Course Instructor: **Prof. Sukumar Srikant**
Modeling Flexible Spine Quadruped

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SC618: Analytical and Geometric Dynamics

This report documents the development of a Simscape model for a flexible spine quadruped, completed through a staged process to build and refine the model's complexity. The aim was to simulate realistic motion and analyze the dynamics and trajectories characteristic of quadrupedal locomotion.

Contents

1 Introduction

The investigation of legged locomotion, particularly in quadruped animals, has gained significant traction due to its potential to inform robotic designs that are both efficient and adaptive. Understanding the biomechanics of quadrupedal movement offers critical insights that can influence the development of robots capable of navigating complex terrains with agility and stability. Quadruped animals, such as cheetahs, demonstrate remarkable adaptations for high-speed movement, employing body flexibility to extend stride length and distribute mechanical forces efficiently. This flexibility reduces peak ground reaction forces (GRF), thereby protecting limbs and minimizing energy expenditure during rapid acceleration and deceleration.

Traditional robotic models often utilize a rigid-body framework when studying gait dynamics. While this approach allows for simplified analysis of motion and control, it fails to capture the nuanced impact of body compliance on performance metrics such as stability, energy efficiency, and load distribution. Observational studies of animal locomotion have long suggested that flexible spines contribute to smoother gait transitions and improved shock absorption, key factors in sustaining high-speed movement over varying terrain.

This report extends findings by presenting a comprehensive Simulink-based simulation study, focusing on the role of body flexibility in the bounding gait of quadrupeds. The study aims to, offer detailed insights into how varying torsional stiffness and compliance levels influence gait characteristics and ground interactions.

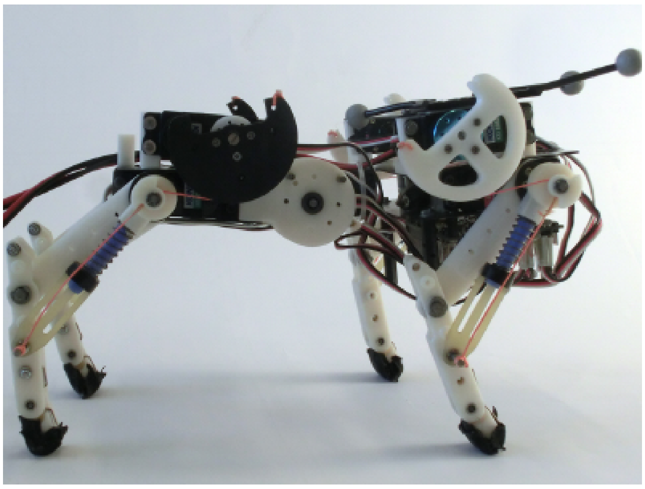

Figure 1: A quadruped robot with flexible spin

2 Model Overview

We used two primary models to analyze the effect of body flexibility on quadruped locomotion:

- **Rigid Body Model**: This simplified model represents the quadruped torso as a single, non-flexible segment. It provides a baseline to evaluate the performance of a standard rigid structure during bounding gaits. The rigid model allows for the analysis of energy transfer and ground reaction forces without any internal movement between body segments

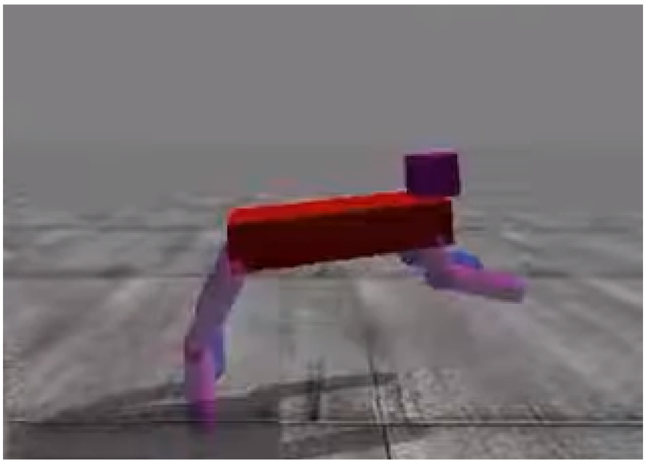

Figure 2: Rigid Body Model

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- **Flexible Body Model**: This more sophisticated model features a torso divided into two segments connected by a torsional spring-damper joint. This joint simulates body flexibility and enables relative rotation between the segments, reflecting how certain quadruped animals use their spines to enhance stride efficiency and adjust to changing terrains. The spring-damper component models the torsional stiffness k_t and damping effects that contribute to energy absorption and distribution during movement.

