Assignment 6

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1 Introduction

The assignment involves implementation of Laplace transform inverse calculation, impulse response etc in analysis of the examples given

2 Question 1 and 2

2.1 Description

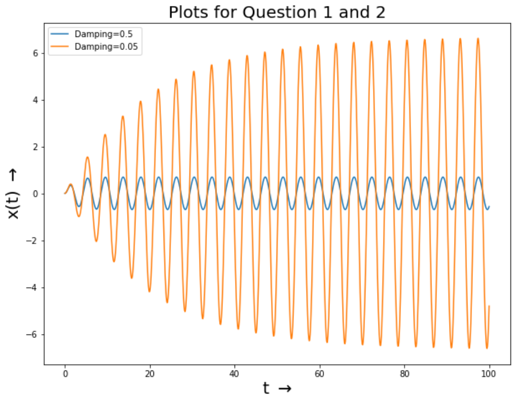
Given a forcing function we have to solve for the dynamics of the system which is a spring mass oscillator. We find the Laplace transform of x(t) and use system.impulse to do the computation.
We have,
f(t) = $\cos(w_d t)e^{-\alpha t}u(t)$
$\therefore$ F(s) = $\dfrac{s+\alpha}{(s+\alpha)^2 + w_d^2}$

Given, $\ddot{x}$ + 2.25x = f(t)
In Laplace domain, X(s) = $\dfrac{F(s)}{s^2 + 2.25}$
Finding impulse response of X(s) will find x(t)

2.2 Code

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.signal as sp
import scipy

#Question 1 and 2
F1 = sp.lti([1,0.5],scipy.polyadd(scipy.polymul([1,0.5],[1,0.5]),[2.25]))

F2 = sp.lti([1,0.05],scipy.polyadd(scipy.polymul([1,0.05],[1,0.05]),[2.25]))

t = np.linspace(0,100,1001)
t,x_2 = sp.impulse(sp.lti(F2.num,scipy.polymul(F2.den,[1, 0, 2.25])),None,T=t)

t = np.linspace(0,100,1001)
t,x_1 = sp.impulse(sp.lti(F1.num,scipy.polymul(F1.den,[1, 0, 2.25])),None,T=t)
plt.figure(figsize=(10,7.5))
plt.title('Plots for Question 1 and 2',size=20)
plt.xlabel('t '+r'$\rightarrow$',size=20)
plt.ylabel('x(t) '+r'$\rightarrow$',size=20)
plt.plot(t,x_1,label='Damping=0.5')
plt.plot(t,x_2,label='Damping=0.05')
plt.legend(prop={'size':10})
plt.savefig('q1and2.png')
plt.show()
```

2.3 Plots



2.4 Inferences

We observe that for smaller damping, the steady state amplitude is more

3 Question 3

3.1 Description

Another way of solving this is to find

$$\frac{X(s)}{F(s)}$$

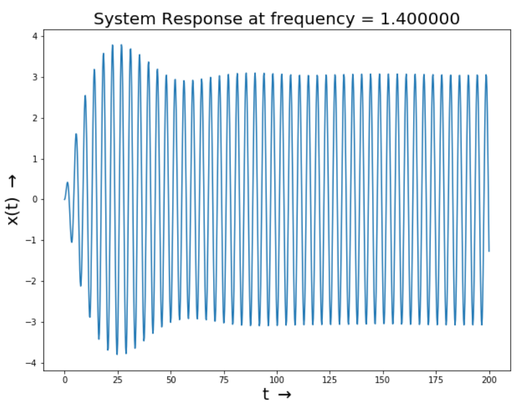
And then feed the input f(t) to the system and find output by signal.lsim. We plot outputs for different values of w_d from 1.4 to 1.6 with damping constant as 0.05. We also plot steady state amplitude vs frequency.

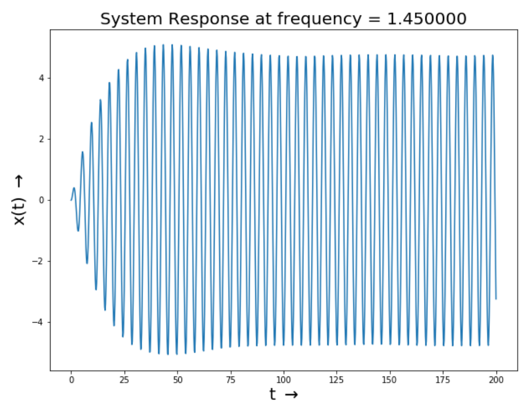
3.2 Codes

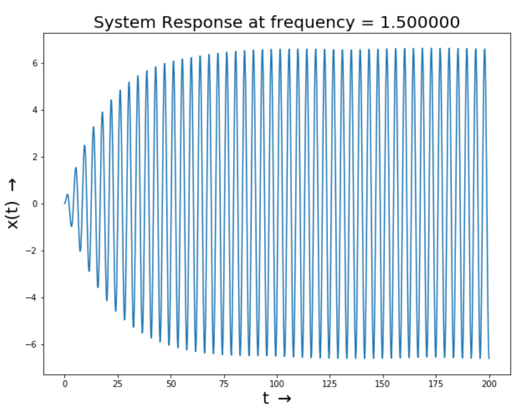
```
#Question 3:

