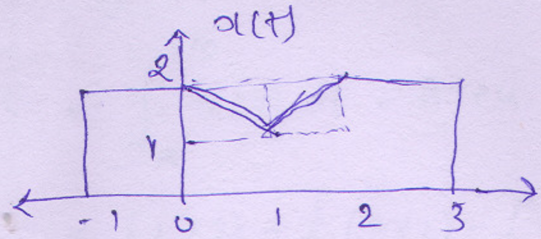
Tutorial - 6 - Que.

(1) For the signal $x(t)$ shown below, find



a) $X(0)$

b) $\int_{-\infty}^{+\infty} X(f)\, df$

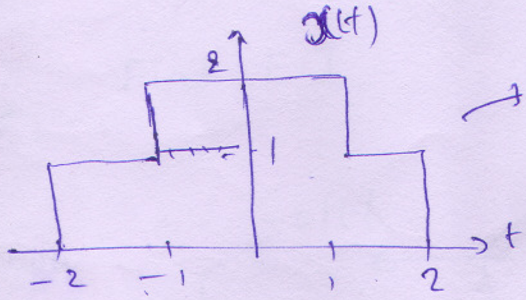
(2) Find the FT of the following signals.

a) ~~$x_1(t) = A\,\text{rect}(t/2T)$~~

b) $x(t) = \dfrac{2a}{a^2 + t^2}$

c) $x(t) = \Pi\left[\dfrac{(t-1)}{2}\right]$

d)



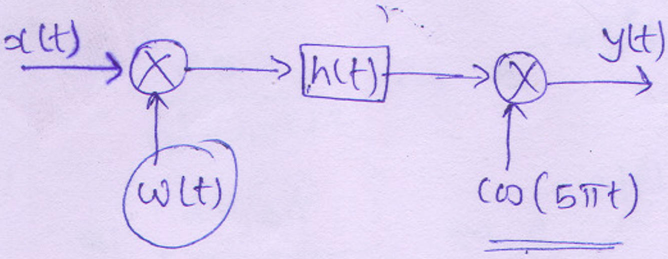
→ $x(t) + x_1(t-1) + x_1(t+1)$

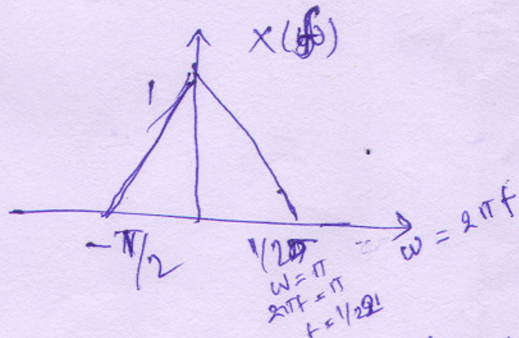
$x(t/2) + x(t)$

$\omega = \pi$

$2\pi f = \pi$
$f = \frac{1}{2}$

(3) Consider the system shown in fig. a). The FT of i/p signal is shown in fig. b). Find the FT of $y(t)$ given that $w(t) = \cos(5\pi t)$ and $h(t) = \dfrac{\sin(6\pi t)}{\pi t}$





(4) Given $y(t) = x(t) * h(t)$, and $g(t) = x(3t) * h(3t)$ such that $g(t) = A\,y(Bt)$, so, what is A & B?

(5) A causal & stable LTI system has the freq. Response $H(\omega) = \dfrac{j\omega + 4}{6 - \omega^2 + 5j\omega}$

a) Find the I.R. b) What is the o/p when the i/p applied is $x(t) = e^{-4t}u(t) - te^{-4t}u(t)$

Tutorial-6 Answers.

Q.1

a) $X(0) = \int_{-\infty}^{+\infty} x(t).e^{-j2\pi(0).t}.dt = \int_{-\infty}^{+\infty} x(t).dt$ = Area under $x(t)$

$= (1\times2) + \{(3\times2) - (\frac{1}{2}\times1\times2)\}$

$= 2+6-1 = \underline{7}$

b) $\int_{-\infty}^{+\infty} X(f).df = \int_{-\infty}^{+\infty} X(f).e^{j2\pi f(0)}.df = x(0) = \underline{2}$

Q.2 $\frac{2a}{\cancel{a^2+t^2}}$ ~~→ F.T~~ $\quad \cancel{e^{-a|t|}} \longleftrightarrow \frac{2a}{\cancel{a^2+\omega^2}}$

$\frac{2a}{a^2+\cancel{4\pi^2}t^2}$ Find F.T.

