common to refer to the transition region as the *depletion region*, since it is depleted (greatly reduced) of mobile charge. When the mobile charge is removed from this region, the immobile ionized impurity atoms are left behind. There are negatively charged

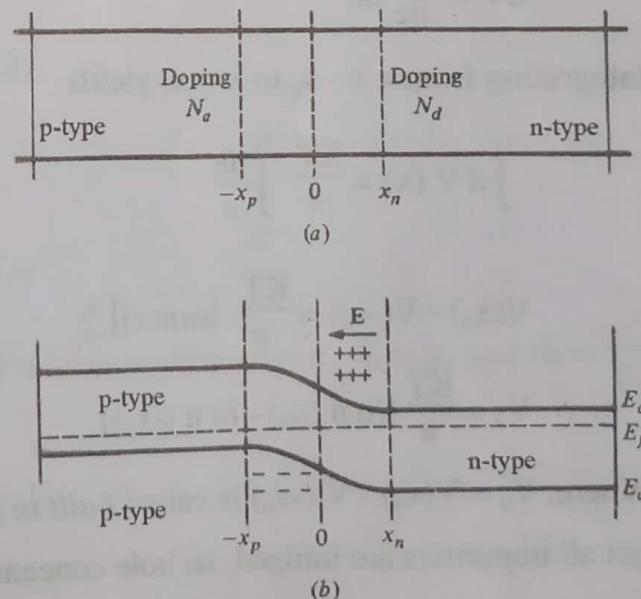


Figure : (a) Geometry of an abrupt pn junction, (b) Band diagram for an abrupt pn junction

acceptors in the p-region from $-x_p$ to 0 and positively charged donors in the n-region from 0 to x_n . For this region, the transition region is also called the *space charge region*. The immobile charges exposed in the transition region result in an electric field directed from the n region to the p region. This internal field tries to drift holes back to the p-region and electrons back to the n-region. In other words the internal field tries to drive the carriers in the direction opposite to their diffusion. The strength of the field will be determined by the equilibrium condition at which the number of carriers diffusing is compensated by the number of carriers drifting in opposite direction.

At equilibrium, the total conduction electron current is zero.

$$\text{i.e. } J = 0 = ne\mu E + e D \frac{dn}{dx}$$

$$-ne\mu E = eD \frac{dn}{dx}$$

$$-E = \frac{D}{n\mu} \frac{dn}{dx}$$

$$\frac{dV}{dx} = \frac{D}{n\mu} \frac{dn}{dx} \quad [\text{Since, } E = -\frac{dV}{dx}]$$

From Einstein's relation, $\frac{D}{\mu} = \frac{KT}{e}$

$$\text{Therefore, } \frac{dV}{dx} = \frac{KT}{ne} \frac{dn}{dx}$$

$$dV = \frac{KT}{ne} dn$$

Integrating from $x = -x_p$ to $x = x_n$ yields

$$\int_{-x_p}^{x_n} dV(x) = \frac{KT}{e} \int_{-x_p}^{x_n} \frac{dn}{n}$$

$$V(x_n) - V(-x_p) = \frac{KT}{e} [\ln n(x)]_{-x_p}^{x_n}$$

$$V_o = \frac{KT}{e} [\ln n(x_n) - \ln n(-x_p)]$$

Where, $V_o = V(x_n) - V(-x_p)$ is called *built in potential or barrier potential*.

Let all impurities are ionized, so hole concentration in p-region is, $p(-x_p) = N_a$

And the electron concentration in n-region is $n(x_n) = N_d$

The electron concentration in p-region $n(-x_p)$ can be calculated as,

$$n(-x_p) \cdot p(-x_p) = n_i^2$$

$$n(-x_p) \cdot N_a = n_i^2$$

$$n(-x_p) = \frac{n_i^2}{N_a}$$

Therefore,

$$V_o = \frac{KT}{e} [\ln N_d - \ln \frac{n_i^2}{N_a}]$$

$$V_o = \frac{KT}{e} \ln \left(\frac{N_d \cdot N_a}{n_i^2} \right) \quad \dots(1)$$

This is the voltage necessary to exactly balance the diffusion current under equilibrium condition.

At $x = -x_p$, hole concentration in p-type material, $p_p = N_a$

$$\text{Therefore electron concentration in p-type material, } n_p = \frac{n_i^2}{N_a}$$

At $x = x_n$, electron concentration in n - region, $n_n = N_d$

$$\text{Therefore, hole concentration in n-region, } p_n = \frac{n_i^2}{N_d}$$

Using the values of N_a and N_d in terms of electron concentration

i.e. $N_a = \frac{n_i^2}{n_p}$ and $N_d = n_n$ in equation (1)

$$V_o = \frac{KT}{e} \ln \left(\frac{n_n}{n_i^2} \cdot \frac{n_i^2}{n_p} \right)$$

$$V_o = \frac{KT}{e} \ln \left(\frac{n_n}{n_p} \right) \quad \dots(2)$$

$$\Rightarrow \ln \left(\frac{n_n}{n_p} \right) = \frac{eV_o}{KT}$$

$$\frac{n_n}{n_p} = \exp \left(\frac{eV_o}{KT} \right) \quad \dots(3)$$

Similarly, using the values of N_a and N_d in terms of hole concentration i.e. $N_a = p_p$ and $N_d = \frac{n_i^2}{p_n}$ in equation (1)

$$V_o = \frac{KT}{e} \ln \left(\frac{n_i^2}{p_n} \cdot \frac{p_p}{n_i^2} \right)$$

$$V_o = \frac{KT}{e} \ln \left(\frac{p_p}{p_n} \right) \quad \dots(4)$$

$$\Rightarrow \ln \left(\frac{p_p}{p_n} \right) = \frac{eV_o}{KT}$$

$$\frac{p_p}{p_n} = \exp\left(\frac{eV_0}{KT}\right) \quad \dots(5)$$

The Width of the Transition Region:

The width of transition region will depend upon the doping concentration on each side of the junction.

The electric field and charge density are related by Poisson's equation as.

$$\begin{aligned} \frac{dE}{dx} &= \frac{\rho}{\epsilon} \Rightarrow \frac{d}{dx} \left(\frac{-dV}{dx} \right) = \frac{\rho}{\epsilon} \\ -\frac{d^2V}{dx^2} &= \frac{\rho}{\epsilon} \end{aligned} \quad \dots(6)$$

The charge density in the space charge region on P-side ($-x_p$ to 0) is given by

$$\rho = -eN_a \quad (N_a \text{ is the number of acceptors per unit volume})$$

Using ' ρ ' in equation (6)

$$\frac{d^2V}{dx^2} = \frac{eN_a}{\epsilon}$$

$$\text{Integrating, } \frac{dV}{dx} = \frac{eN_a x}{\epsilon} + A \quad (A \text{ is integration constant})$$

$$\text{Applying boundary condition, at } x = -x_p, \frac{dV}{dx} = 0$$

$$\Rightarrow 0 = \frac{-eN_a x_p}{\epsilon} + A \Rightarrow A = \frac{eN_a x_p}{\epsilon}$$

$$\text{Therefore, } \frac{dV}{dx} = \frac{eN_a x}{\epsilon} + \frac{eN_a x_p}{\epsilon}$$

$$\text{Again integrating, } V(x) = \frac{eN_a x^2}{2\epsilon} + \frac{eN_a x_p x}{\epsilon} + B \quad (B \text{ is integration constant})$$

$$\text{At, } x = -x_p$$

$$V(-x_p) = \frac{eN_a x_p^2}{2\epsilon} - \frac{eN_a x_p^2}{\epsilon} + B$$

$$V(-x_p) = -\frac{eN_a x_p^2}{2\epsilon} + B$$

Similarly, for the potential distribution in space charge region on n-side (0 to x_n), the charge density is given by,

$$\rho = eN_d \quad (N_d \text{ is the number of donors per unit volume})$$

$$\text{Using ' ρ ' in (6), } \frac{d^2V}{dx^2} = \frac{-eN_d}{\epsilon}$$

$$\text{Integrating, } \frac{dV}{dx} = \frac{-eN_d x}{\epsilon} + C \quad (C \text{ is integration constant})$$

$$\text{At } x = x_p, \frac{dV}{dx} = 0 \Rightarrow C = \frac{e N_d x_n}{\epsilon}$$

$$\text{Therefore, } \frac{dV}{dx} = \frac{-e N_d x}{\epsilon} + \frac{e N_a x_n}{\epsilon}$$

$$\text{Again integrating, } V(x) = \frac{-e N_d x^2}{2\epsilon} + \frac{e N_d x_n x}{\epsilon} + D \quad (\text{D is integration constant})$$

$$\text{At } x = x_n, V(x_n) = \frac{-e N_d x_n^2}{2\epsilon} + \frac{e N_d x_n^2}{\epsilon} + D$$

$$V(x_n) = \frac{e N_d x_n^2}{2\epsilon} + D$$

Now, built in potential, $V_o = V(x_n) - V(-x_p)$

$$V_o = \frac{e N_d x_n^2}{2\epsilon} + \frac{e N_a x_p^2}{2\epsilon} \quad \dots(7)$$

[Since, V must be continuous $B = D$, so cancel each other]

For overall charge neutrality, the total charge per unit area on the left hand side must be equal to that on the right hand side i.e. $N_a x_p = N_d x_n \Rightarrow x_n = \frac{N_a}{N_d} x_p$

$$\text{Using } x_n \text{ in equation (7), } V_o = \frac{e N_d}{2\epsilon} \left(\frac{N_a}{N_d} x_p \right)^2 + \frac{e N_a x_p^2}{2\epsilon}$$

$$= \frac{e N_a}{2\epsilon} \frac{N_a}{N_d} x_p^2 + \frac{e N_a}{2\epsilon} x_p^2$$

$$V_o = \frac{e N_a x_p^2}{2\epsilon} \left(1 + \frac{N_a}{N_d} \right)$$

$$\Rightarrow x_p = \left[\frac{2\epsilon V_o}{e N_a \left(1 + \frac{N_a}{N_d} \right)} \right]^{1/2}$$

Again, substituting $x_p = \frac{N_d}{N_a} x_n$ in equation (7)

$$V_o = \frac{e N_d x_n^2}{2\epsilon} + \frac{e N_a}{2\epsilon} \frac{N_d^2}{N_a^2} x_n^2$$

$$= \frac{e N_d x_n^2}{2\epsilon} \left(1 + \frac{N_d}{N_a} \right)$$

$$x_n = \left(\frac{2\epsilon V_o}{e N_d \left(1 + \frac{N_d}{N_a} \right)} \right)^{1/2}$$

Therefore, total width of transition region

$$W = x_n - (-x_p) = x_n + x_p$$

$$\begin{aligned}
&= \left[\frac{2\epsilon V_o}{eN_a \left(1 + \frac{N_d}{N_a} \right)} \right]^{1/2} + \left[\frac{2\epsilon V_o}{eN_d \left(1 + \frac{N_a}{N_d} \right)} \right]^{1/2} \\
W &= \left(\frac{2\epsilon V_o}{e} \right)^{1/2} \left[\left(\frac{1}{N_d \frac{(N_a+N_d)}{N_a}} \right)^{1/2} + \left(\frac{1}{N_a \frac{(N_a+N_d)}{N_d}} \right)^{1/2} \right] \\
&= \left(\frac{2\epsilon V_o}{e} \right)^{1/2} \frac{1}{(N_a + N_d)^{1/2}} \left[\left(\frac{N_a}{N_d} \right)^{1/2} + \left(\frac{N_d}{N_a} \right)^{1/2} \right] \\
&= \left(\frac{2\epsilon V_o}{e} \right)^{1/2} \frac{1}{(N_a + N_d)^{1/2}} \left(\frac{(N_a + N_d)}{\sqrt{N_a N_d}} \right) \\
&= \left[\frac{2\epsilon V_o (N_a + N_d)}{e N_a N_d} \right]^{1/2} \\
\therefore W &= \left[\frac{2\epsilon V_o}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \quad \dots(8)
\end{aligned}$$

This shows that width of space charge region decreases as the impurity concentration increases.

