Matrix Theory (EE5609) Assignment 3

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***Abstract*—This document provides solution to problem 1.42 of Triangle Exercises.**

Download all latex-tikz codes from

https://github.com/saurabh13002/EE5609/blob/master/Assignment3'

1 Problem

Sides AB and AC of $\triangle ABC$. ABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

2 Solution

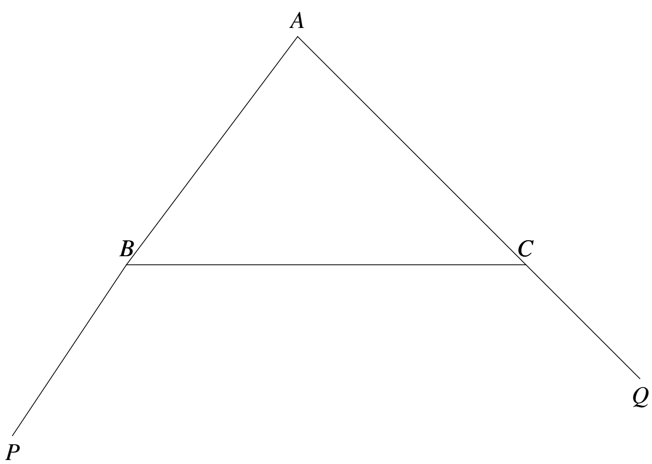

Fig. 1: $\triangle ABC$

Given conditions are : $\triangle ABC$ is a triangle having $\angle PBC < \angle QCB$

$$\frac{(\mathbf{P}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{P}-\mathbf{B}\|\|\mathbf{B}-\mathbf{C}\|} < \frac{(\mathbf{Q}-\mathbf{C})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{Q}-\mathbf{C}\|\|\mathbf{C}-\mathbf{B}\|} \tag{2.0.1}$$

$$\implies \frac{(\mathbf{P}-\mathbf{B}+\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{P}-\mathbf{B}\|} < \frac{(\mathbf{Q}-\mathbf{C}+\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{Q}-\mathbf{C}\|} \tag{2.0.2}$$

As, $\mathbf{P}-\mathbf{B}$,$\mathbf{A}-\mathbf{B}$ and $\mathbf{Q}-\mathbf{C}$,$\mathbf{A}-\mathbf{C}$ are in same direction.

$$\implies \frac{(\mathbf{A}-\mathbf{B}+\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|} < \frac{(\mathbf{A}-\mathbf{C}+\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.3}$$

$$\implies \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|} < \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.4}$$

On, rewriting the equation above;

$$\implies \frac{\|\mathbf{A}-\mathbf{B}\|^2-(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|} < \frac{\|\mathbf{A}-\mathbf{C}\|^2-(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.5}$$

Upon rearranging the terms, we have

$$\implies \|\mathbf{A}-\mathbf{B}\| - \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|} < \|\mathbf{A}-\mathbf{C}\| - \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.6}$$

We know that

$$\cos\angle BAC = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\|\mathbf{A}-\mathbf{C}\|} \tag{2.0.7}$$

Re-writing (2.0.6)

$$\|\mathbf{A}-\mathbf{B}\| - \|\mathbf{A}-\mathbf{C}\|\cos\angle BAC < \|\mathbf{A}-\mathbf{C}\| - \|\mathbf{A}-\mathbf{B}\|\cos\angle BAC \tag{2.0.8}$$

Upon rearranging,

$$\implies \|\mathbf{A}-\mathbf{B}\| + \|\mathbf{A}-\mathbf{B}\|\cos\angle BAC < \|\mathbf{A}-\mathbf{C}\| + \|\mathbf{A}-\mathbf{C}\|\cos\angle BAC \tag{2.0.9}$$

$$\implies \|\mathbf{A}-\mathbf{B}\|(1+\cos\angle BAC) < \\ \|\mathbf{A}-\mathbf{C}\|(1+\cos\angle BAC) \quad (2.0.10)$$

Therefore, $\|\mathbf{A}-\mathbf{B}\| < \|\mathbf{A}-\mathbf{C}\|$
Hence Proved