Tutorial-5

Q.1   For an LTI system

$$y(t) = x(t) * h(t),$$

Let $FT\{y(t)\} = Y(f)$, $FT\{x(t)\} = X(f)$ and $FT\{h(t)\} = H(f)$. Show that if we take Fourier Transform (FT) on both sides then,

$$Y(f) = FT[x(t) * h(t)] = X(f)H(f).$$

(Note: In general, if two signals are convolved in time, then they are multiplied in frequency)

$$Y(f) = FT[x(t) * h(t)] = X(f)H(f),$$

hence

$$y(t) = F^{-1}\{X(f)H(f)\}.$$

i.e. $y(t)$ can be obtained by taking Inverse Fourier Transform of the product of FT of $x(t)$ and FT of $h(t)$.

Q.2   Find the frequency response of the Low pass filter (first order system) by taking the Fourier Transform on the differential equation relating input and output.
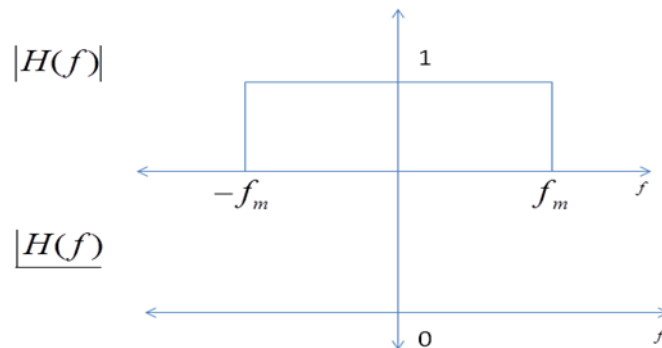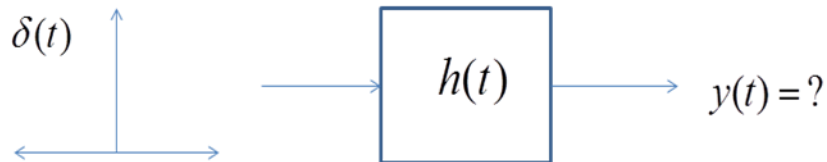
Q.3   Obtain the output of the system given in Q.2 (take RC=1) to the following inputs:

a) $x_1(t) = 10\sin\left(2\pi\left(\dfrac{1}{4\pi}\right)t\right) + 20\cos\left(2\pi\left(\dfrac{1}{10\pi}\right)t\right).$

b) $x_2(t) = 10\sin(2\pi(8\pi)t) + 20\cos(2\pi(10\pi)t).$

c) What is the conclusion that one can draw from this problem?

Q.4 Given magnitude and phase response of the system, an impulse is given as an input to the system(here $f_m = \dfrac{1}{2\pi}$),

$\delta(t)$

$h(t)$

$y(t) = ?$

$|H(f)|$

1

$-f_m$

f_m

f

$\underline{H(f)}$

0

f

a) What is $X(f)$?

b) What is $Y(f)$ and $y(t)$? Is it a causal system?

c) How many frequencies are present in $x(t)$?

d) What is the range of frequencies of $y(t)$ and how many frequencies are present in $y(t)$?

e) Given two signals as inputs to the system, Find the output.

$$x_1(t) = 10\sin\left(2\pi\left(\frac{1}{4\pi}\right)t\right) + 20\cos\left(2\pi\left(\frac{1}{10\pi}\right)t\right)$$

$$x_2(t) = 10\sin\left(2\pi(8\pi)t\right) + 20\cos\left(2\pi(10\pi)t\right)$$

f) What is the conclusion?