Sensor: - The device which change energy from one form to another analysis form.
Transducer: - Sensor which change energy from one form to electrical form.

<u>Signal Condensing:-</u>

Instruments: - Measurement involves the use of instruments as a physical means of determines quantities or variables.
- The instrument serves as an extension of human faculties and enables the man to determine the value of unknown quantity or variable which unaided human faculty can not measure, a measuring instrument exist to provide information about the physical values of some variable being measured.
- In simple cases an instrument consist of a single unit which gives an output reading or signal according to the unknown variable (measured) applied to it.
- In more complex measurement situation a measuring instrument may consist of several separate elements. These separate elements may consist of traducing element which converts the measured to analogies. The analogies signal is then processed by some intermediate means and then fed to the end devices. To presents the results of the measurement for the propose of display and control.
- The above mention component might be contained neither one or more boxes and the boxes holding individual measurement elements might be either close together or physically separate.
- Because of this modular nature of the component contain it; it is common to refer the measuring instrument as a measurement system or instrumentation system.
- The history of development of instrument encompainses three phases:-

(i) Mechanical Instrument:-
- First instrument use by man kind.
- Very reliable for static and stable condition but unable to responds to dynamic and transient conditions.
- having moving parts that are rigid, heavy and bulky has a large mass.

(ii) Electrical Instruments:-
- More rapid than mechanical instrument.
- Depends upon the mechanical meter movement as indicating device.
- Hence have limited time response.

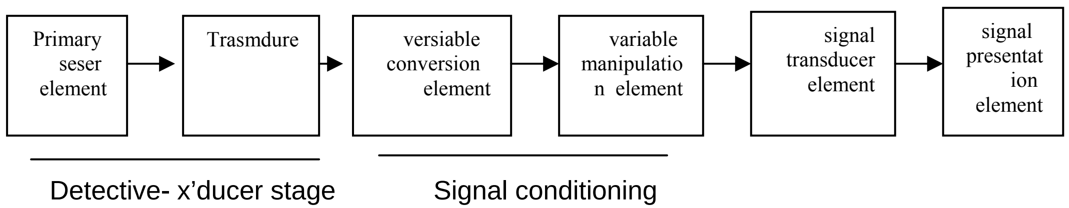
(iii) Electronics instruments:-
- A higher senility.
- A faster response.
- A greater flexibity.
- Lower weight.
- Lower power consumption.

(iv) A higher degree reliability.

Instrumentation System

A system that comprises of input, signal, conditioning and processing and the output is known as instrumentation system.

Components of Instrumentation system:-
Components of instrumentation system can be viewed in below diagram as:-



Primary Sensing elements:-
- Measured is first detected.
- It detect the physical quantity gives the output in different analogous form.
- This analogues output is then converted into electrical signal by a transducer.
-In many cases the physical quantity is directing converted into an electrical quantity by a treasure.
-A transducer is defined as a device which converts a physical quantity into an electrical quantity.

Signal conditioning element:-

(I) variable conversion elements:-

(ii) Variable manipulation element:-
- Change in numerical value of signal.
- Linear processes like amplification, attenuation, integration, differentiation, addition and subtraction.
- Non linear process like modulation, detection sapling and filtering etc.
- The element that follows primary sensing element in any instrumentation system is called system condition element.

Signal transmission element:-

When the element of an instrument are actually physically, separated it become necessary to transmit data from one to another. The element that performs this function is called a signal transmission element. E.g. space craft.
- signal conditioning and transmission stage is commonly known as intermediate stage.

Chapter-2

Measurement:-

Unit and standards of .measurement:-

Unit: - to specify and perform calculations with physical quantities, the physical quantities must be defined both in kind and magnitude. The standard measure of each kind of physical quantity is the unit.

In science and engineering two kinds of unit:-

(Fundamental units:-

(a) Length (l)	-meter (m)	- [L]
(b) Mass (m)	-kilogram (kg)	- [M]
(c) Time (t)	-second (s)	- [T]
(d) Eclectic current	-Ampere (A)	- [I]
(e) Luminous intensity (θ)	-candela (cod)	- [ ]

(ii) **<u>Derived Units</u>**: - All other units which can be expressed in terms of fundamental unit are called derived unit.

e.g.

$$\text{Power} = \frac{\text{work}}{\text{Time}} = \frac{\text{force x distance}}{(\text{Time})}$$

$$= \frac{(\text{mass x acceleration}) \text{ x distance}}{(\text{Time})}$$

$$= \frac{(\text{mass x velocity/time}) \text{ x distance}}{\text{Time}}$$

$$= \frac{(\text{mass x velocity}) \text{ x distance}}{(\text{Time})^2}$$

$$= \frac{\text{Mass x (length/time) x distance}}{(\text{Time})^2}$$

$$= \frac{\text{mass x length x distance}}{(\text{time})^2}$$

$$= \frac{ML^2}{T^3}$$

$$[P] = [ML^2t^{-3}]$$

Conversion of units:-

<u>Abbreviated power of ten:-</u>

Name	**Symbol**	**Equivalent**
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Hector	h	10^2
Decal	d	10
Deco	d	10^{-1}
Cent	c	10^{-2}
Mille	m	10^{-3}
Micro	μ	10^{-6}
Nana	n	10^{-9}
Picot	p	10^{-12}
Tempt	f	10^{-15}
Atom	a	10^{-18}

English into SI conversion:-

Question: - A frequency of signal in communication system is given as 5 MHZ express this term interim of abbreviated powers of 10 as international standard.
Sol^n:-

Given frequency (f) = $5M_{HZ}$
$= 5 \times 10^6 H_Z$

Question: - A flux density in the C.G.S. system is expressed as 20 maxwells/am2. Calculate the flux density in lines/in2. (Note: - 1Maxwell=1line)

$$B = \frac{20 \text{ .Maxwell}}{cm^2}$$
$$= \frac{20 \text{ Maxwell}}{Cm2} \times (2.54 \text{ cm } /1 \text{ inch})2 \quad \times \frac{1 \text{ line}}{1 \text{ maxwell}}$$
$$= 129 \text{ lines } /\text{inch}^2$$

Question: - The velocity of light in free space is given as 2.997925×10^6 m/s express the velocity in km/hr.

Question: - Calculate the velocity of a battery if a charge of 3×10^{-4} c residing on the +ve battery terminal posses 6×10^{-2} j of energy.

Standards of measurement:-

The term standard is applied to a piece of equipment having a known measure of physical quantity. For eg. kg, consisting of platinum iridium hollow cylinder preserved at the international bureau of weighs and measures at severs near peris asx standard of mass in SI system.

There are different types of standard of measurement classified by their function and application in the following categories:-

(I) internal standards
(ii) Primary standards
(iii) Secondary standards
(iv) Working standards

International Standards:-

-Defined on the basis of international agreement.
- maintained at the international al burin of weighs and measures.
-Not available for the ordinary users.
-Height possible accuracy.

Primary Standard:-

- maintained by national standard laboratories.
-Not available for use outside the national laborites.
- One of the main functions is the variation and calibration of secondary standards
- For mass and accuracy of one part in 108

Secondary standers:-

- Used in industrial measuring laborites.
- maintained by peritoneal involve laborites.
-periodically checked by national laborites.
-for mass and accuracy of one pod

Working standards:-

⇒ Used in measurement laborites.
⇒ Used to check general laboratories instrument for their accuracy and performance.
⇒ Used in the quality control department.
⇒ For mass an accuracy of 5 PPM.

And, IEEE standards:-

⇒ Slightly different types of standard.
⇒ Published and maintained by the instate of electrical and electronic Engineers.
⇒ Are not physical item but the standard procedures nomenclature, definition etc.
⇒ Standard test method for testing and evaluating various electronics system and component.
⇒ Specifying of test equipment for examples seminaries in knofes and function of oscilloslscope manufacture by different manufactures.,
⇒ Standard schematric and logic symbols for engineering drawing.

2.2 Measuring instrument for performance parameter:-

There are two types of performance parameters of means using instrument.

1. Accuracy and precision
2. Resolution and sensitivity
3. Drift
4. Static error
5. dead zone

1**. Accuracy and precision:-** Accuracy is the closeness with which an instrument reading approaches the true value of the quantity being measure precision specifies the repeatality of a set of reading is made independently with the same instrument.

Precision is composed of two characteristics:- conformity and the number of significant digit.

The more significant figures, measure the greater precision of measurement.

Question:- A voltmeter is specified as being accurate to 1% of its full scale reading, if the 100v scale is used to measure voltage of @ 12v, how accurate will the reading be ? (Assuming all other error besides the meter reading error is negligible.

Sol$^{n:}$- Accuracy of meter = 1% of it's full scale value.

= 1% of 100v =1v

Thus, the error of the 80v reading = 80± 1v

The possible % error = $\frac{\text{true value} - \text{measure value}}{\text{True value}} \times 100\%$

$= \frac{80-79}{80} \times 100$

$= \cong 1.25\%$

The error of the meter when the 12v measurement is made can still be ± 1v

Then the possible % error $= \frac{12-11}{12} * 100\%$

$= \cong 8\%$

1% of 100v =1v

Question:- A volt meter whose accuracy is 2% of the full scale reading is used on its 0-150v scale. Voltages measured by meter are 15v and 82v. Calculate the possible % error of both reading. Comment upon your answer?

Soln:- Accuracy of meter = 1% of 150

$= \frac{2}{100} \times 150 \qquad = 3$

Thus the error of 15v reading is $= 15 \pm 3$v

The possible % error = $\frac{\text{true value} - \text{measured value}}{\text{True value}} \times 100\%$

$= \frac{15-12}{15} \times 100$

$= \frac{3}{15} \times 100$

$= 20\%$

The error of meter when 82v measurement is made can still be ± 3v.

Then, possible % error $= \frac{82-79}{82} \times 100\%$

$= \frac{3}{82} \times 100$

$= \frac{150}{41}\%$

$= 3.65\%$

Question: A set of independent voltage measurements taken by four observers was recovered as 117.02v, 117.11v, 117.08v and 117.03v calculate the average voltage and the range of error?

(a) The average voltage
(b) The range of error.

Sol^{n}:- (a) $E_{ar} = \frac{E_1 + E_2 + E_3 + E_4}{4}$

$= 117.06$v

(c) Range $= E_{max} - E_{ar}$

$= 117.11 - 117.06$

$= 0.05$v

Also,

$E_{ar} - E_{min} = (117.06-117.02)$v

$= 0.04$v

∴The average range of error $= \frac{0.05 + 0.04}{4}$

$= \pm 0.045$

$= \pm 0.05$

Question:- Given R_1 = 18.7 ohm and R_2 = 3.62 ohm, then calculate, the total resistance with appropriate number of significant figure if,

(i) R_1, R_2 are connected in series.

(ii) R_1, R_2 are connected in parallel.

Soln:- $R_T = R_1 + R_2$

$= 1807 + 3.624$

$= 22.324$ ohm. (five significant figure)

Since, one the resistance is accurate only three significant figure

Hence,

RT = 22.3 ohm (3 significant figure)

(iii) $R_T = \frac{R_1.R_2}{R_1 + R_2}$

$= \frac{18.7 \times 3.62}{18.7+ 3.62}$

$= 3.03569\ \Omega$

RT = 3.03Ω (Three significant figure)

Add 826± 5 to 628±3 with % error.

Soln:- N1 = 826 ± 5

% error = $\frac{5}{826} \times 100$

$= \pm 0.605\%$

N2 = 625 ± 3

% error = $\frac{3}{628} \times 100\%$

$= \pm 0.477\%$

Then,

Sum = N1 + N2

= 1454 ± 8

%error = $\frac{8}{1454} \times 100\%$

$= \pm 0.55\%$

Sum = 1454 ±8 (0.55%)

Resolution and sensitivity:-

Resolution is the smallest increament in input (the quanity being measured) which can be detected with certainly by an instrument.

Sensitivity is defined as the ratio of the change of output to the change of input.

Sensitivity (s) $= \frac{\Delta y}{\Delta x}$

Where, Δx = change in i/p value.

Δy = change in o/p value.

Deflection factor = inverse sensitivity

$= \frac{\Delta x}{\Delta y}$

Question:- A wheat stone bridge require change of 7 ohm in the unknown arm of the bridge to produce a change in deflection of 3mm of the galvanometer. Determine the sensitivity. Also determine the deflection factor.

Soln:-

Sensitivity = $\frac{\text{magnitude o/p response}}{\text{o/p magnitude}}$

$= \frac{3mm}{7\Omega}$

$= 0.429$ mm/Ω

Deflection factor = 7/5

= 2.33 Ω/mm

Question:- A moving coil voltmeter have a scale of 100 division, the full scale reading is 250v and 1/10 of a scale division can be estimated with fair degree of sentainty. Determine the resolution in volt.

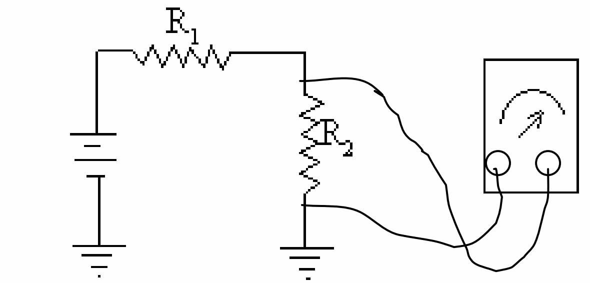
1 scale division $= \frac{200}{100}$

∴Resolution = 1/10 of a scale division

$\frac{1}{10} \times 2v$

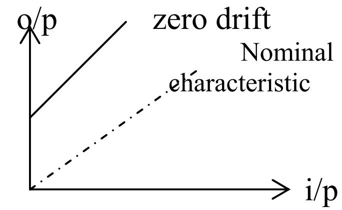
= 0.2v

Question:- What voltage would a 20,000Ω/v meter on a 0-1-v scale show in the circuit of figure given below:-

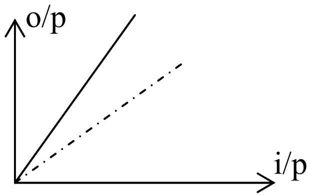


Drift:- drift may be classified into three categories.

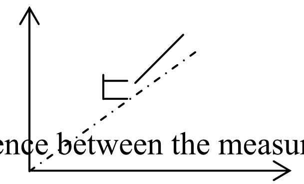
(a) Zero drift



(b) Span drift



(c) Zonal drift



Static error: It is defined as the difference between the measured value and true value of the quantity.

Dade zone : It is defined as the largest change of input quantity for which there is no output of the instrument .

(b) Dynamic characteristic: Dynamic characteristic are
(1) speed of response
(2) Measuring lag
(3) Fidelity
(4) Dynamic error

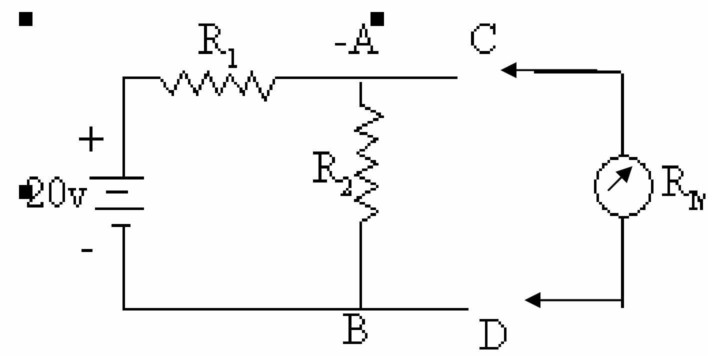
Speed of response: It is defined as the rapidity with which a measuring device response to changes in the measured quantity .