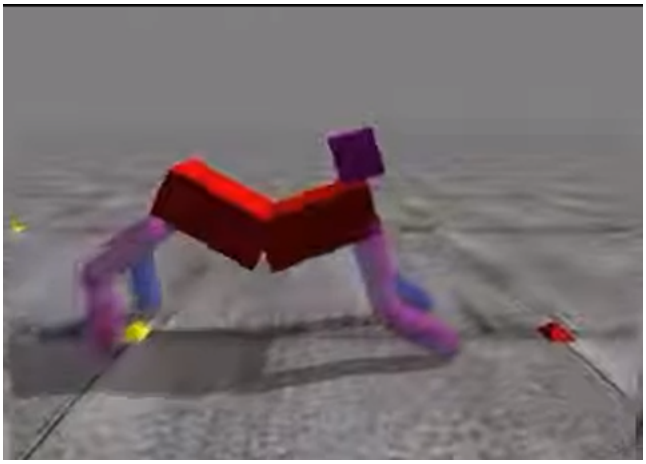

Figure 3: Flexible Body Model

These models are inspired by prior research that demonstrated the significant advantages of incorporating body flexibility into legged robots. Studies have shown that body compliance can improve gait stability, reduce mechanical stress on limbs, and enhance overall energy efficiency. This Simulink study aims to validate these benefits through comparative analysis between rigid and flexible body configurations.

Both models were configured with parameters such as body mass distribution, joint stiffness, and leg spring constants, which align with established biomechanical data to ensure realistic simulation outcomes.

3 Components and Simulation Setup

3.1 Components

The Simulink models were developed using the following core components:

- **Rigid Segment Blocks**: Represent the main body structure in the rigid model, capturing the simplified behavior of a non-flexible torso.
- **Torsional Spring-Damper System**: A key feature in the flexible model that connects the fore and hind body segments. This system simulates rotational compliance, enabling the body to absorb energy and facilitate adaptive motion during bounding.
- **Rotational and Translational Joints**: Simulate realistic leg movements and joint kinematics, allowing for the analysis of limb articulation and ground contact dynamics.
- **Ground Contact Mechanism**: Incorporates ground reaction forces, simulating the interaction between the legs and the surface. This component is essential for studying how GRFs vary with body flexibility and leg positioning.

3.2 Mathematical Formulation

The equations governing the models were derived from kinetic and potential energy principles:

- **Rigid Model**: The dynamics are determined by the variables θ (pitch angle), x (horizontal position), and y (vertical position) of the center of mass (COM). The absence of internal body flexibility simplifies the energy calculations, focusing primarily on external forces and body motion.
- **Flexible Model**: The formulation includes θ_1 and θ_2, representing the pitch angles of the fore and hind body segments. The energy equations incorporate torsional spring energy, defined by the spring constant k_t, which affects how energy is distributed during movement. Potential and kinetic energies were combined to model the oscillatory behavior induced by body compliance.

3.3 Simulation Parameters

- **Segment Mass (m)**: Set at 10.4 kg per segment to represent a realistic distribution of weight.
- **Moment of Inertia (I)**: Calculated as 0.36 kg·m^2, representing the resistance to rotational motion around the COM.
- **Torsional Spring Constant (k_t)**: Varied between 200 and 260 Nm/rad to test the impact of different levels of body stiffness on performance.
- **Leg Spring Constant (k_l)**: Fixed at 6340 N/m to simulate the compliance of the legs, aiding in energy absorption and propulsion.
- **Nominal Leg Length ($\bar{l}$)**: Defined as 0.36 m, providing a consistent base length for leg calculations.

These parameters and components collectively enabled the simulation of realistic quadruped movement, allowing for in-depth analysis of how body flexibility influences gait mechanics and energy efficiency.

4 Setup

The model was designed to replicate realistic bounding gait phases, which include:

- **Double-leg Flight (DF)**: The phase during which all legs are off the ground.
- **Fore-leg Stance (FS)**: When only the fore legs are in contact with the ground.
- **Hind-leg Stance (HS)**: When only the hind legs are in contact with the ground.
- **Double-leg Stance (DS)**: Both fore and hind legs are in contact with the ground simultaneously.

