EE5609: Matrix Theory Assignment-13

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Download codes from

https://github.com/saurabh13002/EE5609/tree/master/Assignment13

1 QUESTION

Let **A**,**B** be n × n matrices. Which of the following equals trace($\mathbf{A}^2\mathbf{B}^2$)?

1) $(\text{trace}(\mathbf{AB}))^2$.
2) $\text{trace}(\mathbf{AB}^2\mathbf{A})$.
3) $\text{trace}((\mathbf{AB})^2)$.
4) $\text{trace}(\mathbf{BABA})$.

2 SOLUTION

Statement	Solution
Definition	The trace of an n × n square matrix **A** is defined as: $$tr(\mathbf{A}) = \sum_{i=1}^{n} a_{ii}$$ where a_{ii} denotes the entry on the ith row and ith column of **A**.
Properties	The properties of the trace : $tr(c\mathbf{A}) = c\ tr(\mathbf{A})$ (2.0.1) $tr(\mathbf{A}^T) = tr(\mathbf{A})$ (2.0.2) $tr(\mathbf{A} + \mathbf{B}) = tr(\mathbf{B} + \mathbf{A})$ (2.0.3) $tr(\mathbf{AB}) = tr(\mathbf{BA})$ (2.0.4) $tr(\mathbf{A}^T\mathbf{B}) = tr(\mathbf{AB}^T)$ (2.0.5) $tr(\mathbf{R}^{-1}\mathbf{AR}) = tr(\mathbf{R}^{-1}(\mathbf{AR}))$ (2.0.6) $= tr((\mathbf{AR})\mathbf{R}^{-1}) = tr(\mathbf{A})$ (2.0.7)
Checking $tr(\mathbf{A}^2\mathbf{B}^2)$.	Upon rewriting and from (2.0.4), $tr(\mathbf{A}^2\mathbf{B}^2) = tr(\mathbf{AABB})$ (2.0.8) $= tr(\mathbf{BAAB})$ (2.0.9) $= tr(\mathbf{BBAA})$ (2.0.10) $= tr(\mathbf{ABBA})$ (2.0.11) $= tr(\mathbf{AABB})$ (2.0.12) $= tr(\mathbf{A}^2\mathbf{B}^2)$ (2.0.13)

Checking $(tr(\mathbf{AB}))^2$.	from (2.0.4), $(tr(\mathbf{AB}))^2 = (tr(\mathbf{BA}))^2$	(2.0.14)
Checking $tr(\mathbf{AB}^2\mathbf{A})$.	from (2.0.4) and (2.0.11), $tr(\mathbf{AB}^2\mathbf{A}) = tr(\mathbf{ABBA})$ i.e from (2.0.13), $tr(\mathbf{AB}^2\mathbf{A}) = tr(\mathbf{A}^2\mathbf{B}^2)$	(2.0.15) (2.0.16) (2.0.17)
Checking $tr(\mathbf{AB})^2$.	from (2.0.4) $tr(\mathbf{AB})^2 = tr(\mathbf{BA})^2$	(2.0.18) (2.0.19)
Checking $tr(\mathbf{BABA})$.	from (2.0.4) $tr(\mathbf{BABA}) = tr(\mathbf{ABAB})$ $= tr(\mathbf{BABA})$	(2.0.20) (2.0.21) (2.0.22)
Conclusion	Hence, from (2.0.4),(2.0.11),(2.0.17) option 2, ie $tr(\mathbf{AB}^2\mathbf{A})$. is the correct answer.	

Table1:Solution