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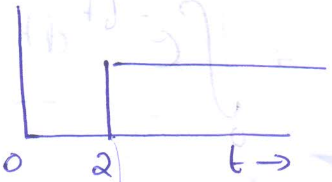
Q. (g)

$\int_{-\infty}^{+\infty} |h(t)|\, dt < +\infty$

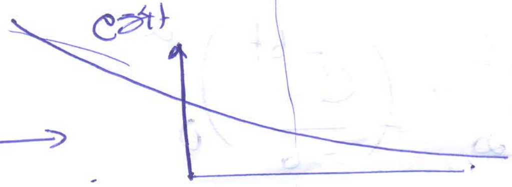
$\boxed{h(t) = 0 \quad t<0}$

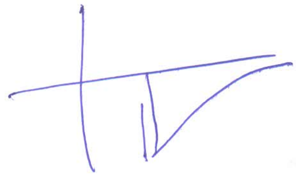
a) $h(t) = e^{-4t} u(t-2)$

$u(t-2) \rightarrow$

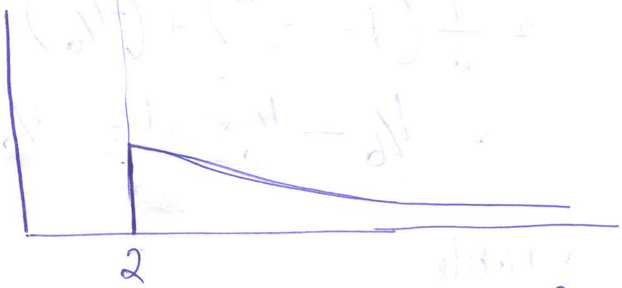


$e^{-4t} \longrightarrow$





$h(t) \longrightarrow$



$h(t) = 0$ for $t<0$ $\longrightarrow$ so causal

$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{2}^{\infty} e^{-4t} dt = \left(\frac{e^{-4t}}{-4}\right)_2^{\infty} = \frac{0 - e^{-8}}{-4} = \frac{e^{-8}}{4} < \infty$

$\longrightarrow$ so stable

$h(-1) = e^{8} u(-1-2) = e^{8} u(-3) = 0 \quad \because (u(-3) = 0)$

$h(0) = e^{0} u(0-2) = \underline{\underline{0}} \quad \because (u(-2) = 0)$

b) $h(t) = e^{-6|t|}$

$h(0) = 1$

$h(-1) = e^{-6 \times |-1|} = e^{-6}$

so $h(t) \neq 0$ for $t<0 \Rightarrow$ Non causal.

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{0} e^{-6(-t)}\,dt + \int_{0}^{\infty} e^{-6t}\,dt$$

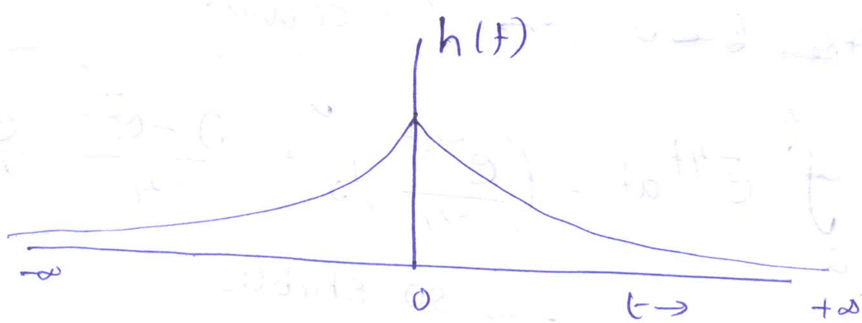
$$= \int_{-\infty}^{0} e^{6t}\,dt + \int_{0}^{\infty} e^{-6t}\,dt$$

$$= \left(\frac{e^{6t}}{6}\right)_{-\infty}^{0} + \left(\frac{e^{-6t}}{-6}\right)_{0}^{\infty}$$

$$= \frac{1}{6}(1 - e^{-\infty}) + (-1/6)(e^{-\infty} - e^{0})$$

$$= 1/6 - 1/6 \times -1 = 1/6 + 1/6 = \underline{\underline{1/3 < \infty}}$$

Stable



c) $h(t) = e^{2t} u(-1-t)$

$h(t)$ for $t < 0$

Let $h(-2) = e^{2\times -2} u(-1-(-2))$

$= e^{-4} u(-1+2) = e^{-4} u(1) \neq 0$

Non causal.

