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Solution - Tut - 12

(1)

a) maximum allowable time

$$= T = \frac{1}{2 \times 5} = 0.1 \text{ sec.}$$

b)
$$v_s(t) = \sum_{k=-\infty}^{+\infty} I_k \, \delta(t - 0.1k)$$

$$= 5 (\dots + \delta(t) + \delta(t-0.1) + \delta(t-0.2) + \dots).$$

$$\times (\cos 5\pi t + 0.5 \cos 10\pi t)$$

$$I_0 = 5 (\cos 5\pi(0) + 0.5 \cos 10\pi(0)) = 5(1+0.5) = \underline{7.5}$$

$$I_1 = 5 (\cos 5\pi(0.1) + 0.5 \cos 10\pi(0.1)) = 5(0 + 0.5(-1)) = \underline{-2.5}$$

$$I_2 = 5 (\cos 5\pi(0.2) + 0.5 \cos 10\pi(0.2))$$

$$= 5 (\cos \pi + 0.5 \cos 2\pi) = 5(-1 + 0.5 \times 1) = 5(-0.5) = \underline{-2.5}$$

$$I_4 = 5 (\cos 5\pi(0.4) + 0.5 \cos 10\pi(0.4))$$

$$= 5 (\cos 2\pi + 0.5 \cos 4\pi) = 5(1 + 0.5 \times 1) = \underline{7.5}$$

~~so~~, ~~b~~

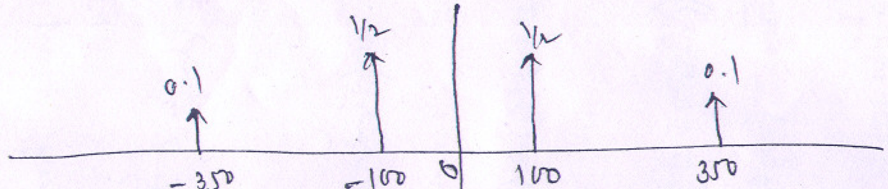
$$I_5 = 5 (\cos 5\pi(0.5) + 0.5 \cos 10\pi(0.5))$$

$$= 5 (\cos \frac{5\pi}{2} + 0.5 \cos 5\pi) = 5(0 + 0.5(-1)) = \underline{-2.5}$$

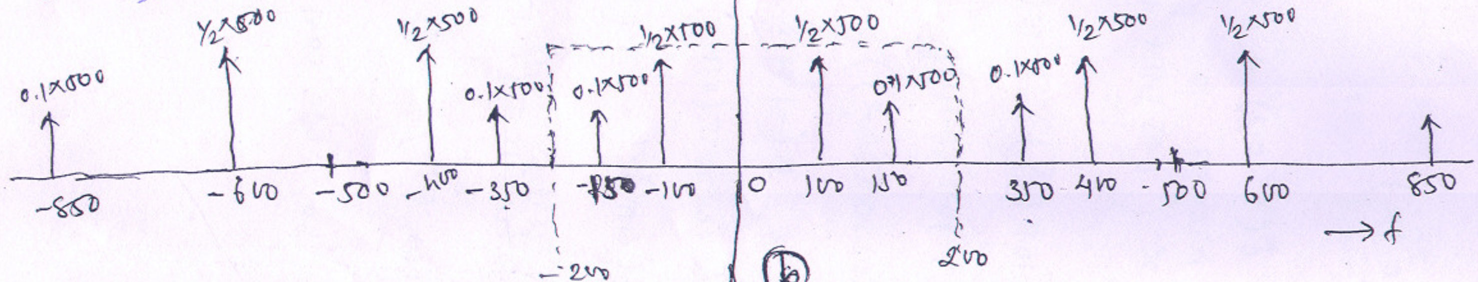
so, $I_0 = I_4$ & $I_1 = I_5$ so, we can say that

in general $\underline{I_k = I_{k+4}}$.