Measuring lag : It is the retardation or delay in the response of a measurement system to change in the measured quantity.
It is of two types:

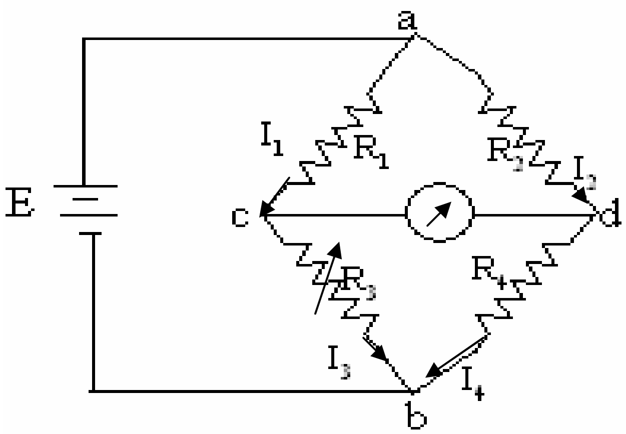
(1) Retardation type :- response begins immediately after a change in measured quantity los ………………..
(2) Time delay:- response begins after a dead time after the applion of the input.
(3) Fidelity:- It is defined as the degree to which a measurement system indicate changes in the measured quantity with out any dynamic error.
(4) Dynamic error:- difference between true value of the quantity under measurement) changing with true. And the value indicated by measurement system if no static error is assume . It is also called measurement error.

What is the true value of voltage across the 500kilo ohm resistor connected between terminals A and B as shown in figure. What would a volt mete4r , with a sensatory of 30kilo ohm/volt read on the following ranges 50, 15, 5v when connected across terminals C and D

Ans:- (10v, 8v, 5,45v, 2.86v)



2.3 Resistance measurement with wheat stone bridge:-



The Bridge said to balanced when the potential difference across the galvanometer is zero volt. So that there is no current is no current through the galvanometer. This condition occurs when

$V_{cA} = V_{dA}$

$I_1R_1 = I_2R_2$ ______________(i)

If the galvanometer current is zero the following condition also exists.

$$I_1 = I_3 = \frac{E}{R_1 + R_3} \quad \text{(ii)}$$

$$\text{Also, } I_2 = I_4 = \frac{E}{R_2 + R_4} \quad \text{(iii)}$$

Combining equation (i), (ii) and (iii) we get

$$\frac{R_1}{R_1 + R_3} = \frac{R_2}{R_2 + R_4}$$

$$R_1(R_2 + R_4) = R_2(R_1 + R_3)$$

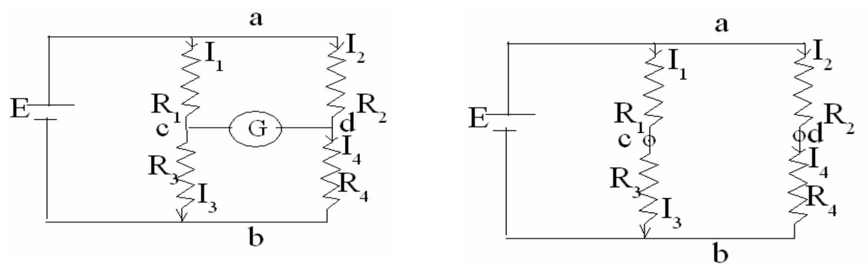
$$R_1R_4 = R_2R_3$$

<u>Note</u>:- If There of the resistance have known values 4th may be determined. Hence if R_4 is the unknown resistor it's resistance let Rx can be determine

$$R_x = R_3 \frac{R_2}{R_4}$$

The resistance R3 is called the standard arm of the bridge and the resistor R_1 and R_2 are called ratio arm.

<u>Thevenin equivalent Circuit</u>:-



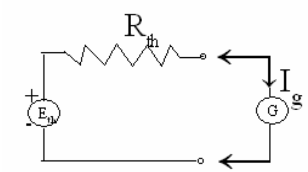
Now, the thevinin are open circuit voltage is found by (fig a)

$E_{cd} = E_{qc} - E_{ad}$

$\quad = I_1R_1 - I_2R_2$

Where, $I_1 = \dfrac{E}{R_1 + R_2}$ and $I_2 = \dfrac{E}{R2 + R4}$

$$\therefore E_{cd} = E_{th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 - R_4}\right)$$



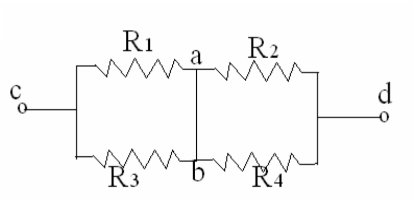
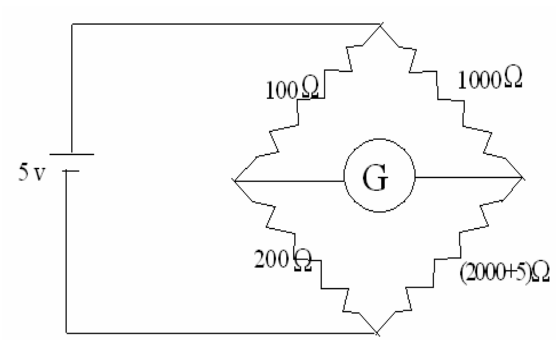


fig (b)



And

$$R_{th} = \frac{R_1R_3}{R_1 + R_3} + \frac{R_2R_4}{R_2 + R_4}$$

$$I_g = \frac{E_{th}}{R_{th}}$$

If the galvanometer has the internal resistance Rg then,

$$I_g = \frac{E_{th}}{R_{th} + R_g}$$

#

Given, $R_1 = 10\ \Omega$

$R_2 = 30\ \Omega$

$R_3 = 5\ \Omega$

In the below figure.

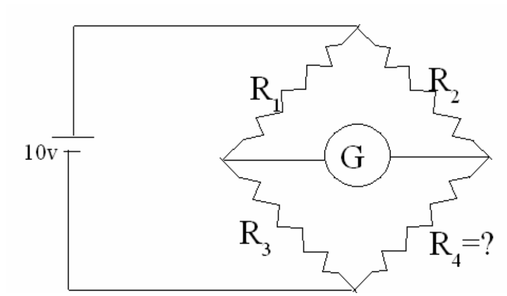
(i) At what value of R4 will the galvanometer show the null deflection.

(ii) If the bridge is balanced, find out the total current I, drawn by bridge circuit.

Req 11.25

Ans:- 0.88A

Question :- Fig below show the schemtic diagram of wheat stone bridge which value of bridge element as shown. The battery voltage is 5v and its internal resistance negligible. The galvanometer have a current sensitivity of 10mm per microamper and internal resistance of 10 ohm. Calculate the deflection of galvanometer caused by the 5 ohm unbalance in arm Be.



Ans:- Eth = 2.77mv

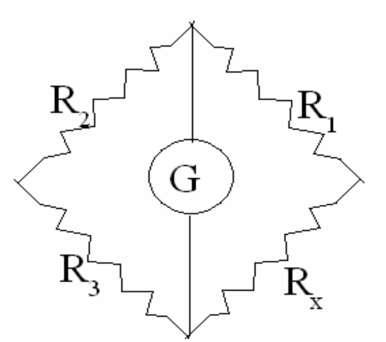
Rth = 734Ω

Ig = 3.32 μA
Dethe = 33.2 mm

Kelvin Bridge :-

- Modification of wheat stone bridge
- Used to measure low value resistance below 1 ohm with increased accuracy.

Consider a bridge circuit given below:



Let, $\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2}$ ________________(i)

When bridge is balanced,

$R_x + R_{np} = \frac{R_1}{R_2}(R_3 + R_{mp})$ _____________(ii)

Now, from equation (i)

$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2}$

$\frac{R_{np} + R_{np}}{R_{mp} + R_{np}} = \frac{R_1 + R_1}{R_2 + R_1}$

or, $\frac{2R_{mp}}{R_y} = \frac{2R_1}{R_1 + R_2}$

$R_{np} = (R_1 / R_1 + R_2)\, R_y$

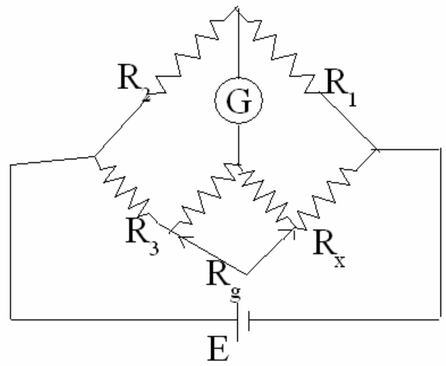
Similarly, $R_{mp} = (R_2/R_1+R_2)R_y$

Putting the value of R_{np} and R_{mp} in equation (ii) we get

$$R_x + \left(\frac{R_1}{R_2}\right)R_y = R_1\left[R_3 + \left(\frac{R_2}{R_1 + R_2}\right)R_y\right]$$

$$\therefore R_x = \frac{R_1}{R_2}.R_3$$

Due to the resistance value of connecting leads of a resister (any other electrical elements)



It is necessary to connect the galvanometer G in such a way such that the bridge circuit is balance as in wheat stone bridge . This development forms the basis for construction of the Kelvin double bridge. The term “double” is used because its circuit contains a second set of ratio arm a , b as shown in figure. These arms connect the galvanometer to a point p at the appropriate potential between m and n and it elements the effect of the resistance of R_y.

An initially established condition is

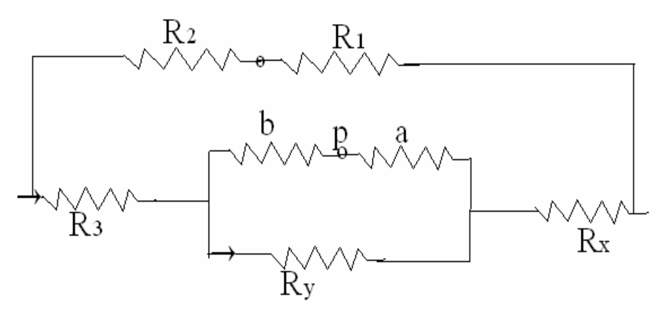
$\frac{a}{} = \frac{R_1}{}$

b. R_2

The 'G' will show the null deflection when the potential at k equals the potential at p.

i.e E_kc = Elmp ________(ii)

When $E_{kl} = E \times \dfrac{R_2}{R_2+R_1}$



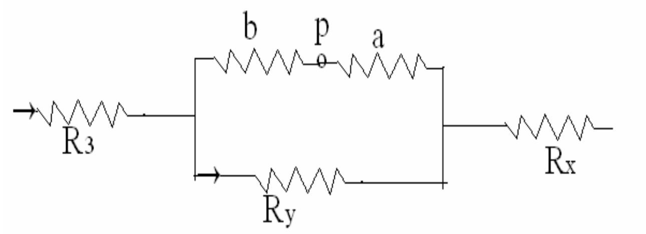
$$E = R_3 + \frac{(a+b)R_y}{a+b+R_y} \times R_x I$$

$$= I[R_3] + R_x + \frac{(a+b)R}{a+b+R_{y\,y}}$$

Where, $E_{kl} = E \times \dfrac{R_2}{R_1+R_2}$

$$E_{kl} = \frac{R_2}{R_1} \times I\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \text{\_\_\_\_\_\_\_\_\_\_(iii)}$$

E_{lmp} = voltage drop across R_3 + voltage drop across b



R_{eq} of a, b, R_y $= \dfrac{(a+b)R_y}{a+b+R_y}$

Voltage drop in b, $= I \times \dfrac{(a+b)R_y}{a+b+R} \times \dfrac{b}{a+b}$

$$\therefore E_{elm} = I\left[R_3 + \frac{b}{a+b} \times \frac{(a+b)R_y}{a+b+R_y}\right]$$

From equation (i) and (ii) we have

$$\frac{R_2}{R_1+R_2} I\left[R_B + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] = I\left[R_3 \frac{b}{a+b} \times \frac{(a+b)R}{a+b+R}\right]$$

Or, $\quad R_3 + R_x + \dfrac{(a+b)R_y}{a+b+R_y} = \dfrac{R_1+R_2}{R_2}\left[R_3 + \dfrac{6R_y}{a+b+R_y}\right]$

Or, $\quad R_3 + R_x + \dfrac{(a+b)R_y}{a+b+R_y} = \dfrac{R_1R_3}{R_2} + R_3 + \dfrac{R_1+R_2}{R_2} \times \dfrac{6R_y}{a+b+R_y}$

$$R_x = \frac{R_1R_3}{R_2} + \frac{R_1R_3}{R_2} + \frac{R_1 6R_y}{R_2(s+b+R_y)} + \frac{6R_y}{a+b+R_y} - \frac{(a+b)R_y}{a+b+R_y}$$

2.4 A.C Bridges (inductance and Capacitance bridge):

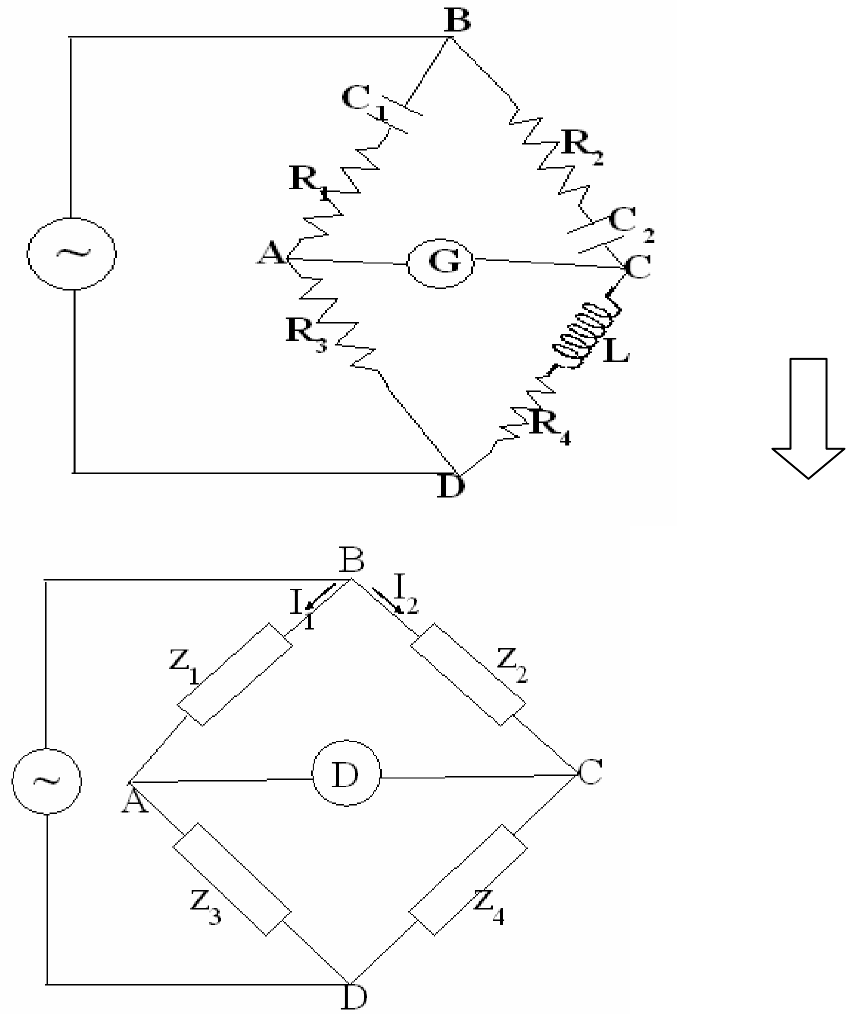


Fig:- The general form of AC bridge.

