EE5609: Matrix Theory Assignment-8

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***Abstract*—This document uses properties of vector spaces and subspaces.**

Download the latex code from

https://github.com/saurabh13002/EE5609/tree/master/Assignment8

1 Problem

Let **V** be the (real) vector space of all functions f from **R** into **R**. Is $f(0) = f(1)$ a subspace of **V**

2 Theorem

A non-empty subset **W** of **V** is a subspace of **V** if and only if for each pair of vectors α β in **W** and each scalar c in $\mathbb{R}$ the vector $c\alpha + \beta$ is again in **W**.

3 Solution

For each of the function to be a subspace, it must be closed with respect to addition and scalar multiplication in **V** defined as, for f g ϵ **W** and c ϵ $\mathbb{R}$

Then,

$$h = cf + g \tag{3.0.1}$$

$$h(0) = cf(0) + g(0) \tag{3.0.2}$$

$$= cf(1) + g(1) \tag{3.0.3}$$

$$= h(1) \tag{3.0.4}$$

Thus, $h(0) = h(1)$. Therefore,**W** is a subset of **V** and also a vector space. Therefore **W** is a subspace of **V**.
Hence, $f(0) = f(1)$ is a subspace of **V**.