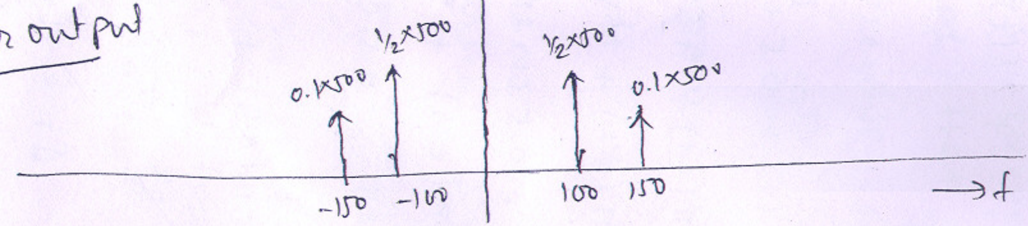
Spectrum of g(t) i.e. G(f)



repeating at every f_s



filter output



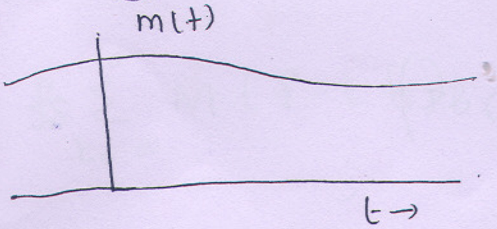
Time domain equation $= \frac{1}{2} \times 500 \times e^{j 2\pi 100 t} + \frac{1}{2} \times 500\, e^{-j 2\pi 100 t}$

$+ 50 \cdot e^{j 2\pi 150 t} + 50 \cdot e^{-j 2\pi 150 t}$

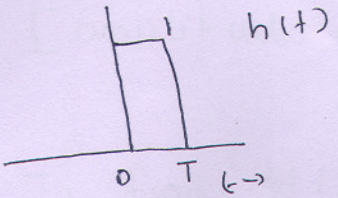
$= 500 \cos(2\pi 100 t) + 50 \cdot \cos(2\pi 150 t)$

2

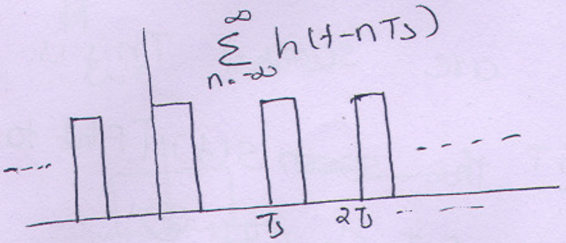
1



– original signal $m(t)$.

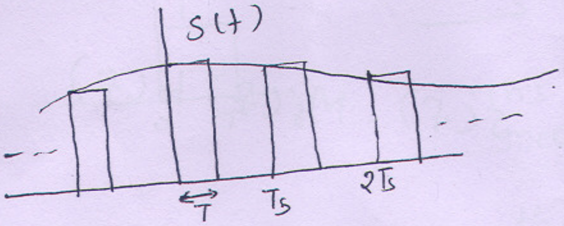


– rectangular pulse of duration T and amplitude 1.



rectangular pulse train

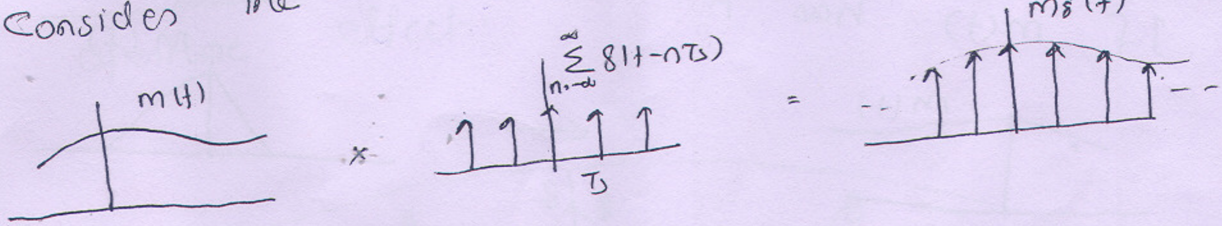
Now do $m(t) \times \sum_{n=-\infty}^{\infty} h(t-nT_s)$



$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t-nT_s) \quad \text{—} \ (1)$$

This is the output of Flat Top sampling.