The condition for bridge balance requires.

$$E_{BA} = E_{BC}$$

$$I_1Z_1 = I_2Z_2 \_\_\_\_\_\_\_\_ \text{(i)}$$

For zero detector current, (the balanced condition), the current are

$$I_1 = \frac{E}{Z_1 + Z_3} \_\_\_\_\_\_\_\_\_\_\_\_\_ \text{(ii)}$$

And $I_2 = \dfrac{E}{Z_2+Z_1}$ ____________(iii)

Putting the values of I_1, I_2 in equation (i) we get,

$Z_1Z_2 = Z_2Z_3$ ___________(iv)

Or when using admittance instead of impedance, we get.

$Y_1Y_x = y2y_3$

Equation (iv) is the general equation for balance of the A.C. bridge.

If the impedance is written in the form

$Z = Z< \theta$

Where, Z = magnitude

And θ = phase angle of the complex impedance then equation (iv) can be written as:-

$$(Z< \theta_1) (Z_x< \theta_x) = (Z_2 < \theta_2) (Z_3< \theta_3)$$

$$Z_1Z_x < \theta_1 \theta'_x \ Z_2Z_3 < \theta_2 \theta_3$$

Equation (v) shows that, two conditions must be simultaneously when balancing an ac bridge conditions,

1. The product of magnitude of opposite arms must be equal. i.e.
   $Z_1Z_x = Z_2Z_3$
2. The sum of the phase angle of the opposite arm must be equal. i.e.
   $< \theta_1 + < \theta x = \theta_2 + < \theta_3$
3. The impedances of the basic A.C. bridges are given as follows:-
   $Z_1 = 100\Omega < 80^0$ (inductive)
   $Z_2 = 250\Omega$ (pure resistive)
   $Z_3 \ 40^0 < 30$ (inductive)

$Z_x = ?$

Determine the constants of the unknown arm if the bridge is balance

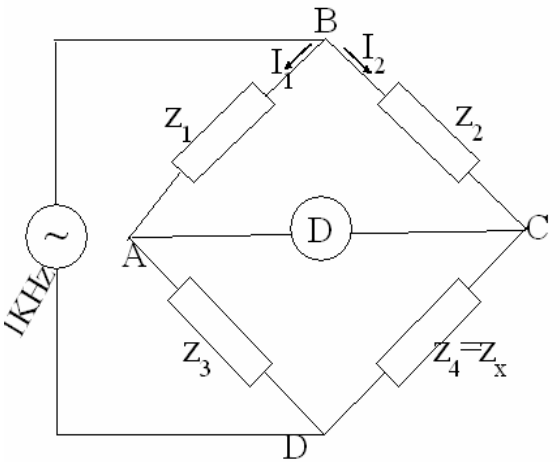
$Z_1 Z_x = Z_2 Z_3$

$Z_x = \dfrac{Z_2 Z_3}{Z_2} = \dfrac{250 \,.\, 400 < 30^0}{100\text{-}2 < 80}$

$= \dfrac{100000<30}{100<80}$

$= 1000< -50^0$

Question:- An A.C. bridge is in balance with following constants: Arm AB:R= 450 Ω; arm QC:R = 300 Ω in series with C = 0.265µF, arm CD unknown; arm DA;R 200 Ω in series with L = 15.9 mH. If the oscillor frequency is 1KHz , find the constant of arm CD.



Sol^n:- $Z_1 = R = 450\ \Omega$

$Z_2 = R - j/wc$

$=320 - j600$

$Z_3 = R + jwl = (200 + j120)\ \Omega$ $\qquad \dfrac{1}{wc} = \dfrac{-1}{2\Pi fc}$

Z_4 = unknown

We have,

$Z_4 = \dfrac{Z_2 Z_3}{Z_1}$

$= \dfrac{(320\text{-}j600)\ (200\ \ j100)}{450}$

$= \dfrac{(670.8 <-63.43^0)(223.6 <26.50)}{450<0^0}$

$=333.31< -36.87^0$

$=266.64 - j200$

$= R - j/wc$

$$= \frac{1}{wc} 200$$

$$\frac{1}{2\pi fc} = 200$$

$$c = \frac{1}{2\pi f \times 200} = \frac{1}{2\pi \times 1 \times 10^3 \times 200}$$

$$c = 0.796 \mu f$$

$$R = 26.64 \Omega$$

<u>Question</u>:- An A.C. bridge circuit working at 1000 hz is in balanced with following constants arm AB = 0.2µf , Arm BC = 500Ω , CD = unknown arm . Arm DA = 300Ω resistance in parallel with a 0.1µf capacitor of arm CD considering it as a series circuit.

Maxwell bridge:-

One of the A.C. bridge which is used in determining the value of unknown inductor the unit the unknown value of capacitor and resistor in the other arm.

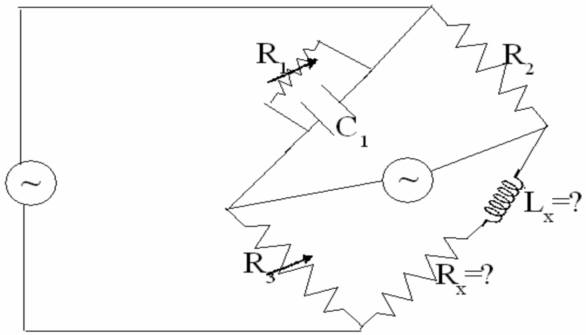


Fig:- Maxwell bridge for inductance measurements

Now, $Z_1 = R_1 // C_1$

$= \frac{R_1 \text{ x } Xc}{R_1 + X_c}$

$= \frac{R_1 \,.\, 1/jwc_1}{R_1 \; 1/jw_1}$

$= \frac{R_1}{1+jw_1R_1}$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1} + 1wc_1$

$Z_2 = R_2$

$Z_3 = R_2 + L_x$

$= R_x + jwl_x$

When bridge is balanced

$Z_1.Zx = Z_2.\, Z_3$

Or, $Z_x = Z_2Z_3Y_1$

$= R_2R_3\,(1/R_1 \;\; jwc1)$

$Rx + jwlx = \frac{R_2R_3}{R_1} + jwR_2R_3C_1$

Comparing real and imaginary parts.

$R_m = \frac{R_2R_3}{R_1}$

And $jwlx = jwR_2R_3C_1$

$Lx = R_2R_3C_1$

Note:- Maxwell bridge is appropriate for measuring medium and factor coil. It does not appropriate fro high Q factor coil. Neither for low Q factor coil.

Hay Bridge:-

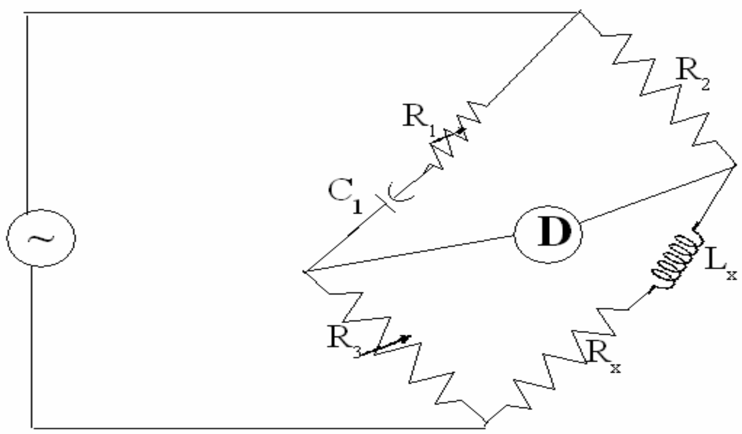


Fig:- Hay bridge for inductance measurement.

Now, $Z_1 = R_1 - \underline{\;\;j}$

WC$_1$

$Z_2 = R_2$

$Z_3 = R_3$

And $Z_x = R_x + jwlx$

For balanced condition

$Z_1 Z_x = Z_2 Z_3$

Substituting the values,

$(R_1 - j/wc1)\ (R_x + jWLx) = R_2 R_3$

Or, $(R_1 WC1 - j)\ (R_x + jwlx) = R_2 R_3 wc1$

$R_1 R_x wc1 + jR_1 LxC_1 W_2 - jR_x - J_2 WLx = R_2 R_3 WLC_1$

Comparing real and imaginary parts.

$R_j R_x WC_1 + WLx = R_2 R_3 WC_1$

$R_1 R_x + Lx = R_2 R_3$ ____________(i)

$jR_1 LxC_1 W_2 - JRx = 0$

$R_x =$ WLxR1 ______________(ii)

WC$_1$

Solving equation (i) and (ii) we get.

$$R_x = \frac{W^2 C_1^2 R_1 R_2 R_3}{1+W^2 C_1^2 R_1^2}$$

And $$L_x = \frac{R_2 R_3 C_1}{1+W^2 C_1^2 R_1^2}$$

When the two phase angle are equal their tangents are also equal when the

Tan = tan

Now from equation (iv) and (v)

Note:- the hey bridge suited for the measurement of high inductors specially for those inductors having a greater than 10. for lesser than 10. Maxwell bridge i.e. more suitable.

Shearing Bridge:-

It is used for the measurement of capacitor.

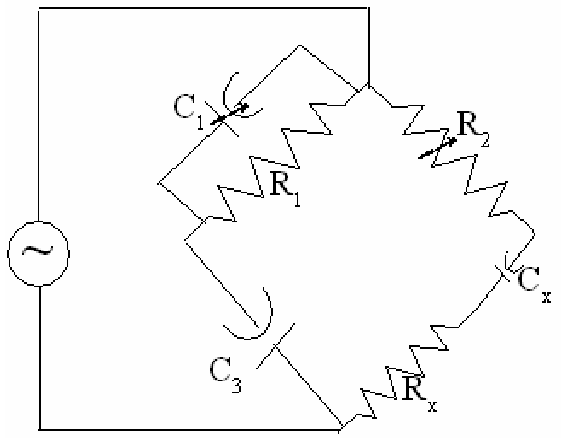


Fig:- shearing bridge.

Now, $Z = R_1 \,//\, C_1 = \dfrac{R_1}{1 + jwc_1 R_1}$

$Y_1 = \dfrac{1}{R_1} + jwc1$

$Z_2 = R_2$

$Z_3 = 0 - j/wc3 = -j/wc3$

$Z_x = R_x - j/wcx$

For the balanced condition of the bridge,

$Z_x = Z_2 Z_3 Y_1$

$R_x - j/Wcx = R_2\,(-j/wc_3)(1/R_1 + jwc_1)$

Or, $R_x - j/wcx = R_2 C_1/R_3 - jR_2/WC_3 R_1$

Equating the real and imaginary term, we get.

$R_x = R_2 \dfrac{C_1}{C_3}$ ____________(i)

And $C_x = C_3 \dfrac{R_1}{R_2}$ ____________(ii)

Dissipation factor (D.F)= 1/Q

$= W_{cx} R_x$ __________(iii)

Or, $D = 1/Q$

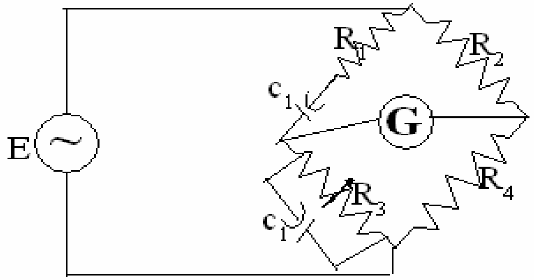
Putting the values of R_x C_x in equation (iii) we get,

$D.F = WR_1C_1$

Note:- If resistor R_1 in the shearing bridge has a fixed value, the dial of capacitor C_1 may be calibrated directly in dissipation factor D.

Wien Bridge:-

It is used to measured frequency.



Now, $Z_1 = R_1 - j/WC_1$

$Z_2 = R_2$

$Z_3 = \dfrac{R_3}{1+JWC_3R_3}$

$Y_3 = \dfrac{1}{R_3} + JWC_3$, $\quad Z_4 = R_4$

For the balanced condition of the bridge.

$Z_1Z_4 = Z_2Z_3$

$Z_2 = Z_1Z_4Y_3$

Or, $R_2 = \left(R_1 - \dfrac{j}{WC_1}\right)R_y\left(\dfrac{1}{R_3} + JWC_3\right)$

Or, $R_2 = \left(R_1R_4 - \dfrac{jR_4}{Wc_1}\right)\left(\dfrac{1}{R_3} + jwc_3\right)$

Or, $R_2 = - + JWC_3R_1R_4 + \dfrac{jR_4}{Wc_1} - \dfrac{R_4C_3}{C_1}$

Equating real and imaginary parts we get,

$R_2 = \dfrac{R_1R_4}{R_3} + \dfrac{R_4C_3}{C_1}$

Or, $\dfrac{R_2}{R_4} = \dfrac{R_1}{R_3} + \dfrac{C_3}{C_1}$

And $WC_3R_1R_4 = \dfrac{R_4}{WC_1R_3}$ _____________(i)

Where, $W = 2\Pi f$

$2\Pi fC_3R_1R_4 = \dfrac{R_4}{2\pi fC_1R_3}$

$f = \dfrac{1}{2\pi\sqrt{C_1C_3R_1R_3}}$

In most wein bridge circuit component choose such that $R_1 = R_3$, and $C_1 = C_3$. This reduces the equation (i) to

$\dfrac{R_2}{R_4} = 2$

And equation (iii) be

$f = \dfrac{1}{2\pi RC}$

2.5 Error in Measurement and error type

Absolute error (δ A):

The difference between the measured value Am and the true valuel A of the unknown u quantity is known as the absolute error of the measurement.

i.e $\delta A = Am^{-A}$

Relative Error($\in_r$):- It is the ratio of absolute error to the true value of the quantity to be measured.

i.e $\in_r = \frac{\delta A}{A}$

The relative error may be expressed as a percentage.

i.e percentage error = r x 100%

<u>Limiting Error</u> ($\pm \delta A$):-

In order assure the purchasure of the quality of the circuit component or measuring instrument, the main feacturs garinty a centrain accuracy. Circuit components such as resistor, inductor and capacitors are garinted to be within the certain percentage of the rated value where as in indicating instruments, the accuracy is mostly a garinted to be with the contain % of full scale reading. For that manufacture have specify deviation from the specified value of a particular quantity in order to enable the purchasure to make proper selection according to his requirement. The limits of these deviation form specified values are defined as limiting or garrrenty error. For example, if the resistance of resistor is given as 500 Ω the manufacturer garrienty that the resistance falls between the limits 450 Ω and 550Ω.

Question:- The measured value of the resistance is 10.25 Ω and where as it's value is 10.22 Ω. Determine the absolute error of measurement.

A_m = 10.25 Ω

A = 10.22 Ω

$\delta A = A_m - A$ 0.003 Ω

the magnitude of the current being measured is 10A the relative error at this current.

$\epsilon r = \frac{\delta A}{Am} = \frac{0.25}{10} = 0.025$

Therefore, the current being measured is between the limits of

$A = A_m(1 \pm \epsilon r)$

$= 10(1 \pm 0.025)$

$= 10 \pm 025$ A

The limiting % error = A x 100%

= 2.5%

\# The current passing through a resistor of 50 ± 0.2 is 4.00 ± 0.02A. Determine the limiting error in the computed value of power dissipation.

