[Calc] Deriving this Wigner distribution using linearity of Wigner transform in density operators

W1ecat $= \frac{1}{\text{Norm}}\left[W_{\alpha,\text{Coh.}} + W_{-\alpha,\text{Coh.}} + W_{+-} + W_{-+}\right]$

$W_{+-} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ipy}\, \psi_\alpha\left(x+\frac{y}{2}\right) \psi_{-\alpha}^{*}\left(x-\frac{y}{2}\right) dy$ (Wikipedia convention)

```
In[•]:= ψ[x0_, p0_, x_] := 1/π^(1/4) e^(i p0 x) e^(-(x - x0)^2/2) e^(-(i x0 p0)/2); (* Coherent state wave function *)
(* For conjugate: use ψ*(x0,p0;x)→ ψ(x0,-p0;x) *)
Ws1s2[α_, θ_, x_, p_, s1_, s2_] :=
  FullSimplify[1/(2 π) ∫_{-∞}^{∞} e^(-i p y) ψ[s1 √2 α Cos[θ], s1 √2 α Sin[θ], x + y / 2] ×
      ψ[s2 √2 α Cos[θ], -s2 √2 α Sin[θ], x - y / 2] dy];
Ws1s2[α, θ, x, p, 1, 1]
Ws1s2[α, θ, x, p, -1, -1]
Ws1s2[α, θ, x, p, 1, -1]
Ws1s2[α, θ, x, p, -1, 1]
```

Out[•]=
$$\frac{e^{-p^2-x^2-2\alpha^2+2\sqrt{2}\,\alpha\,(x\,\text{Cos}[\theta]+p\,\text{Sin}[\theta])}}{\pi}$$

Out[•]=
$$\frac{e^{-p^2-x^2-2\alpha^2-2\sqrt{2}\,\alpha\,(x\,\text{Cos}[\theta]+p\,\text{Sin}[\theta])}}{\pi}$$

Out[•]=
$$\frac{e^{-p^2-x^2-2\,i\,\sqrt{2}\,\alpha\,(p\,\text{Cos}[\theta]-x\,\text{Sin}[\theta])}}{\pi}$$

Out[•]=
$$\frac{e^{-p^2-x^2+2\,i\,\sqrt{2}\,\alpha\,(p\,\text{Cos}[\theta]-x\,\text{Sin}[\theta])}}{\pi}$$

```
In[•]:= FullSimplify[(Ws1s2[α, θ, x, p, 1, 1] + Ws1s2[α, θ, x, p, -1, -1])]
FullSimplify[(Ws1s2[α, θ, x, p, 1, -1] + Ws1s2[α, θ, x, p, -1, 1])]
```

Out[•]=
$$\frac{2\,e^{-p^2-x^2-2\alpha^2}\,\text{Cosh}\left[2\sqrt{2}\,\alpha\,(x\,\text{Cos}[\theta]+p\,\text{Sin}[\theta])\right]}{\pi}$$

Out[•]=
$$\frac{2\,e^{-p^2-x^2}\,\text{Cos}\left[2\sqrt{2}\,\alpha\,(p\,\text{Cos}[\theta]-x\,\text{Sin}[\theta])\right]}{\pi}$$

In[•]:=
$$\text{Wcat}[\alpha\_, \theta\_, x\_, p\_, s\_] := \left(\frac{2\,e^{-p^2-x^2-2\alpha^2}\,\text{Cosh}\left[2\sqrt{2}\,\alpha\,(x\,\text{Cos}[\theta]+p\,\text{Sin}[\theta])\right]}{\pi} + s\,\frac{2\,e^{-p^2-x^2}\,\text{Cos}\left[2\sqrt{2}\,\alpha\,(p\,\text{Cos}[\theta]-x\,\text{Sin}[\theta])\right]}{\pi}\right)\frac{1}{2\left(1+s\,e^{-2\alpha^2}\right)};$$

[Calc] Finding the Wigner distribution after the first beamsplitter

```
(*
   For the current example,
Wigner distribution has a form: W^(2) = 1/N (W++ + W-- + W+- + W-+).
   First, we are interested in finding W++,--,+-,-- separately.

    Careful about using the wavefunction for
  coherent state here. Imaginary part irα is important.
    For conjugate: use ψ*(x0,p0;x)→ ψ(x0,-p0;x)
*)
```

In[∘]:=
```
W2modes1s2[t_, r_, α_, θ_, x1_, p1_, x2_, p2_, s1_, s2_] :=
  Ws1s2[ t α, θ, x1, p1, s1, s2] × Ws1s2[r α, θ + π/2, x2, p2, s1, s2];

(* ++ and -- terms for two modes *)
```

In[∘]:=
```
W2modes1s2[t, r, α, θ, x1, p1, x2, p2, 1, 1] +
 W2modes1s2[t, r, α, θ, x1, p1, x2, p2, -1, -1]
```

Out[∘]=

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2-2 r^2 \alpha^2-2 t^2 \alpha^2+2 \sqrt{2}\, t \alpha (x1 \mathrm{Cos}[\theta]+p1 \mathrm{Sin}[\theta])+2 \sqrt{2}\, r \alpha (p2 \mathrm{Cos}[\theta]-x2 \mathrm{Sin}[\theta])}}{\pi^2} + \frac{e^{-p1^2-p2^2-x1^2-x2^2-2 r^2 \alpha^2-2 t^2 \alpha^2-2 \sqrt{2}\, t \alpha (x1 \mathrm{Cos}[\theta]+p1 \mathrm{Sin}[\theta])+2 \sqrt{2}\, r \alpha (-p2 \mathrm{Cos}[\theta]+x2 \mathrm{Sin}[\theta])}}{\pi^2}$$

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2-2 r^2 \alpha^2-2 t^2 \alpha^2}}{\pi^2} \left(e^{+2 \sqrt{2}\, t \alpha (x1 \mathrm{Cos}[\theta]+p1 \mathrm{Sin}[\theta])+2 \sqrt{2}\, r \alpha (p2 \mathrm{Cos}[\theta]-x2 \mathrm{Sin}[\theta])} + e^{-2 \sqrt{2}\, t \alpha (x1 \mathrm{Cos}[\theta]+p1 \mathrm{Sin}[\theta])+2 \sqrt{2}\, r \alpha (-p2 \mathrm{Cos}[\theta]+x2 \mathrm{Sin}[\theta])}\right)$$

In[∘]:=

$$\mathrm{FullSimplify}\left[e^{+2 \sqrt{2}\, t \alpha (x1 \mathrm{Cos}[\theta]+p1 \mathrm{Sin}[\theta])+2 \sqrt{2}\, r \alpha (p2 \mathrm{Cos}[\theta]-x2 \mathrm{Sin}[\theta])} + e^{-2 \sqrt{2}\, t \alpha (x1 \mathrm{Cos}[\theta]+p1 \mathrm{Sin}[\theta])+2 \sqrt{2}\, r \alpha (-p2 \mathrm{Cos}[\theta]+x2 \mathrm{Sin}[\theta])}, \{r^2 + t^2 == 1\}\right]$$

Out[∘]=

$$2 \mathrm{Cosh}\left[2 \sqrt{2}\, \alpha ((p2\, r + t\, x1) \mathrm{Cos}[\theta] + (p1\, t - r\, x2) \mathrm{Sin}[\theta])\right]$$

```
(* final form w++ + w-- *)
```

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2-2 \alpha^2}}{\pi^2} 2 \mathrm{Cosh}\left[2 \sqrt{2}\, \alpha ((p2\, r + t\, x1) \mathrm{Cos}[\theta] + (p1\, t - r\, x2) \mathrm{Sin}[\theta])\right]$$

```
(* +- and -+ terms for two modes *)
```

```
In[ ]:= W2modes1s2[t, r, α, θ, x1, p1, x2, p2, 1, -1] +
         W2modes1s2[t, r, α, θ, x1, p1, x2, p2, -1, 1]
```

Out[ ]=

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2+2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])-2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])}}{\pi^2} + \frac{e^{-p1^2-p2^2-x1^2-x2^2-2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])+2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])}}{\pi^2}$$

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2}}{\pi^2} \left(e^{+2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])-2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])} + e^{-2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])+2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])}\right)$$

In[ ]:=

$$FullSimplify\left[e^{+2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])-2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])} + e^{-2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])+2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])}, \{r^2+t^2 == 1\}\right]$$

Out[ ]=

$$2 Cos\left[2 \sqrt{2} \alpha ((-p1 t + r x2) Cos[\theta] + (p2 r + t x1) Sin[\theta])\right]$$

(* final form $w_{+-}+w_{-+}$ *)

