

Macroscopic Effects, Polarization, Dielectric Constant and Dielectric Losses

Dielectric

Dielectrics are the materials, which do not conduct electricity. They are used for insulation purpose.

Electric Flux Density (D)

It is defined on free surface charge density $D = \frac{q}{A}$ (unit $\rightarrow \frac{\text{coulomb}}{\text{m}^2}$)

It is also given by, $D = \epsilon E$

Where, E = Electric field strength in Vm^{-1}

ϵ = permittivity of medium

Relative Permittivity (ϵ_r)

It is defined as the ratio of permittivity of the medium to the permittivity of free space.

$$\text{i.e. } \epsilon_r = \frac{\epsilon}{\epsilon_0}, \epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$

Dielectric constant is defined as the ratio of capacitance of a capacitor with dielectric to the capacitance without dielectric,

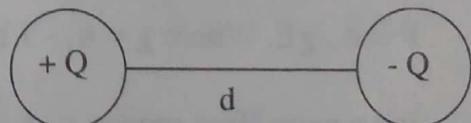
$$K = \frac{C}{C_0}$$

$$\text{Also, for parallel plate capacitor } C = \frac{\epsilon A}{d}, C_0 = \frac{\epsilon_0 A}{d}$$

$$\text{Therefore, } K = \frac{C}{C_0} = \frac{\epsilon}{\epsilon_0} = \epsilon_r$$

Electric Dipole Moment

Two equal and opposite charges separated by finite distance form an electric dipole. The product of magnitude of one of charges and distance between the charges is called dipole moment, i.e. $P = Q.d$



Polarization

- ⇒ When a dielectric is introduced between the plates of capacitors, some charges induce on dielectric. The polarization (P) can be defined as the induced charge within the dielectric per unit area i.e. polarization $P = \frac{Q'}{A}$, Q' = induced charge.
- ⇒ The polarization of the dielectric is defined as the dipole moment per unit volume and represented by polarization vector P

$$\text{i.e. } P = \frac{\sum p_i}{V}, p_i = \text{dipole moment of } i^{\text{th}} \text{ molecule}$$

V = volume

If $+ Q$ and $- Q$ are polarization charges and 'd' be the distance between them, $P = \frac{Q.d}{V} = \frac{Q}{\text{Area}} = \sigma$

Therefore, polarization vector (P) is equivalent to surface polarization charge density.

- ⇒ When dielectric is placed in an external electric field, the electrically charged particles (atoms, ions, molecules) get's arranged in such order that dielectric acquires a certain electric moment. Such phenomenon is called polarization.

The polarization is directly proportional to applied field i.e. $P \propto E$

$$P = \alpha E$$

Where α is constant of proportionality called polarizability.

When $E = 1$ unit, $P = \alpha$

Hence, polarizability (α) is dipole moment produced by field of unit strength.

If there are N dipoles, $P = N\alpha E$

Unit of α is Farad m²

- ⇒ In vacuum, The electric flux density is given by $D_0 = \epsilon_0 E$, when the dielectric of permittivity ϵ is inserted between the plates of capacitor, $D = \epsilon E$. The increase in flux density when dielectric is inserted is called polarization i.e. $P = D - D_0 = \epsilon E - \epsilon_0 E = \epsilon_0 \epsilon E - \epsilon_0 E = \epsilon_0 (\epsilon_r - 1) E = \epsilon_0 \chi E$

$P = \epsilon_0 \chi E$. Where $\chi = \epsilon_r - 1$ is called dielectric susceptibility.

In vacuum (free space), $\epsilon_r = \frac{\epsilon_r}{\epsilon_0} = 1 \Rightarrow \chi = 0, \Rightarrow P = \epsilon_0 \chi E = 0$

Therefore, there is no polarization in free space.

Relation Between Relative Permittivity (ϵ_r) and Polarizability (α)

The induced dipole moment is, $P = \alpha E$

Polarization due to N - molecule, $P = N \alpha E$

Also, the polarization is $P = \epsilon_0 \chi E$

Therefore, $\epsilon_0 \chi E = N \alpha E \Rightarrow \epsilon_0 (\epsilon_r - 1) = N \alpha$

$$\epsilon_r - 1 = \frac{N \alpha}{\epsilon_0} \Rightarrow \epsilon_r = \frac{N \alpha}{\epsilon_0} + 1$$

Types of Polarization

1. Electronic Polarization

In an atom when there is no applied field, the centre of mass of atom and the charge center coincide with each other. So there is no net dipole moment. But when an external field.

is applied the negative charge center (electrons) will be shifted in the direction opposite to the direction of applied field. And the positive charge center (nucleus) will be shifted in the direction of applied field. Hence a dipole is formed in atom called atomic dipole. This phenomenon is called polarization.

Induced dipole moment is proportional to applied electric field.

$$\text{i.e. } P = \alpha E \Rightarrow \alpha = \frac{P}{E} \quad \dots (1)$$

Where α is called polarizability of the atom. The polarization of neutral atom involves, the shifting of electron so α is called electronic polarizability or coefficient of electronic polarization:

$$\text{Since } F = ZeE \quad \dots (2)$$

$$\text{Also } F = \frac{mv^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r \quad \dots (3)$$

Where m = mass of electron, ω = angular frequency

$$\text{Therefore, } ZeE = m\omega^2 r$$

$$E = \frac{m\omega^2 r}{Ze}$$

$$\text{From (1), } \alpha = \frac{P}{E} = \frac{PZe}{m\omega^2 r} = \frac{erZe}{m\omega^2 r} = \frac{Ze^2}{m\omega^2}$$

$$\alpha = \frac{Ze^2}{m\omega^2}, Z = \text{atomic number}$$

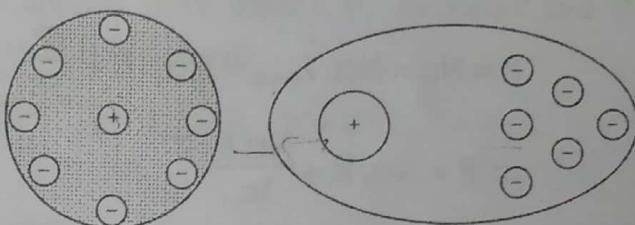


Figure (1): Neutral atom, $P=0$, Figure(2) Polarized atom $P \neq 0$

Clausius - Massotti Equation:

(For electronic polarizability, α_e)

The actual field experienced by a molecule in dielectric is called local field E_{local} . This is the sum of applied field and field due to polarization.