Sol^n:-

% limiting error to resistor = $\pm \frac{0.2}{50}$ x 100% = 0.4%

% limiting error to current = $\pm \frac{0.2}{4.00}$ x 100% = 0.5%

The power dissipated in resistance R due to flow of current I is given by the expression.

$P = I^2R$

In worst possible combination of error the limiting error in the power dissipation is, 2 x 0.5 + 0.4

= 1.4%

So, power dissipation is given by

$P = I^2R = 4^2 \times 50$ w

= 800 w

= 800 ± 1.4%

= 800 ± 11.2 watts.

Questions:- The current passing through a resistor of 100 ± 0.2 ohm is (2.00±0.01) A using the relationship $p=I^2R$. Calculate limiting error in the computate value of power dissipation .

Types of error :-

(1) Gross error

(2) Systematic error
(3) Random error

(1) **Gross error:-**

-Human mistakes
-Misreading of instrument
-Incorrect adjustment and improper application of instruments
-Computational mistakes
- Can not be treated mathematically.

(2) **Systematic error**:-

Error that remain constant or change according to definite low on repeated measurement of the given quantity. It is of two types:

(a)Instrumental error:-
(b) Environmental error

(3) **Random error : -**

These error are of variable magnitude of and sign and do not obey any know low.

The only way to off set these error is increasing the number of reading and using statical means to obtained the best approximation of the true value of the quantity under measurement.

Statistical analysis

1. **Arithmetic mean:-**:

$\bar{x} = (X_1+X_2+X_3+\ldots+X_n)/n$

$= \frac{\sum x}{n}$

Where X = arithmetic mean
$X_1, X_2, \ldots X_n$ = reading taken
n = no. of reading

2. **Deviation from the mean :-**

$d_1 = x_1 - \bar{x}$
$d_2 = x_2 - \bar{x}$
$D = \frac{\sum/d/}{n}$

3. **Average deviation**

$D = /d_1/ + /d_2/ + \ldots\ldots + /d_n/$
$D = \frac{\sum/f/}{n}$

4. **Standard deviation :-**

When the no of readings are infinite.

$$\sigma = \sqrt{\frac{d_1^{\,2} + d_2^{\,2} + \ldots..d_n^{\,2}}{n}}$$

$$= \sqrt{\frac{\Sigma d^2}{n}}$$

When the no of reading are finite.

$$\sigma = \sqrt{\frac{d_1^{\,2} + d_2^{\,2} + \ldots..d_n^{\,2}}{n}}$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}}$$

(iv) variance = σ^2
(v) probable error = 0.67456

Question:- .10 measurements of restance of a registore gave 101.2ohm, 101.7ohm, 101.0ohm, 101.5ohm,101.3ohm,101.2ohm, 101.4ohm,101.3 ohm and 101.1ohm. Assume that only random errors are present . calculate

(a) The arithmetic mean
(b) standard deviation of the reading
(c) the probable error=0.674

Chapter:- 3

Variables and Transducer:

3.1 **Physical variable and its types:**

(i)Electrical

(ii) Mechanical process

(iii) Bio-Physical variables

(i) **Electrica**l:-

-Resistance

-Inductance

-Capacitance

]-Voltage and currents

(ii) **Mechanical process**

-Temperature

-Pressure

-Flow rate etc.

(iii) **Bio-Physical variables**

-Heart beat, pulse beat, body temperature, blood pressure etc.

Transducer and its types

(i) Analog and digital
(ii) Primary and secondary
(iii) Active and passive
(iv) Transducer and inverse resistive
(v) Electrical (capacitive, resistive voltage and current)

Resistive transducers:-

In such a transducers, resistance between the o/p terminals of a transducer gets varied accordingly to the measured. Resistive transducer performed other transducers because dc and ac both are suitable for resistance measurement.

Resistance of any metal conductor is given by the expression.

R = ρl/A

Where, R = resistance Ω

ρ = resistivity of conducture.

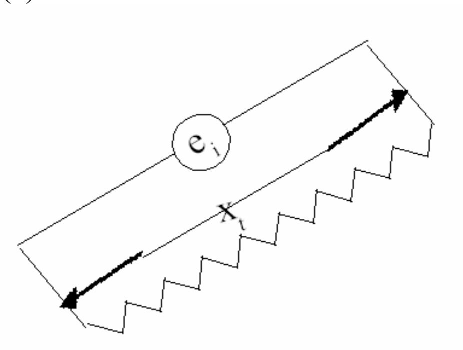
l = length of conductor.

A=crossectional area of conducture m^2

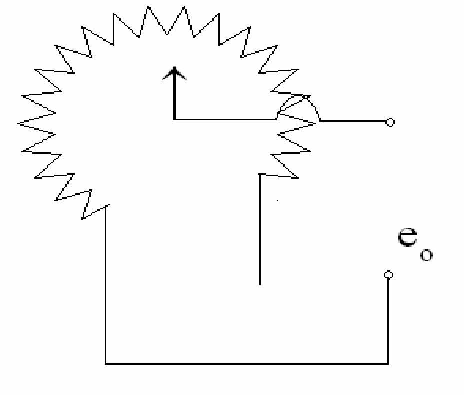
Types:-

- It is a passive transdures.
- Consists of a restive element plconided with a sliding coutent . The sliding contact is called wiper
- According to motion of wiper pontiometer are a folloning type .

(a) Transnational.



(b) Rotational



Rotational

(a) Translational potentiometer:

Let e_i and e_o =i/p and o/p voltage respectively (unit v)

X_t =total length of translational ponmeter.

Xi =displacement of wiper from its zero position; m

R_p =total resistance of the potiontiometer then the o/p voltage under ideal conduction is

$$e_o = \frac{(\text{restance at o/p terminal})}{(\text{restance at i/p terminal})} \times \text{ o/p voltage}$$

$$= \frac{R_p(x_i/x_t)}{(R_p)} \times e_o$$

$$= \frac{X_i}{X_t} \times$$

Under Ideal circumstances the o/p voltage varies linear with displacement as shown in figure below.

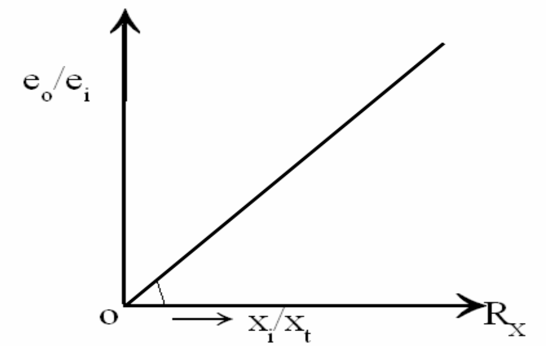


Fig:- characteristics of potentiomerter

(b) Rotational:-

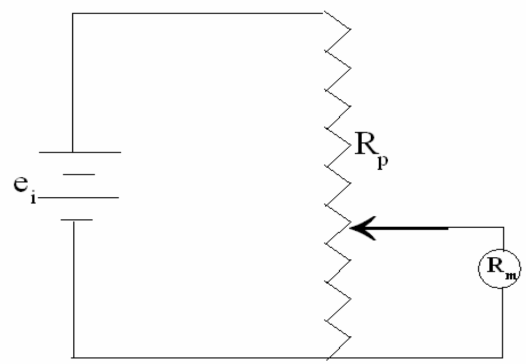
o/p voltage, $e_o = e_i\, \theta_i/\theta_t$ (for single turn potentiometer)

where, θ_1= i/p angular displacement degree

θ_i = total travel at wiper, degree

θ_t = i/p voltage, v

Potentiometer divider:-



It is a device for dividing the potential in a ratio determine by the position of the sliding contact.

Loading Effect:

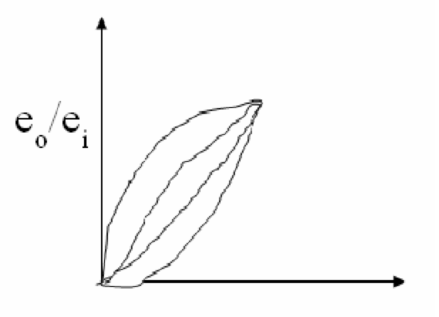
In practice the o/p terminal of the pot. are connected to a device which impedance is finite . When an electrical instrument which form a load for the pot. is connected across o/p terminals , the indicated voltage is less than the actual voltage . This error is caused by the i/p resistance of the o/p device s is called loading effect. Under ideal condition i.e when o/p terminal resistance is infinite (open)

$e_o = e(x_i/x_t)$

$e_o = Ke_i$

where, $K = x_i/x_t$

Under actual conditional R_m is not infinite. Hence the characteristics will non linear as given



Now, The resistance of the parallel combination of load resistance and the portion of the resistance of the potentiometer is

$$\left[\left(\frac{x_i}{x_t}\right)F_p\right]$$

$$= \frac{(x_i/x_t)R_p.R_m}{(x_i/x_t)R_p + R_m} = \frac{K_{Rp}.R_m}{K_{RP} + R_m}$$

The total resistance seen by the source is

$$R = \frac{[Rp - (xi/xt)Rp] + \left\{\left(\frac{x_i}{x_t}R_p\right)R_m\right\}}{(1-K) + \dfrac{K_{Rp}R_m}{K_{Rp}+R_m}}$$

$$= \frac{KR_2p(1-K) + R_pR_m}{K_{Rp} + R_m}$$

$\therefore$ *Current* ,

$$i = ei + R = \frac{e_i(KR_p + R_m)}{K_{R2}p(1-K) + T_qR_m} \text{\_\_\_\_\_\_\_\_\_\_\_\_(i)}$$

The o/p voltage under load condition is

$$e_o = i \times \frac{KR_pR_m}{KR_p + R_m}$$

From equation (i)

$$= \frac{ei(KR_p + R_m)}{kR^2{}_p(1-k) + R_pR_m} \times \frac{KR_pR_m}{KR_p + R_m}$$

$$= ei\frac{K}{K(1-K)(R_p/R_m)+1} \text{\_\_\_\_(ii)}$$

The ratio of o/p voltage to i/p voltage under load condition is

$$e_o = \frac{K}{K(1-K)(e_p/e_m)}$$

The equation to shows that there exists a non linear relationship between o/p voltage e_o and i/p displacement.

x_i such ; $K = x_i/x_t$

In case,

$R_m = \alpha$

$e_o/e_i = K$

Error = o/p voltage under load o/p voltage under non-load.

$$= \frac{e_i - K}{K(1-K)(R_p/R_m)}$$

$$= -ei\left(K2\frac{(K-1)}{K(1-K)+R_m/R_p}\right)$$

the –ve sign indicating that the error is -ve

$$\% \in = \left[\frac{x^2(x-1)}{K(1-K)+(R_m/R_p)}\right]\times 100$$

Strain Gauge:-

- It is a passive transducer.
- Convient mechanical displacement inot charge of resistances.
- The wire in strain Gauge is made of const.

Gauge factor:-

It is defined as the ratio of unit charge in resistance to the unit charge in the length of the wire of strain gauge.

$$K = \frac{\Delta R/R}{\Delta C/l} \quad \_\_\_\_\_\_\_\_\_\_(i)$$

Where, k = gauge factor

R = nominal gauge resistance

ΔR = change in resistance

L = normal length

Δl = change in nominal length

Strain:- It is defined as the ratio of change in length to original length.

Mathematically,

$$\sigma = \frac{\Delta l}{l} \_\_\_\_\_\_\_\_\_\_(ii)$$

Where, σ = strain in lateral direction

From equation (i) and (ii), we can write

$$\text{Gauge factor (K)} = \frac{\Delta R/R}{\sigma} \_\_\_\_\_\_\_\_\_\_(ii)$$

The resistance of the wire is given by

R = ρl/A

$$= \frac{\rho l}{\Pi d^2/4} \_\_\_\_\_\_\_\_\_\_(iv)$$

Now, tenslile on the conductor causes an encrease Δl in the length and a simuntaneous decrease Δd in the it's diameter. The resistance of the conductor the changes to

$$R_\rho = \rho\frac{l+\Delta l}{\pi/4(d-d)^2}$$

$$= \frac{\rho}{\pi/4}\frac{l+\Delta l}{\left[d^2+(\Delta d)^2-2\Delta d.d\right]}$$

$$= \frac{\rho}{\pi/4}\frac{l+\Delta l}{\left[1+\left(\frac{(\Delta d)}{d}\right)^2-\frac{2\Delta d}{d}\right]}$$

Since [Δd/d]2 is very very small

$$R_s = \frac{\rho}{\pi/4d^2}\frac{l+\Delta l}{\left[1-\frac{2\Delta d}{d}\right]}$$

$$R_s = \frac{\rho}{\pi/4l}\frac{l+\Delta l/l}{\left[1-\frac{2\Delta d}{d}\right]} \_\_\_\_\_\_\_\_\_\_(v)$$

Poisons Ratio:-

It is defined as the ratio of strain in the lateral direction to the axial direction.

$$\mu = \frac{\Delta d / d}{\Delta l / l} \_\_\_\_\_\_\_\_\_\_\_\_ (vi)$$

From equation (v) and (vi) we get,

$$R_s = \rho \frac{1}{(\pi/4)d2} \frac{(1+\Delta l/l)}{(1-2\mu\Delta l/l)}$$

$$= R\left[\frac{1+\Delta l/l}{1-2\mu\Delta l/l}\right]$$

$$= R\frac{1+\Delta l/l}{1-2\mu\Delta l/l} \times \frac{1+2\mu\Delta l/l}{1+2\mu\Delta l/l}$$

$$= R\frac{1+2\mu\Delta l/l+\Delta l/l+2\mu(\Delta l/l)^2}{1-4\mu^2(\Delta l/l)^2}$$

Since, $(\Delta l/l)^2$ very very small so neglected,

$$\therefore Rs = R\left[1+(1+2\mu)\frac{\Delta l}{l}\right]$$

$$= R + R(1+2\mu)\Delta l/l$$

$$= R + \Delta R$$

Then

Gauge factor (K) = $\dfrac{\Delta R/R}{\Delta l/l}$

$$= \frac{[R(1+2\mu)\Delta l/l]}{\Delta l/l}$$

K = 1+2μ _________(vii)

Hooks Law:-

It gives the relationship between stress and strain for a linear stress-strain curve. Mathematically,

$\sigma = S/E$ ________(viii)

Where, σ = Strain Δl/l

S = stress kg/cm^2

E = Young's modulus kg/cm^2

Inductive transducers:-

These are passive transducers they operates generally upon one of the following three principle.

1. **variation of self inductance of the coil:-**

   self inductance of coil (L) = N2/R

   where,

   N = no of turns.

   R = reluctance of the magnetic flux

   But,

   R = l/μA

   Then, $L = \dfrac{N^2\mu A}{L} = N^2\mu(A/l)$

   $L = N^2HG$ [G = A/l]

   Where, H = effective permeability of the medium in an around the coil H/m.

   G = A/l = Geometric form factor.

   a. variation in length of coil:-
   b. Variation of permeability
   c. Variation of mutual inductance of the coil:-

   The mutual inductance between the coil is M = $\sqrt{L_1 L_2}$

   Where, L_1 & L_2 = self inductance of the coil.

   K = co-efficient of coupling.