We have from equation (1)

$$V_o = \frac{KT}{e} \ln \frac{N_d N_a}{n_i^2}$$

Using this value of V_o in equation (8)

$$\begin{aligned}
W &= \left[\frac{2\epsilon}{e} \frac{KT}{e} \ln \frac{N_d \cdot N_a}{n_i^2} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \\
W &= \left[\frac{2\epsilon}{e^2} KT \ln \frac{N_d \cdot N_a}{n_i^2} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \quad \dots(9)
\end{aligned}$$

If $N_d \gg N_a$, then we can neglect N_a in the sum $(N_a + N_d)$ then, equation (8) can be rewritten as.

$$W = \left(\frac{2\epsilon V_o}{e} \frac{1}{N_a} \right)^{1/2} = \left(\frac{2\epsilon V_o}{e N_a} \right)^{1/2} \quad \dots(10)$$

$$\Rightarrow V_o = \frac{e N_a W^2}{2\epsilon} \quad \dots(11)$$

Using the value of V_o from equation (1) in equation (10) we get,

$$W = \left(\frac{2\epsilon}{e N_a} \cdot \frac{KT}{e} \ln \frac{N_d \cdot N_a}{n_i^2} \right)^{1/2} = \left[\frac{2\epsilon KT}{e^2 N_a} \ln \frac{N_d \cdot N_a}{n_i^2} \right]^{1/2} \quad \dots(12)$$

In this case almost all of depletion region will fall in p-region as doping concentration of n-region is much heavier than that of p-region.

2. Forward Biased pn Junction:

When an external voltage is applied to the pn-junction in such a manner that the positive terminal of the battery is attached to the p-side and negative terminal to the n-side, the pn junction is said to be forward biased.

When a voltage 'V' is applied, it reduces built in potential ' V_o ' to $V_o - V$. Built in potential is one which acts against diffusion, consequently many holes can now diffuse across the depletion n-region and enter n-side. This results in the injection of excess minority carriers (holes) into the n-region. Similarly, excess electrons can now diffuse towards the p-side and enter the region and thereby become injected minority carriers.

Now the probability that a hole will surmount the potential barrier ($V_o - V$) and diffuse to n-region becomes proportional to $\exp\left[-\frac{e(V_o - V)}{KT}\right]$

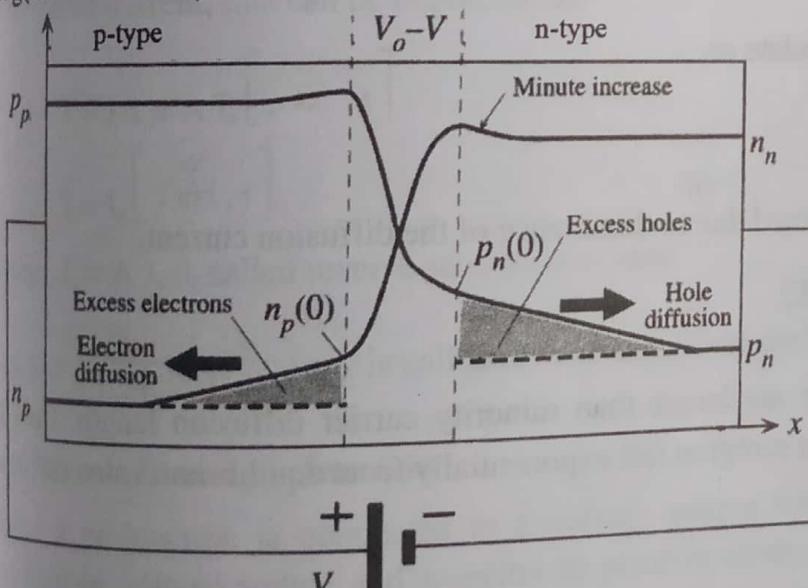
$$\text{i.e. } p_n(0) = p_p \exp\left(-\frac{e(V_o - V)}{KT}\right) \quad \dots(1)$$

Here, $p_n(0)$ is the hole concentration just outside the depletion region at $x = x_{n_0}$, and p_p is the total concentration of hole in p-region.

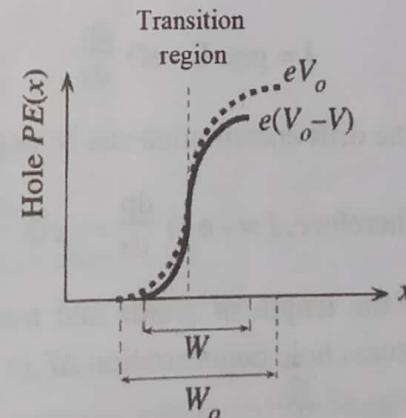
$$\text{We have, } \frac{p_p}{p_n} = e^{\frac{eV_o}{KT}}$$

$$\Rightarrow p_p = p_n e^{\frac{eV_o}{KT}} \quad \dots(2)$$

Log(Concentration)



(a)



(b)

Figure: (a) Carrier concentration profiles across the device under forward bias. (b) The hole potential energy with and without an applied bias. W is the width of junction with forward bias. W_o is the width of junction with no bias.

Here, p_n = concentration of holes in n-region.

Using p_p in equation (1)

$$p_n(0) = p_n e^{\frac{eV_o}{KT}} \cdot e^{\frac{-eV_o + eV}{KT}} = p_n e^{\frac{eV_o}{KT} - \frac{eV_o}{KT} + \frac{eV}{KT}}$$

$$p_n(0) = p_n e^{\frac{eV}{KT}} \quad \dots(3)$$

Which is called *law of junction*. It describes the effect of the applied voltage 'V' on the injected minority carrier concentration just outside the depletion region $p_n(0)$. For no applied voltage, $V = 0$, $p_n(0) = p_n$ which is exactly what we expect.

Similarly, electrons are injected from the n-side to the p-side. The electron concentration $n_p(0)$ Just outside the depletion region towards p-side at $x = -x_p$ is given by

$$n_p(0) = n_p e^{\frac{eV}{KT}}$$

Here, n_p = electron concentration in p-region.

By forward biasing a pn junction, a current through the junction can be maintained. Hole diffusion in n-side and electrons diffusion in p-side will give rise to a current in the same direction. The current flow seems to be surprisingly due to the diffusion of minority charge carriers, there is, however, current due to drift of majority carriers as well.

Now, $\Delta p_n = p_n(0) - p_n$

$$= p_n e^{\frac{eV}{KT}} - p_n$$

$$\Delta p_n = p_n (e^{\frac{eV}{KT}} - 1) \quad \dots(4)$$

The current density for hole is calculate as,

$$J = pe\mu E - eD \frac{dp}{dx}$$

The drift contribution can be neglected due to dominance of the diffusion current.

$$\text{Therefore, } J = -eD \frac{dp}{dx} = -eD \frac{d(\Delta p)}{dx}$$

If the length of p-side and n-side are larger than minority carrier diffusion length, then the excess hole concentration $\Delta P_n(x)$ in n-region fall exponentially from equilibrium value $\Delta P(0)$ as

$$\Delta p_n = \Delta p(0) e^{-\frac{x}{L}}$$

$$\Rightarrow \frac{d(\Delta p_n)}{dx} = -\frac{1}{L} \Delta p(0) e^{-\frac{x}{L}} = -\frac{1}{L} \Delta p_n$$

$$\text{Therefore, current density, } J = \frac{eD}{L} \Delta p_n$$

Substituting Δp_n from equation (4)

$$J = \frac{eD}{L} p_n \left(e^{\frac{eV}{KT}} - 1 \right)$$

But, $p_n = \frac{n_i^2}{n_a} = \frac{n_i^2}{N_d}$ [if all donors are ionized, $n_n \approx N_d$]

$$\text{Therefore, } J = \frac{eD n_i^2}{L N_d} \left(e^{\frac{eV}{KT}} - 1 \right)$$

Putting subscript 'h' for holes on J, D and L

$$J_h = \frac{eD_h n_i^2}{L_h N_d} \left(e^{\frac{eV}{KT}} - 1 \right) \quad \dots(5)$$

Similarly the diffusion current density for electron is

$$J_e = \frac{eD_e n_i^2}{L_e N_a} \left(e^{\frac{eV}{KT}} - 1 \right) \quad \dots(6)$$

The total diffusion current density is,

$$J = J_e + J_h = e n_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) \left(e^{\frac{eV}{KT}} - 1 \right)$$

$$J = J_s \left(e^{\frac{eV}{KT}} - 1 \right) \quad \dots(7)$$

Equation (7) is called *Ideal diode equation*.

In terms of current, this can be expressed as,

$$I = J A = A I_s \left[e^{\frac{eV}{KT}} - 1 \right]$$

$$I = I_s \left[e^{\frac{eV}{KT}} - 1 \right] \quad \dots(8)$$

Where, $I_s = A J_s$ is called reverse saturation current

And $J_s = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) e n_i^2$ is called reverse saturation current density.

3. Reverse Biased pn Junction

When a pn-junction is connected to a voltage source with p-region connected to negative terminal of voltage source and n-region to positive terminal, then the junction is said to be reverse biased.

The negative terminal of the battery will attract holes in p-region to move away from depletion layer, which results in more exposed negative acceptor ions and thus a wider space charge layer.

Similarly, the positive terminal will attract electrons away from the depletion layer which exposes more positively charged donors and consequently widening the depletion region in this side as well.

The movement of electrons in the n-region toward the positive battery terminal cannot be sustained because there is no electron supply to this n-side. The p-side also cannot supply electrons to the n-side because it has almost no electrons.

The electrons from the battery cannot enter the p-side and cannot move to n-side because before they get a chance to do so, they will be recombined with holes in p-region. So, a current cannot be sustained through pn junction in reverse biased condition. The applied voltage adds to built-in potential as shown in figure (1b). Similarly the electric field in the depletion layer is the sum of internal field and applied field.

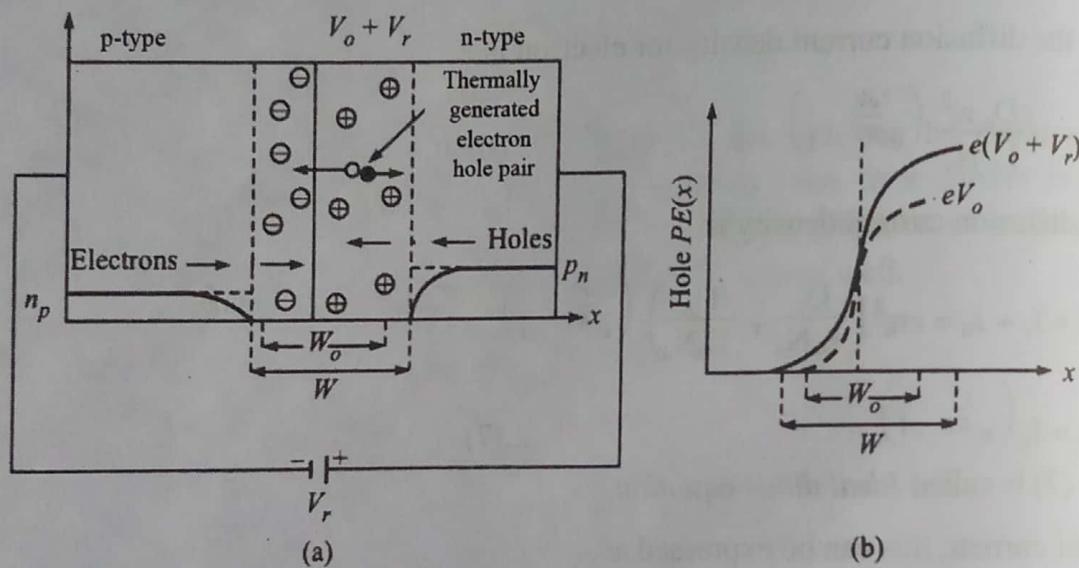


Figure 1: (a) Reversed-biased pn junction; minority carrier profiles and the origin of the reverse current
(b) The hole PE across the junction under reverse bias.