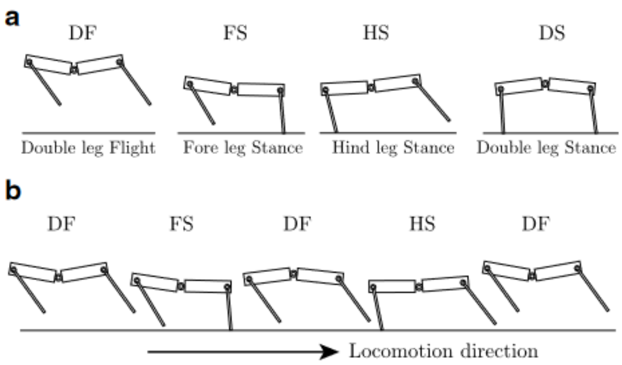

Figure 4: Four phases for the stance condition, DF, HS, FS, and DS

These phases were programmed to simulate transitions in a bounding gait cycle to ensure that the motion dynamics closely mimic biological observations.

4.1 Simulation Configuration

Initial Conditions: The simulations were initialized at a standard apex height, ensuring consistent starting positions across trials. This apex height reflects the position where the vertical velocity of the center of mass is momentarily zero, a critical point in the bounding gait cycle.

Solver Settings: The simulations utilized the ODE45 solver, chosen for its adaptive step-sizing capability, which enhances computational efficiency while maintaining precision. This solver is well-suited for handling the non-linear dynamics and hybrid systems typical of biomechanical simulations involving alternating contact phases. Additional parameters, such as error tolerances, were fine-tuned to ensure accurate results without excessive computational load.

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5 Modeling

A variety of gaits exist for a four-legged robot. In these gaits, there is a possibility that at each time step, one, two, three, or four legs will be on the ground. So in this paper, we first model the free system, which includes the robot in the flight mode, and then we add the constraints according to the number of legs that are on the ground. There are numerous methods for modeling dynamical systems, such as Newton-Euler formulation, Hamiltonian, and Lagrangian approaches. In this paper, we model a quadruped robot using the **Lagrangian Formulation**. The modeling procedure includes the following steps:

1. Deriving the Lagrangian and extracting the equations of motion.
2. Adding the constraints according to the number of legs on the ground.
3. Contact modeling when a foot collides with the ground.

To check the authenticity of the derived equations of motion, the MATLAB SimMechanics toolbox is used. To verify the equations, the pinned model of the system (one foot stuck to the ground) was released during simulation by the toolbox, and another time, the equations of motion were integrated in time to produce the motion. Both results were equal. Thus, it was concluded that the equations were obtained correctly. In the subsequent sections, we will present the details of each of these steps.

A. Unconstrained Dynamics Extraction

In the first stage, we choose a set of generalized coordinates that fully specify the position of the system as shown in Fig. 1 in the flight mode, which includes the full dynamics of the system. To utilize the Lagrangian Formulation, we need to write down the kinetic and potential energies based on the generalized coordinates. To do so, we first derive the positions of the center of mass (CM) for each of the robot's links:

$$P_{cm_i} = \begin{bmatrix} X_{cm_i} \\ Y_{cm_i} \end{bmatrix} = f_i(q) \tag{1}$$

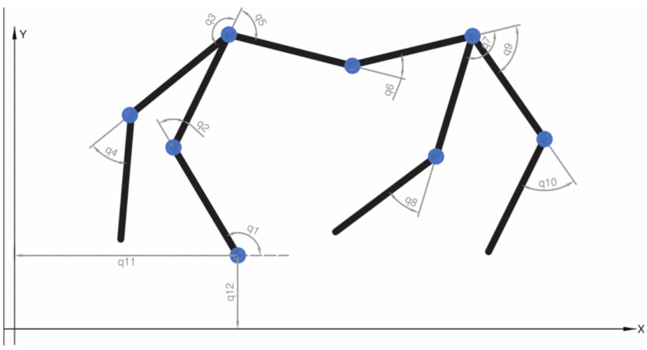

Figure 5: The generalized coordinates for the quadruped robot

where P_{cm_i} is the position of the ith center of mass and $f_i(q)$ represents the forward kinematics equations. q is a vector containing generalized coordinates:

$$q_s = [q_i, q_2, \dots, q_8, q_9, q_{10}]^T \tag{2}$$

$$q = [q_s; q_{11}; q_{12}] \tag{3}$$

Here, $q_i, i = 1, \dots, 10$ are the robot's relative joint angles. Additionally, q_{11} and q_{12} represent the horizontal and vertical positions of the first leg's tip, respectively. Taking the time derivative of the positions results in their linear velocity:

$$V_{cm_i} = \dot{P}_{cm_i} = \frac{\partial P_{cm}}{\partial q}\dot{q} \tag{4}$$