Principal of production of eddy current :

These transdures operets on the principle that when a conducting plate near acoil carrying alternating currents , Eddy current are induced in the conducting plate , the producing its own magnetic fild in opposition to the main fild creating by the coil .This eddy current induced in the conducting plate reduce the net flux linking of the coil and inductance of the coil is reduced . i.e the inductance of the coil changes with the movement of the plate .

Linear varibal differential transformer :

The most widdly used inductive transdure to translate the linear motion into electrical singal is called LVST .The basics construction of LVST is shown below.

Fig take ok

Transdure consists of single primary winding p and two secondary winding s1 and s2 wound on cylindrical former . Both S1 and S2 are identical and have equal of tens. Since the primary winding is excited by an alternating currents source . It produces an alternating magnetic field which in turn induces AC voltages in two secondary winding S_1 and S_2

Take fig. ok

Inoder to convert the output from S1 and S2 into a single voltage , the two secondary S1 and S2 are connected in series . The output voltage of the transdure is the difference of two voltage.

Take fig ok

When the core is at a position voltage induced in S1 and S2 are equal Vot=0

When the core is at position A (to the left) then the voltage induced in S1 is more then S2 . Hence

When the core is moved at B to the right then the voltage induced in S2 will be more then S1.

Therefore two differential voltages are 180 deg. Of phase with each other .

Capactive transdure :

These are passive transdures . the capcantance of a parallel plate capacitor is given by

Where A= overlapping area pf plate sq. of n1.

d = distance b/w two plates m .

= permittivity of medium f/m

= Relative permittivity .

= permittivity of free space .

In capacitive transdures capacitance of capacitor is very by any of the following 3 methods .

1. By varying overlapping area of plate
2. By varying distance b/w the plates .
3. By varying relative permittivity of dielectric b/w two plates .

Differential capacitor arrangement:-

In order to achieve linear characteristics differential capacitor arrangement is used. This arrangement using three plates as shown in the figure below:-

Fig:- differential arrangement of capacitors.

P_1 and P_2 are fixed plate and M is the movable plate to which the dicplacement to be measured is applied . thus we have two capacitors whose differential output is taken.

When the plate M is mid way between the plate P_1 and P_2 then their response capacitor be

$$C_1 = C_2 = \frac{A}{d} \quad \text{\_\_\_\_\_\_\_(i)}$$

Similarly,

$$E_1 = E_2 = E/2 \quad \text{\_\_\_\_\_(ii)}$$

And $E = E_1 - E_2 = 0$

Voltage across C_1 is, $E_1 = E \times \frac{C_2}{C_1+C_2}$ _______(iii)

Let the movable plate be moved up due to displacement x, then,

$$C1 = \frac{\varepsilon A}{d-x}, \qquad C_2 = \frac{\varepsilon A}{d+x}$$

$$E_1 = \frac{C_2 E}{C_1+C_2} \qquad \text{[from equation (iii)}$$

$$= E \frac{\in A/(d+x)}{\varepsilon A/(d-x) + \varepsilon A/(d+x)}$$

$$= E \frac{\varepsilon A/(d+x)}{\frac{\varepsilon Ad + \varepsilon AD - \varepsilon Ax}{(d-x)(d+x)}} \varepsilon$$

$$= E \frac{.\varepsilon A/(d+x)}{2\varepsilon Ad/[E(d-x)(d+x)]}$$

$$E \frac{\varepsilon A}{(d+x)} \times \frac{(d-x)(d+x)}{2\varepsilon Ad}$$

$$= E_1 = E\frac{d-x}{2d} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_(vi)}$$

Similarly,

$$E_2 = E\frac{d+x}{2d} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(v)}$$

Differential o/p voltage,

$$\Delta E = E_2 - E_1$$

$$= E\left(\frac{d+x}{2d} - \frac{d-x}{2d}\right)$$

$$\Delta E = E\, x/d$$

Therefore, the o/p voltage varies linearly as the displacement x.

$$\therefore \text{Sensitivity(s)} = \frac{\Delta}{x} = \frac{E}{d}$$

Note:- The differential method can be used for displacement of 10-8mm to 10mm with an accuracy of 0.1%.

Piezoelectric x'ducer's :-

There are active transducer, these transducer use piezo electric materials. A piezoelectric matrial is one in which an electric potential appears across certain surfaces of a cryotals if the dimensios of the crystal are change by the application of the mechanical force. This potential is produced by the dicplacement of charges. The effect is reversible i.e. conversely, if a potential difference is applied across the opposite faces of the material it charges it's physical dimension.s This effect is known as piezoelectric effect.

Common piezoelectric material include Rochellle salte, Amonanium dihydrogen phosphate, lithium sulphate, dipotasium tartarate, quartz, and caramacs A & B

When the force is applied to the crystal surface charges are induced. The magnitude and polarity of such charges are proportional to the magnitude and direction of applied force. So,

$$Q = d.F \quad \text{\_\_\_\_\_\_\_(i)}$$

Where

D = crystal charge sensitivity, c/N

F = Applied force, N

Therefore F causes a charge in thickness of the crystal by Δt in meter.

So, $f = \frac{AE}{T} \Delta t$

Where ,

A = Area of crystals in m2

E = young's modulus of elasticity.

T = thickness of the crystals.

The charge at the electrodes give raise to an O/P voltage E_0 and given by

$$E_0 = \frac{Q}{Cp}$$

Where, Cp = capacitance between electrical and since

$$Cp = \frac{\epsilon_o \epsilon_r A}{T} \quad \text{\_\_\_\_\_\_(iv)}$$

From equation (iii)

$$Eo = \frac{Q}{C_p} = \frac{d.f}{C_p} \qquad \text{from equation (i)}$$

$$= \frac{d.\ F}{\epsilon_o \epsilon_r A/t}$$

$$= \frac{d}{\epsilon_o \epsilon_r} \times \frac{F}{A} \times t$$

E_o = gpt

Where, g = d = Crystal voltage susitivity, (Vm/N) constant

$P = \frac{F}{A}$ = pressure in pascal.

Question:- A strain guage is banded to a beam 0.1m long and has a cros sectional area 4cm2. Young's modulus for steel is 207 x 109 N/m2. The strain gauge has an unstrain resistance of 240 ohm and a gauge factor 2 ohm when a load is applied the resistance of gauge changes by 0.3ohm . calculate the change in length of steel beam and the amount of force applied to the beam. [PU 2003]

Soln:- Gauge factor(k) = 2.2

Nominal length (l) 0.1m

Nominal gauge resistance ® = 240Ω

Change in resistance ® = 0.13 Ω

Young's modulus (E) = $207*10^9$ N/m^2

Cross- sectional are (A) = 4 cm^2

Δl = ?

Now, we have,

$$K = \frac{\Delta R/R}{\Delta l/l}$$

$$= \frac{0.13 * 240}{2.2} * 0.1m$$

$$= 2.46*10^{-5} \text{ m}$$

Now, stress(S) = σE

$$= \frac{\Delta l E}{L}$$

$$= 50.922 \times 106 \times 4 \times \times 10^{-4} \text{ N}$$

And force (f) = SA

$$= 50.922 \times 106 \times 4 \times 10^{-4} \text{ N}$$

20.37 KN

Question:-A quartz piezo electric crystal having a thichness 12.5 mm and voltage sensitivity of a 0.065 v-m/N is subjected to a pressure of 1.8 MN/M^2 . Calculate the voltage output if the permeability of quarz is 40.6 x 10^{-12} F/m. Calculate it's charge density.

Sol^n:- Given thickness of crystal (t) = 2.5 mm

$= 2.5 \times 10^{-3}$m

Voltage sensitivity (g) = 0.065 V-m/N

Pressure (P) = 1.8 MN/m2

$= 1.8 \times 106\ N/m^2$

Permittivity of quartz (ε) = $\varepsilon_o\varepsilon_r = 40.6 * 10^{-6}$ f/N

Now, Vo = gpt

$=0.065*1.8*106*2.5*10^{-3}$

= 292.5v

Now, charge density(d) $= \varepsilon_o\varepsilon_r g$

$= 40.6*10^{-12}*0.065$ C/N

$= 2.64*10^{-12}$ C/N

= 2.64 Pc/N

Resistance Thermometers:-

All most all metal conductor has a +ve tempreture coefficient of resistance. So that their resistance increase with an increase in tempreture. So the material such as carbon and germanium have a –ve tempreture coefficient of resistance.

Equivalent circuit of piezo-electric transducer:-

The basic equivalent circuit of a piezo- electric transducer is shown below:-

Fig:- equivalent circuit of piezo-electic transducer

The source is a charge generator. The value of charge is Q = d.F

Where, d = charge sensitivity of a crystals. (constant for a given crystal)

F = force applied N.

The generated is across the capacitance of the crystals Cp and it's leakage resistance Rp.

The charge generator can be replaced by an equivalent voltage source having a voltage of

$$E_o = \frac{Q}{C_p} = \frac{d.F}{C_p}$$

In series with a capacitance C_p and resistance R_p.

Resistance thermometer:-

Resistance tempreture detectors (RTDs) or resistance thermometers employ a sensitive element of extremely pure platinum, copper or nickel wire that provides a definite resistance value at each temperature with in it's range. The relationship between tempreture and resistance of conductors in the tempreture range near 0^oc can be calculated from the equation.

All most all metallic conductor have +ve temperature coefficient of resistance so that their resistance increase with an increase in tem. Same materials such as carbon and Gernaum have –ve tem coefficient of resistance that signifies that the resistance decrease with increase in temp.

Resistance thermometer are generally of the preamble type for immersion in the medium which temp is to be measured or controls fig shows the variation of resistance with temp for several, commonly used materials. The graphs indicates that the resistance platinum and copper increases all most linear with increasing temp. which the characteristic for nickel is non-linear.

Thermocouple:-

1821 thomas seeback discoued that when two dissimilar metal were in contact a voltage was generated where the voltage has a function of temperature, the device, consisting of two dissimilar metals joint together is called a thermocouple and the voltage is called seeback voltage in the honour of discover.

Thermistors:-

Thermistor is a centrak of of a term "thermal resistor" themistor are generally composed of semiconductors materials most thermistore have a –ve coefficient of temp. i.e. their resistance cases the resistance of a thermistor at room temperature may decrease as muchas 6% for each 1^0c rise in temp. This allows the thermistors circuit to detect very small charges in tempreture which could not be observed with an RTD or thermocouple. Thermistors are widly used in applications which involve measurements in the range of -600c to 150c. the resistance of thermistors range from 0.5ohm tol 0.75 M ohm.

Characteristic of thermistors

Do yourself

Characteristic of transducer:-

Chapter – 4

4. Signal conditioning and processing:-

Measurement of dynamic physical quantity require faithfull represention of their analog or digital output obtained from the intermediate stage i.e signal conditioning stage and this places a severe strain on the signal conditioning equipment. The signal conditioning equipment may be required to do linear process like

- Amplification
- Attenuation
- Integration
- Differetation
- Addition
- Substration

And non-linear processes like modulation demodulation, shamping , filtering , clipping, squaring, linearing and multiplication by another function etc.

These tasks are by no means simple. They require ingennity improper selection of most faithfull method of reproduction of output signals for the final data presentation stage.

The signal conditioning in many situation is an exciation and amplification system for passive transducer. For active transducers, it may an amplification system. In both the application, the transducer output is brough upto sufficient level to make it useful for conversion, processing indicating and recording.

Excitation is needed for passive transducer because these transducers don't generate their own voltage or current depending upon the excitation sources, a signal conditioning circuit may have A.C or D.C. voltage sources and according to these sources, signal conditioning circuit may be classified as:-

(i) D.C. signal conditioning system

(ii) A.C. signal conditioning system.

D.C. Signal conditioning System.

D.C. system are generally used for fommon resistance transducers. Such as potentiometer and strain gauges.

.

Calibration and Zeroing h/W:-

In this unit, the calibration of desired or required parameters like voltage, current , resistance is calibratdred in terms of measurand for example strain gauge is used in the dc bridge whose parameters found in terms of resistanced but is calibrated in terms of force pressure displacement i.e measurenad

Zeroing n/w:- It is a circuit which processes for calibration such that only the linear portion of the teranducer characteristice is genered. Therefore zeroing n/w fixes the zero point and calibration starts from there.

D.C. n/w:- In D.C. n/w we may have amplification, integration, addition substraction etc. unit. This unit works only when it is supplied by power supply.

Low pass filter:- It is used for filtering high frequency signal.

(ii) **A.C. signal conditioning System:-**

A.C. system have to be used for variable reactance transducer and for systems where signals have to be transmitted via long cables to connect the trasducer to the signal conditioning equipment.

Carrier oscillator:- This unit provides oscillating signal with certain frequency to run the A.C. bridge. This carrier signal is also provided to phase sensitive demodulator circuit to multiply the signal coming from the bridge via A.C amplification.

Phase sensitive demodulator:- It senses the phase or the signal and demodulates the incoming signal.

Filters:- For faithfcull reproduction fo signal it becomes necessary to elements any kind of unwanted signal which may get introduced into the system either at the transduction stage or at the signal conditioning stage. The filters are thus design to pass the signals of wanted frequncies and to reject the signal of unwanted frequncies.

An electrical filter is a frequency selective circuit that passes a specified band of frequency, blocks or alternate signal of frequency ouside this band. The basic electrical filters are of two forms as regards the components constituting them. They are

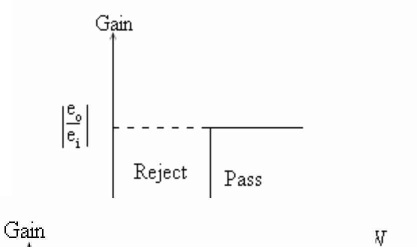
(i) passive filters.
(ii) Active filters.

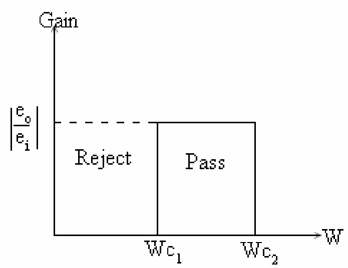
(i) **Passive filters**:- These filters only used passive circuit elements like resistors, capacitors and inductors.

(ii) **Active filters**:- There filters use active element like operationals amplifier in addition to passive element like resitance, inductance and capacitance.

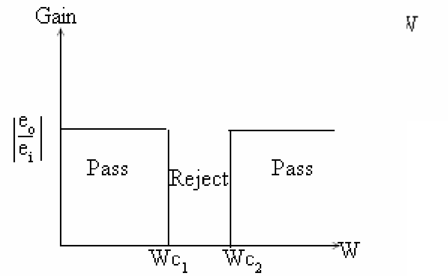
Both passive and active filters may classifie further as

(i) low pass filter
(ii) High pass filter





(c) Band pass filter



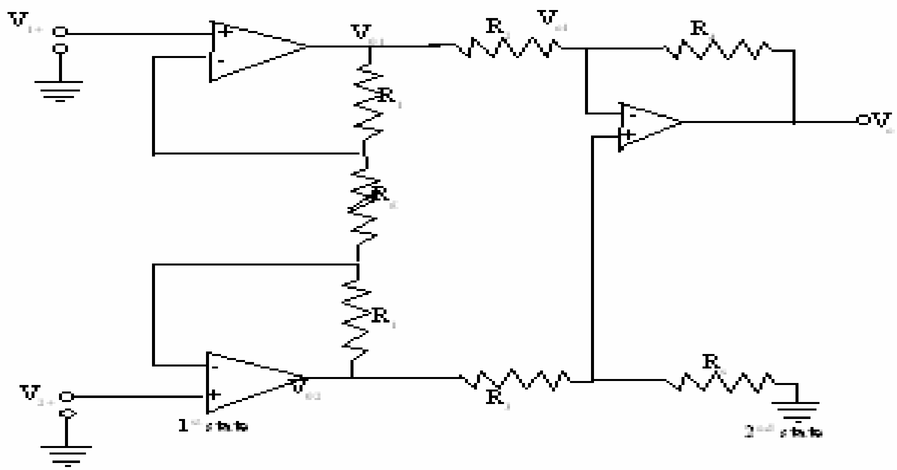
(d) band stop filter

(iii) Band pass filter
(iv) Band stop filter.

