

✓ 1 / 1  
points

1.

$p(x|\theta)p(\theta)$  is a distribution over:

☒  $(x, \theta)$



**Correct**

$p(x|\theta)p(\theta) = p(x, \theta)$  which is a distribution over vector  $(x, \theta)$

☐  $\theta$

☐  $x$

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points

2.

Choose correct statements:

☐  $p(a|b) = \int p(a|b, c)dc$



**Un-selected is correct**

☐  $p(a|b) = \int p(a, c|b)dc$



**Correct**

The sum rule.

☐  $p(a|b) = \int p(a|b, c)p(c)dc$ , when  $b$  and  $c$  are independent



**Correct**

$p(c|b) = p(c)$  when  $c$  and  $b$  are independent

☐  $p(a, b|c) = p(a|b, c)p(b|c)$



**Correct**

✓ 1 / 1  
points

3.

Choose correct statements:

☐  $p(a | b, c) = p(a | b)p(a | c)$  when  $b$  and  $c$  are independent



Un-selected is correct

☐  $p(a | b, c) = \frac{p(b | a, c)p(a | c)}{\int p(b | a', c)p(a' | c)da'}$



Correct

☐  $p(a | b) = \frac{p(a, c | b)}{p(c | a, b)}$



Correct

Apply Bayes rule to  $p(c | a, b)$

☐  $p(a | b)p(b) + p(a | \bar{b})p(\bar{b}) = p(a)$ , for binary  $b$



Correct

The law of total probability

☐  $p(a | b) + p(a | \bar{b}) = p(a)$ , for binary  $b$



Un-selected is correct

✓ 1 / 1  
points

4.

Let joint probability over random variables  $a, b, c$  be  $p(a, b, c) = p(a|b)p(b|c)p(c)$ . Are random variables  $a$  and  $c$  independent?

☐ Yes

☒ No



Correct

Let's marginalize joint probability by  $b$  and we get  $\int p(a, b, c)db = \int p(a|b)p(b|c)p(c)db = p(c) \int p(a|b)p(b|c)db$ . Unfortunately, integral contain inside both  $a$  and  $c$  and it can't be decomposed into two integrals  $\int f(a, b)db$  and  $\int g(c, b)db$ , so  $a$  and  $c$  is dependent.

✓ 1 / 1  
points

5.

Let joint probability over random variables  $a, b, c, d$  be  $p(a, b, c, d) = p(a|b)p(b)p(c|d)p(d)$ . Are random variables  $a$  and  $c$  independent?

☐ Yes



**Correct**

Let's marginalize joint probability by  $b$  and  $d$ , so we get  $\int p(a, b, c, d) db dd = \int p(a|b)p(b)p(c|d)p(d) db dd = \left( \int p(a|b)p(b) db \right) \left( \int p(c|d)p(d) dd \right)$ . So we decomposed it into two integrals  $\int f(a, b)db$  and  $\int g(c, d)dd$ , so  $a$  and  $c$  is independent.

☐ No

✓ 1 / 1  
points

**6.**

Recall the probabilistic regression setting. In the lecture, we have proved that solving the least-squares problem with L2 regularizer  $L(w) = \sum_{i=1}^n (w^T x_i - y_i)^2 + C \sum_{i=1}^N w_i^2$  is equivalent to finding the MAP estimate for  $w$  with prior distribution  $\mathcal{N}(w | 0, \gamma I)$ . Let us now choose a prior distribution to be Laplace distribution:  $p(w|0, b) = \frac{1}{(2b)^n} \prod_{i=1}^n \exp\left(-\frac{|w_i|}{b}\right)$  instead of Normal. Adding which of the following regularizers to the least-squares problem is equivalent to finding a MAP-estimate for such a model?

☐  $\sum_{i=1}^N w_i^{\frac{1}{2}}$

☒  $\sum_{i=1}^N |w_i|$

**Correct**

Let's get minus logarithm of Laplace distribution and we will get  $C \sum_{i=1}^n |w_i| + D$ , where  $C$  and  $D$  are some constants. We can forget about  $D$  constant because it's not important when we will minimize loss function.

☐  $\sum_{i=1}^N \frac{1}{|w_i|}$

☐  $\sum_{i=1}^N w_i$

✓ 1 / 1  
points

**7.**

For linear regression with loss function  $L(w) = \sum_{i=1}^n (w^T x_i - y_i)^2 + C \sum_{i=1}^N w_i^2$  prior distribution for weights  $w$  was Normal distribution  $\mathcal{N}(w|0, \gamma I)$ . Which prior distribution on weights is right for loss function  $L(w) = \sum_{i=1}^n (w^T x_i - y_i)^2$  if each component of weights should be in some predefined range:  $w_i \in [l, r]$ ?

☒ Uniform distribution with the same limits for each component.

$$\begin{cases} \frac{1}{(b-a)^n}, & \text{if } \forall w_i \in [l, r] \\ 0, & \text{else} \end{cases}$$

$$p(w|a, b) = \frac{1}{(b-a)^n}$$

$$1, \text{ if } \forall w_i \in [l, r] \text{ else } 0.$$

**Correct**

We can formulate question using equivalent loss function  $L(w) = \sum_{i=1}^n (w^T x_i - y_i)^2 + \sum_{i=1}^N r(w_i)$  without limits on weights component but function

$$\begin{cases} C, & \text{if } w_i \in [l, r] \\ +\infty, & \text{else} \end{cases}$$

$r(w_i) = \begin{cases} C, & \text{if } w_i \in [l, r] \\ +\infty, & \text{else} \end{cases}$ . The same regularisation we will have if we find minus logarithm of Uniform distribution.