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2}}{\pi^2} 2 Cos\left[2 \sqrt{2} \alpha ((-p1 t + r x2) Cos[\theta] + (p2 r + t x1) Sin[\theta])\right]$$

```
In[ ]:= (* General cat two mode B={{t, i r},{i r, t}} *)
        (* Two-mode Wigner distribution just before heralding *)
        (*W2mode[t_,r_,α_,θ_,x1_,p1_,x2_,p2_,s_]:=
```

$$\frac{e^{-p1^2-p2^2-x1^2-x2^2}}{\pi^2\ 2\left(1+s\ e^{-2\ \alpha^2}\right)} 2 \left(e^{-2\ \alpha^2} Cosh\left[2 \sqrt{2}\ \alpha\ ((p2\ r+t\ x1)\ Cos[\theta]+(p1\ t-r\ x2)\ Sin[\theta])\right] + s\ Cos\left[2 \sqrt{2}\ \alpha\ ((-p1\ t+r\ x2)\ Cos[\theta]+(p2\ r+t\ x1)\ Sin[\theta])\right]\right);*)$$

$$W2mode[t\_, r\_, \alpha\_, \theta\_, x1\_, p1\_, x2\_, p2\_, s\_] := \frac{1}{2\left(1+s\,e^{-2\alpha^2}\right)} \frac{1}{\pi^2}$$

$$\left(e^{-p1^2-p2^2-x1^2-x2^2-2 r^2 \alpha^2-2 t^2 \alpha^2+2 \sqrt{2} t \alpha (x1 Cos[\theta]+p1 Sin[\theta])+2 \sqrt{2} r \alpha (p2 Cos[\theta]-x2 Sin[\theta])} + e^{-p1^2-p2^2-x1^2-x2^2-2 r^2 \alpha^2-2 t^2 \alpha^2-2 \sqrt{2} t \alpha (x1 Cos[\theta]+p1 Sin[\theta])+2 \sqrt{2} r \alpha (-p2 Cos[\theta]+x2 Sin[\theta])} + \left(s\,e^{-p1^2-p2^2-x1^2-x2^2+2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])-2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])} + s\,e^{-p1^2-p2^2-x1^2-x2^2-2 i \sqrt{2} r \alpha (x2 Cos[\theta]+p2 Sin[\theta])+2 i \sqrt{2} t \alpha (p1 Cos[\theta]-x1 Sin[\theta])}\right)\right)$$

In[ ]:=

```
(* Real Variables *)
GaussianIntegralMatrix[M_] := (2 E^((M[[1,3]] M[[2,1]]^2 - M[[1,2]] M[[2,1]] M[[2,2]] + M[[1,1]] M[[2,2]]^2 + M[[1,2]]^2 M[[3,1]] - 4 M[[1,1]] M[[1,3]] M[[3,1]]) / (M[[2,2]]^2 - 4 M[[1,3]] M[[3,1]])) π) / (Sqrt[-4 M[[1, 3]] + M[[2,2]]^2/M[[3,1]]] Sqrt[-M[[3, 1]]]);

ExpCoefficientsToMatrix[expr_, x_, p_] :=
  CoefficientList[Exponent[expr, E], {x, p}, {3, 3}];
GaussIntR1[expr_, x_] := Module[{a, b, c, d, expo},
   expo = Exponent[expr, E];
   d = Coefficient[expr, E, expo];
   {c, b, a} = CoefficientList[expo, x, 3];
   (d E^(-b^2/(4 a) + c) Sqrt[π]) / Sqrt[-a]
  ];
GaussIntR2[expr_, var1_, var2_] := Module[{expo, coef},
   expo = Exponent[expr, E];
   coef = Coefficient[expr, E, expo];
   coef GaussianIntegralMatrix[CoefficientList[expo, {var1, var2}, {3, 3}]]
  ];
(* Trunated Gaussian integrals: ∫_{-Δ/2}^{Δ/2} E^(c x^2 + b x + a) dx *)
ClearAll[TruncatedGaussInt];
TruncatedGaussInt[expr_, x_, Δ_] := Module[{a, b, c, d, expo, coef},
   expo = Exponent[expr, E];
   d = Coefficient[expr, E, expo];
   {c, b, a} = CoefficientList[expo, {x}, {3}];
   d (E^(-b^2/(4 a) + c) Sqrt[π] (-Erfi[(b - a Δ)/(2 Sqrt[a])] + Erfi[(b + a Δ)/(2 Sqrt[a])])) / (2 Sqrt[a])];
```

[Calc] δ Gaussian filtering to account for loss

In[ ]:=

```
(1/(π 2 δ) E^(-1/(2 δ) (x^2 + x2^2 - 2 x x2 + p^2 + p2^2 - 2 p p2))) W2mode[t, r, α, θ, x1, p1, x, p, s]
```

Out[ ]=

$$\frac{1}{4\,\pi^3\,\left(1+e^{-2\,\alpha^2}\,s\right)\,\delta}\,e^{-\frac{p^2-2\,p\,p2+p2^2+x^2-2\,x\,x2+x2^2}{2\,\delta}}\left(e^{-p^2-p1^2-x^2-x1^2-2\,r^2\,\alpha^2-2\,t^2\,\alpha^2+2\,\sqrt{2}\,t\,\alpha\,(x1\,\mathrm{Cos}[\theta]+p1\,\mathrm{Sin}[\theta])+2\,\sqrt{2}\,r\,\alpha\,(p\,\mathrm{Cos}[\theta]-x\,\mathrm{Sin}[\theta])}+\right.$$
$$e^{-p^2-p1^2-x^2-x1^2-2\,r^2\,\alpha^2-2\,t^2\,\alpha^2-2\,\sqrt{2}\,t\,\alpha\,(x1\,\mathrm{Cos}[\theta]+p1\,\mathrm{Sin}[\theta])+2\,\sqrt{2}\,r\,\alpha\,(-p\,\mathrm{Cos}[\theta]+x\,\mathrm{Sin}[\theta])}+$$
$$e^{-p^2-p1^2-x^2-x1^2+2\,i\,\sqrt{2}\,r\,\alpha\,(x\,\mathrm{Cos}[\theta]+p\,\mathrm{Sin}[\theta])-2\,i\,\sqrt{2}\,t\,\alpha\,(p1\,\mathrm{Cos}[\theta]-x1\,\mathrm{Sin}[\theta])}\,s+$$
$$\left.e^{-p^2-p1^2-x^2-x1^2-2\,i\,\sqrt{2}\,r\,\alpha\,(x\,\mathrm{Cos}[\theta]+p\,\mathrm{Sin}[\theta])+2\,i\,\sqrt{2}\,t\,\alpha\,(p1\,\mathrm{Cos}[\theta]-x1\,\mathrm{Sin}[\theta])}\,s\right)$$

```
(* GaussFiltered =
```
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{2\ \pi\ \delta}e^{-\frac{1}{2\ \delta}(x^2+x2^2-2x\ x2+p^2+p2^2-2p\ p2)}\right)\text{W2mode}[t,r,\alpha,\theta,x1,p1,x,p,s]\,dp\,dx\ \text{*)}$$

In[ ]:=
$$\text{integrandsun}=\frac{1}{\pi^3\ 2\ \delta}e^{-\frac{p^2-2\,p\,p2+p2^2+x^2-2\,x\,x2+x2^2}{2\,\delta}}$$
$$\{e^{-p^2-p1^2-x^2-x1^2-2\,r^2\,\alpha^2-2\,t^2\,\alpha^2+2\sqrt{2}\,t\,\alpha\,(x1\,\text{Cos}[\theta]+p1\,\text{Sin}[\theta])+2\sqrt{2}\,r\,\alpha\,(p\,\text{Cos}[\theta]-x\,\text{Sin}[\theta])},$$
$$e^{-p^2-p1^2-x^2-x1^2-2\,r^2\,\alpha^2-2\,t^2\,\alpha^2-2\sqrt{2}\,t\,\alpha\,(x1\,\text{Cos}[\theta]+p1\,\text{Sin}[\theta])+2\sqrt{2}\,r\,\alpha\,(-p\,\text{Cos}[\theta]+x\,\text{Sin}[\theta])},$$
$$e^{-p^2-p1^2-x^2-x1^2+2\,i\sqrt{2}\,r\,\alpha\,(x\,\text{Cos}[\theta]+p\,\text{Sin}[\theta])-2\,i\sqrt{2}\,t\,\alpha\,(p1\,\text{Cos}[\theta]-x1\,\text{Sin}[\theta])}\,s,$$
$$e^{-p^2-p1^2-x^2-x1^2-2\,i\sqrt{2}\,r\,\alpha\,(x\,\text{Cos}[\theta]+p\,\text{Sin}[\theta])+2\,i\sqrt{2}\,t\,\alpha\,(p1\,\text{Cos}[\theta]-x1\,\text{Sin}[\theta])}\,s\};$$

```
integralsun = Simplify[GaussIntR2[#, x, p], {r² + t² == 1}] & /@ integrandsun
```