$$E_{\text{local}} = E + \frac{P}{3\epsilon_0}$$

And induced polarization is now given by,

$$p = \alpha_e E_{\text{local}}$$

Hence, polarization due to N, molecules

$$P = Np = N\alpha_e E_{\text{local}} = N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right)$$

$$\Rightarrow P = N\alpha_e E + \frac{N\alpha_e P}{3\epsilon_0}$$

$$P \left(1 - \frac{N\alpha_e}{3\epsilon_0} \right) = N\alpha_e E$$

$$P = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}} \quad \dots(1)$$

Also, $P = \epsilon_0 \chi E \dots\dots(2)$

$$\text{Comparing equation (1) and (2)} \quad \epsilon_0 \chi E = \frac{N\alpha_e E}{1 - \frac{N\alpha_e}{3\epsilon_0}}$$

$$\epsilon_0 \chi \left(1 - \frac{N\alpha_e}{3\epsilon_0} \right) = N\alpha_e$$

$$\epsilon_0 \chi - \frac{N\alpha_e \chi}{3} = N\alpha_e$$

$$N\alpha_e + \frac{N\alpha_e \chi}{3} = \epsilon_0 \chi \Rightarrow N\alpha_e \left(1 + \frac{\chi}{3} \right) = \epsilon_0 \chi$$

$$\Rightarrow N\alpha_e \left(\frac{\chi + 3}{3} \right) = \epsilon_0 \chi$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\chi}{\chi + 3} = \frac{\epsilon_r - 1}{\epsilon_r - 1 + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\chi}{\chi + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \dots(3)$$

Equation (3) is called Clausius - Massotti equation, which relates relative permittivity and electronic polarizability more accurately.

In terms of optical frequency, the Clausius-Massotti equation is given by

$$\frac{N\alpha}{3\epsilon_0} = \frac{n^2 - 1}{n^2 + 2} \quad [\text{Since, } \epsilon_r = n^2, n \text{ is the optical frequency}]$$

2. Ionic Polarization:

The ionic crystal (crystal having ionic bonding) exhibits ionic polarization. Examples in NaCl, KCl, LiBr etc. The ionic crystal have well defined lattice site in which positive and negative ions are located. Each pairs of positive and negative ion form a dipole. Since these dipoles are lined one after another so there is no net dipole moment. $P_+ - P_- = 0$

When an external field is applied, the negative ions move in the direction opposite to applied field and positive ions move in the direction of applied field. Hence P_+ increases and P_- decreases so that $P_+ - P_- \neq 0$. In this way the ionic crystal is polarized.

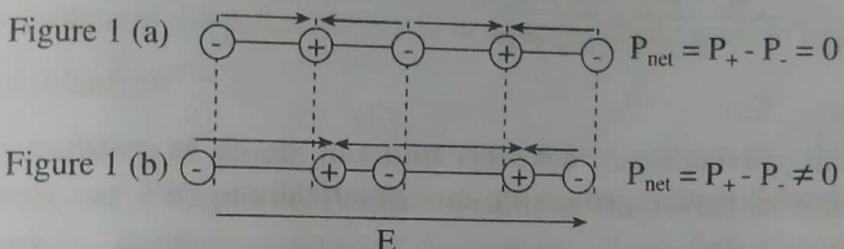


Figure 1 (a): Na^+Cl^- chain in the absence of field, Figure 1 (b): Na^+Cl^- chain in the presence of field

The actual field experienced by a molecule in dielectric is called local field E_{local} , which is the sum of applied field and field due to polarization.

$$E_{\text{local}} = E + \frac{P}{3\epsilon_0}$$

And induced polarization is now given by,

$$p = \alpha_i E_{\text{local}}$$

Hence, polarization due to N, molecules

$$P = Np = N\alpha_i E_{\text{local}} = N\alpha (E + \frac{P}{3\epsilon_0})$$

$$\Rightarrow P = N\alpha_i E + \frac{N\alpha P}{3\epsilon_0}$$

$$P(1 - \frac{N\alpha_i}{3\epsilon_0}) = N\alpha_i E$$

$$P = \frac{N\alpha_i E}{1 - \frac{N\alpha_i}{3\epsilon_0}} \quad \dots(1)$$

$$\text{Also, } P = \epsilon_0 \chi E \quad \dots(2)$$

Comparing equation (1) and (2) $\epsilon_0 \chi E = \frac{N\alpha_i E}{1 - \frac{N\alpha_i}{3\epsilon_0}}$

$$\epsilon_0 \chi \left(1 - \frac{N\alpha_i}{3\epsilon_0}\right) = N\alpha_i$$

$$\epsilon_0 \chi - \frac{N\alpha_i \chi}{3} = N\alpha_i$$

$$N\alpha_i + \frac{N\alpha_i \chi}{3} = \epsilon_0 \chi \Rightarrow N\alpha_i \left(1 + \frac{\chi}{3}\right) = \epsilon_0 \chi$$

$$\Rightarrow N\alpha_i \left(\frac{\chi + 3}{3}\right) = \epsilon_0 \chi$$

$$\frac{N\alpha_i}{3\epsilon_0} = \frac{\chi}{\chi + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 1 + 3} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$

$$\frac{N\alpha_i}{3\epsilon_0} = \frac{X_i}{X_i + 3} = \frac{\epsilon_{r-1}}{\epsilon_{r+2}} \quad \dots(3)$$

Which is the Clausius-Massotti equation for ionic polarization.

Note: α_i is more than ten times α_e , so ionic solids have larger dielectric constant.

3. Orientational (Dipolar) Polarization:

Certain molecules like HCl have permanent dipole moments. These dipoles are randomly oriented due to thermal agitation and hence the net dipole moment is zero. When an external field is applied they tend to align in the direction of applied field producing polarization. In these molecules there is no possibility of ionic polarization because the bond between +ve and -ve ions is very strong.

The torque experienced by the dipole is

$$\tau = a F \sin \theta, \quad a \rightarrow \text{dipole separation}$$

$$= a Q E \sin \theta \quad F \rightarrow \text{Force}$$

$$[\text{Since, } \tau = r \times F = r F \sin \theta]$$

$$= P_o E \sin \theta, \quad [P_o = Q.a]$$

$$\text{The energy of dipole, } U = \int \tau d\theta = \int P_o E \sin \theta$$

$$U = -P_o E \cos \theta \quad [\text{Integration constant is assumed to be zero}]$$

$$U_{\max} = P_o E \text{ for } \theta = \pi$$

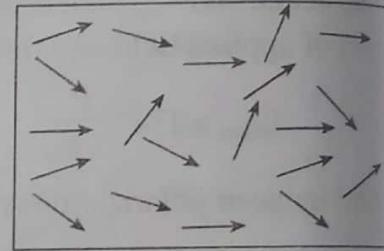
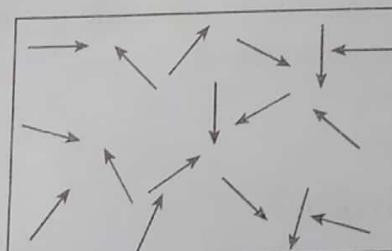


Figure 1: In the absence of field ($P_{av} = 0$), Figure 2: In the presence of field ($P_{av} \neq 0$)

