EE5609: Matrix Theory Assignment-7

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***Abstract*—This document use elementary row operations to find invertiblile matrix,and hence the inverse.**

Download the latex code from

https://github.com/saurabh13002/EE5609/tree/master/Assignment7

1 Problem

For each of the two matrices use elementary row operations to discover whether it is invertible, and to find the inverse in case it is invertible.

$$\mathbf{A} = \begin{pmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix}$$

2 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix} \tag{2.0.1}$$

By applying row reductions on $\mathbf{A}$

$$\begin{pmatrix} 2 & 5 & -1 \\ 4 & -1 & 2 \\ 6 & 4 & 1 \end{pmatrix} \xleftrightarrow{R_2=R_2-2R_1} \mathbf{A} = \begin{pmatrix} 2 & 5 & -1 \\ 0 & -11 & 4 \\ 6 & 4 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow{R_3=R_3-3R_1} \begin{pmatrix} 2 & 5 & -1 \\ 0 & -11 & 4 \\ 0 & -11 & 4 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1=\frac{R_1}{2}} \begin{pmatrix} 1 & \frac{5}{2} & \frac{-1}{2} \\ 0 & -11 & 4 \\ 0 & -11 & 4 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_3=R_3-R_2} \begin{pmatrix} 1 & \frac{5}{2} & \frac{-1}{2} \\ 0 & -11 & 4 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_2=\frac{-R_2}{11}} \begin{pmatrix} 1 & \frac{5}{2} & \frac{-1}{2} \\ 0 & 1 & \frac{-4}{11} \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_1=R_1-\frac{5}{2}R_2} \begin{pmatrix} 1 & 0 & \frac{9}{22} \\ 0 & 1 & \frac{-4}{11} \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.7}$$

For a matrix to be invertible, it has to be a matrix of full rank. However the matrix $\mathbf{A}$ is not of full rank ($Rank(\mathbf{A}) < 3$).Therefore $\mathbf{A}$ is not invertible.
Let us now consider augmented matrix $\mathbf{B}|\mathbf{I}$, By applying row reductions on $\mathbf{B}|\mathbf{I}$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 3 & 2 & 4 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_2=R_2-3R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 5 & -2 & -3 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{2.0.8}$$

$$\xleftrightarrow{R_2=\frac{R_2}{5}} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{-3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{2.0.9}$$

$$\xleftrightarrow{R_1=R_1+R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{8}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{-3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right) \tag{2.0.10}$$

$$\xleftrightarrow{R_3=R_3-R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{8}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{-2}{5} & \frac{-3}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{-8}{5} & \frac{3}{5} & \frac{-1}{5} & 1 \end{array}\right) \tag{2.0.11}$$

$$\xleftrightarrow{R_1=R_1+R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & \frac{-2}{5} & \frac{-3}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{-8}{5} & \frac{3}{5} & \frac{-1}{5} & 1 \end{array}\right) \tag{2.0.12}$$

$$\xleftrightarrow{R_3=\frac{-5}{8}R_3}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & \frac{-2}{5} & \frac{-3}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{-3}{8} & \frac{1}{8} & \frac{-5}{8} \end{array}\right) \tag{2.0.13}$$

$$\xleftrightarrow{R_3=R_2+\frac{2}{5}R_3}\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & \frac{-3}{4} & \frac{1}{4} & \frac{-1}{4} \\ 0 & 0 & 1 & \frac{-3}{8} & \frac{1}{8} & \frac{-5}{8} \end{array}\right) \tag{2.0.14}$$

For a matrix to be invertible, it has to be a matrix of full rank. Here, the matrix **B** is of full rank ($Rank(\mathbf{B}) = 3$). Therefore **B** is invertible and the inverse matrix $\mathbf{B}^{-1}$ can be written from (2.0.14):

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ \frac{-3}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{-3}{8} & \frac{1}{8} & \frac{-5}{8} \end{pmatrix} \tag{2.0.15}$$

3 Observation

1) For a matrix to be invertible, it has to be a matrix of full rank.
2) For a given matrix **A**, if the augmented matrix **A|I** on applying elementary row operations transforms into a matrix of the form **I|B**. Hence,the matrix **A** is invertible, and the inverse matrix $\mathbf{A}^{-1}$ is given by **B**.
3) If the reduced row echelon form matrix for **A|I** is not of the form **I|B**, then the matrix **A** is not invertible.