Consider the case of ideal sampling.



$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t-nT_s)$$

convolve $m_\delta(t)$ with $h(t)$

$$m_\delta(t) * h(t) = \int_{-\infty}^{\infty} m_\delta(t)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t-nT_s) \right) h(t-\tau)\, d\tau$$

$$\sum_{n=-\infty}^{\infty} m(nT_s) \left(\int_{-\infty}^{\infty} \delta(t-nT_s)\, h(t-\tau)\, d\tau \right)$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s) \left(\delta(t-nT_s) * h(t) \right)$$

$$= \sum_{n=-\infty}^{\infty} m(nT_s)\, h(t-nT_s) \quad \text{— ②}$$

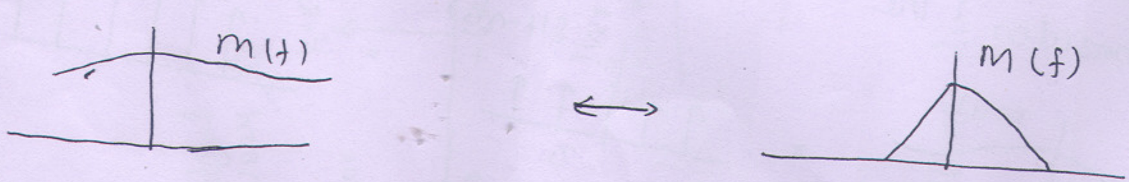
The equations ① and ② are same. This is done to Find the F.T of the ~~speech~~ s(t) [Flat top sampled signal] by using the FT of equ ②.

So

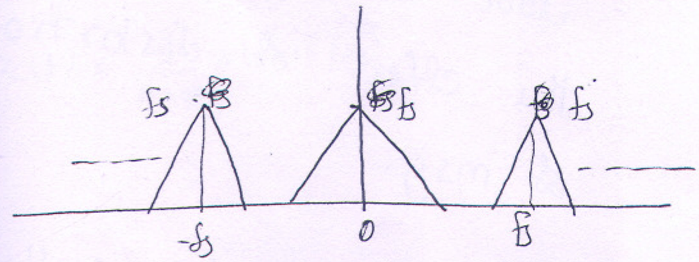
$$s(t) = m_\delta(t) * h(t) \Rightarrow S(f) = M_\delta(f)\, H(f)$$

$$= f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)\, H(f)$$

If m(t) has F.T M(f) as shown



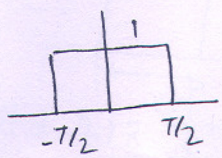
$$f_s \sum_{k=-\infty}^{\infty} M(f - k f_s) \longrightarrow$$



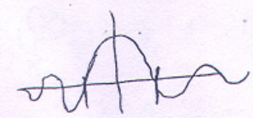
[amplitudes will be multiplied by fs].

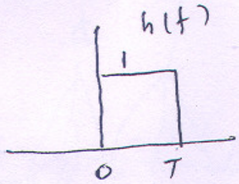
To find H(f)

we know



$\longrightarrow$ T sinc(f T)

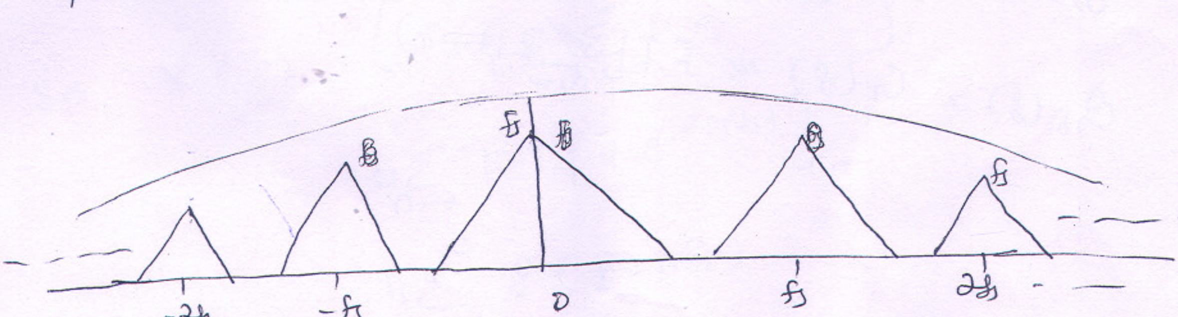




$\longrightarrow$ $T \operatorname{sinc} fT \, e^{-j2\pi f \frac{T}{2}}$ [This is a sinc fun, the gain of