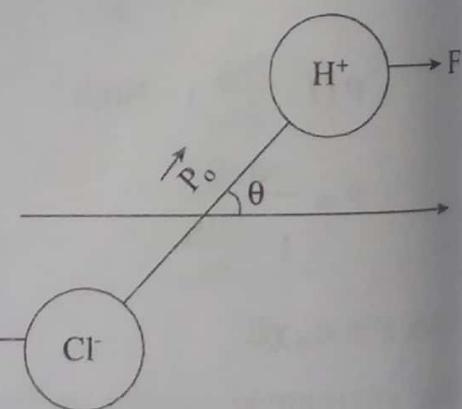


Figure 3: Dipole experiencing a torque due to field E

$$U_{\min} = -P_o E \text{ for } \theta = 0$$

From fig (3), The effective dipole moment to align the dipole along the direction of field is its component along the field direction i.e. $\langle P_o \cos \theta \rangle$ [Average of cosine components of all P_o]
Using Boltzmann statistics we obtain,

$$P_{av} = \langle P_o \cos \theta \rangle = \frac{P_o^2 E}{3KT}, K \text{ is Boltzman constant, } T \text{ is temperature}$$

$$\text{Therefore, Total Polarization } P = N P_{av} = \frac{N P_o^2 E}{3KT} \dots\dots(1)$$

Also, $P = N\alpha_o E$, α_o = Orientational polarizability

Comparing equation (1) and (2).

$$\left[\alpha_o = \frac{P_o^2}{3KT} \right]$$

Therefore for polar molecule the polarizability is inversely proportional to temperature.

4. Interfacial Polarization

It is due to the accumulation of charge at an interface between two materials or two regions of same materials. There are always some defects and impurities within the dielectric material such as lattice vacancies, impurity centre and dislocation. These defects contribute free charge carriers such as electrons and holes.

When an external field is applied the positive ions migrate towards cathode. Since those ions cannot leave dielectric (cannot enter into cathode), so they form a positive space charge at the interface near by cathode. The positive charges accumulate at the interface near by cathode and negative charges remain in the bulk. This creates dipoles and hence polarization.

This type of interfacial polarization generally occurs in dielectric materials such as ceramic or glass.

In some dielectric, the trapping of electrons and holes in the crystal defects at the interface near to electrodes give rise to this type of polarization.

Interfacial polarization also takes place at the boundary between crystals.

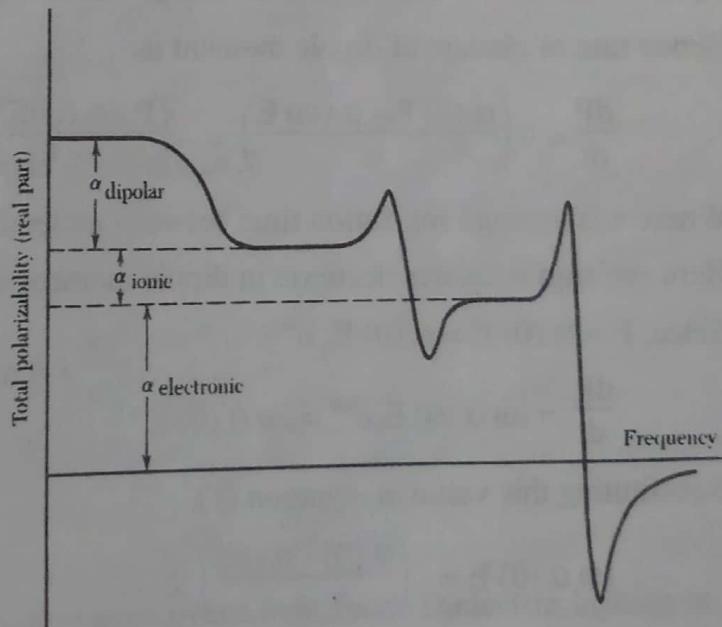


Figure : Variation of different polarizability with frequency

Dielectric Losses, Frequency and Temperature Effect

Dependence of Dipolar Polarizability on Temperature

We have, orientational polarizability, $\alpha = \frac{P^2}{3KT}$. This shows that polarizability is inversely proportional to temperature. Hence Dielectric constant (or permittivity) also depends on temperature in the same manner as α . As temperature increases, Thermal agitation of molecules within the material increases, so the dielectric constant hence dielectric strength is lower at higher temperature than at lower temperature.

Dependence of Dipolar (Orientational) Polarizability on Frequency

When a dielectric material is placed in an alternating electric field, a part of energy is lost, which is known as *dielectric loss*. This is because; the reversing nature of a.c. frequency causes the direction of dipoles to reverse.

When a dielectric is applied by a d.c. field E_0 then the dipole moment is $\alpha(0) E_0$. Here $\alpha(0)$ is polarizability at zero frequency (for d.c.)

When d.c. field E_0 is suddenly changed by a.c. field $E = E_0 e^{i\omega t}$, then the dipole moment is $P = \alpha(0) E$.

After the a.c field has switched on, there occur collision between molecules which reduces the dipole moment to $\alpha(\omega) E$. Here $\alpha(\omega)$ is polarizability at frequency (ω) [i.e. for a.c]

Hence rate of change of dipole moment is,

$$\frac{dP}{dt} = - \left(\frac{\alpha(0) E - \alpha(\omega) E}{\tau} \right) = - \left(\frac{P - \alpha(\omega) E}{\tau} \right) \quad \dots(1)$$

Where τ is average relaxation time between molecular collisions.

Here -ve sign indicates decrease in dipole moment with time.

Since, $P = \alpha(0) E = \alpha(0) E_0 e^{i\omega t}$

$$\frac{dP}{dt} = i\omega \alpha(0) E_0 e^{i\omega t} = i\omega \alpha(0) E$$

Substituting this value in equation (1)

$$i\omega \alpha(0) E = - \left(\frac{\alpha(0) - \alpha(\omega)}{\tau} \right) E$$

$$\Rightarrow -\alpha(0) + \alpha(\omega) = i\omega \tau \alpha(0)$$

$$\alpha(\omega) = \alpha(0) + i\omega \tau \alpha(0)$$

$$\alpha(\omega) = \alpha(0) (1 + i\omega \tau)$$

$$\alpha(0) = \frac{\alpha(\omega)}{1 + i\omega \tau}$$

... (2)

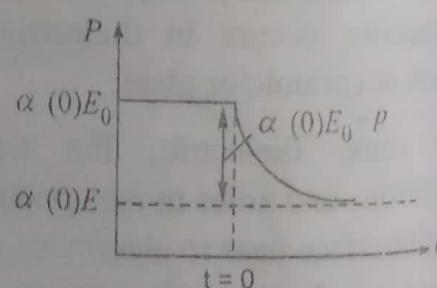


Figure : When d.c. field is suddenly changed by a.c. field the dipole moment changes from $\alpha(0)E_0$ to $\alpha(0)E$

Hence, At high frequency, the relaxation time is very small (since, $\tau = 2\pi/\omega$). This means the time of collision between molecules is small and they make large number of collision in short interval of time, which randomize dipole orientation to reduce polarization. However at low frequency, the relaxation time is very large. This means it takes, large time for a single collision. Hence polarization is not disturbed by collision and hence has larger value.