In the above equation, $\dot{q}$ is the time derivative of each joint's angle. The angular velocity of each link, ω_i, is the sum of all angular velocities that account for the rotation of the ith link.

Then the Kinetic energy, T, is derived as follows:

$$T = \sum_{i=1}^{10} T_i \tag{5}$$

$$T_i = \frac{1}{2} m_i V_{cm_i}^2 + \frac{1}{2} I_i \omega_i^2 \tag{6}$$

where I_i is the moment of inertia for each link. The Potential energy, U, is derived as follows:

$$U = \sum_{i=1}^{10} m_i g Y_{cm_i} \tag{7}$$

Now that we have both Kinetic and Potential energies, we may derive and form the Lagrangian, L:

$$L = T - U \tag{8}$$

The equations of motion are derived as follows:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \tag{9}$$

Here, τ_i is the generalized force, which is the torque applied by the motor to that link.

B. Constraint Modeling and Solving the Equations of Motion

Knowing the unpinned model of the system in the flight mode is advantageous, although it will be computationally more expensive to solve the equations of motion. When the system is modeled with constraints, it becomes solvable by adding a few terms and solving the equations mathematically. For example, consider the case when only the first leg is on the ground. The velocities of that leg along horizontal and vertical directions are zero. These constraints can be written as:

$$J_1 \dot{q} = 0 \tag{10}$$

where J_1 is a matrix that corresponds to the linear Jacobian of the first leg:

$$J_1 = \begin{bmatrix} 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & 0 & 1 \end{bmatrix}_{2\times 12} \tag{11}$$

Each leg on the ground adds two constraints, forming the augmented Jacobian matrix for all legs:

$$J = \begin{bmatrix} J_1^T \\ \vdots \\ J_N^T \end{bmatrix}_{2N\times 12} \tag{12}$$

where N is the number of feet on the ground. The final form of the constraints is:

$$J\dot{q} = 0 \tag{13}$$

For the constrained system, the equations of motion are:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + J^T F \tag{14}$$

$$J\ddot{q} = -\dot{J}\dot{q} \tag{15}$$

Here, F is a vector representing the ground reaction forces in horizontal and vertical directions.

To solve these equations numerically, we use:

$$\begin{bmatrix} D & -J^T \\ J & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -C(q,\dot{q})\dot{q} - G + \tau \\ -\dot{J}\dot{q} \end{bmatrix} \tag{16}$$

C. Contact Modeling

A normal quadruped robot that walks is a model that switches between different phases of a certain gait. So far, we have introduced what happens when a robot is in each phase. Now, we determine the effect of ground contact on the system's states. Suppose there are a number of feet on the ground; for instance, the third and fourth legs strike the ground. In this small time, large forces are applied to the system, which can be considered as impulse forces. Consequently, the velocities will change discretely. Considering a short time before and after collision, we have:

$$\int_{t^-}^{t^+} \left(D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)\right) dt = \int_{t^-}^{t^+} \left(\tau + J^T F\right) dt \tag{17}$$

Since positions do not change rapidly, the integral simplifies to:

$$D(q)\dot{q}^+ - D(q)\dot{q}^- = J^T F_{\text{integrated}} \tag{18}$$

Considering the constraint equations, finding $\dot{q}^+$ from the above and solving for both $F_{\text{integrated}}$ and $\dot{q}^+$ results in:

$$F_{\text{integrated}} = -\left(JD(q)^{-1}J^T\right)^{-1} J\dot{q}^- \tag{19}$$

$$\dot{q}^+ = D(q)^{-1}\left(J^T F_{\text{integrated}}\right) + \dot{q}^- \tag{20}$$

The only key point is that the ground normal forces must be positive. If any normal force turns out negative, the corresponding constraint should be removed, and the equations must be solved again with the updated conditions.