The holes in the n-side near the depletion layer can be extracted by the field and driven across depletion layer to the p-region giving rise to a small current. Similarly, the electrons in p-side near the depletion region are extracted and driven across the depletion layer towards n-side and again give rise to a small current. There will be small diffusion current through the junction due to holes diffusing from depletion layer to n-region, and electrons diffusing from depletion layer to p-region. Therefore there is a small reverse current due to these two causes discussed above,

The reverse current is given by

$$J = -J_s \left[e^{\frac{eV}{KT}} - 1 \right] \quad \dots(1)$$

Where, $J_s = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) e n_i^2 \dots(2)$ is called reverse saturation current density. Here equation (2) is called Shockley equation.

Considering the effect of thermal generation the equation for reverse current will be modified to

$$J_{s,rev} = \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right) en_i^2 + \frac{en_i W}{\tau_g} \quad \dots(3)$$

Here, current density due to thermal generation, J_{gen} $= \frac{en_i W}{\tau_g}$ and τ_g is the mean thermal generation time to generate electron hole pair.

The width of depletion region with no bias is.

$$W = \left[\frac{2\epsilon V_0}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

When the reverse voltage V is applied, the total potential of the Junction becomes $(V_0 + V)$, so the total width of depletion region for reverse biased is given by

$$W = \left[\frac{2\epsilon (V_0 + V)}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \quad \dots(4)$$

Diffusion or Storage Capacitance (C_D)

When the pn junction is forward biased we get diffusion capacitance. If the forward voltage across the pn junction is increased, the more holes will diffuse towards n-region and more electrons will diffuse towards p-side. Electrons are minority carriers in p-region and holes are minority carriers in n-region. *The rate of increase of diffused minority charge with applied potential difference is defined as diffusion capacitance.*

$$\text{i.e. } C_D = \frac{dQ}{dV}$$

Depletion Layer or Transition Capacitance (C_T)

When the pn junction is reverse biased we get depletion layer capacitance. For reverse biased, the depletion region acts like an insulator or as a dielectric material. The p and n-regions on either side have low resistance and acts as plates of parallel plate capacitor. *This junction capacitance is called transition capacitance or depletion layer capacitance.* It is also termed as space charge layer capacitance or barrier capacitance.

It is given by,

$$C_T = \frac{\epsilon A}{W}$$

Where, ϵ = Permittivity of depletion region

A = Cross - sectional area of p or n-region

W = Width of transition region.

When the reverse biased is increased, it will increase W and C_T will decreases.

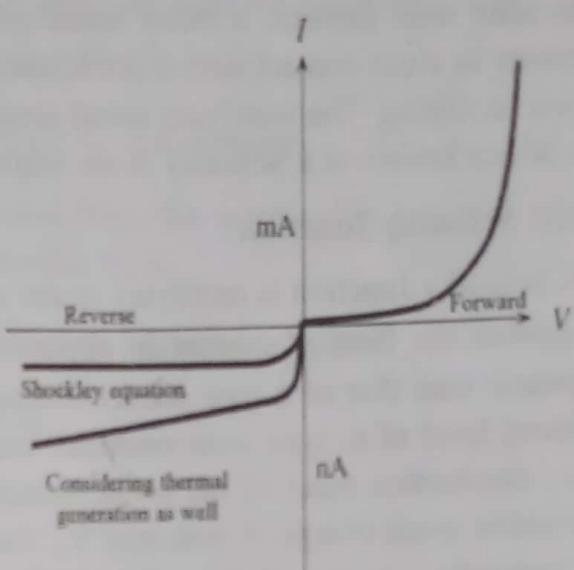


Figure 2: Forward and reverse I-V characteristics of a pn junction

Metal Semiconductor Junction

In solid state physics, a metal semiconductor junction is a type of junction in which a metal comes in close contact with a semiconductor material. M-S Junction can either be rectifying or non rectifying. The rectifying metal semiconductor Junction forms a Schottky Junction, making a device known as a Schottky diode while the non rectifying junction is called Ohmic contact.

1. Schottky Junction

A Schottky Junction is rectifying in the sense that it allows flow of current in one direction and opposes the flow of current in opposite direction. When the work function of metal (ϕ_m) is greater than that of n-type semiconductor (ϕ_n), a Schottky Junction is formed. In this case the Fermi level of n-type semiconductor lies above that of metal. The more energetic electrons in the conduction band of the semiconductor can easily tunnel into metal, leaving behind a net positive space charge. A potential V_o , therefore develops between the metal and semiconductor. Eventually, at equilibrium this built in potential reaches a value that prevents further accumulation of electrons at the metal surface. At equilibrium, the Fermi level throughout the solid is uniform. Thus E_{Fm} and E_{Fn} line up.

In case of Schottky junction the flow of electrons takes place from semiconductor side to metal side. The electrons will accumulate in metal and the positive charges were left in the region of semiconductor near to contact region. So the gap $E_c - E_F$ increases in that region. The Fermi level is at equilibrium position or at constant position. So to maintain the increase in $E_c - E_F$ gap the conduction band of semiconductor near to contact region bends upward as shown in figure. This phenomenon is called *band bending*.

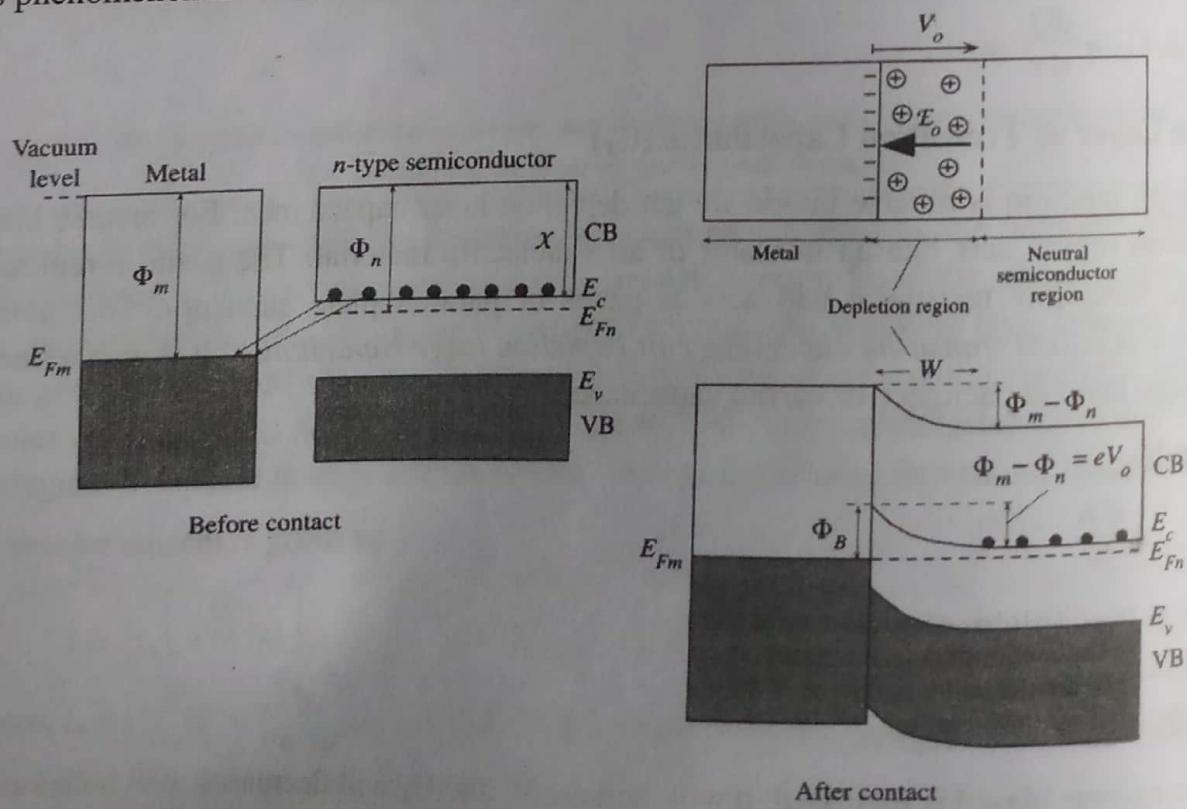


Figure (1): Formation of a Schottky junction between metal and an n-type semiconductor when $\phi_m > \phi_n$

There are two current components due to electrons flowing through the junction.

1. The current due to electrons being thermally emitted from metal to the conduction band of the semiconductor which faces the Schottky barrier $\phi_B = \phi_m - \phi_n$ is.

$$J_1 = C_1 \exp\left(\frac{-\phi_B}{KT}\right)$$

...(1), where C_1 is a constant

2. The current due to electron being thermally emitted from the conduction band of the semiconductor to the metal at the cost of built in potential V_o is,

$$J_2 = C_2 \exp\left(-\frac{eV_o}{KT}\right)$$

...(2), where C_2 is another constant.

At equilibrium, these currents in reverse direction are equal.

$$\text{i.e. } J_{\text{equilibrium}} = J_2 - J_1 = 0$$

$$J_1 = J_2 \dots \text{(3)}$$

Case - I

Under forward biased condition the semiconductor side is connected to negative terminal of supply. It assists the flow of electron from semiconductor to metal. The applied potential (V) reduces the built in potential (V_o) to $V_o - V$. The potential energy barrier for the emission of electrons from the semiconductor to the metal is now $e(V_o - V)$.

Therefore, the forward current J_2^{for} is given by

$$J_2^{\text{for}} = C_2 \exp\left[\frac{-e(V_o - V)}{KT}\right]$$

Since ϕ_B is the same, so J_1 remains unchanged. The net current is then,

$$\begin{aligned} J &= J_2^{\text{for}} - J_1 = J_2^{\text{for}} - J_2 \quad [\text{Since, } J_1 = J_2] \\ &= C_2 \exp\left[\frac{-e(V_o - V)}{KT}\right] - C_2 \exp\left(\frac{-eV_o}{KT}\right) \\ &= C_2 \exp\left(\frac{-eV_o}{KT}\right) \cdot \left[\exp\left(\frac{eV}{KT}\right) - 1 \right] \end{aligned}$$

$$J = J_o \left[\exp\left(\frac{eV}{KT}\right) - 1 \right] \quad \dots(4)$$

Where, $J_o = C_2 \exp\left(-\frac{eV_o}{KT}\right)$ is called reverse saturation

current depends on the material and surface properties of two solid. It is equal to J_2 as given by equation (2).