We know $e^{-a|t|} \longleftrightarrow \frac{2a}{a^2+4\pi^2f^2}$ for all t

By duality $\frac{2a}{a^2+\cancel{4}\pi^2t^2} \longleftrightarrow e^{-a|f|}$ for all f

$e^{-a|f|}$

$\frac{2a}{a^2+4\pi^2t^2} \leftrightarrow e^{-a|f|}$

$\frac{1}{2\pi}\cdot\frac{2a/2\pi}{(a/2\pi)^2+t^2} \leftrightarrow e^{-a|f|}$

taking $\frac{a}{2\pi} = \tilde{a}$

$\frac{2\tilde{a}}{\tilde{a}^2+t^2} \Leftrightarrow 2\pi\cdot e^{-2\pi\tilde{a}|f|}$

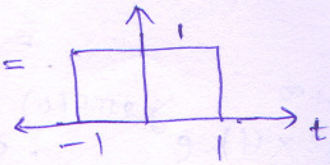
$\Rightarrow$ so, $\frac{2a}{a^2+t^2} \Leftrightarrow 2\pi\cdot e^{-2\pi a|f|}$

$\frac{t-1}{2} = \tau$

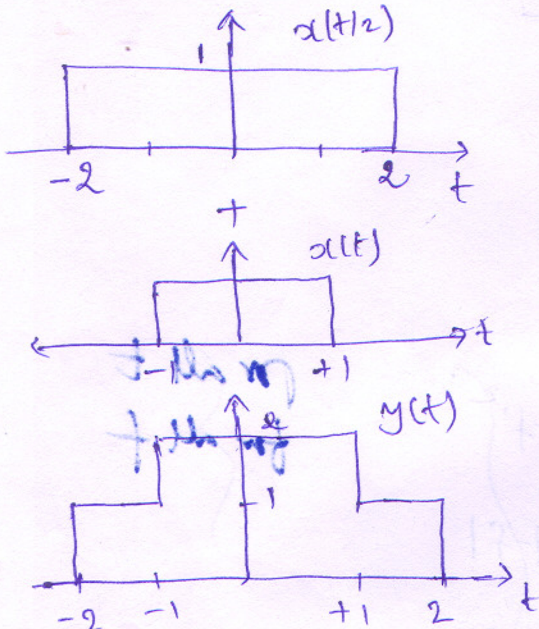
b) $x(t) = \Pi\left[\left(\frac{t-1}{2}\right)\right]$ = rectangular f^{n}.

we know $\Pi(t/2) \leftrightarrow 2\,\mathrm{sinc}(2f)$, now by time shifting property $\Pi\left[\left(\frac{t-1}{2}\right)\right] \leftrightarrow e^{-j2\pi f(1)} \cdot 2\,\mathrm{sinc}(2f)$

c) Here, if we take $x(t) =$



then we can represent y(t) as follows.



so, $y(t) = x(t/2) + x(t)$

we know $x(t) = \mathrm{rect}(t/2) \leftrightarrow 2\,\mathrm{sinc}(2f)$

so, $y(t) = x(t/2) + x(t)$

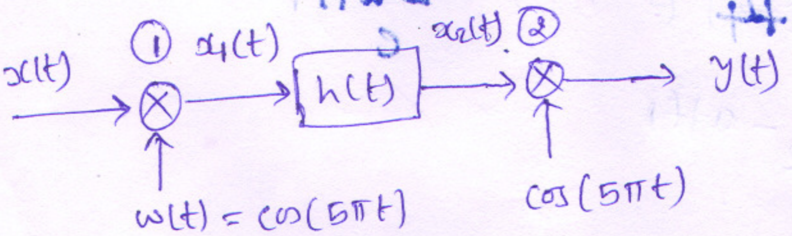
$$Y(f) = \frac{1}{(1/2)} X\left(\frac{f}{1/2}\right) + X(f)$$

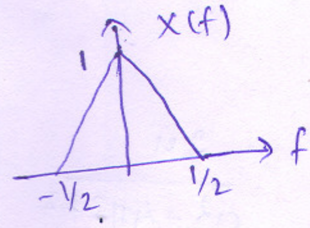
$$= 2X(2f) + X(f)$$

$$= 4\,\mathrm{sinc}(4f) + 2\,\mathrm{sinc}(2f)$$

we can take $y(t) = x(t) + x(t-1) + x(t+1)$ also.

