CT 203 TUTORIAL - I

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I a) Calculate the period of the mentioned signals

1) $x_1(t) = e^{j3t}$

2) $x_2(t) = e^{3t}$

b) Determine which of the following signals are periodic, if periodic, find the fundamental frequency

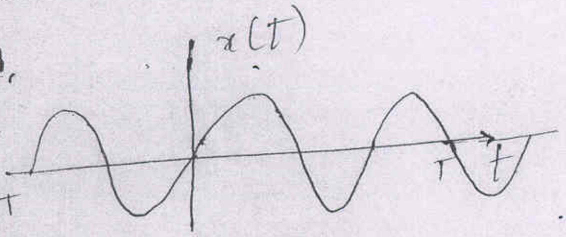
1) $x(t) = \cos(18\pi t) + \sin(12\pi t)$

2) $x(t) = \sin\left(\frac{2\pi}{3}t\right)\cos\left(\frac{4\pi}{5}t\right)$

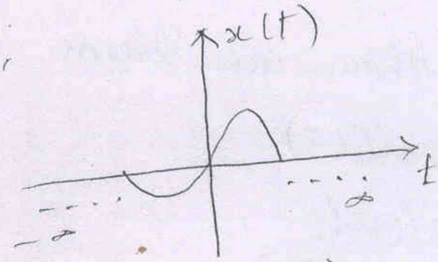
3) $x(t) = \cos 3t + \sin 5\pi t$

c) Determine whether the signals are periodic

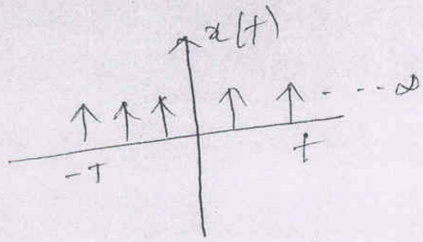
i,



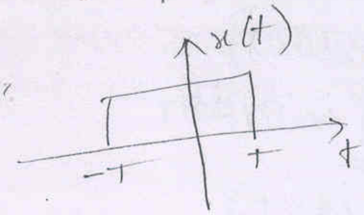
ii,



iii,



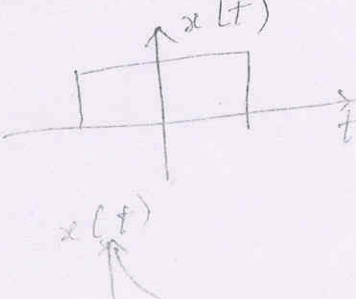
iv,



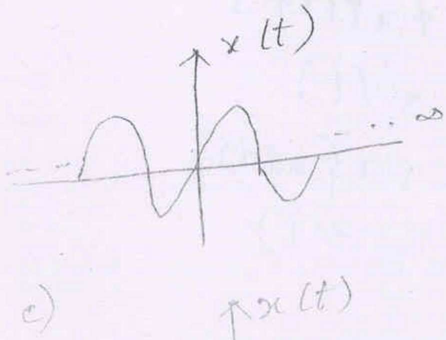
d) How many cycles will be generated in one hour when the signal is $x(t) = \cos 2000\pi t$

II Determine which of the signals are energy and power signals

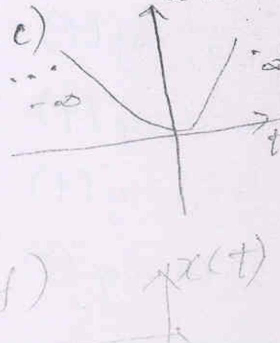
a)



b)



c)



d)

e)

f)

2) Determine which of them are energy and power signals.

i, $x(t) = e^{-t}$ where $t \geq 0$.

