Q.1 Here we need to prove,

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 . df$$

$$LHS = \int_{-\infty}^{+\infty} |x(t)|^2 . dt = \int_{-\infty}^{+\infty} x(t) . x^*(t) . dt$$

$$= \int_{-\infty}^{+\infty} x(t) \underbrace{\left[\int_{-\infty}^{+\infty} X^*(f) . e^{-j2\pi ft} . df \right]}_{\left[\int_{-\infty}^{+\infty} X(f) . e^{j2\pi ft} . df \right]^*} . dt$$

now, reversing the order of integration.

$$= \int_{-\infty}^{+\infty} X^*(f) . \left[\int_{-\infty}^{+\infty} x(t) . e^{-j2\pi ft} . dt \right] . df$$

$$= \int_{-\infty}^{+\infty} X^*(f) . X(f) . df$$

$$so, \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 . df .$$

Q.2 i) $\int_T \cos n\omega_0 t . \cos m\omega_0 t . dt$ (for $n \neq m$)

$$= \int_T \frac{\cos(n+m)\omega_0 t + \cos(n-m)\omega_0 t}{2} . dt$$

$$= \left[\frac{\sin(n+m)\omega_0 t}{2(n+m)\omega_0} + \frac{\sin(n-m)\omega_0 t}{2(n-m)\omega_0} \right]_0^T$$

now, here $\omega_0 = \frac{2\pi}{T}$

$$= \left[\frac{\sin (n+m) \frac{2\pi}{T} . t}{2(n+m) \frac{2\pi}{T}} + \frac{\sin (n-m) \frac{2\pi}{T} . t}{2(n-m) \frac{2\pi}{T}} \right]_0^T$$

$$= \left[\frac{\sin (n+m) \frac{2\pi}{T} . T}{2(n+m) \frac{2\pi}{T}} + \frac{\sin (n-m) \frac{2\pi}{T} . T}{2(n-m) \frac{2\pi}{T}} \right] - [0 - 0]$$

$$= [0 + 0] - [0 - 0]$$

(for $n \neq m$)

$$= \underline{\underline{0}}$$

now, $\int_T \cos n w_0 t . \cos m w_0 t . dt$ (for ~~n=m~~ . $n = m$)

$$= \int_T \frac{\cos (n+m) w_0 t + \cos (n-m) w_0 t}{2} . dt$$

$$= \int_T \frac{\cos (2m) w_0 t + \cos (0) w_0 t}{2} . dt$$

$$= \int_T \frac{\cos (2m w_0 t) + 1}{2} . dt$$

$$= \left[\frac{\sin (2m w_0 t)}{4 m w_0} + \frac{1}{2} t \right]_0^T$$

$$= \left[0 + \frac{1}{2} T \right] - [0 + 0] = \underline{\underline{\frac{T}{2}}}$$

ii) $\int_T \sin (n w_0 t) . \sin (m w_0 t) . dt$

$$= \int_T \frac{\cos (n-m) w_0 t - \cos (n+m) w_0 t}{2} . dt$$

a). For $n \neq m$

$$= \left[\frac{\sin(n-m)\omega_0 t}{2(n-m)\omega_0} - \frac{\sin(n+m)\omega_0 t}{2(n+m)\omega_0} \right]_0^T$$

putting limits we get

$$= \underline{\underline{0}}$$

b) For $n = m$,

$$= \int_T \frac{1 \cancel{+} \cos(2n\cancel{t}\omega_0 t)}{2} \cdot dt = \left[\frac{1}{2}t \cancel{+} \frac{\sin(2n\omega_0 t)}{4n\omega_0} \right]_0^T$$

$$= \underline{\underline{\frac{T}{2}}}$$

iii) $\int_T \sin n\omega_0 t \cdot \cos m\omega_0 t \cdot dt$

$$= \int_T \frac{\sin(n+m)\omega_0 t + \sin(n-m)\omega_0 t}{2} \cdot dt$$

$$= \left[-\frac{\cos(n+m)\omega_0 t}{2(n+m)\omega_0} \cancel{+} \frac{\cos(n-m)\omega_0 t}{2(n-m)\omega_0} \right]_0^T$$

$$= -\left[\left(\frac{\cos(n+m)\frac{2\pi}{T}\cdot T}{2(n+m)\omega_0} + \frac{\cos(n-m)\frac{2\pi}{T}\cdot T}{2(n-m)\omega_0} \right) - \left(\frac{\cos(n+m)\frac{2\pi}{T}\cdot 0}{2(n+m)\omega_0} + \frac{\cos(n-m)\frac{2\pi}{T}\cdot 0}{2(n-m)\omega_0} \right) \right]$$

$$= -\left[\left(\frac{1}{2(n+m)\omega_0} + \frac{1}{2(n-m)\omega_0} \right) - \left(\frac{1}{2(n+m)\omega_0} + \frac{1}{2(n-m)\omega_0} \right) \right]$$

$$= \underline{\underline{0}}$$

→ so, from given signal set, If we take any pair of signals, then they will be orthogonal to each other. (~~[illegible]~~)

If we take pair cosines (or) sines signals only, then, $m = n$ is not possible and as shown for $m \neq n$ they are orthogonal. For one sine and one cosine signal, they will be always orthogonal whether it is $n \neq m$ (or) $n = m$.