Case II:

When the Schottky junction is reverse biased, that is the positive terminal is connected to the semiconductor, It resists (opposes) the flow of electron from semi conductor to metal. The

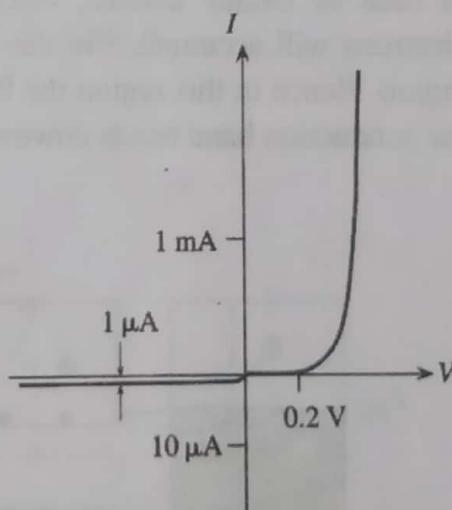


Figure 2: $I - V$ characteristics of Schottky Junction exhibits rectifying properties.

applied potential (V_r) increases the built in potential V_o to $V_o + V_r$. The corresponding current density is

$$J_2^{\text{rev}} = C_2 \exp\left[-\frac{e(V_o + V_r)}{KT}\right] \ll J_1 \quad \dots(5)$$

Then under reverse bias condition the current is primarily due to the emission of electrons over the barrier ϕ_B from the metal to the conduction band of the semiconductor as determined by equation (1).

Thus the I - V characteristics of Schottky junction exhibit rectifying property as shown in figure (2).

2. Ohmic Contact

An Ohmic contact means, it allows the flow of current in both forward and reverse direction. It is formed when a metal having smaller work function is brought in contact with n-type semiconductor having larger work function. That is the Fermi level of metal lies above to that of semiconductor, which means electrons tunnel into the semi conductor from metal side. An equilibrium is reached when the accumulated electrons in the CB of the semiconductor prevent further electrons tunneling from the metal that is the Fermi level is uniform across the whole system from one end to another.

In case of Ohmic contact, electrons move from metal side to semiconductor side. These electrons will accumulate in the region of semiconductor near to contact called accumulation region. Hence in this region the $E_C - E_F$ gap decreases. To maintain the decrease in $E_C - E_F$ gap the conduction band bends downward. This phenomenon is called *band bending*.

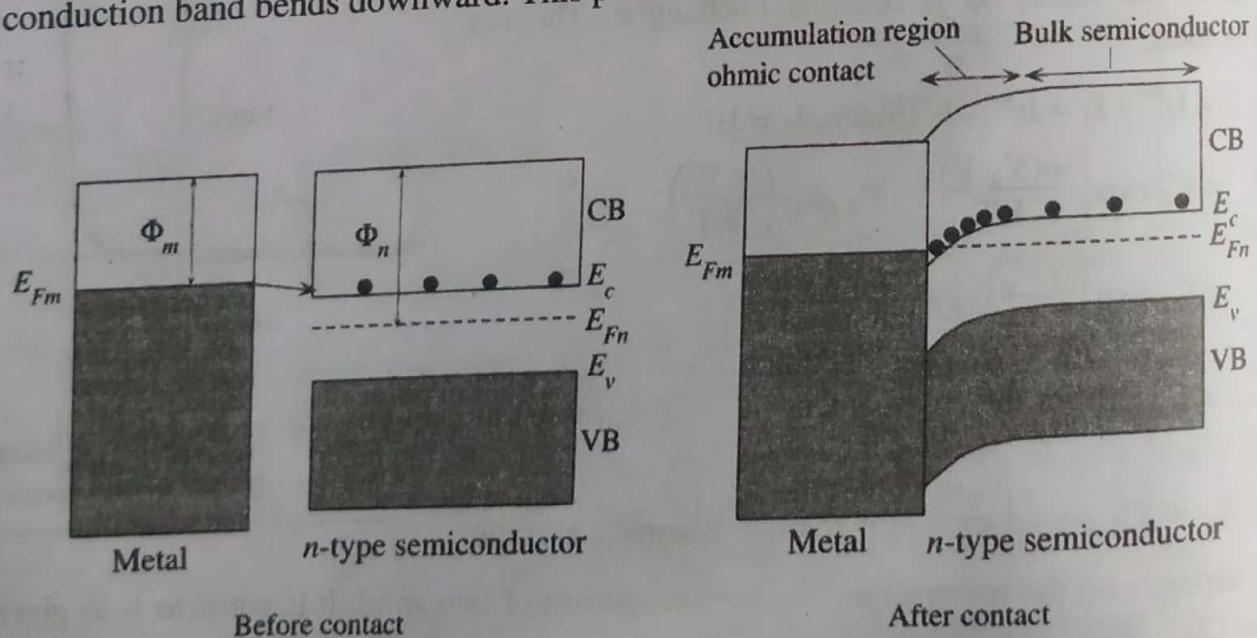


Figure 3: When a metal with a smaller work function than an n-type semiconductor is put into contact with the n-type semiconductor, the resulting junction is an Ohmic contact in the sense that it does not limit the current flow.

The semi conductor region near the Junction in which there are excess of electron is called the accumulation region. It can be seen from figure (3) that the conduction electrons on either side

of contact region have about the same energy and therefore there is no barrier involved to cross the junction by electrons in either direction under applied field.

The Ohmic contact is a junction between a metal and a semiconductor that does not limit current flow in either direction. The current is limited by the resistance of semiconductor outside the contact region rather than the thermal emission of carriers across the potential barrier at contact. Both the metal and the accumulation region have comparatively high concentrations of electrons compared with the bulk of semiconductor. The current is therefore determined by the resistance of the bulk region. The current density is simply $J = \sigma E$

Where, σ = conductivity of semiconductor

E = applied field.

Solved Examples

1. A heavily doped p-side with acceptor concentration of 10^{18} cm^{-3} is connected to n-side with donor concentration of 10^{16} cm^{-3} . Calculate the built in potential, depletion width in n-side and p-side and overall depletion width. Assume $T = 300 \text{ K}$ and $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$.

Solution:

Here, $N_a = 10^{18} \text{ cm}^{-3}$, $N_d = 10^{16} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, $T = 300 \text{ K}$.

1. $V_o = ?$

$$\begin{aligned} \text{We have, } V_o &= \frac{KT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) \\ &= \left(\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \right) \cdot \ln \left[\frac{10^{18} \times 10^{16}}{(1.45 \times 10^{10})^2} \right] \\ &= 0.815 \text{ volt} \end{aligned}$$

2. The total depletion width is given by,

$$\begin{aligned} w &= \sqrt{\frac{2\epsilon V_o}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)} \\ &= \sqrt{\frac{2\epsilon_r \epsilon_0 V_o}{e} \left(\frac{N_a + N_d}{N_a \cdot N_d} \right)} \\ &= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.815}{1.6 \times 10^{-19}} \left(\frac{10^{24} + 10^{22}}{10^{24} \times 10^{22}} \right)} \\ &= 3.3 \times 10^{-7} \text{ m} \end{aligned}$$

3. Now the depletion width in n-region is,

$$x_n = \left[\frac{2 \epsilon V_o}{e N_d \left(1 + \frac{N_d}{N_a} \right)} \right]^{1/2}$$

$$= \left[\frac{2 \epsilon_r \epsilon_0 V_o}{e N_d} \frac{N_a}{(N_a + N_d)} \right]^{1/2}$$

$$= \left[\sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.815}{1.6 \times 10^{-19}}} \frac{10^{24}}{10^{22} (10^{24} + 10^{22})} \right]$$

$$= 3.26 \times 10^{-7} \text{ m}$$

end

$$n = \frac{n_i^2}{P}$$

Now the depletion width in p-region is,

$$x_p = w - x_n$$

$$= 3.3 \times 10^{-7} - 3.26 \times 10^{-7}$$

$$= 0.04 \times 10^{-7} \text{ m}$$

$$= 4 \times 10^{-9} \text{ m}$$

2. Calculate the diffusion coefficient of electrons at 300 K in n-type silicon semiconductor doped with 10^{15} arsenic atoms per cm^3 ($\mu = 1300 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$)

Solution:

Here, $T = 300 \text{ K}$, $\mu = 1300 \text{ cm}^2 \text{ V}^{-1} \text{s}^{-1}$

$$\mu = 1300 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{s}^{-1} = 0.13 \text{ m}^2 \text{ V}^{-1} \text{s}^{-1}$$

From Eistein relation,

$$\frac{D}{\mu} = \frac{KT}{e}$$

$$D = \frac{\mu KT}{e} = \frac{0.13 \times 1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}$$

$$= 3.36 \text{ m}^2 \text{ s}^{-1}$$

3. The steady state excess hole concentration of $\Delta P = 10^{16} \text{ cm}^{-3}$ (at $x = 0$) is injected to semi infinite silicon bar of cross section $A = 10^{-3} \text{ cm}^2$. If hole diffusion length $L_h = 10^{-3} \text{ cm}$ and hole life time $\tau = 10^{-6} \text{ s}$. Then derive equation of excess hole distribution and calculate. 1) Steady state storage charge ii) Hole current $I_p (x = 0)$

Solution:

The continuity equation for holes is

$$\frac{d^2 (\Delta P_h)}{dx^2} - \frac{\Delta P_h}{L_h^2} = 0 \quad \dots(1)$$

Here, $L_h = \sqrt{D_h \tau_h}$ is called diffusion length for holes. Equation (1) describes the steady state behavior of minority carrier concentration in a semiconductor.

The solution of equation (1) is

$$\Delta P_n(x) = A e^{x/L} + B e^{-x/L} \quad \dots(2)$$

Where 'A' and 'B' are constants that have to be found from the boundary conditions. For a semi infinite bare, at $x \rightarrow \infty/2 (= \infty)$, $\Delta P(\infty) = 0$

$$\text{From (2)} \quad 0 = A + 0 \Rightarrow A = 0$$

Therefore, equation (2) can be written as,

$$\Delta P(x) = B e^{-x/L} \quad \dots(3)$$

$$\text{Again at } x = 0, \Delta P(x) = \Delta P(0)$$

$$\text{From (3), } B = \Delta P(0)$$

$$\text{Therefore, } \Delta P(x) = \Delta P(0) e^{-x/L} \quad \dots(4)$$

Equation (4) gives the spatial distribution of excess holes.

II) The hole current density due to diffusion is

$$\begin{aligned} J &= -eD \frac{d(\Delta P)}{dx} \\ &= -eD \frac{d}{dx} [\Delta P(0) e^{-x/L}] \\ &= \frac{eD}{L} \Delta P(x) \end{aligned}$$

$$\text{Therefore, hole current, } I_p(x) = J.A. = \frac{AeD}{L} \Delta P(x)$$

$$\text{At, } x = 0, I_p(x=0) = \frac{A e D}{L} \Delta P(0)$$

$$\text{Since, } L = \sqrt{D\tau} \Rightarrow D = \frac{L^2}{\tau} = \frac{(10^{-3} \times 10^{-2})^2}{10^{-6}}$$

$$D = 10^{-4} \text{ m}^2 \text{s}^{-1}$$

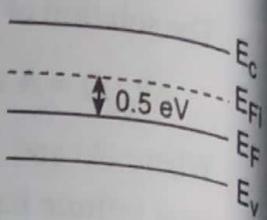
$$\text{Hence, } I_p(x=0) = \frac{10^{-3} \times 10^{-4} \times 1.6 \times 10^{-19} \times 10^{-4} \times 10^{16} \times 10^6}{10^{-3} \times 10^{-2}} \\ = 1.6 \times 10^{-3} \text{ A} = 1.6 \text{ mA}$$

I) The steady state charge storage

$$\begin{aligned} Q &= I\tau \\ &= 1.6 \times 10^{-3} \times 10^{-6} \\ &= 1.6 \times 10^{-9} \text{ C} \end{aligned}$$

$$Q = 1.6 \text{ nC}$$

4. A semiconductor crystal is characterized by following energy band diagram at 300 K.
1. Find the type of dopant and doping level.
 2. Calculate the conductivity of semiconductor at room temperature.
 3. If it is desired that the Fermi level is to be raised to 0.1 eV above intrinsic Fermi level, what type of dopant is to be used? Determine its doing level. ($\mu_h = 450 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1}$ at 300 K)