ii, $x(t) = e^{-|t|}$ $\forall t$

iii, $x(t) = 2 \cos 2\pi 100t + 5 \cos 2\pi 50t$

III Which of the following are Deterministic and Random Signals

i, $x(t) = e^{-t}$ $t \geq 0$

ii, Thermal noise

iii, Temperature at 10:00 AM on everyday

IV Which of them are linear

i, $y(t) = x(t)\, x(t-2)$

ii, $y(t) = x(t) \cos 6t$

iii, $y(t) = \sin\{x(t)\}$

iv, $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

v, $y(t) = 2x(t) + 3$

vi, $y(t) = |x(t)|$

V Classify which of the following are linear and Time-invariant

i, $y(t) = t\, x(t) + 3$

ii, $y(t) = x^3(t)$

iii, $y(t) = \sin\{x(t)\}$

iv, $y(t) = e^{-x(t)}$

TUTORIAL - I - ANSWERS   CT203

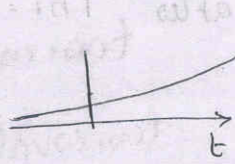
I

1.) $x_1(t) = e^{j3t} = e^{j\omega_0 t}$ [compare with general representation]

$\omega_0 = 3$

$2\pi f = \frac{2\pi}{T} = 3 \Rightarrow T = \underline{\underline{\frac{2\pi}{3}}}$ period $= \frac{2\pi}{3}$ sec

$x_2(t) = e^{3t} \rightarrow$



$\rightarrow$ Not periodic.

2

A $x(t) = \cos 18\pi t + \sin 12\pi t$

period of signal = LCM of ~~perioc~~ individual periods

= Reciprocal of GCD of frequencies

$\cancel{2\pi/T_1 = 18\pi}$ $2\pi f_1 = 18\pi \Rightarrow f_1 = 9$ Hz

$\cancel{\frac{1}{T_1} = 9}$ $2\pi f_2 = 12\pi \Rightarrow f_2 = 6$ Hz

GCD of f_1 & f_2 = $\underline{\underline{3 \text{ Hz}}} = f_0$

B $x(t) = \sin\left(\frac{2\pi t}{3}\right) \cos\left(\frac{4\pi t}{5}\right)$

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$= \frac{1}{2}\left[\sin\left(\frac{22\pi t}{15}\right) + \sin\left(-\frac{2\pi t}{15}\right)\right]$

$= \frac{1}{2}\left[\sin\left(\frac{22\pi t}{15}\right) - \sin\left(\frac{2\pi t}{15}\right)\right]$

Take GCD as above $f_0 = \underline{\underline{\frac{1}{15}}}$

c) $x(t) = \cos 3t + \sin 5\pi t$ No GCD So not periodic

$f_1 = 3/2\pi$ $f_2 = 5/2$

3)
(i) Non periodic (ii) Non periodic (iii) Periodic (iv) Non periodic

→ Defn of periodic signal → $x(t) = x(t+T)$ for any t ranging from $-\infty$ to $+\infty$.

4 No of cycles in 1 sec = frequency = $\frac{2000\pi}{2\pi} = 1000$

No of cycles elapsed after 1 hr = $60 \times 60 \times 1000$

II

1) (i) Energy → [finite duration signal] ✓ $E = \int_{-\infty}^{\infty} e^{-2t} dt = \int_{0}^{\infty} e^{-2t} dt = -\frac{1}{2}(e^{-2t})_{0}^{\infty} = -\frac{1}{2}(0-1) = \frac{1}{2}$ finite value

(ii) ~~Power → [all periodic signals are power signals]~~

(iii) ~~NENP → [as $t \to \infty$ amp $\to \infty$]~~

(iv) Energy → [as $t \to \infty$ amp $\to 0$] ✓

(v) ~~power → const amp signals are power signals~~ (iii) Power signal

(vi) ~~power → combination of P+E signal is Power signal~~

2)
~~(i) Energy~~
~~(ii) NENP~~
~~(iii) Energy~~

(2) (i) Energy

(ii) Energy

(iii) Power

III

1) Deterministic

2) Random

3) Random

IV

1) non linear ~~TI~~ Time Invariant

2) linear ~~TV~~ Time variant

3) Non linear Time Invariant

4) Linear Time variant

5 Non linear Time Invariant

6 Non linear Time Invariant

7 Non linear Time Invariant

8 Non linear Time Invariant

V

1) Non linear Time variant

2) Non linear Time Invariant

3) Non linear Time Invariant

4 Non linear Time Invariant

5 linear Time variant

IV

1. $y(t) = x(t)x(t-2)$

$x_1(t) \Rightarrow y_1(t) = x_1(t)x_1(t-2)$

$x_2(t) \Rightarrow y_2(t) = x_2(t)x_2(t-2)$

$x_1(t)+x_2(t) \Rightarrow \left(x_1(t)+x_2(t)\right)\left(x_1(t-2)+x_2(t-2)\right) = y_3(t)$