The response of various filters is shown below. There are ideal responses but can not be achieved in actual practice.

Instrumentation amplifier:- The instrumentation amplifier is a dedicated differential amplifier with extremely high input impedance. It's gain can be precisely set by a single internal or external resistor. The high common mode rejection makes this amplifier very useful in recovering small signals buried in large common mode off sets and nose. It is a closed loop device with carefully set gain.

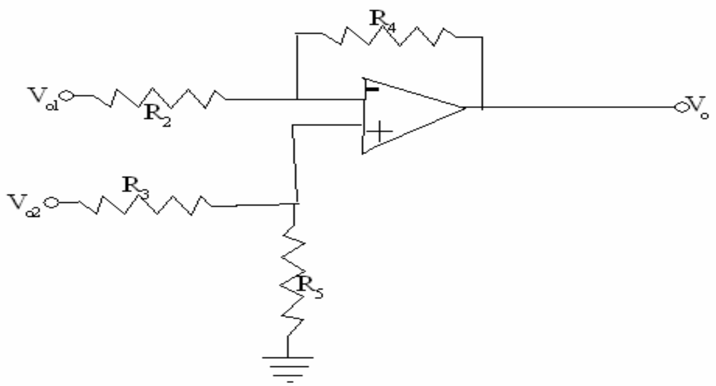
It consists of two stages. The first stage offers very high input impedance to both input signal and allows to set the gain with a single resistor. The second stage is a differential amplifiers with a o/p –ve feedback and ground connections all brought out.



$$V_{01} = \left(1+\frac{R_1}{R_g}\right)V_1 - \left(\frac{R_1}{R_g}\right)V_2$$

$$V_{02} = \left(1+\frac{R_1}{R_g}\right)V_2 - \left(\frac{R_1}{R_g}\right)V_1$$

Now for 2nd stage,



$$V_0 = \left(1+\frac{R_y}{R_2}\right)\left(\frac{R_5}{R_3+R_5}\right)V_{02} - \frac{R_4}{R_2}V_{01} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(iii)}$$

Putting the value of V_{01} and V_{02} in equation (iii), we get.

$$V_0 = \left\{\left(1+\frac{R_4}{R_2}\right)\left(\frac{R_5}{R_3+R_5}\right)\left[\left(1+\frac{R_1}{R_g}\right)V_2 - \left(\frac{R_1}{R_h}\right)V_1\right]\right)$$

For simplify it is assume that,

$R_2 = R_3 = R_4 = R_5 = R_1$, then,

$$V_0 = \left\{1+1\left(\frac{1}{1+1}\right)\left[\left(\frac{1+R_1}{R_h}\right)V_2 - \left(\frac{R}{R_h}\right)V_1\right]\right\} - \left\{\left(\frac{1}{1}\right)\left(1+\frac{R_1}{R_g}\right)V_1 - \left(\frac{R_1}{R_g}\right)V_2\right\}$$

$$= \left(1+\frac{R_1}{R_h}\right)V_2 - \left(\frac{R_1}{R_g}\right)V_1 - \left(1+\frac{R_1}{R_h}\right)V_1 + \left(\frac{R_1}{R_g}\right)V_2$$

$$= \left\{1+\frac{R_1}{R_g}+\frac{R_1}{R_g}\right\} - V_1\left\{1+\frac{R_1}{R_g}+\frac{R_1}{R_g}\right\}$$

$$= \left(1+2\frac{R_1}{R_g}\right)(V_2 - V_1)$$

$$= V_0 = (V_2 - V_1)\left(1+\frac{2R_2}{R_g}\right)$$

Force on moving Charge:-

. A charge particle in $\vec{F} = Q\vec{E}$ In electric field the force is defined as

is found experimentally to $\hat{B}$ motion in magnetic field of flux density experience a force whose mag is proportional to the product of mag. of

flux density and to the sign sine of the $\hat{B}$ and $\hat{V}$ charge Q_1 . It's velocity

$\vec{F} = Q\vec{E} \times \vec{B}$ angle between the vector and. Therefore $\therefore$

The force on a moving particle due to combine electric and magnetic field is obtained easily by superposition.

Called loventz's force equation. $\vec{F} = Q(\vec{E} + \vec{V} \times \vec{B})$

The force on a charge particle moving through a steady magnetic field, in differential form can be represented as

$d\vec{F} = dQ(\vec{V} \times \vec{B})$

(in terms of velocity) $\vec{J} = \rho_v \overrightarrow{\times V}$

And $dQ = \rho_v dv$

$\therefore$ $df = \rho_v dv\ (\vec{V} \times \vec{B})$

$\overrightarrow{(J} \times \vec{B})dv =$

And, previously, we say that,

$\vec{J}.dv = \vec{K}ds = Id\vec{L}$

$d\vec{F} = (\vec{K} \times \vec{B})ds$

$d\vec{F} = (Id\vec{L} \times \vec{B})$

Force on moving charge:-

$$\vec{F} = \int_{vol} (\vec{J} \times \vec{B})dv$$

$$\vec{F} = \int_{sur} (\vec{K} \times \vec{B})dv$$

$$\vec{F} = \int_{Line} (Id\vec{L} \times \vec{B})$$

$$\vec{F} = \int Id\vec{L} \times \vec{B}$$

$$\vec{F} = \int IdL.BSin\theta$$

$= L.BSin\theta$

$\therefore F = BILSin\theta$

Permeability( ):- The permeability or absolute permeability is the property of a material denoted by . show how well the material conduct the

$\mu_o = 4\pi \times 10^{-7}$ magnetic flux through it.

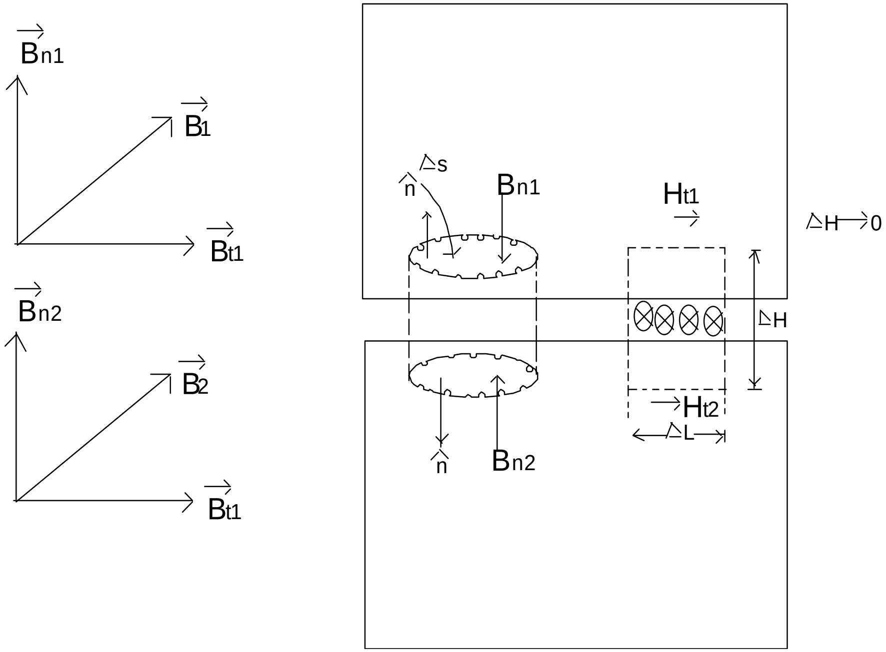
where, A quantity called relative permeability, is defined as

$\mu_r = \mu / \mu_o$

= is the permeability of given material. μ

= permeability of vaccum. μ_o

Boundary Condition:-



Boundary condition in magneto statics.
From Maxwell last equation in static electric field and steady magnetic field.

$$\nabla.\vec{B} = 0$$

$$\int_{vol} \nabla.\vec{B} = 0$$

$$\int_{sur} \vec{B}.ds = 0$$

$$\int_{Top} \vec{B}.d\vec{s} + \int_{botom} \vec{B}.d\vec{s} + \int_{Side} \vec{B}.ds = 0$$

$$\vec{B}x_1 S - \vec{B}x_2 S = 0$$

__________(i) $\vec{B}x_1 - \vec{B}x_2$

$$\mu_1 \vec{H}n_1 = \mu_2 \vec{H}n_2$$

__________(ii) $\frac{\vec{H}n_1}{\vec{H}n_2} = \frac{\mu_2}{\mu_1}$

The normal component of magnetic flux is continueous that is it does not charge in crossing from one to another medium and from equation (ii) it shows that the normal component of magnetic flux intensity is discontinuous.
Now from Ampere circuital law,

$$\oint H.d\vec{L} = I$$

$$\int_{upper} H.dL + \int_{hight} H.dL + \int_{Lower} H.dL + \int_{Lect} H.dL = I$$

$$\vec{H}t_1.\Delta L - \vec{H}t_2.\Delta L = K\Delta L$$

$$Ht_1 - Ht_2 = K$$

$\frac{Current}{Width}$ Where, K =

K as is usually zero for no current exist.
At the boundary, ∴

$Ht_1 - Ht_2 = 0$

$Ht_1 = Ht_2$ __________(iii)

$$\frac{Bt_1}{\mu_1} = \frac{Bt_2}{\mu_2}$$

__________(iv) $\frac{\vec{B}t_1}{\vec{B}t_2} = \frac{\mu_1}{\mu_2}$

From equation (iii) it is clear that, tangential component of magnetic field intensity is continuous at the boundary but the tangential component of magnetic flux density is discontinuous.

Chapter:- 12
<u>Faraday Law:-</u>

This law states that an emf and induced potential difference that appear at the two terminals of open ckt is equal to the time rate of change of magnetic fluxes linking the ckt.

Where, - (V or wb/sec) ___(i) $-\frac{d\phi}{dt}$ Mathematically, e. m. f =

ve sign comes from lenses law that the induced voltage acts to produce an opposing flux.

The e. m f appear at the two terminals of an open ckt in three ways.

(i) The two terminals of an open ckt moves in a static magnetic field, the motional induction.

(ii) The static open ckt is under the changing (Time variant) magnetic field, the transformer induction.

(iii) The two terminals of an open ckt moves in time variant magnetic field, the net induction.

If the closed path is that taken by an n turn filamentary conductor.

$\frac{d\phi}{dt}$ emf = - N

is the voltage about a specific closed path. $\oint \vec{E}.d\vec{L}$ And emf =

Under the time varying flux (Transformer induction).

$-\frac{d\phi}{dt} = \oint E.d\vec{L} \therefore$ emf =

__________(3) $-\frac{d\left(\int_s \vec{B}.d\vec{S}\right)}{dt} = \oint \vec{E}.d\vec{L}$ or,

Where B is in the direction of ds.

Here, the magnetic flux is the only time variant quantity on the right side of the equation (3) and the partial derivative may taken under integral sign.

$\int_s \frac{\partial B}{\partial t}.d\vec{S} = \oint \vec{E}.d\vec{L} \therefore$

By applying stoke theorem,

$\int_{Sur} (\nabla \times E).d\vec{S} = -\int_{sur} \frac{\partial \vec{B}}{\partial t} d\vec{S}$

Now differentiating the about equation.

$\left(\nabla \times \vec{E}\right) d\vec{S} = -\frac{\partial \vec{B}}{\partial t}.d\vec{S}$

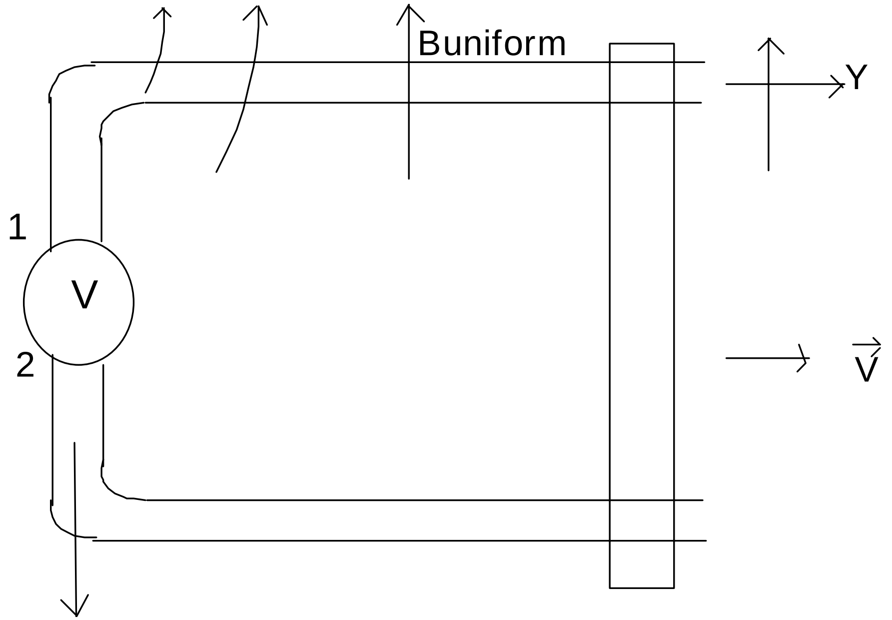
$\therefore \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

One of the Maxwell four equation,

Where, B is not in function time

$0 \therefore \nabla \times \vec{E} =$

Now, let us consider the case of a time constant flux and moving closed path.



Let us consider the sliding bar is moving at velocity and magnetic flux density is constant is normal to the plane containing the closed path.

ϕ = BA = B.yd

$$\therefore emf = -\frac{d\phi}{dt} = -\frac{d(Byd)}{dt}$$

d$\vec{u}$ = - B

Again, from lentz equation , for magnetic field,

$$\vec{F} = Q\overrightarrow{u} \times \overrightarrow{B}$$

$$\frac{F}{Q} = \vec{u} \times \vec{B}$$

is the motional electric field intensity, $\vec{u} \times \vec{B}$ E_m=

$\oint \vec{E}_m . d\vec{L} = \oint (\vec{u} \times \vec{B}) dL \therefore$ e. m. f =

And for above configuration,

$\oint (\vec{u} \times \vec{B}) dL = \int_0^d VBSin90^0 . dx$ e. m. f =

= - BVd

$\oint (\vec{u} \times B) . dL$ e. m. f =

Again, under the influence of both motional induction and transformer induction we have,

$$\therefore emf = \oint E.dL = -\int \frac{\partial B}{\partial t} . d\vec{S} + \oint (\vec{U} \times \vec{B}) . d\vec{L}$$

Inadquency of Ampere's law derived for direct current:-

The point form of Ampere's circuital law is . it is derived with a conductor carrying direct current. It is only applicable to time invariant situation. However time invariant situation are demenate in practice. Hence there is inadquancy in the amperes circuital law regardless of point or integrant form.