Solution:

1. Here the Fermi level is below the intrinsic Fermi level, therefore the given material is of p-type.

The hole concentration or the acceptor concentration is given by, $p = n_i e^{\frac{(E_{F_i} - E_{F_p})}{kT}}$

$$p = 1.145 \times 10^{16} \times e^{\left(\frac{0.5 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} \right)}$$

$$p = 2.825 \times 10^{24} \text{ m}^{-3}$$

2. We have, $n.p = n_i^2$

$$\text{So the electron concentration, } n = \frac{n_i^2}{p} = \frac{(1.145 \times 10^{16})^2}{2.825 \times 10^{24}}$$

$$n = 4.64 \times 10^6 \text{ m}^{-3}$$

Here, $n \ll p$. So the conductivity, $\sigma = p e \mu_h$

$$= 2.825 \times 10^{24} \times 1.6 \times 10^{-19} \times 450 \times 10^{-4}$$

$$= 20340 \Omega^{-1} \text{m}^{-1}$$

3. If it is desired to raise Fermi level 0.1 eV above the intrinsic Fermi level, the donors must be doped. The electron concentration is given by,

$$n = n_i e^{\frac{(E_{F_n} - E_{F_i})}{kT}}$$

$$N_d - N_a = 1.145 \times 10^{16} e^{\left(\frac{0.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300} \right)}$$

$$= 5.46 \times 10^{17} \text{ m}^{-3}$$

5. An n-type Si semiconductor containing 10^{16} Phosphorous (donor) atoms per cm^{-3} has been doped with 10^{17} boron (acceptor) atoms cm^{-3} . Calculate electron and hole concentration in this semi conductor ($n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$)

Solution:

Here, $N_d = 10^{16} \text{ cm}^{-3}$, $N_a = 10^{17} \text{ cm}^{-3}$.

Thus the semiconductor has been compensation doped with excess acceptors over donor, so

$$p = N_a - N_d = 10^{17} - 10^{16} = 9 \times 10^{16} \text{ cm}^{-3}$$

The electron concentration

$$n = \frac{n_i^2}{p} = \frac{(1.45 \times 10^{10})^2}{9 \times 10^{16}} = 2.34 \times 10^3 \text{ cm}^{-3}$$

- b) Find the resistance of a 1cm^3 pure Silicon crystal. What is the resistance when the crystal is doped with one Arsenic in 10^9 Silicon atoms? Given: Atomic concentration is Si is $5 \times 10^{22} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, $\mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ and $\mu_h = 450 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$.

Solution:

Case I, for pure silicon, $\sigma = ne\mu_e + pe\mu_h$

$$\begin{aligned}\sigma &= n_i e (\mu_e + \mu_h) \\ &= 1.45 \times 10^{16} \times 1.6 \times 10^{-19} (0.1350 + 0.0450) \\ &= 4.176 \times 10^{-4} \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

Since, $L = 1 \text{ cm} = 10^{-2} \text{ m}$ and $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$,

$$\text{The resistance is, } R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{10^{-2}}{4.176 \times 10^{-4} \times 10^{-4}} = 210 \times 10^3 \Omega = 210 \text{ k}\Omega$$

Case II,

When the crystal is doped with 1 As in 10^9 Si atoms,

Then, 10^9 silicon atoms are doped with 1 As

1 silicon atom is doped with $\frac{1}{10^9}$ As.

5×10^{22} silicon's atoms are doped with $\frac{5 \times 10^{22}}{10^9}$ As

Therefore, $N_d = 5 \times 10^{13} \text{ cm}^{-3}$

At, room temperature all the donors are ionized so,

$$n = N_d = 5 \times 10^{13} \text{ cm}^{-3}$$

The hole concentration is now,

$$p = \frac{n_i^2}{N_d} = \frac{(1.45 \times 10^{10})^2}{5 \times 10^{13}} = 4.2 \times 10^6 \text{ cm}^{-3}$$

Here $p \ll n_i$ or $p \ll n$

Therefore,

$$\begin{aligned}\sigma &= ne\mu_e \\ &= 5 \times 10^{13} \times 1.6 \times 10^{-19} \times 0.1350 \\ &= 1.08 \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

$$\text{The resistance, } R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{10^{-2}}{1.08 \times 10^{-4}} = 92.6 \Omega$$

This shows the drastic fall in resistance when the crystal is doped with only one As in 10^9 Si atoms.

7. Find the resistance of a 1 cm^3 pure Silicon crystal. What is the resistance when the crystal is doped with one Boron in 10^9 Silicon atoms? Given: Atomic concentration in Si is $5 \times 10^{22} \text{ cm}^{-3}$, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$, $\mu_e = 1350 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1}$ and $\mu_h = 450 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1}$.

Solution:

$$\text{Here, } n_i = 1.45 \times 10^{10} \text{ cm}^{-3} = 1.45 \times 10^{16} \text{ m}^{-3}$$

$$\mu_e = 1350 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1} = 0.1350 \text{ m}^2 \text{V}^{-1} \text{S}^{-1}, \mu_h = 450 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1} = 0.0450 \text{ m}^2 \text{V}^{-1} \text{S}^{-1}$$

$$\text{Case I: for pure silicon, } \sigma = ne\mu_e + pe\mu_h = n_i e(\mu_e + \mu_h)$$

$$\sigma = 1.45 \times 10^{16} \times 1.6 \times 10^{-19} (0.1350 + 0.0450)$$

$$= 4.176 \times 10^{-4} \Omega^{-1} \text{m}^{-1}$$

Since, $L = 1 \text{ cm} = 10^{-2} \text{ m}$ and $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, The resistance is, $R =$

$$\rho L = \frac{L}{\sigma A} = \frac{10^{-2}}{4.176 \times 10^{-4} \times 10^{-4}} = 210 \times 10^3 \Omega = 210 \text{ k}\Omega$$

Case II, when the crystal is doped with 1 B in 10^9 Si atoms Then,

10^9 Silicon atoms are doped with 1 Boron atom

1 Silicon atom is doped with $\frac{1}{10^9}$ Boron atom

5×10^{22} Silicon's atoms are doped with $\frac{5 \times 10^{22}}{10^9}$ Boron atoms.

Therefore, $N_a = 5 \times 10^{13} \text{ cm}^{-3}$

At room temperature, all the acceptors are ionized, So,

$$p = N_a = 5 \times 10^{13} \text{ cm}^{-3}$$

The electron concentration is,

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a} = \frac{(1.45 \times 10^{10})^2}{(5 \times 10^{13})} = 4.2 \times 10^6 \text{ cm}^{-3}$$

Here, $n \ll n_i$ or $n \ll p$

Therefore,

$$\begin{aligned} \sigma &= p e \mu_h \\ &= 5 \times 10^{19} \times 1.6 \times 10^{-19} \times 0.0450 \\ &= 0.36 \Omega^{-1} \text{m}^{-1} \end{aligned}$$

Therefore, Resistance, $R = \frac{\rho L}{A} = \frac{L}{\sigma A}$

$$= \frac{10^{-2}}{0.36 \times 10^{-4}} = 278 \Omega$$

This shows the drastic fall in resistance when the crystal is doped with only one B-atom in 10^9 Si atoms.

8. Given that the density of states related effective masses of electrons and holes in Si are approximately $1.08 m_e$ and $0.60 m_e$ respectively, and the electron and hole drift mobilities at room temperature are 1350 and $450 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1}$ respectively. Calculate the intrinsic concentration and intrinsic resistivity of Si. The energy band gap for Silicon is 1.10 eV . ($T = 300 \text{ K}$)

Solution:

Here

$$m_e^* = 1.08 m_e \text{ and } m_h^* = 0.60 m_e, \mu_e = 1350 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1} = 0.1350 \text{ m}^2 \text{V}^{-1} \text{S}^{-1},$$

$$\mu_h = 450 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1} = 0.0450 \text{ m}^2 \text{V}^{-1} \text{S}^{-1}$$

$$E_g = 1.10 \text{ eV, Room Temperature, } T = 300 \text{ K}$$

$$\text{Now, } N_c = 2 \left(\frac{2\pi m_e^* K T}{h^2} \right)^{3/2}$$

$$= 2 \left[\frac{2 \times 3.14 \times 1.08 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.624 \times 10^{-34})^2} \right]^{3/2}$$

$$N_c = 2.81 \times 10^{25} \text{ m}^{-3}$$

$$\text{and, } N_v = 2 \left(\frac{2\pi m_e^* K T}{h^2} \right)^{3/2}$$

$$N_v = 2 \left[\frac{2 \times 3.14 \times 0.6 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.624 \times 10^{-34})^2} \right]$$

$$N_v = 1.16 \times 10^{25} \text{ m}^{-3}$$

The intrinsic concentration is,

$$\begin{aligned} n_i &= (N_c N_v)^{1/2} e^{-\frac{E_g}{2KT}} \\ &= (2.81 \times 10^{25} \times 1.16 \times 10^{25})^{1/2} \exp \left[-\frac{1.1 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300} \right] \\ &= 1 \times 10^{16} \text{ m}^{-3} \end{aligned}$$

The conductivity is,

$$\begin{aligned} \sigma &= ne\mu_e + pe\mu_h = n_i e (\mu_e + \mu_h) \\ &= 1 \times 10^{16} \times 1.6 \times 10^{-19} (0.1350 + 0.0450) \\ &= 2.9 \times 10^{-4} \Omega^{-1} \text{m}^{-1} \end{aligned}$$

$$\therefore \text{Resistivity, } \rho = \frac{1}{\sigma} = (3.45 \times 10^5) \Omega \text{m}$$

9. An n-type Si wafer has been doped uniformly with 10^{16} antimony (Sb) atoms cm^{-3} . Calculate the position of the Fermi energy with respect to the Fermi energy E_{Fi} in intrinsic Si. The above n-type Si sample is further doped with 2×10^{17} Boron atoms

cm^{-3} . Calculate the position of the Fermi energy with respect to the Fermi energy E_{F_i} in intrinsic Si ($T = 300 \text{ K}$) ($n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$)

Case I, Sb gives n - type doping with $N_d = 10^{16} \text{ cm}^{-3}$ and since $N_d \gg n_i$ ($\approx 1 \times 10^{10} \text{ cm}^{-3}$), we have,

$$n = N_d = 10^{16} \text{ cm}^{-3} = 10^{22} \text{ m}^{-3}$$

$$\text{For intrinsic Si, } n = N_c \exp \left[-\frac{(E_c - E_{F_i})}{KT} \right]$$

Since for intrinsic Si, $n_i = n$

$$\text{So, } n_i = N_c \exp \left[-\frac{(E_c - E_{F_i})}{KT} \right] \quad \dots(1)$$

$$\text{Where as for n-type doped Si, } n = N_c \exp \left[-\frac{(E_c - E_{Fn})}{KT} \right]$$

According to question, $N_d \gg n_i$ So, $n \approx N_d$

Therefore,

$$N_d = N_c \exp \left[-\frac{(E_c - E_{Fn})}{KT} \right] \quad \dots(2)$$

Where, E_{F_i} and E_{Fn} are the Fermi energies in the intrinsic and n-type Si.

