TUTORIAL - 5 ANSWERS

Q.1) we know, $y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$

Taking Fourier Transform of $y(t)$,

$$FT\{y(t)\} = \int_{-\infty}^{+\infty} y(t) \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau \right] \cdot e^{-j2\pi ft} \cdot dt$$

$$= \int_{-\infty}^{+\infty} x(\tau) \underbrace{\left[\int_{-\infty}^{+\infty} h(t-\tau) \cdot e^{-j2\pi ft} \cdot dt \right]}_{FT\{h(t-\tau)\} = e^{-j2\pi f\tau} \cdot H(f)} \cdot d\tau$$

(using shifting property)

$$= \int_{-\infty}^{+\infty} x(\tau) \cdot H(f) \cdot e^{-j2\pi f\tau} \cdot d\tau$$

$$= H(f) \cdot \int_{-\infty}^{+\infty} x(\tau) \cdot e^{-j2\pi f\tau} \cdot d\tau$$

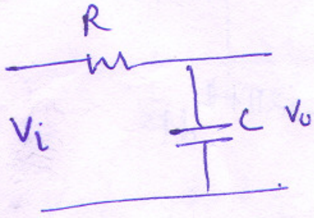
$$\boxed{Y(f) = H(f) \cdot X(f)}$$

so, $Y(f) = FT\{x(t) * h(t)\} = X(f) \cdot H(f)$.

inverse

Q.2 As we solved the same Question i.e. Low pass filter Impulse Response in Tutorial-2, ~~Impacts~~.

Differential Equation we are getting as below



$$\frac{dy(t)}{dt} = \frac{1}{RC}\left(x(t) - y(t)\right)$$

Taking Fourier Transform on both sides

$$FT\left\{\frac{dy(t)}{dt}\right\} = FT\left\{\frac{1}{RC}\left(x(t) - y(t)\right)\right\}$$

$$= \frac{1}{RC}\left(FT\{x(t)\} - FT\{y(t)\}\right)$$

$$(j2\pi f)\,Y(f) = \frac{1}{RC}\left(X(f) - Y(f)\right)$$

$$\Rightarrow \left(1 + j(2\pi f)RC\right)Y(f) = \cancel{b}\,X(f)$$

$$\Rightarrow \frac{Y(f)}{X(t)} = \frac{1}{1 + j2\pi f RC}$$

$$\Rightarrow H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1 + j2\pi f RC}.$$

We have got $h(t)$ as $\frac{1}{\tau} e^{-t/\tau} u(t)$

From this also one can get $H(f)$

3) a. $x(t) = 10 \sin 2\pi \left(\frac{1}{4\pi}\right) t + 20 \cos 2\pi \left(\frac{1}{10\pi}\right) t$

The impulse response of the low pass filter is

$$H(f) = \frac{1}{1 + j2\pi f RC}$$

here $RC = 1$

$$\Rightarrow H(f) = \frac{1}{1+j2\pi f} \quad ; \text{Let } f_1 = \frac{1}{4\pi} \; ; f_2 = \frac{1}{10\pi}$$

The response of the system will be

$$y(t) = 10 \left|H(f_1)\right| \sin\left(2\pi \left(\frac{1}{4\pi}\right) t + \angle H(f_1)\right)$$
$$+ 20 \left|H(f_2)\right| \cos\left(2\pi \left(\frac{1}{10\pi}\right) t + \angle H(f_2)\right)$$

Now

$$H(f_1) = \frac{1}{1+j2\pi\left(\frac{1}{4\pi}\right)} = \frac{1}{1+0.5j} = 0.89 \angle -26.56^\circ$$

$$H(f_2) = \frac{1}{1+j2\pi\left(\frac{1}{10\pi}\right)} = \frac{1}{1+0.2j} = 0.98 \angle -11.31^\circ$$

$$\Rightarrow y(t) = 10(0.89) \sin\left(2\pi\left(\frac{1}{4\pi}\right)t - 26.56^\circ\right)$$
$$+ 20(0.98) \cos\left(2\pi\left(\frac{1}{10\pi}\right)t - 11.31^\circ\right)$$

$$= 8.9 \sin\left(2\pi\left(\frac{1}{4\pi}\right)t - 26.56^\circ\right) + 19.6 \cos\left(2\pi\left(\frac{1}{10\pi}\right)t - 11.31^\circ\right)$$

