EE5609: Matrix Theory
Assignment-11

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***Abstract*—This document solves problem on Eigen values and properties .**

Download the latex code from

https://github.com/saurabh13002/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbf{A}$ be a real symmetric matrix and $\mathbf{B} = \mathbf{I} + i\mathbf{A}$, where $i^2 = -1$. Then
1. $\mathbf{B}$ is invertible if and only if $\mathbf{A}$ is invertible.
2. All eigenvalues of $\mathbf{B}$ are necessarily real.
3. $\mathbf{B} - \mathbf{I}$ is necessarily invertible.
4. $\mathbf{B}$ is necessarily invertible.

2 Solution

Given, $\mathbf{A}$ is a symmetric matrix,
Let us assume λ be the eigen value of $\mathbf{A}$

$\implies \lambda \epsilon \mathbb{R}$

$\implies i\lambda$ is eigen value of $i\mathbf{A}$

$\implies 1+i\lambda$ is eigen value of $\mathbf{I} + i\mathbf{A}$

Given $\mathbf{B} = \mathbf{I} + i\mathbf{A}$

Therefore, $1 + i\lambda$ is eigen value of $\mathbf{B}$

We know that, $\det \mathbf{B}$ is equals to product of eigen values of $\mathbf{B}$

Hence 0, can not be the eigen value of $\mathbf{B}$

$\implies \det \mathbf{B} \neq 0$

Therefore, $\mathbf{B}$ is necessarily invertible.

Hence **4**. is the correct answer.