Q.3





~~At first sum~~ At point ①.

Use $x(t)\,y(t) \Leftrightarrow X(f) * Y(f)$

$x(t) * y(t) \Leftrightarrow X(f)\,Y(f)$

we have $x_1(t) = x(t).\cos(5\pi t)$

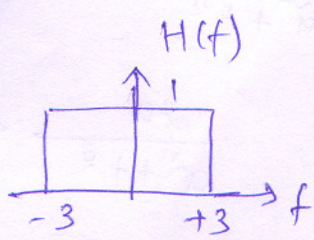
$$X_1(f) = X(f) * \left[\frac{1}{2}\left(\delta(f-5/2) + \delta(f+5/2)\right)\right]$$

$$= \frac{1}{2}\left[X(f-5/2) + X(f+5/2)\right]$$

then at point ②

$x_2(t) = x_1(t) * h(t) \Rightarrow X_2(f) = X_1(f).H(f)$

Now, $h(t) = \frac{\sin(6\pi t)}{\pi t} \longleftrightarrow H(f) = \mathrm{rect}.(f/6)$



...re $x_1(f)$ freq. range is within the $H(f)$ freq. range, so $x_1(f)$ will be multiplied by 1, and as an output we are getting $x_1(f)$ only.

→ At ~~point~~ $y(t)$

$$y(t) = x_2(t) \cdot \cos 5\pi t$$

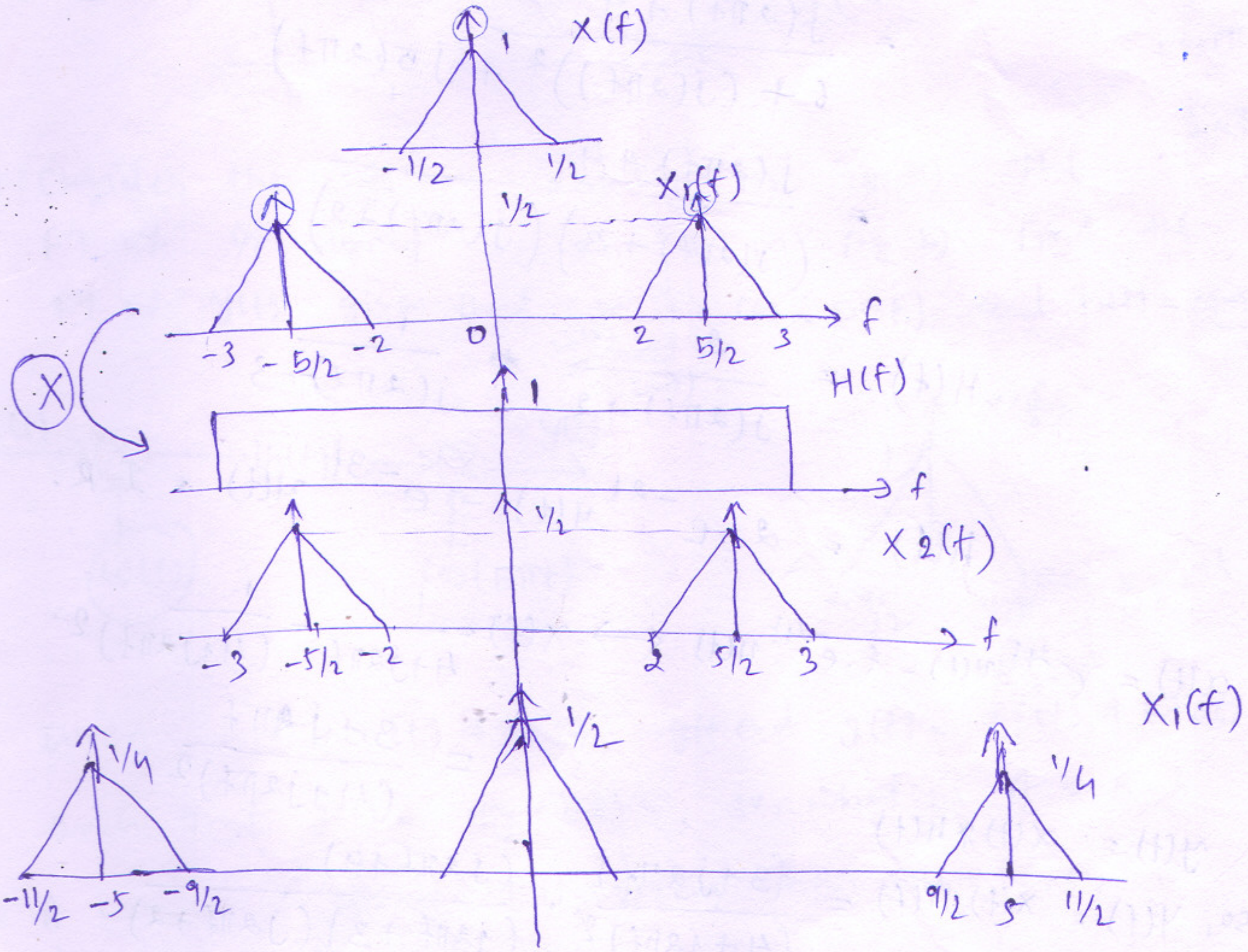
$$Y(f) = X_2(f) * \left[\frac{1}{2} \left(\delta(f - 5/2) + \delta(f + 5/2) \right) \right]$$