Power Loss in Dielectric with Frequency

The dielectric constant when AC is applied can be written in the form of real and imaginary part as, $\epsilon_r' = \epsilon_r - i\epsilon_r''$

$$\text{Where } \epsilon_r' = \epsilon_h + \frac{\epsilon_l - \epsilon_h}{1 + \omega^2 \tau^2} \dots (1) \text{ and } \epsilon_r'' = \omega \tau \left(\frac{\epsilon_l - \epsilon_h}{1 + \omega^2 \tau^2} \right) \dots (2)$$

Here, ϵ_h is the dielectric constant at high frequency and ϵ_l is the dielectric constant at low frequency. Equations (1) and (2) are called *Debye equations*.

For a capacitor when AC applied, the admittance is calculated as,

$$Y = \frac{\epsilon_0 A \omega \epsilon_r''}{d} + \frac{i \epsilon_0 A \omega \epsilon_r'}{d} = \frac{1}{R} + i \omega C$$

$$\text{Here, } \frac{1}{R} = \frac{\epsilon_0 A}{d} \omega \epsilon_r'' \text{ and } C = \frac{\epsilon_0 A}{d} \epsilon_r'$$

Here the dielectric can be considered as an ideal capacitor with capacitance 'C' and resistor with resistance R.

$$\text{The power loss is } P = \frac{V^2}{R}$$

$$P = V^2 \cdot \frac{\epsilon_0 A \omega \epsilon_r''}{d} = \frac{V^2}{d^2} \cdot Ad \omega \epsilon_0 \epsilon_r'' = E^2 (Ad) \omega \epsilon_0 \epsilon_r''$$

$$\frac{\text{Power}}{\text{volume}} \left(\frac{P}{Ad} \right) = E^2 \omega \epsilon_0 \epsilon_r''$$

$$\text{The loss tangent or loss factor is defined as } \tan \delta = \frac{\epsilon_r''}{\epsilon_r'}$$

$$\epsilon_r'' = \epsilon_r' \tan \delta$$

$$\frac{\text{power loss}}{\text{volume}} = E^2 \omega \epsilon_0 \epsilon_r' \tan \delta$$

Dielectric loss is the mechanism by which microwaves ovens heat food. Dielectric heating at high frequencies is used in industrial applications. Such as heating plastics and drying wood.

Dielectric Breakdown

When an electric field is applied to a dielectric, various phenomena such as electrical conduction, polarization and dielectric losses occur. When the voltage applied to insulation

increases the leakage current increases. If the voltage applied is sufficiently increased there takes place a break down of insulation. In gases and liquid, the break down is not permanent but in solids. There can be permanent damage due to breakdown. The maximum electric field that can be applied without causing breakdown is called dielectric strength E_{br} .

Breakdown of Gaseous Dielectric:

- The breakdown in gases depends not only in chemical composition of the gas but also in pressure, temperature and the distance between the electrodes.
- The breakdown in gaseous dielectric begins with ionization caused by collision of electrons. If the field is sufficiently large, these electrons can be accelerated to sufficiently large kinetic energy and produce more electrons and positively charged ion.
- *Corona discharge*: If the field is not uniform, an increase in voltage may cause discharge at the points of high intensity of electric field. Such a local discharge is called corona discharge. This phenomenon is attended by a characteristic sound, bluish luminescence. The appearance of corona is due to

- i. *Rate of loss of energy with increase in voltage.*

According to Peek's law.

$$P_c = Af(V - V_c)^2$$

P_c = Power loss during corona discharge

A = Constant depends upon geometry of electrodes

f = frequency

V = Applied voltage

V_c = Voltage at which corona appears.

For corona discharge $V > V_c$

- ii. *Chemical transformation of gas in the region of corona.*

Pressure dependence of gaseous discharge:

At high pressure the concentration of molecules increases and the separation between molecule decreases. So to ionize the gas molecules the free electrons should be accelerated by larger potential. Hence dielectric strength of gas increases with increases in pressure.

Breakdown of Liquid Dielectrics:

Though breakdown in liquid is not so clear as in solid and gases. Some of the cases of liquid breakdown are as follows.

- ⇒ Moisture and gas absorption reduces dielectric strength of liquid (such as paper, cotton fabric absorbs moisture in transformer oil)

- ⇒ During long course of time oxidation may produce acidic products due to which some parts may become more conducting causing discharge.
- ⇒ When high voltage is applied, the emission of electrons from electrodes may cause discharge.

Breakdown of Solid Dielectrics:

There are various mechanisms of dielectric breakdown in solids such as electronic break down. Thermal breakdown, electro mechanical breakdown, breakdown due to voids and cracks, breakdown due to insulation aging etc.

Electronic (intrinsic) Breakdown:

This is due to application of electric field on dielectric material. On the application of electric field, free charged particles are accelerated and produce more charged particles during collision with the molecules of dielectric. This can set up an avalanche of ions causing breakdown.

Thermal Breakdown:

When a voltage is applied to the dielectric, it liberates heat according to Joule's law and hence temperature rises. If this heat generated cannot be removed quickly, there become hot spots. These spots can be permanent conducting channel, between two electrodes. This causes breakdown of solid called thermal breakdown.

Electro-mechanical Breakdown:

When an electric field is applied to dielectric, it gets deformed mechanically because it experience mechanical compressive force. On the other hand, when compressive force is applied to a dielectric an electric field is set up across it due to polarization. This effect is called piezo-electricity.

When the applied voltage is increased, the thickness of dielectric reduced which increases charge between the electrodes. This will lead to further increase in compressive force [$F = qE$]. This process continues to deform dielectric medium plastically.

The increase in ' E ' and decrease in ' d ' produces more Joule heat, this may also lead to thermal breakdown.

Note: The amount of heat produced due to flow of current through conductor. $H = I^2Rt = IVt$ (in Joule). Where, I = current, R = resistance, t = time

Breakdown due to Voids and Cracks: (voids → empty space)

The electric field in the voids and cracks are stronger than in dielectric material itself, this may lead partial discharge around voids and cracks. During the course of time, this partial discharge erodes the internal surface of voids by locally melting the insulator. This leads to an electrical tree type discharge and can lead to major physical and chemical breakdown if not controlled in proper time.

Breakdown due to Insulation Aging

With time, the insulating strength of dielectric goes on degrading, and eventually dielectric breakdown take place even at lower value of applied field.

Ferroelectricity and Piezoelectricity

Ferro - electricity

The materials having permanent polarization even in the absence of applied field are called ferroelectric materials and the phenomenon is called Ferro electricity. The temperature above which Ferro electricity is lost is called Curie temperature T_c .

The variation of polarization with electric field is not linear for Ferro-electric material but forms a closed loop called hysteresis loop which is similar to as that for Ferro- magnetic material.

Consider a specimen with no initial polarization. When an increasing field E is applied, the polarization increases along OA. When the applied field reaches to E_1 , all dipoles lie in direction of applied field.

