EE5609: Matrix Theory Assignment-14

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Download codes from

https://github.com/saurabh13002/EE5609/tree/master/Assignment14

1 Question

Let $\mathbf{M_n(K)}$,denote the space of all n $\times$ n matrices with entries in a field $\mathbb{K}$. Fix a non singular matrix $\mathbf{A} = (\mathbf{A_{ij}})\epsilon\mathbf{M_n(K)}$, and consider the linear map $\mathbf{T} : \mathbf{M_n(K)} \to \mathbf{M_n(K)}$given by $\mathbf{T(X)} = \mathbf{AX}$.Then

1) trace($\mathbf{T}$) = $n\sum_{i=1}^{n}\mathbf{A}_{ii}$.
2) trace($\mathbf{T}$) = $\sum_{i=1}^{n}\sum_{j=1}^{n}\mathbf{A}_{ij}$.
3) $rank(\mathbf{T})$ =n^2.
4) $\mathbf{T}$ is non singular.

2 Solution

Statement	**Solution**
Definition	$\mathbf{T} : \mathbb{F}^{nxm} \to \mathbb{F}^{nxm}$ defined as: $$\mathbf{T(X)} = \mathbf{AX}$$ where $\mathbf{A}$ n x n is fixed and is a linear transformation.
Properties	If $\mathbf{A}$ is the matrix representation of a linear transformation $\mathbf{T}$, then $$nullity(\mathbf{T}) = \text{m.}nullity(\mathbf{A}) \quad (2.0.1)$$ $$rank(\mathbf{T}) = \text{m.}rank(\mathbf{A}) \quad (2.0.2)$$ $$tr(\mathbf{T}) = \text{m.}tr(\mathbf{A}) \quad (2.0.3)$$ $$\text{Also,rank of a non singular n x n matrix,} \mathbf{A} = \text{n} \quad (2.0.4)$$ $$tr(\mathbf{A}) = \sum_{i=1}^{n}\mathbf{A}_{ii} \quad (2.0.5)$$
Checking $tr(\mathbf{T})$.	$$\text{from (2.0.3), } tr(\mathbf{T}) = \text{m.}tr(\mathbf{A}), \quad (2.0.6)$$ $$\text{Since, } \mathbf{A} \text{ is a square matrix } \therefore \text{ m=n} \quad (2.0.7)$$ $$\text{also, from (2.0.5),} \quad (2.0.8)$$ $$\implies tr(\mathbf{T}) = \text{n}\sum_{i=1}^{n}\mathbf{A}_{ii} \quad (2.0.9)$$ $$\text{Hence it is a correct option.} \quad (2.0.10)$$

Checking tr($\mathbf{T}$) = $\sum_{i=1}^{n}\sum_{j=1}^{n}\mathbf{A}_{ij}$.	$$\text{from (2.0.9), } tr(\mathbf{T}) = \text{n}\sum_{i=1}^{n}\mathbf{A}_{ii}, \text{Hence, discarding the option.} \tag{2.0.11}$$
Checking $rank(\mathbf{T})$ =n^2.	$$\text{from (2.0.2) and (2.0.4), } rank(\mathbf{T}) = \text{m}.rank(\mathbf{A}) \tag{2.0.12}$$ $$\text{from (2.0.7), } rank(\mathbf{T}) = \text{n}.rank(\mathbf{A}) \tag{2.0.13}$$ $$\implies rank(\mathbf{T}) = \text{n.n} = \text{n}^2 \tag{2.0.14}$$ $$\therefore \text{Option 3 is also correct.} \tag{2.0.15}$$
Checking $\mathbf{T}$ is non singular.	$$\text{from the given data } \mathbf{T}(\mathbf{X}) = \mathbf{AX} \text{ is a linear map and } \mathbf{A} \text{ is non singular .} \tag{2.0.16}$$ $$\text{Hence, } \mathbf{T} \text{ is non singular.} \tag{2.0.17}$$
Conclusion	Hence, from (2.0.9),(2.0.11),(2.0.14) and (2.0.17) option 1,3 and 4 are the correct answers.

Table1:Solution