Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)
First insemester Examination
CT203 (Signals and Systems)
Date of Examination: 30-08-2012
Duration: 2:00 hrs
Maximum Marks: 30

Instructions:

1. Attempt all questions.
2. Use of scientific non programmable calculator is permitted.
3. Figures in brackets indicate full marks.
4. All the acronyms carry their usual meaning.
5. You may make valid assumptions wherever required.

Q1: Using the definition of power signal, determine whether the following signals belong to power signal catagory. Also determine power contained in each signal, if it is a power signal (5 marks)

(a) $10\cos(100t+\frac{\pi}{3}),\quad -\infty < t < \infty$ 1

(b) $10\cos(5t)\cos(10t),\quad -\infty < t < \infty$ 2

(c) $10\cos(100t+\frac{\pi}{3}),\quad -5 < t < 5$ 2

Q2: Simplify the following expressions using the properties of impulse function. Write down the property used. (Here $-\infty < t, f < \infty$) (4 marks)

(a) $\left(\frac{\sin(t)}{t^2+2}\right)\delta(t)$

(b) $\delta(f)\left(\frac{\sin(kf)}{f}\right)$, where k is a constant 2+2

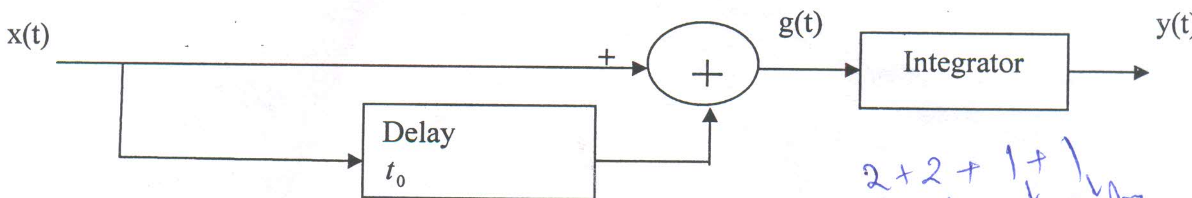
Q3: Prove that for all $k \neq 0$, $\delta(kt) = \frac{1}{|k|}\delta(t)$ (2 marks) 2

Q4: Consider the first order system, or equivalently the RC low pass filter as discussed in the tutorial. The unit step response of this system is given by $\left[1-\exp(\frac{-t}{\tau_c})\right]u(t)$,

where $RC = \tau_c$.

Now consider that input is changed to rectangular pulse of width τ with unit height. (i.e., the pulse exists for times between $-\tau/2$ and $\tau/2$ with amplitude =1). Use the available unit step response to determine the response of the system for the rectangular pulse and plot the same for all t (time). (7 marks) 5+2

Q5: Consider the system shown below as a block diagram where x(t) is the input and y(t) is the output. Here the Delay block delays the input by an amount of t_0 seconds. Determine the impulse response for the system and plot it for all t (time). Is this a causal system? If yes, why? (6 marks)



2+2+1+1
imp plt caus why

Q6: Find the frequency components and their amplitudes present in the following signal $x(t) = \cos^2(2\pi 10t + \theta) + 2\cos^3(2\pi 20t)$, where θ is the phase shift. What is the condition on the time interval for which the signal should exist if your answer has to be correct? How many number of frequency components will be there in $x(t)$ if this condition is not satisfied? Can you give justification for the answer you wrote for the last part of the question (I have not discussed about the justification in the class, but I would like to know it, if you can think and write).
(6 marks)

3+1+1+1

BEST OF LUCK

ANSWERS I TEST CT903

30-08-2012

Q.1. Yes, the first two belong to power signal category.

(a) Power in (a) $10\cos(100t + \pi/3)$, $-\infty < t < \infty$

Here $2\pi f_0 = 100$ $\therefore$ $f_0 = \frac{100}{2\pi}$ $\therefore$ $T_0 = \frac{2\pi}{100}$ is the fundamental period or period of the signal.

