EE5609: Matrix Theory
Assignment-11

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Abstract

This document solves problem on Eigen values and properties.

Download all solutions from

https://github.com/saurabh13002/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbf{A}$ be a real symmetric matrix and $\mathbf{B} = \mathbf{I} + i\mathbf{A}$, where $i^2 = -1$. Then choose the correct option.

1) $\mathbf{B}$ is invertible if and only if $\mathbf{A}$ is invertible.
2) All Eigenvalues of $\mathbf{B}$ are necessarily real.
3) $\mathbf{B} - \mathbf{I}$ is necessarily invertible.
4) $\mathbf{B}$ is necessarily invertible.

2 Explanation

Statement 1.	$\mathbf{B}$ is invertible if and only if $\mathbf{A}$ is invertible.
False statement	Matrix $\mathbf{B}$ is invertible even if $\mathbf{A}$ is non invertible.
Example:	Consider a matrix $$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (2.0.1)$$ a real non invertible,symmetric matrix. $$\implies \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + i\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1+i & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.2)$$ is invertible even if $\mathbf{A}$ is non invertible.

Statement 2.	All Eigenvalues of $\mathbf{B}$ are necessarily real.
False statement	Matrix $\mathbf{B}$ can have complex Eigenvalues.
Proof :	Eigen values of $\mathbf{B}$ = Eigen values of ($\mathbf{I}$) + i (Eigen values of $\mathbf{A}$). Clearly from (2.0.2) above Eigen values of $\mathbf{B}$ are 1 and $1+i$ respectively. Hence $\mathbf{B}$ can also have complex Eigen value.
Statement 3.	$\mathbf{B}-\mathbf{I}$ is necessarily invertible.
False statement	$\mathbf{B}-\mathbf{I}=i\mathbf{A}$ will be invertible if $\mathbf{A}$, is invertible.
Proof:	We have $\mathbf{B}-\mathbf{I}=i\mathbf{A}$ $\implies \mathbf{B}-\mathbf{I}=i\mathbf{A}=\begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}$,from (2.0.1) Hence $\mathbf{B}-\mathbf{I}$ is not invertible,unless $\mathbf{A}$ is invertible.
Statement 4.	$\mathbf{B}$ is necessarily invertible.
Correct Statement:	Matrix $\mathbf{B}$ has non zero Eigen values corresponding to Eigenvector X .
Proof:	Let X be an Eigen vector of $\mathbf{A}$ corresponding to Eigen value λ also,$\lambda \epsilon \mathbb{R}$ $\implies \mathbf{A}X=\lambda X$ $\therefore \mathbf{B}X=(\mathbf{I}+i\mathbf{A})X=\mathbf{I}X+\mathrm{i}\mathbf{A}X=X+i\lambda X$ $\implies \mathbf{B}X=(1+i\lambda)X$ Therefore, $1+i\lambda$ is an Eigen value of $\mathbf{B}$, corresponding to Eigen vector X,which are non zero. Hence, $\mathbf{B}$ is necessarily invertible.

TABLE 1: Solution summary