24/10 CT:203 Signals & Systems:

1. find the L.T of the following signals with R.O.C?

a) $x_1(t) = e^{-t}u(t) + e^{-3t}u(t)$

b) $x_2(t) = e^{-2t}u(t) + e^{4t}u(-t)$

c) $x_3(t) = e^{-t}u(-t) + e^{5t}u(t)$

d) $x_4(t) = 1 \quad \forall t$

e) ~~$x_5(t) = sgn(t)$~~

f) $x_6(t) = \delta(3t) + u(2t)$

2. Consider the signal $x(t) = e^{-5t} + e^{-\beta t}u(t)$ & its L.T is $X(s)$. What are the constraints placed on the real & imaginary parts of β if the R.O.C of $X(s)$ is $Re\{s\} > -3$?

3. find the L.T of the following signals

a) unit ramp starting at $t=a$

b) $x(t) = u(t-5)$

c) $y(t) = e^{5t}u(-t+3)$

4. find the I.L.T of $Y(s) = \frac{e^{-3s}}{(s+1)(s+2)}$ $\sigma > -1$

5. find the L.T of

1) $x_1(t) = \cos\omega_0 t\, u(t)$

2) $x_2(t) = te^{-3t}u(t)$

3) ~~$x_3(t) = e^{-at}\sin\omega_0 t\, u(t)$~~

$s = \sigma + j\omega$

6. Let $x(t)$ be a signal that has a rational L.T with exactly 2 poles located at $s=-1$ and $s=-3$ if $g(t) = e^{2t}x(t)$ & $G(j\omega)$ converges, determine whether $g(t)$ is

a) Left-sided b) right-sided c) two-sided d) finite duration

7. An LTI Causal continuous time system has a rational transfer function with simple poles at $s=-2$ and $s=-4$ and one of the simple zero at $s=-1$. A unit step $u(t)$ is applied as the input of the system. The steady state, output has a constant value of 1. find the impulse response

8. Given $H(s) = \dfrac{s-1}{(s+1)(s-2)}$, find $h(t)$ for each of the following cases

i, Stable
ii, Causal
iii, neither causal nor stable.

I

a. $x_1(t) = e^{-t}u(t) + e^{-3t}u(t)$

$= \frac{1}{s+1}\ ;\sigma > -1 \qquad \frac{1}{s+3}\ ;\sigma > -3$

$\therefore X_1(s) = \frac{1}{s+1} + \frac{1}{s+3}\ ;\sigma > -1$

b) $x_2(t) = e^{-2t}u(t) + e^{4t}u(-t)$

$\frac{1}{s+2}\ ;\sigma > -2 \qquad \frac{-1}{s-4}\ ;\sigma < 4$

$\therefore X_2(s) = \frac{1}{s+2} - \frac{1}{s-4}\ ;-2 < \sigma < 4$

c) $x_3(t) = e^{-t}u(-t) + e^{5t}u(t)$

$\frac{1}{s+1}\ ;\sigma < -1 \qquad \frac{1}{s-5}\ ;\sigma > 5$

No- Common R.O.C

$\therefore$ No- Laplace Transform

d) $x_4(t) = 1 \quad \forall t$

$\Rightarrow x_4(t) = u(t) + u(-t)$

$= \frac{1}{s}\ ;\sigma > 0 \ + \ \frac{-1}{s}\ ;\sigma < 0$

No-common R.O.C

$\therefore$ No- Laplace Transform

Don't include

f) $x_6(t) = \delta(3t) + u(2t)$

$= \frac{1}{3}\delta(t) + u(t)$

$= \frac{1}{3}\ ;(\text{s plane}) \qquad \frac{1}{s}\ ,\sigma > 0$

$\therefore X_6(s) = \frac{1}{3} + \frac{1}{s}\ ;\sigma > 0$

2\. Given

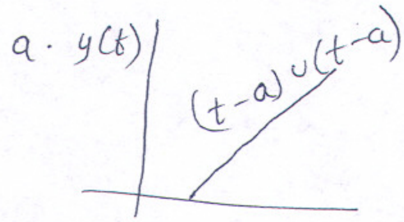
$$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$$

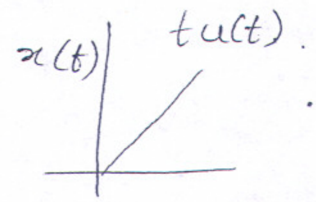
and R.O.C of $X(s)$ is $Re\{s\} > -3$

$$= \frac{1}{s+5} \;; \sigma > -5 \quad + \quad \frac{1}{s+\beta} \;; \sigma > Re(-\beta)$$

$$\therefore Re(\beta) = 3$$

3\. a.





