# EE5609: Matrix Theory Assignment-6

Major Saurabh Joshi
MTech Artificial Intelligence
AI20MTECH13002

***Abstract*—This document explains a system of linear equation in two variables with no solutions.**

Download the latex code from

https://github.com/saurabh13002/EE5609/tree/master/Assignment6

## 1 Problem

Give an example of a system of two linear equations in two unknowns which has no solution.

## 2 Solution

Let us assume two equations as given below $\begin{pmatrix}5 & 2\end{pmatrix}\mathbf{x} = 7$ and $\begin{pmatrix}10 & 4\end{pmatrix}\mathbf{x} = -3$

Let the coefficient matrix be given as

$$\mathbf{A} = \begin{pmatrix}5 & 2\\10 & 4\end{pmatrix}, \mathbf{b} = \begin{pmatrix}7\\-3\end{pmatrix} \tag{2.0.1}$$

the augmented matrix be given as matrix be given as

$$\mathbf{A}|\mathbf{B} = \begin{pmatrix}5 & 2 & 7\\10 & 4 & -3\end{pmatrix} \tag{2.0.2}$$

Applying row reduction

$$\begin{pmatrix}5 & 2 & 7\\10 & 4 & -3\end{pmatrix} \xleftrightarrow{R_2=R_2-2R_1} \begin{pmatrix}5 & 2 & 7\\0 & 0 & -17\end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_1=\frac{R_1}{5}} \begin{pmatrix}1 & \frac{2}{5} & \frac{7}{5}\\0 & 0 & -17\end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_2=\frac{R_2}{-17}} \begin{pmatrix}1 & \frac{2}{5} & \frac{7}{5}\\0 & 0 & 1\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_1=R_1-\frac{7}{5}R_2} \begin{pmatrix}1 & \frac{2}{5} & 0\\0 & 0 & 1\end{pmatrix} \tag{2.0.6}$$

$$\tag{2.0.7}$$

Clearly,On comparing the ranks of matrix **A** and **A|B**, we find that rank of matrix $\mathbf{A}|\mathbf{B} \neq \mathbf{A}$ Hence the system of linear equation have no solutions.

## 3 Observation

Consider the system $\mathbf{Ax} = \mathbf{b}$, with coefficient matrix **A** and augmented matrix **A|B**.
As above, the sizes of **b**, **A**, and **A|B** are m × 1, m × n, and m × (n + 1), respectively; in addition, the number of unknowns is n.
**Ax** is inconsistent (i.e., no solution exists) if and only if rank **A** < rank **A|B**.