Nonlinear Constrained Control

In this section, assuming we have a trajectory for each joint, we design a controller to reduce the tracking error exponentially. Considering the reference trajectory for actuated joints as $q_b(t)$, the tracking error is:

$$e = B^T q - q_b(t) \tag{21}$$

The considered gait is a slow trot in which, during each phase, two legs are on the ground. This results in four constraint equations per phase, leaving only 8 coordinates of the robot controllable (active joints). The matrix B maps $\tau_1, \ldots, \tau_8$ to their corresponding coordinates: $q_2, q_3, q_4, q_5, q_7, q_8, q_9, q_{10}$. Other joints are passive and have no actuators. By taking the first and second derivatives of e, we get:

$$\dot{e} = B^T\dot{q} - \dot{q}_b(t) \tag{22}$$

$$\ddot{e} = B^T\ddot{q} - \ddot{q}_b(t) \tag{23}$$

The error dynamics are desirable as they decrease exponentially:

$$\ddot{e} + K_d\dot{e} + K_p e = 0 \tag{24}$$

Substituting (23) into (24):

$$B^T\ddot{q} - \ddot{q}_b(t) + K_d\dot{e} + K_p e = 0 \tag{25}$$

However, directly adding this to the solver matrix in Equation (16) results in a singular matrix. Therefore, the equation is rewritten as:

$$B^T D(q)^{-1}\left(-C(q,\dot{q})\dot{q} - G(q) + B\tau + J^T F\right) - \ddot{q}_b(t) + K_d\dot{e} + K_p e = 0 \tag{26}$$

Now, the required torque to achieve the desired dynamics must be obtained. Since F is unknown, these equations are not directly solvable. However, by combining them with the previously derived equations, all desired variables, including $\ddot{q}$, F, and τ, can be solved. Rewriting the equations yields:

$$\begin{bmatrix} D(q) & J^T & 0 \\ J & 0 & B^T \\ 0 & B & B^T D(q)^{-1} B \end{bmatrix} \begin{bmatrix} \ddot{q} \\ F \\ \tau \end{bmatrix} = \begin{bmatrix} -C(q,\dot{q})\dot{q} - G(q) \\ -\dot{J}\dot{q} \\ -K_p e - K_d \dot{e} + \ddot{q}_b + B^T D(q)^{-1} \left(C(q,\dot{q})\dot{q} + G(q) \right) \end{bmatrix} \tag{27}$$

To achieve the desired tracking performance, this linear system of equations is solved and integrated. The tracking error will asymptotically converge to zero.

Changing Ground Reaction Forces Using Torques in Zero Velocity

To change ground reaction forces with zero acceleration, torques must be adjusted. Based on the constraint equations:

$$J\ddot{q} + \dot{J}\dot{q} = 0 \tag{28}$$

Substituting $\ddot{q}$, we get:

$$JD^{-1}\left(-G + J^T F + B\tau\right) + \dot{J}\dot{q} = 0 \tag{29}$$

For $\dot{q} = 0$:

$$JD^{-1}J^T F = JD^{-1}\left(B\tau - G\right) \tag{30}$$

$$F = \left(JD^{-1}J^T\right)^{-1} JD^{-1}\left(B\tau - G\right) \tag{31}$$

If zero acceleration is desired for all generalized coordinates:

$$G - \tau = J^T F \tag{32}$$

Substituting (31) into (32):

$$\left(G - B\tau\right)\left(I - J^T\left(JD^{-1}J^T\right)^{-1} JD^{-1}\right) = 0 \tag{33}$$

There are two possible cases for the above equation:

1. $G - B\tau = 0$: This suggests that vector G must be eliminated using $B\tau$, which is typically infeasible due to actuator limitations.
2. $G - B\tau$ lies in the null space of $\left(I - J^T\left(JD^{-1}J^T\right)^{-1} JD^{-1}\right)$.

If vectors satisfying the second case are found, any linear combination of them satisfies the equation:

$$\tau = \alpha(\tau_2 - \tau_1) + \tau_1 \tag{34}$$

where α is a scalar.

6 Simulation Results for Different Scenarios

6.1 Rigid Model Baseline

The rigid body model simulations served as a control, displaying the following characteristics:

- **COM Trajectory**: The center of mass (COM) exhibited noticeable vertical oscillations throughout the gait cycle. These oscillations highlighted the limitations of a non-compliant body in dissipating forces effectively, resulting in more abrupt and high-magnitude movements during stance phases.

