EE5609: Matrix Theory Assignment-8

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***Abstract*—This document uses properties of vector spaces and subspaces.**

Download the latex code from

https://github.com/saurabh13002/EE5609/tree/master/Assignment8

1 Problem

Let $\mathbf{V}$ be the (real) vector space of all functions f from $\mathbf{R}$ into $\mathbf{R}$. Is $f(0) = f(1)$ a subspace of $\mathbf{V}$

2 Theorem

A non-empty subset $\mathbf{W}$ of $\mathbf{V}$ is a subspace of $\mathbf{V}$ if and only if for each pair of vectors α β in $\mathbf{W}$ and each scalar c in $\mathbb{R}$ the vector $c\alpha + \beta$ is again in $\mathbf{W}$.

3 Solution

For each of the function to be a subspace, it must be closed with respect to addition and scalar multiplication in $\mathbf{V}$ defined as, for f g ϵ $\mathbf{W}$ and c ϵ $\mathbb{R}$

Then,

$$h = f + g \tag{3.0.1}$$
$$\implies h(0) = f(0) + g(0) \tag{3.0.2}$$
$$= f(1) + g(1) \tag{3.0.3}$$
$$= h(1) \tag{3.0.4}$$

Also,

$$ch(0) = cf(0) + cg(0) \tag{3.0.5}$$
$$= cf(1) + cg(1) \tag{3.0.6}$$
$$= ch(1) \tag{3.0.7}$$

Thus, $\mathbf{W}$ is a subset of $\mathbf{V}$ and also a vector space. Therefore $\mathbf{W}$ is a subspace of $\mathbf{V}$.
Hence, $f(0) = f(1)$ is a subspace of $\mathbf{V}$.