This will get multiplied by spectrums

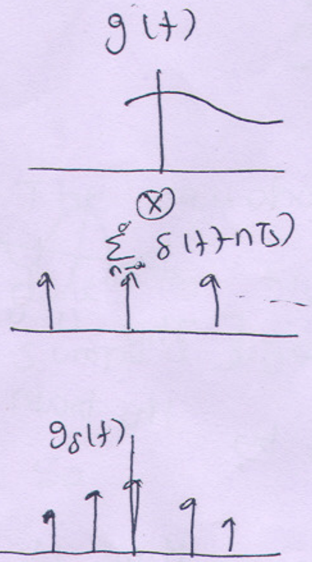
So by using this FT sampling there is an amplitude distortion of T sinc fT and phase distortion / phase delay of T/2. This is called aperture effect.

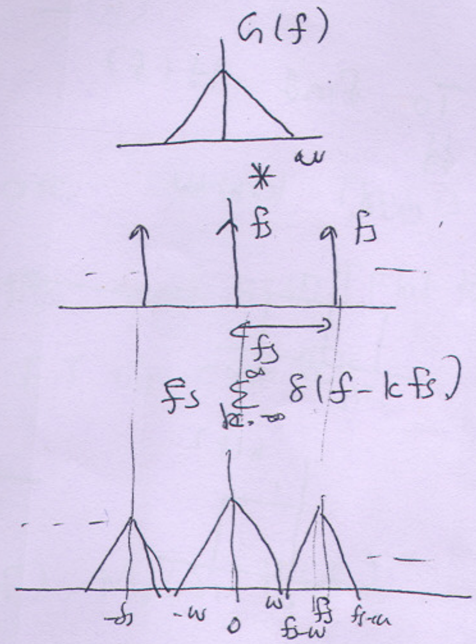


This is the spectrum of resultant signal

As the width of pulse reduces, the first lobe of sinc fn widens and it will reduce the effect of distortion. to the ~~midd~~ spectrum of msg.

Note: → To find F.T of ideally sampled signal ms(t)





$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s) \qquad g(t) \longleftrightarrow G(f)$$

$$g_\delta(t) = g(t) \times \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$G_\delta(f) = G(f) * FT\left[\sum_{n=-\infty}^{\infty} \delta(t-nT_s)\right]$$

$F.T\left[\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right]$; $x(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT_s)$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_s t}$$

$$C_n = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} x(t)\, e^{-j2\pi n f_s t}\, dt$$

$$= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \sum_{n=-\infty}^{\infty}\delta(t-nT_s)\, e^{-j2\pi n f_s t}\, dt$$

$$= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2}\left[\cdots + \delta(t+2T_s) + \delta(t+T_s) + \underline{\delta(t)} + \delta(t-T_s) + \delta(t-2T_s) \cdots\right]$$

In the interval $-T_s/2$ to $T_s/2$ only $\delta(t)$ will come

$$= \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \delta(t)\, e^{0}\, dt = \underline{\underline{1/T_s}}$$

$$x(t) = \sum_{n=-\infty}^{\infty}\frac{1}{T_s} e^{jn\omega_s t} = f_s \sum_{n=-\infty}^{\infty} e^{j2\pi n f_s t}.$$