$\frac{1}{1+\frac{1}{2}j}$ $\frac{1}{\sqrt{1+1/4}}$, $\frac{1}{\sqrt{5/4}}$ $\frac{2}{\sqrt{5}}$

b. $x(t) = 10 \sin 2\pi (8\pi) t + 20 \cos 2\pi (10\pi) t$

Now, Let $f_1 = 8\pi$; $f_2 = 10\pi$

$$H(f_1) = \frac{1}{1+j2\pi(8\pi)} = \frac{1}{1+j16\pi^2} = 6.33 \times 10^{-3} \angle -89.63^\circ$$

$$H(f_2) = \frac{1}{1+j2\pi(10\pi)} = \frac{1}{1+j20\pi^2} = 5.06 \times 10^{-3} \angle -89.70^\circ$$

$$\Rightarrow y(t) = 10(6.33 \times 10^{-3}) \sin(2\pi(8\pi)t - 89.63^\circ) + \cancel{20\cos}\; 20(5.06 \times 10^{-3}) \cos(2\pi(10\pi)t - 89.70^\circ)$$

c) ~~Here the 3-db frequency is $\frac{1}{2\pi}$ hz~~

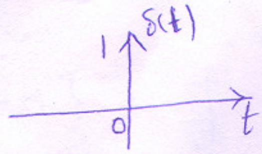
When the input signal had frequency less than $\frac{1}{2\pi}$ hz i.e $\frac{1}{4\pi}$ hz & $\frac{1}{10\pi}$ hz, the signal is passed through the system with much less attenuation

Whereas, if the input signal had frequencies ~~greater~~ much greater than ~~much~~ than $\frac{1}{2\pi}$ hz i.e 8π hz & 10π hz, the signal is passed through the system with great attenuation

Since the system is passing lower frequencies with less attenuation and passing higher frequencies with greater attenuation, it has got the name LOW PASS FILTER

4) Given

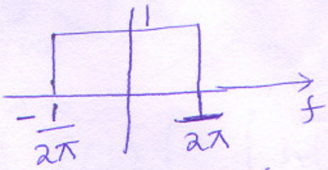
$x(t)$ is a impulse signal



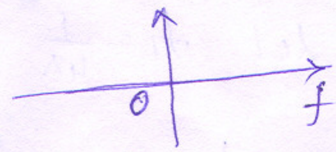
$$\frac{\sin t}{t} = \frac{\sin(\pi t/\pi)}{\frac{\pi t}{\pi}} = \frac{1}{\pi} \cdot \frac{\sin \pi (t/\pi)}{\pi (t/\pi)} = \operatorname{sinc}(t/\pi)$$

The impulse response of the system is given by $H(f)$ whose magnitude & phases are given below

$|H(f)| =$



$\angle H(f) = \theta(f)$



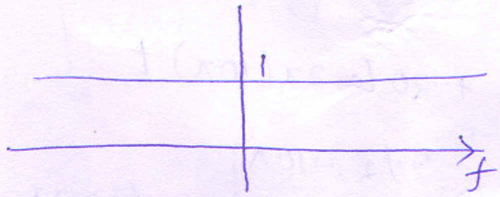
a) $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$