Consider $x(t) = tu(t)$

$$X(s) = \int_0^\infty t e^{-st} dt$$

$$= t\frac{e^{-st}}{-s}\Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} dt$$

$$= \frac{te^{-st}}{-s}\Big|_0^\infty + \left[\frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= 0 + \frac{1}{s^2}$$

$$= \frac{1}{s^2} \;; \sigma > 0$$

Now consider,

$$y(t) = x(t-a)$$

$$\Rightarrow Y(s) = \frac{e^{-as}}{s^2} \;; \sigma > 0.$$

3) b) $x(t) = u(t-5)$

$$X(s) = \frac{1}{s} e^{-5s}.$$

c) $y(t) = e^{5t} u(-t+3)$

$$= e^{5t} u(-(t-3))$$

$\Rightarrow$ ~~C~~onsider

$$x(t) = \cancel{u(-(t-3))}\, u(t+3)$$

$$\Rightarrow X(s) = \frac{1}{s} e^{-3s}$$

Now $x(-t) \leftrightarrow X(-s)$

$\Rightarrow$ L.T of $u(-t+3)$

is $-\frac{1}{s} e^{3s}$

Now consider $e^{5t} u(-t+3)$ as $z(t)$

$$\Rightarrow Z(s) = \frac{-1}{(s-5)} e^{3(s-5)} \quad //$$

4) Given $Y(s) = \frac{e^{-3s}}{(s+1)(s+2)}$

$$= e^{-3s} \left[\frac{1}{s+1} - \frac{1}{s+2} \right]$$

$\cancel{\Rightarrow y(t) = e}$

Consider $X(s) = \frac{1}{s+1} - \frac{1}{s+2}$

$$\Rightarrow x(t) = e^{-t} u(t) - e^{-2t} u(t)$$

Since $Y(s) = e^{-3s} X(s)$

$$\Rightarrow y(t) = x(t-3) //$$

5)

1) $x_1(t) = \cos\omega_0 t\, u(t)$

$= \left(\dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) u(t)$

$\Rightarrow X_1(s) = \dfrac{1}{2}\left[\dfrac{1}{s-j\omega_0} + \dfrac{1}{s+j\omega_0}\right]; \sigma > 0 \quad \left[\because U(s) = \dfrac{1}{s} \atop \sigma > 0\right]$

$= \dfrac{1}{2}\left[\dfrac{s+j\omega_0 + s - j\omega_0}{s^2 + \omega_0^2}\right]$

$= \dfrac{s}{s^2+\omega_0^2}$ //

2) $x_2(t) = t e^{-3t} u(t)$

Consider

$x(t) = t\,u(t)$

$\Rightarrow X(s) = \dfrac{1}{s^2}; \sigma > 0$

Now $x_2(t) = e^{-3t} x(t)$

$\Rightarrow X_2(s) = X(s+3)$

$= \dfrac{1}{(s+3)^2}; \sigma > 0 + (-3)$

$; \sigma > -3$

6. Given

$$X(s) = \frac{1}{(s+1)(s+3)}$$

Given
$$g(t) = e^{2t} x(t)$$

$$\Rightarrow G(s) = X(s-2) = \underline{\frac{1}{(s-1)(s+1)}} \Rightarrow$$

∴ R.O.C. should contain jω axis.

∴ $g(t)$ is two-sided signal

$\cancel{\frac{2}{s-1}} = \frac{1}{}$

7. Given

$$H(s) = \frac{s+1}{(s+2)(s+4)}$$

and $y(t)|_{t\to\infty} = 1$ $\frac{1}{2}\left[\frac{1}{s-1} + \frac{1}{s}\right.$

$$x(t) = u(t)$$
$$\Rightarrow X(s) = \frac{1}{s}$$

$$\therefore Y(s) = X(s)\, H(s)$$
$$= \frac{s+1}{(s+2)(s+4)} \times \frac{1}{s}$$

$$y(\infty) = \lim_{s\to 0} sY(s)$$
$$= \frac{1}{8}$$

$$\therefore H(s) = \frac{8(s+1)}{(s+2)(s+4)}$$
$$= \frac{-4}{s+2} + \frac{12}{s+4}$$

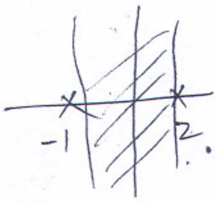
$$\therefore h(t) = -4e^{-2t}u(t) + 12e^{-4t}u(t)$$

8. Given

$$H(s) = \frac{s-1}{(s+1)(s-2)}$$

Case i Stable:

A system is stable if R.O.C includes $j\omega$ axis



$$H(s) = \frac{s-1}{(s+1)(s-2)}$$

$$= \frac{2/3}{s+1} + \frac{1/3}{s-2}$$

$$\Rightarrow h(t) = \frac{2}{3}e^{-t}u(t) + \frac{1}{3}e^{-2t}(-u(-t))$$

Case ii Causal:

for the system to be causal, impulse response is right sided and the associated R.O.C is right of the right most pole

$$H(s) = \frac{\frac{2}{3}}{s+1} + \frac{1/3}{s-2}$$

$$\Rightarrow h(t) = \frac{2}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$

Case iii neither causal nor stable

$$\Rightarrow h(t) = \frac{2}{3}e^{-t}(-u(-t)) + \frac{1}{3}e^{-2t}(-u(-t))$$