Out[ ]=
$$\left\{\frac{e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\alpha^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,t^2\,\alpha^2\,\delta-2\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(x1\,\text{Cos}[\theta]+p1\,\text{Sin}[\theta])+2\sqrt{2}\,r\,\alpha\,(-p2\,\text{Cos}[\theta]+x2\,\text{Sin}[\theta])}{1+2\,\delta}}}{\pi^2\,(1+2\,\delta)},\right.$$
$$\frac{e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\alpha^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,t^2\,\alpha^2\,\delta+2\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(x1\,\text{Cos}[\theta]+p1\,\text{Sin}[\theta])+2\sqrt{2}\,r\,\alpha\,(p2\,\text{Cos}[\theta]-x2\,\text{Sin}[\theta])}{1+2\,\delta}}}{\pi^2\,(1+2\,\delta)},$$
$$\frac{e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\alpha^2\,\delta-4\,t^2\,\alpha^2\,\delta-2\,i\sqrt{2}\,r\,\alpha\,(x2\,\text{Cos}[\theta]+p2\,\text{Sin}[\theta])+2\,i\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(p1\,\text{Cos}[\theta]-x1\,\text{Sin}[\theta])}{1+2\,\delta}}\,s}{\pi^2\,(1+2\,\delta)},$$
$$\left.\frac{e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\alpha^2\,\delta-4\,t^2\,\alpha^2\,\delta+2\,i\sqrt{2}\,r\,\alpha\,(x2\,\text{Cos}[\theta]+p2\,\text{Sin}[\theta])-2\,i\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(p1\,\text{Cos}[\theta]-x1\,\text{Sin}[\theta])}{1+2\,\delta}}\,s}{\pi^2\,(1+2\,\delta)}\right\}$$

In[ ]:=
```
(* Checking normalization *)
Simplify[
 Total[GaussIntR2[GaussIntR2[#, x2, p2], x1, p1] & /@ integralsun], {r² + t² == 1}]
```

Out[ ]=
$$2+2\,e^{-2\,\alpha^2}\,s$$

[Rslt] δ Gaussian Filtered 2 mode normalized Wigner

$$\text{(*}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{2\pi\ \delta}e^{-\frac{1}{2\ \delta}(x^2+x2^2-2x\ x2+p^2+p2^2-2p\ p2)}\right)\text{W2mode}[t,r,\alpha,\theta,x1,p1,x,p,s]\,dp\,dx\ \text{*)}$$

$$\text{Wn}[\delta\_,t\_,r\_,\alpha\_,\theta\_,x1\_,p1\_,x2\_,p2\_,s\_]:=\frac{1}{2\left(1+s\,e^{-2\,\alpha^2}\right)\pi^2\,(1+2\,\delta)}$$
$$\left(e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\alpha^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,t^2\,\alpha^2\,\delta-2\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(x1\,\text{Cos}[\theta]+p1\,\text{Sin}[\theta])+2\sqrt{2}\,r\,\alpha\,(-p2\,\text{Cos}[\theta]+x2\,\text{Sin}[\theta])}{1+2\,\delta}}+\right.$$
$$e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\alpha^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,t^2\,\alpha^2\,\delta+2\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(x1\,\text{Cos}[\theta]+p1\,\text{Sin}[\theta])+2\sqrt{2}\,r\,\alpha\,(p2\,\text{Cos}[\theta]-x2\,\text{Sin}[\theta])}{1+2\,\delta}}+$$
$$e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\alpha^2\,\delta-4\,t^2\,\alpha^2\,\delta-2\,i\sqrt{2}\,r\,\alpha\,(x2\,\text{Cos}[\theta]+p2\,\text{Sin}[\theta])+2\,i\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(p1\,\text{Cos}[\theta]-x1\,\text{Sin}[\theta])}{1+2\,\delta}}\,s+$$
$$\left.e^{-\frac{p1^2+p2^2+x1^2+x2^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\alpha^2\,\delta-4\,t^2\,\alpha^2\,\delta+2\,i\sqrt{2}\,r\,\alpha\,(x2\,\text{Cos}[\theta]+p2\,\text{Sin}[\theta])-2\,i\sqrt{2}\,t\,\alpha\,(1+2\delta)\,(p1\,\text{Cos}[\theta]-x1\,\text{Sin}[\theta])}{1+2\,\delta}}\,s\right);$$

The analysis is common for both dyne detectors up to this point

[Calc] Homodyne heralding

(* Leaving out the pre factor "$\frac{1}{2\left(1+s\ e^{-2\ \alpha^2}\right)}$"for now *)

(* Tracing out p2 and projecting on a Δ interval on the x2 *)

In[∘]:=
```
p2traced  = Simplify[GaussIntR1[#, p2]] &/@integralsun
```

Out[∘]=

$$\left\{\frac{e^{-\frac{p1^2+x1^2+x2^2+2\alpha^2+2\,p1^2\delta+2\,x1^2\delta+4\,t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]-2\,r^2\alpha^2\,\mathrm{Cos}[\theta]^2-2\sqrt{2}\,\alpha\,(p1\,t-r\,x2+2\,p1\,t\,\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\sqrt{\frac{1}{1+2\delta}}}{\pi^{3/2}},\right.$$

$$\frac{e^{-\frac{p1^2+x1^2+x2^2+2\alpha^2+2\,p1^2\delta+2\,x1^2\delta+4\,t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]-2\,r^2\alpha^2\,\mathrm{Cos}[\theta]^2+2\sqrt{2}\,\alpha\,(p1\,t-r\,x2+2\,p1\,t\,\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\sqrt{\frac{1}{1+2\delta}}}{\pi^{3/2}},$$

$$\frac{1}{\pi^{3/2}}\,e^{-\frac{p1^2+x1^2+x2^2+2\,p1^2\delta+2\,x1^2\delta+4\alpha^2\delta-4\,t^2\alpha^2\delta+2\,i\sqrt{2}\,\alpha\,(-r\,x2+p1\,(t+2\,t\,\delta))\,\mathrm{Cos}[\theta]-2\,i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]+2\,r^2\alpha^2\,\mathrm{Sin}[\theta]^2}{1+2\delta}}\,s\,\sqrt{\frac{1}{1+2\delta}},$$

$$\left.\frac{1}{\pi^{3/2}}\,e^{-\frac{p1^2+x1^2+x2^2+2\,p1^2\delta+2\,x1^2\delta+4\alpha^2\delta-4\,t^2\alpha^2\delta-2\,i\sqrt{2}\,\alpha\,(-r\,x2+p1\,(t+2\,t\,\delta))\,\mathrm{Cos}[\theta]+2\,i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]+2\,r^2\alpha^2\,\mathrm{Sin}[\theta]^2}{1+2\delta}}\,s\,\sqrt{\frac{1}{1+2\delta}}\right\}$$

In[∘]:=
```
Simplify[TruncatedGaussInt[#, x2, Δx], (δ) > 0] &/@p2traced
```

Out[∘]=

$$\left\{\frac{1}{2\pi}\,e^{-\frac{p1^2+x1^2+2\alpha^2-2\,r^2\alpha^2+2\,p1^2\delta+2\,x1^2\delta+4\,t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\right.$$

$$\left(\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{2\pi}\,e^{-\frac{p1^2+x1^2+2\alpha^2-2\,r^2\alpha^2+2\,p1^2\delta+2\,x1^2\delta+4\,t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}$$

$$\left(\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{2\pi}\,i\,e^{-\frac{p1^2+x1^2+2\,r^2\alpha^2+2\,p1^2\delta+2\,x1^2\delta+4\alpha^2\delta-4\,t^2\alpha^2\delta+2\,i\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]-2\,i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\,s$$

$$\left(\mathrm{Erfi}\left[\frac{-i\,\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]-\mathrm{Erfi}\left[\frac{i\,\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{2\pi}\,e^{-\frac{p1^2+x1^2+2\,r^2\alpha^2+2\,p1^2\delta+2\,x1^2\delta+4\alpha^2\delta-4\,t^2\alpha^2\delta-2\,i\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]+2\,i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\,s$$

$$\left.\left(\mathrm{Erf}\left[\frac{\Delta x-2\,i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\,i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\right\}$$

In[ ]:=

$$\text{unHOMwoPrefactor} = \Big\{\frac{1}{2\pi} e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{2\pi} e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{2\pi} e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta+2i\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Cos}[\theta]-2i\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}}\, s \left(\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{2\pi} e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta-2i\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Cos}[\theta]+2i\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}}\, s \left(\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\Big\};$$

Now multiply this by the normalization prefactor: $\frac{1}{2\left(1+s\,e^{-2\alpha^2}\right)}$ to give the proper “un”normalized output of the homodyne heralding.