- Now the field is reduced so that polarization reduces from it's initial value A to along the curve ABCD. At B it is seen that there is permanent polarization even if $E = 0$.
- The electric field E_c at which polarization is zero is called coercive field.
- If the electric field is made further negative to $-E_1$, the polarization occur in opposite direction and hence negative.
- Again on increasing E from $-E_1$ to $+E_1$, the polarization moves along the curve DFA. The closed curve ABCDFA is called hysteresis curve.

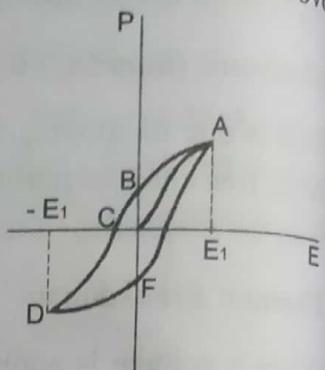


Figure : Hysteresis loop for ferroelectric material

Properties:

1. The variation of polarization is not linear but forms a closed loop called hysteresis loop.
2. The Ferro-electricity is lost above Curie temperature. Above Curie temperature (T_c) dielectric constant follows Curie - Weiss law.

$$\epsilon_r = \frac{C}{T - T_o}, \text{ where } C \rightarrow \text{Curie constant}, T_o \rightarrow \text{Curie - Weiss temp.}$$

Piezoelectricity:

In certain crystals, mechanical stress produces an electric polarization and conversely an applied field produces mechanical strain. This phenomenon is called Piezo-electricity.

In such crystals, in the absence of external force, the center of mass of both positive and negative charges coincides. But when an external force is applied, the centres of masses of charges are shifted causing polarization. The unit cell of such crystal is called *non centro symmetric*.

If T_j is applied mechanical stress along J direction then

$$P_i = d_{ij} T_j$$

Where, P_i is induced polarization and d_{ij} is piezoelectric coefficient

Symmetrically, if E_i is applied electric field along i-direction, then induced strain S_j is given by

$$S_j = d_{ij} E_i$$

Solved Examples

1. The electronic polarizability of the Ar atom is $1.7 \times 10^{-40} \text{ Fm}^2$. What is the static dielectric constant of solid Ar if its density is 1.8 gcm^{-3} ? Given $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$, Atomic mass of Ar = 39.95 g mol^{-1} .

Solution:

Here, $\alpha_e = 1.7 \times 10^{-40} \text{ Fm}^2$, density $d = 1.8 \text{ gcm}^{-3}$

$$\text{Number of atoms per unit volume, } N = \frac{dN_A}{\text{Mat}}$$

$$= \frac{6.02 \times 10^{23} \times 1.8}{39.95} = 2.71 \times 10^{22} \text{ cm}^3 = 2.71 \times 10^{28} \text{ m}^{-3}$$

From Calusius-Massotti equation,

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \Rightarrow \frac{N\alpha_e \epsilon_r}{3\epsilon_0} + \frac{2N\alpha_e}{3\epsilon_0} = \epsilon_r - 1$$

$$\epsilon_r \left(\frac{N\alpha_e}{3\epsilon_0} - 1 \right) = - \left[1 + \frac{2N\alpha_e}{3\epsilon_0} \right]$$

$$\epsilon_r = \frac{1 + \frac{2N\alpha_e}{3\epsilon_0}}{1 - \frac{N\alpha_e}{3\epsilon_0}} = 1.63$$

2. Consider a pure Si crystal that has $\epsilon_r = 11.9$ a). What the electronic polarizability due to valence electrons per Si atom b). Suppose that a voltage is applied across Si crystal sample. By how much is the local field greater than the applied field? Given, $\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$, $N = 5 \times 10^{28} \text{ atoms per m}^3$.

Solution:

Here, $\epsilon_r = 11.9$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$, $N = 5 \times 10^{28} \text{ m}^{-3}$

- a. From Clausius - Mossotti equation

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \Rightarrow \alpha_e = \frac{3\epsilon_0}{N} \left(\frac{\epsilon_r - 1}{\epsilon_r + 2} \right)$$

$$\alpha_e = \frac{3(8.85 \times 10^{-12})}{(5 \times 10^{28})} \left(\frac{11.9 - 1}{11.9 + 2} \right) = 4.17 \times 10^{-40} \text{ Fm}^2$$

b. The local field is

$$E_{\text{local}} = E + \frac{P}{3\epsilon_0}$$

$$\text{Also, } P = \epsilon_0 \chi E = \epsilon_0 (\epsilon_r - 1) E$$

Substituting for P,

$$E_{\text{local}} = E + \frac{1}{3} (\epsilon_r - 1) E$$

$$\frac{E_{\text{local}}}{E} = \left(1 + \frac{\epsilon_r}{3} - \frac{1}{3}\right) = \frac{\epsilon_r + 2}{3} = 4.63$$

3. Consider the CsCl crystal which has one (Cs^+ - Cl^-) pair per unit cell and a lattice parameter a of 0.412 nm. The electronic polarizability of Cs^+ and Cl^- ions are $3.35 \times 10^{-40} \text{ Fm}^2$ and $3.40 \times 10^{-40} \text{ Fm}^2$ respectively and the mean ionic polarizability per ion pair is $6 \times 10^{-40} \text{ Fm}^2$. What is dielectric constant at low frequencies and that at optical frequencies?

Solution:

Here, $a = 0.412 \text{ nm}$, $\alpha_e(\text{Cs}^+) = 3.35 \times 10^{-40} \text{ Fm}^2$, $\alpha_e(\text{Cl}^-) = 3.40 \times 10^{-40} \text{ Fm}^2$, Ionic polarizability per ion pair, $\alpha_i = 6 \times 10^{-40} \text{ Fm}^2$.

Since, one unit cell contains one ion pair, so number of ions pair (N_i) per unit volume = $\frac{1}{a^3}$

$$= \frac{1}{(0.412 \times 10^{-9})^3} = 1.43 \times 10^{28} \text{ m}^{-3}$$

Therefore, concentration of Cs^+ = concentration of Cl^- = concentration of Cs^+ - Cl^- ion pair = $N_i = 1.43 \times 10^{28} \text{ m}^{-3}$