$\therefore$ By def^n of power for periodic signal

$$P = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} 100\cos^2(100t + \pi/3)\,dt = \frac{100}{2}\left[\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} 1\,dt + \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\cos 2(100t + \pi/3)\,dt\right]$$

Consider $\frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\cos 2(100t + \pi/3)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\cos(200t + 2\pi/3)\,dt$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{T_0/2}\left(\cos 200t \cos 2\pi/3 - \sin 200t \cdot \sin 2\pi/3\right)dt\right.$$

After integration & simplifying

$= 0$.

$$\therefore P = 100/2\left[\frac{T_0}{2} + T_0/2\right]\frac{1}{T_0} = \frac{100}{2} = 50.$$

(b) $10\cos(5t)\cos(10t)$, $-\infty < t < \infty$

This signal can be rewritten as

$\frac{10}{2}\left[\cos 5t + \cos(15t)\right]$, This is periodic signal

Period of first component $2\pi/5$, Period of second component $\frac{2\pi}{15}$

So period of the added signal ~~is lcm of the two~~ $\frac{2\pi}{5} = T_0$

(So the freq of given signal with which it repeats is $\frac{5}{2\pi}$)

So power is $\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \cancel{100}\,25(\cos 5t + \cos 15t)^2\,dt$

or $P = \frac{\cancel{100}}{25} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \left(\cos^2 5t + \cos^2 15t + 2 \cos 5t \cos 15t \right) dt$

$= \frac{\cancel{100}}{25} \left[\frac{1}{2} + \frac{1}{2} \right] + 0$ the last term integration is zero.

$= 25 \,[1] + 0 = 25$

(c) This signal is not periodic since t is between -5 and 5. But it is an energy signal.
Hence for an energy signal, power $= 0$.

Q 2.

(a) $\left[\frac{\sin(t)}{t^2+2} \right] \delta(t) = 0$, Use $x(t)\,\delta(t-\tau) = x(\tau)\,\delta(t-\tau)$ with $\tau = 0$

(b) $\delta(f) \left[\frac{\sin kf}{f} \right] = k\,\delta(f)$, Use $x(f)\,\delta(f) = x(0)\,\delta(f)$

Q 3.

Let $kt = x$, Let $k > 0$.

$\int_{-\infty}^{\infty} \phi(t)\,\delta(kt)\,dt = \frac{1}{a} \int_{-\infty}^{\infty} \phi\left(\frac{x}{k}\right) \delta(x)\,dx$
$= \frac{1}{k} \phi(0)$

$|||^{ly}$ for $k < 0$,

$\int_{-\infty}^{\infty} \phi(t)\,\delta(kt)\,dt = -\frac{1}{k} \phi(0)$

$\therefore \int_{-\infty}^{\infty} \phi(t)\,\delta(kt)\,dt = \frac{1}{|k|} \phi(0)$

$= \frac{1}{|k|} \int_{-\infty}^{\infty} \phi(t)\,\delta(t)\,dt$

$= \int_{-\infty}^{\infty} \phi(t) \frac{1}{|k|} \delta(t)\,dt$

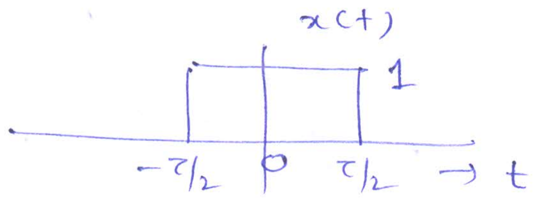
$\therefore \delta(kt) = \frac{1}{|k|} \delta(t)$