Dividing equation (2) by (1)

$$\frac{N_d}{n_i} = \exp \left[\frac{-E_c + E_{Fn} + E_c - E_{F_i}}{KT} \right]$$

$$\frac{N_d}{n_i} = \exp \left[\frac{(E_{Fn} - E_{F_i})}{KT} \right]$$

$$\left\{ N_d = n_i \exp \left(\frac{(E_{Fn} - E_{F_i})}{KT} \right) \text{ Therefore, } n = N_d = n_i \exp \left(\frac{(E_{Fn} - E_{F_i})}{KT} \right) \right\}$$

$$\text{Taking ln on both sides, } \frac{E_{Fn} - E_{F_i}}{KT} = \ln \left(\frac{N_d}{n_i} \right)$$

$$E_{Fn} - E_{F_i} = KT \ln \left(\frac{N_d}{n_i} \right)$$

$$= 1.38 \times 10^{-23} \times 300 \ln \left(\frac{10^{22}}{10^{16}} \right)$$

$$= 5.72 \times 10^{-20} \text{ J} = 0.36 \text{ eV.}$$

Case - II

Now the wafer is further doped with Boron, the acceptor concentration is, $N_a = 2 \times 10^{17} \text{ cm}^{-3}$. Here $N_a > N_d$ so the semiconductor is compensation doped and compensation converts the semiconductor to p-type Si, Thus

$$p = N_a - N_d = (2 \times 10^{17} - 10^{16}) = 1.9 \times 10^{17} \text{ cm}^{-3}$$

$$= 1.9 \times 10^{23} \text{ m}^{-3}$$

For intrinsic Si, $p = N_v \exp\left[-\frac{(E_{Fi} - E_v)}{KT}\right]$

Since, for intrinsic Si, $n_i = p$

$$\text{So, } n_i = N_v \exp\left[-\frac{(E_{Fi} - E_v)}{KT}\right] \quad \dots(3)$$

Whereas for p-type doped Si,

$$p = N_v \exp\left[-\frac{(E_{FP} - E_v)}{KT}\right]$$

For compensation p-type doping, $p = N_a - N_d$

$$\therefore N_a - N_d = N_v \exp\left[-\frac{(E_{FP} - E_v)}{KT}\right] \quad \dots(4)$$

Where, E_{Fi} and E_{FP} are the Fermi energies in the intrinsic and p-type Si respectively.

Dividing equation (4) by (3).

$$\frac{N_a - N_d}{n_i} = \exp\left[\frac{-E_{FP} + E_v + E_{Fi} - E_v}{KT}\right]$$

$$\frac{N_a - N_d}{n_i} = \exp\left[-\frac{(E_{FP} - E_{Fi})}{KT}\right] \quad \left\{ p = N_a - N_d = n_i \exp\left(\frac{(E_{Fi} - E_{FP})}{KT}\right) \right\}$$

Taking \ln on both sides,

$$-\frac{(E_{FP} - E_{Fi})}{KT} = \ln\left(\frac{N_a - N_d}{n_i}\right)$$

$$E_{FP} - E_{Fi} = -KT \ln\left(\frac{N_a - N_d}{n_i}\right)$$

$$= -1.38 \times 10^{-23} \times 300 \ln\left(\frac{1.9 \times 10^{23}}{1 \times 10^{10}}\right)$$

$$E_{FP} - E_{Fi} = -0.43 \text{ eV}$$

$$E_{Fi} - E_{FP} = 0.43 \text{ eV}$$

10. A Si sample has been doped with 10^{17} arsenic atoms cm^{-3} . Calculate the conductivity of the sample at 27°C and at 127°C . $\mu_e = 800 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ at 27°C and $\mu_e = 420 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ at 127°C . $n_i = 1.145 \times 10^{10} \text{ cm}^{-3}$

Solution:

Here, $N_d = 10^{17} \text{ cm}^{-3}$. That is it is the heavily doped case. So $n = N_d = 10^{17} \text{ cm}^{-3}$,

and $p = \frac{n_i^2}{N_d} = \frac{(1.145 \times 10^{10})^2}{10^{17}} = 1.3 \times 10^3 \text{ cm}^{-3}$ which is very less than n so can be neglected.

1. Therefore, the total conductivity at 27°C is

$$\sigma = ne\mu_e$$

$$= 10^{17} \times 10^6 \times 1.6 \times 10^{-19} \times 800 \times 10^{-4}$$

$$\sigma = 1280 \Omega^{-1} m^{-1}$$

2. Similarly, the total conductivity at 127°C is,

$$\begin{aligned}\sigma &= n_e e \mu_e \\ &= 10^{17} \times 10^6 \times 1.6 \times 10^{-19} \times 420 \times 10^{-4} \\ &= 672 \Omega^{-1} m^{-1}\end{aligned}$$

11. A Si sample has been doped with 10^{17} arsenic atoms cm^{-3} . It is further doped with 10^{16} boron atoms cm^{-3} . Calculate the conductivity of the sample at 27°C and 127°C (Given, $\mu_e = 600 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ at 27°C and $\mu_e = 400 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ at 127°C), $n_i = 1.145 \times 10^{10} \text{ cm}^{-3}$.

Solution:

$$\text{Here, } N_d = 10^{17} \text{ cm}^{-3} = 10^{23} \text{ m}^{-3}$$

$N_a = 9 \times 10^{16} \text{ cm}^{-3} = 9 \times 10^{22} \text{ cm}^{-3}$, So we have n - type material by compensation effect.

$$n = N_d - N_a = (1 \times 10^{23} - 9 \times 10^{22}) = 1 \times 10^{22} \text{ m}^{-3}$$

$$\text{And, } p = \frac{n_i^2}{n} = \frac{(1.145 \times 10^{16})^2}{1 \times 10^{22}} = 1.31 \times 10^{10}$$

Since, $p \ll n$ so conductivity is only due to n

$$\text{At, } T = 27^\circ\text{C} = (273 + 27) = 300 \text{ K}$$

$$\begin{aligned}\sigma &= ne \mu_e = 1 \times 10^{22} \times 1.6 \times 10^{-19} \times 600 \times 10^{-4} \\ &= 96 \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

$$\text{At, } T = 127^\circ\text{C} = (273 + 127) = 400 \text{ K}$$

$$\begin{aligned}\sigma &= ne \mu_e = 1 \times 10^{22} \times 1.6 \times 10^{-19} \times 400 \times 10^{-4} \\ &= 48 \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

12. Consider a Schottky junction diode between tungsten (W) and n-type Si. The electron affinity of Si is 4.01 eV and the work function of W is 4.55 eV 1) What will be the barrier height ϕ_B from metal to the semiconductor? 2) Using the Richardson's equation for the Thermionic emission, $J_o = A_o T^2 \exp(-\phi_B/KT)$, calculate reverse saturation current. Where $A_o = 110 \text{ AK}^{-2} \text{ cm}^{-2}$, $T = 300 \text{ K}$, cross sectional area $A = 0.01 \text{ cm}^2$ and J_o represents reverse saturation current density. 3) Calculate the forward current if the forward bias of 0.2 V is applied across the diode.

Solution:

- 1) Here, work function of W, $\phi = 4.55 \text{ eV}$

Electron affinity of Si, $\chi = 4.01 \text{ eV}$

The barrier height, $\phi_B = \phi - \chi = 4.55 - 4.01 = 0.54 \text{ eV}$

- 2) Here, $A_o = 110 \text{ AK}^{-2} \text{ cm}^{-2} = 110 \times 10^4 \text{ AK}^{-2} \text{ m}^{-2} = 1.1 \times 10^6 \text{ AK}^{-2} \text{ m}^{-2}$

$$T = 300 \text{ K}, \phi_B = 0.54 \text{ eV} = 0.54 \times 1.6 \times 10^{-19} = 8.64 \times 10^{-20} \text{ J}$$

Given,

$$J_o = A_o T^2 \exp(-\phi_B/KT)$$

Therefore, Reverse saturation current, $I_o = J_o A$ (A = area)

$$I_o = A A_o T^2 \exp(-\phi_B/KT)$$

$$= 0.01 \times 10^{-4} \times 1.1 \times 10^6 \times (300)^2 \times \exp\left(\frac{-8.64 \times 10^{-20}}{1.38 \times 10^{-23} \times 300}\right)$$

$$= 8.55 \times 10^{-5} \text{ A}$$

- 3) When the applied voltage $V = 0.2$ volt is applied, the forward current is,

$$I = I_o \left[\exp\left(\frac{eV}{KT}\right) - 1 \right]$$

$$= 8.55 \times 10^{-5} \left[\exp\left(\frac{1.6 \times 10^{-19} \times 0.2}{1.38 \times 10^{-23} \times 300}\right) - 1 \right]$$

$$I = 0.2 \text{ A}$$

13. In an abrupt Si p+n junction, the mobility for minority electrons and holes are $120 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ and $440 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$ at $T = 300 \text{ K}$. The life time of holes in n-region is 417 ns , whereas that of electrons in the p-region is 5 ns . Calculate the diffusion coefficients and minority carrier diffusion lengths.

Solution:

$$\text{Here, } \mu_e = 120 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1} = 0.012 = 1.2 \times 10^{-2} \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}$$

$$\mu_h = 440 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1} = 4.4 \times 10^{-2} \text{ m}^2 \text{ V}^{-1} \text{ S}^{-1}, T = 300 \text{ K}$$

Thus, from Einstein's relation

$$\frac{D_e}{\mu_e} = \frac{KT}{e} \Rightarrow D_e = \frac{KT \mu_e}{e}$$

$$D_e = \frac{1.38 \times 10^{-23} \times 300 \times 0.012}{1.6 \times 10^{-19}} = 3.10 \times 10^{-4} \text{ m}^2 \text{ S}^{-1}$$

$$\text{And, } D_h = \frac{KT \mu_h}{e} = \frac{1.38 \times 10^{-23} \times 300 \times 4.4 \times 10^{-2}}{1.6 \times 10^{-19}} = 1.139 \times 10^{-3} \text{ m}^2 \text{ S}^{-1}$$

$$\text{Here, } \tau_e = 5 \text{ ns} = 5 \times 10^{-9} \text{ sec, } \tau_h = 417 \text{ ns} = 4.17 \times 10^{-7} \text{ s}$$

The diffusion lengths are given by,

$$L_e = \sqrt{D_e \tau_e} = \sqrt{3.10 \times 10^{-4} \times 5 \times 10^{-9}} = 1.2 \times 10^{-6} \text{ m}$$

$$\text{and } L_h = \sqrt{D_h \tau_h} = \sqrt{1.139 \times 10^{-3} \times 4.17 \times 10^{-7}} = 2.18 \times 10^{-5} \text{ m}$$

14. An abrupt Si p+n junction has a cross sectional area of 1 mm^2 , an acceptor concentration of $5 \times 10^{18} \text{ cm}^{-3}$ on the p-side and a donor concentration of 10^{16} cm^{-3} on the n-side. What is the built in potential across the Junction. What is the reverse

thermally generated current when the diode is reverse biased by a voltage $V_r = 5 \text{ V}$. Given $T = 300 \text{ K}$, $n_i = 1.145 \times 10^{10} \text{ cm}^{-3}$ and mean thermal generation time is $1 \mu\text{s}$.