From the Clausius - Mossotti equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0} [N_i \alpha_e(\text{Cs}^+) + N_i \alpha_e(\text{Cl}^-) + N_i \alpha_i]$$

$$= \frac{N_i}{3\epsilon_0} [\alpha_e(\text{Cs}^+) + \alpha_e(\text{Cl}^-) + \alpha_i]$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1.43 \times 10^{28} [3.35 + 3.4 + 6] \times 10^{-40}}{3 \times 8.85 \times 10^{-12}} = 0.69$$

$$\epsilon_r - 1 = 0.69 \epsilon_r + 0.69 \times 2$$

$$\epsilon_r (1 - 0.69) = 1 + (0.69 \times 2)$$

$$\epsilon_r = \frac{1 + (0.69 \times 2)}{1 - 0.69} = 7.6$$

At high frequencies that is, near optical frequencies, the ionic polarization is too sluggish to allow ionic polarization to contribute to ϵ_r' . Thus, ϵ_{rop} , relative permittivity at optical frequencies is given by,

$$\frac{\epsilon_{rop} - 1}{\epsilon_{rop} + 2} = \frac{1}{3\epsilon_0} [N_i \alpha_e (Cs^+) + N_i \alpha_e (Cl^-)]$$

$$= \frac{N_i}{3\epsilon_0} [\alpha_e (Cs^+) + \alpha_e (Cl^-)]$$

$$\frac{\epsilon_{rop} - 1}{\epsilon_{rop} + 2} = \frac{1.43 \times 10^{28} (3.35 + 3.4) \times 10^{-40}}{3 \times 8.85 \times 10^{-12}} = 0.36$$

$$\epsilon_{rop} - 1 = 0.36 \epsilon_{rop} + (2 \times 0.36)$$

$$\epsilon_{rop} = \frac{1 + (2 \times 0.36)}{1 - 0.36} = 2.69$$

4. Calculate the heat generated per second due to dielectric loss per cm^3 of XLPE (typical power cable insulator) and alumina (Al_2O_3) at 60 Hz and 1MHz at a field of 100 kV cm^{-1} . Their properties are give in table below.

Material	$f = 60 \text{ Hz}$		$f = 1 \text{ MHz}$	
	ϵ_r'	$\tan \delta$	ϵ_r'	$\tan \delta$
XLPE	2.3	3×10^{-4}	2.3	4×10^{-4}
Alumina	8.5	1×10^{-3}	8.5	1×10^{-3}

a. For XLPE

i. at $f = 60 \text{ Hz}$

$$\begin{aligned} P/V &= \omega E^2 \epsilon_0 \epsilon_r' \tan \delta \\ &= (2\pi \times 60) \times (100 \times 10^3 \times 10^2)^2 \times 8.85 \times 10^{-12} \times 2.3 \times 3 \times 10^{-4} \\ &= 230 \text{ W m}^{-3} \end{aligned}$$

ii. At $f = 1 \text{ MHz} = 1 \times 10^6 \text{ Hz}$

$$\begin{aligned} P/V &= \omega E^2 \epsilon_0 \epsilon_r' \tan \delta \\ &= (2\pi \times 10^6) \times (100 \times 10^3 \times 10^2)^2 \times 8.85 \times 10^{-12} \times 2.3 \times 4 \times 10^{-4} \\ &= 5.53 \times 10^6 \text{ W m}^{-3} \end{aligned}$$

b. For Alumina

i. At $f = 60 \text{ Hz}$

$$\begin{aligned} P/V &= \omega E^2 \epsilon_0 \epsilon_r' \tan \delta \\ &= 2\pi \times 60 \times (10^7)^2 \times 8.85 \times 10^{-12} \times 8.5 \times 1 \times 10^{-3} \\ &= 2834.5 \text{ W m}^{-3} \end{aligned}$$

ii. At $f = 1\text{MHz}$

$$\begin{aligned} P/V &= 2\pi \times 1 \times 10^6 \times (10^7)^2 \times 8.85 \times 10^{-12} \times 8.5 \times 1 \times 10^{-3} \\ &= 4.72 \times 10^7 \text{ W/m}^3 \end{aligned}$$

5. Design a circuit at 25°C ($\tan \delta = 0.02$) at 1 kHz for 560 PF capacitor that uses thick polyester film.

Solution:

$$\text{We have, } \frac{1}{R} = \frac{\epsilon_0 A}{d} \omega \epsilon_r' \tan \delta$$

$$\text{And } C = \frac{\epsilon_0 A}{d} \epsilon_r'$$

$$\begin{aligned} \text{Therefore, } \frac{1}{R} &= C \omega \tan \delta = 560 \times 10^{12} \times 2\pi \times 1 \times 10^3 \times 0.02 \\ &= 7.04 \times 10^{-9} \Omega^{-1} \end{aligned}$$

$$R = 1.42 \times 10^8 \Omega = 142 \text{ M}\Omega$$

Therefore the equivalent circuit is an ideal capacitor of 560 PF in parallel with a $142\text{ M}\Omega$ resistance.

6. A ceramics low frequency and high frequency dielectric constants are 3.6 and 2.58 respectively. The Debye peak occurs at $\tau = 26.5\text{ }\mu\text{s}$. Calculate ϵ_r' and the dielectric loss factor $\tan \delta$ at 29 kHz .

Solution:

Here, $\epsilon_l = 3.6$, $\epsilon_h = 2.58$, $\tau = 26.5\text{ }\mu\text{s}$, $f = 29\text{ kHz}$

$$\begin{aligned} \text{We have, } \epsilon_r' &= \epsilon_h + \frac{\epsilon_l - \epsilon_h}{1 + \omega^2 \tau^2} \\ &= 2.58 + \frac{3.6 - 2.58}{1 + (2\pi \times 29 \times 10^3 \times 26.5 \times 10^{-6})^2} = 2.62 \\ \epsilon_r'' &= \omega \tau \left(\frac{\epsilon_l - \epsilon_h}{1 + \omega^2 \tau^2} \right) \\ &= \frac{2\pi \times 29 \times 10^3 \times 26.5 \times 10^{-6} (3.6 - 2.58)}{1 + (2\pi \times 29 \times 10^3 \times 26.5 \times 10^{-6})^2} = 0.202 \end{aligned}$$

$$\text{And hence, } \tan \delta = \frac{\epsilon_r''}{\epsilon_r'} = \frac{0.202}{2.62} = 0.077$$

7. A glass dielectric has dielectric constant 2.6 and loss tangent 7×10^{-5} at 1MHz. Calculate the loss of power per unit volume if the signal applied to the glass has peak amplitude of 0.71 V/m.

Solution:

Here, $\epsilon_r = 2.6$, $\tan \delta = 7 \times 10^{-5}$, $f = 1\text{MHz}$, $E = 0.71 \text{ V/m}$

$$\begin{aligned}\therefore P/V &= \omega E^2 \epsilon_0 \epsilon_r \tan \delta \\ &= 2\pi \times 1 \times 10^6 \times (0.71)^2 \times 8.85 \times 10^{-12} \times 2.6 \times 7 \times 10^{-5} \\ &= 5.1 \times 10^{-9} \text{ W/m}^3\end{aligned}$$

8. The optical index of refraction and the dielectric constant for glass are 1.45 and 6.5 respectively. Calculate the percentage of ionic polarizability.