- **Ground Reaction Force (GRF)**: The GRFs recorded in the rigid model simulations showed significant peak forces during the stance phases. These peaks indicated a sharp transfer of energy between the robot and the ground, consistent with the absence of body compliance and limited capacity for energy distribution [**?**].

6.2 Flexible Body Model Analysis

The flexible body model simulations revealed several advantageous characteristics compared to the rigid model:

- **Reduced GRFs**: The maximum GRF observed in the flexible body model was consistently lower than in the rigid body model. This suggested that the compliant joint effectively dissipated forces more evenly, reducing the impact stress on the limbs during stance phases.

- **Energy Redistribution**: The integration of a torsional spring-damper joint in the flexible model enabled energy to be temporarily stored and released, smoothing vertical oscillations and reducing gravitational potential energy at the apex. This led to a more fluid and energy-efficient motion.

6.3 Simulation Plots

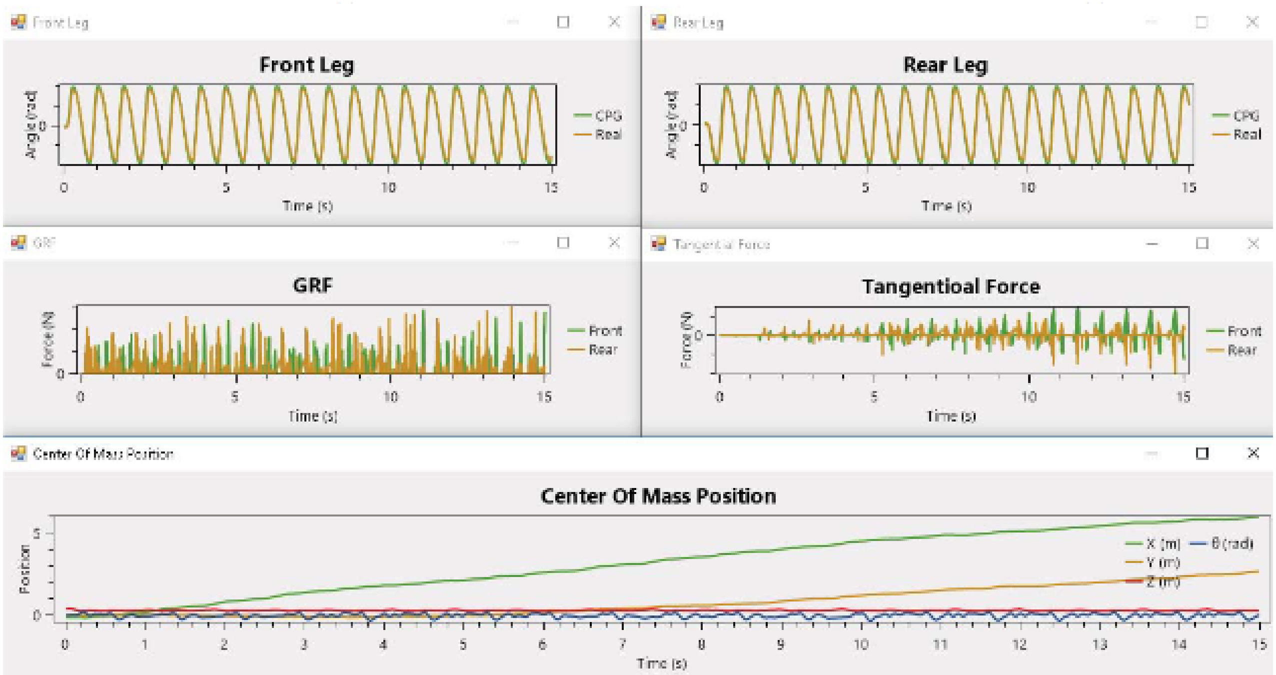

Figure 6: The desired and real CPG trajectories for front and rear legs are shown in the top plots. In the middle row, the ground reaction force (in vertical direction) and tangential force are shown for the front and rear legs. Due to the symmetric in the structure, the results are only shown for one of the front and rear legs. The last plot in the bottom row, indicates the position of center of mass in x y z directions and the alignment of the body in proportion to the horizontal line

6.4 Comparative Analysis

- kt Dependence The analysis revealed a clear dependence of the GRF on the torsional spring constant k_t. As k_t increased, the peak GRFs decreased, which corresponds to better energy absorption and smoother transitions between gait phases. However, beyond a certain point, further increases in k_t led to diminishing returns, with no significant reduction in GRFs observed. This suggests that while body flexibility plays a crucial role in improving performance, there is an optimal range of flexibility, and excessive stiffness may result in less adaptive movement, similar to a more rigid body configuration.

