Computer Graphics (L09)

EG678EX

3-D Transformations

Translation

$x' = x + t_x$

$y' = y + t_y$

$z' = z + t_z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = T.P$$

Rotation (about co-ordinate axes)

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_z(\theta).P$$

$$y' = y\cos\theta - z\sin\theta$$
$$z' = y\sin\theta + z\cos\theta$$
$$x' = x$$

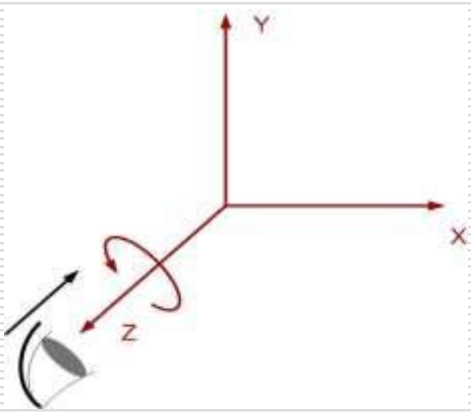
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

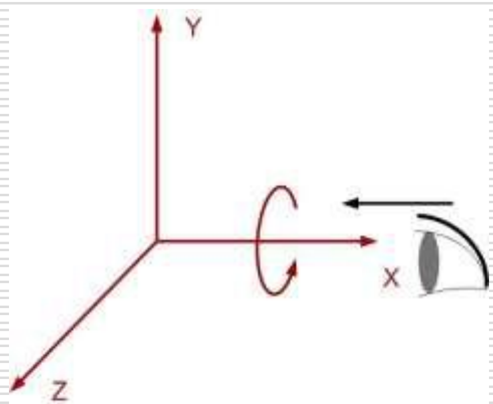
$$P' = R_x(\theta).P$$

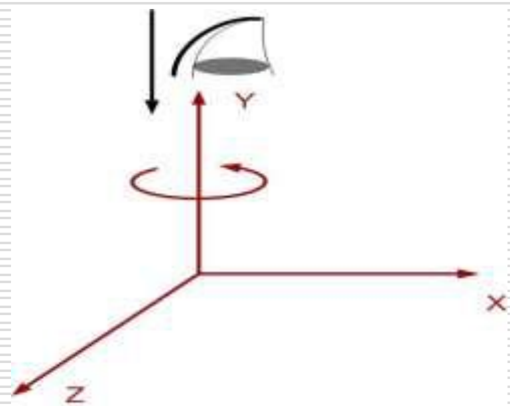
$$z' = z\cos\theta - x\sin\theta$$
$$x' = z\sin\theta + x\cos\theta$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = R_y(\theta).P$$



