EE5609: Matrix Theory
Assignment-11

Major Saurabh Joshi
MTech Artificial Intelligence
AI20MTECH13002

Abstract

This document solves problem on Eigen values and properties.

Download all solutions from

https://github.com/saurabh13002/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbf{A}$ be a real symmetric matrix and $\mathbf{B} = \mathbf{I} + i\mathbf{A}$, where $i^2 = -1$. Then

1. $\mathbf{B}$ is invertible if and only if $\mathbf{A}$ is invertible.
2. All eigenvalues of $\mathbf{B}$ are necessarily real.
3. $\mathbf{B} - \mathbf{I}$ is necessarily invertible.
4. $\mathbf{B}$ is necessarily invertible.

2 solution

Given	Let $\mathbf{A}$ be a real symmetric matrix, and $\mathbf{B} = \mathbf{I} + i\mathbf{A}$, where $i^2 = -1$.
Checking Option 1	Lets assume, $\mathbf{A}$ is non invertible, $\implies \det \mathbf{A} = 0$ $\implies \det \mathbf{B} = \det \mathbf{I}$ $\implies$ $\mathbf{B}$ is invertible even if $\mathbf{A}$ is non invertible. since,$\det \mathbf{I} = 1$ Thus Option 1 is incorrect.
Checking Option 2	Eigen values of $\mathbf{B}$ = Eigen values of $\mathbf{I}$ + i (Eigen values of $\mathbf{A}$). Clearly, Eigen values of $\mathbf{B}$ are real only if $\mathbf{A}$, has zero Eigen values, as $\mathbf{A}$ is a real symmetric matrix. Thus, Option 2 is incorrect.

Checking Option 3	$\mathbf{B} - \mathbf{I} = i\mathbf{A}$ $\implies \det(\mathbf{B} - \mathbf{I}) = \det i\mathbf{A} = \det \mathbf{A}$ Hence, $\mathbf{B} - \mathbf{I}$ is invertible only if $\mathbf{A}$ is invertible Thus option 3 is also incorrect.
Checking Option 4 **Property**: $\det \mathbf{B}$ is equals to product of eigen values of $\mathbf{B}$	Let us assume, λ be the eigen value of $\mathbf{A}$, as $\mathbf{A}$ is symmetric matrix. $\implies \lambda \epsilon \mathbb{R}$ Then, $i\lambda$ is an eigen value of $i\mathbf{A}$ $\implies 1 + i\lambda$ is an eigen value of $\mathbf{I} + i\mathbf{A}$ Given, $\mathbf{B} = \mathbf{I} + i\mathbf{A}$ Therefore, $1 + i\lambda$ is an eigen value of $\mathbf{B}$. Hence, 0 can not be the eigen value of $\mathbf{B}$ $\implies \det \mathbf{B} \neq 0$ Therefore, $\mathbf{B}$ is necessarily invertible.
Correct option	The correct option is **4**.