$$\text{WunHOM}[\Delta x\_, \delta\_, t\_, r\_, \alpha\_, \theta\_, x1\_, p1\_, s\_] :=$$

$$\frac{1}{4\pi\left(1+s\,e^{-2\alpha^2}\right)} \Bigg(e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)+$$

$$e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)+$$

$$s\,e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta+2i\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Cos}[\theta]-2i\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta x+i\,2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x-i\,2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

$$+\,s\,e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta-2i\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\text{Cos}[\theta]+2i\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\Bigg)$$

In[ ]:=

$$\text{Simplify}\Bigg[\Bigg(e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\text{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}}$$

$$\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)+$$

$$e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\text{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}}$$

$$\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)+$$

$$s\,e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta+2i\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\text{Cos}[\theta]-2i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}}$$

$$\left(\text{Erf}\left[\frac{\Delta x+i\,2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x-i\,2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

$$+\,s\,e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta-2i\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\text{Cos}[\theta]+2i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}}$$

$$\left(\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\Bigg)\Bigg]$$

Out[ ]=

$$e^{-\frac{x1^2+2\alpha^2+2r^2\alpha^2+2x1^2\delta+4\alpha^2\delta+4t^2\alpha^2\delta+p1^2\,(1+2\delta)+2\sqrt{2}\,t\,(i\,p1+x1)\,\alpha\,(1+2\delta)\,\text{Cos}[\theta]}{1+2\delta}}$$

$$\Bigg(\Bigg(e^{\frac{2\alpha\left(\alpha+4t^2\alpha\delta+\sqrt{2}\,t\,(2i\,p1+x1)\,(1+2\delta)\,\text{Cos}[\theta]-i\sqrt{2}\,t\,x1\,(1+2\delta)\,\text{Sin}[\theta]\right)}{1+2\delta}}+e^{\frac{2\alpha\left(\alpha+4t^2\alpha\delta+\sqrt{2}\,t\,x1\,(1+2\delta)\,\text{Cos}[\theta]+i\sqrt{2}\,t\,x1\,(1+2\delta)\,\text{Sin}[\theta]\right)}{1+2\delta}}\Bigg)$$

$$s\,\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+$$

$$\left(e^{\frac{2\alpha\left(\alpha+4t^2\alpha\delta+\sqrt{2}\,t\,(2i\,p1+x1)\,(1+2\delta)\,\text{Cos}[\theta]-i\sqrt{2}\,t\,x1\,(1+2\delta)\,\text{Sin}[\theta]\right)}{1+2\delta}}+e^{\frac{2\alpha\left(\alpha+4t^2\alpha\delta+\sqrt{2}\,t\,x1\,(1+2\delta)\,\text{Cos}[\theta]+i\sqrt{2}\,t\,x1\,(1+2\delta)\,\text{Sin}[\theta]\right)}{1+2\delta}}\right)$$

$$s\,\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+$$

$$\left(e^{2\alpha\left(\frac{2\alpha\,(r^2+\delta)}{1+2\delta}+i\sqrt{2}\,p1\,t\,\text{Cos}[\theta]-\sqrt{2}\,p1\,t\,\text{Sin}[\theta]\right)}+e^{2\alpha\left(\frac{2\alpha\,(r^2+\delta)}{1+2\delta}+\sqrt{2}\,t\,(i\,p1+2\,x1)\,\text{Cos}[\theta]+\sqrt{2}\,p1\,t\,\text{Sin}[\theta]\right)}\right)$$

$$\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\Bigg)$$

Now let’s find the success probability for this process

In[ ]:=
```
(* trace out mode 1o find the success probability *)
pSterms = 1/(2 (1 + s e^(-2 α^2))) Simplify[GaussIntR2[#, x1, p1], {δ > 0, r^2 + t^2 == 1}] & /@
  unHOMwoPrefactor
```

Out[ ]=

$$\left\{\frac{\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]}{4\left(1+e^{-2\alpha^2}s\right)},\ \frac{\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]}{4\left(1+e^{-2\alpha^2}s\right)},\right.$$
$$\frac{e^{-2\alpha^2}s\left(\mathrm{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)}{4\left(1+e^{-2\alpha^2}s\right)},$$
$$\left.\frac{e^{-2\alpha^2}s\left(\mathrm{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)}{4\left(1+e^{-2\alpha^2}s\right)}\right\}$$

$$\left\{\frac{\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]}{4\left(1+e^{-2\alpha^2}s\right)},\right.$$
$$\frac{\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]}{4\left(1+e^{-2\alpha^2}s\right)},$$
$$\frac{e^{-2\alpha^2}s\left(\mathrm{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)}{4\left(1+e^{-2\alpha^2}s\right)},$$
$$\left.\frac{e^{-2\alpha^2}s\left(\mathrm{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)}{4\left(1+e^{-2\alpha^2}s\right)}\right\}$$

In[ ]:=
```
PsHOM[Δx_, δ_, r_, α_, θ_, s_] :=
  1/(2 (1 + e^(-2 α^2) s)) ((Erf[(Δx - 2 Sqrt[2] r α Sin[θ])/(2 Sqrt[1 + 2 δ])] + Erf[(Δx + 2 Sqrt[2] r α Sin[θ])/(2 Sqrt[1 + 2 δ])]) +
    e^(-2 α^2) s (Erf[(Δx - 2 i Sqrt[2] r α Cos[θ])/(2 Sqrt[1 + 2 δ])] + Erf[(Δx + 2 i Sqrt[2] r α Cos[θ])/(2 Sqrt[1 + 2 δ])]));
δHOM[η_] := (1 - η)/(2 η);
WnHOM[Δx_, δ_, t_, r_, α_, θ_, x1_, p1_, s_] :=
  Re[WnHOM[Δx, δ, t, r, α, θ, x1, p1, s]/PsHOM[Δx, δ, r, α, θ, s]];
```

In[ ]:=
```
PsHOM[2, δHOM[0.85], Sqrt[0.25], 2, 0, 1]
```

Out[ ]=

$0.80805 + 0.\,i$

[Calc] Heterodyne heralding

(* Leaving out the pre factor "$\frac{1}{2\left(1+s\,e^{-2\,\alpha^2}\right)}$"for now *)

(* Projecting on a Δp interval on the p2 *)

In[∘]:= projectp2 = Simplify[TruncatedGaussInt[#, p2, Δp], (δ) > 0] & /@ integralsun

Out[∘]=

$$\Bigg\{\frac{1}{2\,\pi^{3/2}\,\sqrt{1+2\,\delta}}\,e^{-\frac{p1^2+x1^2+x2^2+2\,\alpha^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,t^2\,\alpha^2\,\delta-2\,\sqrt{2}\,t\,x1\,\alpha\,(1+2\,\delta)\,\mathrm{Cos}[\theta]-2\,r^2\,\alpha^2\,\mathrm{Cos}[\theta]^2-2\,\sqrt{2}\,\alpha\,(p1\,t-r\,x2+2\,p1\,t\,\delta)\,\mathrm{Sin}[\theta]}{1+2\,\delta}}\left(\mathrm{Erf}\left[\frac{\Delta p-2\,\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\,\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]\right),$$

$$\frac{1}{2\,\pi^{3/2}\,\sqrt{1+2\,\delta}}\,e^{-\frac{p1^2+x1^2+x2^2+2\,\alpha^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,t^2\,\alpha^2\,\delta+2\,\sqrt{2}\,t\,x1\,\alpha\,(1+2\,\delta)\,\mathrm{Cos}[\theta]-2\,r^2\,\alpha^2\,\mathrm{Cos}[\theta]^2+2\,\sqrt{2}\,\alpha\,(p1\,t-r\,x2+2\,p1\,t\,\delta)\,\mathrm{Sin}[\theta]}{1+2\,\delta}}\left(\mathrm{Erf}\left[\frac{\Delta p-2\,\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\,\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]\right),$$

$$\frac{1}{2\,\pi^{3/2}\,\sqrt{1+2\,\delta}}\,e^{-\frac{p1^2+x1^2+x2^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,\alpha^2\,\delta-4\,t^2\,\alpha^2\,\delta+2\,i\,\sqrt{2}\,\alpha\,(-r\,x2+p1\,(t+2\,t\,\delta))\,\mathrm{Cos}[\theta]-2\,i\,\sqrt{2}\,t\,x1\,\alpha\,(1+2\,\delta)\,\mathrm{Sin}[\theta]+2\,r^2\,\alpha^2\,\mathrm{Sin}[\theta]^2}{1+2\,\delta}}\,s\left(\mathrm{Erf}\left[\frac{\Delta p+2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]-i\,\mathrm{Erfi}\left[\frac{i\,\Delta p+2\,\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]\right),$$

$$\frac{1}{2\,\pi^{3/2}\,\sqrt{1+2\,\delta}}\,e^{-\frac{p1^2+x1^2+x2^2+2\,p1^2\,\delta+2\,x1^2\,\delta+4\,\alpha^2\,\delta-4\,t^2\,\alpha^2\,\delta-2\,i\,\sqrt{2}\,\alpha\,(-r\,x2+p1\,(t+2\,t\,\delta))\,\mathrm{Cos}[\theta]+2\,i\,\sqrt{2}\,t\,x1\,\alpha\,(1+2\,\delta)\,\mathrm{Sin}[\theta]+2\,r^2\,\alpha^2\,\mathrm{Sin}[\theta]^2}{1+2\,\delta}}\,s\left(\mathrm{Erf}\left[\frac{\Delta p-2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]\right)\Bigg\}$$

```
In[•]:= unHETwoPrefactor = Simplify[TruncatedGaussInt[#, x2, Δx], (δ) > 0] & /@ projectp2
```