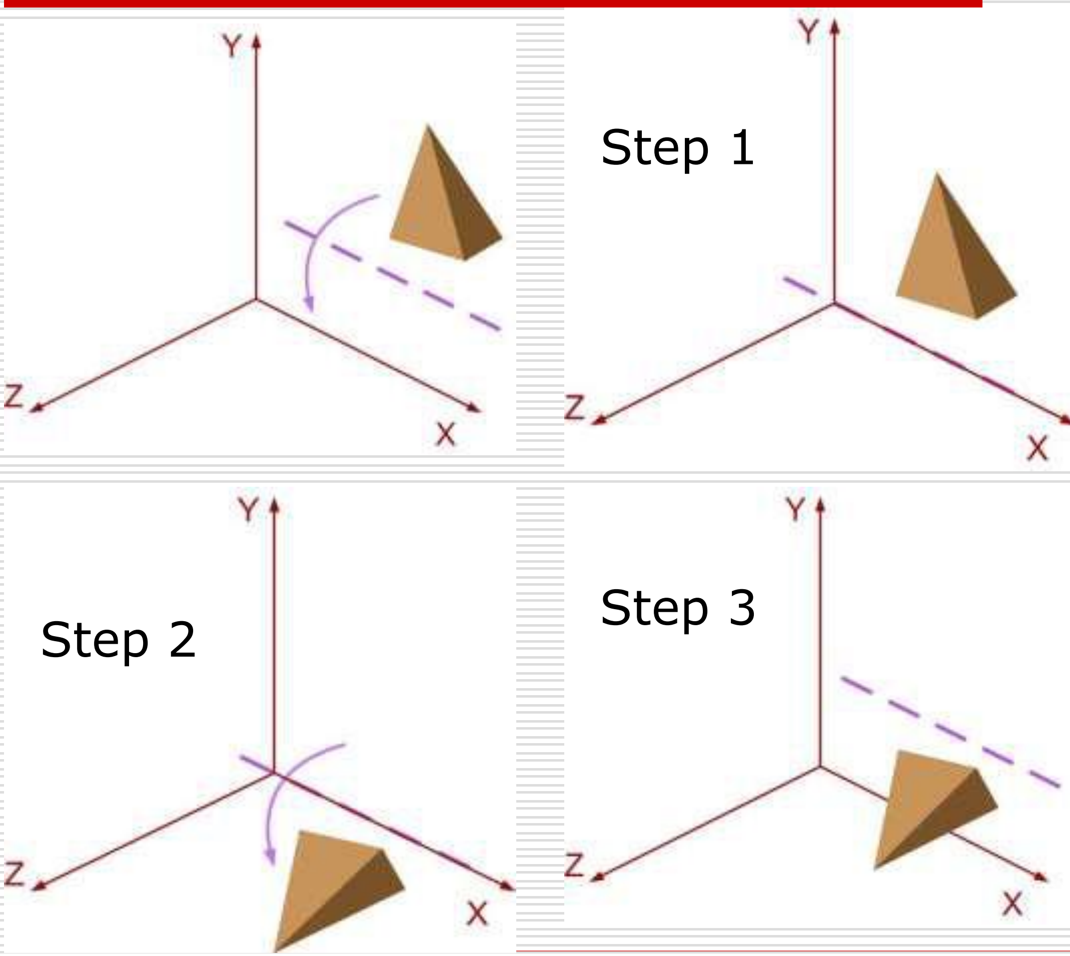




Remember The cyclic order:

$$x \rightarrow y \rightarrow z \rightarrow x$$

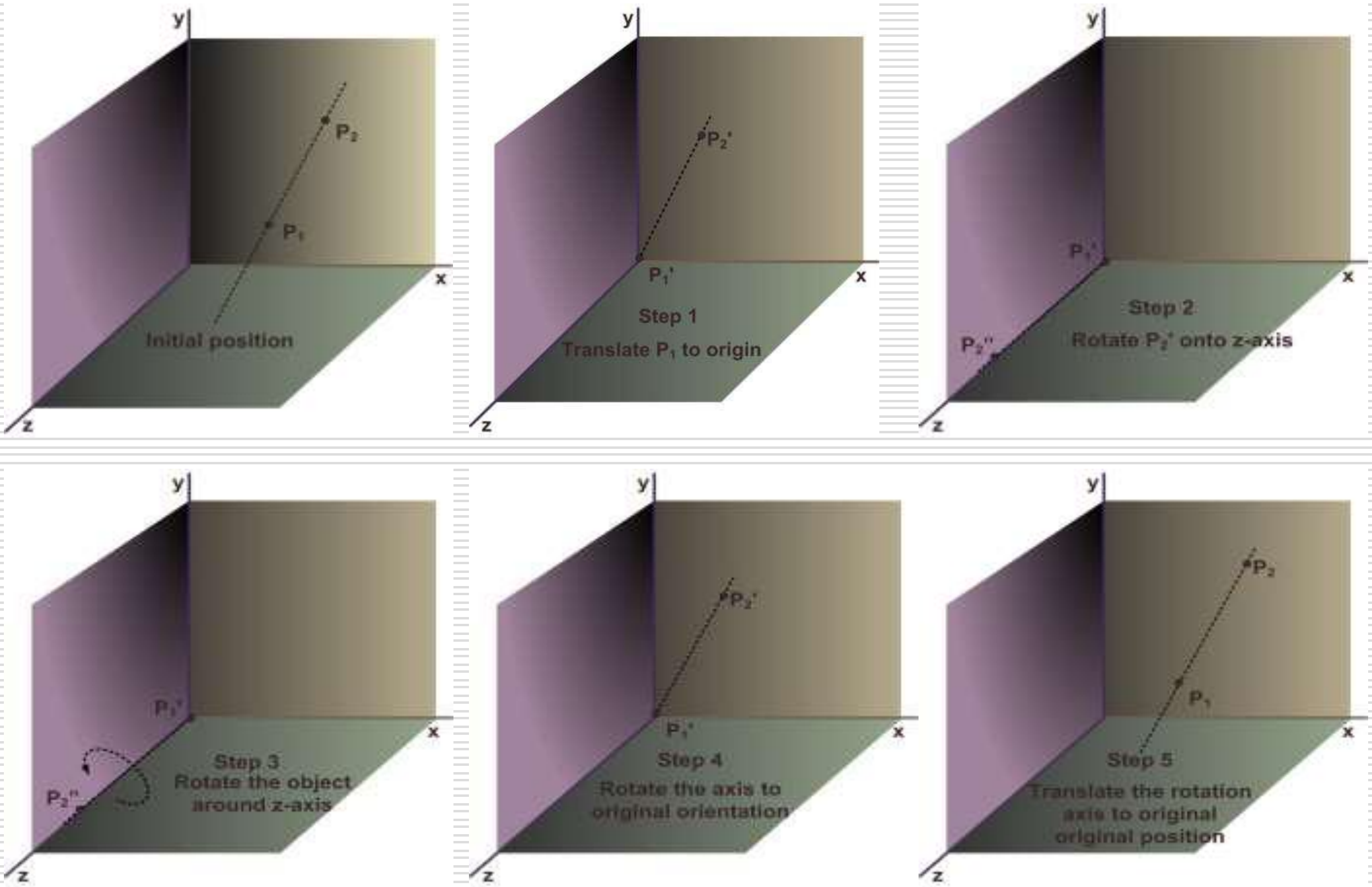
Rotation about axis parallel to co-ordinate axis