Q.3 Here for the real signal $x(t)$ we need to show

$$\int_{-\infty}^{+\infty} x(t) \cdot \hat{x}^*(t)\, dt = 0. \qquad \int_{-\infty}^{\infty} x(t) \left(\hat{x}(t)\right)^* dt$$

→ We know from Parseval's Theorem

$$\int_{-\infty}^{+\infty} x(t) \cdot y^*(t) \cdot dt = \int_{-\infty}^{+\infty} X(f) \cdot Y^*(f) \cdot df$$

Applying this to $x(t)$ and $\hat{x}(t)$

$$\int_{-\infty}^{+\infty} x(t) \cdot \hat{x}(t) \cdot dt = \int_{-\infty}^{+\infty} x(t) \cdot \hat{x}^*(t) \cdot dt = \int_{-\infty}^{+\infty} X(f) \cdot \hat{X}^*(f) \cdot df$$

$$= \int_{-\infty}^{+\infty} X(f) \left[j \operatorname{sgn}(f)\, X^*(f) \right] \cdot df$$

$$= j \int_{-\infty}^{+\infty} \operatorname{sgn}(f)\, |X(f)|^2 \cdot df$$

Here $\operatorname{sgn}(f)$ is an odd function of 'f', while $|X(f)|^2$ is an even function, the integrand ~~in the last integrand~~ is odd and hence the integral is zero.

$$\int_{-\infty}^{+\infty} x(t) \cdot \hat{x}(t) \cdot dt = 0$$

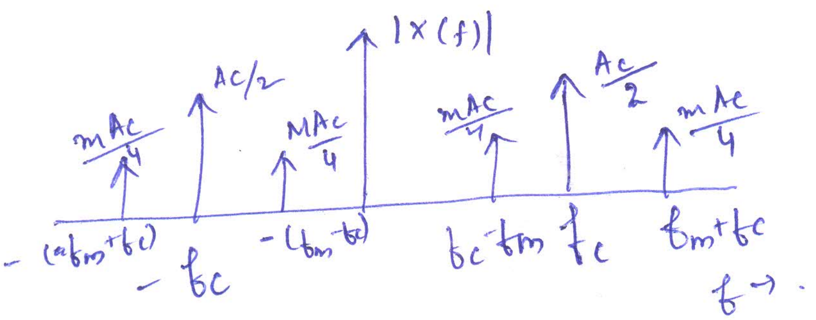
$x(t) = A_c(1 + m\cos\omega_m t)\cos\omega_c t$

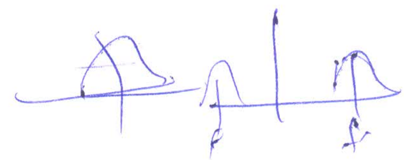
$x_p(t) = x(t) + j\hat{x}(t)$

Let's first obtain the freq response of $x(t)$.

$$x(t) = A_c\cos\omega_c t + \frac{mA_c}{2}\left[\cos(\omega_m+\omega_c)t + \cos(\omega_m-\omega_c)t\right]$$

$$\therefore X(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \frac{mA_c}{4}\Big[\delta(f+(f_m+f_c))$$
$$+\,\delta(f-(f_m+f_c)) + \delta(f+(f_m-f_c)) + \delta(f-(f_m-f_c))\Big]$$

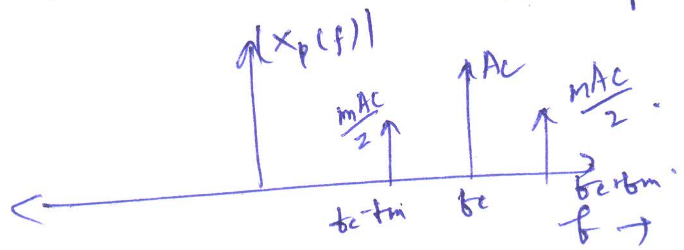




(a) $X_p(f) = X(f) + j\hat{X}(f)$

But $\hat{X}(f) = -j\,\mathrm{sgn}(f)\cdot X(f)$

$\therefore X_p(f) = 2X^+(f)$ [where $X^+(f)$ is the +ve part of the spectrum of $X(f)$].



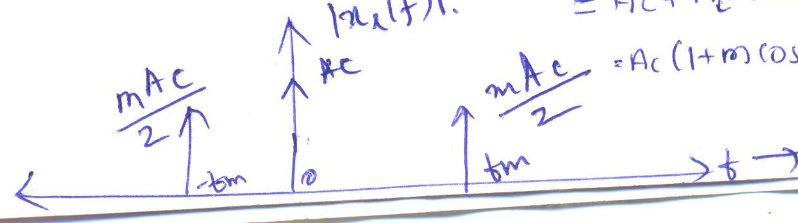
(b) $x_p(t) = A_c e^{j2\pi f_c t} + \frac{mA_c}{2}\left[e^{j2\pi(f_c+f_m)t} + e^{j2\pi(f_c-f_m)t}\right]$.

(c) Complex Signal. (Since its not symmetric about the axis)

(d) $x_p(t)\,e^{-j2\pi f_c t} = A_c + \frac{mA_c}{2}e^{j2\pi f_m t} + \frac{mA_c}{2}e^{-j2\pi f_m t} = x_l(t)$

$= A_c + A_c m\cos 2\pi f_m t$ = complex envelope of signal

$= A_c(1 + m\cos 2\pi f_m t)$



e) Real signal. (Since its symmetric about the axis)

f) low pass signal

g) Bandwidth is f_m.