$\Rightarrow y_3(t) \neq y_1(t)+y_2(t)$ $\Rightarrow$ Non linear

2. $y(t) = x(t)\cos 6t$

$x_1(t) \Rightarrow x_1(t)\cos 6t = y_1(t)$

$x_2(t) \Rightarrow x_2(t)\cos 6t = y_2(t)$

$x_1(t)+x_2(t) \Rightarrow \left(x_1(t)+x_2(t)\right)\cos 6t = y_3(t)$

$\Rightarrow y_3(t) = y_1(t)+y_2(t)$

$\Rightarrow$ Linear

3) $y(t) = \sin\{x(t)\}$

$x_1(t) \Rightarrow y_1(t) = \sin\{x_1(t)\}$

$x_2(t) \Rightarrow y_2(t) = \sin\{x_2(t)\}$

$x_1(t) + x_2(t) \Rightarrow y_3(t) = \sin\{x_1(t) + x_2(t)\}$

$\Rightarrow y_3(t) \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-linear

4) $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$x_1(t) \Rightarrow \int_{-\infty}^{t} x_1(\tau)\, d\tau = y_1(t)$

$x_2(t) \Rightarrow \int_{-\infty}^{t} x_2(\tau)\, d\tau = y_2(t)$

$x_1(t) + x_2(t) \Rightarrow \int_{-\infty}^{t} (x_1(\tau) + x_2(\tau))\, d\tau = y_3(t)$

$\Rightarrow y_3(t) = y_1(t) + y_2(t)$

$\Rightarrow$ linear

5) $y(t) = 2x(t) + 3$

$x_1(t) \Rightarrow y_1(t) = 2x_1(t) + 3$

$x_2(t) \Rightarrow y_2(t) = 2x_2(t) + 3$

$x_1(t) + x_2(t) \Rightarrow y_3(t) = 2(x_1(t) + x_2(t)) + 3$

$\Rightarrow y_3(t) \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-linear

6) $y(t) = |x(t)|$

$y_1(t) = |x_1(t)|$

$y_2(t) = |x_2(t)|$

$T[x_1(t) + x_2(t)] = |x_1(t) + x_2(t)| \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-linear

V

1. $y(t) = tx(t) + 3$

$x_1(t) \Rightarrow t x_1(t) + 3 = y_1(t)$

$x_2(t) \Rightarrow t x_2(t) + 3 = y_2(t)$

$x_1(t) + x_2(t) \Rightarrow t[x_1(t) + x_2(t)] + 3 \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-linear

delay in $x(t)$ by t_0

$\Rightarrow y_1(t) = t x(t - t_0) + 3$

delay in $y(t)$ by t_0

$\Rightarrow y_2(t) = (t - t_0) x(t - t_0) + 3$

$\Rightarrow y_2(t) \neq y_1(t)$

$\Rightarrow$ Time-variant

2. $y(t) = x^3(t)$

$x_1(t) \Rightarrow y_1(t) = x_1^3(t)$

$x_2(t) \Rightarrow y_2(t) = x_2^3(t)$

$x_1(t) + x_2(t) \Rightarrow y_3(t) = (x_1(t) + x_2(t))^3 \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-linear

