EE5609: Matrix Theory
Assignment-11

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Abstract

This document solves problem on Eigen values and properties.

Download all solutions from

https://github.com/saurabh13002/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbf{A}$ be a real symmetric matrix and $\mathbf{B} = \mathbf{I} + i\mathbf{A}$, where $i^2 = -1$. Then choose the correct option.

1. $\mathbf{B}$ is invertible if and only if $\mathbf{A}$ is invertible.

2. All eigenvalues of $\mathbf{B}$ are necessarily real.

3. $\mathbf{B} - \mathbf{I}$ is necessarily invertible.

4. $\mathbf{B}$ is necessarily invertible.

2 Results used

Result 1	Eigenvalues of real symmetric matrix are real
Result 2	If a square matrix $\mathbf{A}$ is lower or upper triangular then Eigen values of $\mathbf{A}$ are entries on the main diagonal.
Result 3	If the eigenvalue of a matrix $\mathbf{A}$ is λ,corresponding to the Eigen vector X, then $\mathbf{A} + c\mathbf{I}$ has Eigen value $\lambda + c$, corresponding to the Eigen vector X. c can be scalar or complex.
Result 3	$\det \mathbf{A}$=product of Eigen values of $\det \mathbf{A}$

3 Solution

Given	Let $\mathbf{A}$ be a real symmetric matrix, and $\mathbf{B} = \mathbf{I} + i\mathbf{A}$, where $i^2 = -1$.
Checking Option 1	Lets assume, $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ a real symmetric and non invertible matrix,as the rank of $\mathbf{A} < 2$. $\implies$ Eigen values of $\mathbf{A}$ are 1 and 0 $\implies \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + i \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1+i & 0 \\ 0 & 1 \end{pmatrix}$, is invertible even if $\mathbf{A}$ is non invertible. Thus Option 1 is incorrect.
Checking Option 2	Eigen values of $\mathbf{B}$ = Eigen values of $\mathbf{I}$ + i (Eigen values of $\mathbf{A}$). Clearly, Eigen values of $\mathbf{B}$ are, 1 and $1 + i$, Hence Eigen values of $\mathbf{B}$ are necessarily real is wrong. Thus, Option 2 is incorrect.
Checking Option 3	$\mathbf{B} - \mathbf{I} = i\mathbf{A}$ $\implies (\mathbf{B} - \mathbf{I}) = i\mathbf{A} = \begin{pmatrix} i & 0 \\ 0 & 0 \end{pmatrix}$ Hence, $\mathbf{B} - \mathbf{I}$ is not invertible Thus option 3 is also incorrect.
Checking Option 4	Let X be an Eigen vector of $\mathbf{A}$ corresponding to Eigen value λ. $\lambda \epsilon \mathbb{R}$ $\implies \mathbf{A}X = \lambda X$ $\therefore \mathbf{B}X = (\mathbf{I} + i\mathbf{A})X = \mathbf{I}X + \mathrm{i}\mathbf{A}X = X + i\lambda X$ $\implies \mathbf{B}X = (1 + i\lambda)X$ Therefore, $1 + i\lambda$ is an Eigen value of $\mathbf{B}$ corresponding to Eigen vector X,which are non zero. Therefore, all Eigen values of $\mathbf{B}$ are non zero Therefore, $\mathbf{B}$ is necessarily invertible.
Correct option	The correct option is **4**.