Solution:

Here, optical index of refraction, $n = 1.45$

Dielectric constant, $\epsilon_r = 6.5$

We have from Clausius - Massotti equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0} \quad \dots(1)$$

At optical frequencies, ϵ_r in Clausius-Massotti relation for electronic polarization is replaced by n^2

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N \alpha_e}{3\epsilon_0} \quad \dots(2)$$

Now, dividing equation (2) by (1)

$$\frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} = \frac{\alpha_e}{\alpha_e + \alpha_i}$$

So, percentage of ionic polarizability is

$$\begin{aligned}\frac{\alpha_i}{\alpha_e + \alpha_i} \times 100 &= \left[1 - \frac{\alpha_e}{\alpha_e + \alpha_i} \right] \times 100 \\ &= \left[1 - \left(\frac{n^2 - 1}{n^2 + 2} \right) \times \left(\frac{\epsilon_r + 2}{\epsilon_r - 1} \right) \right] \times 100 \\ &= \left[1 - \left(\frac{1.45^2 - 1}{1.45^2 + 2} \right) \times \left(\frac{6.5 + 2}{6.5 - 1} \right) \right] \times 100 \\ &= 58.47\%\end{aligned}$$

9. The isolated water molecule has a permanent dipole P_o of 6.1×10^{-30} Cm (Coulombmeter). What is the dielectric constant of steam at a pressure of 10 atm and at a temperature of 400°C .

Solution:

Here, $P_o = 6.1 \times 10^{-30}$ cm. $P = 10$ atm

$T = 400^\circ\text{C}$

From gas law, $PV = nRT$

$P = \frac{N}{N_A} RT$, N = number of molecules per unit volume. N_A = number of molecules per mole.

$$\Rightarrow N = \frac{PN_A}{RT} = \frac{10 \times 10^5 \times 6.02 \times 10^{23}}{8.31 \times (400 + 273)} = 1.08 \times 10^{26}/\text{m}^3$$

$$\text{and } \alpha_o = \frac{P_o^2}{3KT} = \frac{(6.1 \times 10^{-30})^2}{3 \times 1.38 \times 10^{-23} \times (400 + 273)} = 1.34 \times 10^{-39} \text{ Fm}^2$$

The Clausius - Mossotti equation does not apply to orientational polarization, since N is small, using equation.

$$\begin{aligned}\epsilon_r &= \frac{N \alpha_o}{\epsilon_o} + 1 \\ &= \frac{1.08 \times 10^{26} \times 1.34 \times 10^{-39}}{8.85 \times 10^{-12}} + 1 \\ &= 1.02\end{aligned}$$

10. A solid contains 5×10^{28} identical atoms per m^3 , each with a polarizability of 2×10^{-40} Fm^2 . Assuming that the internal field is given by the Lorentz formula, calculate the ratio of the internal field to the applied field.

Solution:

Here, $N = 5 \times 10^{28}/\text{m}^3$, $\alpha = 2 \times 10^{-40} \text{ Fm}^2$

We have, from Lorentz formula

$$E_{local} = E + \frac{P}{3\epsilon_o}$$

Since, $P = N \alpha E_{local}$

$$E_{local} = E + \frac{N\alpha E_{local}}{3\epsilon_o}$$

$$E_{local} \left(1 - \frac{N\alpha}{3\epsilon_o}\right) = E$$

$$\frac{E_{local}}{E} = \left(\frac{1}{1 - \frac{N\alpha}{3\epsilon_0}} \right) = \frac{1}{\left(1 - \frac{5 \times 10^{28} \times 2 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}} \right)}$$

$$\frac{E_{local}}{E} = 1.6$$

11. Assuming that the polarizability of an argon atom is equal to 1.43×10^{-40} Fm², calculate the relative dielectric constant at 0°C and 1 atmosphere.

Solution:

Here, $\alpha_e = 1.43 \times 10^{-40}$ Fm², $\epsilon_r = ?$, T = 0°C = 273 K

$$P = 1 \text{ atm} = 1 \times 10^5 \text{ N/m}^2$$

We have from Clausius-Mossotti equation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0}$$

$$3\epsilon_r\epsilon_0 - 3\epsilon_0 = N\alpha_e \epsilon_r + 2N\alpha_e$$

$$\epsilon_r (3\epsilon_0 - N\alpha_e) = 2N\alpha_e + 3\epsilon_0$$

$$\epsilon_r = \frac{2N\alpha_e + 3\epsilon_0}{3\epsilon_0 - N\alpha_e}$$

From gas equation PV = nRT

$$P = \frac{\text{Number of atoms per unit volume (N)}}{\text{Number of atoms per mole (N}_A\text{)}} \times RT$$

$$P = \frac{N}{N_A} \cdot RT$$

$$N = \frac{PN_A}{RT} = \frac{1 \times 10^5 \times 6.02 \times 10^{23}}{8.31 \times 273} = 2.65 \times 10^{25}/\text{m}^3$$

$$\text{Therefore, } \epsilon_r = \frac{(2 \times 2.65 \times 10^{25} \times 1.43 \times 10^{-40}) + (3 \times 8.85 \times 10^{-12})}{(3 \times 8.85 \times 10^{-12}) - (2.65 \times 10^{25} \times 1.43 \times 10^{-40})}$$

$$\epsilon_r = 1.00043$$

12. A parallel plate capacitor of area 20 cm² and separation of 1mm. The space between plates is filled with Al₂O₃. When potential difference of 10V is applied, calculate field strength and dipole moment induced in oxide layer. Assume relative permittivity of Al₂O₃ = 8

Solution:

Here, A = 20cm², d = 1mm, V = 10V, $\epsilon_r = 8$, E = ?, P = ?

We have,

$$E = \frac{V}{d} = \frac{10}{1 \times 10^{-3}} = 1 \times 10^4 \text{ N/C}$$

$$\text{Since, } C = \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\text{Also, } q = C \cdot V = \frac{\epsilon_0 \epsilon_r A \cdot V}{d}$$

The dipole moment, $P = q \cdot d$

$$\begin{aligned} P &= \epsilon_0 \epsilon_r A \cdot V \\ &= 8.85 \times 10^{-12} \times 8 \times 20 \times 10^{-4} \times 10 \\ &= 1.416 \times 10^{-12} \text{ Coulomb-meter} \end{aligned}$$

13. A parallel plate capacitor has an area of 10 cm^2 and a separation of 0.1 mm . The space between the plates is filled with polyethylene. An alternative voltage with amplitude of 2 volts is applied at a frequency of 1 MHz . At this frequency the real part of the relative permittivity is 2.25 and the loss tangent is 4×10^{-4} . Calculate the energy dissipated per second.

