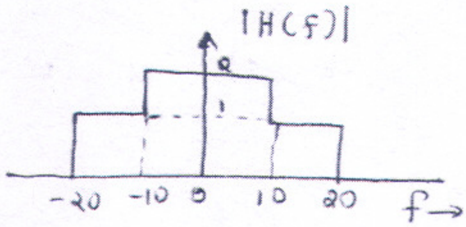
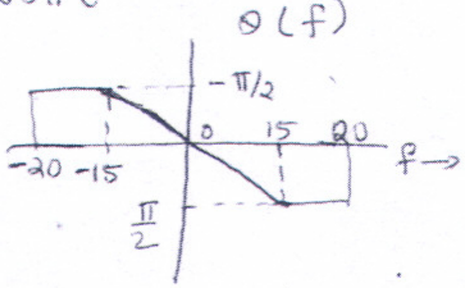
TUTORIAL QUESTIONS - 7

1) Consider the system $H(f)$ with magnitude and phase response shown in figure below. Find output for the given inputs and check what type of distortion is present at the output?. Find the delay introduced by the system to the inputs?.

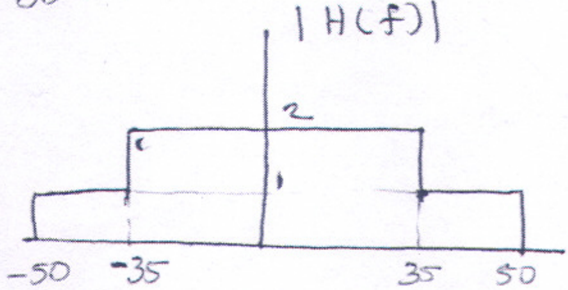
1) $x_1(t) = 2\cos 10\pi t + \sin 26\pi t$

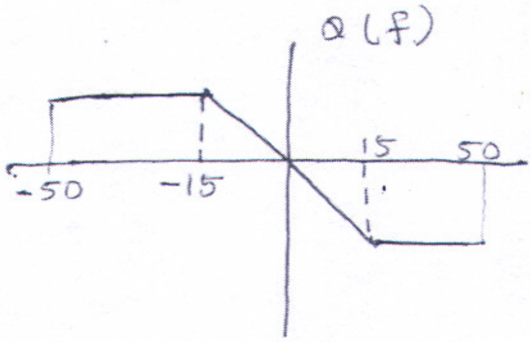
2) $x_2(t) = 4\cos 5\pi t + \sin 60\pi t$



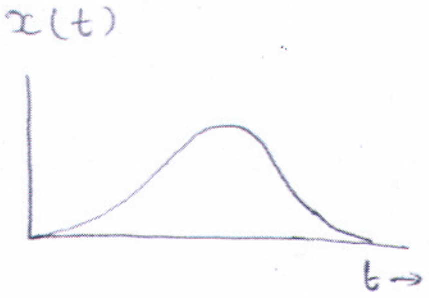


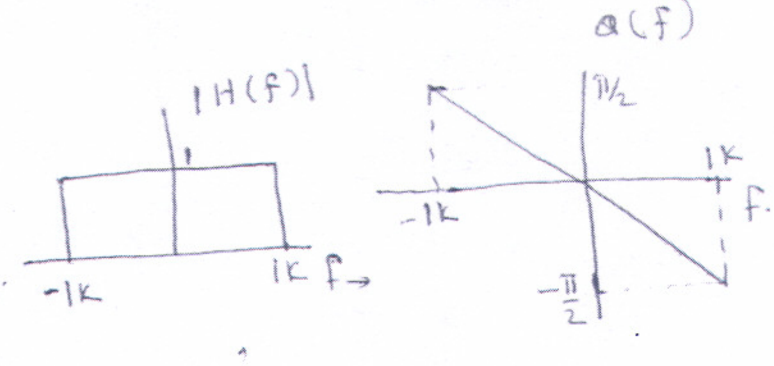
2) Find the outputs for the same set of inputs with a different system whose $|H(f)|$ and $\theta(f)$ are shown below. Find the delay introduced by the system to both the inputs.