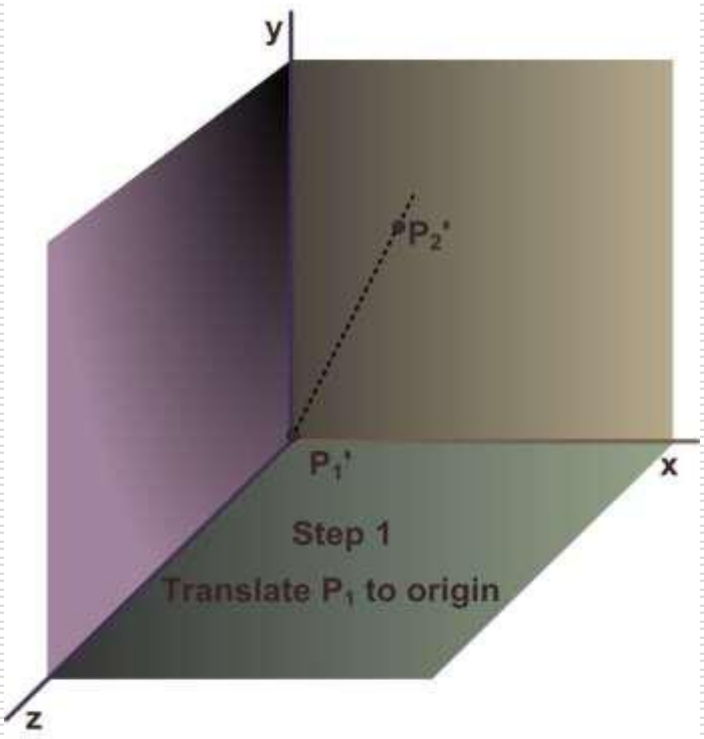
Steps

- Translate the object so as to coincide rotation axis to parallel co-ordinate axis
- Perform the rotation about the axis
- Translate back the object so as to move rotation axis to original position