Q4

unit step response $= (1 - e^{-t/\tau_c})\,u(t)$

where $RC = \tau_c$

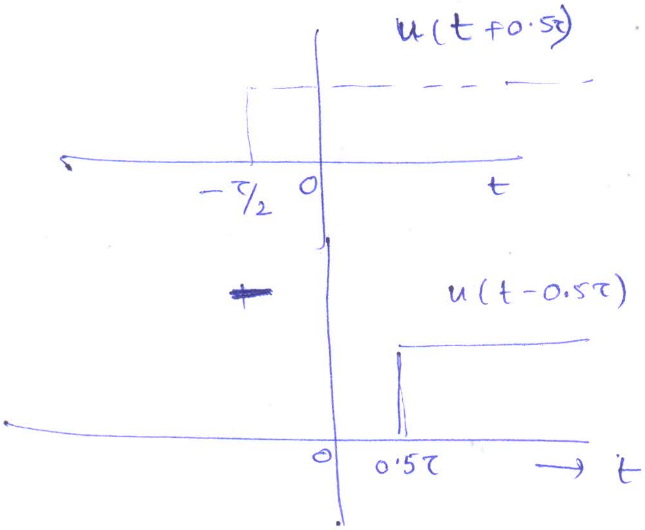
Now the input applied is



what is the output in terms of unitstep response.

Ans:

x(t) can be written as sum of two step functions



So $x(t) = u(t+\tau/2) - u(t-\tau/2) = x_1(t) + x_2(t)$

∴ Hence the ~~input~~ o/p has to be sum of responses due to each ~~input~~ part in $x(t)$ ie sum of response due to $x_1(t)$ and $x_2(t)$

Since timeinvariance system is used the response to delayed or advanced step input has to be delayed or advanced response by the same amount

So the response is given by

$$\cancel{y(t) = 0 \quad \text{for } t < -0.5\tau}$$
$$\cancel{= (1 - e^{-t})}$$

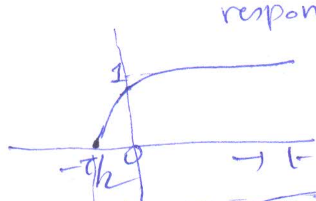
$$y(t) = \left(1 - e^{-\frac{(t+0.5\tau)}{\tau_c}}\right) u(t+0.5\tau)$$
$$- \left(1 - e^{-\frac{(t-0.5\tau)}{\tau_c}}\right) u(t-0.5\tau)$$
$$\forall t$$

This can be written as

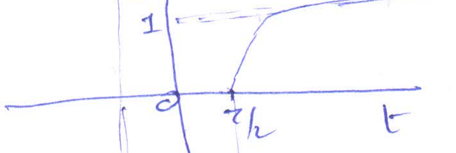
$$y(t) = 0 \qquad t < 0.5\tau$$
$$= \left(1 - e^{-\frac{(t+0.5\tau)}{\tau_c}}\right) \qquad -0.5\tau \le t \le 0.5\tau$$
$$= \left(1 - e^{-\frac{\tau}{\tau_c}}\right) e^{-\frac{(t-0.5\tau)}{\tau_c}}, \qquad 0.5\tau \le t$$

This is plotted as below

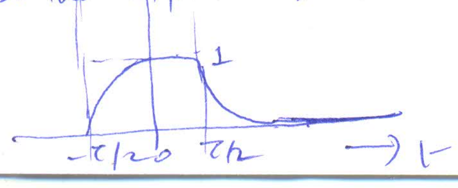
response due to $u(t+0.5\tau)$



response due to $u(t-0.5\tau)$



So the response $y(t)$



Q5.

If + sign is considered after delay (before the sum block)