Solution:

$$\text{Here } T = 300 \text{ K}, N_a = 5 \times 10^{18} \text{ cm}^{-3} = 5 \times 10^{24} \text{ m}^{-3},$$

$$N_d = 10^{16} \text{ cm}^{-3} = 10^{22} \text{ m}^{-3}, n_i = 1.145 \times 10^{10} \text{ cm}^{-3} = 1.145 \times 10^{16} \text{ m}^{-3}.$$

The built in potential is,

$$V_o = \frac{KT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln \left(\frac{5 \times 10^{24} \times 10^{22}}{(1.145 \times 10^{16})^2} \right)$$

$$V_o = 0.87 \text{ V}$$

When a reverse bias of V_r is applied, the potential difference across the depletion region becomes $V_o + V_r$, and the width of depletion region is

$$\begin{aligned} W &= \left[\frac{2\epsilon(V_o + V_r)}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} \\ &= \left[\frac{2\epsilon_r \epsilon_o (V_o + V_r)}{e} \frac{(N_a + N_d)}{N_a N_d} \right]^{1/2} \\ &= \left[\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times (0.87 + 5)}{1.6 \times 10^{-19}} \left(\frac{5 \times 10^{24} + 10^{22}}{5 \times 10^{24} \times 10^{22}} \right) \right]^{1/2} \end{aligned}$$

$$W = 8.8 \times 10^{-7} \text{ m} = 0.88 \mu\text{m}$$

The thermal generation current is

$$I_{gen} = J_{gen} \cdot A = \frac{en_i W \cdot A}{\tau_g}$$

$$I_{gen} = \frac{1.6 \times 10^{-19} \times 1.145 \times 10^{16} \times 8.8 \times 10^{-7} \times 1 \times 10^{-6}}{1 \times 10^{-6}}$$

$$= 1.61 \times 10^{-9} \text{ A}$$

$$= 1.61 \text{ nA}$$

15. An abrupt p⁺n Junction diode has a cross sectional area of 1 mm^2 , an acceptor concentration of $5 \times 10^{18} \text{ cm}^{-3}$ on the p-side and donor concentration of 10^{16} cm^{-3} on the n-side. What is the current when there is a forward bias of 0.6 V across the diode at 27°C and at 100°C? Given, at 27°C, $D_e = 3.10 \text{ cm}^2 \text{ S}^{-1}$, $D_h = 11.39 \text{ cm}^2 \text{ S}^{-1}$, $L_e = 1.2 \mu\text{m}$, $L_h = 21.8 \mu\text{m}$, $n_i = 1.145 \times 10^{10} \text{ cm}^{-3}$. At 100°C, $n_i = 1.145 \times 10^{12} \text{ cm}^{-3}$

Solution:

The forward current is given by (For $T = 27^\circ\text{C} = 300 \text{ K}$)

$$I = I_s \left[\exp \left(\frac{eV}{KT} \right) - 1 \right]$$

$$\text{Where, } I_s = A \cdot J_s = A e n_i^2 \left[\frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_a} \right]$$

$$= 1 \times 10^{-6} \times 1.6 \times 10^{-19} \times (1.145 \times 10^{16})^2 \left[\frac{11.39 \times 10^{-4}}{21.8 \times 10^{-6} \times 10^{22}} + \frac{3.1 \times 10^{-4}}{1.2 \times 10^{-6} \times 5 \times 10^{24}} \right]$$

$$= 2.1 \times 10^{-6} \times [5.22 \times 10^{-21} + 5.167 \times 10^{-23}]$$

$$= 1.1 \times 10^{-14} \text{ A}$$

Therefore,

$$I = 1.1 \times 10^{-14} \left[\exp \left(\frac{1.6 \times 10^{-19} \times 0.6}{1.38 \times 10^{-23} \times 300} \right) - 1 \right]$$

$$= 1.29 \times 10^{-4} \text{ A}$$

We know that,

$$I \propto n_i^2$$

$$\text{Therefore, } \frac{I_s(27^\circ\text{C})}{I_s(100^\circ\text{C})} = \frac{n_i^2(27^\circ\text{C})}{n_i^2(127^\circ\text{C})}$$

$$I_s(\text{at } 100^\circ\text{C}) = \frac{n_i^2(\text{at } 127^\circ\text{C})}{n_i^2(\text{at } 27^\circ\text{C})} \times I_s(\text{at } 27^\circ\text{C})$$

$$= \left(\frac{1.145 \times 10^{12}}{1.145 \times 10^{10}} \right)^2 \times 1 \times 10^{-14}$$

$$I_s(\text{at } 100^\circ\text{C}) = 1.1 \times 10^{-10} \text{ A}$$

Now, At 100°C , the forward current with 0.6 V across the diode is

$$I = I_s \left[\exp \left(\frac{eV}{KT} \right) - 1 \right]$$

$$= 1.1 \times 10^{-10} \left[\exp \left(\frac{1.6 \times 10^{-19} \times 0.6}{1.38 \times 10^{-23} \times 373} \right) - 1 \right]$$

$$= 0.014 \text{ A}$$

A pn junction Semiconductor has resistivity of $5\Omega\text{-cm}$. If mobility of hole is $450 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and electron mobility is three times the mobility of hole. At room temperature, find (i) Built in potential (ii) depletion width that lies in n-region and p-region and (iii) built in electric field at $x = 0$ (Given, $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$ at $T = 300 \text{ K}$, $\epsilon_r = 11.9$ for Si)

tion:

$$\text{Here, } \rho = 5\Omega\text{-cm} = 5 \times 10^{-2} \Omega\text{-m}$$

$$\text{Therefore } \sigma = \frac{1}{\rho} = \frac{1}{5 \times 10^{-2}} = 20 \Omega^{-1}\text{m}^{-1}$$

$$\mu_h = 450 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} = 450 \times 10^{-4} \text{ m}^2\text{V}^{-1}\text{s}^{-1}$$

$$\mu_e = 3 \times 450 = 1350 \text{ cm}^2\text{V}^{-1}\text{s}^{-1} = 1350 \times 10^{-4} \text{ m}^2\text{V}^{-1}\text{s}^{-1}$$

$$T = 300 \text{ K}$$

$$\text{For n-side, } \sigma = N_d e \mu_e$$

$$\Rightarrow N_d = \frac{\sigma}{e \mu_e} = \frac{20}{1.6 \times 10^{-19} \times 1350 \times 10^{-4}} = 9.26 \times 10^{20} \text{ m}^{-3}$$

For p - side, $\sigma = N_a e \mu_h$

$$N_a = \frac{\sigma}{e \mu_h} = \frac{20}{1.6 \times 10^{-19} \times 450 \times 10^{-4}} = 2.78 \times 10^{21} \text{ m}^3$$

1) The built in potential is given by

$$V_o = \frac{KT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

$$V_o = \left(\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \right) \ln \left[\frac{2.78 \times 10^{21} \times 9.26 \times 10^{20}}{(1.45 \times 10^{16})^2} \right] = 0.6 \text{ Volts}$$

2) The total depletion width is given by

$$\begin{aligned} w &= \sqrt{\frac{2\epsilon V_o}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right)} \\ &= \sqrt{\frac{2\epsilon_r \epsilon_0 V_o}{e} \left(\frac{N_a + N_d}{N_a \cdot N_d} \right)} \\ &= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.6}{1.6 \times 10^{-19}} \left(\frac{2.78 \times 10^{21} + 9.26 \times 10^{20}}{2.78 \times 10^{21} \times 9.26 \times 10^{20}} \right)} \\ &= 1.07 \times 10^{-6} \text{ m} = 1.07 \mu\text{m} \end{aligned}$$

Now the depletion width in n -region is

$$\begin{aligned} x_n &= \left[\frac{2\epsilon V_o}{e N_d \left[1 + \frac{N_d}{N_a} \right]} \right]^{1/2} \left[\frac{2\epsilon_r \epsilon_0 V_o}{e N_d} \left(\frac{N_a}{N_a + N_d} \right) \right]^{1/2} \\ x_n &= \left[\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.6 \times 2.78 \times 10^{21}}{1.6 \times 10^{-19} \times 9.26 \times 10^{20} (2.78 \times 10^{21} + 9.26 \times 10^{20})} \right]^{1/2} \\ &= 0.8 \times 10^{-6} \text{ m} = 0.8 \mu\text{m} \end{aligned}$$

Therefore, depletion width in p-region is,

$$x_p = w - x_n = 1.07 - 0.8 = 0.27 \mu\text{m}$$

3) The built in electric field is given by

$$E_o = \frac{V_o}{w} = \frac{0.6}{1.07 \times 10^{-6}} = 5.6 \times 10^5 \text{ V/m}$$

17. In an n-type semiconductor, the Fermi level lies 0.4 eV below the conduction band. If the concentration of donor atoms is doubled find the new position of the Fermi level. Assume $K_B T = 0.03 \text{ eV}$.

Solution:

For intrinsic semiconductor we have, $n = N_c \exp \left[-\frac{(E_c - E_{Fi})}{KT} \right]$

Where as for n-type doped semiconductor, $n = N_c \exp \left[-\frac{(E_c - E_{Fn})}{KT} \right]$

$$i.e. N_d = N_c \exp\left[\frac{-(E_c - E_{Fn})}{KT}\right]$$

Here, $E_c - E_{Fn} = 0.4 \text{ eV}$

$$\text{Therefore, } N_d = N_c \exp\left(\frac{-0.4 \text{ eV}}{KT}\right) \quad \dots(1)$$

$$\text{For second case, } 2 N_d = N_c \exp\left[\frac{-(E_c - E_{Fn})}{KT}\right] \quad \dots(2)$$

Dividing equation (2) by (1)

$$2 = \exp\left[-\frac{(E_c - E_{Fn})}{KT} + \frac{0.4 \text{ eV}}{KT}\right]$$

$$-\frac{(E_c - E_{Fn})}{KT} + \frac{0.4 \text{ eV}}{KT} = \ln(2)$$

$$-(E_c - E_{Fn}) + 0.4 \text{ eV} = KT \ln(2)$$

$$\begin{aligned} (E_c - E_{Fn}) &= 0.4 \text{ eV} - KT \ln(2) \\ &= 0.4 \text{ eV} - [0.03 \times \ln(2)] \\ &= 0.379 \text{ eV.} \end{aligned}$$

18. In an n-type Ge semiconductor, the donor impurity concentration is 1 atom per 10^8 Ge atoms. At what temperature will the Fermi level coincide with the edge of the conduction band? (Given: concentration of the Ge atoms is 4.4×10^{22} per cm^3 , effective mass of electron is 0.8 times its true mass.)

Solution:

Here, 10^8 Ge atoms contain 1 donor atom

1 Ge atom contains $\frac{1}{10^8}$ donor atom

4.4×10^{22} Ge atoms contain $\frac{1}{10^8} \times 4.4 \times 10^{22}$ donor atoms

Therefore, $N_d = \frac{4.4 \times 10^{22}}{10^8} \text{ cm}^{-3} = 4.4 \times 10^{14} \text{ cm}^{-3} = 4.4 \times 10^{20} \text{ m}^{-3}$

Temperature, $T = ?$, $E_c - E_{Fn} = 0$, $m_e^* = 0.8 m_e$

We have,

$$n = N_c e^{-\frac{(E_c - E_{Fn})}{KT}} \Rightarrow n = N_c$$

$$N_d = 2 \left(\frac{2\pi m_e^* K T}{h^2} \right)^{3/2}$$

$$\frac{2\pi m_e^* K T}{h^2} = \left(\frac{N_d}{2} \right)^{2/3}$$

$$T = \frac{h^2}{2\pi m_e K} \left(\frac{N_d}{2} \right)^{2/3}$$

$$= \frac{(6.624 \times 10^{-34})^2}{2 \times 3.14 \times 0.8 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23}} \left(\frac{4.4 \times 10^{20}}{2} \right)^{2/3}$$

$$T = 0.25 \text{ K}$$

19. p and n side of a semiconductor are characterized by

Doping level in p side = 10 times of doping level of n-side. Mobility in n side, $\mu_e = 1200 \text{ cm}^2/\text{V.S.}$, $\mu_h = 275 \text{ cm}^2/\text{V.S.}$ Mobility in p-side, $\mu_e = 1000 \text{ cm}^2/\text{V.S.}$, $\mu_h = 225 \text{ cm}^2/\text{V.S.}$ Mean diffusion life time $\tau_n = 0.1 \mu\text{s}$ (n-side), $\tau_e = 1 \mu\text{s}$ (p - side) calculate at $T = 300 \text{ K}$.

- Conductivity in bulk n and p region if built in potential is 0.816 V.
- Width of depletion layer
- Reverse saturation current, if area of cross section is 6 mm^2 .