#Transfer function definition:
H = sp.lti([1],[1,0,2.25])

def lti_system(H,f,t):
    t,x,svec = sp.lsim(H,f,t)
    return t,x

freq = np.arange(1.4,1.65,0.05)
t = np.linspace(0,200,1001)

Freq, Time = np.meshgrid(freq,t)

f = np.exp(-0.05*Time)*np.cos(Freq*Time)

steadystate = []
x = []

for i in range(len(freq)):
    t,x1 = lti_system(H,f[:,i],t)
    x2 = x1[int(0.9*len(x1)):-1]
    steadystate.append(x2.max())     #Find steady state value for resonance plotting
    x.append(x1)
```

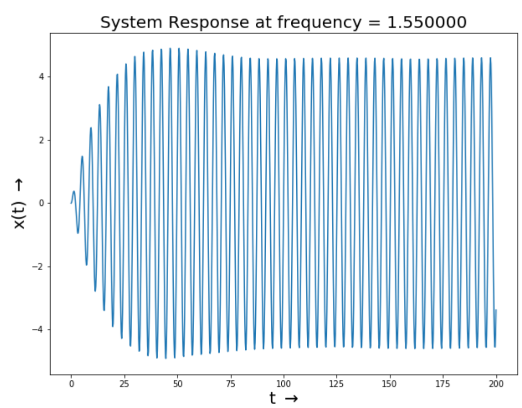
```
    plt.figure(figsize=(10,7.5))
    plt.plot(t,x1)
    plt.title('System Response at frequency = %f' %freq[i],size = 20)
    plt.xlabel('t '+r'$\rightarrow$',size=20)
    plt.ylabel('x(t) '+r'$\rightarrow$',size = 20)
    plt.savefig('frequency_%f.png' %freq[i])
    plt.show()