Ampere's law conflicting with the continuity equation:-

We have,

$$\nabla \times \vec{h} = \vec{J}$$

$$\nabla (\nabla \times H) = \nabla . \vec{J}$$

_____(i) divergence of curl of any vector is zero. $\nabla . \vec{J} = 0$

However, continuity equation is given by,

_________(ii) $\nabla . \vec{J} = -\frac{\partial \rho_v}{\partial t}$

Here, result obtained from equation (i) is valid and it is conflecting with equation (ii) that involves, sometime variant quantity. Hence, they are conflecting.

Displacement Current:-

As we saw ear tiler the amperes circuital law conflecting with the continuity equation, Maxwell succeeded in modifying amperes circuital law. So that it may be used for time variant situation. As well as it not conflecting with the continuity equation.

is added in ampere circuital law so that, $\vec{G}$ A vector

$$\nabla \times \vec{H} = \vec{J} + \vec{G}$$

Taking divergence on both side of above equation we have,

$$\nabla.(\nabla \times \vec{H}) = \nabla(\vec{J} + \vec{G})$$

$$0 = \nabla.\vec{J} + \nabla\vec{G}$$

$$\nabla.\vec{J} = -\nabla\vec{G}$$

And from continuity equation,

$$\nabla.\vec{J} = -\frac{\partial \rho_v}{\partial t}$$

) $\rho_v = \nabla.D$ (Since, $\quad \nabla.\vec{G} = \frac{\partial \rho_v}{\partial t}$

$$\nabla.\vec{G} = \frac{\partial \nabla \vec{D}}{\partial t}$$

$$\nabla.\vec{G} = \frac{\partial \nabla \vec{D}}{\partial t} = \nabla..\frac{\partial \vec{D}}{\partial t}$$

$$\vec{G} = \frac{\partial \vec{D}}{\partial t}(A/m^2)$$

$\therefore$ The equation (i) becomes:

Maxwell 1st equation in time varying field. $\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{G}}{\partial t}}$

Hence Ampere circuital law can be used for time variant situatin and obeys the continuity equation. And if we take divergence yields the continuity equation.

The term is refered to the displacement current density and denoted by

(A/m^2) $\quad \boxed{\vec{J}d = \frac{\partial \vec{G}}{\partial t}}$

$\oint \vec{J}d.dS = \int \frac{\partial \vec{D}}{\partial t}.d\vec{S}$ And Id =

Called the displacement current and suggest that how time varying magnetic field (in term time varying electric field) may exist in free space even there is no conduction current at all. Since it results from a time varying electric flux density (or displacement density) termed as displacement current density.

The conduction current density is the motion of charge usually electron in a region of zero net charge density i.e.

$$\vec{J} = \sigma E$$

Convection current density is the motion of volume charge density

and the third in non conducting medium in which no $\vec{J} = \rho_v \vec{V}$ denoted by

= 0. $\vec{J}$ volume charge density is present

= 0) is the displacement current. $\vec{J}$ (if $\quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$

Again, we may obtain a time varying version of ampere circuital law by integrating over the surfaces is given by.

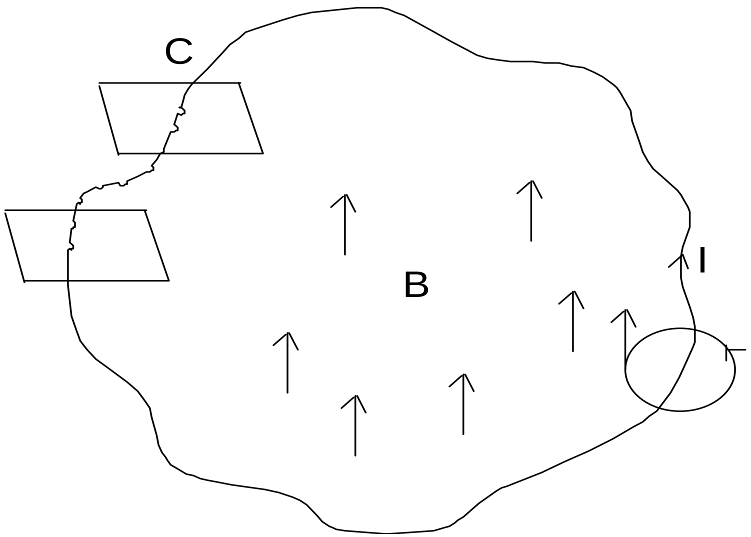
$$\int_S (\nabla \times \vec{H}).d\vec{S} = \int \vec{J}.d\vec{S} + \int \vec{D}.d\vec{S}$$

By applying stokes theorem.

$$\oint \vec{H}.d\vec{L} = I + Id = I + \int_S \frac{\partial \vec{D}}{\partial t}.d\vec{S}$$

Let us consider a filamentary conductor forms a loop, connecting the two plate of a parallel plate capacitor. Now varying the magnetic field

Emf = V_oCoswt $\quad$ sinusodialy the induced emf is given by



$$\therefore I = \frac{\partial \theta}{\partial t} = \frac{dCv}{dt}$$

$$\frac{d}{dt}(V_o Coswt) = \text{C.}$$

$= -W_c V_o$Sinwt

$$\in \frac{S}{d} V_o Sinwt = \text{W.}$$

$$\oint_K H.d\vec{L} = I_k \text{ and}$$

The path and value of along the path are both definite quality and is definite quantity and I_L is the current through any $\oint_K H.d\vec{L}$ integration closed surface and in conductor it is conduction surface.

Now we used to consider the displacement current, within the capacitor then,

$$\vec{D} = \in \vec{E}$$

$$\text{V/d} \in =$$

$$\frac{\text{V}_o\text{Sinwt}}{\text{d}} \in =$$

$$\frac{\partial \bar{D}}{\partial t}.S \ \ \text{Id} = \therefore$$

$$\frac{\partial}{\partial t}.\frac{\in (V_o Coswt)}{d}.S =$$

$$\frac{W \in V_o}{d} Sinwt \ \text{Id} = -$$

This is same as the conduction current in filametnryu loop and it is due to this current propagation is possible in free space.

<u>Maxwell's third and fourth equations for time variant fields:-</u>

Maxwell's 3^{rd} and 4^{th} equations for tie variant fields are identical to these listed for time inverient as it deals with the charge enclosed.

$$\nabla.\vec{D} = \rho_v$$

$$\nabla.\vec{B} = 0$$

Maxwell's equations in point form, Maxwell's equation in integral form:-

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \vec{B} = \rho_v$$

$$\nabla \times \vec{B} = 0$$

are $\vec{E}$ and $\vec{D}$ And auxiliary equations relating

$$\vec{D} = \in \vec{E}$$

$\vec{H}$ and $\vec{B}$ And

$\vec{B} = \mu\vec{H}$

If we do not have nice materials to work with, then,

$\vec{D} = \in\vec{E} + \vec{P}$

$\vec{B} = \mu(\vec{H} + \vec{M})$

is the magnetization field. $\vec{M}$ is the polarization and $\vec{P}$ Where,

$X_e E_o \vec{E} = \vec{P}$

$X_m \vec{H}$ and M =

$$\nabla \times H = \vec{J} + \frac{\partial\vec{D}}{\partial t} \rightarrow \oint \vec{H}.d\vec{L} = \int_S \vec{J}.d\vec{S} + \int_S \frac{\partial\vec{D}}{\partial t}.d\vec{S}$$

$$\nabla \times E = -\frac{\partial\vec{B}}{\partial t} \rightarrow \oint \vec{H}.d\vec{L} = \int_S \vec{E}.d\vec{L} = -\int \frac{\partial B}{\partial t}.d\vec{S}$$

$$\nabla.\vec{D} = \rho_v \rightarrow \int_S \vec{D}.d\vec{S} = \int_{vol} \rho_v.dV$$

$$\nabla.\vec{B} = 0 \rightarrow \int_S \vec{B}.d\vec{S} = 0$$

Time invariant Integral form	**Time invariant point form**
$\oint \vec{H}.d\vec{L} = \int_S \vec{J}.d\vec{S}$	$\nabla \times H = \vec{J}$
$\int \vec{E}.d\vec{L} = 0$	$\nabla \times E = -\frac{\partial\vec{B}}{\partial t}$
$\int_S \vec{D}.d\vec{S} = \int_{vol} \rho_v.dV$	$\nabla.\vec{D} = \rho$
$\oint \vec{B}.d\vec{S} = 0$	$\nabla.\vec{B} = 0$

Time invariant Integral form	**Time invariant point form**
$\oint \vec{H}.d\vec{L} = \int_S \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) d\vec{S}$	$\nabla \times H = \vec{J} + \frac{\partial\vec{D}}{\partial t}$
$\int_S \vec{E}.d\vec{L} = -\int_S \frac{\partial B}{\partial t}.d\vec{S}$	$\nabla \times E = -\frac{\partial\vec{B}}{\partial t}$
$\int_S \vec{D}.d\vec{S} = \int_{vol} \rho_v.dV$	$\nabla.\vec{D} = \rho_v$
$\int_S \vec{B}.d\vec{S} = 0$	$\nabla.\vec{B} = 0$

<u>Retarted Potential:-</u>

Scalar electric potential V at a point cause by line charge with a linear charge density is defined as

__________(i) $\int \frac{\rho_L dL}{4\pi \in r}$ V =

Where, r is the distance between dL and point of interest. Similarly, magnetic vector potential is defined as

_________(ii) $\vec{A} = \int \frac{\mu dL}{4\pi r}$

Equation (i) and (ii) the ρ_L and I do not charge with time and therefore V and A at a the point of interest are fixed for all the time. But if ρ_L and I very withy time then their values seen at the time of measurement cannot be used to calculate V and A at a distance point.

Because it takes time to reach the effect from source to the point of interest. The values of ρ_L and I has been changed to some other new values. Hence, the values of ρ_L and I which occurred, before the time required for the effect to be reach from the source top point of interest must be read out to calculate the effecty at the point of interest. Hence equation (i) and (ii) are modified to

___________(iii) $\int \frac{[\rho_L]dL}{4\pi \in r}$ V =

$\vec{A} = \int \frac{\mu[I]dL}{4\pi r}$

are called retarted scalar electric and retarted vector $\vec{A}$ Hence, V and magnetic potential. The symbol [.] represents that the corresponding quantity has been retared in time inorder encompass the lapled in propagating the effect from source to the point where quantity is being calculated.

has to be calculated at point P for time t $\vec{A}$ If retarted magnetic potential then,

_ _ _ _ r _ _ _ _ _ _ .P

$\vec{V}$

Velocity

)]$\vec{V}$ [I] I_oCos(W(t- t')] = I_oCos(W(t – t'/

Chapter:- 13

Wave equations:-

<u>Wave propagation free space:-</u>

In free space Maxwell equation can be written in the form of

$$\nabla \times \overrightarrow{H} = \frac{\partial D}{\partial t} = \in \frac{\partial \overrightarrow{E}}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial \overrightarrow{H}}{\partial t}$$

$$\nabla . \overrightarrow{D} = 0$$

$$\nabla . \overrightarrow{H} = 0$$

If the expression for time varying x component of the electric field then, $\vec{E}_x$

$_o$ Cos(Wt + Ψ) $\vec{E}_x$ (x,y,z) Cos (wt +Ψ)= $\vec{E}_x$ = $\vec{E}_x$

(in phasor expression) $E_{xs} = E_{xo}e^{j\psi}$ and

= Coswt + Jsinwt $e^{jw\psi}$ and

$\therefore \vec{E}_x = \mathrm{Re}\{\vec{E}_{xs}.e^{jwt}\}$

$\mathrm{Re}\{\vec{E}_{xo}.e^{j\psi}.e^{jwt}\}=$

$= \mathrm{Re}\{\vec{E}_{xo}.e^{j(wt+\psi)} + JSin(Wt+\psi)\}$

$= \vec{E}_{xo}Sin(Wt+\psi)$

$\vec{E}_x = \vec{E}_{xo}Sin(Wt+\psi)$ Again,

$\frac{\partial E_x}{\partial t} = -W\vec{E}_{xo}.Sin(Wt+\psi)$

$$\mathrm{Re}\{(Jw.\vec{E}_{xs}).e^{jwt}\}$$
$$= \mathrm{Re}\{Jw.\vec{E}_{xo}.e^{jw}e^{jwt}\}$$
$$= \mathrm{Re}\{Jw.\vec{E}_{xo}.e^{j(wt+\psi)}\}$$
$$= \mathrm{Re}\{Jw.\vec{E}_{xo}\{Cos(wt+\psi) + JSin(wt+\psi)\}.\}$$
$$= \mathrm{Re}\{Jw.\vec{E}_{xo}.Cos(wt+\psi) - JSin(wt+\psi)\}$$
$$= w\vec{E}_{xo}Sin(wt+\psi)$$

Hence, the multiplication of a phaser quantity by jw is equivalent to the differentiation of that quantity in time domain. Thus, phasor has replaced the differentiation with a multiplication. Hence Maxwell's four equation in the point forms for time variant field can be rewritten as.

$$\therefore \nabla \times \vec{H}_s = JW \in_o \vec{E}_s$$
$$\nabla \times \vec{E}_s = -JW\mu_o \vec{H}_s$$
$$\nabla.\vec{E}_s = 0$$
$$\nabla.\vec{H}_s = 0$$

$\nabla \times \vec{E}_s = -JW\mu_o \vec{H}_s$ Again,

Taking curl on both sides, we have

$$\nabla \times \nabla \times \vec{E}_s = \nabla(\nabla.E_o)\nabla^2\vec{E}_s = -JW\mu_o\nabla\vec{H}_s$$
$$-\nabla^2\vec{E}_s = -JW\mu_o(jw\nabla \in_o \vec{E}_s = W^2 \in_o \mu_o\vec{E}_s$$
$$\nabla^2\vec{E}_s = W^2 \in_o \mu_o \vec{E}_s$$

___________Helmtoltz's equation $\boxed{\nabla^2\vec{E}_s = K^2{}_o\vec{E}_s}$

is called the wave number. $k_o = w\sqrt{\in_o \mu_o}$ Where,

$$\therefore \nabla^2\vec{E}_{xs} = -K^2{}_o\vec{E}_{xs}$$

The x component of above equation is

$$\frac{\nabla^2\vec{E}_{xs}}{\partial x^2} + \frac{\nabla^2\vec{E}_{xs}}{\partial y^2} + \frac{\nabla^2\vec{E}_{xs}}{\partial z^2} = -K^2{}_o\vec{E}_{xs}$$

does not vary with x or y, then, $\vec{E}_{xs}$ Assuming

$$\frac{\nabla^2\vec{E}_{xs}}{\partial z^2} = -K^2{}_o\vec{E}_{xs}$$
$$or, \frac{\nabla^2\vec{E}_{xs}}{\partial z^2} + K^2{}_o\vec{E}_{xs} = 0$$

This is the second order differential equation whose sometime is given by.

__________(a) $\quad \vec{E}_{xs} = \vec{E}_{xs}e^{-JK_oz}$

Next we insert, e^{jat} factor and take real part.

in +z direction $\therefore E_x(z,t) = \vec{E}_{xs}Cos(wt - K_oz)$

in -z direction $E_x(z,t) = \vec{E}_{xs}Cos(wt + K_oz)$

Where K_o will be interpreted as a spatial frequency. Which in the present case measures a phase shift per unit distance along z-direction i.e radia per meter.

$$C = \frac{1}{\sqrt{\mu_o \in_o}}$$
$$Vp = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_r \in_r}}$$

Thus, we can write,

$K_o = W/c$

$$E_x(z,t) = \vec{E}_{xs}Cos[w(t - z/c)] \therefore$$

And our requirement is that, the entire cosine argument be the same multiple of 2π for al the time in order to keep trap of chosen point.