- ☐ Laplace distribution with zero mean and the same divergence for each component.

$$p(w | 0, b) = \frac{1}{(2b)^\alpha} \prod_{i=1}^n \exp^{-\frac{|w_i|}{b}}$$

- ☐ Gamma distribution with the same parameters for each component.

$$p(w | \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \prod_{i=1}^n w_i^{\alpha-1} \exp^{-\beta w_i}$$



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8.

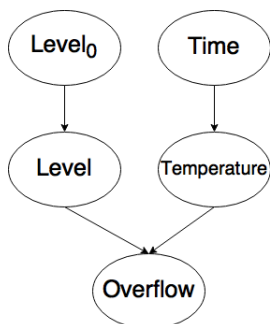
For the **remaining** problems we will use the following story:

You have a kettle that boils water. You pour water up to level  $L_0$  and turn the kettle on. Over time, temperature  $Temp$  starts to increase. At time  $T$ , level of water is  $L$ . Since water is boiling, water level slightly oscillates and so can be considered random. You also know that the height of a kettle is limited. If at some point water level exceeds this value, water will split on a table. We will denote this event as a binary random variable  $O$  (overflow). Our goal is to determine the maximum allowed initial water level  $L_{max}$  so that we can write it down in a kettle manual. Normally we would like to find  $L_{max}$  for which, for example,  $P(O | L_0 = L_{max}) = 0.001$  if you pour this amount of water, overflow will occur with a fairly low probability.

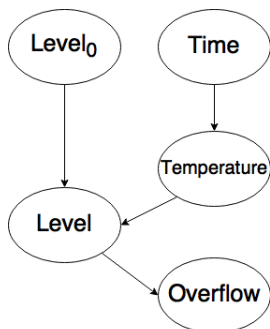
In these tasks we will construct a Bayesian network and select probability distributions needed for the model.

Our first step is to choose the correct Bayesian network.

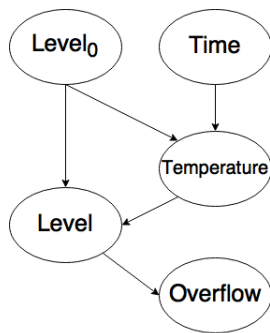
- ☐ a)



- ☐ b)

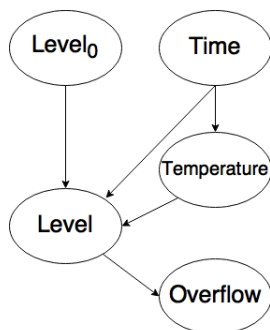


- ☒ c)



Correct  
Correct!

☐ d)



✓ 1 / 1  
points

9.

Write joint distribution for this situation.

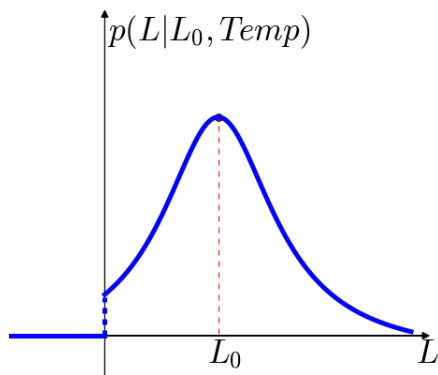
- ☐  $p(L_0|L, Temp)p(T|Temp)p(Temp|L)p(L|O)p(O)$
- ☒  $p(O|L)p(L|L_0, Temp)p(Temp|L_0, T)p(L_0)p(T)$

Correct

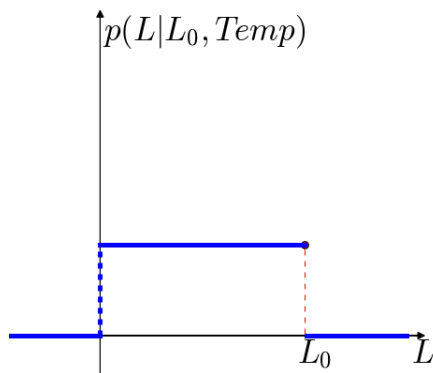
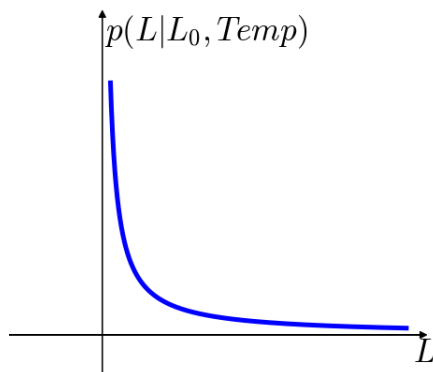
- ☐  $p(L|O)p(Temp|L_0, L)p(Temp|L_0, T)p(L_0)p(T)$
- ☐  $p(O|L)p(L|L_0, Temp)p(L_0, T|Temp)p(Temp)$

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points

10.

Which distribution can you use for  $p(L|L_0, Temp)$ ?☒ a)

Correct

☐ b)☐ c)

☐ d)