- Stability Observations One of the most notable findings from the simulations was the improvement in dynamic stability at moderate levels of k_t. When k_t was tuned to a value that allowed for sufficient flexibility without excessive compliance, the model showed enhanced stability in the bounding gait. This result aligns with biological studies of quadrupedal animals like cheetahs, which utilize spine flexibility to maintain stable and efficient locomotion. Moderate flexibility allows the body to better absorb and distribute forces, leading to smoother transitions and reduced mechanical stress on the limbs.

7 Discussion

7.1 Key Findings

- **Flexibility Advantages**: The introduction of body flexibility significantly reduced peak GRFs, enhancing gait efficiency and minimizing the risk of mechanical wear on robotic joints and limbs. This confirms previous findings that body compliance improves performance by aiding energy absorption and reducing abrupt force transfers between the body and the ground.

- **Trade-offs**: While body flexibility offers substantial benefits, there are trade-offs to consider. Excessive stiffness, leading to a 'rigid-like' behavior, diminishes the advantages of flexibility, as the robot would fail to properly absorb and redistribute energy. On the other hand, excessive compliance, or too much flexibility, may lead to increased energy losses, as the body may oscillate excessively or fail to maintain stable motion.

- **Biological Correlation**: The simulation results align with the hypothesis that certain quadrupedal animals, such as cheetahs, use spine flexibility to optimize stride length and minimize foot loading. The flexible model demonstrated reduced GRFs and more fluid energy distribution, reflecting how real-world animals use spinal compliance to improve speed and efficiency during high-speed movement over varied terrain [?] [?].

8 Conclusion

The Simulink-based simulations effectively modeled the impact of body flexibility on quadruped locomotion, validating prior findings and offering new insights into how flexibility influences gait dynamics and ground interactions. The study confirmed that moderate body compliance, particularly through torsional spring-damper systems, can significantly improve energy efficiency, reduce ground reaction forces, and enhance overall gait stability.

Future work will focus on further optimizing the torsional spring-damper system and investigating how adaptive control strategies could dynamically alter stiffness during different phases of the gait cycle. This would allow legged robots to adapt to varying terrain conditions, more closely mimicking biological locomotion and enhancing real-world applicability.

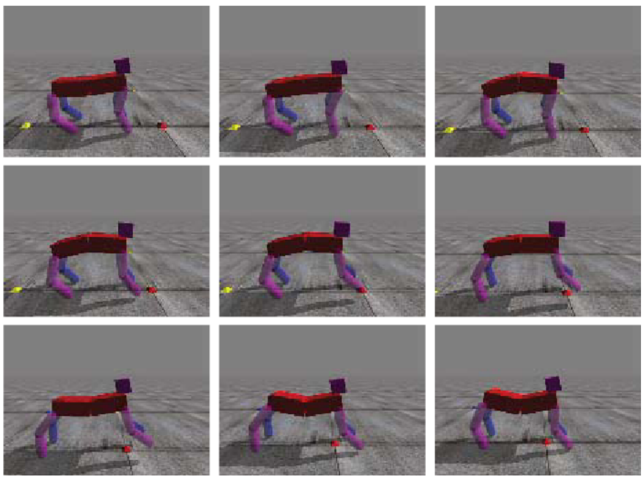

Figure 7: Robot's movement in a bounding cycle for a sample simulation. The above frames are from left to right and up to down

9 Appendices

Attached Files:

- Simulation models (.slx) for the body configurations.
- Raw data outputs, including COM trajectories and GRF time series, are available for detailed review and further analysis.

10 References

1 Kamimura, T., et al. (2015). *Body flexibility effects on foot loading based on quadruped bounding models.*

2 IEEE Robotics Letters (2018). *Body flexibility and quadruped running.*

3 Related analytical and simulation studies on flexible body dynamics.