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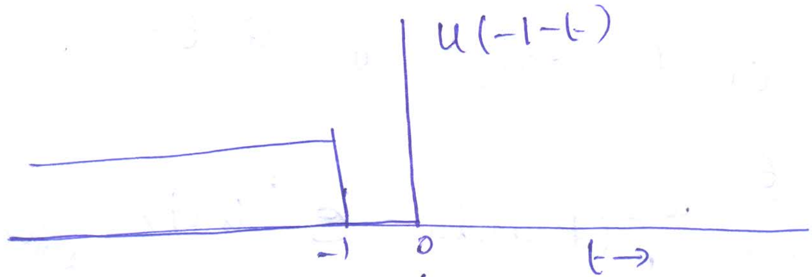

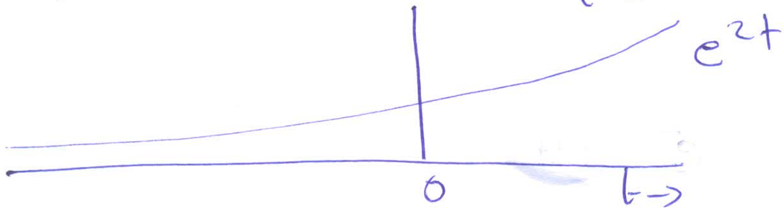

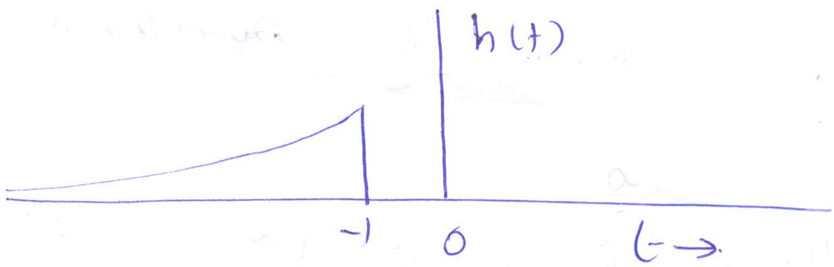

→ $h(t) \neq 0$ for $t < 0$ ⇒ Non causal.

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{-1} e^{2t}\,dt = \left(\frac{e^{2t}}{2}\right)_{-\infty}^{-1}$$

$$= \frac{1}{2}\left(e^{-2} - e^{-\infty}\right)$$

$$= \underline{\underline{e^{-2}/2}} < \infty.$$

<u>Stable</u>

d) Here the respons for $x(t)$ is given. So we need to find the Impulse Response. So replace $x(t)$ by $\delta(t)$ and $y(t)$ by $h(t)$

$$h(t) = \int_{0}^{t} e^{-a}\,\delta(t-a)\,da$$

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$\delta(t-a)$ exists only at $a=t$.

So $\int_0^t e^{-a}\,\delta(t-a)\,da = e^{-t}u(t)$ $\qquad x(t_0) = \int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt$