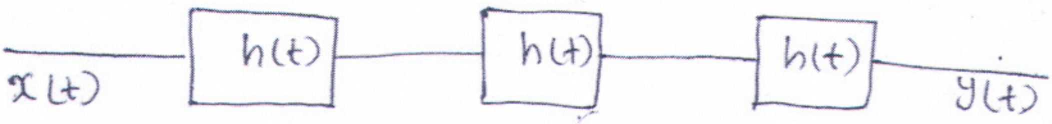


3) The signal $x(t)$ is given as shown in figure. ~~If~~ The maximum frequency present in the signal is 100Hz. Find the ~~o/t~~ output when the signal is passed through a system $H(f)$ with $|H(f)|$ and $\theta(f)$ as shown below.





4. For the system shown in figure if $x(t) = \cos t$ and $h(t) = \frac{1}{\pi t}$, then find the output $y(t)$.



5. Find the ~~Frequency Response~~ Hilbert Transform of $\frac{1}{1+t^2}$

F.T. of

Tutorial - 7 - Answers.

Q.1

1) For $x_1(t) = 2\cos 10\pi t + \sin 26\pi t$

Here we have two frequency components

$$f_1 = 5\text{ Hz}, \quad f_2 = 13\text{ Hz}$$

so, for given $H(f)$ we have output as

$$x_1(t) = 2|H(f_1)|\cos(10\pi t + \theta(f_1)) + |H(f_2)|\sin(26\pi t + \theta(f_2))$$

where,

$|H(f_1)| = 2$ & $|H(f_2)| = 1$

~~$\theta(f_1)$~~

$15 \rightarrow -\pi/2$

$5 \rightarrow ?$ $\Rightarrow$ $\dfrac{\cancel{5}\times -\pi/2}{\cancel{15}\,3} = -\pi/6 = \theta(f_1)$

$15 \rightarrow -\pi/2$

$13 \rightarrow ?$ $\Rightarrow$ $\dfrac{13\times -\pi/2}{15} = \dfrac{-13\pi}{30} = \theta(f_2)$

so, final output is

$$y_1(t) = 2(2)\cos(10\pi t - \pi/6) + \sin\left(26\pi t - \frac{13\pi}{30}\right)$$

$$= 4.\cos 10\pi\left(t - \frac{\pi}{6\times 10\pi}\right) + \sin 26\pi\left(t - \frac{13\pi}{30\times 26\pi}\right)$$

$$y_1(t) = 4.\cos 10\pi\left(t - \frac{1}{60}\right) + \sin 26\pi\left(t - \frac{1}{60}\right)$$

→ so, here there will be Amplitude distortion but not phase distortion As delay is same for both frequencies.

ii) similarly, we can get o/p for $x_2(t)$ as below

$$y_2(t) = 8\cos\left(5\pi t - \frac{\pi}{12}\right) + 0.\sin(60\pi t)$$

$$y_2(t) = 8\cos(5\pi t - \pi/12)$$

Q.2.

1) For $x_1(t) = 2\cos 10\pi t + \sin 26\pi t$

$|H(f_1)| = 2$, $|H(f_2)| = 2$

$\theta(f_1) = -\pi/6$, $\theta(f_2) = \frac{-13\pi}{30}$

So, $y_1(t) = 2|H(f_1)|\cos(10\pi t - \pi/6) + 2\cdot\sin\left(26\pi t - \frac{13\pi}{30}\right)$

$= 4\cos 10\pi\left(t - \frac{\pi}{6\times 10\pi}\right) + 2\cdot\sin 26\pi\left(t - \frac{13\pi}{30\times 26\pi}\right)$

$= 4\cdot\cos 10\pi\left(t - \frac{1}{60}\right) + 2\sin 26\pi\left(t - \frac{1}{60}\right)$

Here, there is no phase and no amplitude distortion.

ii) for $x_2(t) = 4\cos 5\pi t + \sin 60\pi t$

$|H(f_1)| = 2$, $|H(f_2)| = 2$

$\theta(f_1) = -\pi/12$, $\theta(f_2) = -\pi/2$

So, $y_2(t) = 2|H(f_1)|\cos(5\pi t + \theta(f_1)) + |H(f_2)|\sin(60\pi t + \theta(f_2))$

$= 4\cdot\cos(5\pi t - \pi/12) + 2\cdot\sin(60\pi t - \pi/2)$

$= 4\cdot\cos 5\pi\left(t - \frac{\pi}{12\times 5\pi}\right) + 2\sin 60\pi\left(t - \frac{\pi}{2\times 60\pi}\right)$

$y_2(t) = 4\cdot\cos 5\pi\left(t - \frac{1}{60}\right) + 2\sin 60\pi\left(t - \frac{1}{120}\right)$

So, Here there will be phase distortion and there wont be Amplitude distortion.