Out[•]=

$$\left\{\frac{1}{4\pi} e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\,\text{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right) \left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),\right.$$

$$\frac{1}{4\pi} e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\,\text{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}} \left(\text{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right) \left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{4\pi} e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta+2i\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\,\text{Cos}[\theta]-2i\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}}\, s \left(\text{Erfi}\left[\frac{-i\,\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]-\text{Erfi}\left[\frac{i\,\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right) \left(i\,\text{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erfi}\left[\frac{i\,\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\left.\frac{1}{4\pi} e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta-2i\sqrt{2}\,p1\,t\,\alpha(1+2\delta)\,\text{Cos}[\theta]+2i\sqrt{2}\,t\,x1\,\alpha(1+2\delta)\,\text{Sin}[\theta]}{1+2\delta}}\, s \left(\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right) \left(\text{Erf}\left[\frac{\Delta p-2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\right\}$$

In[•]:= **unHETwoPrefactor** =

$$\left\{\frac{1}{4\pi}\,e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta-2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\right.$$

$$\left(\mathrm{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

$$\left(\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{4\pi}\,e^{-\frac{p1^2+x1^2+2\alpha^2-2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4t^2\alpha^2\delta+2\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}$$

$$\left(\mathrm{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

$$\left(\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{4\pi}\,e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta+2i\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]-2i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\,s$$

$$\left(\mathrm{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

$$\left(\mathrm{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p-2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$

$$\frac{1}{4\pi}\,e^{-\frac{p1^2+x1^2+2r^2\alpha^2+2p1^2\delta+2x1^2\delta+4\alpha^2\delta-4t^2\alpha^2\delta-2i\sqrt{2}\,p1\,t\,\alpha\,(1+2\delta)\,\mathrm{Cos}[\theta]+2i\sqrt{2}\,t\,x1\,\alpha\,(1+2\delta)\,\mathrm{Sin}[\theta]}{1+2\delta}}\,s$$

$$\left(\mathrm{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

$$\left.\left(\mathrm{Erf}\left[\frac{\Delta p-2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\right\};$$

In[ ]:= `Total[Simplify[unHETwoPrefactor, {δ > 0, r^2 + t^2 == 1}]]`

Out[ ]=

$$\frac{1}{4\pi} e^{-p1^2 - x1^2 - 2\alpha^2 + 2t^2\alpha^2 + 2i\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Cos}[\theta] - 2i\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Sin}[\theta]}$$
$$s\left(\mathrm{Erf}\left[\frac{\Delta x - 2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta x + 2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$
$$\left(\mathrm{Erf}\left[\frac{\Delta p - 2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta p + 2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right) +$$
$$\frac{1}{4\pi} e^{-p1^2 - x1^2 - 2\alpha^2 + 2t^2\alpha^2 - 2i\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Cos}[\theta] + 2i\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Sin}[\theta]}\, s$$
$$\left(\mathrm{Erf}\left[\frac{\Delta x - 2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta x + 2i\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$
$$\left(\mathrm{Erf}\left[\frac{\Delta p - 2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta p + 2i\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right) +$$
$$\frac{1}{4\pi} e^{-p1^2 - x1^2 - 2t^2\alpha^2 - 2\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Cos}[\theta] - 2\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Sin}[\theta]}$$
$$\left(\mathrm{Erf}\left[\frac{\Delta p - 2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta p + 2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$
$$\left(\mathrm{Erf}\left[\frac{\Delta x - 2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta x + 2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right) +$$
$$\frac{1}{4\pi} e^{-p1^2 - x1^2 - 2t^2\alpha^2 + 2\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Cos}[\theta] + 2\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Sin}[\theta]}$$
$$\left(\mathrm{Erf}\left[\frac{\Delta p - 2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta p + 2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$
$$\left(\mathrm{Erf}\left[\frac{\Delta x - 2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right] + \mathrm{Erf}\left[\frac{\Delta x + 2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$

In[•]:=
$$\text{WunHET}[\Delta x\_, \Delta p\_, \delta\_, t\_, r\_, \alpha\_, \theta\_, x1\_, p1\_, s\_] := \frac{1}{8\pi\left(1+s\,e^{-2\alpha^2}\right)}\Bigg(\left(e^{-p1^2-x1^2-2t^2\alpha^2-2\sqrt{2}\,t\,x1\,\alpha\,\text{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,\text{Sin}[\theta]}+e^{-p1^2-x1^2-2t^2\alpha^2+2\sqrt{2}\,t\,x1\,\alpha\,\text{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,\text{Sin}[\theta]}\right)\left(\text{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)+s\left(e^{-p1^2-x1^2-2r^2\alpha^2+2i\sqrt{2}\,p1\,t\,\alpha\,\text{Cos}[\theta]-2i\sqrt{2}\,t\,x1\,\alpha\,\text{Sin}[\theta]}+e^{-p1^2-x1^2-2r^2\alpha^2-2i\sqrt{2}\,p1\,t\,\alpha\,\text{Cos}[\theta]+2i\sqrt{2}\,t\,x1\,\alpha\,\text{Sin}[\theta]}\right)\left(\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\text{Erf}\left[\frac{\Delta p-2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\Bigg);$$

Now let’s find the success probability for this process

In[•]:=
```
(* trace out mode 1o find the success probability *)
```
$$\text{pStermsHET} = \frac{1}{2\left(1+s\,e^{-2\alpha^2}\right)}\,\text{Simplify}\left[\text{GaussIntR2}[\#, x1, p1], \{\delta>0, r^2+t^2==1\}\right]\ \&\ /@\ \text{unHETwoPrefactor}$$

Out[•]=
$$\Bigg\{\frac{1}{8\left(1+e^{-2\alpha^2}s\right)}\left(\text{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$
$$\frac{1}{8\left(1+e^{-2\alpha^2}s\right)}\left(\text{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$
$$\frac{1}{8\left(1+e^{-2\alpha^2}s\right)}e^{-2\alpha^2}s\left(\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\text{Erf}\left[\frac{\Delta p-2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right),$$
$$\frac{1}{8\left(1+e^{-2\alpha^2}s\right)}e^{-2\alpha^2}s\left(\text{Erf}\left[\frac{\Delta x-2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\text{Erf}\left[\frac{\Delta p-2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta p+2i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\Bigg\}$$

In[ ]:=
```
Simplify[Total[pStermsHET]]
```
Out[ ]=

$$\frac{1}{4\left(1+e^{-2\alpha^2}s\right)}\left(e^{-2\alpha^2}s\left(\mathrm{Erf}\left[\frac{\Delta x-2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\mathrm{Erf}\left[\frac{\Delta p-2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\,i\,\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)+\left(\mathrm{Erf}\left[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\mathrm{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\left(\mathrm{Erf}\left[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\mathrm{Erf}\left[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\mathrm{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\right)$$

In[ ]:=
```
PsHET[Δx_, Δp_, δ_, r_, α_, θ_, s_] :=
  1/(4 (1 + e^(-2 α^2) s)) (e^(-2 α^2) s (Erf[(Δx - 2 i Sqrt[2] r α Cos[θ])/(2 Sqrt[1 + 2 δ])] + Erf[(Δx + 2 i Sqrt[2] r α Cos[θ])/(2 Sqrt[1 + 2 δ])])
     (Erf[(Δp - 2 i Sqrt[2] r α Sin[θ])/(2 Sqrt[1 + 2 δ])] + Erf[(Δp + 2 i Sqrt[2] r α Sin[θ])/(2 Sqrt[1 + 2 δ])]) +
    (Erf[(Δp - 2 Sqrt[2] r α Cos[θ])/(2 Sqrt[1 + 2 δ])] + Erf[(Δp + 2 Sqrt[2] r α Cos[θ])/(2 Sqrt[1 + 2 δ])])
     (Erf[(Δx - 2 Sqrt[2] r α Sin[θ])/(2 Sqrt[1 + 2 δ])] + Erf[(Δx + 2 Sqrt[2] r α Sin[θ])/(2 Sqrt[1 + 2 δ])]));
δHET[η_] := (2 - η)/(2 η);

WnHET[Δx_, Δp_, δ_, t_, r_, α_, θ_, x1_, p1_, s_] :=
  Re[WnHET[Δx, Δp, δ, t, r, α, θ, x1, p1, s]/PsHET[Δx, Δp, δ, r, α, θ, s]];
```