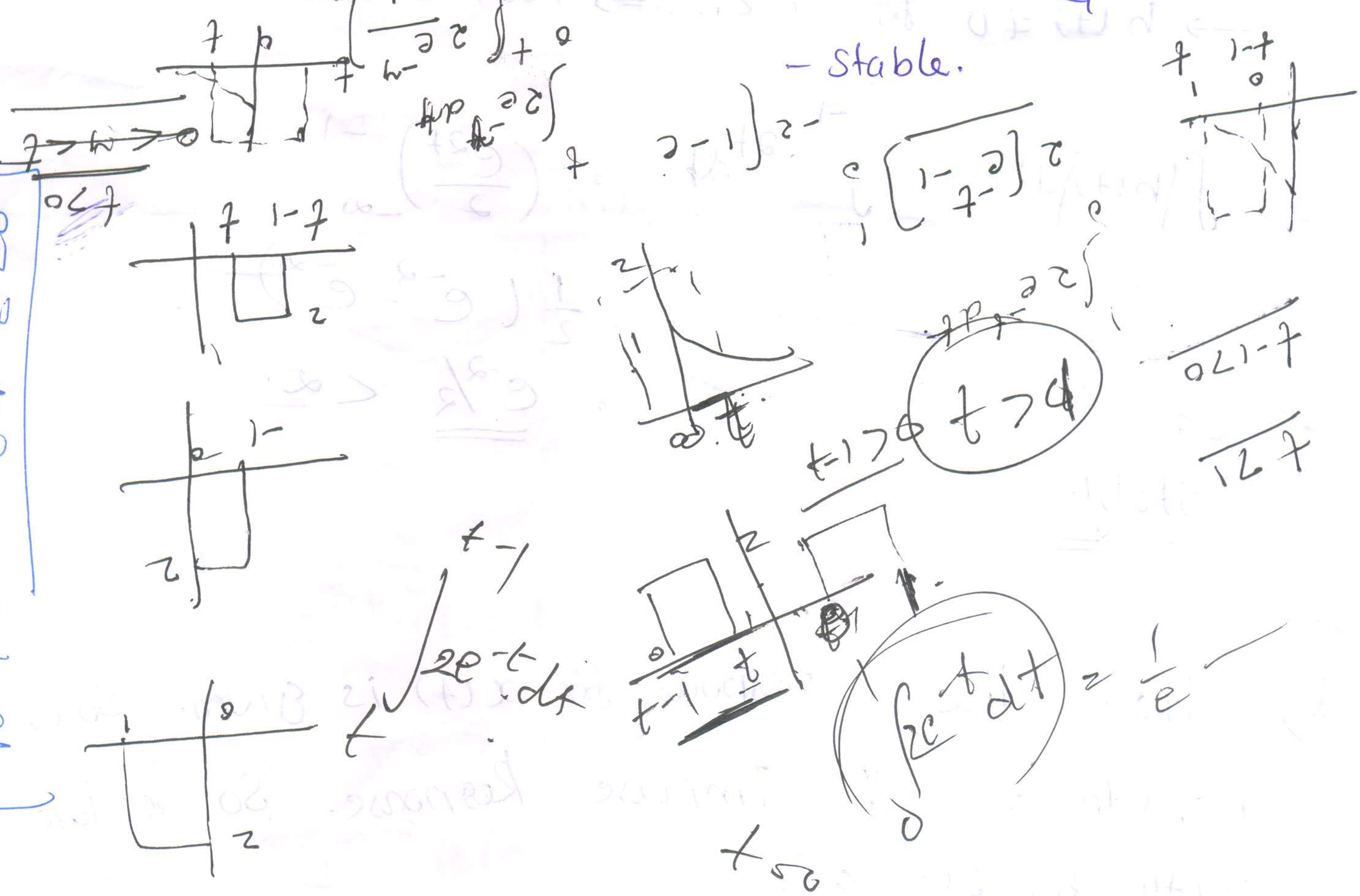
$h(t) = e^{-t}u(t)$

$h(t) = 0$ for $t<0$ So causal

$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} e^{-t}u(t) = \int_0^{\infty} e^{-t} = \left(\frac{e^{-t}}{-1}\right)_0^{\infty} = \frac{e^{-\infty}-e^{0}}{-1} = 1 < \infty$

– Stable.