General 3-D Rotation



General 3-D rotation Mathematics

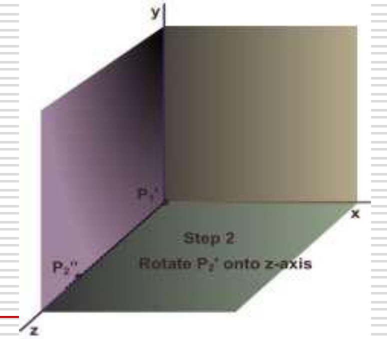


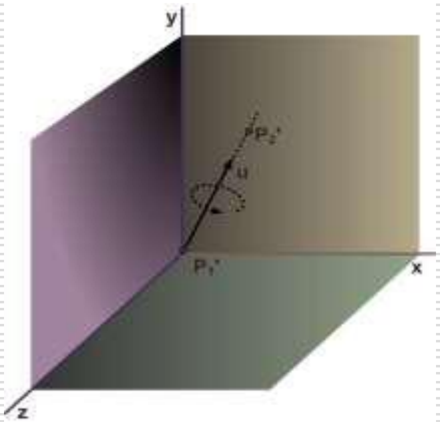
Step 1

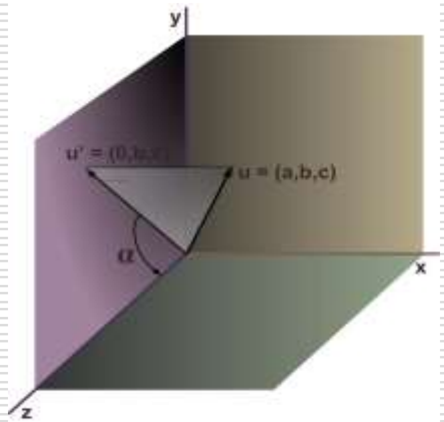
Translate object so as to coincide P_1 to origin

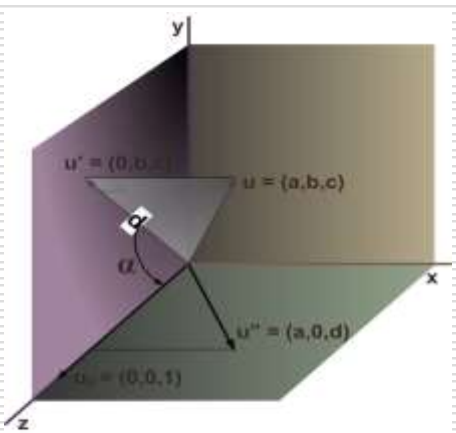
$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

General 3-D Rotation Mathematics









Step 2

- ❑ Accomplished in two steps

i. Perform x axis rotation with angle α so as to bring rotation axis to zx plane

ii. Perform y axis rotation with angle ß so as to coincide the rotation axis with z-axis

X- Axix rotation (α)

$$\vec{V} = \vec{p}_2 - \vec{p}_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

$$\vec{u} = \frac{\vec{V}}{|\vec{V}|} = (a, b, c)$$

$$a = \frac{x_2 - x_1}{|\vec{V}|}, \quad b = \frac{y_2 - y_1}{|\vec{V}|}, \quad c = \frac{z_2 - z_1}{|\vec{V}|}$$

$$\cos\alpha = \frac{\vec{u}'.\vec{u}_z}{|\vec{u}'||\vec{u}_z|} = \frac{c}{d}$$

$$d = \sqrt{b^2 + c^2}$$

$$\vec{u}' \times \vec{u}_z = \vec{u}_x |\vec{u}'||\vec{u}_z| \sin\alpha$$

$$\vec{u}' \times \vec{u}_z = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ 0 & b & c \\ 0 & 0 & 1 \end{vmatrix} = \vec{u}_x.b$$

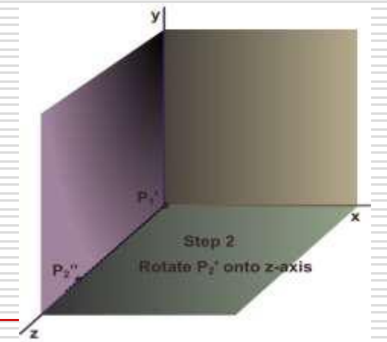
$|\vec{u}'| = d$
and
$|\vec{u}_z| = 1$

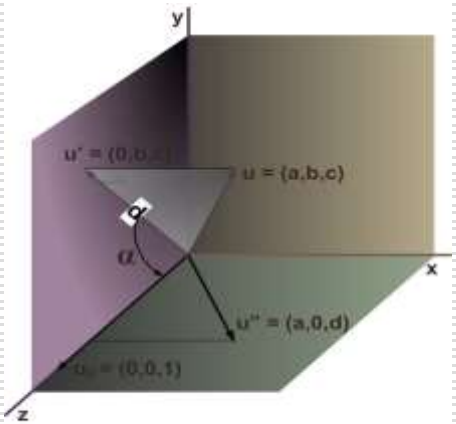
$$d\sin\alpha = b$$

$$\sin\alpha = \frac{b}{d}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

General 3-D Rotation Mathematics





Y-Axis Rotation (β)