[Calc] Threshold detection of zero

```
(* For threshold - use calcs from Wigner Cat -
 v2.nb and iPad: Cat via noisy Threshold ZPS *)
```

Out[ ]=

$$\frac{e^{-p^2-x^2-2\alpha^2+2\sqrt{2}\,\alpha\,(x\,\mathrm{Cos}[\theta]+p\,\mathrm{Sin}[\theta])}}{\pi}$$

Out[ ]=

$$\frac{e^{-p^2-x^2-2\alpha^2-2\sqrt{2}\,\alpha\,(x\,\mathrm{Cos}[\theta]+p\,\mathrm{Sin}[\theta])}}{\pi}$$

Out[ ]=

$$\frac{e^{-p^2-x^2-2\,i\,\sqrt{2}\,\alpha\,(p\,\mathrm{Cos}[\theta]-x\,\mathrm{Sin}[\theta])}}{\pi}$$

Out[ ]=

$$\frac{e^{-p^2-x^2+2\,i\,\sqrt{2}\,\alpha\,(p\,\mathrm{Cos}[\theta]-x\,\mathrm{Sin}[\theta])}}{\pi}$$

In[ ]:=

```
ClearAll[V, Vfunc, v];
Vfunc[μ_, s_, η_] :=
```

$$\frac{e^{t^2\alpha^2}}{2\pi}\,s^{\mu}\left(\frac{\mathrm{FullSimplify}\left[\mathrm{Cosh}\left[\alpha^2\,(1-\eta)\,r^2\right]+(-1)^{\mu}\,\mathrm{Sinh}\left[\alpha^2\,(1-\eta)\,r^2\right]\right]}{e^{\alpha^2\,(1-\eta\,r^2)}+s\,e^{-\alpha^2\,(1-\eta\,r^2)}}\right);$$

```
V[s_, η_] := {Vfunc[0, s, η], Vfunc[0, s, η], Vfunc[1, s, η], Vfunc[1, s, η]};
```

$$v=\left\{e^{-p1^2-x1^2-2t^2\alpha^2+2\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Cos}[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Sin}[\theta]},\right.$$
$$e^{-p1^2-x1^2-2t^2\alpha^2-2\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Cos}[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Sin}[\theta]},$$
$$e^{-p1^2-x1^2+2\,i\,\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Cos}[\theta]-2\,i\,\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Sin}[\theta]},$$
$$\left.e^{-p1^2-x1^2-2\,i\,\sqrt{2}\,p1\,t\,\alpha\,\mathrm{Cos}[\theta]+2\,i\,\sqrt{2}\,t\,x1\,\alpha\,\mathrm{Sin}[\theta]}\right\};$$

In[ ]:=

$$\mathrm{Simplify}\left[\sum_{i=1}^{4}\mathrm{GaussIntR2}[V[s,\eta][[i]]\times v[[i]],\,x1,\,p1]\;/.\;t^2\to 1-r^2\right]$$

Out[ ]=

1

In[∘]:= $\sum_{i=1}^{4} V[s, \eta][[i]] \times v[[i]]$

Out[∘]=

$$\frac{e^{-p1^2-x1^2-t^2\alpha^2-r^2\alpha^2(-1+\eta)-2\sqrt{2}\,t\,x1\,\alpha\,Cos[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,Sin[\theta]}}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}+\frac{e^{-p1^2-x1^2-t^2\alpha^2-r^2\alpha^2(-1+\eta)+2\sqrt{2}\,t\,x1\,\alpha\,Cos[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,Sin[\theta]}}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}+\frac{e^{-p1^2-x1^2+t^2\alpha^2+r^2\alpha^2(-1+\eta)+2i\sqrt{2}\,p1\,t\,\alpha\,Cos[\theta]-2i\sqrt{2}\,t\,x1\,\alpha\,Sin[\theta]}\,s}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}+\frac{e^{-p1^2-x1^2+t^2\alpha^2+r^2\alpha^2(-1+\eta)-2i\sqrt{2}\,p1\,t\,\alpha\,Cos[\theta]+2i\sqrt{2}\,t\,x1\,\alpha\,Sin[\theta]}\,s}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}$$

In[∘]:= **WnTHR[η_, t_, r_, α_, θ_, x1_, p1_, s_] :=**

$$\frac{e^{-p1^2-x1^2-t^2\alpha^2-r^2\alpha^2(-1+\eta)-2\sqrt{2}\,t\,x1\,\alpha\,Cos[\theta]-2\sqrt{2}\,p1\,t\,\alpha\,Sin[\theta]}}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}+\frac{e^{-p1^2-x1^2-t^2\alpha^2-r^2\alpha^2(-1+\eta)+2\sqrt{2}\,t\,x1\,\alpha\,Cos[\theta]+2\sqrt{2}\,p1\,t\,\alpha\,Sin[\theta]}}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}+\frac{e^{-p1^2-x1^2+t^2\alpha^2+r^2\alpha^2(-1+\eta)+2i\sqrt{2}\,p1\,t\,\alpha\,Cos[\theta]-2i\sqrt{2}\,t\,x1\,\alpha\,Sin[\theta]}\,s}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)}+\frac{e^{-p1^2-x1^2+t^2\alpha^2+r^2\alpha^2(-1+\eta)-2i\sqrt{2}\,p1\,t\,\alpha\,Cos[\theta]+2i\sqrt{2}\,t\,x1\,\alpha\,Sin[\theta]}\,s}{2\pi\left(e^{\alpha^2(1-r^2\eta)}+e^{-\alpha^2(1-r^2\eta)}\,s\right)};$$

In[∘]:=

$$\mathrm{Simplify}\left[2\pi\sum_{i=1}^{4}\sum_{j=1}^{4}(V[s,\eta][[i]]\times V[s,\eta][[j]]\times \mathrm{GaussIntR2}[v[[i]]\times v[[j]], x1, p1])\right]$$

Out[∘]=

$$\frac{1}{2\left(1+e^{2\alpha^2(-1+r^2\eta)}\,s\right)^2} e^{2(-1+r^2-3t^2)\alpha^2}\left(e^{4t^2\alpha^2}+e^{8t^2\alpha^2}+4\,e^{2\alpha^2(3t^2+r^2(-1+\eta))}\,s+e^{4\alpha^2(t^2+r^2(-1+\eta))}\,s^2+e^{4\alpha^2(2t^2+r^2(-1+\eta))}\,s^2\right)$$

In[∘]:= $$\mathrm{Simplify}\left[e^{2(-1+r^2-3t^2)\alpha^2}\left(e^{4t^2\alpha^2}+e^{8t^2\alpha^2}+4\,e^{2\alpha^2(3t^2+r^2(-1+\eta))}\,s+e^{4\alpha^2(t^2+r^2(-1+\eta))}\,s^2+e^{4\alpha^2(2t^2+r^2(-1+\eta))}\,s^2\right) /.\ t^2\to 1-r^2,\ \{s^2==1,\ 0\le r\le 1\}\right]$$

Out[∘]=

$$1+e^{4(-1+r)(1+r)\alpha^2}+e^{4r^2\alpha^2(-1+\eta)}+e^{4\alpha^2(-1+r^2\eta)}+4\,e^{2\alpha^2(-1+r^2\eta)}\,s$$

In[∘]:=

```
purityTHR[r_, α_, θ_, η_, s_] :=
```

$$\frac{1+e^{4(-1+r)(1+r)\alpha^2}+e^{4r^2\alpha^2(-1+\eta)}+e^{4\alpha^2(-1+r^2\eta)}+4\,e^{2\alpha^2(-1+r^2\eta)}\,s}{2\left(1+e^{2\alpha^2(-1+r^2\eta)}\,s\right)^2};$$

```
(* Directly taken from calcs. *)
```

$$\text{PsTHR[r\_, α\_, η\_, s\_]} := \frac{\text{Exp}\left[\alpha^2\left(1-\eta\,r^2\right)\right]+s\,\text{Exp}\left[-\alpha^2\left(1-\eta\,r^2\right)\right]}{\text{Exp}\left[\alpha^2\right]+s\,\text{Exp}\left[-\alpha^2\right]};$$

In[∘]:=

$$\text{Simplify}\left[2\pi\sum_{i=1}^{4}\sum_{j=1}^{4}\left(\text{V[s, η][[i]]}\times\text{V[s, 1][[j]]}\times\text{GaussIntR2[v[[i]]}\times\text{v[[j]], x1, p1]}\right)\right]$$