Solution:

Here, $A = 20 \text{ cm}^2$, $d = 0.1 \text{ mm}$, $V = 2 \text{ Volts}$, $f = 1 \text{ MHz}$, $\epsilon_r' = 2.25$, $\tan \delta = 4 \times 10^{-4}$

We have, power loss per unit volume, $\frac{P}{V} = \omega E^2 \epsilon_0 \epsilon_r' \tan \delta$

Therefore, power loss = $\omega E^2 \epsilon_0 \epsilon_r' \tan \delta \times \text{volume}$

Therefore, Energy dissipated per second (P) = $\omega E^2 \epsilon_0 \epsilon_r' \tan \delta \times \text{volume}$

$$= 2\pi f \left(\frac{V}{d} \right)^2 \cdot \epsilon_0 \epsilon_r' \tan \delta \times \text{volume}$$

$$= 2\pi f \left(\frac{V}{d} \right)^2 \cdot \epsilon_0 \epsilon_r' \tan \delta \times A \times d$$

$$= 2 \times 3.14 \times 1 \times 10^6 \times \left(\frac{2}{0.1 \times 10^{-3}} \right)^2 \times 8.85 \times 10^{-12} \times 2.25 \times 4 \times 10^{-4} \times 20 \times 10^{-4} \times (0.1) \times 10^{-3}$$

$$= 4 \times 10^{-6} \text{ W}$$

14. If the refractive index of a medium with respect to free space, n , is defined as the ratio of the velocity of electromagnetic wave in free space to that in the medium, show that the Clausius - Mosotti equation becomes, $\frac{N\alpha}{3\epsilon_0} = \frac{n^2 - 1}{n^2 + 2}$

Solution:

We have Clausius-Mosotti equation

$$\frac{N\alpha}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \quad \dots(1)$$

The velocity of electromagnetic wave in free space and in medium is given by

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

And in medium it is given by

$$\text{And } V = \frac{1}{\sqrt{\mu \epsilon}}$$

For a material that is not magnetic, the permeability is μ_0

$$\text{Therefore, } V = \frac{1}{\sqrt{\mu_0 \epsilon}}$$

$$\text{Refractive index, } n = \frac{C}{V} = \sqrt{\frac{\mu_0 \epsilon}{\mu_0 \epsilon_0}} = \sqrt{\epsilon_r}$$

$$\epsilon_r = n^2$$

Hence, Clausius - Mossotti equation becomes

$$\frac{N\alpha}{3\epsilon_0} = \frac{n^2 - 1}{n^2 + 2}$$

15. KCl has crystal structure with a lattice parameter of 0.629 nm. Calculate the relative permittivity of KCl crystal at optical frequencies given that the electronic polarizability of K^+ is $1.264 \times 10^{-40} \text{ Fm}^2$ and that of Cl^- is $3.048 \times 10^{-40} \text{ Fm}^2$. How does this compare with the measured value of 2.19?

Solution:

Here lattice parameter, $a = 0.629 \times 10^{-9} \text{ m}$

Since, one unit cell contains one ion pair, So number of ions pair per unit volume, (N_i)

$$= \frac{1}{a^3} = \frac{1}{(0.629 \times 10^{-9})^3} = N_i = 4 \times 10^{27}/\text{m}^3$$

Therefore, concentration of K^+ = concentration of Cl^- = concentration of $K^+ - Cl^-$ ion pair

$$= N_i = 4 \times 10^{27}/\text{m}^3.$$

Now at optical frequency, the Clausius Massotti equation is given by, $\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{1}{3\epsilon_0} (N_i \alpha_e (K^+) + N_i \alpha_e (Cl^-))$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N_i}{3\epsilon_0} [\alpha_e (K^+) + \alpha_e (Cl^-)]$$

$$= \frac{4 \times 10^{27}}{3 \times 8.85 \times 10^{-12}} [(1.264 \times 10^{-40}) + (3.048 \times 10^{-40})]$$

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = 0.065$$

$$\Rightarrow \epsilon_r - 1 = 0.065 \epsilon_r + 0.13 \Rightarrow (1 - 0.065) \epsilon_r = 1.13$$

$$\Rightarrow \epsilon_r = 1.21$$

Which is smaller by $2.19 - 1.21 = 0.98$ with the measured value.

16. An electrolytic condenser consisting of an oxidized Al sheet with an effective surface area of 400 cm^2 has capacitance of $8 \mu\text{F}$. The dielectric constant of Al_2O_3 is 8. A potential difference of 10V is applied between the Al and the electrolyte. What is the electric field strength and what is the total dipole moment induced in the oxide layer.

Solution:

Here, $A = 400 \text{ cm}^2$, $C = 8 \mu\text{F}$, $\epsilon_r = 8$, $V = 10\text{V}$

$$\text{We have, } C = \frac{\epsilon A}{d} = \frac{\epsilon_r \epsilon_0 A}{d} \Rightarrow d = \frac{\epsilon_r \epsilon_0 A}{C}$$

$$d = \frac{8 \times 8.85 \times 10^{-12} \times 400 \times 10^{-4}}{8 \times 10^{-6}} = 3.54 \times 10^{-7} \text{ m}$$

$$\text{Now the electric field strength, } E = \frac{V}{d} = \frac{10}{3.54 \times 10^{-7}} = 2.825 \times 10^7 \text{ V/m}$$

The induced dipole moment, $P = \epsilon_0 \chi E = \epsilon_0 (\epsilon_r - 1) E$

$$\begin{aligned} P &= 8.85 \times 10^{-12} \times (8-1) \times 2.825 \times 10^7 \\ &= 1.75 \times 10^{-3} \text{ Coulomb/m}^2 \end{aligned}$$

17. The water has static dielectric constant of 8.1 and optical index of refraction 1.33. Calculate the percentage contribution of ionic polarizability.

Solution:

Here optical index of refraction $n = 1.33$, Dielectric constant, $\epsilon_r = 8.1$ we have from Clausius Massotti equation.

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0}$$

At optical frequencies, ϵ_r in Clausius - Massotti relation for electronic polarization is replaced by n^2

$$\text{i.e. } \frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0} \quad \dots(2)$$

[For optical frequency the ionic polarizability is negligible)

Dividing equation (2) by (1)

$$\frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} = \frac{\alpha_e}{\alpha_e + \alpha_i}$$

The percentage of ionic polarizability is

$$\begin{aligned}\frac{\alpha_i}{\alpha_e + \alpha_i} \times 100\% &= \left[1 - \frac{\alpha_e}{\alpha_e + \alpha_i} \right] \times 100\% \\ &= \left[1 - \left(\frac{n^2 - 1}{n^2 + 2} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} \right) \right] \times 100\% \\ &= \left[1 - \left(\frac{(1.33^2 - 1)}{(1.33^2 + 2)} \times \frac{(8.1 + 2)}{(8.1 - 1)} \right) \right] \times 100\% \\ &= 70.98\%\end{aligned}$$

18. Calculate the dielectric constant of Barium Strontium Titanate at 100°C if its Curie-Weiss temperature is 71°C and curie constant 1.3×10^5 .

Solution:

Here $T = 100^\circ\text{C} = 373\text{K}$, $T_C = 71^\circ\text{C} = 344\text{ K}$

$$C = 1.3 \times 10^5$$

We have from Curie- Weiss law

$$\chi = \frac{C}{T - T_C} = \frac{1.3 \times 10^5}{(373 - 344)} = 4482.76$$

Since, $\chi = \epsilon_r - 1$

$$\epsilon_r = \chi + 1 = 4482.76 + 1$$

$$\epsilon_r = 4483.76$$

Exercise

1. What is polarization? Derive an expression showing relationship between dielectric susceptibility and electronic polarizability of an atom.
2. Define polarization. Show that "there will be no polarization if $\epsilon_r = 1$ ".
3. Explain polarization in detail. Show that there is no polarization in free space or vacuum.
4. What are the different types of polarization mechanism in dielectric medium?
5. Derive Clausius-Massotti equation showing the relation between electronic polarization and relative permittivity.