x = np.transpose(x) #Final matrix containing outputs for different frequencies

plt.figure(figsize=(10,7.5))
plt.plot(freq,steadystate,'ro')
plt.title('Steady State values as a function of frequency',size=20)
plt.xlabel('t '+r'$\rightarrow$',size=20)
plt.ylabel('Steady State Amplitudes '+r'$\rightarrow$',size=20)
plt.savefig('steadystatefreqdep.png')
plt.show()
```

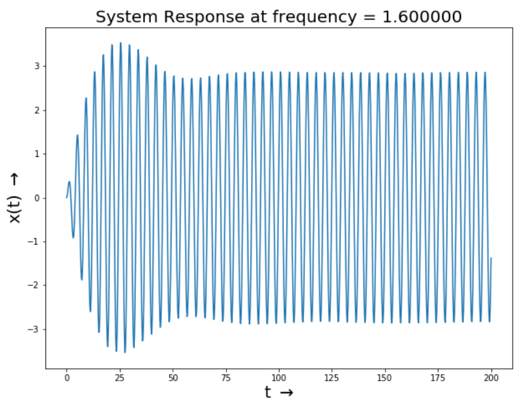
3.3 Plots

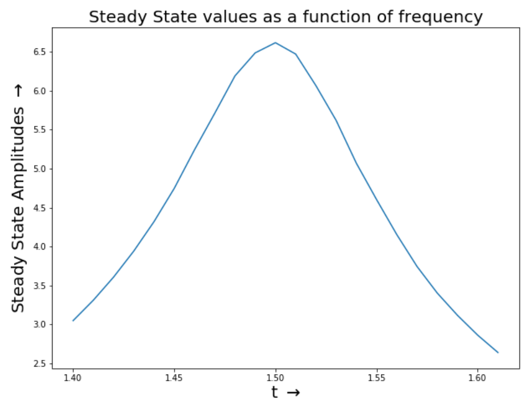












3.4 Inference

We observe that steady state amplitude is maximum at natural frequency from the plot. Also for frequencies around it there is a rise in amplitude followed by decay to steady state value(Peaking)

4 Question 4

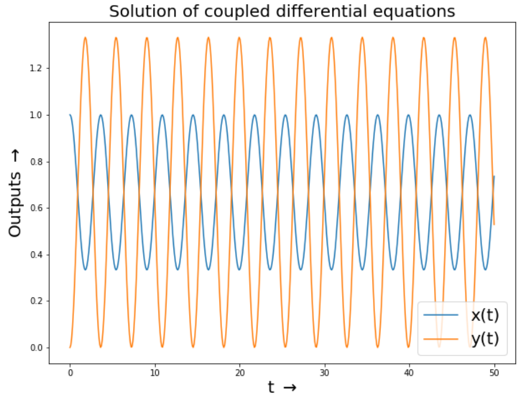
4.1 Description

To solve for $\ddot{x} + (x - y) = 0$
$\ddot{y} - (x - y) = 0$
This translates to:
x””+3x” = 0
y””+3y” = 0
Initial conditions can be found as:
x(0) = 1, x’(0) = 0, x”(0) = -1, x”’(0) = 0
y(0) = y’(0) = y”’(0) = 0, y”(0) = 2
We solve this too by signal.lsim

4.2 Codes

```

#Question 4:
H_X = sp.lti([1],[1,0,3,0,0])
H_Y = sp.lti([1],[1,0,3,0,0])
t = np.linspace(0,50,500)
t,x, svec = sp.lsim(H_X,0,t,X0 = [0,-1,0,1])
t,y, svec = sp.lsim(H_Y,0,t,X0 = [0,2,0,0])
plt.figure(figsize=(10,7.5))
plt.title('Solution of coupled differential equations',size=20)
plt.plot(t,x,label='x(t)')
plt.plot(t,y,label='y(t)')
plt.xlabel('t '+r'$\rightarrow$',size=20)
plt.ylabel('Outputs '+r'$\rightarrow$',size=20)
plt.legend(prop={'size':20})
plt.savefig('Coupled.png')
plt.show()
```

4.3 Plots



4.4 Inference

The two outputs are 180 degrees out of phase with different amplitudes. But their DC values are same. This system is in what we call coupled oscillations with antisymmetric mode.

5 Question 5

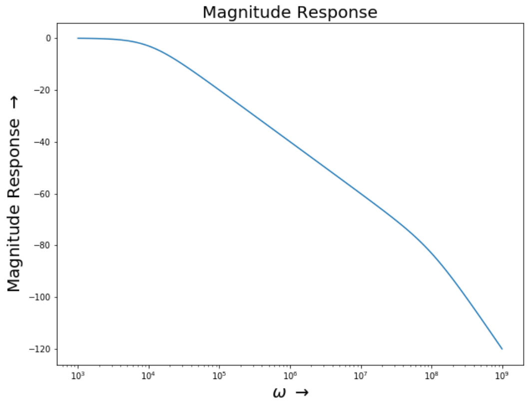
5.1 Description

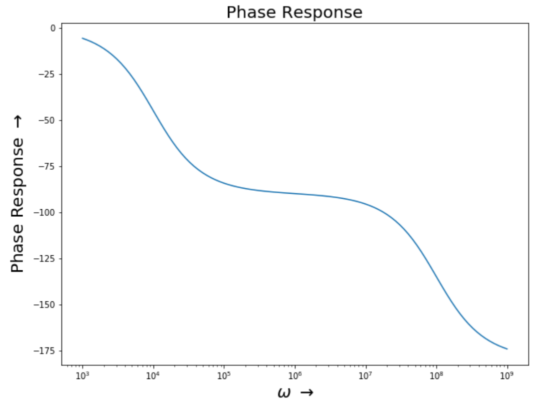
We have to plot magnitude and phase response for LCR circuit which behaves like a lowpass filter.

5.2 Codes