$h(t) = 0$ for $t<0$



$t>0$

$\int_0^{t-1} 2e^{-t}\,dt$

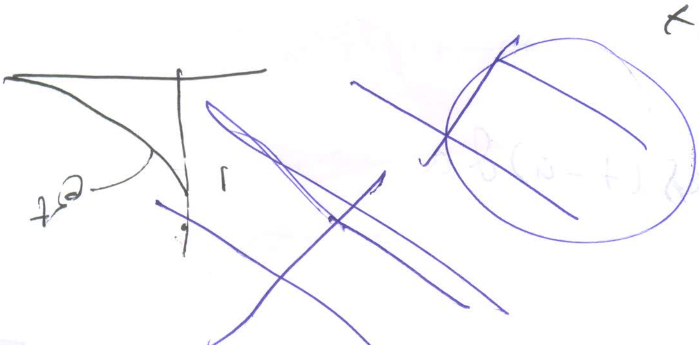
$t-1>0$ $\quad t>1$

$\int_0^{1} 2e^{-t}\,dt = \frac{1}{e}$

$t<0$

$t-1<0$

$t<1$



Q. 1(4)

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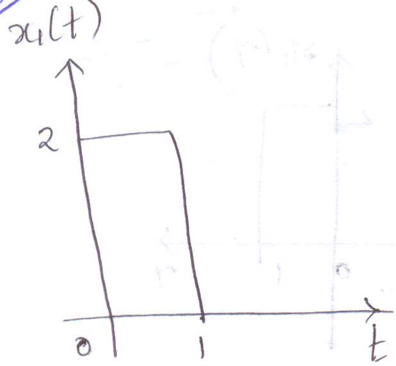
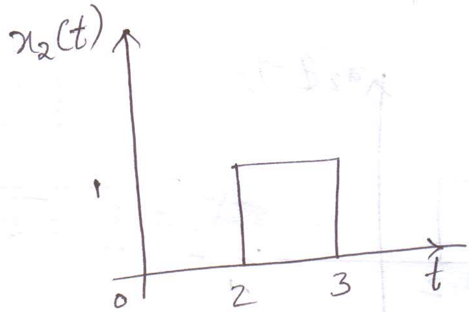
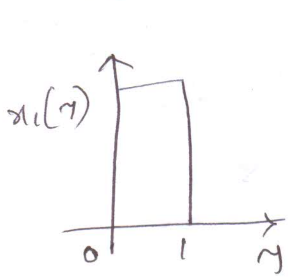


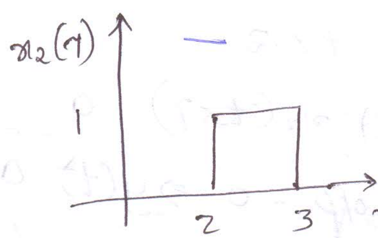


Conventional approach

consider → change axis from t to τ

$x_1(\tau)$ & $x_2(\tau)$

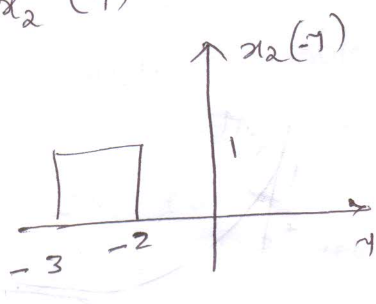




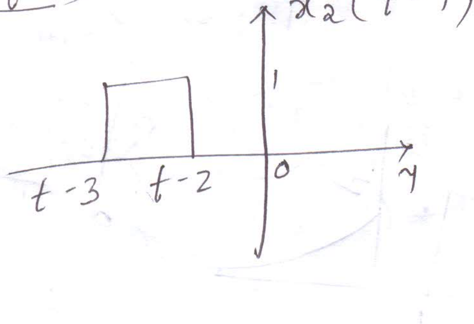
Folding

Consider

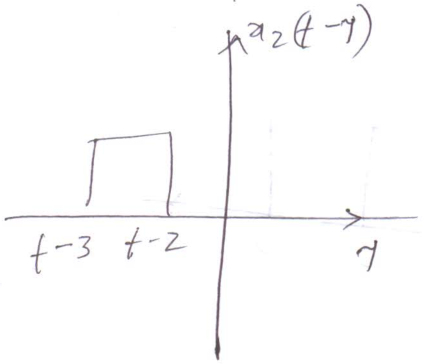
$x_2(\tau)$

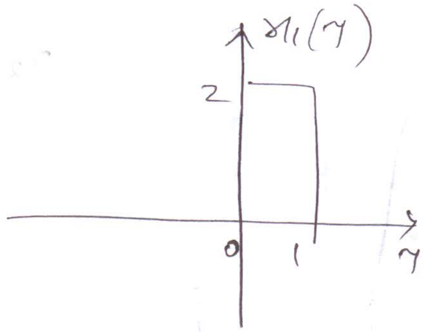


Shifting



Multiplication:





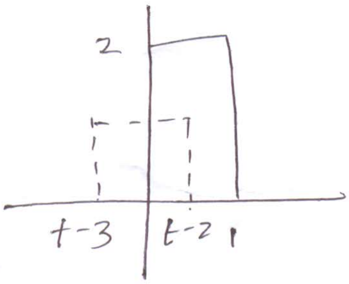
Now if $t-2<0$

$\Rightarrow t<2$

$x_1(\tau)\, x_2(t-\tau) = 0$

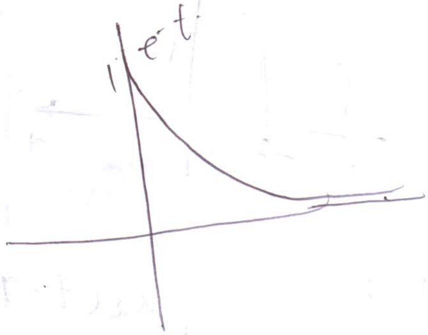
$\Rightarrow$ o/p $= 0 \Rightarrow y(t) = 0$

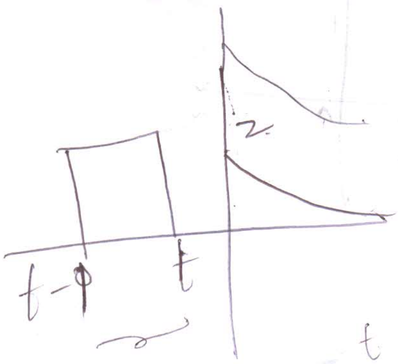
Consider $0<t-2<1 \Rightarrow 2<t<3$

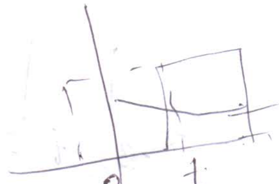


$$y(t) = \int_0^{t-2} (1\times 2)\, d\tau$$

$$= 2(t-2)$$





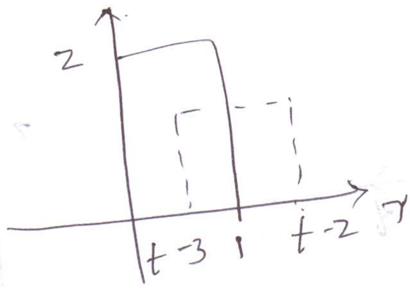


0 1

$t-1>0$

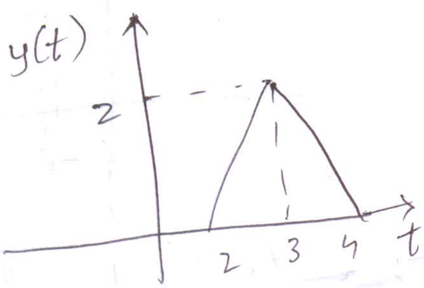
$< t-3 < 1$

$3 < t < 4$



$t) = x_1(t) * x_2(t)$

$$= \int_{t-3}^{1} 2(1)\,d\tau = 2(-t+4) \quad ; \; 3<t<4$$



$$y(t) = \begin{cases} 0 & ; t<2 \\ 2(t-2) & ; 2<t<3 \\ 2(-t+4) & ; 3<t<4 \\ 0 & ; t>4 \end{cases}$$

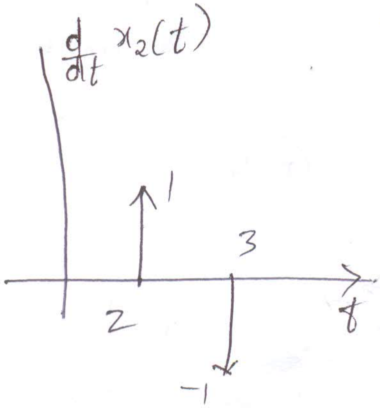
lternative Method: using properties:

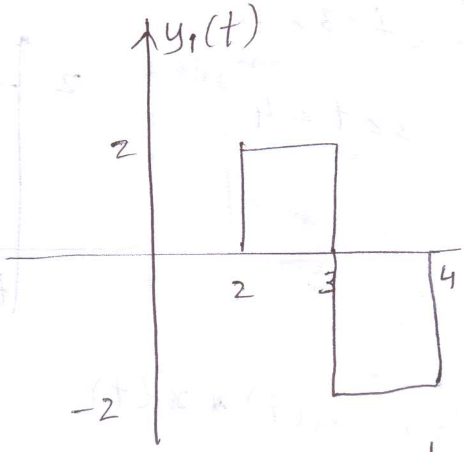
n convolution problems, any given signal which can be converted to impulse would be of great help. as convolution involved with impulse is nothing but using ~~shifting~~ sifting property

$$y(t) = x_1(t) * x_2(t)$$

$$\frac{d}{dt} y(t) = \boxed{x_1(t) * \frac{d}{dt} x_2(t)}$$

$$\Rightarrow y(t) = \int_{-\infty}^{t} x_1(t) * \frac{d}{dt} x_2(t)$$

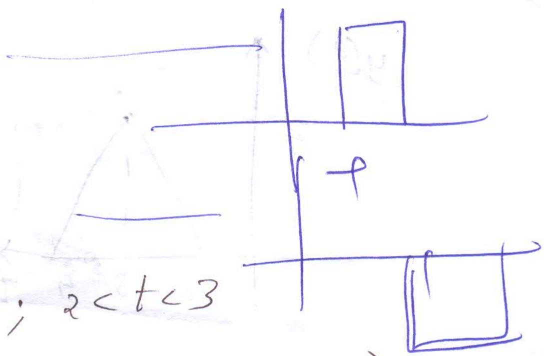




$$\frac{d}{dt}x_2(t) = \delta(t-2) - \delta(t-3)$$

$$y_1(t) = x_1(t) * \frac{d}{dt}x_2(t)$$

$$y(t) = \int_{-\infty}^{t} y_1(t)\,dt$$



Case i,

$2<t<3$

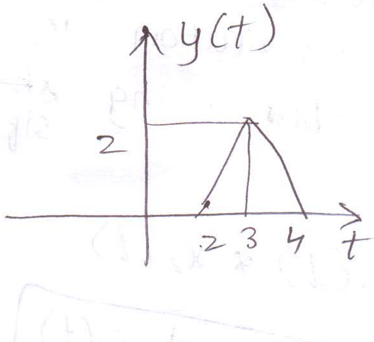
$$y(t) = \cancel{x_3(t)} = \int_{2}^{t} 2\,dt = 2(t-2); \quad 2<t<3$$

At $t=3$ $y(3) = 2(3-2) = 2$

Case ii : $3<t<4$ because of discontinuity at $t=3$

$$y(t) = 2 + \int_{3}^{t} -2\,dt$$

$$= -2t+8$$



(5) $h(t) = e^{\alpha t} u(t) + e^{\beta t} u(-t)$

$$\int_{-\infty}^{\infty} |h(t)| \, dt < \infty.$$

$$\int |h(t)| dt = \int_{-\infty}^{\infty} e^{\alpha t} u(t) dt + \int_{-\infty}^{\infty} e^{\beta t} u(-t) \, dt$$

$$= \int_{0}^{\infty} e^{\alpha t} \cancel{u(t)} dt + \int_{-\infty}^{0} e^{\beta t} dt$$

The above value should be finite. So take it individually

$$\int_{0}^{\infty} e^{\alpha t} dt < \infty \quad \text{if } \alpha \text{ is -ve ie } \underline{\alpha < 0}$$

and

$$\int_{-\infty}^{0} e^{\beta t} dt < \infty \quad \text{if } \beta \text{ is +ve } \underline{\beta > 0}$$