$= \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi f t} dt$

$= \cancel{e^{-j2\pi(f)0}}\ e^{-j2\pi f(0)}$

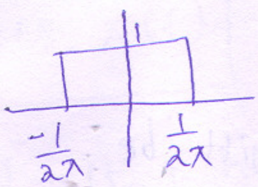
$= 1$

$X(f) =$



b) $\cancel{Y}$, $Y(f) = H(f) X(f)$

$=$



Let $f_m = \frac{1}{2\pi}$

b) $y(t) = \int_{-\infty}^{\infty} Y(f) e^{j2\pi f t} df$

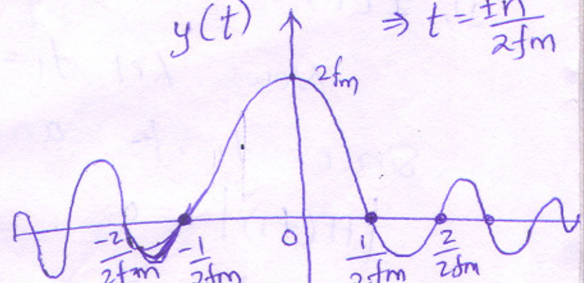
$= \int_{-f_m}^{f_m} 1\, e^{j2\pi f t} df$

$= \left. \frac{e^{j2\pi f t}}{j2\pi t} \right|_{-f_m}^{f_m}$

$= \frac{\sin 2\pi f_m t}{\pi t} \times \frac{2f_m}{2f_m}$

$= 2f_m \operatorname{sinc}(2f_m t)$

$2\pi f_m t = \pm n\pi$
$\Rightarrow t = \frac{\pm n}{2f_m}$



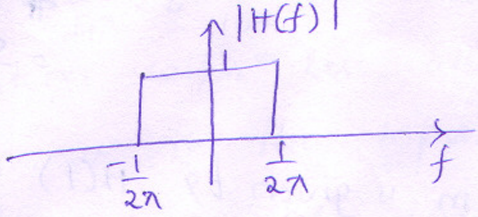
Since $h(t) = y(t)$
& $h(t) \neq 0$ for $t < 0$
∴ It is non-causal system

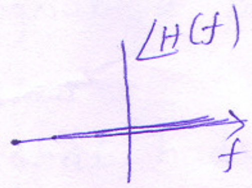
c) ~~By~~ ~~k~~ The spectrum of $x(t)$ extends from $-\infty$ to ∞ hence it has infinite number of frequencies

d) from the frequency spectrum of $y(t)$, we conclude that there are infinite number of frequencies in the range of $\frac{-1}{2\pi} < f < \frac{1}{2\pi}$

e) i, $x(t) = 10 \sin 2\pi \left(\frac{1}{4\pi}\right) t + 20 \cos 2\pi \left(\frac{1}{10\pi}\right) t$

The magnitude & phase response of the system is given by





Let $f_1 = \frac{1}{4\pi}$ $\quad f_2 = \frac{1}{10\pi}$

$|H(f_1)| = 1 \qquad \angle H(f_1) = 0^\circ$

$|H(f_2)| = 1 \qquad \angle H(f_2) = 0^\circ$

The response of the system will be

$y(t) = 10|H(f_1)| \sin\left(2\pi\left(\frac{1}{4\pi}\right)t + \angle H(f_1)\right) + 20|H(f_2)| \cos\left[2\pi\left(\frac{1}{10\pi}\right)t + \angle H(f_2)\right]$

$= (1) 10 \sin\left(2\pi\left(\frac{1}{4\pi}\right)t + 0^\circ\right) + 20(1) \cos\left(2\pi\left(\frac{1}{10\pi}\right)t + 0^\circ\right)$

$= x(t)$

ii, $x(t) = 10 \sin 2\pi (8\pi) t + 20 \cos 2\pi (10\pi) t$

Now Let $f_1 = 8\pi \quad f_2 = 10\pi$

Since f_1, f_2 are outside the magnitude spectrum

$|H(f_1)| = 0 \qquad \angle H(f_1) = 0$

$|H(f_2)| = 0 \qquad \angle H(f_2) = 0$

The response of the system will be

$y(t) = 10(0) \sin\left(2\pi\left(8\pi\right)t + 0^\circ\right) + 20(0) \cos\left(2\pi(10\pi)t + 0^\circ\right)$

$= 0$

f) Since the frequencies which are less than $\frac{1}{2\pi}$ hz are passed without attenuation & the frequencies which are greater than cut off frequency $\frac{1}{2\pi}$ hz are completely attenuated, hence the system is ideal LOW PASS FILTER. As already seen ~~in Q4~~, an ideal LPF is a noncausal filter (Practically non realizable)

f)