Out[∘]=

$$\left(e^{-2\alpha^2\left(-1+3t^2+r^2\eta\right)}\left(e^{4t^2\alpha^2}+e^{8t^2\alpha^2}+2\,e^{6t^2\alpha^2}\,s+2\,e^{2\alpha^2\left(3t^2+r^2(-1+\eta)\right)}\,s+e^{2\alpha^2\left(2t^2+r^2(-1+\eta)\right)}\,s^2+e^{2\alpha^2\left(4t^2+r^2(-1+\eta)\right)}\,s^2\right)\right)\Big/\left(2\left(e^{-2\left(-1+r^2\right)\alpha^2}+s\right)\left(e^{-2\alpha^2\left(-1+r^2\eta\right)}+s\right)\right)$$

In[∘]:=

$$\text{Simplify}\Big[\left(e^{-2\alpha^2\left(-1+3t^2+r^2\eta\right)}\left(e^{4t^2\alpha^2}+e^{8t^2\alpha^2}+2\,e^{6t^2\alpha^2}\,s+2\,e^{2\alpha^2\left(3t^2+r^2(-1+\eta)\right)}\,s+e^{2\alpha^2\left(2t^2+r^2(-1+\eta)\right)}\,s^2+e^{2\alpha^2\left(4t^2+r^2(-1+\eta)\right)}\,s^2\right)\right)\ /.\ t^2\to 1-r^2,\ \left\{s^2==1,\ 0\le r\le 1\right\}\Big]$$

Out[∘]=

$$e^{-2r^2\alpha^2(2+\eta)}\left(e^{2r^2\alpha^2}+e^{2r^2\alpha^2\eta}\right)\left(e^{4\alpha^2}+e^{4r^2\alpha^2}+2\,e^{2\left(1+r^2\right)\alpha^2}\,s\right)$$

In[∘]:=

```
fidelityZPSTHR[r_, α_, θ_, η_, s_] :=
```

$$\frac{e^{-2r^2\alpha^2(2+\eta)}\left(e^{2r^2\alpha^2}+e^{2r^2\alpha^2\eta}\right)\left(e^{4\alpha^2}+e^{4r^2\alpha^2}+2\,e^{2\left(1+r^2\right)\alpha^2}\,s\right)}{\left(2\left(e^{-2\left(-1+r^2\right)\alpha^2}+s\right)\left(e^{-2\alpha^2\left(-1+r^2\eta\right)}+s\right)\right)};$$

In[∘]:=

$$\text{Simplify}\Big[\text{Simplify}\Big[\text{Total}\Big[2\pi\,\text{Table}\left[\text{Simplify}\left[\text{V[s, 1][[i]]}\times\text{GaussIntR2[v[[i]]}\times\text{unHOMwoPrefactor[[j]], x1, p1]},\ \left\{\delta>0,\ s^2==1,\ t^2==1-r^2,\ 0<r<1\right\}\right],\ \{i,\ 1,\ 4\},\ \{j,\ 1,\ 4\}\right]\ /.$$

$$\left\{\left(\text{Erf}\left[\frac{\Delta x-2\sqrt{2}\ r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\sqrt{2}\ r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\to f_{\delta}^{\Delta x}[\mathrm{i}\,\alpha_i],\right.$$

$$\left.\left(\text{Erf}\left[\frac{\Delta x-2\,\mathrm{i}\sqrt{2}\ r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\,\mathrm{i}\sqrt{2}\ r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\to f_{\delta}^{\Delta x}[\alpha_r]\right\},\ 2\Big]\Big]\ /.\ t^2\to 1-r^2\Big]$$

Out[∘]=

$$\frac{e^{-2r^2\alpha^2}\left(1+e^{4\left(-1+r^2\right)\alpha^2}+2\,e^{2\left(-1+r^2\right)\alpha^2}\,s\right)\left(e^{2r^2\alpha^2}\,f_{\delta}^{\Delta x}[\mathrm{i}\,\alpha_i]+f_{\delta}^{\Delta x}[\alpha_r]\right)}{2+2\,e^{2\left(-1+r^2\right)\alpha^2}\,s}$$

In[∘]:= fidelityZPSHOM [Δx_, δ_, r_, α_, θ_, s_] :=

$$\frac{1+e^{4\,(-1+r^2)\,\alpha^2}+s\,2\,e^{2\,(-1+r^2)\,\alpha^2}}{\text{PsHOM}[\Delta x,\,\delta,\,r,\,\alpha,\,\theta,\,s]\;4\,\left(1+s\,e^{-2\,\alpha^2}\right)\,\left(1+\,s\,e^{2\,(-1+r^2)\,\alpha^2}\,\right)}$$

$$\left(\left(\text{Erf}\left[\frac{\Delta x-2\,\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\,\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]\right)+\right.$$

$$\left.e^{-2\,r^2\,\alpha^2}\,\left(\text{Erf}\left[\frac{\Delta x-2\,i\,\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]+\text{Erf}\left[\frac{\Delta x+2\,i\,\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\,\sqrt{1+2\,\delta}}\right]\right)\right);$$

In[∘]:= fidelityZPSHOM $\left[2,\,\delta\text{HOM}[0.85],\,\sqrt{0.25},\,2,\,0,\,1\right]$

fidelityZPSTHR $\left[\sqrt{0.25},\,2,\,0,\,0.3,\,1\right]$

Out[∘]=

0.653259 + 0. i

Out[∘]=

0.624462

In[∘]:= PsHOM $\left[2,\,\delta\text{HOM}[0.85],\,\sqrt{0.25},\,2,\,0,\,1\right]$

PsTHR $\left[\sqrt{0.25},\,2,\,0.3,\,1\right]$

Out[∘]=

0.80805 + 0. i

Out[∘]=

0.741022

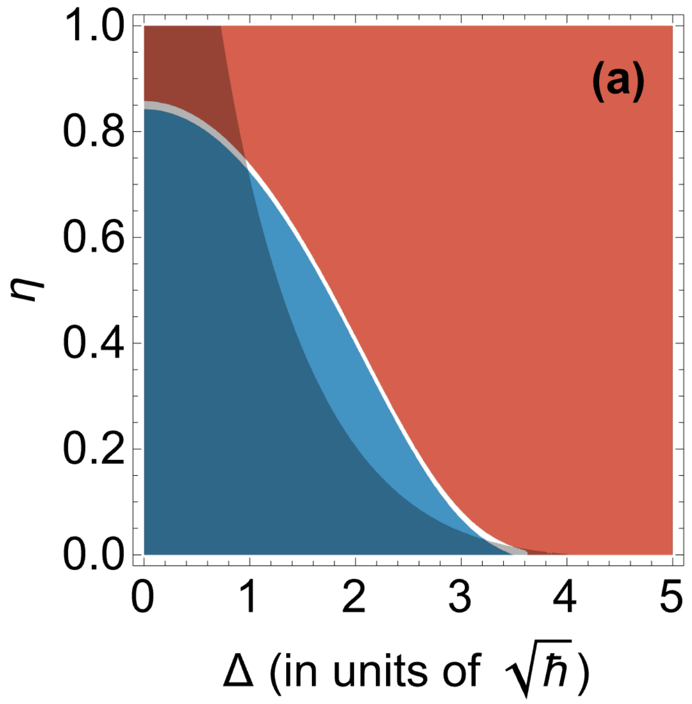
In[ ]:=
```
(* Plotting overlap and prob. improvement plots on
 top of each other. Intersecting lines mean that dyning
 is better to simulate certain threshold efficiencies. *)
ClearAll[Δx, α, r, t, θ, η, x1, p1, x2, p2, s, plotpoints];
t = √0.75 ;
r = √(1 - t^2) ;
θ = 0;
α = 2;
s = 1;
"Dyne efficiency chosen:"
ηH = 0.85
plotpoints = 60;
cols = RGBColor /@ {(*"#053061","#2166ac",*)"#4393c3", "#92c5de", "#d1e5f0",
    "#f7f7f7", "#fddbc7", "#f4a582", "#d6604d"(*,"#b2182b","#67001f"*)};
"HOMODYNE:"
text = Text[Style["(a)", {FontSize → Large, FontFamily → "Arial", Black, Bold}],
   {4.5, .9}, {0, 0}];
txt = Graphics[{text}];
plotROHOM = DensityPlot[HeavisideTheta[
    Re[-fidelityZPSHOM [Δ, δHOM[ηH], r, α, θ, s] + fidelityZPSTHR[r, α, θ, η, s]]],
   {Δ, 0, 5}, {η, 0, 1}, PlotPoints → plotpoints,
   ColorFunction → (Blend[cols, #] &), ColorFunctionScaling → True,
   PlotRange → All, FrameLabel → {"Δ (in units of √ħ)", "η"}, LabelStyle →
    {FontSize → Large, FontFamily → "Arial", Black}, ImageSize → Medium];
plotRIHOM = DensityPlot[
   HeavisideTheta[Re[PsHOM[Δ, δHOM[ηH], r, α, θ, s] - PsTHR[r, α, η, s]]],
   {Δ, 0, 5}, {η, 0, 1}, PlotPoints → plotpoints,
   ColorFunction → (Blend[{RGBColor[0, 0, 0, 0.3], RGBColor[1, 1, 1, 0]}, #] &),
   ColorFunctionScaling → False, PlotRange → All,
   FrameLabel → {"Δ (in units of √ħ)", "η"}, LabelStyle →
    {FontSize → Large, FontFamily → "Arial", Black}, ImageSize → Medium];
Show[{plotROHOM, plotRIHOM, txt}]
ClearAll[Δx, α, r, t, θ, η, x1, p1, x2, p2, s, plotpoints];
```

