Tutorial-4

Q.1 Prove that the area of a Gaussian function given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}, \forall x$$

is unity.

Q.2 Given that $H(f) = \int_{-\infty}^{+\infty} h(t) e^{-j2\pi ft} dt$, where $h(t)$ is a function of time and $H(f)$ is a function of frequency. Show that $h(t) = \int_{-\infty}^{+\infty} H(f) e^{j2\pi ft} df$. Here $h(t)$ and $H(f)$ are the Fourier Transform pair. This applies to signals as well i.e. $x(t)$ is a signal in time domain, then its fourier transform is $X(f)$. These are written as below:

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

Q.3 A signal $A\sin(2\pi f_1 t + \phi_1)$ is applied as input to an LTI system.

a) Derive the expression for the output.

b) Write down the frequency of the input.

c) Write down the frequency of the output.

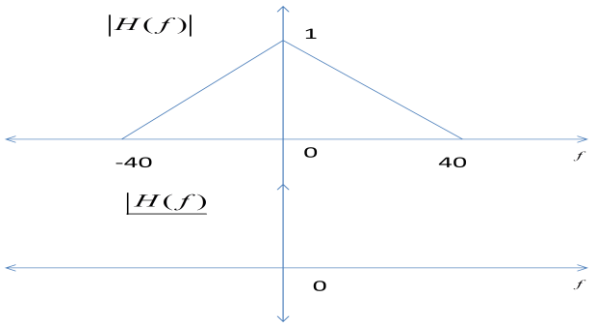
d) Write the amplitude and the phase of the output.

What is the conclusion one can draw from this problem?

Q.4 A signal

$$x(t) = 10\sin(2\pi 10t + 45^\circ) + 20\sin(2\pi 20t + 90^\circ)$$

is applied as an input to an LTI system which has the following magnitude response ($|H(f)|$ vs. f) and the phase response ($\underline{|H(f)|}$ vs. f or $\theta(f)$ vs. f)



Write the output response of the system $y(t)$.

Q.5 A signal $x(t) = 10\cos(2\pi 50t)$ is applied as an input to an LTI system. The system behaviour is such that it passes 50 Hz signal without any attenuation i.e. $|H(f)|_{f=50} = 1$. But it changes phase component of $x(t)$, i.e. $\underline{|H(f)|}_{f=50} \neq 0$. Let $\underline{|H(f)|} = -\frac{\pi}{2}$.

a) What is the delay introduced by the system on the 50 Hz signal.

b) Draw the input and Output waveform.