Q.3 Here, the maximum frequency present in the input signal is 100 Hz.
Here from the given ~~phas~~ Amplitude spectrum, we can say that there wont be any amplitude distortion as magnitude spectrum is constant for given range of frequencies

But from phase distortion for 100 Hz there will be phase delay of $-\pi/20$.

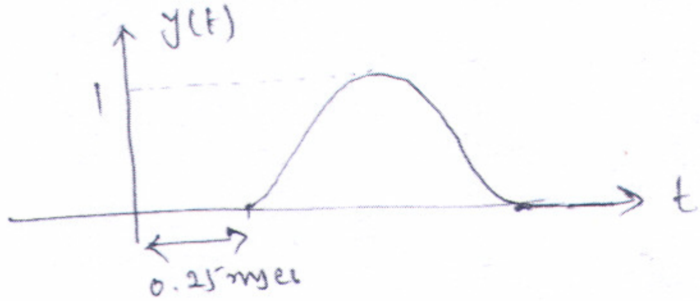
$$1000 \rightarrow -\pi/2$$
$$100 \rightarrow ?$$
$$\Rightarrow \frac{100 \times -\pi/2}{1000} = -\pi/20$$

So, for ~~add~~ time delay in the o/p signal

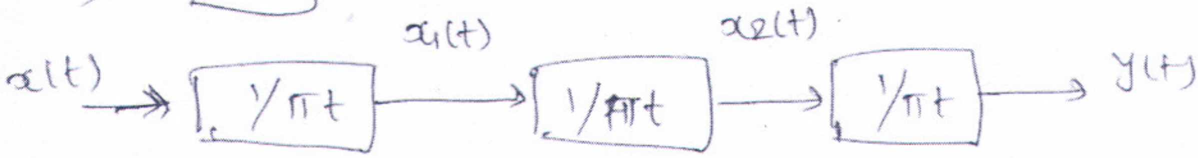
$$2\pi f \tau = +\pi/20$$

$$\tau = + \frac{1}{40 \times 100} = 0.25 \text{ msec.}$$

the o/p is



Q.4 Here



$$x_1(t) = x(t) * \frac{1}{\pi t} = \cos t * \frac{1}{\pi t} = \cos(t - \pi/2)$$

$$x_1(t) = \sin t$$

$$\rightarrow x_2(t) = \sin t * \frac{1}{\pi t} = \sin(t - \pi/2) = -\cos t$$

$$\rightarrow y(t) = -\cos t * \frac{1}{\pi t} = -\cos(t - \pi/2)$$

$$\boxed{y(t) = -\sin t}$$

Q.5 $x(t) = \frac{1}{1+t^2}$ $h(t) = \frac{1}{\pi t}$

$$\hat{H}(f) = H(f) \cdot X(f)$$

For $X(f)$

for $\frac{2a}{a^2+t^2} \leftrightarrow 2\pi e^{-2\pi a|f|}$

$\left(\because \frac{2a}{a^2+4\pi^2 f^2} \leftrightarrow e^{-a|t|} \right.$

so, from Duality

$$\frac{2a}{a^2+4\pi^2 t^2} \leftrightarrow e^{-a|f|}$$

$$\frac{1}{2\pi} \frac{2a/2\pi}{(a/2\pi)^2 + t^2} \leftrightarrow e^{-a|f|}$$

taking $a/2\pi = \hat{a}$

$$\frac{2\hat{a}}{\hat{a}^2+t^2} \leftrightarrow 2\pi e^{-a|f|} = 2\pi e^{-2\pi\hat{a}|f|}$$

so, for $\frac{2(1)}{(1)^2+t^2} \leftrightarrow 2\pi e^{-2\pi(1)|f|}$

$$\frac{1}{1+t^2} \leftrightarrow \pi e^{-2\pi|f|}$$

$$\hat{H}(f) = (-j\,\mathrm{sgn}(2\pi f)) \cdot \pi e^{-2\pi|f|}$$