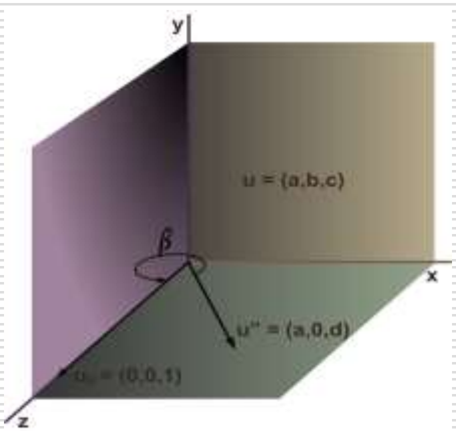
$$\cos\beta = \frac{\vec{u}''.\vec{u}_z}{|\vec{u}''||\vec{u}_z|} = d$$

$$\vec{u}''\times\vec{u}_z = \vec{u}_y|\vec{u}''||\vec{u}_z|\sin\beta$$

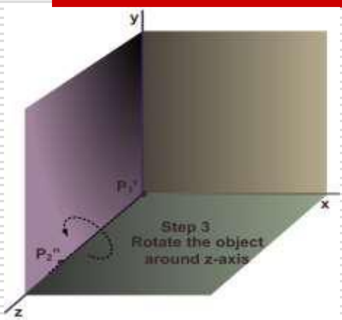
$$\vec{u}''\times\vec{u}_z = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ a & 0 & d \\ 0 & 0 & 1 \end{vmatrix} = \vec{u}_y.(-a)$$

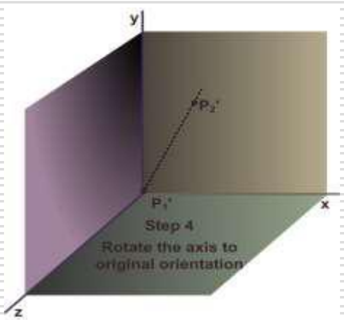
$$\sin\beta = -a$$

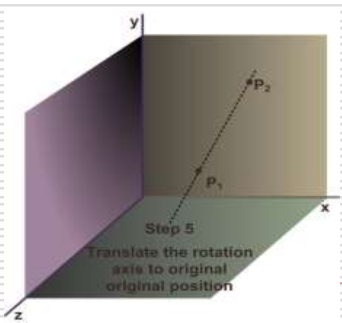
$$R_y(\beta) = \begin{bmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



General 3-D Rotation Mathematics







Step 3

❑Perform z-axis Rotation with angle θ (the angle which the object is to be rotated about the given axis)

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 4

❑Perform inverse Rotation and i.e $\mathbf{R_x^{-1}(\alpha)}$ and $\mathbf{R_y^{-1}(\beta)}$

Step 5

❑Perform inverse Translation i.e $\mathbf{T^{-1}}$

Final composite Matrix is obtained as:

$$R(\theta) = T^{-1}.R_x^{-1}(\alpha).R_y^{-1}(\beta).R_z(\theta).R_y(\beta).R_x(\alpha).T$$

3-D Scaling

Scaling about origin

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fixed Point Scaling

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & (1-s_x)x_f \\ 0 & s_y & 0 & (1-s_y)y_f \\ 0 & 0 & s_z & (1-s_z)z_f \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$S(x_f, y_f, z_f, s_x, s_y, s_z) = T(x_f, y_f, z_f).S(s_x, s_y, s_z).T(-x_f, -y_f, -z_f)$$

3-D Reflection

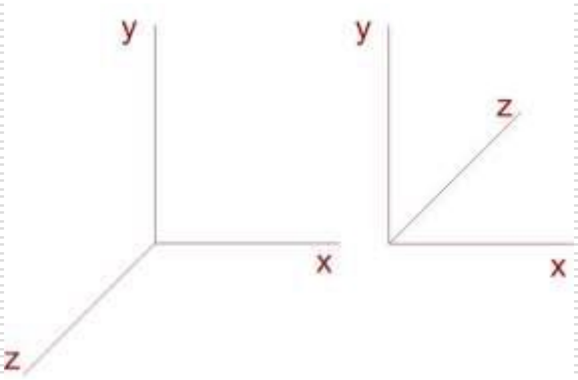
- Performed relative to a reflection axis or reflection plane
- Axis reflection→ equivalent to 180 degree rotation about the axis in 3-D space
- Plane reflection → equivalent to 180 degree rotation in 4-D space
  - 4-D space ?? → not visualized in euclidian space
- Reflection about a plane converts right handed co-ordinate system to left handed co-ordinate system and vice versa

- Reflection in xy plane

  $x' = x$
  $y' = y$
  $z' = -z$

  - Matrix is as

$$RF_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



3-D Shear

- Z-axis shear

  x′ = x + a.z

  y′ = y + b.z

  z′ = z

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$SH_z = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$