$$= \frac{1}{2} \left[X_2(f - 5/2) + X_2(f + 5/2) \right]$$

$$= \frac{1}{2} \left[X_1(f - 5/2) + X_1(f + 5/2) \right]$$

$$= \frac{1}{4} \left[X(f - 5/2 - 5/2) + X(f + 5/2 + 5/2) + X(f + 5/2 - 5/2) + X(f + 5/2 + 5/2) \right]$$

$$= \frac{1}{4} \left[X(f-5) + X(f+5) \right] + \frac{1}{2} X(f)$$



Q.4 $g(t) = x(3t) * h(3t)$

$$G(f) = \frac{1}{3}X\left(\frac{f}{3}\right)\cdot\frac{1}{3}H\left(\frac{f}{3}\right) = \frac{1}{9}X(f/3).H(f/3) = \frac{1}{9}Y\left(\frac{f}{3}\right) \quad \text{--- (1)}$$

$$g(t) = \cancel{\tfrac{1}{9}}.\ \cancel{x(3t)}.\ A\cdot y(Bt)$$

$$G(f) = \frac{A}{B}Y\left(\frac{f}{B}\right) \quad \text{-- (2)}$$

Comparing (1) & (2) we get $A = 1/3$, $B = 3$

Q.5

$$H(f) = \frac{j(2\pi f)+4}{6-(4\pi^2+2)+10j\pi f}$$

$$= \frac{j(2\pi f)+4}{6-(j(2\pi f))^2+j(10\pi f)}$$

$$= \frac{j(2\pi f)+4}{6+(j(2\pi f))^2+j5(2\pi f)}$$

$$= \frac{j(2\pi f)+4}{(j(2\pi f)+3)(j(2\pi f)+2)}$$

$$H(f) = \frac{2}{j(2\pi f)+2} - \frac{1}{j(2\pi f)+3}$$

$$h(t) = 2.e^{-2t}.u(t) - e^{-3t}.u(t) = I.R.$$

Here $x(t) = e^{-4t}.u(t) - t.e^{-4t}.u(t) \longleftrightarrow X(f) = \frac{1}{4+j2\pi f} - \frac{1}{(4+j2\pi f)^2}$

$$= \frac{3+j2\pi f}{(4+j2\pi f)^2}$$

Now, $y(t) = x(t) * h(t)$

so, $Y(f) = X(f).H(f) = \frac{(3+j2\pi f)}{(4+j2\pi f)^{\cancel{2}}}\cdot\frac{(j2\pi f+4)}{(j2\pi f+3)(j2\pi f+2)}$

$$Y(f) = \frac{1}{(j2\pi f+2)(j2\pi f+4)} = \frac{1/2}{j2\pi f+2} - \frac{1/2}{j2\pi f+4}$$

so, $\boxed{y(t) = \frac{1}{2}e^{-2t}.u(t) - \frac{1}{2}e^{-4t}.u(t)}$