EE5609: Matrix Theory Assignment-12

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Download codes from

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https://github.com/saurabh13002/EE5609/tree/master/Assignment12
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1 Question

Which of the following subsets of $\mathbb{R}^4$ is a basis of $\mathbb{R}^4$?
$\mathbf{B_1}$=$\{1000, 1100, 1110, 1111\}$
$\mathbf{B_2}$=$\{1000, 1200, 1230, 1234\}$
$\mathbf{B_3}$=$\{1200, 0011, 2100, -5500\}$

1) $\mathbf{B_1}$ and $\mathbf{B_2}$ but not $\mathbf{B_3}$.
2) $\mathbf{B_1}$,$\mathbf{B_2}$, and $\mathbf{B_3}$.
3) $\mathbf{B_1}$ and $\mathbf{B_3}$ but not $\mathbf{B_2}$.
4) Only $\mathbf{B_1}$.

2 Solution

Statement	**Solution**
Definition	Let $\mathbf{V}$ be a vector space. Then $\{\mathbf{v}_1, \cdots, \mathbf{v}_n\}$ is called a basis for $\mathbf{V}$ if the following conditions hold. $$\text{span}\{\mathbf{v}_1, \cdots, \mathbf{v}_n\} = \mathbf{V} \tag{2.0.1}$$ $$\{\mathbf{v}_1, \cdots, \mathbf{v}_n\} \text{ is linearly independent} \tag{2.0.2}$$
Given	$$\mathbf{B_1} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathbf{B_2} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{pmatrix}, \mathbf{B_3} = \begin{pmatrix} 1 & 0 & 2 & -5 \\ 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{2.0.3}$$
Checking $\mathbf{B_1}$	$$\text{Checking for linear independence.Upon row reducing } \mathbf{B_1} \tag{2.0.4}$$ $$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xleftrightarrow{R_1 \to R_1 - R_2, R_2 \to R_2 - R_3, R_3 \to R_3 - R_4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$ Clearly Rank of $\mathbf{B_1}$ is 4,ie full rank.Hence it forms a Basis.

Checking $\mathbf{B_2}$	Checking for linear independence.Upon row reducing $\mathbf{B_2}$ (2.0.6) $$\begin{pmatrix}1 & 1 & 1 & 1\\ 0 & 2 & 2 & 2\\ 0 & 0 & 3 & 3\\ 0 & 0 & 0 & 4\end{pmatrix} \xleftrightarrow{R_2\to\frac{R_2}{2},R_1\to R_1-R_2,R_3\to\frac{R_3}{3},R_2\to R_2-R_3,R_4\to\frac{R_4}{4},R_3\to R_3-R_4} \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \quad (2.0.7)$$ Rank of $\mathbf{B_2}$ is 4, ie full rank.Hence it also forms a Basis.
Checking $\mathbf{B_3}$	Checking for linear independence.Upon row reducing $\mathbf{B_3}$ (2.0.8) $$\begin{pmatrix}1 & 0 & 2 & -5\\ 2 & 0 & 1 & 5\\ 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0\end{pmatrix} \xleftrightarrow{R_2\to R_2-2R_1,R_4\to R_4-R_2,R_3\to-\frac{R_3}{3},R_1\to R_1-2R_3} \begin{pmatrix}1 & 0 & 0 & 5\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & -5\\ 0 & 0 & 0 & 0\end{pmatrix} \quad (2.0.9)$$ Rank of $\mathbf{B_3}$ is 3, ie not full rank.Hence it does not forms a Basis.
Conclusion	Hence option 1, ie $\mathbf{B_1}$,$\mathbf{B_2}$ and not $\mathbf{B_3}$ is the correct answer.

Table1:Solution