EE5609: Matrix Theory
Assignment-13

Major Saurabh Joshi
MTech Artificial Intelligence
AI20MTECH13002

Download codes from

https://github.com/saurabh13002/EE5609/tree/master/Assignment13

1 Question

Let $\mathbf{A}$,$\mathbf{B}$ be n $\times$ n matrices.Which of the following equals trace($\mathbf{A}^2\mathbf{B}^2$)?

1) (trace($\mathbf{AB}$))^2.
2) trace($\mathbf{AB}^2\mathbf{A}$).
3) trace(($\mathbf{AB}$)^2).
4) trace($\mathbf{BABA}$).

2 Solution

Statement	**Solution**
Definition	The trace of an n $\times$ n square matrix $\mathbf{A}$ is defined as: $$tr(\mathbf{A}) = \sum_{i=1}^{n} a_{ii}$$ where a_{ii} denotes the entry on the ith row and ith column of $\mathbf{A}$.
Properties	The properties of the trace : $tr(c\mathbf{A}) = c\ tr(\mathbf{A}) \quad (2.0.1)$ $tr(\mathbf{A}^T) = tr(\mathbf{A}) \quad (2.0.2)$ $tr(\mathbf{A}+\mathbf{B}) = tr(\mathbf{B}+\mathbf{A}) \quad (2.0.3)$ $tr(\mathbf{AB}) = tr(\mathbf{BA}) \quad (2.0.4)$ $tr(\mathbf{A}^T\mathbf{B}) = tr(\mathbf{AB}^T) \quad (2.0.5)$ $tr(\mathbf{R}^{-1}\mathbf{AR}) = tr(\mathbf{R}^{-1}(\mathbf{AR})) \quad (2.0.6)$ $= tr((\mathbf{AR})\mathbf{R}^{-1}) = tr(\mathbf{A}) \quad (2.0.7)$
Checking $tr(\mathbf{A}^2\mathbf{B}^2)$.	Upon rewriting and from (2.0.4), $tr(\mathbf{A}^2\mathbf{B}^2) = tr(\mathbf{AABB}) \quad (2.0.8)$ $= tr(\mathbf{BAAB}) \quad (2.0.9)$ $= tr(\mathbf{BBAA}) \quad (2.0.10)$ $= tr(\mathbf{ABBA}) \quad (2.0.11)$ $= tr(\mathbf{AABB}) \quad (2.0.12)$ $= tr(\mathbf{A}^2\mathbf{B}^2) \quad (2.0.13)$

Checking $(tr(\mathbf{AB}))^2$.	$$\text{from (2.0.4)}, (tr(\mathbf{AB}))^2 = (tr(\mathbf{BA}))^2 \tag{2.0.14}$$
Checking $tr(\mathbf{AB}^2\mathbf{A})$.	$$\text{Rewriting}, tr(\mathbf{AB}^2\mathbf{A}) = tr(\mathbf{ABBA}) \tag{2.0.15}$$ $$\text{from (2.0.4)}, tr(\mathbf{AB}^2\mathbf{A}) = tr(\mathbf{AABB}) = tr(\mathbf{A}^2\mathbf{B}^2) \tag{2.0.16}$$
Checking $tr(\mathbf{AB})^2$.	$$\text{from (2.0.4)}, tr(\mathbf{AB})^2 = tr(\mathbf{BA})^2 \tag{2.0.17}$$
Checking $tr(\mathbf{BABA})$.	$$\text{from (2.0.4)} \tag{2.0.18}$$ $$tr(\mathbf{BABA}) = tr(\mathbf{ABAB}) \tag{2.0.19}$$ $$= tr(\mathbf{BABA}) \tag{2.0.20}$$
Conclusion	Hence, from (2.0.4), and (2.0.16) option 2, ie $tr(\mathbf{AB}^2\mathbf{A})$. is the correct answer.

Table1:Solution