1) For the energy signals prove parseval's theorem

$$\int_{-\infty}^{\infty} \left|x(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|X(f)\right|^2 df$$

2) Consider a signal set

$\{ 1,\ \cos 2\pi f_0 t,\ \cos 2\pi 2 f_0 t,\ \cos 2\pi 3 f_0 t,\ \cdots, \sin 2\pi f_0 t\ ,\ \sin 2\pi 2 f_0 t,\ \sin 2\pi 3 f_0 t,\ \cdots \}$

Then show that

$$\int_T \cos n\omega_0 t\ \cos m\omega_0 t\ dt = \begin{cases} 0\ for\ n \neq m \\ \frac{T}{2}\ for\ n = m \neq 0 \end{cases}$$

$$\int_T \sin n\omega_0 t\ \sin m\omega_0 t\ dt = \begin{cases} 0\ for\ n \neq m \\ \frac{T}{2}\ for\ n = m \neq 0 \end{cases}$$

$$\int_T \sin n\omega_0 t\ \cos m\omega_0 t\ dt = 0\ e^{-j2\pi f_c t}\ \ for\ all\ \text{n and m}$$

3) Show that if we consider a real signal $x(t)$, it is orthogonal is $\hat{x}(t)$

$i.e\ \int_{-\infty}^{\infty} x(t)\ \hat{x}(t)\ dt = 0$

4) One of the applications of Hilbert transform is to form an analytical signal $x(t) + j\ \hat{x}(t)$ and this representation helps us in the analysis of band pass signal in terms of low pass signals (the details will be taught in your higher classes in communication)

The question is as follows here

Consider an AM signal

$x(t) = A_c (1 + m\cos\omega_m t)\cos\omega_c t\ \ \forall t\ , \omega_c = 2\pi f_c,\ \omega_m = 2\pi f_m\ ,\ c\ for\ carrier,\ m\ for$

$\mathrm{mod}\, ulating signal$

a) Is this low pass signal or band pass signal and write the band width of the signal

b) For this AM signal find the spectrum of the analytical signal denoted by $X_p(f)$
c) Write down the time domain expression $x_p(t)$
d) Is $x_p(t)$ real or complex
e) Now multiply $x_p(t)$ by $e^{-j2\pi f_c t}$ and find the spectrum and time domain representation for the same
f) Is this real or complex
g) Is this low pass signal or band pass signal
h) Write the bandwidth of the signal found in (e)