Solution:

$$1. \text{ We have, } V_o = \frac{KT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) \Rightarrow \ln \left(\frac{N_a N_d}{n_i^2} \right) = \frac{eV_o}{KT}$$

$$\frac{N_a N_d}{n_i^2} = \exp \left(\frac{eV_o}{KT} \right) \Rightarrow N_a \cdot N_d = n_i^2 \exp \left(\frac{eV_o}{KT} \right)$$

$$\text{According to question } N_a = 10 N_d$$

$$\text{Therefore, } 10 N_d \cdot N_d = n_i^2 \exp \left(\frac{eV_o}{KT} \right)$$

$$N_d^2 = \frac{n_i^2}{10} \exp \left(\frac{eV_o}{KT} \right)$$

$$N_d = \frac{n_i}{\sqrt{10}} \exp \left(\frac{eV_o}{2KT} \right)$$

$$= \frac{1.45 \times 10^{16}}{\sqrt{10}} \exp \left(\frac{1.6 \times 10^{-19} \times 0.816}{2 \times 1.38 \times 10^{-23} \times 300} \right)$$

$$N_d = 3.23 \times 10^{22} \text{ m}^{-3}$$

$$\text{and } N_a = 10 N_d = 3.23 \times 10^{23} \text{ m}^{-3}$$

Now, conductivity in n-side, $\sigma_n = N_d e \mu_e$ (in n-side)

$$\sigma_n = 3.23 \times 10^{22} \times 1.6 \times 10^{-19} \times 1200 \times 10^{-4}$$

$$\sigma_n = 620.16 \Omega^{-1} \text{ m}^{-1}$$

Again, conductivity in p-side, $\sigma_p = N_a e \mu_h$ (in p-side)

$$\sigma_p = 3.23 \times 10^{23} \times 1.6 \times 10^{-19} \times 225 \times 10^{-4} = 1162.8 \Omega^{-1} \text{ m}^{-1}$$

2. The width of depletion layer is given by

$$W = \left[\frac{2eV_o}{e} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2} = \left[\frac{2e_r \epsilon_0 V_o}{e} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

$$W = \left[\frac{2 \times 11.9 \times 8.85 \times 10^{-12} \times 0.816}{1.6 \times 10^{-19}} \left(\frac{3.23 \times 10^{23} + 3.23 \times 10^{22}}{3.23 \times 10^{23} \times 3.23 \times 10^{22}} \right) \right]^{1/2}$$

$$W = 1.913 \times 10^{-7} \text{ m} = 0.19 \mu\text{m}$$

3. Here, $A = 6 \text{ mm}^2 = 6 \times 10^{-6} \text{ m}^2$

$$\text{We have, } I_s = A \cdot J_s = A e n_i^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right)$$

$$\text{We know, } \frac{D_e}{\mu_e} = \frac{KT}{e} \Rightarrow D_e = \frac{KT}{e} \mu_e \text{ (in p-side)}$$

$$D_e = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times 1000 \times 10^{-4} = 2.59 \times 10^{-3} \text{ m}^2 \text{ S}^{-1}$$

$$\text{Similarly, } \frac{D_h}{\mu_h} = \frac{KT}{e} \Rightarrow D_h = \frac{KT}{e} \mu_h \text{ (in n-side)}$$

$$D_h = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times 275 \times 10^{-4} = 7.12 \times 10^{-4} \text{ m}^2 \text{ S}^{-1}$$

$$\text{Again, } L_e = \sqrt{D_e \tau_e} = \sqrt{2.59 \times 10^{-3} \times 1 \times 10^{-6}} = 5 \times 10^{-5} \text{ m}$$

$$\text{And, } L_h = \sqrt{D_h \tau_h} = \sqrt{7.12 \times 10^{-4} \times 0.1 \times 10^{-6}} = 8.44 \times 10^{-6} \text{ m}$$

Hence,

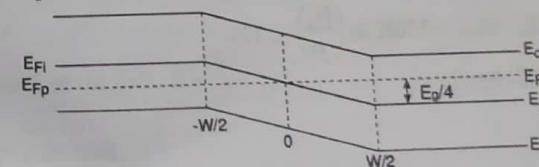
$$I_s = 6 \times 10^{-6} \times 1.6 \times 10^{-19} \times (1.45 \times 10^{16})^2 \left(\frac{D_e}{L_e N_a} + \frac{D_h}{L_h N_d} \right)$$

$$= 2.018 \times 10^6 \left(\frac{2.59 \times 10^{-3}}{5 \times 10^{-5} \times 3.23 \times 10^{23}} + \frac{7.12 \times 10^{-4}}{8.44 \times 10^{-6} \times 3.23 \times 10^{23}} \right)$$

$$= 2.018 \times 10^6 (1.6 \times 10^{-22} + 2.61 \times 10^{-21})$$

$$I_s = 5.59 \times 10^{-15} \text{ A}$$

20. A semiconductor junction is characterized by the energy band diagram given below.



If the material is Si at 300 K, determine the resistivity of the region $x > \frac{W}{2}$. Given, $n_i = 1.181 \times 10^{10} \text{ cm}^{-3}$, $Eg = 1.12 \text{ eV}$, $\mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ S}^{-1}$.

Solution:

$$\text{Here, from figure, } E_{Fn} - E_{Fi} = \frac{E_s}{4} = \frac{1.12}{4} = 0.28 \text{ eV}$$

We have, $N_d = n = n_i \exp\left(\frac{E_{F0} - E_F}{KT}\right)$

$$N_d = 1.181 \times 10^{16} \exp\left(\frac{0.28 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}\right)$$

$$= 5.9 \times 10^{20} \text{ m}^{-3}$$

Given, $\mu_e = 1350 \text{ cm}^2 \text{V}^{-1} \text{S}^{-1} = 1350 \times 10^{-4} \text{ m}^2 \text{V}^{-1} \text{S}^{-1}$

Therefore, $\sigma = N_d e \mu_e$

$$= 5.9 \times 10^{20} \times 1.6 \times 10^{-19} \times 1350 \times 10^{-4}$$

$$= 12.74 \Omega^{-1} \text{m}^{-1}$$

And, $\rho = \frac{1}{\sigma} = 0.0785 \Omega \text{m}$

$\rho = 7.85 \Omega \cdot \text{cm}$.

21. In an n-type semi conductor, the Fermi level lies 0.3 eV below the conduction band at 300 K. If the temperature is increased to 330 K, find the new position of the Fermi level.

Solution:

We have, $n = N_c e^{\frac{-(E_c - E_{Fn})}{KT}}$

$$e^{\frac{(E_c - E_{Fn})}{KT}} = \frac{N_c}{n}$$

$$(E_c - E_{Fn}) = KT \ln\left(\frac{N_c}{n}\right)$$

For first case, 0.3 eV = 300K $\ln\left(\frac{N_c}{n}\right)$... (1)

For second case, $(E_c - E_{Fn}) = 330K \ln\left(\frac{N_c}{n}\right)$... (2)

Dividing equation (2) by (1)

$$\frac{E_c - E_{Fn}}{0.3 \text{ eV}} = \frac{330}{300}$$

$$E_c - E_{Fn} = 1.1 \times 0.3$$

$$E_c - E_{Fn} = 0.33 \text{ eV}$$

22. The minority carrier life time in p-type material is 10^{-7} sec. The mobility of electron in silicon is $0.15 \text{ m}^2 \text{V}^{-1} \text{S}^{-1}$ at 300K. a) What is the diffusion length L_e b) If 10^{20} electrons / m^3 are injected at $x = 0$, what is the diffusion current density at $x = 0$.

Solution:

- a. The diffusion coefficient D_e is given by

$$\frac{D_e}{\mu_e} = \frac{KT}{e}$$

$$D_e = \frac{KT \mu_e}{e} = \frac{1.38 \times 10^{-23} \times 300 \times 0.15}{1.6 \times 10^{-19}}$$

$$= 3.88 \times 10^{-3} \text{ m}^2 \text{S}^{-1}$$

Thus the diffusion length is

$$L_e = \sqrt{D_e \tau_e}$$

$$= \sqrt{3.88 \times 10^{-3} \times 10^{-7}}$$

$$= 1.97 \times 10^{-5} \text{ m}$$

- b. The diffusion current density at $x = 0$ is given by,

$$J_e = \frac{e D_e \Delta n(x)}{L_e} = \frac{e D_e \Delta n(0)}{L_e}$$

$$= \frac{1.6 \times 10^{-19} \times 3.88 \times 10^{-3} \times 10^{20}}{1.97 \times 10^{-5}}$$

$$= 3.15 \times 10^3 \text{ A/m}^2$$

23. Find the resistance of p-n junction Germanium diode if temperature is 27°C and $I_o = 1\text{ }\mu\text{A}$ for an applied forward bias of 0.2 Volt.

Solution:

Here, $T = 27^\circ\text{C} = 300 \text{ K}$, $I_o = 10^{-6} \text{ A}$, $V = 0.2 \text{ Volt}$

We have,

$$I = I_o \left[\exp\left(\frac{eV}{KT}\right) - 1 \right]$$

$$= 10^{-6} \left[\exp\left(\frac{1.6 \times 10^{-19} \times 0.2}{1.38 \times 10^{-23} \times 300}\right) - 1 \right]$$

$$= 2.27 \times 10^{-3} \text{ A}$$

Now the resistance of the bulk, $R = \frac{V}{I}$

$$R = \frac{0.2}{2.27 \times 10^{-3}} = 88.10 \Omega$$

24. Find the value of the applied forward voltage for a p-n junction diode if $J_o = 30 \text{ micro amp/cm}^2$ and $J = 2 \text{ ampere/cm}^2$, $T = 300 \text{ K}$.

Solution:

The general formula is,

$$J = J_o \left[\exp\left(\frac{eV}{KT}\right) - 1 \right]$$

$$\exp\left(\frac{eV}{KT}\right) - 1 = \frac{J}{J_o}$$

$$\exp\left(\frac{eV}{KT}\right) = 1 + \frac{J}{J_o}$$

$$\frac{eV}{KT} = \ln\left(1 + \frac{J}{J_o}\right)$$

$$V = \frac{KT}{e} \ln\left(1 + \frac{J}{J_o}\right)$$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln\left(1 + \frac{2}{30 \times 10^6}\right)$$

$$V = 0.287 \text{ Volt}$$

Exercise

1. Prove that the position of Fermi level is near the middle of band gap in pure silicon semiconductor.
2. Derive the relation for finding the concentration of electron in an extrinsic semiconductor.
3. Explain how many are there electrons available in conduction band in n type semiconductor even if average thermal energy is in sufficient to surmount the electrons from valence band to conduction band.
4. Derive the relation for intrinsic concentration and explain how temperature effects intrinsic concentration.
5. Explain about intrinsic Fermi level of pure semiconductor and derive a relationship of the intrinsic Fermi level assuming that intrinsic carrier concentration is known.
6. Explain how carrier concentration of a semiconductor depends on temperature.
7. Show that the electron concentration in CB is equal to hole concentration in VB in intrinsic semiconductor. But conductivity due to electrons and due to holes is not equal to each other in the same semiconductor. Use necessary mathematical expression to justify your answer.
8. Explain the effect of energy gap and temperature on the electron and hole concentration of intrinsic semiconductor.
9. Starting from Fermi-Dirac distribution function prove that the Fermi level in intrinsic semiconductor lies midway between the conduction band and valence band.
10. With necessary derivation show that the Fermi level in intrinsic semiconductor lies between conduction band and valence band.
11. Derive the relation for electron concentration in intrinsic semiconductor.
12. Derive the relation for hole concentration in intrinsic semiconductor.