$g(t) = x(t) + x(t - t_0)$

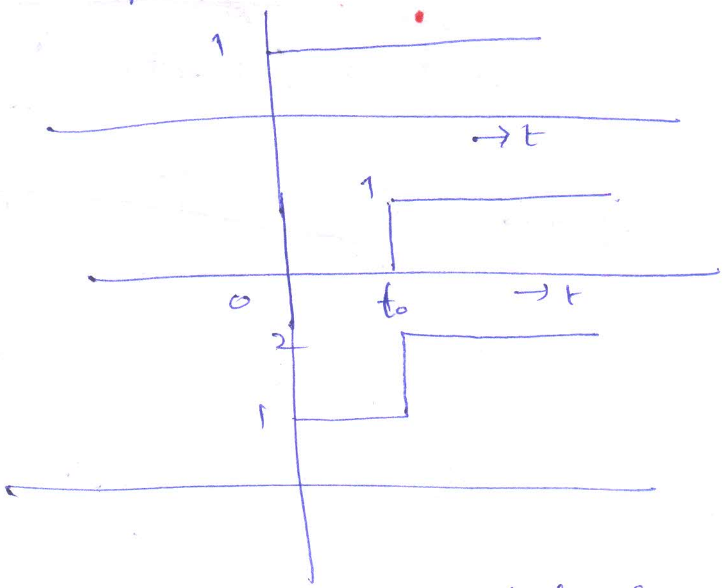
when impulse is applied

$g(t) = \delta(t) + \delta(t - t_0)$

the o/p is $h(t) = \int_{-\infty}^{t} \left(\delta(\tau) + \delta(\tau - t_0)\right) d\tau$

$= [u(t) + u(t - t_0)]$

So the plot of $h(t)$ is

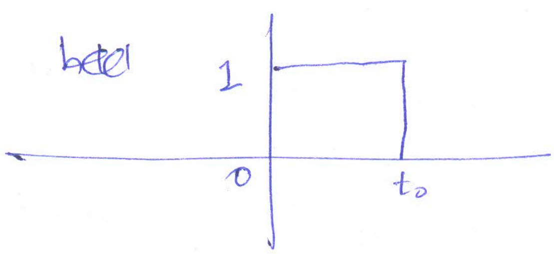


Any one can be attempted

If we consider −ve sign before sum block

$h(t) = u(t) - u(t - t_0)$

So the plot of $h(t)$ is



Both cases the system is causal since $h(t) = 0$ for $t < 0$

Q6] Use $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Consider $x(t) = \cos^2(2\pi 10t + \theta) + 2\cos^3(2\pi 20t)$

Frequency components & amplitudes?

$$x(t) = \frac{1 + \cos 2(2\pi 10t + \theta)}{2} + \frac{2}{4}\left[\cos 3(2\pi 20t) + 3\cos 2\pi 20t\right]$$

$$= \frac{1}{2} + \frac{1}{2}\cos(2\pi(20t) + 2\theta)$$

$$+ \frac{1}{2}\cos(2\pi 60t) + \frac{3}{2}\cos 2\pi 20t$$

dc component ($f = 0$) with amplitude $= \frac{1}{2}$

$f = 20$ Hz " " $= \frac{1}{2}$ (1)

$f = 60$ Hz " $= \frac{1}{2}$

The signal should exist betw $-\infty$ and ∞

ie $-\infty < t < \infty$

If this condition is not satisfied then infinite number of frequencies will be present

The reason - As soon as you limit the time the discontinuities are created. We know from Fourier series theory that a square wave has infinite frequencies though at discrete intervals. This is due to discontinuities present in the square wave.

Marks are awarded for both

This can be written as $\frac{1}{2}\left[\cos(2\pi 20t)\cos 2\theta - \sin 2\pi 20t \sin 2\theta\right]$

$= \cos 2\pi 20t\left[\frac{\cos 2\theta}{2}\right] + \sin 2\pi 20t\left(-\frac{\sin 2\theta}{2}\right)$ (2)

Finally Ans: $\frac{1}{2} + \frac{1}{2}\cos(2\pi 60t) + \cos 2\pi 20t\left[\frac{\cos 2\theta}{2} + \frac{3}{2}\right] + \sin 2\pi 20t\left(-\frac{\sin 2\theta}{2}\right)$

$\therefore$ Amplitude for frequency $= 20$hz is $\sqrt{\left(\frac{\cos 2\theta + 3}{2}\right)^2 + \left(\frac{\sin 2\theta}{2}\right)^2} = \sqrt{\frac{5 + 3\cos 2\theta}{2}}$