F.T of $1 = \delta(f)$

so $$X(f) = F.T\left[f_s\sum_{n=-\infty}^{\infty} 1\cdot e^{j2\pi n f_s t}\right]$$

$$= f_s\sum_{n=-\infty}^{\infty} F.T\left[1\cdot e^{j2\pi n f_s t}\right]$$

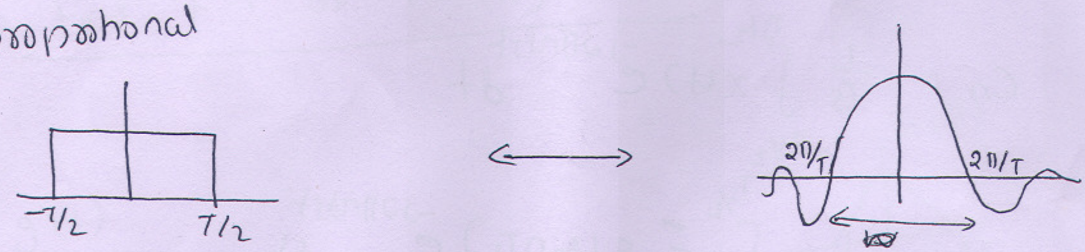
$$= f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s)$$

$$= f_s\sum_{k=-\infty}^{\infty}\delta(f-kf_s)$$

But the aperture effect should be considered.

This is determined by the width of rect pulse.

The width of central lobe of sinc function is determined by width of rect pulse. They are inversely proportional



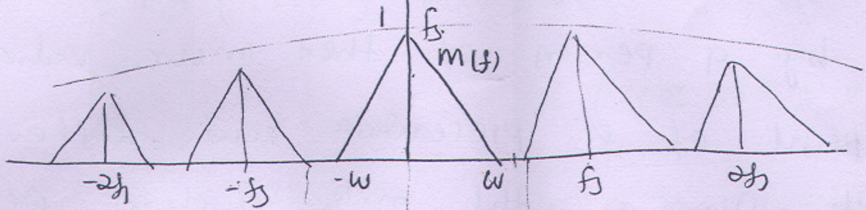
As T ↑ the width of central lobe reduces and will distort the msg spectrum corresponding to $k=0$ in the $f_s\sum_{k=-\infty}^{\infty} M(f-kf_s)\,H(f)$. This will make reconstruction impossible. So the width of rect pulse should be as less as possible.

In practice $T \le 0.1T_s$ is the condition where the distortion is very less and even the use of equaliser may ~~also~~ not be needed under this condition.

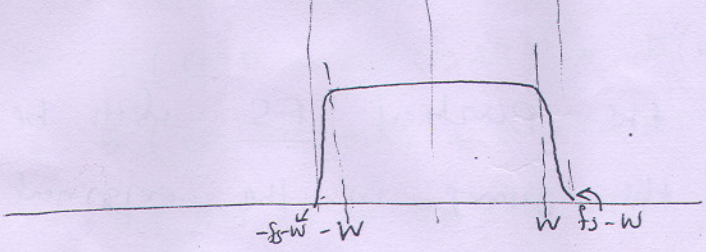
$$G_\delta(f) = G(f) * f_s \sum_{k=-\infty}^{\infty} \delta(f - kf_s)$$

$$= f_s \sum_{k=-\infty}^{\infty} G(f - kf_s).$$

b) Spectrum of ~~MS~~ $(M_\delta(f)) = f_s \sum_{k=-\infty}^{\infty} M(f-kf_s)H(f)$

