(2 ½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1   Attempt All (Each of 5Marks) **(15M)**

(a)   **Select correct answer from the following:**

1) In which of the following method, we approximate the curve of solution by the tangent in each interval .
a) Simpson's Method
b) Euler's method
c) Newton's method
d) None of the above

2) $\int 1/(9x^2 + 25) \, dx =$
a) $(3/5) \tan^{-1}(3x/5) + c$
b) $(1/9) \tan^{-1}(3x/5) + c$
c) $(3/5) \tan^{-1}(5x/3) + c$
d) $(1/15) \tan^{-1}(3x/5) + c$

3) A function is said to be invertible if and only if it is_____
a) Bijective   b) injective   c) Inflexion   d) Surjective

4) $\lim\limits_{x\to\infty} 7/2x =$
a) 1   b) infinite   c) zero   d) None

5) If $f(x, y) = x^3y^3 + y^3 + 1$ then $f_x(x, y)$ is
a) $3x^2$ b) $3xy$ c) y^3x d) None

(b)   **Fill in the blanks:**
($continuous$, ∞, $(4i+5j)/41$, $(4i+5j)/31$, $-\infty$, e^x, derivative, $x - 3 \log|x + 3| + c$)
1. $\lim\limits_{x\to\infty}(5 - 2x) =$ _____.
2.   The derivative of e^x is _____.
3.   Unit vector of $4i+5j$ is _____.
4. $\int x/(x +3) \, dx =$ _____
5.   The rate of change of one variable with respect to another is called _____.

(c)   **Answer the following in one line**
1. Define Tangent Plane
2. Define Critical Point
3. Define the term Definite Integral
4. Evaluate $\int_{\pi/3}^{2\pi} \sin x \ dx$
5. Linearization of a function

Q. 2   Attempt the following (Any THREE)   **(15M)**
(a)   Show that $\lim_{x \to 1} 2x^2 + 3x - 4 = 1$
(b)   Discuss the continuity of the function $f(x) = \sqrt{4 - x^2}$
(c)   Show that the function $f(x) = x^3 - 9x^2 + 30x + 7$ is always increasing.
(d)   Find the relative extrema of $f(x) = 4xy - x^4 - y^4$ using both first and second derivative test.
(e)   Using Newton's method find the approximate root for the equation $f(x) = x - \cos x$
(f)   Divide 100 into two parts such that sum of their square is minimum.

Q. 3   Attempt the following (Any THREE)   **(15M)**
(a)   Evaluate $\int \sin^{-1}\sqrt{x} \ dx$

(b)   Evaluate $\int_{\frac{\pi}{6}}^{\frac{n}{3}} \frac{1}{(1+\cot x)} \ dx$

(c)   Estimate $\int_0^4 x^2 \ dx$ using simpson's rule and n = 4.

(d)   Solve the differential equation
$\sec^2 x \tan y \ dx + \sec^2 y \tan x \ dy = 0$

(e)   Solve dy/dx = 1 − y; y(0) = 0, find y(0.1) and y(0.3) using Euler's method. Taking h = 0.1.
(f)   Solve the differential equation
$(x + 1)\frac{dy}{dx} - y = e^x (x + 1)^2$

Q. 4   Attempt the following (Any THREE)   **(15)**
(a)   Show that f(x, y) = 2x² +3xy is continuous at (2, 3)
(b)   Find the second order derivatives of $f(x,y) = x^2 y^3 + x^4 y$
(c)   If z=x²y, x=t² and y=t³ Use chain rule to find $\frac{dz}{dt}$.
(d)   Find the directional derivative of f(x, y)=x³ +2xy² at the point (-2, -3) in the direction of the vector a= $i + j$
(e)   Find the gradient vector of f(x, y) if $f(x, y) = 10 - 8x^2 - 2y^2$. Evaluate it at (2, 3)
(f)   Find the equation for the tangent plane and parametric equations for normal line to the surface z=x²y at the point (2, 1, 4)

Q. 5   Attempt the following (Any THREE)   **(15)**

(a)   Locate all relative extrema and saddle points of
       $f(x, y)= 3x^2 - 2xy + y^2 - 8y$

(b)   Solve the differential equation
       $\frac{dy}{dx} = (4x + y + 1)^2$

(c)   Draw the graph of $y= 4 - 3x^2 + x^3$ and find the intervals on which the function y is
       increasing and decreasing(draw the graph on the answer sheet itself)

(d)   Find the asymptotes of the function $y= \frac{x}{(x+1)(x+2)^2}$

(e)   Solve the differential equation
       $dy/dx = (4x + y + 1)^2$

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