Out[ ]=
```
Dyne efficiency chosen:
```

Out[ ]=
```
0.85
```

Out[ ]=
```
HOMODYNE:
```

Out[ ]=



In[ ]:=

$$\text{Simplify}\Big[\text{Simplify}\Big[\text{Total}\Big[2\pi\,\text{Table}\Big[\text{Simplify}\big[V[s,1][[i]]\times\text{GaussIntR2}[v[[i]]\times\text{unHETwoPrefactor}[[j]],x1,p1],\{\delta>0,\ s^2==1,\ t^2==1-r^2,\ 0<r<1\}\big],\{i,1,4\},\{j,1,4\}\Big]\ /.$$

$$\Big\{\Big(\text{Erf}\Big[\frac{\Delta x-2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\Big]+\text{Erf}\Big[\frac{\Delta x+2\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\Big]\Big)\to f_\delta^{\Delta x}[i\,\alpha_i],$$

$$\Big(\text{Erf}\Big[\frac{\Delta x-2\,i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\Big]+\text{Erf}\Big[\frac{\Delta x+2\,i\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\Big]\Big)\to f_\delta^{\Delta x}[\alpha_r],$$

$$\Big(\text{Erf}\Big[\frac{\Delta p-2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\Big]+\text{Erf}\Big[\frac{\Delta p+2\sqrt{2}\,r\,\alpha\,\text{Cos}[\theta]}{2\sqrt{1+2\delta}}\Big]\Big)\to f_\delta^{\Delta p}[i\,\alpha_r],$$

$$\Big(\text{Erf}\Big[\frac{\Delta p-2\,i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\Big]+\text{Erf}\Big[\frac{\Delta p+2\,i\sqrt{2}\,r\,\alpha\,\text{Sin}[\theta]}{2\sqrt{1+2\delta}}\Big]\Big)\to f_\delta^{\Delta p}[\alpha_i]$$

$$\Big\}$$

$$,2\Big]\Big]$$

$$/.\ t^2\to 1-r^2\Big]$$

Out[ ]=

$$\frac{e^{-2r^2\alpha^2}\left(1+e^{4(-1+r^2)\alpha^2}+2\,e^{2(-1+r^2)\alpha^2}s\right)\left(e^{2r^2\alpha^2}f_\delta^{\Delta p}[i\,\alpha_r]\,f_\delta^{\Delta x}[i\,\alpha_i]+f_\delta^{\Delta p}[\alpha_i]\,f_\delta^{\Delta x}[\alpha_r]\right)}{4+4\,e^{2(-1+r^2)\alpha^2}s}$$

In[ ]:=
$$\text{fidelityZPSHET}[\Delta x\_, \Delta p\_, \delta\_, r\_, \alpha\_, \theta\_, s\_] := \frac{1 + e^{4\left(-1+r^2\right)\alpha^2} + s\, 2\, e^{2\left(-1+r^2\right)\alpha^2}}{\text{PsHET}[\Delta x, \Delta p, \delta, r, \alpha, \theta, s]\; 8 \left(1 + s\, e^{-2\alpha^2}\right)\left(1 + s\, e^{2\left(-1+r^2\right)\alpha^2}\right)}$$
$$\left(\left(\text{Erf}\left[\frac{\Delta p - 2\sqrt{2}\, r\, \alpha\, \text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right] + \text{Erf}\left[\frac{\Delta p + 2\sqrt{2}\, r\, \alpha\, \text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\right.$$
$$\left(\text{Erf}\left[\frac{\Delta x - 2\sqrt{2}\, r\, \alpha\, \text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right] + \text{Erf}\left[\frac{\Delta x + 2\sqrt{2}\, r\, \alpha\, \text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right) +$$
$$e^{-2 r^2 \alpha^2}\left(\text{Erf}\left[\frac{\Delta p - 2 i\sqrt{2}\, r\, \alpha\, \text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right] + \text{Erf}\left[\frac{\Delta p + 2 i\sqrt{2}\, r\, \alpha\, \text{Sin}[\theta]}{2\sqrt{1+2\delta}}\right]\right)$$
$$\left.\left(\text{Erf}\left[\frac{\Delta x - 2 i\sqrt{2}\, r\, \alpha\, \text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right] + \text{Erf}\left[\frac{\Delta x + 2 i\sqrt{2}\, r\, \alpha\, \text{Cos}[\theta]}{2\sqrt{1+2\delta}}\right]\right)\right);$$

In[ ]:=
$$\text{fidelityZPSHET}\left[2, 2, \delta\text{HOM}[0.85], \sqrt{0.25}, 2, 0, 1\right]$$
$$\text{fidelityZPSTHR}\left[\sqrt{0.25}, 2, 0, 0.3, 1\right]$$

Out[ ]=

0.918688 + 0. i

Out[ ]=

0.624462

In[ ]:=
$$\text{PsHET}\left[2, 2, \delta\text{HOM}[0.85], \sqrt{0.25}, 2, 0, 1\right]$$
$$\text{PsTHR}\left[\sqrt{0.25}, 2, 0.3, 1\right]$$

Out[ ]=

0.23768 + 0. i

Out[ ]=

0.741022

In[ ]:=
$$\text{fidelityZPSTHR}\left[\sqrt{0.25}, 2, 0, 0.3, 1\right]$$

Out[ ]=

0.624462

```
(* Plotting overlap and prob. improvement plots on
 top of each other. Intersecting lines mean that dyning
 is better to simulate certain threshold efficiencies. *)
ClearAll[Δx, α, r, t, θ, η, x1, p1, x2, p2, s, plotpoints];
t = √0.75 ;
r = √(1 - t^2) ;
θ = 0;
α = 2;
s = 1;
"Dyne efficiency chosen:"
ηH = 0.85
plotpoints = 60;
cols = RGBColor /@ {(*"#053061","#2166ac",*)"#4393c3", "#92c5de", "#d1e5f0",
    "#f7f7f7", "#fddbc7", "#f4a582", "#d6604d"(*,"#b2182b","#67001f"*)};
"HETERODYNE:"
text = Text[Style["(b)", {FontSize → Large, FontFamily → "Arial", Black, Bold}],
   {4.5, .9}, {0, 0}];
txt = Graphics[{text}];
plotROHET = DensityPlot[HeavisideTheta[Re[
     -fidelityZPSHET[Δ, Δ, δHET[ηH], r, α, θ, s] + fidelityZPSTHR[r, α, θ, η, s]]],
   {Δ, 0, 5}, {η, 0, 1}, PlotPoints → plotpoints,
   ColorFunction → (Blend[cols, #] &), ColorFunctionScaling → True,
   PlotRange → All, FrameLabel → {"Δ (in units of √ℏ)", "η"}, LabelStyle →
    {FontSize → Large, FontFamily → "Arial", Black}, ImageSize → Medium];
plotRIHET = DensityPlot[
   HeavisideTheta[Re[PsHET[Δ, Δ, δHET[ηH], r, α, θ, s] - PsTHR[r, α, η, s]]],
   {Δ, 0, 5}, {η, 0, 1}, PlotPoints → plotpoints,
   ColorFunction → (Blend[{RGBColor[0, 0, 0, 0.3], RGBColor[1, 1, 1, 0]}, #] &),
   ColorFunctionScaling → False, PlotRange → All,
   FrameLabel → {"Δ (in units of √ℏ)", "η"}, LabelStyle →
    {FontSize → Large, FontFamily → "Arial", Black}, ImageSize → Medium];
Show[{plotROHET, plotRIHET, txt}]
ClearAll[Δx, α, r, t, θ, η, x1, p1, x2, p2, s, plotpoints];
```

Out[ ]=

```
Dyne efficiency chosen:
```

Out[ ]=

```
0.85
```

Out[ ]=

```
HETERODYNE:
```

Out[ ]=