```
#Question 5:
Hlowpass = sp.lti([1],[1e-12,1e-4,1])
w, S, phi = Hlowpass.bode()

plt.figure(figsize=(10,7.5))
plt.plot(w,S)
plt.xscale('log')
plt.title('Magnitude Response',size=20)
plt.xlabel(r'$\omega$'+' '+r'$\rightarrow$',size=20)
plt.ylabel('Magnitude Response '+r'$\rightarrow$',size=20)
plt.savefig('Magnituderesp.png')
plt.show()

plt.figure(figsize=(10,7.5))
plt.plot(w,phi)
plt.xscale('log')
plt.title('Phase Response',size=20)
plt.xlabel(r'$\omega$'+' '+r'$\rightarrow$',size=20)
plt.ylabel('Phase Response '+r'$\rightarrow$',size=20)
plt.savefig('Phaseresp.png')
plt.show()
```

5.3 Plots





5.4 Inference

We observe that the system is a second order lowpass filter with resonance angular frequency of 1MHzrad. There is no peaking, it is overdamped

6 Question 6

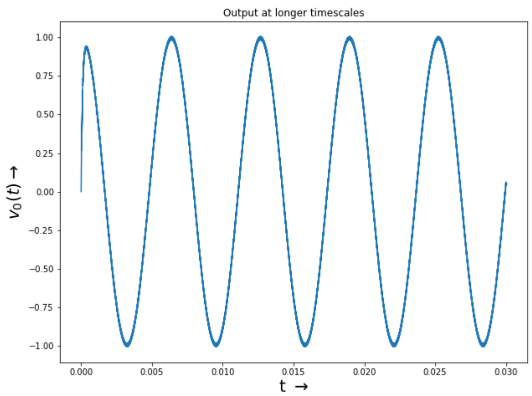
6.1 Description

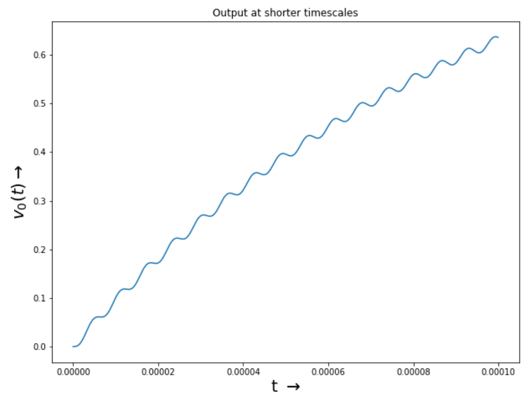
To provide input to the earlier low pass filter with frequencies of 1KHzrad and 1MHzrad and observe the output.

6.2 Codes

```
t = np.arange(0,0.03,1e-7)
v_input = np.cos(1e3*t)-np.cos(1e6*t)
t, v_output, svec = sp.lsim(Hlowpass,v_input,t,X0=None)

plt.figure(figsize=(10,7.5))
plt.plot(t,v_output)
plt.title('Output at longer timescales')
plt.xlabel('t '+r'$\rightarrow$',size=20)
plt.ylabel(r'$v_{0}(t) \rightarrow$',size=20)
plt.savefig('zoomout.png')
plt.show()

plt.figure(figsize=(10,7.5))
plt.plot(t[0:1000],v_output[0:1000])
plt.title('Output at shorter timescales')
plt.xlabel('t '+r'$\rightarrow$',size=20)
plt.ylabel(r'$v_{0}(t) \rightarrow$',size=20)
plt.savefig('zoomin.png')
plt.show()
```

6.3 Plots





6.4 Inference

We observe that the filter allows the angular frequency 1KHz rad to pass while the higher frequency ripples can only be seen at shorter timescales. This effect comes into play in practical design of switching regulators etc