delay in $x(t)$ by t_0

$\Rightarrow y_1(t) = x^3(t - t_0)$

delay in $y(t)$ by t_0

$\Rightarrow y_2(t) = x^3(t - t_0)$

$\Rightarrow y_2(t) = y_1(t)$

$\Rightarrow$ Time-invariant

3. $y(t) = \sin\{x(t)\}$

$x_1(t) \Rightarrow y_1(t) = \sin\{x_1(t)\}$

$x_2(t) \Rightarrow y_2(t) = \sin\{x_2(t)\}$

$x_1(t)+x_2(t) \Rightarrow y_3(t) = \sin\{(x_1(t)+x_2(t))\}$

$\Rightarrow y_3(t) \neq y_1(t) + y_2(t)$

$\Rightarrow$ Non-linear

delay in $x(t)$ by t_0 $\Rightarrow y_1(t) = \sin\{x_1(t-t_0)\}$

delay in $y(t)$ by t_0 $\Rightarrow y_2(t) = \sin\{x(t-t_0)\}$

$\Rightarrow y_2(t) = y_1(t)$

$\Rightarrow$ Time-invariant

4. $y(t) = e^{-x(t)}$

$x_1(t) \Rightarrow y_1(t) = e^{-x_1(t)}$

$x_2(t) \Rightarrow y_2(t) = e^{-x_2(t)}$

$x_1(t)+x_2(t) \Rightarrow y_3(t) = e^{-(x_1(t)+x_2(t))}$

$\Rightarrow y_3(t) \neq y_1(t) + y_2(t)$

Non linear

Delay in $x(t)$ by t_0 $\Rightarrow y_1(t) = e^{-x(t-t_0)}$

Delay in $y(t)$ by t_0 $\Rightarrow y_2(t) = e^{-x(t-t_0)}$

$\Rightarrow y_2(t) = y_1(t)$

$\Rightarrow$ Time invariant

II
2)
i) $x(t) = e^{-t} \quad t \geq 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} e^{-2t} dt = \left[\cancel{\frac{e^{-2t}}{-2}} \right]$$

$$= \int_{0}^{\infty} e^{-2t} dt = \left[\frac{e^{-2t}}{-2}\right]_0^{\infty} = -1/2 \left[e^{-2\times\infty} - e^{0} \right] = \underline{\underline{1/2}} = \text{finite}$$

(ii) $x(t) = e^{-|t|} \quad \forall t$

$$E = \int_{-\infty}^{\infty} e^{-|t|} dt = \int_{-\infty}^{0} e^{+t} dt + \int_{0}^{\infty} e^{-t} dt$$

$$= \left[e^{t}\right]_{-\infty}^{0} + \left[\frac{e^{-t}}{-1}\right]_0^{\infty}$$

$$= (1-0) + \left(\frac{e^{-\infty}-1}{-1}\right)$$

$$= 1 + 1 = \underline{\underline{2}} \quad \text{finite}$$

(iii) $x(t) = 2\cos 2\pi 100t + 5\cos 2\pi 50t \qquad T = \frac{1}{50}$

$$E = \cancel{\int_{-\infty}^{\infty} |x(t)|^2 dt}$$

$$= \int_{-\infty}^{\infty} \cancel{(4\cos^2 2\pi 100t + 25\cos^2 2\pi 50t + 20 \cos 2\pi 100t \cos 2\pi 50t)}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 = 4\cos^2 2\pi 100t + 25\cos^2 2\pi 50t + 20 \cos 2\pi 100t \cos 2\pi 50t$$

$$= \int_{-\infty}^{\infty}$$

$$P_{av} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \Big[4(1 + \cos 2\pi 200t) + \frac{25}{2}(1 + \cos 2\pi 100t) + \frac{20}{2}\big[\cos 2\pi 150t + \cos 2\pi 50t\big]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \Big[2 + 2\cos 2\pi 200t + \frac{25}{2} + \frac{25}{2}\cos 2\pi 100t + 10\cos 2\pi 150t + 10\cos 2\pi 50t \Big]$$

~~T = 1/50~~ period is 1/50

$$= \lim_{T\to\infty} \frac{1}{2T} \Big[2t + 2\frac{\sin 2\pi 200t}{2\pi 200t} + \frac{25}{2}t + \frac{25}{2}\frac{\sin 2\pi 100t}{2\pi 100} + 10\frac{\sin 2\pi 150t}{2\pi 150} + 10\frac{\sin 2\pi 50t}{2\pi 50} \Big]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \Big[2(2T) + \frac{25}{2}(2T) \Big]$$

$$= 2 + \frac{25}{2} = \frac{29}{2}$$ finite power signal