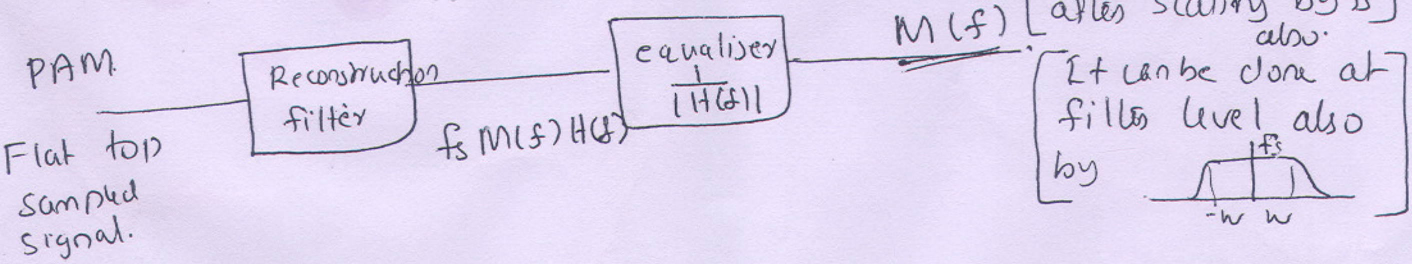


we can use a reconstruction filter of BW $(f_s - W)$ to get back the signal



Now the output will be $f_s M(f) H(f)$.

~~Now~~ But we require only $M(f)$. So to remove the effect of $H(f)$ we use an equaliser



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$$G_\delta(f) = G(f) * f_s \sum_{k=-\infty}^{\infty} \delta(f - kf_s)$$

$$= f_s \sum_{k=-\infty}^{\infty} G(f - kf_s).$$

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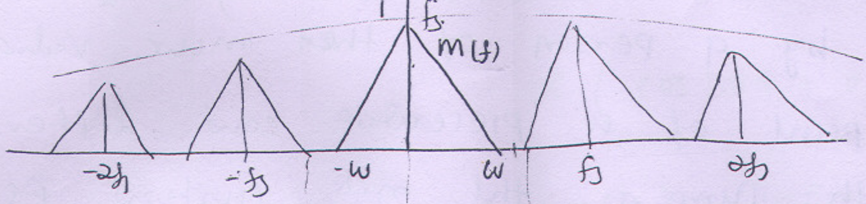

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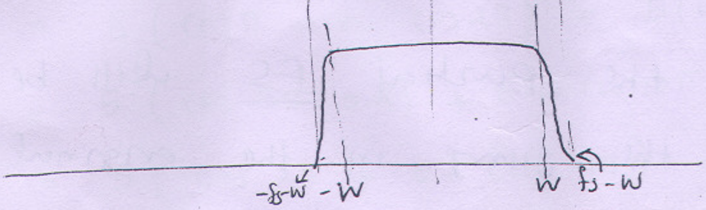

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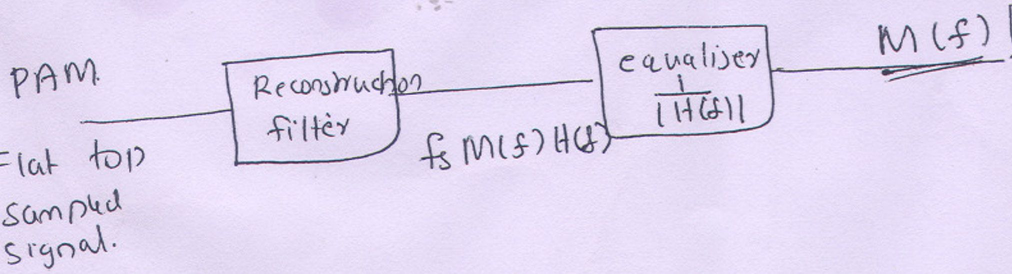
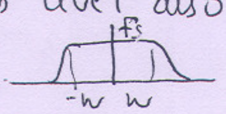


$M(f)$ [after scaling by f_s]

[It can be done at filter level also by



]

Gibbs phenomena:

→ n^{th} partial sum of the ~~jump~~ FS has large discontinuities near the jump, which might increase the max. of the partial sum above that of the function itself. The overshoot does not die out as the freq. increases, but reaches a finite limit.

→ It turns out that the FS exceeds the hight of the square by 9 person of that max value

→ so, at any point of a piecewise cont. differentiable function with jump a, the n^{th} partial FS will overshoot this jump by approximately $a.(0.09)$ at one end and undershoot by the same amount at the other end.

→ so, the jump in the partial FS. will be about 18% larger than the jump in the original function.

→ It reflects the difficulty inherent in approximating a discontinuous function by a finite series of continuous sine and cosine waves.