$Wt - K_o z = W(t\text{-}z/c)\ 2m\pi$

Which signifies that as time increases the position z must also increases in order to satisfy above equation and also called as traveling wave.

Again from maxwell's second equation.

$$\nabla \times \vec{E}_s = jw\mu_o \vec{H}_s$$

component varying only with z, then, $\vec{E}_{xs}$ And for

$$\frac{\partial E_{xs}}{\partial z} = -jw\mu_o \vec{H}_{ys}$$

$$\therefore \vec{H}_{ys} = -\frac{1}{jw\mu_o}(-jko)\vec{E}_{xo}.e^{-jk_o z}$$

$$= -\frac{K_o}{w\mu_o}\vec{E}_{xo}.e^{-jk_o z}$$

$$= -\frac{w\sqrt{\mu_o \in_o}}{w\mu_o}\vec{E}_{xs}.e^{-jk_o z}$$

$$= \frac{\sqrt{\in_o}}{\mu_o}\vec{E}_{xs}.e^{-jk_o z}$$

$$\_\_\_\_\_\_\_\_(6) \therefore \vec{H}_y(z,t) = \sqrt{\frac{\in_o}{\mu_o}}\vec{E}_{xo}Cos(wt - K_o z)$$

Here, the ratio,

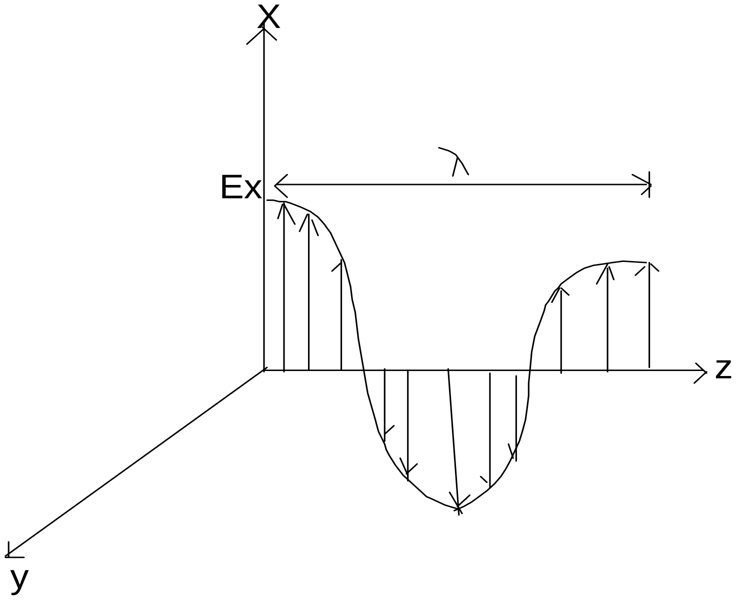
$$\frac{\vec{E}_x}{\vec{H}_y}\sqrt{\frac{\in_o}{\mu_o}} = Cons\tan t$$

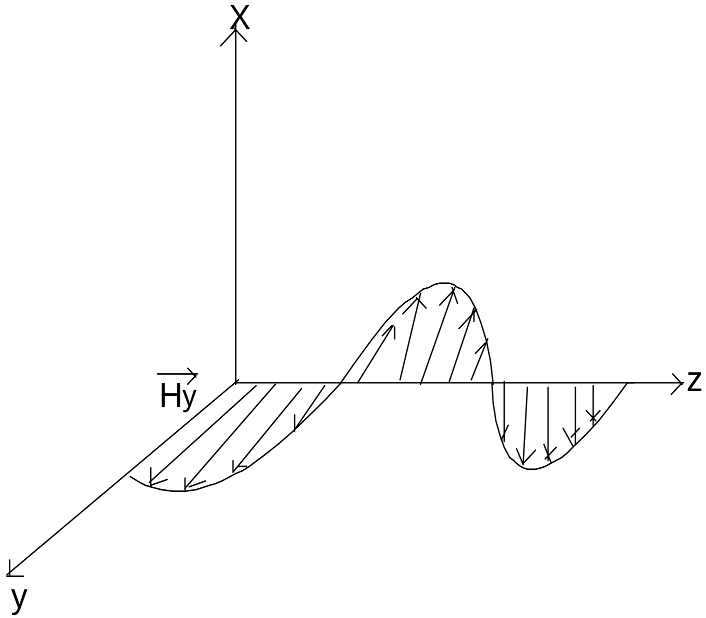
occurs when $W(t - z/c)$ is an integral $\vec{H}_y$ or $\vec{E}_x$ The maximum value of multiple or 2π radian i.e neither field is maximum every where at same instant. The ratio both changes in space and time should be every where constant and is called instransic impedasnce denoted by η (etc)

for free space $\sqrt{\frac{\in}{\mu}}\ \eta =$

$= 120\pi \quad 377 \ \cong \ \sqrt{\frac{\in_o}{\mu_o}}\ \eta_o =$

And is called uniform plain wave became it's value is uniform through out any plain. Both the electric and magnetic fields are perpendicular to the direction of propagation or both lie in plane that is transverse to the direction of propagation and is called transverse electromagnetic wave (TEM).





Wave propagation in dielectrics:-

Let us non extend our analytical treatment of the uniform plane wave to propagation in a dielectric of permeability ( ) and permeability ( ). Medium is isotropic and homonous.

From helmtoltz equation

$$\nabla^2 E_s = -K^2 E_s$$

And the wave number is now a function of material property.

$\sqrt{\mu \in} = K_o \sqrt{\mu_r \in_r}$ K = W

we have, $\vec{E}_{xs}$ And for

________(ii) $\frac{\partial^2 \vec{E}_{xs}}{\partial z^2} = K^2 \vec{E}_{xs}$

An important feature of wave propagation in a dielectric that K can be complex value and is referred to as complex propagation constant given by,

Jk = α+ jβ

And the solution of equation (ii) will be

$\vec{E}_{xs} = \vec{E}_{xs} e^{-j\beta z}$

__________(3) $\vec{E}_{xo} e^{-\alpha z} e^{-j\beta z} =$

Multiplying equation (3) by e^{jwt} and taking real part.

$E_{xs} e_{jwt} = E_{xo} e^{-z} e^{j(wt-\beta z)}$

$E_x = E_{xo} e^{-\alpha z} Cos(wt-\beta z)$

We say that uniform plane wave propage in forward z direction with phase constant β (rad/m). But losses amplitude with increasing z, according to the factor $e^{-\alpha z}$ for +ve α. Thus the general effect of a complex value K is to yield a traveling wave that changes it's amplitude with distance.

If α is +ve then it is called attention coefficient. If α is –ve then it is called gain coefficient as in laser amplifier.

Phase velocity:-

When α = 0 then it's equation (4) becomes:

)) $\sqrt{\mu \in}$ E_x = Ex_0 Cos(W(t-z

$\sqrt{\mu \in}$ Where, β = Co

)) is constant for any point on the wave provided $\sqrt{\mu \in}$ The quantity (W(t-z the distance of the point is measured from certain reference line

) = constant $\sqrt{\mu \in}$ W(t-z

Again differenciating w.r.t time. Above equation will reduces to

) = 0 $\sqrt{\mu \in}\ \dfrac{dZ}{dt}$ W(1-

$\dfrac{dZ}{dt} - \dfrac{1}{\sqrt{\mu \in}}$ or,

is the rate of change fo distance w.r.t time or velocity of the $\dfrac{dZ}{dt}$ where,

$\sqrt{\mu \in}$ constant phase point called phase velocity defined by V_p = 1/

$\dfrac{1}{\sqrt{\mu_o \in_o}} = 3 \times 10^8 m/s$ for free space, Vop =

$\dfrac{C}{\sqrt{\mu_r \in_r}} =$

Now form Maxwell's second equation ,

$$\nabla \times E_s = -jw\mu \vec{H}_s$$

for x component of electric field, we have y component of magnetic field and the above equation is represented as:-

$$\frac{\partial \vec{E}_{xs}}{\partial z} = -jw\mu \vec{H}_s$$

$$\vec{E}_{xo} e - jw\sqrt{\mu \in z} - jw\sqrt{\mu \in} = -jw\mu \vec{H}_{ys}$$

$$\vec{H}_{ys} = \vec{E}_{xo}\sqrt{\frac{\in}{\mu}} Cos(wt - \beta z)$$

in time domain,

$$\vec{H}_{ys} = \vec{E}_{xo}\sqrt{\frac{\in}{\mu}} Cos(wt - w\sqrt{\mu \in} z)$$

$$\vec{E}_{xo}\sqrt{\frac{\in}{\mu}}Cos(wt-\beta z)=$$

$$\eta=\sqrt{\frac{\mu}{\in}}$$ where, intrinsic impedance

Hence, two fields are again perpendicular to each other hence TEM (transverse electromagnetic medium).

Wave equation in Dissipation (Lossy) media:-

In dielectric, conductivities are zero and only the displacement current was considered in perfet conductor oly the conduction current exist but most of the media in real life fall in between these two extreme and therefore both the conduction and displacement currents are considered.

$$\therefore \nabla\times\left(\nabla\times\vec{E}_s\right)=\nabla\left(\nabla.\vec{E}_s\right)-\nabla^2\vec{E}_s$$

$\nabla\times\vec{E}_s = jw\mu\vec{H}_z$ But,

$\nabla.\vec{E}_s=0 \therefore \nabla^2\vec{E}_s=\nabla(-jw\mu\vec{H}_s)$ And

_________(ii) $-jw\mu\left(\nabla\times\vec{H}_s\right)=-\nabla^2\vec{E}_s$ or,

Now, from maximum 1st equation,

$$\nabla\times\vec{H}_s=\vec{j}+jw\in\vec{E}_s$$

$$\sigma\vec{E}+jw\in\vec{E}_s=$$

$$\vec{E}(\sigma+jw\in)=$$

Putting this value in equation (ii) ,

$$-jw\mu(\sigma+jwt)\vec{E}_s=-\nabla^2\vec{E}_s$$

$$\nabla^2\vec{E}_s=(jw\sigma-w^2\sigma t).\vec{E}_s$$

_____(iii) $-(w^2\mu t-jw\mu\sigma).\vec{E}_s=$

Now taking only the x component varying along z direction ,

____(iv) $\therefore\dfrac{\partial^2E_{xs}}{\partial z^2}=-(w^2\mu\in-jw\mu\sigma)\vec{E}_{xs}$

Form the previous section, since the medium is dissipating in time, the wave attenuates experimentally,

$$\vec{E}_{xs}=\vec{E}_{xo}e^{-z}e^{-j\beta z}$$

$$\vec{E}_{xo}e^{-(\alpha+j\beta)z}e^{-j\beta z}=$$

$$\vec{E}_{xo}e^{\gamma z}\quad=\vec{E}_{xs}$$

and is called as peropagation constant $\gamma=\alpha+j\beta$ Where,

= attemation constant. α

= phase constant in complex form. β

Substituting this value in equation (iv)

$$\therefore\ \gamma^2e^{xo}e^{-\gamma}z=-(w^2\mu\in-jw\mu\sigma)\vec{E}_{xo}e^{-\gamma z}$$

$$\gamma^2=-(w^2\mu\in-jw\mu\sigma)$$

Where only the +ve value of has meaning. Thus,

___________(5) $\gamma^2=\sqrt{-w^2\mu\in-jw\mu\sigma}=\alpha+j\beta\ \therefore$

$$\gamma=\sqrt{j^2w^2\left(\frac{1}{j^2w^2}\right)jw\mu\sigma-\frac{1}{j^2w^2}w^2\mu\in}$$

$$jw\sqrt{\frac{\mu\sigma}{jw}+\mu\in}=$$

$$jw\sqrt{-j\frac{\mu\sigma}{w}+\mu\in}=$$

$$jw\sqrt{\mu\in\left(1-j\frac{\sigma}{wt}\right)}=$$

$$\gamma=jw\sqrt{\mu\in\left(1-j\frac{\sigma}{wt}\right)}$$

Again for magnetic field,

$$\nabla\times\vec{E}_s=-jw\mu\vec{H}_s$$

$$\frac{\partial\vec{E}_s}{\partial z}=-jw\mu\vec{H}_{ys}$$

$$\frac{\partial}{\partial z}\left\{E_{xo}e^{-\gamma z}\right\}=-jw\mu\vec{H}_{ys}$$

$$\gamma E_{xo}e^{-\gamma z}=-jw\mu\vec{H}_{ys}$$

$$\vec{H}_{ys}=\frac{\gamma}{jw\mu}E_{xo}e^{-\gamma z}$$

$$\text{\_\_\_\_\_\_\_\_(vi)}\ \vec{H}_{ys}=E_{xo}e^{-\gamma z}\frac{1}{jw\mu/\gamma}$$

Again from equation (5)

$$\gamma=\sqrt{jw\mu\sigma-w^2\mu\in}=\sqrt{jw\mu\left(\sigma-\frac{w\in}{j}\right)}$$

$$\sqrt{jw\mu(\sigma-jw\in)}=$$

$$\therefore\vec{H}_{ys}=E_{xo}e^{-\gamma z}\frac{1}{\dfrac{jw\mu}{\sqrt{jw\mu(\sigma+jw\in)}}}$$

$$=E_{xo}e^{-\gamma z}\frac{1}{\sqrt{\dfrac{jw\mu}{\sigma+jw\in}}}$$

$$\therefore\vec{H}_{ys}=\frac{E_{xo}e^{-\gamma z}}{\sqrt{\dfrac{jw\mu}{\sigma+jw\in}}}$$

Where, intrinsic impedance,

$$\sqrt{\frac{jw\mu}{\sigma+jw\in}}\ \eta=$$

Skin Effects:-

The electric field intensity in a dissipative medium attennates experimentially as discussed in the previous section is given by.

$$\vec{E}_x=E_{xo}e^{-\alpha z}Cos(wt-\beta z)$$

Since the displacement current is negligible.

$$\vec{E}_x=E_{xo}e^{-\alpha z\sqrt{\pi f\mu\sigma}}Cos(wt-z\sqrt{\pi f\mu\sigma})$$

$$J_{xo}=\sigma\vec{E}_x$$

$$e-^{z\sqrt{\pi f\mu\sigma}}\ Cos(wt-z\sqrt{\pi f\mu\sigma})\ \ \sigma\vec{E}_x=$$

The exponential factor is unify at z = 0 and decreases to e^{-1} = 0.368

$$\frac{1}{\sqrt{\pi f \mu \sigma}}\, Z =$$

AC Resistance:-

The4 resistance of a conductor to the direct current is called the dc resistance and is same throughout the material and hence the current flow uniformly through out the cross section of the conductor.

In contrary the resistance of a conductor to the alternating current a.c. at any point and current is not uniform throughout the cross section of the conductor.

The skin effect has already shown that the conduction current decreases experimentally at it penetrates into the conductor and dies out within the few skin depths. Thus it may be said that the a.c resistance of a conductor increases exponentially in the direction towards it's centre hence a.c flows through the thin outer layer having a width of a few skin depth leaving the inner portion of the conductor unused. IN order to make use of whole cross section of a conductor a bundle of conductors with small diameter held tightly together held instead of using a single layer diameter conductor.